Step Functions

A special case of : Basis Functions.

Using Polynomial Regression which impose all of our **features** to be non-linear function of X, we can instead use **step functions** to avoid applying the **polynomial** to the global structure of our data set.

Here we convert a continuous variable X into **ordered categorical variable**, by creating **cutpoints** $c_1, c_2, \ldots, c_p$ in the range of X now we construct :

$$
\begin{aligned}
&C_0(X) = I(X < c_1) \\
&C_1(X) = I(c_1 \leq X < c_2) \\
&C_2(X) = I(c_2 \leq X < c_3) \\
&\vdots \\
&C_{p-1}(X) = I(c_{p-1} \leq X < c_p) \\
&C_p(X) = I(c_p \leq X)
\end{aligned}
$$

- Where $I(.)$ is an **indicator function** returns 1 if the condition is **true**, otherwise $zero$
- So for any value of X $C_1(x_i) + C_2(x_i) + \cdots + C_p(x_i) = 1$, since X must be in one of those **cutpoints**

$$y_i = \beta_0 + \beta_1 C_1(x_i) + \beta_2 C_2(x_i) + \cdots + \beta_p C_p(x_i) + \epsilon_i$$

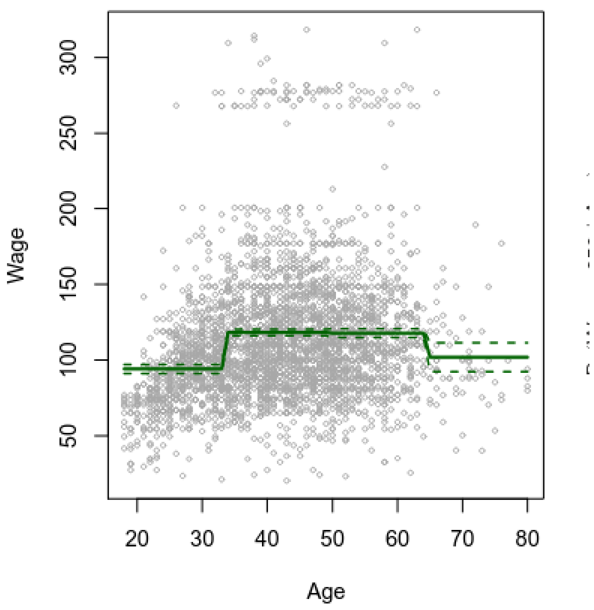
It's also called **piece-wise constant model**, at most one of $C_1, C_2, \ldots, C_p$ can be non-zero with other's being zero, when $X < c_1$ all of the predictors are $zero$ except β_0

The matrix form of **Step function Regression** is given :

$$Y = X\beta + \epsilon$$

With X being :

$$
\mathbf{X} = \begin{bmatrix}
1 & C_1(x_1) & C_2(x_1) & \ldots & C_p(x_1) \\
1 & C_1(x_2) & C_2(x_2) & \ldots & C_p(x_2) \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & C_1(x_n) & C_2(x_n) & \ldots & C_p(x_n)
\end{bmatrix}
$$



Limits of Step Function Regression

Unfortunately, unless there are natural breakpoints in the predictors (features) , **piecewise-constant** functions can miss the action, they are very popular in bio-statistics and epidemiology and some niche disciplines.

- **Discontinuous** : **Step functions** create discontinuous jumps at each cutpoint
- **Choice of cutpoints** : the Results depend heavily on where u place boundaries

- **Too ridged** : Since it create breaks on the data it may miss sooth trends on the data

Note : **Step Function Regression** is still linear and Ordinary Least Squares apply to it , also it can be applied to any linear model we discuss before .