**Department of Statistics,**

**Modern college of Arts, Science and Commerce, Pune-05**

**M.Sc.I (Statistics) Semester II**

**ST-28                                                                                                       Date:**

**Practical No. 16                                                                            Submission date:**

**Title: Model Sampling  from Multivariate Normal distribution and computation of M.L.E.’s of**

**parameters.**

**\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\***

Q.1 a) Draw a random sample of size 15 from Np, distribution with p=2, where

                                                =1  2' and   =   4 1     1 4   .

b) Compute M.L.E. of , and    based on the sample data generated in (a).

c) Find the distribution of Y= (2X1+2X2 , X1-X2)’. Compute M.L.E.’s of parameters of

                the corresponding distribution based on the generated sample of size 15 from it.

d) Find the distribution of (2X1+3X2 | X1-X2=10). Generate 15 observations from it.

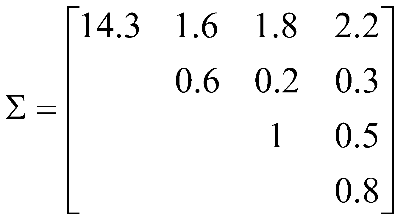
Q.2 a) Draw a random sample of size 15 from N3, distribution, where =4   5   6'

    and  =5 3 1  4 3   3 .

b) Compute M.L.E. of '+b ,  where =3   3  3 '   b=1  2  4' and    based on

                the sample data generated in (a) where is a population correlation matrix.

c) Find the distribution of  X1+3X2+5X3.

Q.3 Consider  X ~ N4, where =4   6  1   5' and  

               Partition X  as X1=X1, X2' and   X2=X3, X4'

1. Find the distribution of   X1|X2=(4  5)’.
2. Find the distribution of X2|X1=(1  2)'
3. Compute the M.L.E.’s of the corresponding parameters based on the r.s. of size 15 for (a) and (b) above.

> #1a

> n=15

> p=2

> mu = matrix(c(1,2),nrow=2)

> mu

[,1]

[1,] 1

[2,] 2

> Si = matrix(c(4,1,1,4),ncol = 2)

> Si

[,1] [,2]

[1,] 4 1

[2,] 1 4

> C=chol(Si)

> C

[,1] [,2]

[1,] 2 0.500000

[2,] 0 1.936492

> c=t(C)

> c

[,1] [,2]

[1,] 2.0 0.000000

[2,] 0.5 1.936492

> Y = matrix(rep(0,30),nrow=2)

> E1n=matrix(rep(1,n),nrow=1)

> for(i in 1:2)

+ {

+ Y[i,]=rnorm(15,0,1)

+ }

> Y

[,1] [,2] [,3] [,4] [,5] [,6]

[1,] -3.21570578 0.004173273 1.0558277 -0.9994661 0.6607246 -0.3447780

[2,] -0.05746072 -1.905823194 0.6219657 -0.9665116 -1.0402225 0.6139829

[,7] [,8] [,9] [,10] [,11] [,12] [,13]

[1,] 1.5168977 0.7474534 0.3541057 -1.092915 0.2120367 -0.5225635 -0.3537281

[2,] 0.4828386 -0.7179568 -0.7528264 -1.246800 1.7220473 0.9877537 0.2437025

[,14] [,15]

[1,] 1.3903983 0.2254774

[2,] 0.7993193 -1.3415112

> X = (c%\*%Y) + (mu%\*%E1n)

> X

[,1] [,2] [,3] [,4] [,5] [,6] [,7]

[1,] -5.4314116 1.008347 3.111655 -0.9989322 2.321449 0.310444 4.033795

[2,] 0.2808749 -1.688524 3.732345 -0.3713746 0.315980 3.016584 3.693462

[,8] [,9] [,10] [,11] [,12] [,13] [,14]

[1,] 2.4949067 1.7082113 -1.1858292 1.424073 -0.04512692 0.2925437 3.780797

[2,] 0.9834094 0.7192107 -0.9608743 5.440749 3.65149511 2.2950638 4.243074

[,15]

[1,] 1.4509547

[2,] -0.4850866

>

> #1b

> #X follows multivariate normal distribution (p- variate) with parameter mu and variance covariance matrix Si

> #to calculate M.L.E. of mu based on sample

> En1 = t(E1n)

> Xbar = (1/n)\*(X%\*%En1)

> Xbar

[,1]

[1,] 0.9517251

[2,] 1.6577592

> #Xbar is the M.L.E of mu

> Enn = matrix(rep(1,225),nrow=15)

> I = diag(15)

> B = (1/n)\*Enn

> A1 = I-B

> S = (1/n)\*(X%\*%A1%\*%t(X))

> S

[,1] [,2]

[1,] 5.228773 2.004922

[2,] 2.004922 4.497749

> SI = (n/(n-1))\*S

> SI

[,1] [,2]

[1,] 5.602256 2.148130

[2,] 2.148130 4.819016

> #Here SI is the M.L.E of sigma

>

> #1c

> A = matrix(c(2,2,1,-1),nrow=2,byrow=T)

> mu1= A%\*%mu

> Si1= A%\*%Si%\*%(t(A))

> #Thus Y1 = (2X1+2X2,X1-X2) with parameter mu1 and Si1

> Y1 = A%\*%X

> #Y1 follows multivariate normal distribution (p- variate) with parameter mu1 and variance covariance matrix Si1

> #to calculate M.L.E. of mu1 based on sample

> En1 = t(E1n)

> Xbar1 = (1/n)\*(Y1%\*%En1)

> Xbar1

[,1]

[1,] 5.218969

[2,] -0.706034

> #Xbar1 is the M.L.E of mu1

> Enn = matrix(rep(1,225),nrow=15)

> I = diag(15)

> B = (1/n)\*Enn

> A1 = I-B

> S1 = (1/n)\*(Y1%\*%A1%\*%t(Y1))

> S1

[,1] [,2]

[1,] 54.945458 1.462048

[2,] 1.462048 5.716678

> #Here S1 is the M.L.E of sigma

>

> #1d

> #to find distribution of (2X1+3X2|X1-X2=10)

> A2 = matrix(c(2,3,1,-1),nrow=2,byrow=T)

> A2

[,1] [,2]

[1,] 2 3

[2,] 1 -1

> mu2= A2%\*%mu

> mu2

[,1]

[1,] 8

[2,] -1

> Si2= A2%\*%Si%\*%(t(A2))

> Si2

[,1] [,2]

[1,] 64 -3

[2,] -3 6

> #Thus Y2 = (2X1+3X2,X1-X2) follows multivariate normal distribution with parameter mu2 and Si2

> mu3 = mu2[1,]+(Si2[1,2]\*(1/Si2[2,2])\*(10-mu2[2,]))

> mu3

[1] 2.5

> Si3 = Si2[1,1]-(Si2[1,2]\*(1/Si2[2,2])\*Si2[2,1])

> Si3

[1] 62.5

> #Thus (2X1+3X2|X1-X2=10) follows normal distribution with parameter mu3 and Si3

> Y2 = rnorm(15,mu3,Si3)

> Y2

[1] 34.604181 -24.828059 23.591249 -1.922399 -29.823855 -5.523604

[7] 162.571905 2.822039 91.481034 -60.499998 61.690142 6.470572

[13] -91.841930 42.531486 47.997458

>

> #2a

> n=15

> p=3

> mu = matrix(c(4,5,6),nrow=3)

> mu

[,1]

[1,] 4

[2,] 5

[3,] 6

> Si = matrix(c(5,3,1,3,4,3,1,3,3),ncol = 3,byrow=T)

> Si

[,1] [,2] [,3]

[1,] 5 3 1

[2,] 3 4 3

[3,] 1 3 3

> C=chol(Si)

> C

[,1] [,2] [,3]

[1,] 2.236068 1.341641 0.4472136

[2,] 0.000000 1.483240 1.6180797

[3,] 0.000000 0.000000 0.4264014

> c=t(C)

> c

[,1] [,2] [,3]

[1,] 2.2360680 0.00000 0.0000000

[2,] 1.3416408 1.48324 0.0000000

[3,] 0.4472136 1.61808 0.4264014

> Y = matrix(rep(0,45),nrow=3)

> E1n=matrix(rep(1,n),nrow=1)

> for(i in 1:3)

+ {

+ Y[i,]=rnorm(15,0,1)

+ }

> Y

[,1] [,2] [,3] [,4] [,5] [,6]

[1,] -0.9232671 0.6498282 -0.01857017 -0.09263239 -0.75502745 -0.6288050

[2,] 0.3861805 -1.0038218 -0.54641793 -1.58880966 0.03006941 -0.5183459

[3,] 0.9807609 0.1800552 0.38542578 -0.82130231 -1.57216152 0.7941741

[,7] [,8] [,9] [,10] [,11] [,12]

[1,] 1.9816780 -1.1613318 0.9593290 -0.8562555 1.27072254 1.0025969

[2,] 0.5056723 0.7624181 1.5673944 -1.2787026 -0.92808458 -0.7819279

[3,] 0.6731836 -1.6731414 -0.3744073 2.0244382 0.03239677 0.1611685

[,13] [,14] [,15]

[1,] 0.9634089 1.1831516 -0.2854728

[2,] 0.6460682 -0.5610166 -0.2135227

[3,] 0.3013901 0.4074148 -1.5946324

> X = (c%\*%Y) + (mu%\*%E1n)

> X

[,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8]

[1,] 1.935512 5.453060 3.958476 3.792868 2.311707 2.593949 8.431167 1.403183

[2,] 4.334105 4.382928 4.164617 2.519135 4.031625 3.387538 8.408733 4.572759

[3,] 6.630171 4.743124 5.271894 3.037548 5.040624 5.218702 7.991498 6.000860

[,9] [,10] [,11] [,12] [,13] [,14] [,15]

[1,] 6.145125 2.085355 6.841422 6.241875 6.154248 6.645607 3.361663

[2,] 8.611897 1.954590 5.328281 5.185338 7.250823 5.755242 4.300293

[3,] 8.805546 4.411252 5.080384 5.251876 7.604753 5.795074 4.846882

>

> #2b

> a = matrix(c(3,3,3),nrow=3,byrow=T)

> a

[,1]

[1,] 3

[2,] 3

[3,] 3

> b = matrix(c(1,2,4),nrow=3,byrow=T)

> b

[,1]

[1,] 1

[2,] 2

[3,] 4

> at = t(a)

> at

[,1] [,2] [,3]

[1,] 3 3 3

> mu1= (at%\*%mu) + t(b)

Error in (at %\*% mu) + t(b) : non-conformable arrays

> mu1

[,1]

[1,] 6

[2,] -1

> Si1= t(a)%\*%Si%\*%a

> Si1

[,1]

[1,] 234

> #Thus Y1 = aX + b follows normal with parameter mu1 and Si1

> Y1 = a%\*%X + b

Error in a %\*% X : non-conformable arguments

> #Y1 follows normal distribution with parameter mu1 and variance covariance matrix Si1

> #to calculate M.L.E. of mu1 based on sample

> En1 = t(E1n)

> Xbar1 = (1/n)\*(Y1%\*%En1)

> Xbar1

[,1]

[1,] 5.218969

[2,] -0.706034

> #Xbar1 is the M.L.E of mu1

> Enn = matrix(rep(1,225),nrow=15)

> I = diag(15)

> B = (1/n)\*Enn

> A1 = I-B

> S = (1/n)\*(X%\*%A1%\*%t(X))

> S

[,1] [,2] [,3]

[1,] 4.495475 2.917452 1.304787

[2,] 2.917452 3.437412 2.390315

[3,] 1.304787 2.390315 2.087472

> #Here S is the M.L.E of sigma

> r = cov2cor(S)

> r

[,1] [,2] [,3]

[1,] 1.0000000 0.7421645 0.4259332

[2,] 0.7421645 1.0000000 0.8923370

[3,] 0.4259332 0.8923370 1.0000000

> #Here r is M.L.E of rho

>

>

> #2c

> A = matrix(c(1,3,5),nrow=1)

> mu2= A%\*%mu

> mu2

[,1]

[1,] 49

> Si2= A%\*%Si%\*%(t(A))

> Si2

[,1]

[1,] 234

> #Thus Y1 = (X1+3X2+5X3) with parameter mu2 and Si2

> Y1 = A%\*%X

> Y1

[,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8]

[1,] 48.08868 42.31746 42.81179 26.53802 39.6097 38.85007 73.61486 45.12576

[,9] [,10] [,11] [,12] [,13] [,14] [,15]

[1,] 76.00855 30.00538 48.22818 48.05727 65.93048 52.88671 40.49695

> #Y1 follows normal distribution with parameter mu2 and variance covariance matrix Si2

>

> #3a

> p=2

> mu = matrix(c(4,6,1,5),nrow=4,byrow=T)

> mu

[,1]

[1,] 4

[2,] 6

[3,] 1

[4,] 5

> Si = matrix(c(14.3,1.6,1.8,2.2,1.6,0.6,0.2,0.3,1.8,0.2,1,0.5,2.2,0.3,0.5,0.8),ncol = 4,byrow=T)

> Si

[,1] [,2] [,3] [,4]

[1,] 14.3 1.6 1.8 2.2

[2,] 1.6 0.6 0.2 0.3

[3,] 1.8 0.2 1.0 0.5

[4,] 2.2 0.3 0.5 0.8

> mu1 = matrix(c(4,6),nrow=2,byrow=T)

> mu2 = matrix(c(1,5),nrow=2,byrow=T)

> S11 = matrix(c(14.3,1.6,1.6,0.6),nrow=2,byrow=T)

> S11

[,1] [,2]

[1,] 14.3 1.6

[2,] 1.6 0.6

> S12 = matrix(c(1.8,2.2,0.2,0.3),nrow=2,byrow=T)

> S12

[,1] [,2]

[1,] 1.8 2.2

[2,] 0.2 0.3

> S21 = matrix(c(1.8,0.2,2.2,0.3),nrow=2,byrow=T)

> S21

[,1] [,2]

[1,] 1.8 0.2

[2,] 2.2 0.3

> S22 = matrix(c(1,0.5,0.5,0.8),nrow=2,byrow=T)

> S22

[,1] [,2]

[1,] 1.0 0.5

[2,] 0.5 0.8

> #X(1) subset follows bivariate normal with parameter mu1 and S11

> #X(2) subset follows bivariate normal with parameter mu2 and S22

> X2 = matrix(c(4,5),nrow=2,byrow=T)

> mu3 = mu1+(S12%\*%solve(S22)%\*%(X2-mu2))

> mu3

[,1]

[1,] 5.854545

[2,] 6.054545

> Si3 = S11-(S12%\*%solve(S22)%\*%S21)

> Si3

[,1] [,2]

[1,] 7.9872727 0.7672727

[2,] 0.7672727 0.4872727

> #Thus X(1)|X(2)=(4,5)' follows bivariate normal distribution with parameter mu3 and Si3

>

>

> #3b

> X1 = matrix(c(1,2),nrow=2,byrow=T)

> mu4 = mu2+(S21%\*%solve(S11)%\*%(X1-mu1))

> mu4

[,1]

[1,] 0.6345515

[2,] 4.0697674

> Si4 = S22-(S21%\*%solve(S11)%\*%S12)

> Si4

[,1] [,2]

[1,] 0.7734219 0.2232558

[2,] 0.2232558 0.4546512

> #Thus X(2)|X(1)=(1,2)' follows bivariate normal distribution with parameter mu4 and Si4

>

> #3c

> mu3 = mu1+(S12%\*%solve(S22)%\*%(X2-mu2))

> Si3 = S11-(S12%\*%solve(S22)%\*%S21)

> n=15

> p=2

> C1=chol(Si3)

> C1

[,1] [,2]

[1,] 2.826176 0.2714879

[2,] 0.000000 0.6430918

> c1=t(C1)

> c1

[,1] [,2]

[1,] 2.8261763 0.0000000

[2,] 0.2714879 0.6430918

> Y1 = matrix(rep(0,30),nrow=2)

> E1n=matrix(rep(1,n),nrow=1)

> for(i in 1:2)

+ {

+ Y1[i,]=rnorm(15,0,1)

+ }

> Y1

[,1] [,2] [,3] [,4] [,5] [,6] [,7]

[1,] 1.6458085 0.09526325 -0.8402874 -0.3496847 1.073374 2.2298720 0.84299906

[2,] 0.6554805 -0.54609545 -2.0623120 -1.0908112 -1.006812 0.7155584 0.09553828

[,8] [,9] [,10] [,11] [,12] [,13] [,14]

[1,] 0.7398479 0.2289870 -0.4295725 0.6955018 -1.416705 0.101013 -0.06275071

[2,] 1.0866764 0.4757718 0.2227399 -0.7295733 -2.341286 1.440708 -0.22646985

[,15]

[1,] 0.4990810

[2,] -0.5108997

> X11 = (c1%\*%Y1) + (mu3%\*%E1n)

> X11

[,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8]

[1,] 10.505890 6.123776 3.479745 4.866275 8.888091 12.156557 8.237009 7.945486

[2,] 6.922897 5.729219 4.500162 5.258119 5.698481 7.120099 6.344849 6.954238

[,9] [,10] [,11] [,12] [,13] [,14] [,15]

[1,] 6.501703 4.640498 7.820156 1.850686 6.140026 5.677201 7.265036

[2,] 6.422678 6.081164 5.774183 4.164265 7.008476 5.891868 5.861484

> #X11 is a random sample from the distribution X(1)|X(2)=(4,5)'

> #to calculate M.L.E. of mu based on sample

> En1 = t(E1n)

> Xbar = (1/n)\*(X11%\*%En1)

> Xbar

[,1]

[1,] 6.806542

[2,] 5.982145

> #Xbar is the M.L.E of mu3

> Enn = matrix(rep(1,225),nrow=15)

> I = diag(15)

> B = (1/n)\*Enn

> A1 = I-B

> S = (1/n)\*(X11%\*%A1%\*%t(X11))

> S

[,1] [,2]

[1,] 6.505664 1.661578

[2,] 1.661578 0.716618

> #Here S is the M.L.E of sigma

> mu4 = mu2+(S21%\*%solve(S11)%\*%(X1-mu1))

> Si4 = S22-(S21%\*%solve(S11)%\*%S12)

> C2=chol(Si4)

> C2

[,1] [,2]

[1,] 0.8794441 0.2538602

[2,] 0.0000000 0.6246649

> c2=t(C2)

> c2

[,1] [,2]

[1,] 0.8794441 0.0000000

[2,] 0.2538602 0.6246649

> Y2 = matrix(rep(0,30),nrow=2)

> E1n=matrix(rep(1,n),nrow=1)

> for(i in 1:2)

+ {

+ Y2[i,]=rnorm(15,0,1)

+ }

> Y2

[,1] [,2] [,3] [,4] [,5] [,6]

[1,] 1.6122374 -0.8751284 -0.01661236 -1.0082774 0.9956351 -0.3210435

[2,] 0.9662465 -1.0474019 -0.38343111 0.3998348 -1.9502533 1.0593883

[,7] [,8] [,9] [,10] [,11] [,12]

[1,] -0.04538418 -1.29844354 -0.09749454 2.7230983 -0.6817202 -0.2802816

[2,] 1.05147085 0.07018926 0.42600740 0.4462644 -1.3834644 0.4917014

[,13] [,14] [,15]

[1,] 1.5148398 -0.5361065 -1.337010

[2,] 0.8167604 0.1002341 1.707163

> X22 = (c2%\*%Y2) + (mu4%\*%E1n)

> X22

[,1] [,2] [,3] [,4] [,5] [,6] [,7]

[1,] 2.052424 -0.135075 0.6199419 -0.2521721 1.510157 0.3522117 0.5946386

[2,] 5.082631 3.193332 3.8260343 4.0635687 3.104265 4.6500299 4.7150631

[,8] [,9] [,10] [,11] [,12] [,13] [,14]

[1,] -0.507357 0.5488105 3.029364 0.03501667 0.3880595 1.966768 0.1630758

[2,] 3.783989 4.3111293 5.039819 3.03250428 4.3057637 4.964526 3.9962841

[,15]

[1,] -0.5412741

[2,] 4.7967583

> #X22 is a random sample from the distribution X(2)|X(1)=(1,2)'

> #to calculate M.L.E. of mu based on sample

> En1 = t(E1n)

> Xbar = (1/n)\*(X22%\*%En1)

> Xbar

[,1]

[1,] 0.6549727

[2,] 4.1910465

> #Xbar is the M.L.E of mu4

> Enn = matrix(rep(1,225),nrow=15)

> I = diag(15)

> B = (1/n)\*Enn

> A1 = I-B

> S = (1/n)\*(X22%\*%A1%\*%t(X22))

> S

[,1] [,2]

[1,] 1.0051474 0.3074559

[2,] 0.3074559 0.4576797

> #S is M.L.E of sigma Si4