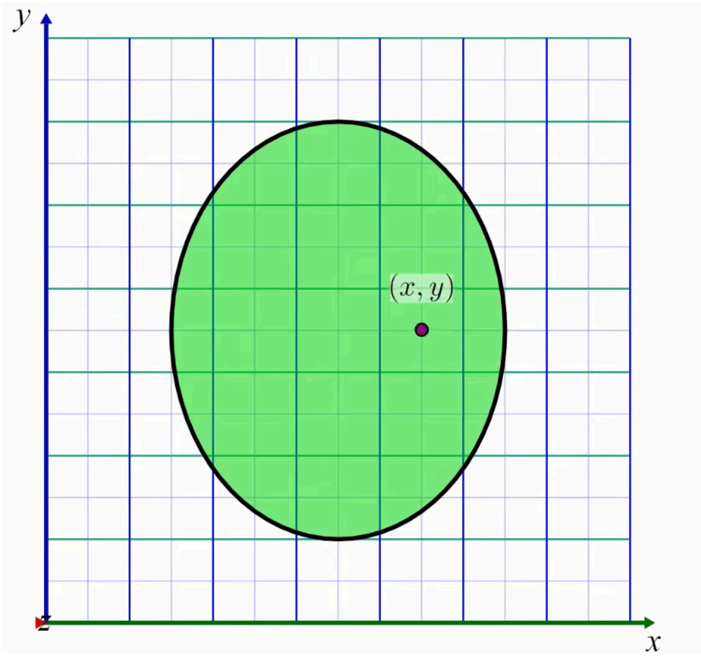
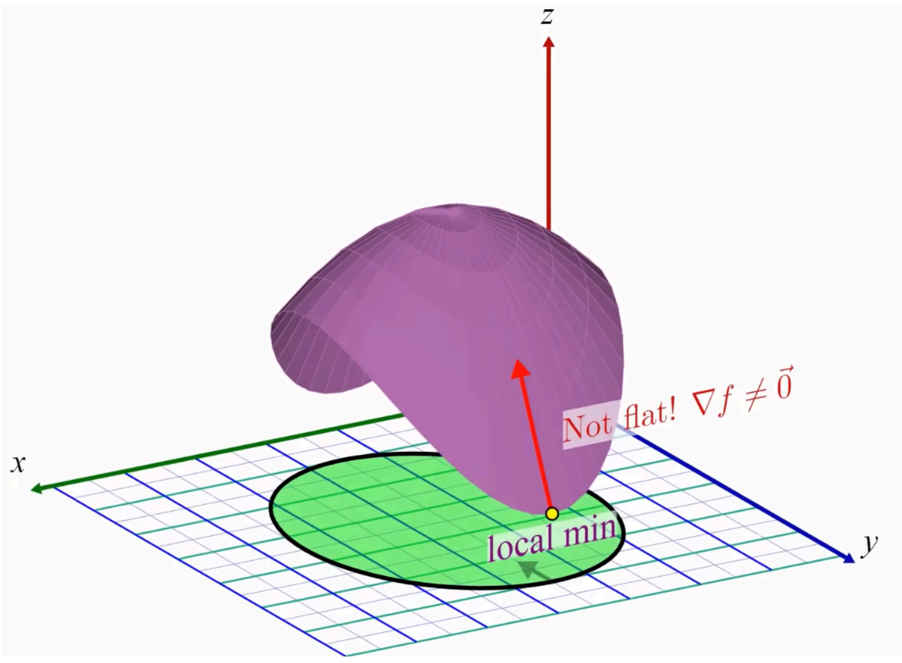
Lagrange Multipliers

in a simple single or multivariate functions calculating the **local max/min** is done simply by calculating the derivative or the gradient of the function $\nabla f = 0$

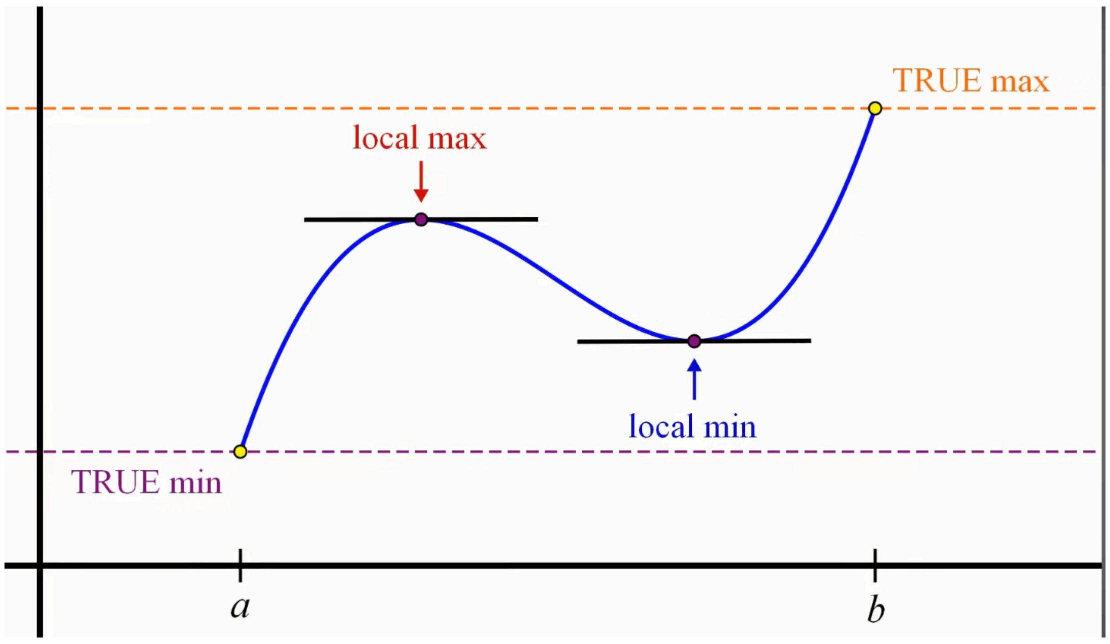


- In many real world problems our variables are restricted with constraints and they are not allowed to take values out side the constraint region x *and* y *are both independent variables*



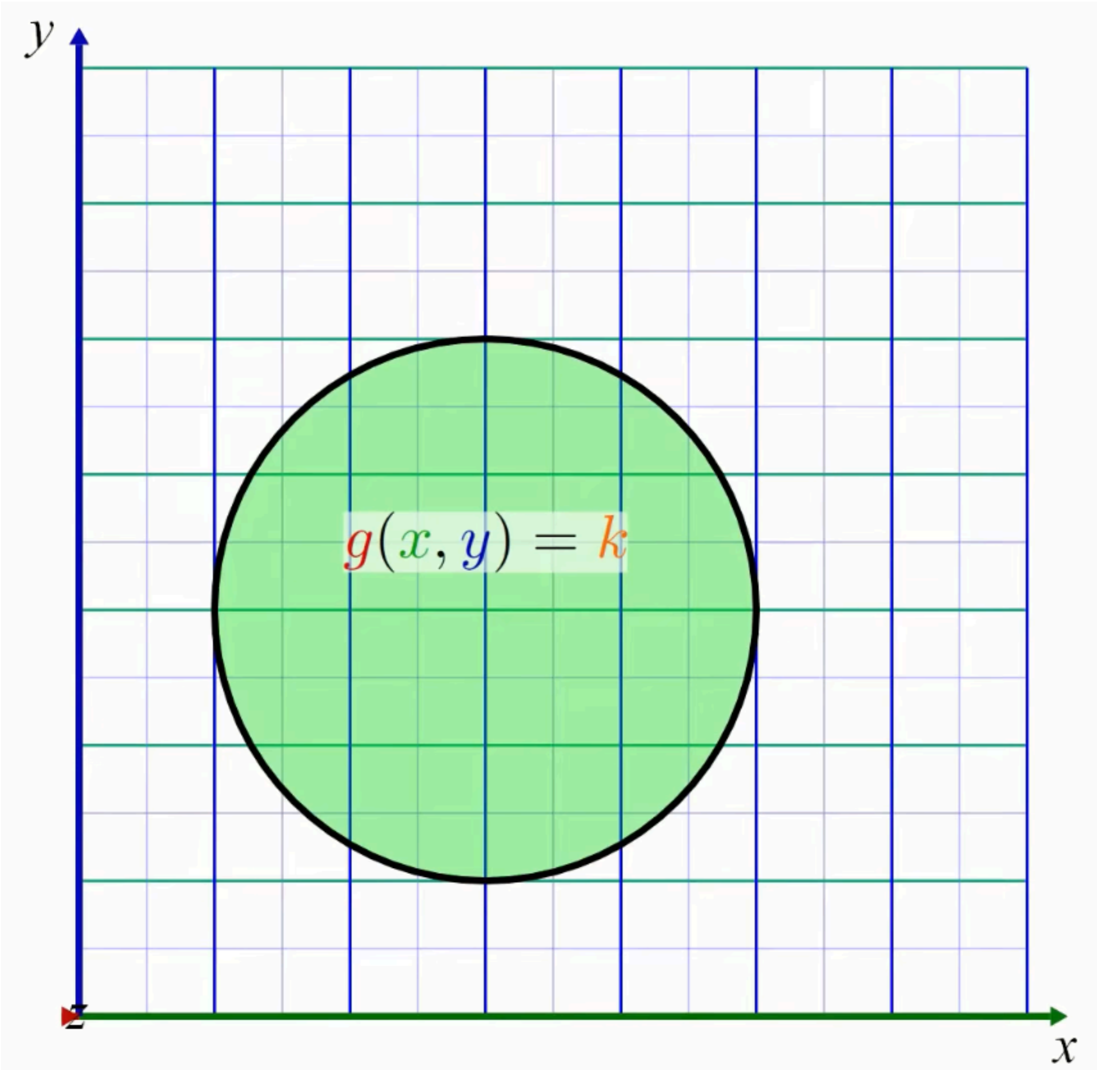
- Often times the boundary is what contains the local min/max even tho the gradient isn't flat $\nabla f \neq 0$

In a single variable function it's pretty straight forward to check the **left and right** boundary of a function curve to determine the **True min/max**

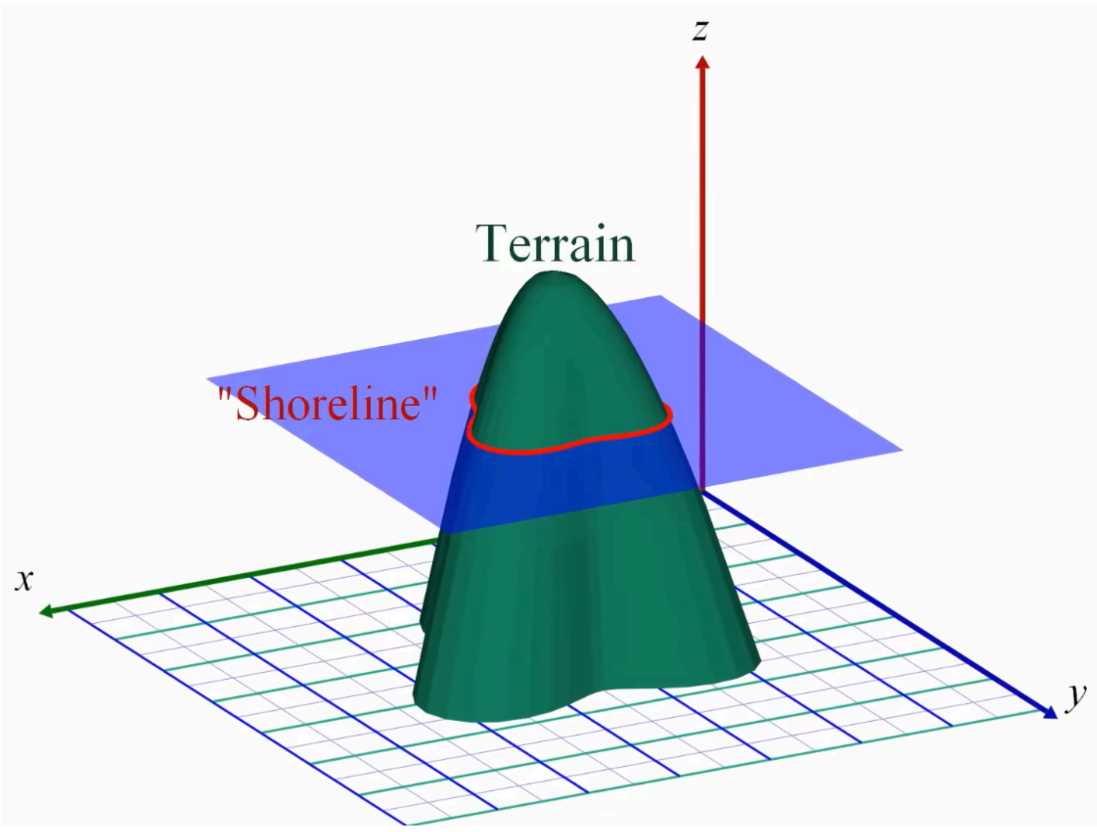


- In a **multivariate function** the boundary is a curve which contain infinite points, so checking manually is out of the table

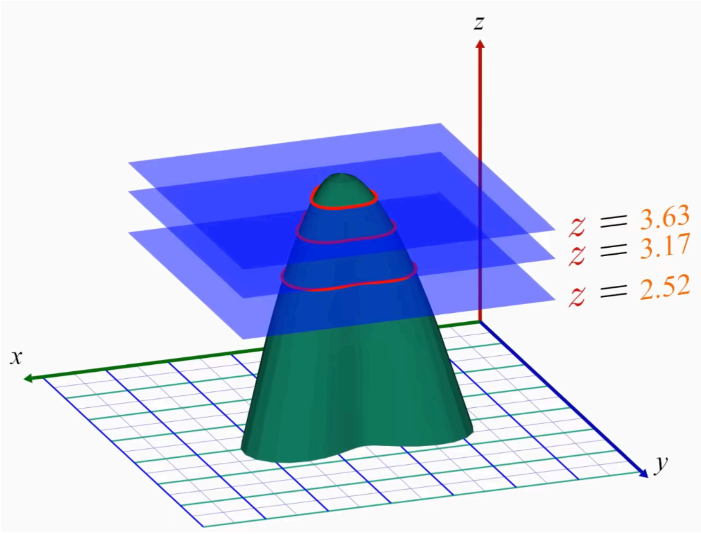
Taking a look at the constraint area and trying to give it a formal formula, gets us $g(x, y) = k$, with k is some constant



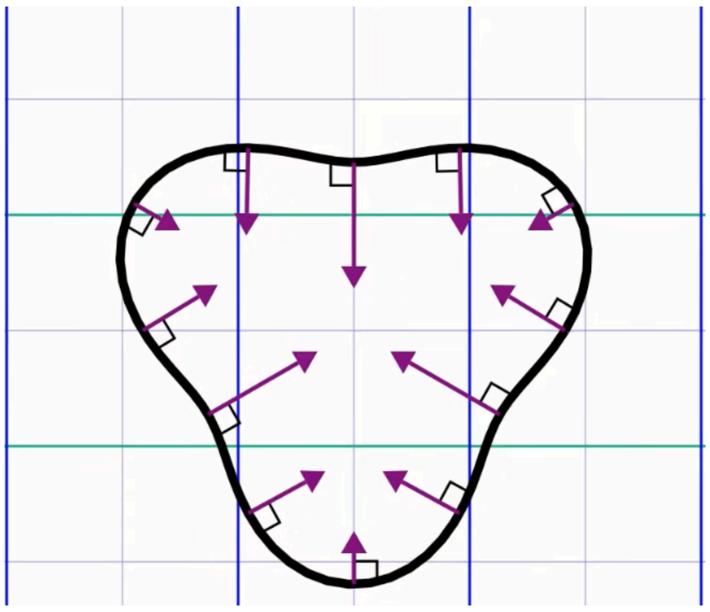
With $f(x, y)$ and $g(x, y)$ both defining a surfaces and k a constant which can be thought of as a **level curve** and $g(x, y)$ as a **Terrain**

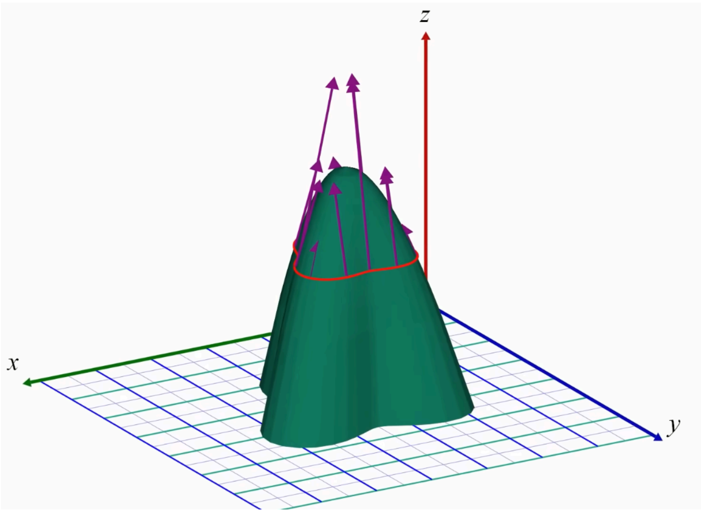


- Changing the values of k will give us different **level curves**

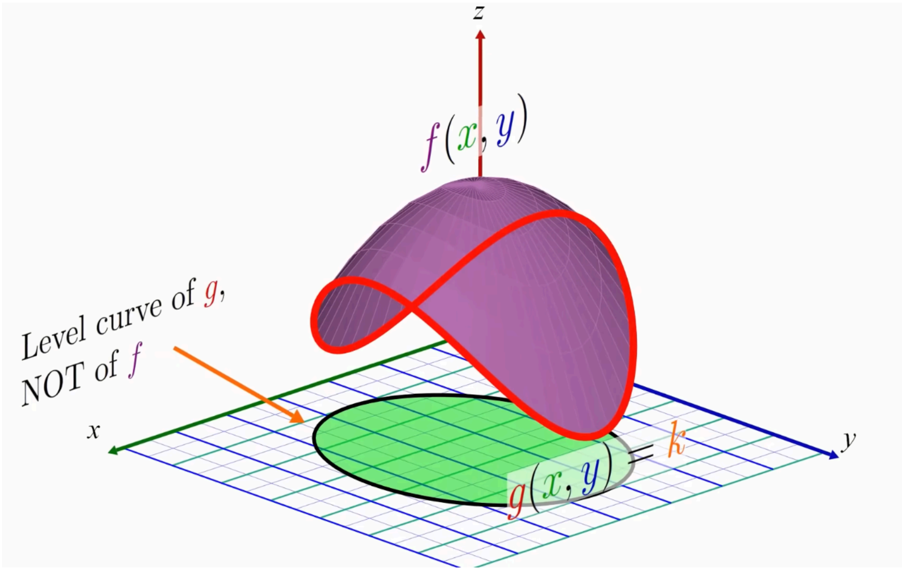


- Looking at it from above the gradient vector to a surface at any point of this **level curve** is perpendicular, Which means they all point out to the point with the steepest ascent at a point on the surface while being perpendicular





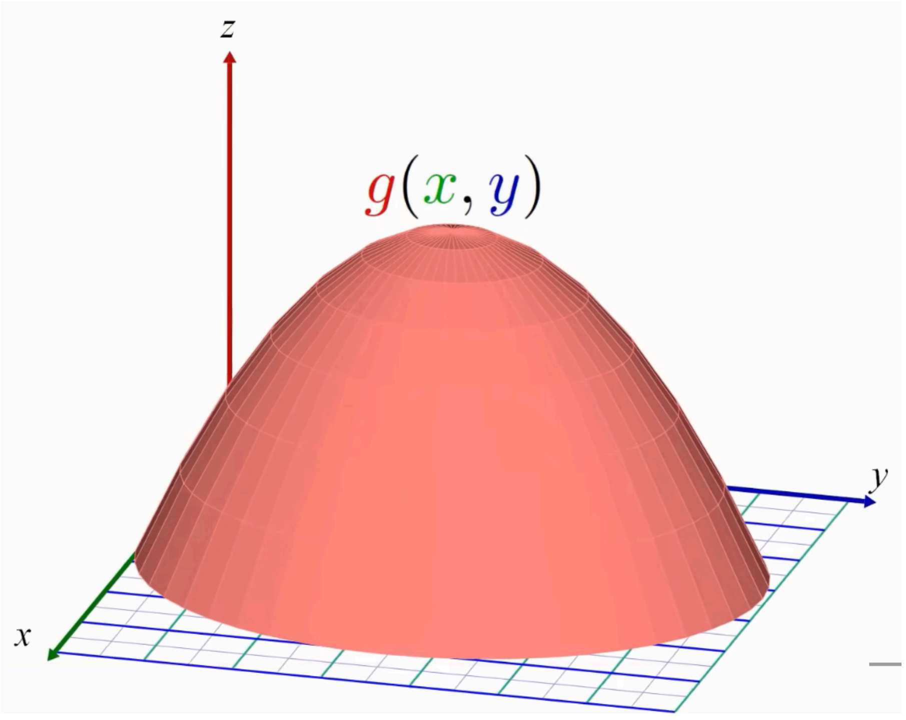
going back to the constraint region of $g(x, y) = k$, while keep in mind the boundary curve isn't the $f(x, y)$ cause it's not a surface **level curve** as the **Terrain** shown above

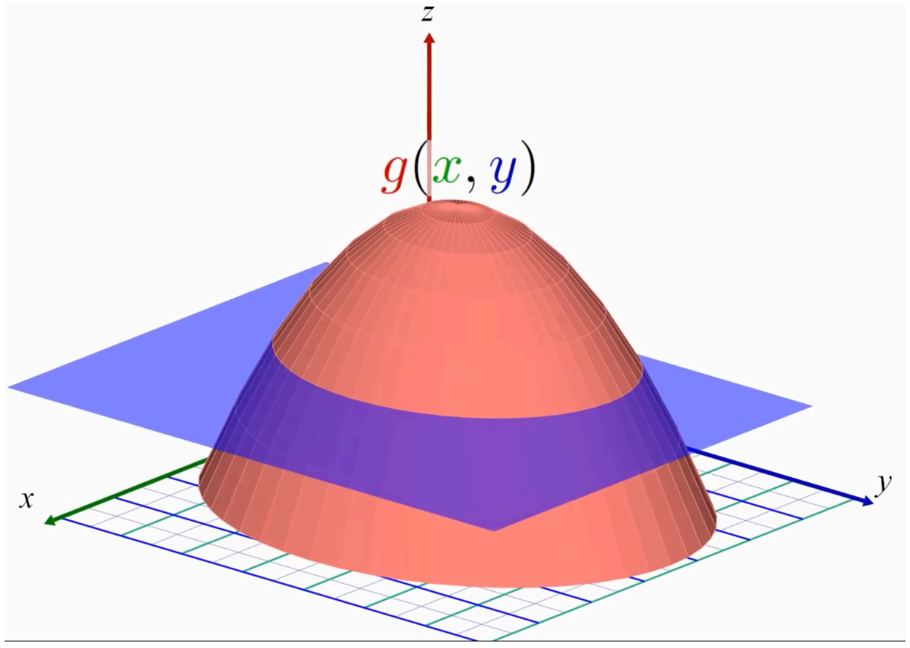


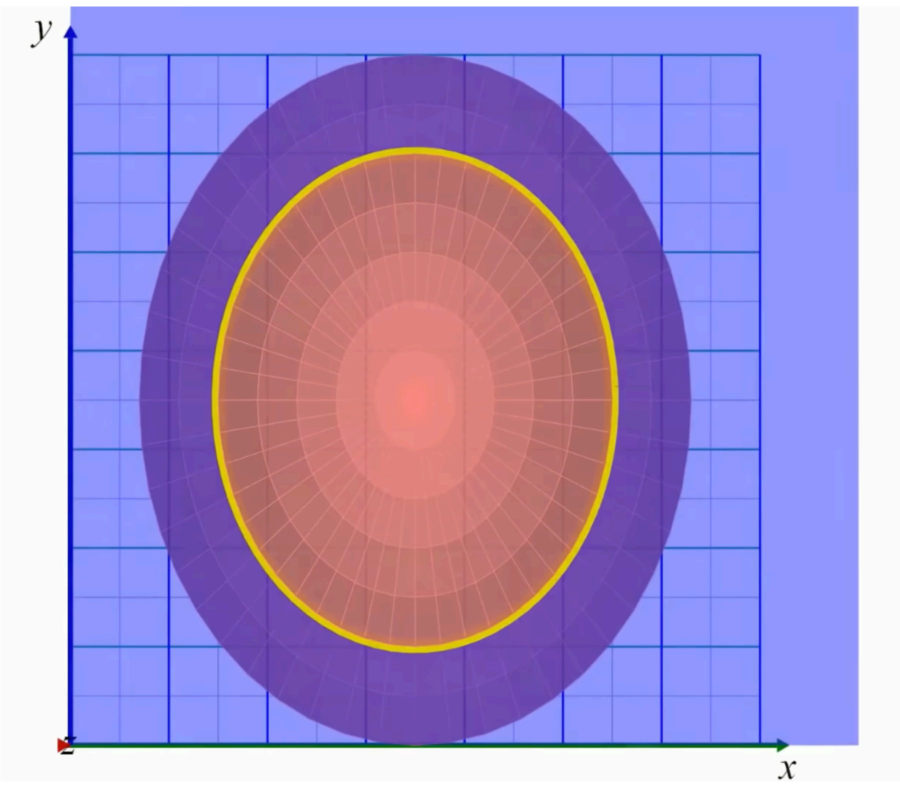
In this problem there is two surfaces we dealing with :

- The $f(x, y)$ which we wanna find the *max/min*

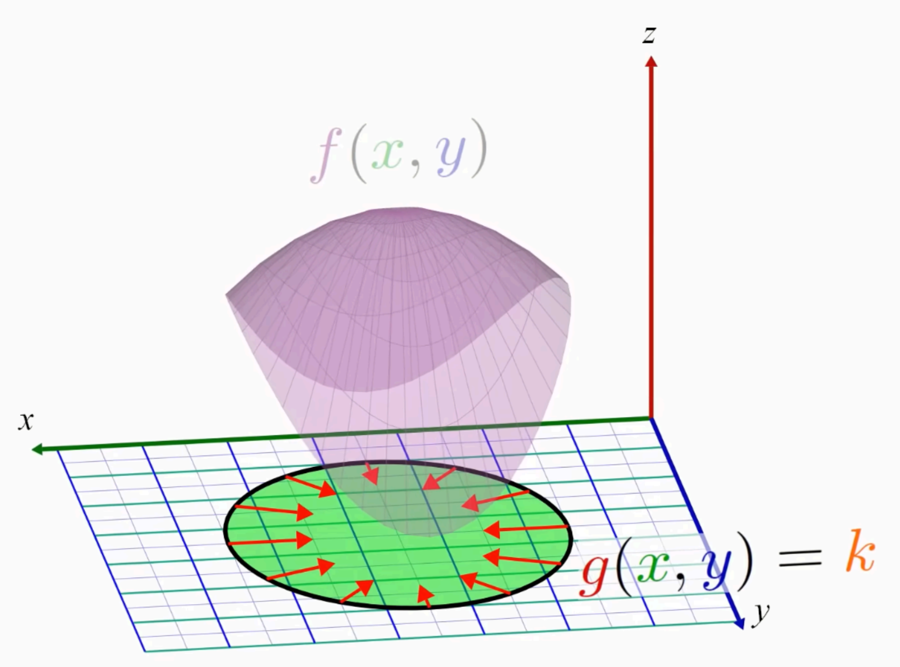
- The constraint $g(x,y)=k$ the one that gives us the **surface level curve**





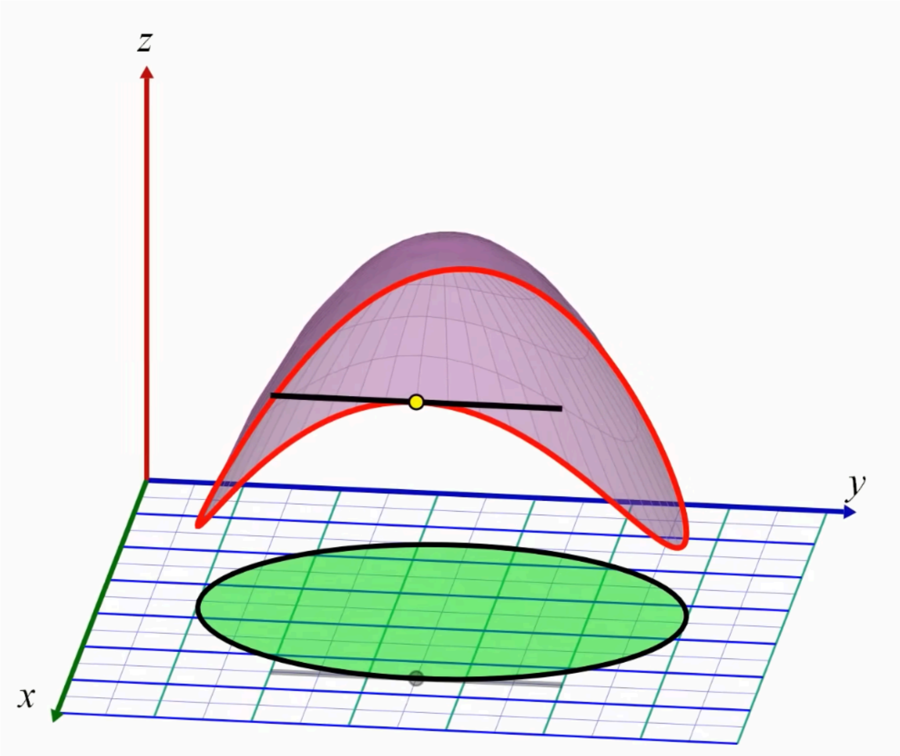


- This is how the $g(x,y)$ surface looks like, and one of it's **level curves** *the water and terrain* is our **constraint region** we have on the main problem above
- So that means the gradient level of the ∇g looks perpendicular to the constraint curve at all points

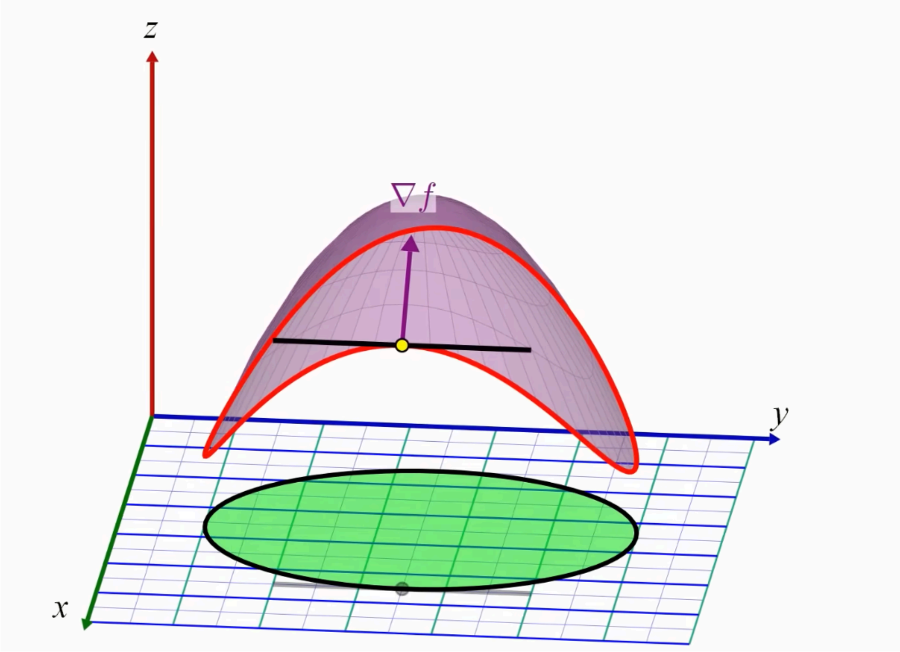


Now to the main problem : *finding the max/min of the* $f(x, y)$ *that follows the constraint of* $g(x, y) = k$.

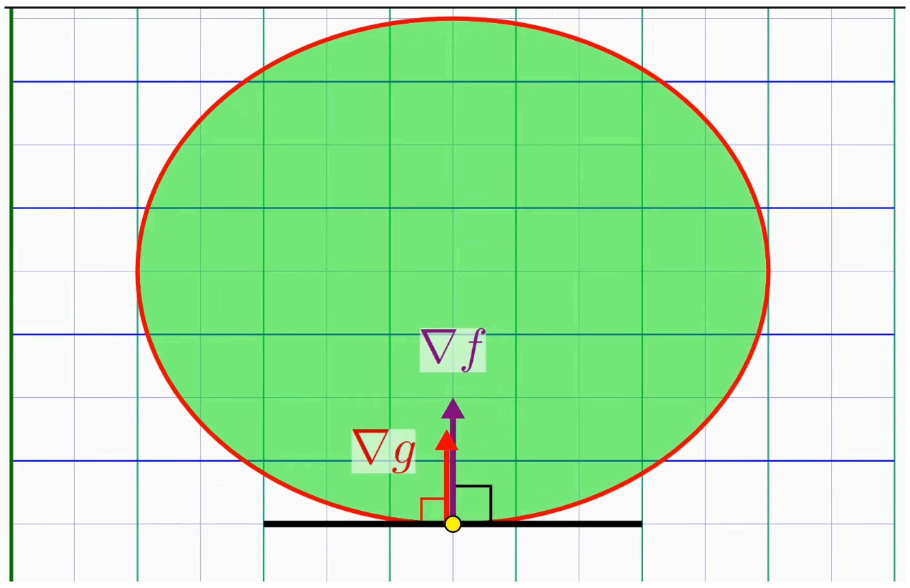
By just looking a the curve we can say that this point is the **local max** of the boundary curve $f(x, y)$, while following the rules of calculus the point is sitting where the curve is flat which indicate *max/min*



- So if the **boundary curve** is flat on that point this means that the gradient $\nabla f(x, y)$ is **perpendicular**



- and since the **boundary curve** of $f(x, y)$ came from the **level curve** of $g(x, y) = k$ which all the gradient points on that surface are perpendicular



- Which means both ∇f and ∇g are perpendicular to the constraint curve on that point(∇f is parallel to ∇g on the xy plane)

Putting it more formally and clear : **The max/min of a function $f(x, y)$ subject to a constraint $g(x, y) = k$ must occur where ∇f is parallel to ∇g** .

In linear algebra two parallel vectors (which in our case gradient vectors) are simply as scalar of the other one

$$\nabla f = \lambda \nabla g$$

- The λ is called the **Lagrange Multiplier**