Cross-Entropy Loss Function

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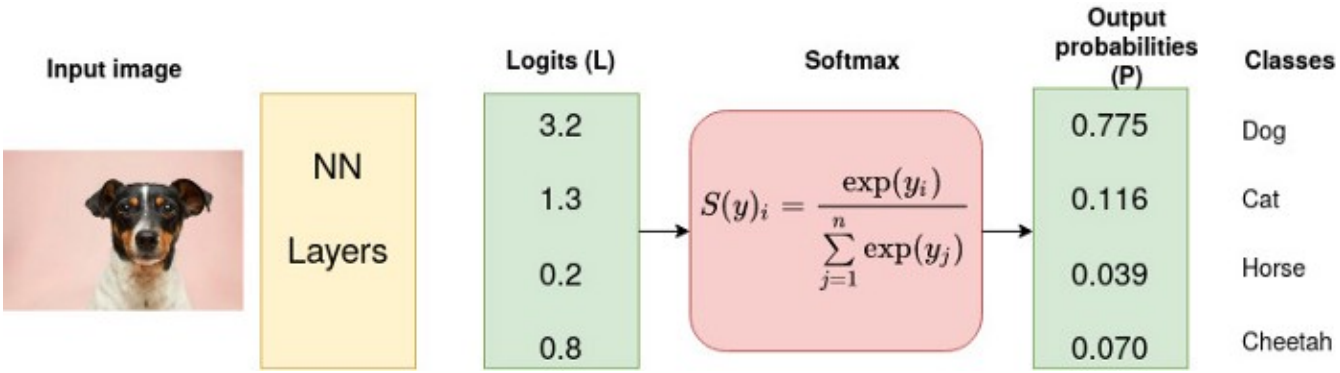
Photo by Fatos Bytyqi on Unsplash

When working on a Machine Learning or a Deep Learning Problem, loss/cost functions are used to optimize the model during training. The objective is almost always to

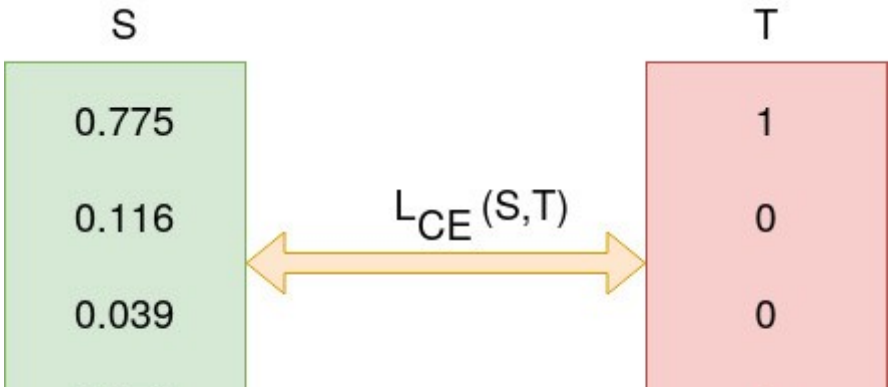
understanding of Cross-Entropy is pegged on understanding of Softmax activation function. I have put up another article below to cover this prerequisite

Consider a 4-class classification task where an image is classified as either a dog, cat, horse or cheetah.



Input image source: Photo by Victor Grabarczyk on Unsplash . Diagram by author.

In the above Figure, Softmax converts logits into probabilities. The purpose of the Cross-Entropy is to take the output probabilities (P) and measure the distance from the truth values (as shown in Figure below).



Cross Entropy (L) (Source: Author).

For the example above the desired output is `[1,0,0,0]` for the class `dog` but the model outputs `[0.775, 0.116, 0.039, 0.070]` .

The objective is to make the model output be as close as possible to the desired output (truth values). During model training, the model weights are iteratively adjusted accordingly with the aim of minimizing the Cross-Entropy loss. The process of adjusting the weights is what defines *model training* and as the model keeps training and the loss is getting minimized, we say that the model is *learning*.

The concept of cross-entropy traces back into the field of Information Theory where Claude Shannon introduced the concept of entropy in 1948. Before diving into Cross-Entropy cost function, let us introduce entropy .

Entropy

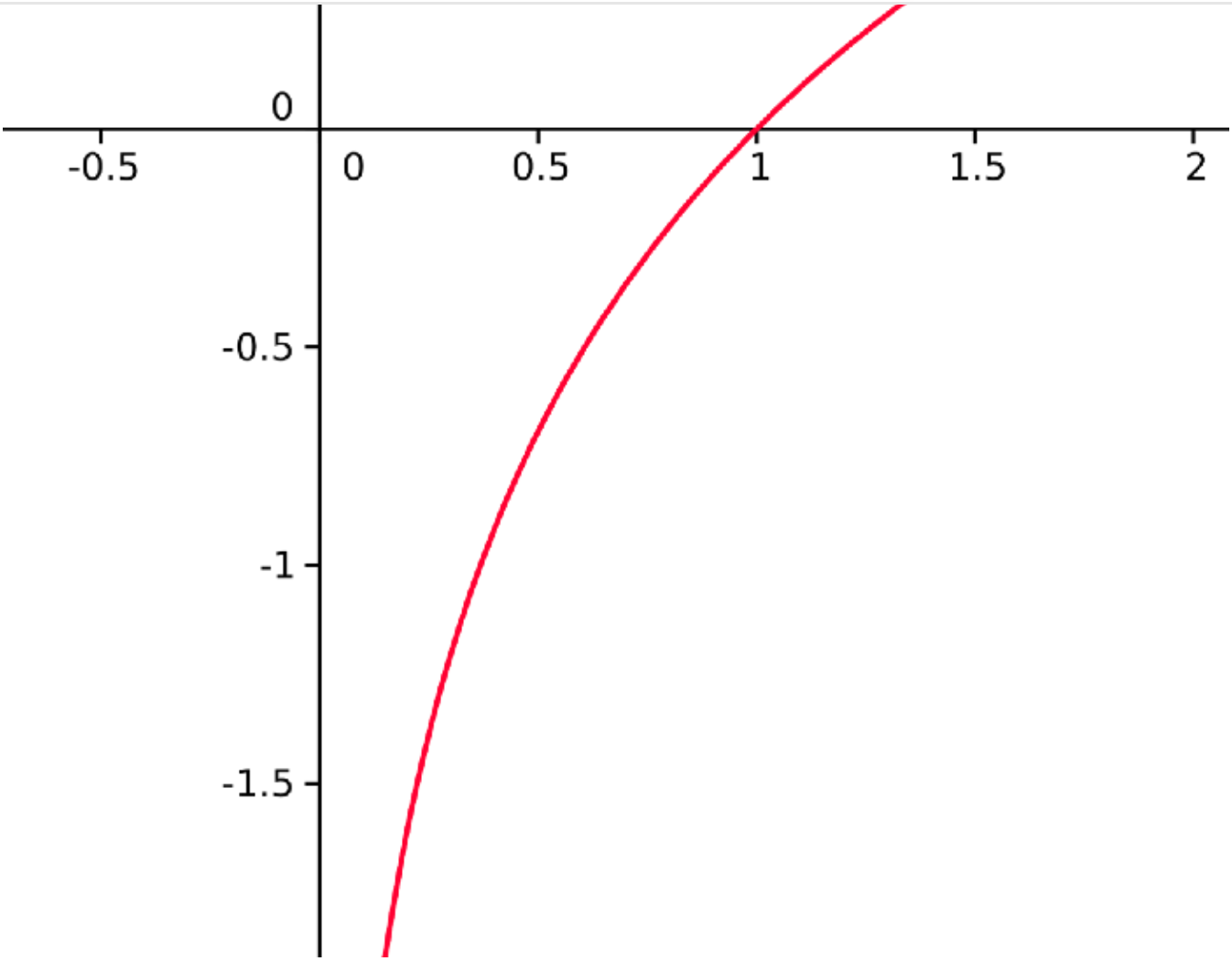
Entropy of a random variable X is the level of uncertainty inherent in the variables possible outcome.

For `p(x)` — probability distribution and a random variable X, entropy is defined as follows

$$H(X) = \begin{cases} -\int_x p(x)\log p(x), & \text{if } X \text{ is continous} \\ \sum_x p(x)\log p(x), & \text{if } X \text{ is discrete} \end{cases}$$

Definition of Entropy. Note log is calculated to base 2.

Reason for negative sign: `log(p(x))<0` for all `p(x)` in `(0,1)` . p(x) is a probability distribution and therefore the values must range between 0 and 1.

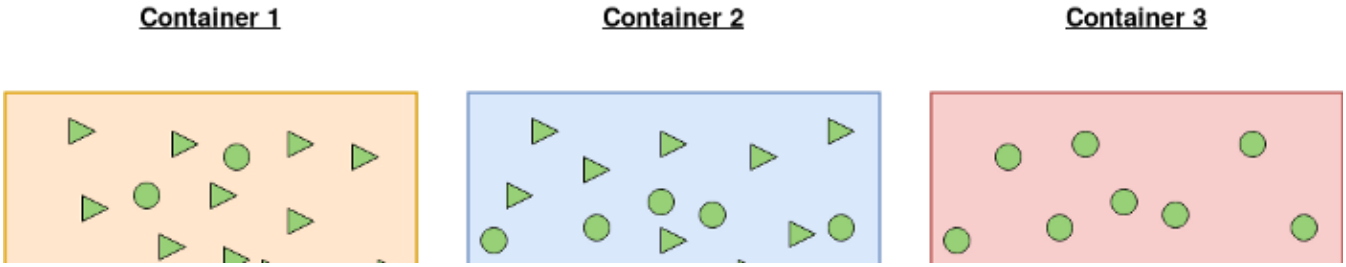


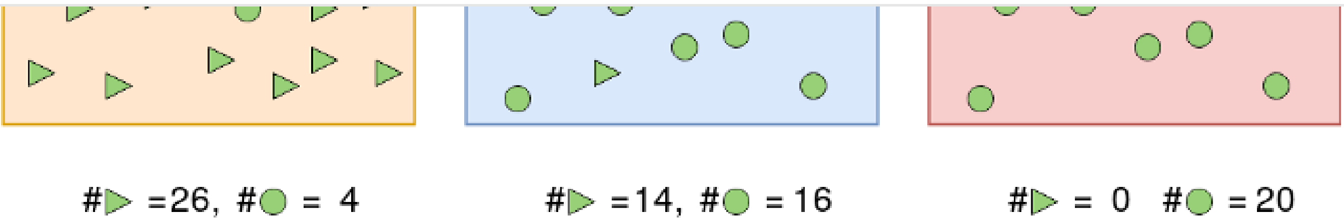
A plot of log(x). For x values between 0 and 1, log(x) <0 (is negative). (Source: Author).

The greater the value of entropy, `H(x)` , the greater the uncertainty for probability distribution and the smaller the value the less the uncertainty.

Example

Consider the following 3 “containers” with shapes: triangles and circles





3 containers with triangle and circle shapes. (Source: Author).

Container 1: The probability of picking a triangle is 26/30 and the probability of picking a circle is 4/30. For this reason, the probability of picking one shape and/or not picking another is more certain.

Container 2: Probability of picking the a triangular shape is 14/30 and 16/30 otherwise. There is almost 50–50 chance of picking any particular shape. Less certainty of picking a given shape than in 1.

Container 3: A shape picked from container 3 is surely a circle. Probability of picking a circle is 1 and the probability of picking a triangle is 0. It is perfectly certain than the shape picked will be circle.

Let us calculate the entropy so that we ascertain our assertions about the certainty of picking a given shape.

Entropy for container 1

$$
\begin{aligned}
H(X) &= -\sum_{x} p(x)\log(p(x)) \\
&= -[p(x_1)\log_2(p(x_1)) + p(x_2)\log_2(p(x_2))] \\
&= -\left[\frac{26}{30}\log_2\left(\frac{26}{30}\right) + \frac{4}{30}\log_2\left(\frac{4}{30}\right)\right] \\
&= 0.5665
\end{aligned}
$$

Entropy for container 2

$$
\begin{aligned}
H(X) &= -\sum_{x} p(x)\log(p(x)) \\
&= -[p(x_1)\log_2(p(x_1)) + p(x_2)\log_2(p(x_2))] \\
&= -\left[\frac{14}{30}\log_2\left(\frac{14}{30}\right) + \frac{16}{30}\log_2\left(\frac{16}{30}\right)\right]
\end{aligned}
$$

Entropy for container 3

$$
\begin{aligned}
H(X) &= -\sum_{x} p(x)\log(p(x)) \\
&= -[p(x_1)\log_2(p(x_1)) + p(x_2)\log_2(p(x_2))] \\
&= -\left[\frac{20}{20}\log_2\left(\frac{20}{20}\right) + \frac{20}{20}\log_2\left(\frac{20}{20}\right)\right] \\
&= 0
\end{aligned}
$$

As expected the entropy for the first container is smaller than the second one. This is because probability of picking a given shape is more certain in container 1 than in 2. The entropy for the third container is 0 implying perfect certainty.

Cross-Entropy Loss Function

Also called **logarithmic loss**, **log loss** or **logistic loss**. Each predicted class probability is compared to the actual class desired output 0 or 1 and a score/loss is calculated that penalizes the probability based on how far it is form the actual expected value. The penalty is logarithmic in nature yielding a large score for large differences close to 1 and small score for small differences tending to 0.

Cross-entropy loss is used when adjusting model weights during training. The aim is to minimize the loss, i.e, the smaller the loss the better the model. A perfect model has a cross-entropy loss of 0.

Cross-entropy is defined as

$$
L_{\text{CE}} = -\sum_{i=1}^{n} t_i \log(p_i), \quad \text{for n classes,}
$$

where t_i is the truth label and p_i is the Softmax probability for the i^{th} class.

Binary Cross-Entropy Loss

For binary classification, we have binary cross-entropy defined as

$$L = -\sum_{i=1}^{2} t_i \log(p_i)$$
$$= -\left[t \log(p) + (1-t)\log(1-p)\right]$$

where t_i is the truth value taking a value 0 or 1 and p_i is the Softmax probability for the i^{th} class.

Mathematical definition of Binary Cross-Entopy.

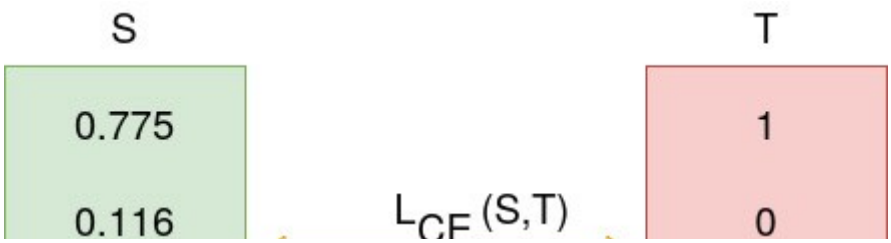
Binary cross-entropy is often calculated as the average cross-entropy across all data examples

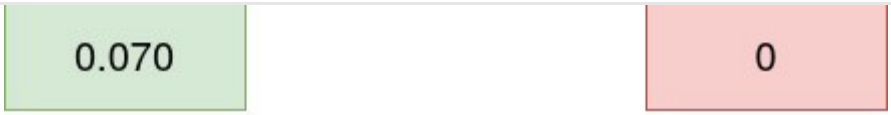
$$L = -\frac{1}{N}\left[\sum_{j=1}^{N} \left[t_j \log(p_j) + (1-t_j)\log(1-p_j)\right]\right]$$

for N data points where t_i is the truth value taking a value 0 or 1 and p_i is the Softmax probability for the i^{th} data point.

Example

Consider the classification problem with the following Softmax probabilities (S) and the labels (T). The objective is to calculate for cross-entropy loss given these information.





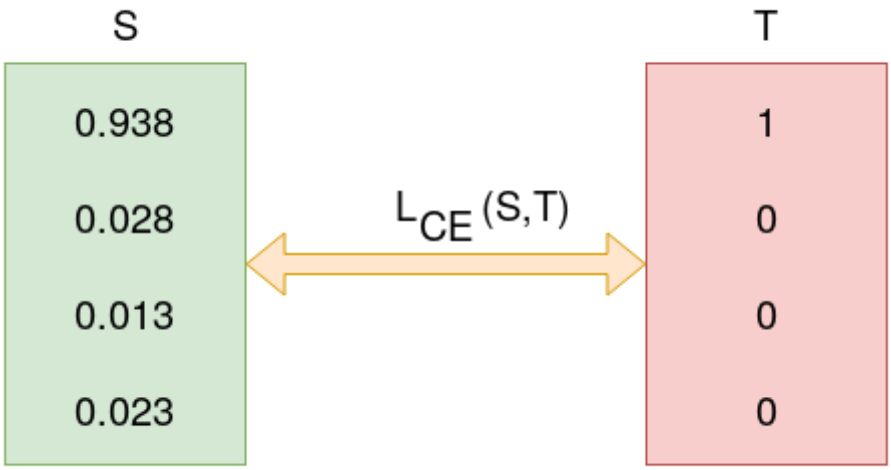
Logits(S) and one-hot encoded truth label(T) with Categorical Cross-Entropy loss function used to measure the 'distance' between the predicted probabilities and the truth labels. (Source: Author)

The categorical cross-entropy is computed as follows

$$
\begin{aligned}
L_{CE} &= -\sum_{i=1} T_i \log(S_i) \\
&= -\left[1 \log_2(0.775) + 0 \log_2(0.126) + 0 \log_2(0.039) + 0 \log_2(0.070)\right] \\
&= -\log_2(0.775) \\
&= 0.3677
\end{aligned}
$$

Softmax is continuously differentiable function. This makes it possible to calculate the derivative of the loss function with respect to every weight in the neural network. This property allows the model to adjust the weights accordingly to minimize the loss function (model output close to the true values).

Assume that after some iterations of model training the model outputs the following vector of logits



$$L_{CE} = -1 \log_e(0.936) + 0 + 0 + 0$$

`0.095` is less than previous loss, that is, `0.3677` implying that the model is learning. The process of optimization (adjusting weights so that the output is close to true values) continues until training is over.

Keras provides the following cross-entropy loss functions: binary, categorical, sparse categorical cross-entropy loss functions.

Categorical Cross-Entopy and Sparse Categorical Cross-Entropy

Both categorical cross entropy and sparse categorical cross-entopy have the same loss function as defined in Equation ??. The only difference between the two is on how truth labels are defined.

- Categorical cross-entropy is used when true labels are one-hot encoded, for example, we have the following true values for 3-class classification problem `[1,0,0]`, `[0,1,0]` and `[0,0,1]`.
- In sparse categorical cross-entropy , truth labels are integer encoded, for example, `[1]`, `[2]` and `[3]` for 3-class problem.

I hope this article helped you understand cross-entropy loss function more clearly.

Softmax Activation Function — How It Actually Works

Softmax is a function placed at the end of deep learning network to convert logits into classification probabilities.

towardsdatascience.com

Thanks for reading :-)