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Practical NO. 11

Multivariate Analysis (Contour Plot)

Q-1) Consider $X \sim N_2(\mu, \Sigma)$ where $\mu = (0, 2)'$ and

$$\Sigma = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$$

1. Write down bivariate normal density $(x_1, x_2) \sim BN(0, 2, 4, 4, \rho = 1/4)$

$$f(x_1, x_2) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} e^{-\left\{\frac{1}{2(1-\rho^2)}\left[\left(\frac{x_1-\mu_1}{\sigma_1}\right)^2 + \left(\frac{x_2-\mu_2}{\sigma_2}\right)^2 - 2\rho\left(\frac{x_1-\mu_1}{\sigma_1}\right)\left(\frac{x_2-\mu_2}{\sigma_2}\right)\right]\right\}}$$

$-\infty < x_1, x_2 < \infty$, $\sigma_1^2, \sigma_2^2 > 0$

$-\infty < \mu_1, \mu_2 < \infty$, $|\rho| < 1$

$$= \frac{1}{2\pi(16)\sqrt{\frac{15}{16}}} \exp\left\{\frac{-1}{2(15/16)}\left[\left(\frac{x_1}{4}\right)^2 + \left(\frac{x_2-2}{4}\right)^2 - 2\left(\frac{15}{16}\right)\left(\frac{x_1}{4}\right)\left(\frac{x_2-2}{4}\right)\right]\right\}$$

2) $(\underline{x} - \underline{\mu})' \Sigma^{-1} (\underline{x} - \underline{\mu})$

$$\begin{bmatrix} x_1 - 0 \\ x_2 - 2 \end{bmatrix}' \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}^{-1} \begin{bmatrix} x_1 - 0 \\ x_2 - 2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 & x_2 - 2 \end{bmatrix} \frac{1}{15} \begin{bmatrix} 4 & -1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 - 2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 & x_2 - 2 \end{bmatrix} \begin{bmatrix} 0.02667 & -0.0667 \\ -0.0667 & 0.02667 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 - 2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 & x_2 - 2 \end{bmatrix} \begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 - 2 \end{bmatrix}$$

$[ax_1+b(x_2-2)' \quad bx_1+a(x_2-2)] \begin{bmatrix} x_1 \\ x_2-2 \end{bmatrix}$

$\therefore ax_1^2 + bx_1(x_2-2) + bx_1(x_2-2) + a(x_2-2)^2$

$ax_1^2 + a(x_1-2)^2 + 2bx_1x_2 - 4bx_1$

$0.02667x_1^2 + 0.02667(x_2-2)^2 - 0.1334x_1x_2 + 0.2668x_1$

3] To determine & sketch constant density contour with 50% of probability

$\mu(0,2)' \quad \Sigma = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$

Here:

$\cancel{\sigma_{11} = \sigma_{12} = 4}$ ~~and~~ $\cancel{\sigma_{12} = \sigma_{21}}$

$\sigma_{11} = \sigma_{22} = 4$ and $\sigma_{12} = \sigma_{21} = 1$

Covariance is positive

Major axis $= c\sqrt{\sigma_{11}+\sigma_{12}}$

$\alpha = 50\% = 0.5$

Using minitab find probability of c for α.

$P(X \le x)$	x	for 2 d.f
0.5	1.38629	

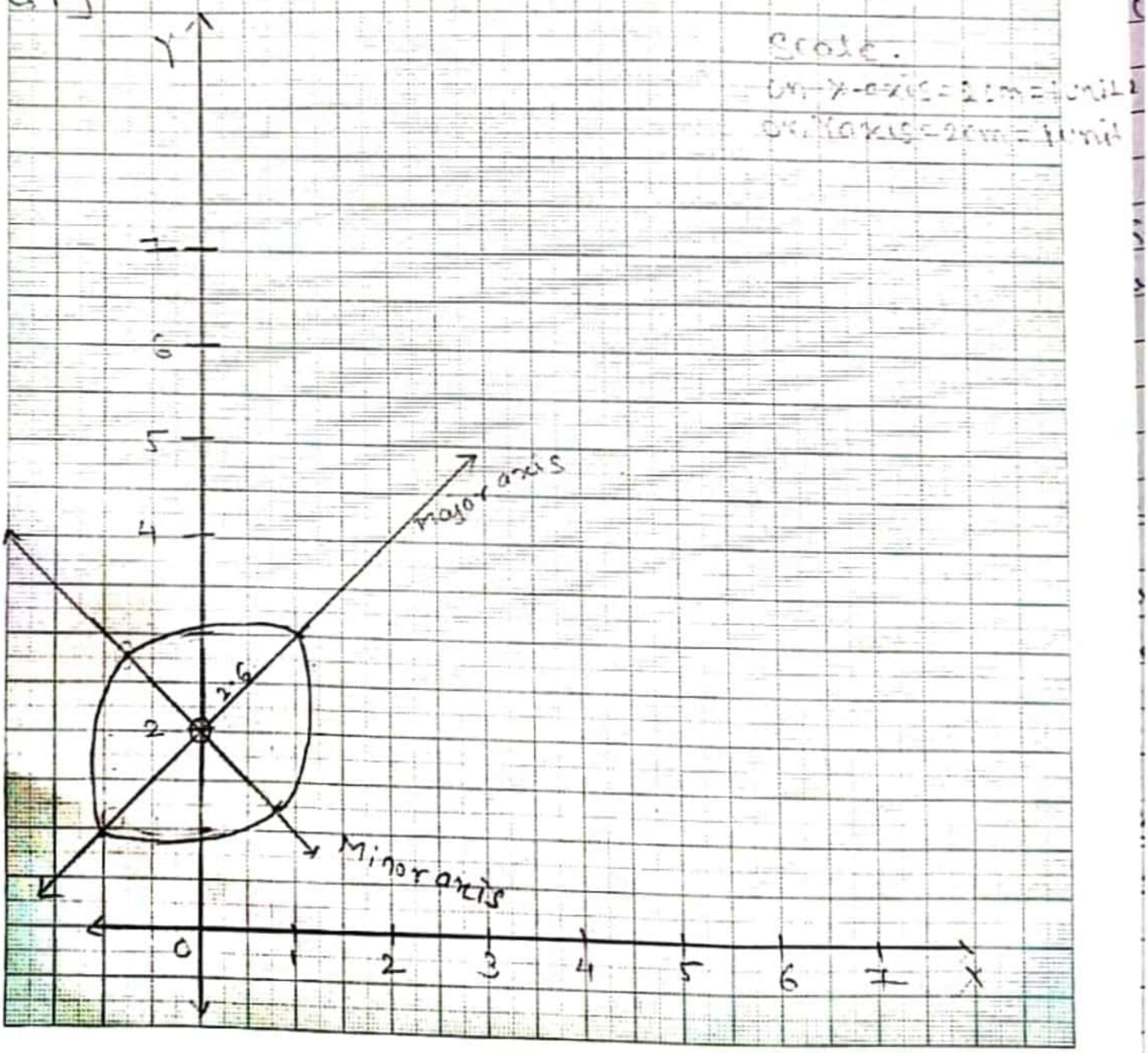
$c^2 = 1.38629$, $c = 1.17741$

Major axis $= c\sqrt{\sigma_{11}+\sigma_{12}} = 1.1774\sqrt{5} = 2.63277$

minor axis $= c\sqrt{\sigma_{11}-\sigma_{12}} = 1.1774\sqrt{3} = 2.03933$

then we plot this axis on graph from point

Q1]



Scale.
On x-axis = 2 cm = 1 unit
On y-axis = 2 cm = 1 unit
Y
X
0
1
2
3
4
5
6
7
Major axis
Minor axis
2.6

Q-2) Let $X \sim N_2(\mu, \Sigma)$ where $\mu = (3\ 2)'$ and $\Sigma = \begin{bmatrix} 7 & -4 \\ -4 & 5 \end{bmatrix}$, Sketch the constant density contour that contains 70% of the probability

→ $\mu = (3\ 2)'$, $\Sigma = \begin{bmatrix} 7 & -4 \\ -4 & 5 \end{bmatrix}$

Here,

$\sigma_{11} = 7$, $\sigma_{12} = \sigma_{21} = -4$, $\sigma_{22} = 5$

covariance is negative,

major axis $= c\sqrt{\sigma_{11} - \sigma_{12}}$, $\alpha = 70\% = 0.7$

Using minitab find prob. of c for α

$p(X < x)$	x for 2 d.f.
0.7	2.40795

$c^2 = 2.40795$ $\quad c = 1.55175$

major axis $= c\sqrt{\sigma_{11} - \sigma_{12}} = 1.55175\sqrt{11} = 5.14659$

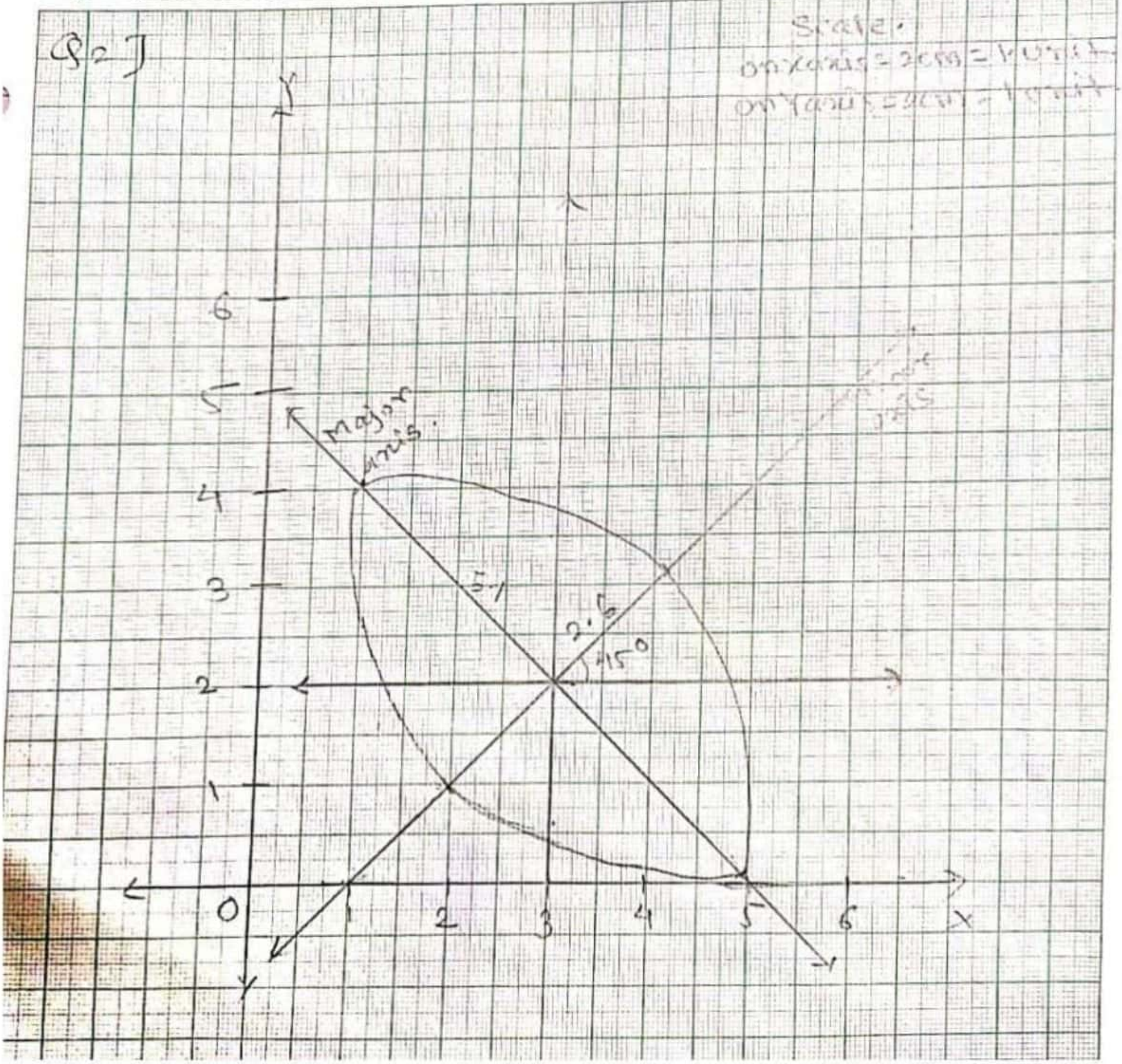
minor axis $= c\sqrt{\sigma_{11} + \sigma_{12}} = 1.55175\sqrt{3} = 2.6877$

then we plot this on graph from point μ.

Scanned with CamScanner

Origin : (________________) Slope : ________________

Q2]



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Q-3] Let $\underline{X} \sim N_2(\mu, \Sigma)$ where $\mu = (4\ 2)'$ and $\Sigma = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

sketch the constant density contours that contains 50%, 70% and 90% of probability

$\mu = (4\ 2)'$, $\Sigma = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

Here,

$\sigma_{11} = 2$, $\sigma_{22} = 2$, $\sigma_{12} = \sigma_{21} = 1$

Covariance is positive

major axis $= c\sqrt{\sigma_{11} + \sigma_{12}}$

$\alpha_1 = 50\% = 0.5$

$\alpha_2 = 70\% = 0.7$

$\alpha_3 = 90\% = 0.9$

Using minitab we find prob. for c_i & α_i, $i = 1,2,3$

$P(X \le x)$	x	for 2 d.f.	
0.5	1.38629	$c_1^2 = 1.38629$	$c_1 = 1.17740$
0.7	2.40995	$c_2^2 = 2.40995$	$c_2 = 1.55175$
0.9	4.60517	$c_3^2 = 4.60517$	$c_3 = 2.1459$

1] Major axis $= c_1\sqrt{\sigma_{11} + \sigma_{12}} = 1.17741\sqrt{3} = 2.0393$

Minor axis $= c_1\sqrt{\sigma_{11} - \sigma_{12}}$ $1.17741\sqrt{1} = 1.17741$

2] Major axis $= c_2\sqrt{\sigma_{11} + \sigma_{12}} = 1.55175\sqrt{3} = 2.68770$

Minor axis $= c_2\sqrt{\sigma_{11} - \sigma_{12}} = 1.55175\sqrt{1} = 1.55175$

3] Major axis $= c_3\sqrt{\sigma_{11} + \sigma_{12}} = 2.1459\sqrt{3} = 3.71691$

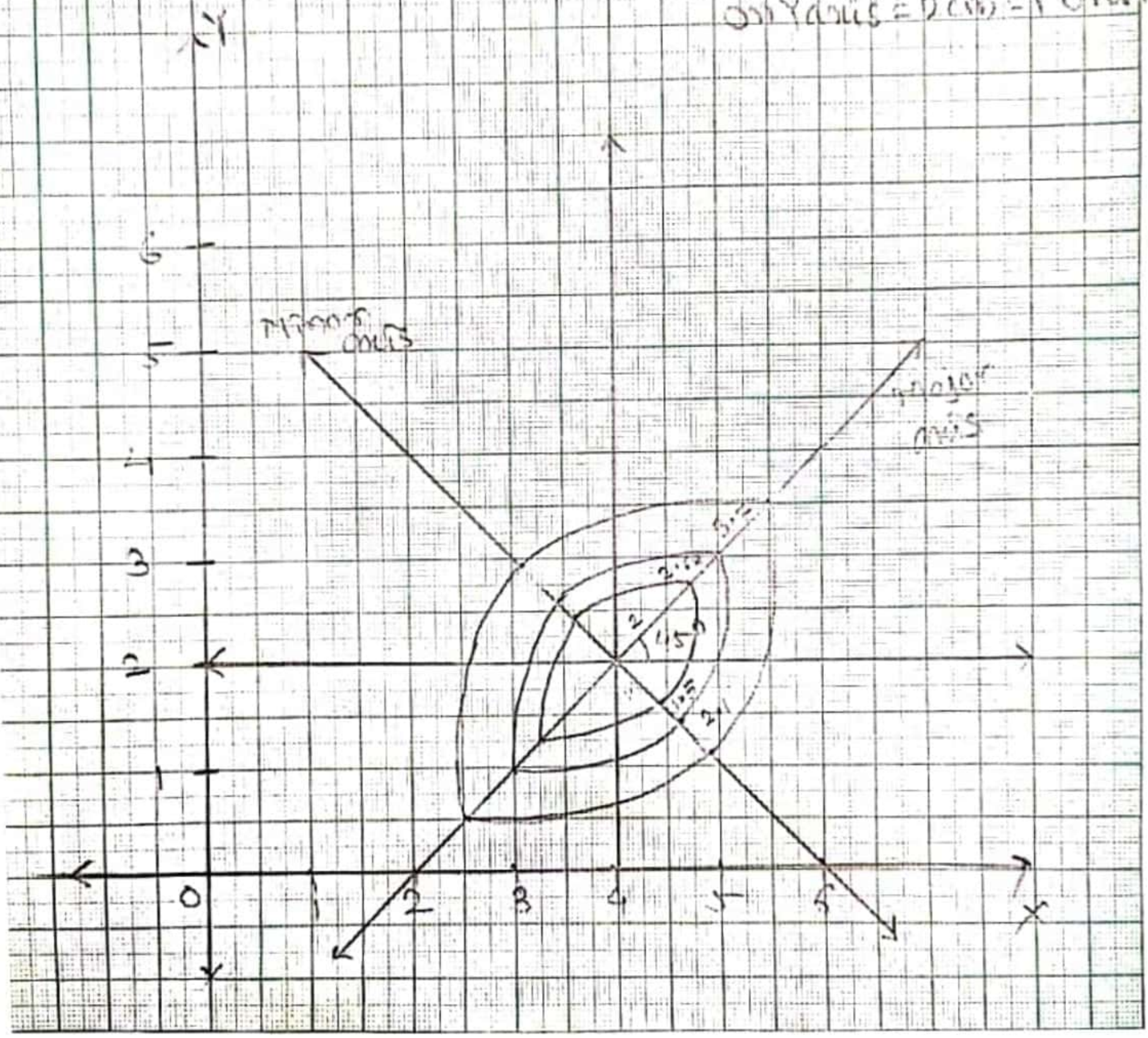
Minor axis $= c_3\sqrt{\sigma_{11} - \sigma_{12}} = 2.1459\sqrt{1} = 2.14596$

Then we plot this on graph from point μ.

Origin : (______________) Slope : ______________ on y - axis -

Q3]

Scale:
on x axis = 2cm = 1 unit
on Y axis = 2cm = 1 unit



Q4 answer:-

X_1 : Measurement involves Sending a shock wave down the board.

X_2 : Measurement is determined while vibrating the board.

X_3: Measurements are obtained from static tests.

X_4: Measurements are obtained from static tests.

For checking the normality assumption:

Program:

```
x=[ 1889   1651   1561   1778 ;      % Given X =( X1, X2 , X3 ,X4 )
2403   2048   2087   2197;
2119   1700   1815   2222;
1645   1627   1110   1533;
1976   1916   1614   1883;
1712   1712   1439   1546;
1943   1685   1271   1671;
2104   1820   1717   1874;
2983   2794   2412   2581;
1745   1600   1384   1508;
1710   1591   1518   1667;
2046   1907   1627   1898;
1840   1841   1595   1741;
1867   1685   1493   1678;
1859   1649   1389   1714;
1954   2149   1180   1281;
1325   1170   1002   1176;
1419   1371   1252   1308;
1828   1634   1602   1755;
1725   1594   1313   1646;
2276   2189   1547   2111;
1899   1614   1422   1477;
1633   1513   1290   1516;
2061   1867   1646   2037;
1856   1493   1356   1533;
1727   1412   1238   1469;
2168   1896   1701   1834;
1655   1675   1414   1597;
```

```
2326   2301   2065   2234;
1490   1382   1214   1284]

n=30
xbar=(1/n)*x'*ones(n,1)   % Sample mean vector of vector X
var=(1/n)*x'*(eye(n)-((1/n)*ones(n,n)))*x    % Sample Variance-Covariance matrix
```

% To check the normality of vector X we use $(X-\mu)'\Sigma^{-1}(X-\mu) \le \chi^2_{\alpha,p}$

```
for i=1:n
  s(i)=(x(i,:)-xbar')*(inv(var))*(x(i,:)-xbar')';  % this gives (X − μ)Σ⁻¹(X − μ)'
end
s'   % (X − μ)'Σ⁻¹(X − μ)
```

Output:

x =

1889	1651	1561	1778
2403	2048	2087	2197
2119	1700	1815	2222
1645	1627	1110	1533
1976	1916	1614	1883
1712	1712	1439	1546
1943	1685	1271	1671
2104	1820	1717	1874
2983	2794	2412	2581
1745	1600	1384	1508
1710	1591	1518	1667
2046	1907	1627	1898
1840	1841	1595	1741
1867	1685	1493	1678
1859	1649	1389	1714
1954	2149	1180	1281
1325	1170	1002	1176
1419	1371	1252	1308
1828	1634	1602	1755
1725	1594	1313	1646

2276	2189	1547	2111
1899	1614	1422	1477
1633	1513	1290	1516
2061	1867	1646	2037
1856	1493	1356	1533
1727	1412	1238	1469
2168	1896	1701	1834
1655	1675	1414	1597
2326	2301	2065	2234
1490	1382	1214	1284

```
n =

  30

xbar =

 1.0e+003 *

  1.9061

  1.7495

  1.5091

  1.7250

var =

 1.0e+005 *

  1.0210  0.9146  0.8438  0.9109

  0.9146  0.9813  0.7360  0.7836

  0.8438  0.7360  0.8885  0.8734

  0.9109  0.7836  0.8734  1.0075

ans =

  0.6207

  5.6659

  7.8793

  5.3872

  1.4463
```

2.2957

5.1604

1.5390

12.6877

0.7930

1.9974

0.4795

2.7889

0.1340

1.1165

17.4284

3.6226

4.1276

1.4102

1.5155

10.2394

5.2301

0.8237

2.6261

4.7346

3.5152

2.4637

3.0985

6.5004

2.6729

The value of $\chi^2_{\alpha,p} = \chi^2_{0.05,4} = 9.48773$

Most of the above values of $(X - \mu)'\Sigma^{-1}(X - \mu) = s'$ is less than $\chi^2_{0.05,4} = 9.48773$

Therefore, the four variables in data follows normality assumption.