Discriminant Functions

Its a function $\delta_k(x)$ we compute for each class k and we **assign a new** Observation x to the class with the largest $\delta_k(x)$

$$\text{Classify } x \text{ to the class } \hat{k} = argmax_k\ \delta_k(x)$$

Derivation

Generative Models Discriminant Function

in Bayes' theorem assume with know or **estimate** the **prior probability** $P(Y = K) = \pi_k$ and the **update** (Class conditional density) $f_k(x) = P(X = x|Y = k)$

$$P(Y = k|X = x) = \frac{\pi_k f_k(x)}{\sum_i \pi_i f_i(x)}$$

- since the **denominator** is the same for all classes k, the **numerator** is what decide the value of the posterior by maximizing it

$$\delta_k(x) = \log(\pi_k) + \log(f_k(x))$$

- This is the **Discriminant Function** in Generative Models for Classification like **LDA** With :
  - $\log(\pi_k)$ $\log P(x|Y = k)$ the prior which represents how likely class k is before observing the data
  - $f_k(x)$ $P(Y = k)$which is the log likelihood of x belonging to class k
  - In **LDA** Since we assume that all classes **variance** is equal it results in a linear function of x

Decision Boundaries

The **decision boundary** between two classes k and j is the set of points x where :

$$\delta_k(x) = \delta_j(x)$$

- This is where the **model** cant decide between the class k or j

Intuition

Discriminant function take an input x and returns a score for each class k , the class with the highest value the observation x get assigned to.