

PROBABILITIES

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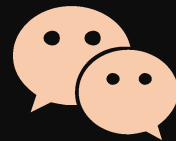
Statistics for Management
Fall 2016

Plan for today

1. Quiz
2. Probabilities
3. *Presentations

PROBABILITIES

What is it important to learn probabilities?

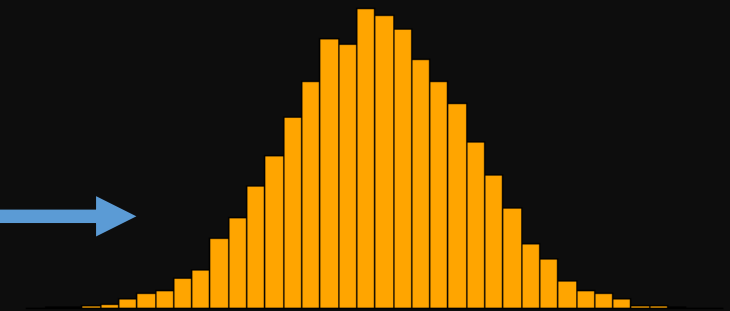


Probabilities and Stats



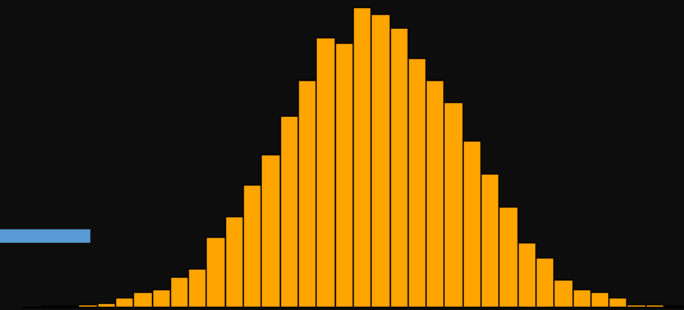
Probability

Predict data/distribution using a model



Statistics

Predict probability, using data/distribution



So, why do we need probabilities?

Answer business problems

To be

What is the probability of x given y ?

Match relevant modeling technique

Linear regression, logit, poisson, neg binomial, nonparametric models....

Recognize the limit of our data
(and model)

Sampling, "margin of error"

Basic terms

Sample space: All possible outcomes in an experiment.

Event: An outcome in a sample space.

Probability: Likelihood that an event will occur ($0 \leq P \leq 1$)

Car dealership example

Purchased		
Planned	Yes	No
Yes	100	25
No	50	325

What is the sample space?

What are the events?

What is the probability to buy a car at the dealership?

Car dealership example

Purchased			
Planned	Yes	No	Total
Yes	100	25	125
No	50	325	375
Total	150	350	500

Marginal probability
 $P(\text{purchased})$
Mutually exclusive

Sample space

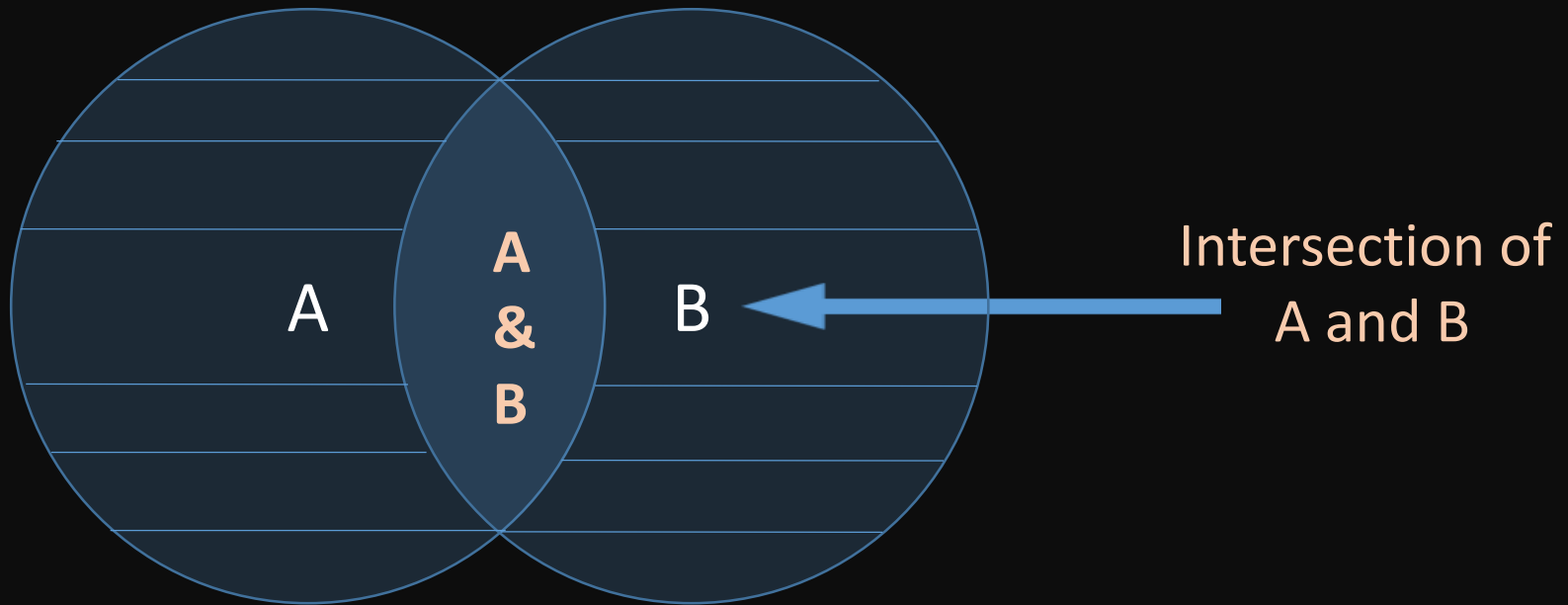
What is the sample space? **500**

What are the events? **Buying/not buying a car (totaling 500)**

What is the probability to buy a car at the dealership? **150/500 (30%)**

Joint probability

Finding the probability of event A and event B



Car dealership example

Purchased			
Planned	Yes	No	Total
Yes	100	25	125
No	50	325	375
Total	150	350	500

Joint probability
 $P(\text{planned \& didn't purchase})$



Sample space



What is the probability of a planned, non-purchase? **25/500 (5%)**

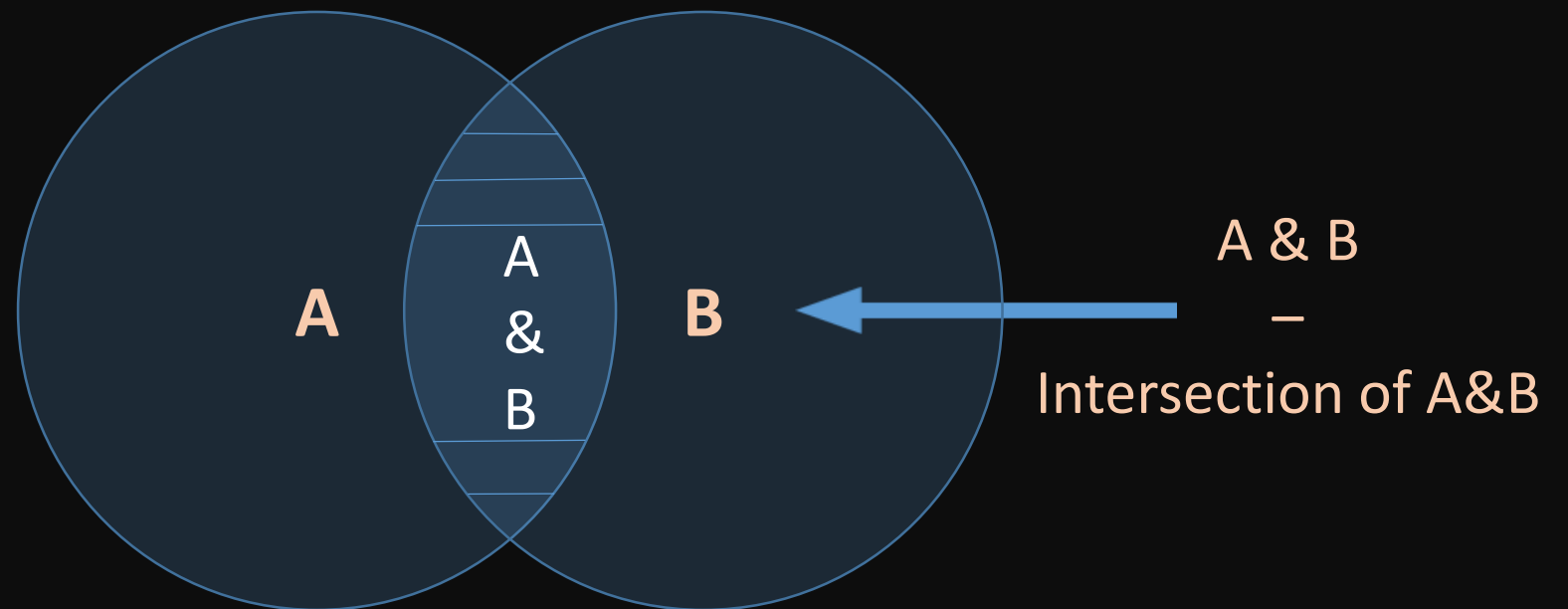
HR example

Quit after 24 months		
Better Offer	Yes	No
Yes	15	10
No	5	70

- ✓ What is the sample space?
- ✓ What is the probability of an employee quitting after 24 months?
- ✓ What is the probability of an employee quitting after 24 months without receiving a better offer?

General addition rule

Finding the probability of event A or event B



Car dealership

		Purchased		
Planned <u>and</u> purchased	Planned	Yes	No	Total
	Yes	100	25	125
	No	50	325	375
	Total	150	350	500

Planned purchase

Actual purchase

What is probability that a customer plans a purchase or actually purchases a car?

$$125/500 + 150/500 - 100/500 = 175/500 = 35\%$$

HR example

Quit after 24 months			
Better Offer	Yes	No	Total
Yes	15	10	25
No	5	70	75
Total	20	80	100

✓ What is the probability of quitting or not quitting after 24 months?

Conditional Probability

The probability of event A given (conditional) B

$$P(A|B) = P(A \& B) / P(B)$$

Car dealership

$$P(A|B) = P(A \& B) / P(B)$$

**Planned and
purchased**

Purchased			
Planned	Yes	No	Total
Yes	100	25	125
No	50	325	375
Total	150	350	500

**Sample
space**

**Didn't plan and
purchased**

Of the customers who planned purchasing a car, what is the probability of actually purchasing a car? Is this probability higher than customers who haven't planned a purchase? **Yes!**

$$100/125 = \mathbf{80\%}$$

$$25/125 = \mathbf{20\%}$$

Decision Trees

Present probabilities as a "tree" with branches indicating disjoint events



HR example

Quit after 24 months			
Better Offer	Yes	No	Total
Yes	15	10	25
No	5	70	75
Total	20	80	100

- ✓ Of those who did not receive a better offer, what is the probability of quitting after 24 months?
- ✓ Given this probability do you think that there is a relation between better offer and quitting?
- ✓ Draw a decision tree to describe the probabilities in the HR example

Statistical independence

The conditional probability of event A given B equals marginal probability A

$$P(A|B) = P(A)$$

Which means that the outcome of one event **does not affect** the other

Car dealership

Is there a difference between shoppers of premium to standard cars?
Follow-up with customers asking whether they are satisfied with their purchase.

$$P(A|B) = P(A)$$

Car type	Satisfied		
	Yes	No	Total
Premium	45	5	50
Standard	90	10	100
Total	135	15	150

$$P(\text{satisfied} \mid \text{premium}) = (45/150)/(50/150) = \mathbf{90\%}$$

$$P(\text{satisfied} \mid \text{standard}) = (90/150)/(100/150) = \mathbf{90\%}$$

There is no difference. Yes. Satisfaction is statistically independent from car type.

HR example

Quit after 24 months			
Better Offer	Yes	No	Total
Yes	15	10	25
No	5	70	75
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- ✓ Of those who did not receive a better offer, what is the probability of quitting after 24 months?
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