Part II

The Quantum Inverse Scattering Method

Introduction to Part II

The quantum inverse scattering method relates the Bethe Ansatz to the theory of classical completely integrable differential equations. These are sometimes called soliton equations. The modern way to solve them is called the classical inverse scattering method. In a sense this is a nonlinear generalization of the Fourier transform.

In this Part the quantum inverse scattering method is expounded. The main statements of the classical inverse scattering method necessary for quantization are given in Chapter V where the Lax representation is introduced. The Hamiltonian structure of integrable models is also discussed along with the infinite number of integrals of motion. The most convenient method of analyzing the Hamiltonian structure relies on the classical r-matrix. Some concrete models will be considered. Chapter VI is devoted in particular to the quantum inverse scattering method. The R-matrix, which is the main object of this method, is introduced. The Yang-Baxter equation for the R-matrix is discussed. The main statements of the method are given and a number of examples are presented. The algebraic formulation of the Bethe Ansatz, one of the main achievements of the quantum inverse scattering method, is presented in Chapter VII. The notion of the determinant of the transition matrix in the quantum case is introduced in this chapter. (This is closely related to the concept of the antipode in quantum groups.)

Integrable models of quantum field theory on the lattice are presented in Chapter VIII. The quantum inverse scattering method provides a mechanism for transferring continuous models of quantum field theory to the lattice while preserving the R-matrix. For classical models this means that the structure of the action-angle variables remains the same [2]. In the quantum case, this leads to conservation of the scattering matrix and critical exponents that drive the long distance asymptotics of the correlation functions. For relativistic models of field theory (such as the sine-Gordon model), the lattice variant provides a rigorous solution to

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the problem of ultraviolet divergences. We can study continuous models as condensed lattice models (i.e., the lattice spacing $\Delta \to 0$). Our construction guarantees that there will be no phase transition. The explicit form of the L-operator allows the classification of all integrable models having a given R-matrix.

The close relationship of the quantum inverse scattering method to other methods of contemporary mathematical physics should be mentioned. Firstly, it is connected with quantum groups [1] and the theory of knots [5]. It is also related to the method used in classical statistical physics for solving two-dimensional lattice models. We shall abbreviate the name "quantum inverse scattering method" to QISM. We must note that the physical characteristics of particles—the dressed energy, momentum and S-matrix—are all calculated within QISM exactly as for the coordinate Bethe Ansatz.

It is interesting to mention that later in the book (see Chapter XIV) we shall obtain classical completely integrable differential equations for quantum correlation functions. We shall study them from a different point of view. The most important problem in this case is determining the long distance asymptotics of the correlation functions. We shall do this by means of the Riemann-Hilbert problem; see Chapters XV and XVI and, for example, [3], [4].