TIME SERIES FORECASTING

EXPONENTIAL SMOOTHING FORECAST

Predictive power of a model is estimated by comparing its forecasting performance on a Test Data

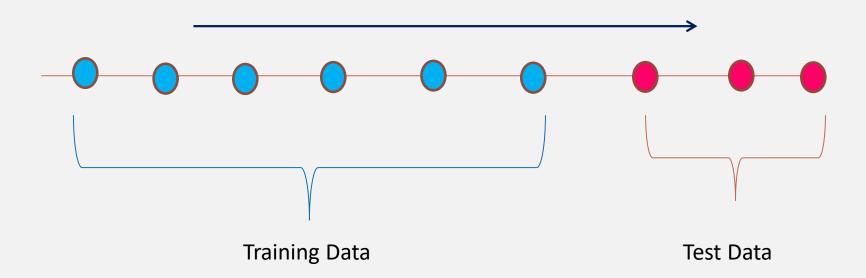
A part of sample data is used to train (develop) the model: Training Data

A part of sample is withheld from estimation process: Test Data

The model which gives smallest measure of error between forecast and actual series is the 'best'

- Training Data is used to identify a few working models
- The forecasts for training data are called **fitted values**
- Each of the models is tested against the observed values of the series for hold-out period
- The model is selected to be the best where observed and forecasted values are the closest

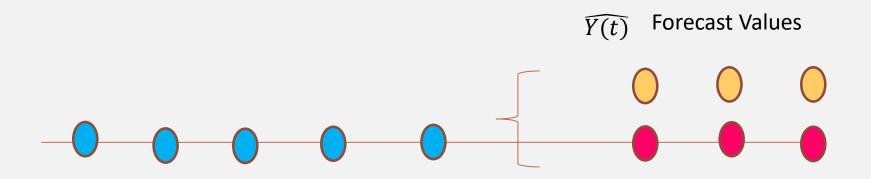
- For other predicting models hold-out sample is randomly chosen from the total sample
- Usually in 80:20 ratio
- For time series hold-out sample has to be the most recent period because of the ordered nature of the data



- If there is seasonality in the data, the at least one full season's data need to be held out for validation
- Once a model is selected, held-out period must be incorporated into the sample for final forecasts for future

Measures of Forecast Accuracy

• Performance of forecast method is tested by comparing the forecast values with the test sample observations



Y(t) Observed Values

Compare the observed and forecast values through various methods

Measures of Forecast Accuracy

Residual sum of squares

$$RSS = \sum (Y(t) - \widehat{Y(t)})^2$$

Mean sum of squares

$$MSS = \frac{1}{T} \sum (Y(t) - \widehat{Y(t)})^{2}$$

Mean absolute deviation

$$MAD = \frac{1}{T} \sum (|Y(t) - \widehat{Y(t)}|)$$

Mean absolute percent error

$$MAPE = \frac{1}{T} \sum \left[\left(|Y(t) - \widehat{Y(t)}| \right) / Y(t) \right] \times 100$$

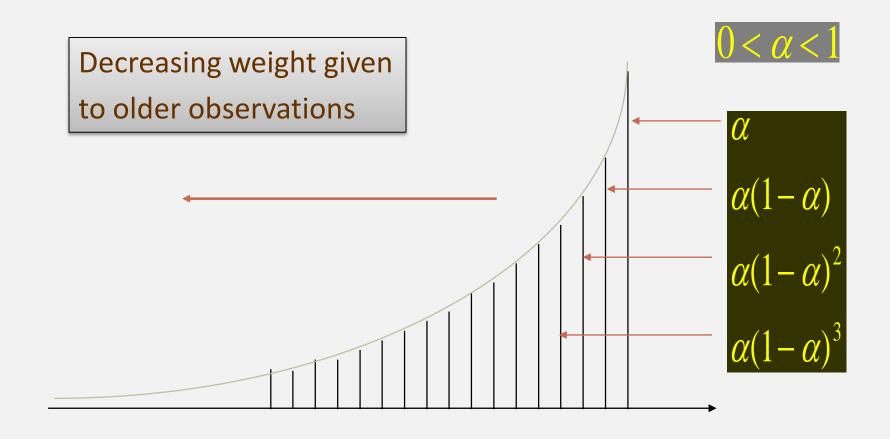
Forecast by Average

- Using mean of all past observations easiest, intuitive, naïve
- Naturally does not work well!!
- Can this scheme be modified to get useful forecasts?

Exponential Smoothing

- Weighted averages of past observations
- Weights decaying as observations get older
- Practically speaking, only the recent observations matter
- One or more parameters control how fast the weights decay
- These parameters have values between 0 and 1

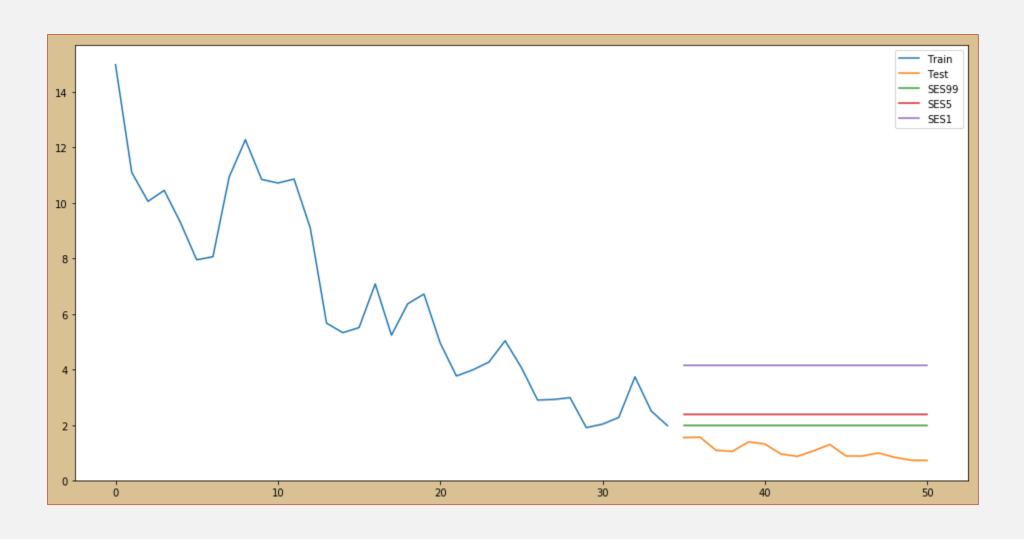
Exponential Smoothing



Simple Exponential (SES)

If the time series neither has a pronounced trend nor seasonality: Almost non-available!

Petrol Consumption



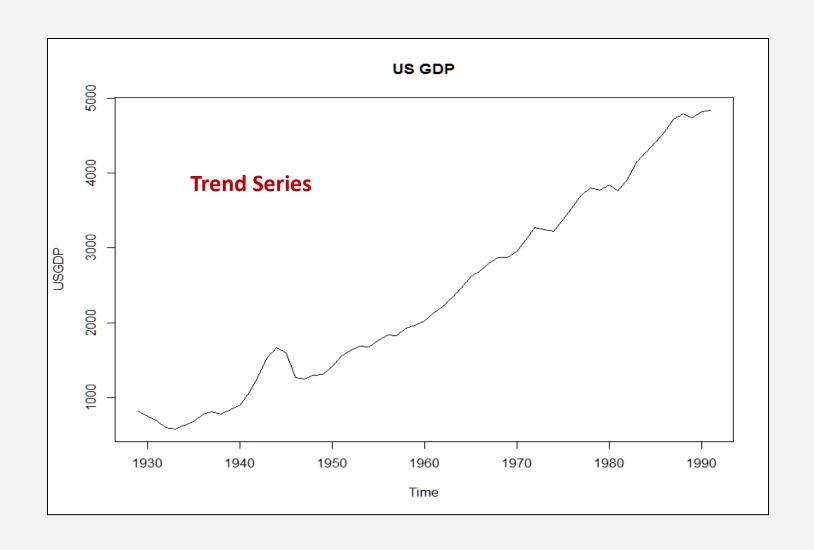
Exponential Smoothing

- Performance of the smoothing parameter α controls performance of the method
- If α is closer to 1, forecasts follow the actual observations more closely
- If α is closer to 0, forecasts are farther from the actual observations and the line is smooth

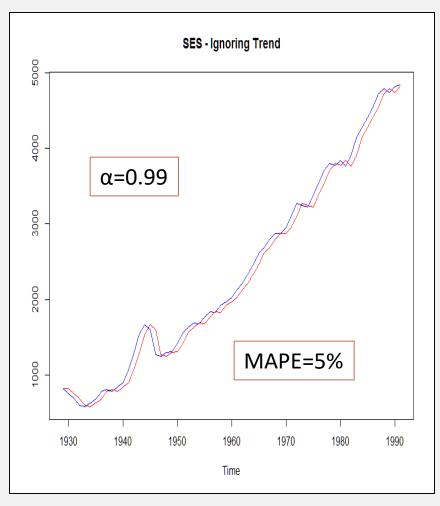
Double Exponential

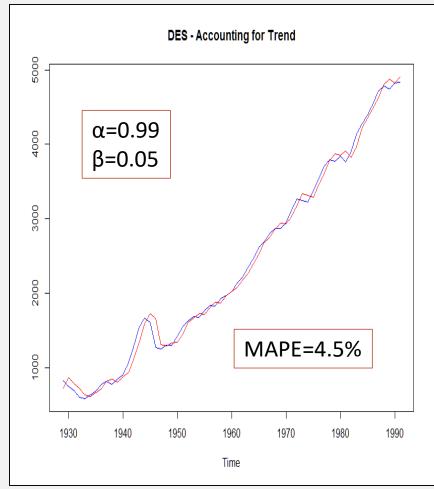
- Applicable when data has Trend but no seasonality
- An extension of SES
- Two separate components are considered: Level and Trend
- Level is the local mean
- One smoothing parameter α corresponds to the level series
- A second smoothing parameter β corresponds to the trend series
- Also known as Holt model

Caselet V: US GDP

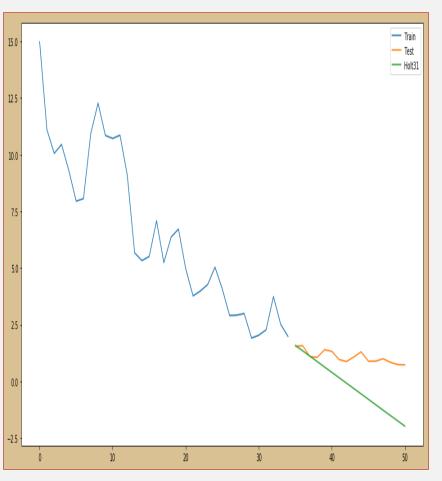


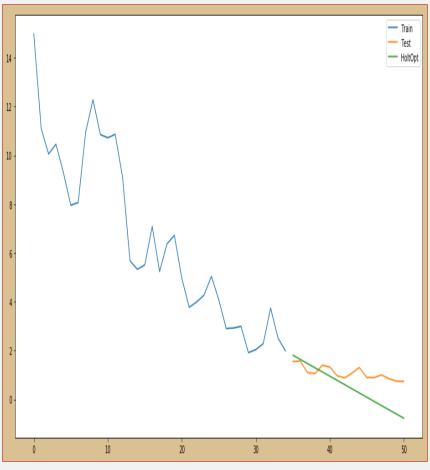
Caselet V: US GDP



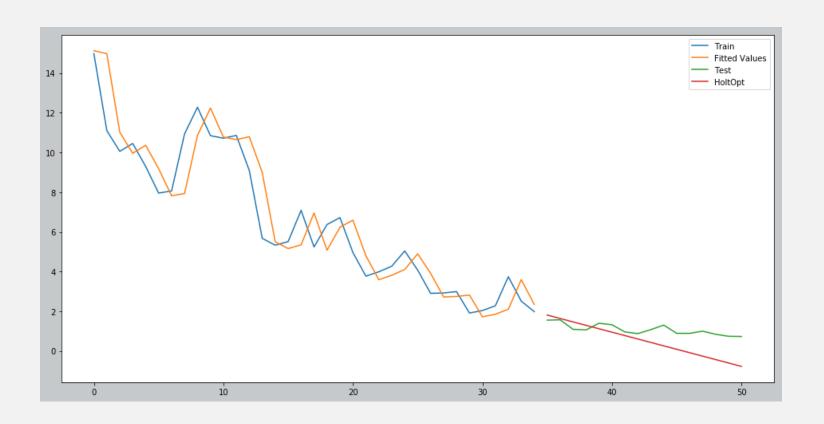


Petrol Consumption





Petrol Consumption



Exponential Smoothing with Seasonality: Holt-Winters' Model

- Because Seasonality can be additive or multiplicative, HW model can be additive or multiplicative
- Simultaneously smooths the level, trend and seasonality
- Three separate smoothing parameters

```
    α: Smooths level; 0 < α < 1</li>
    β: Smooths trend; 0 < β < 1</li>
    γ : Smooths seasonality; 0 < γ < 1</li>
```

Champagne Sales

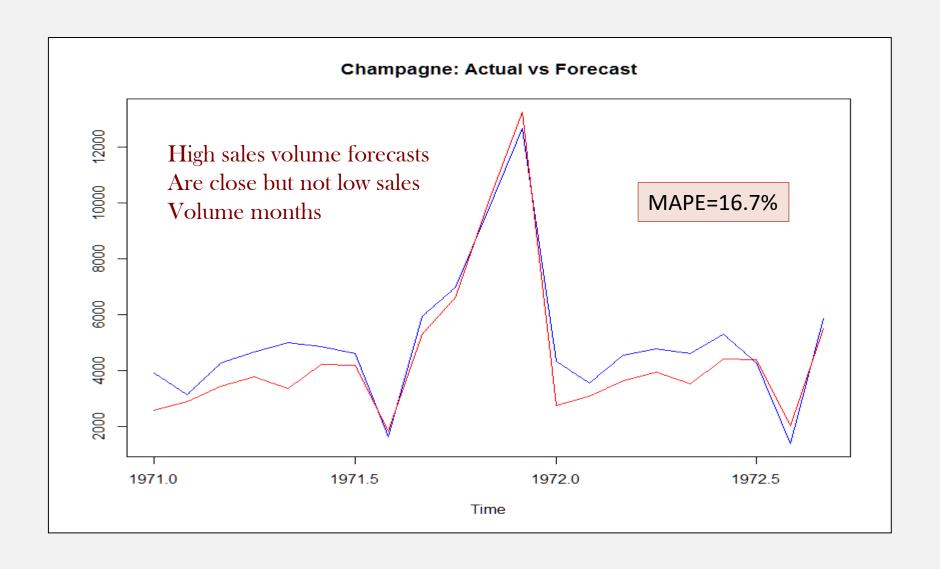
Model Estimates

Smoothing parameters:

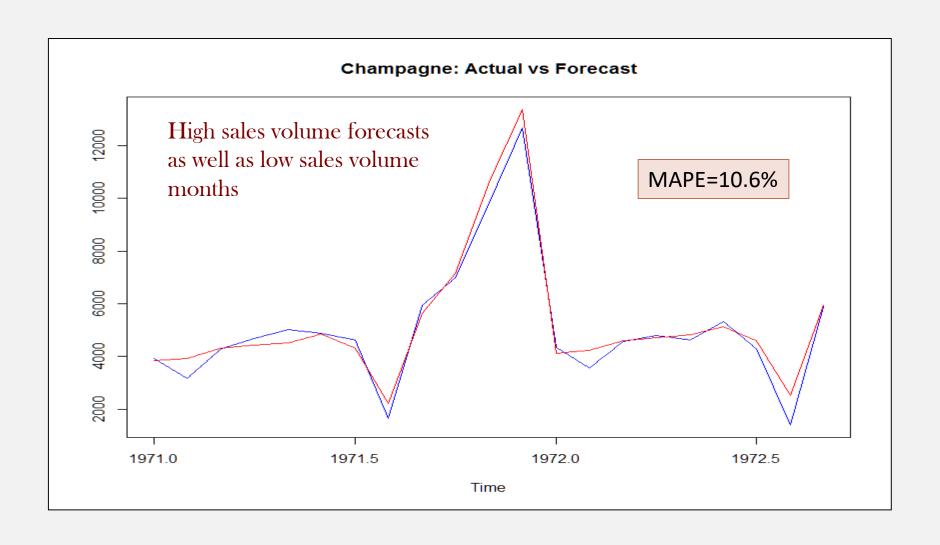
```
alpha = 0.0842
beta = 1e-04
gamma = 0.7608
```

- Smoothing parameter for trend (β) almost 0 corroborates well with insignificant YOY movement
- Smoothing parameter for seasonality(γ) fairly high that almost all fluctuations are due to seasonality

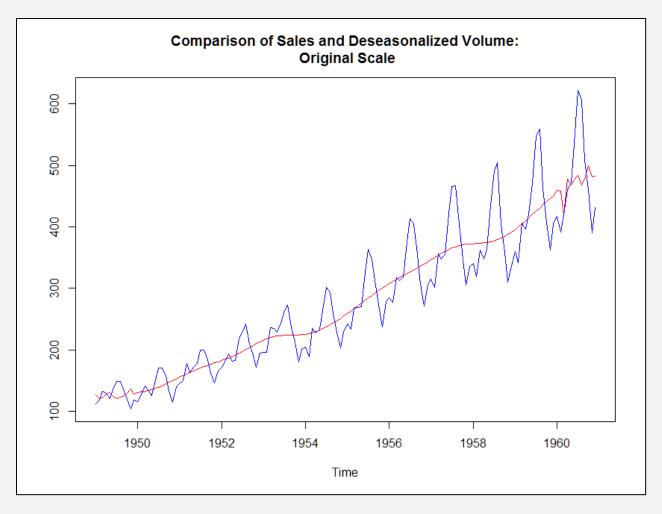
Champagne Sales



Champagne Sales



Passenger Volume



Significant trend and seasonality both

Passenger Volume

Model Estimates

Smoothing parameters:

```
alpha = 0.3515
beta = 0.0147
gamma = 0.6481
```

- Smoothing parameter for trend (β) small compared to other parameters
- Indicates almost a straight line trend
- Smoothing parameter for seasonality(γ) fairly high

Passenger Volume

