INFX 573: Problem Set 5 - Learning from Data

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Due: Tuesday, November 8, 2016

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Instructions: Before beginning this assignment, please ensure you have access to R and RStudio.

1. Download the `problemset5.Rmd` file from Canvas. Open `problemset5.Rmd` in RStudio and supply your solutions to the assignment by editing `problemset5.Rmd`.
2. Replace the "Insert Your Name Here" text in the `author:` field with your own full name. Any collaborators must be listed on the top of your assignment.
3. Be sure to include well-documented (e.g. commented) code chucks, figures and clearly written text chunk explanations as necessary. Any figures should be clearly labeled and appropriately referenced within the text.
4. Collaboration on problem sets is acceptable, and even encouraged, but each student must turn in an individual write-up in his or her own words and his or her own work. The names of all collaborators must be listed on each assignment. Do not copy-and-paste from other students' responses or code.
5. When you have completed the assignment and have **checked** that your code both runs in the Console and knits correctly when you click `Knit PDF`, rename the R Markdown file to `YourLastName_YourFirstName_ps5.Rmd`, knit a PDF and submit the PDF file on Canvas.

Setup: In this problem set you will need, at minimum, the following R packages.

```
# Load standard libraries

library(tidyverse)
library(Sleuth3)
# Contains data for problemset
library(UsingR)
# Contains data for problemset
library(MASS)
# Modern applied statistics functions
#Sleuth3Manual()
```

1. Davis et al. (1998) collected data on the proportion of births that were male in Denmark, the Netherlands, Canada, and the United States for selected years. Davis et al. argue that the proportion of male births is declining in these countries. We will explore this hypothesis. You can obtain this data as follows:

   (a) Use the `lm` function in **R** to fit four (one per country) simple linear regression models of the yearly proportion of males births as a function of the year and obtain the least squares fits. Write down the estimated linear model for each country.

   ```
   countryBirth <- ex0724
   #copying the data
   View(countryBirth)
   #observing the dataset
   summary(countryBirth)
   ```

```
##       Year         Denmark         Netherlands         Canada
##  Min.   :1950   Min.   :0.5108   Min.   :0.5087   Min.   :0.5120
##  1st Qu.:1961   1st Qu.:0.5127   1st Qu.:0.5120   1st Qu.:0.5128
##  Median :1972   Median :0.5141   Median :0.5129   Median :0.5136
##  Mean   :1972   Mean   :0.5142   Mean   :0.5130   Mean   :0.5137
##  3rd Qu.:1983   3rd Qu.:0.5153   3rd Qu.:0.5139   3rd Qu.:0.5145
##  Max.   :1994   Max.   :0.5175   Max.   :0.5160   Max.   :0.5153
##                                                   NA's   :24
##       USA
##  Min.   :0.5120
##  1st Qu.:0.5122
##  Median :0.5126
##  Mean   :0.5126
##  3rd Qu.:0.5128
##  Max.   :0.5134
##  NA's   :24
```

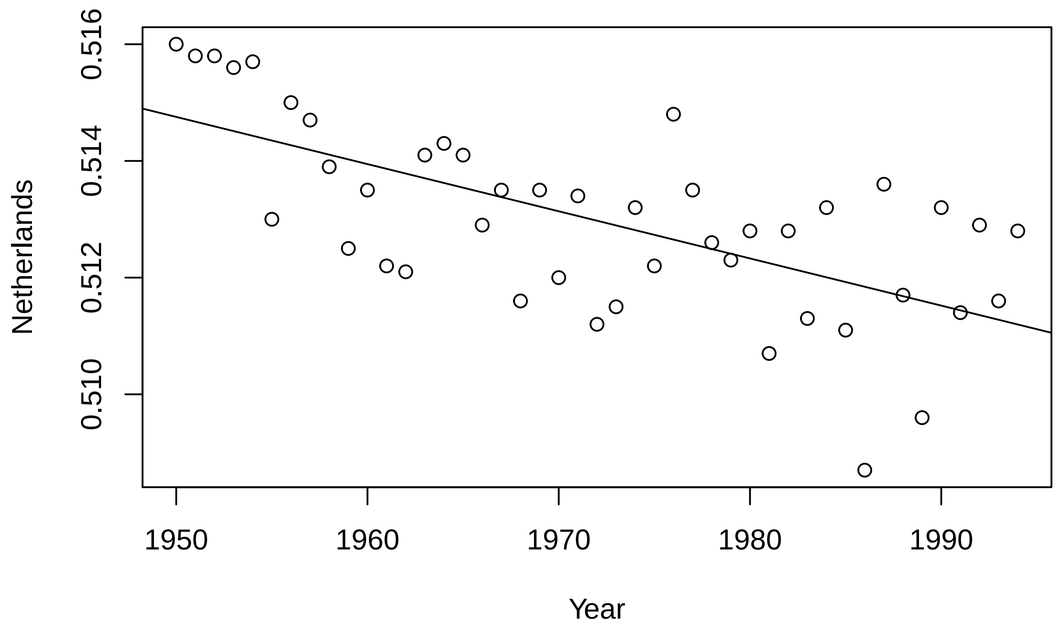
```
#summarizing the data
```

Now, using the lm() function to fit four simple linear regression models.

```
attach(countryBirth)
# attaching the dataset
birthNetherlands<-lm(Netherlands ~ Year)
#fitting the linear regression model
birthNetherlands
```

```
##
## Call:
## lm(formula = Netherlands ~ Year)
##
## Coefficients:
## (Intercept)         Year
##   6.724e-01   -8.084e-05
```

```
#observing the result
plot(Netherlands ~ Year)
#plotting the data
abline(birthNetherlands)
```



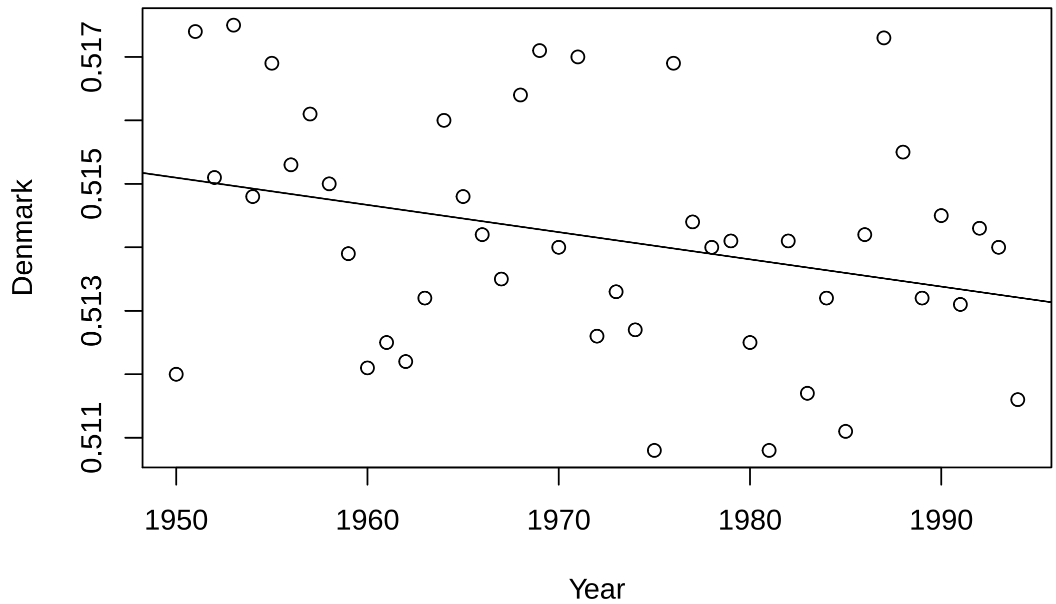

```
#Drawing the line of best fit
```

Male Birth Rate for Netherlands= (-8.084e-05)year+6.724e-01

```
birthDenmark<-lm(Denmark ~ Year)
#fitting the linear regression model
birthDenmark
```

```
##
## Call:
## lm(formula = Denmark ~ Year)
##
## Coefficients:
## (Intercept)         Year
##   5.987e-01   -4.289e-05
```

```
#observing the result
plot(Denmark ~ Year)
#plotting the data
abline(birthDenmark)
```



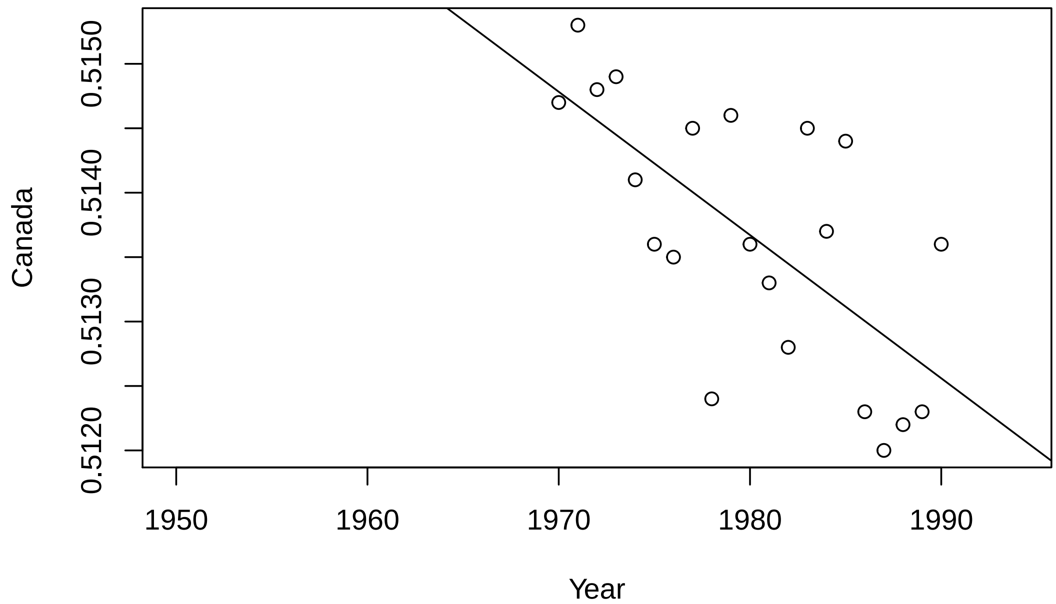

```r
#Drawing the line of best fit
```

Male Birth Rate for Denmark= (-4.289e-05)year+5.987e-01

```r
birthCanada<-lm(Canada~ Year)
#fitting the linear regression model
birthCanada
```

```
##
## Call:
## lm(formula = Canada ~ Year)
##
## Coefficients:
## (Intercept)         Year
##   0.7337857   -0.0001112
```

```r
#observing the result
plot(Canada ~ Year)
#plotting the data
abline(birthCanada)
```



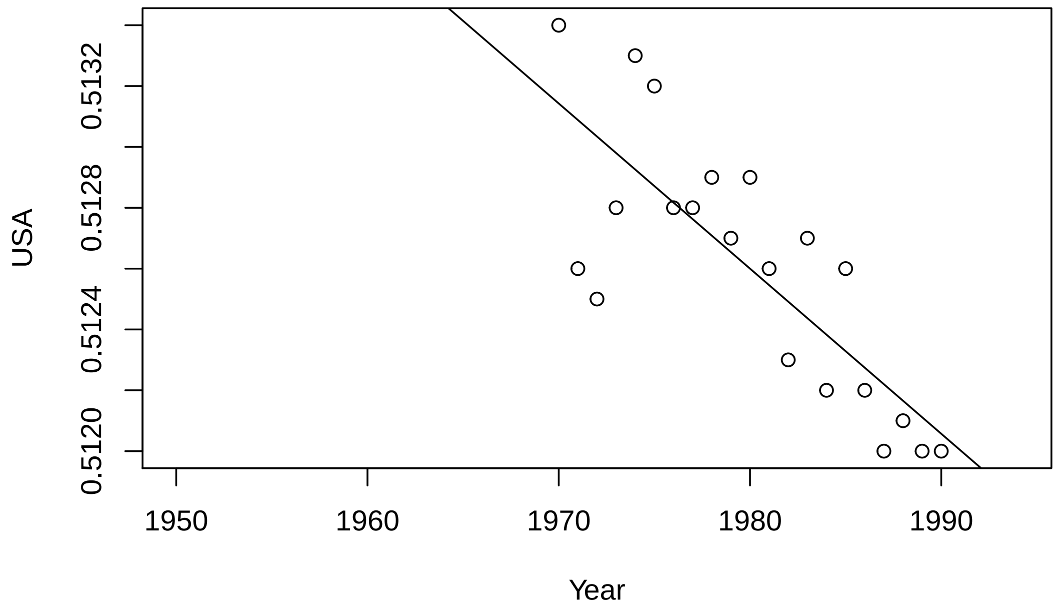

```
#Drawing the line of best fit
```

Male Birth Rate for Canada= (-0.0001112)year+0.7337857

```
birthUSA<-lm(USA ~ Year)
#fitting the linear regression model
birthUSA
```

```
##
## Call:
## lm(formula = USA ~ Year)
##
## Coefficients:
## (Intercept)         Year
##   6.201e-01   -5.429e-05
```

```
#observing the result
plot(USA ~ Year)
#plotting the data
abline(birthUSA)
```




```r
#Drawing the line of best fit
```

Male Birth Rate for USA= (-5.429e-05)year+6.201e-01

(b) Obtain the t-statistic for the test that the slopes of the regression lines are zero, for each of the four countries. Is there evidence that the proportion of births that are male is truly declining over this period?

```r
summary(birthNetherlands)
```

```
##
## Call:
## lm(formula = Netherlands ~ Year)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0031437 -0.0008246  0.0002819  0.0009287  0.0021478
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  6.724e-01  2.792e-02   24.08  < 2e-16 ***
## Year        -8.084e-05  1.416e-05   -5.71 9.64e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.001233 on 43 degrees of freedom
## Multiple R-squared:  0.4313, Adjusted R-squared:  0.418
## F-statistic: 32.61 on 1 and 43 DF,  p-value: 9.637e-07
```

p value: 9.64e-07 t value: -5.71

```r
summary(birthDenmark)
```

```
##
## Call:
## lm(formula = Denmark ~ Year)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.003225 -0.001339  0.000089  0.001119  0.003790
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  5.987e-01  4.080e-02  14.673   <2e-16 ***
## Year        -4.289e-05  2.069e-05  -2.073   0.0442 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.001803 on 43 degrees of freedom
## Multiple R-squared:  0.09083,    Adjusted R-squared:  0.06968
## F-statistic: 4.296 on 1 and 43 DF,  p-value: 0.04424
```

p value: 0.0442 t value: -2.073

```r
summary(birthCanada)
```

```
##
## Call:
## lm(formula = Canada ~ Year)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -1.494e-03 -6.161e-04 -8.312e-05  4.951e-04  1.284e-03
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  7.338e-01  5.480e-02  13.390 3.98e-11 ***
## Year        -1.112e-04  2.768e-05  -4.017 0.000738 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.000768 on 19 degrees of freedom
##   (24 observations deleted due to missingness)
## Multiple R-squared:  0.4592, Adjusted R-squared:  0.4307
## F-statistic: 16.13 on 1 and 19 DF,  p-value: 0.0007376
```

p value: 0.000738 t value: -4.017

```r
summary(birthUSA)
```

```
##
## Call:
## lm(formula = USA ~ Year)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
```

```
## -5.343e-04 -1.800e-04 -1.714e-05  2.571e-04  3.743e-04
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)  6.201e-01  1.860e-02  33.340  < 2e-16 ***
## Year        -5.429e-05  9.393e-06  -5.779 1.44e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.0002607 on 19 degrees of freedom
##   (24 observations deleted due to missingness)
## Multiple R-squared:  0.6374, Adjusted R-squared:  0.6183
## F-statistic:  33.4 on 1 and 19 DF,  p-value: 1.439e-05
```

p value: 1.44e-05 t value: -5.779

As you can see, the p-value for all four countries are less than 0.05, we can say that the relationship between proportion of male birth and year is statistically significant and the proortion of male birth is declining over the years.

2. Regression was originally used by Francis Galton to study the relationship between parents and children. One relationship he considered was height. Can we predict a man's height based on the height of his father? This is the question we will explore in this problem. You can obtain data similar to that used by Galton as follows:

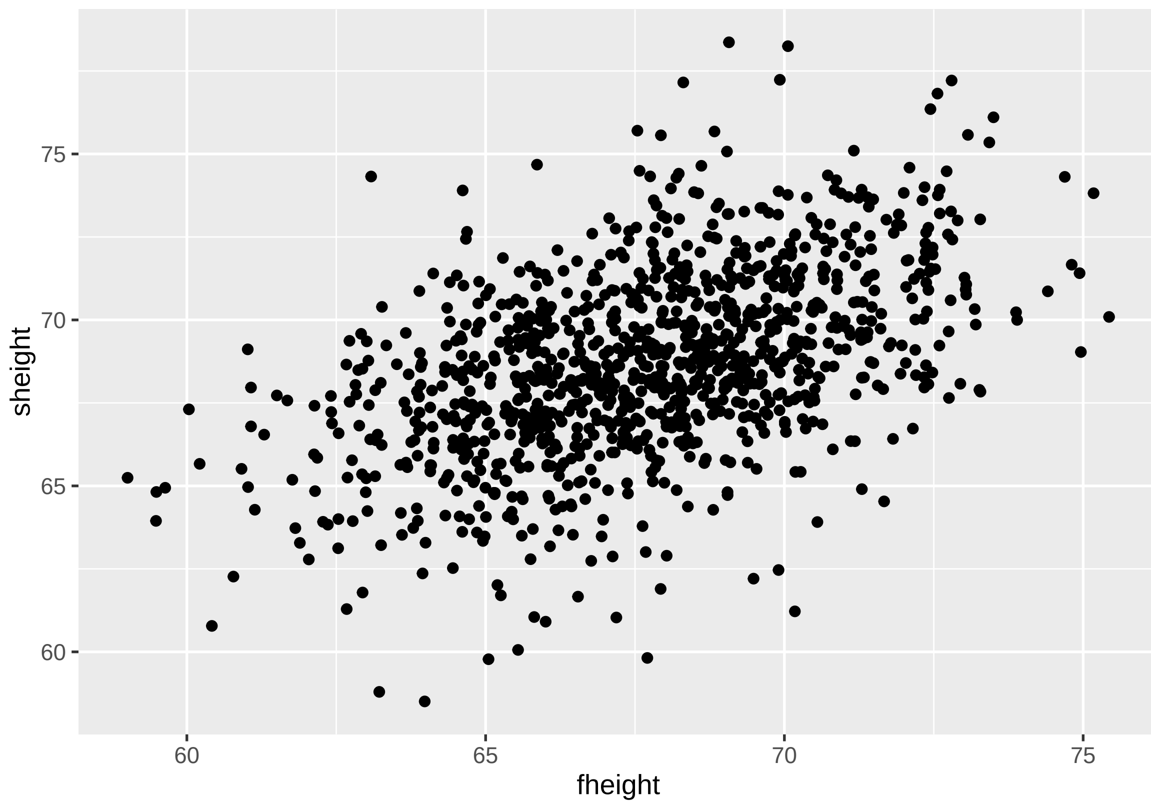
```r
# Import and look at the height data
heightData <- tbl_df(get("father.son"))
str(heightData)
```

```
## Classes 'tbl_df', 'tbl' and 'data.frame':    1078 obs. of  2 variables:
##  $ fheight: num  65 63.3 65 65.8 61.1 ...
##  $ sheight: num  59.8 63.2 63.3 62.8 64.3 ...
```

```r
summary(heightData)
```

```
##     fheight         sheight
##  Min.   :59.01   Min.   :58.51
##  1st Qu.:65.79   1st Qu.:66.93
##  Median :67.77   Median :68.62
##  Mean   :67.69   Mean   :68.68
##  3rd Qu.:69.60   3rd Qu.:70.47
##  Max.   :75.43   Max.   :78.36
```

```r
ggplot(heightData, aes(fheight, sheight))+geom_point()
```




```r
# get visualization
```

(a) Perform an exploratory analysis of the dataset. Describe what you find. At a minimum you should produce statistical summaries of the variables, a visualization of the relationship of interest in this problem, and a statistical summary of that relationship.

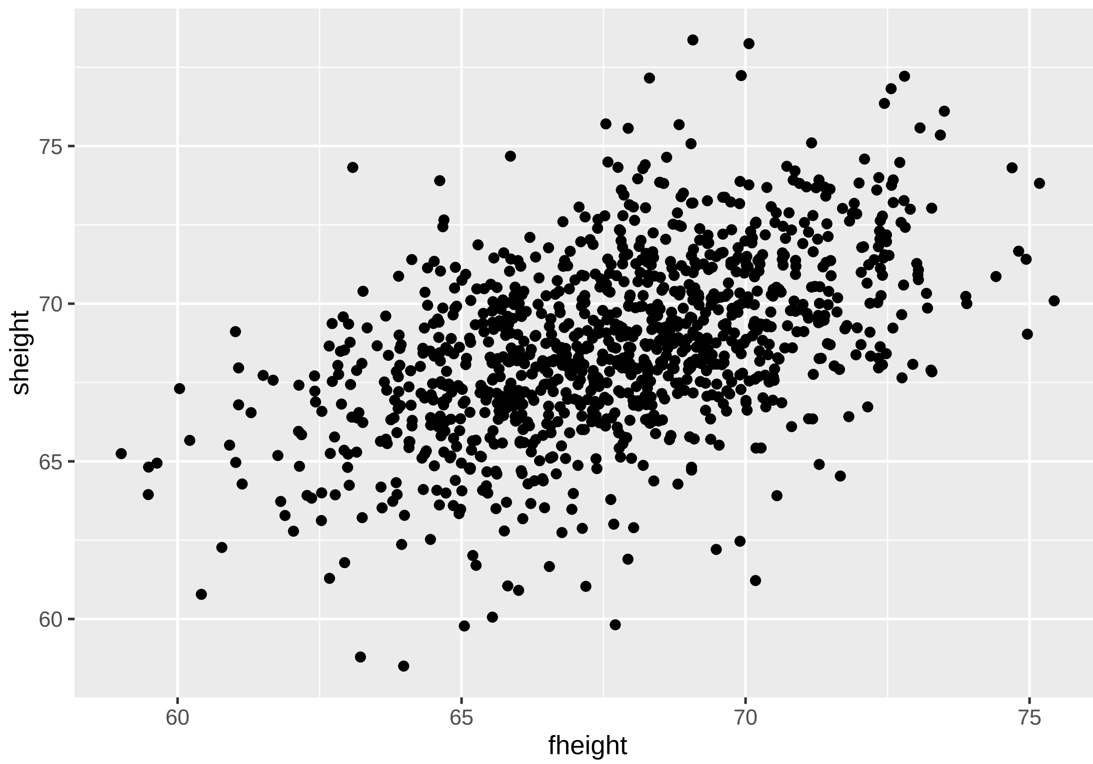
```r
str(heightData)
```

```
## Classes 'tbl_df', 'tbl' and 'data.frame':    1078 obs. of  2 variables:
##  $ fheight: num  65 63.3 65 65.8 61.1 ...
##  $ sheight: num  59.8 63.2 63.3 62.8 64.3 ...
```

```r
summary(heightData)
```

```
##     fheight         sheight
##  Min.   :59.01   Min.   :58.51
##  1st Qu.:65.79   1st Qu.:66.93
##  Median :67.77   Median :68.62
##  Mean   :67.69   Mean   :68.68
##  3rd Qu.:69.60   3rd Qu.:70.47
##  Max.   :75.43   Max.   :78.36
```

```r
ggplot(heightData, aes(fheight, sheight))+geom_point()
```




```
# get visualization
```

As per the visualizations, as the father's height increases, you can see the increase in the son's height too.

(b) Use the `lm` function in R to fit a simple linear regression model to predict son's height as a function of father's height. Write down the model,

$$\hat{y}_{\texttt{sheight}} = \hat{\beta}_0 + \hat{\beta}_i \times \texttt{fheight}$$

filling in estimated coefficient values and interpret the coefficient estimates.

```
attach(heightData)
# attach the data frame
sonsHeightLM <- lm(sheight ~ fheight)
# fitting a simple linear regression model
summary(sonsHeightLM)
```

```
##
## Call:
## lm(formula = sheight ~ fheight)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.8772 -1.5144 -0.0079  1.6285  8.9685
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 33.88660    1.83235   18.49   <2e-16 ***
## fheight      0.51409    0.02705   19.01   <2e-16 ***
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.437 on 1076 degrees of freedom
## Multiple R-squared:  0.2513, Adjusted R-squared:  0.2506
## F-statistic: 361.2 on 1 and 1076 DF,  p-value: < 2.2e-16
```

```r
sonsHeightLM
```

```
##
## Call:
## lm(formula = sheight ~ fheight)
##
## Coefficients:
## (Intercept)      fheight
##     33.8866       0.5141
```

As per the above statistical summary, we can write down the model as: 33.8866+(0.5141)*fheight. Here, Response variable is son's height and father's height is the predictor variable. The value of Beta coefficient is 0.5141 which means that with every inch increase in the father's height, son's height tend to increase by 0.5141 inches. But, this is possibe within the natural range of the height for the human beigns and beyond that, it won't hold the relationship.

(c) Find the 95% confidence intervals for the estimates. You may find the `confint()` command useful.

```r
confint(sonsHeightLM)
```

```
##                  2.5 %     97.5 %
## (Intercept) 30.2912126 37.4819961
## fheight      0.4610188  0.5671673
```

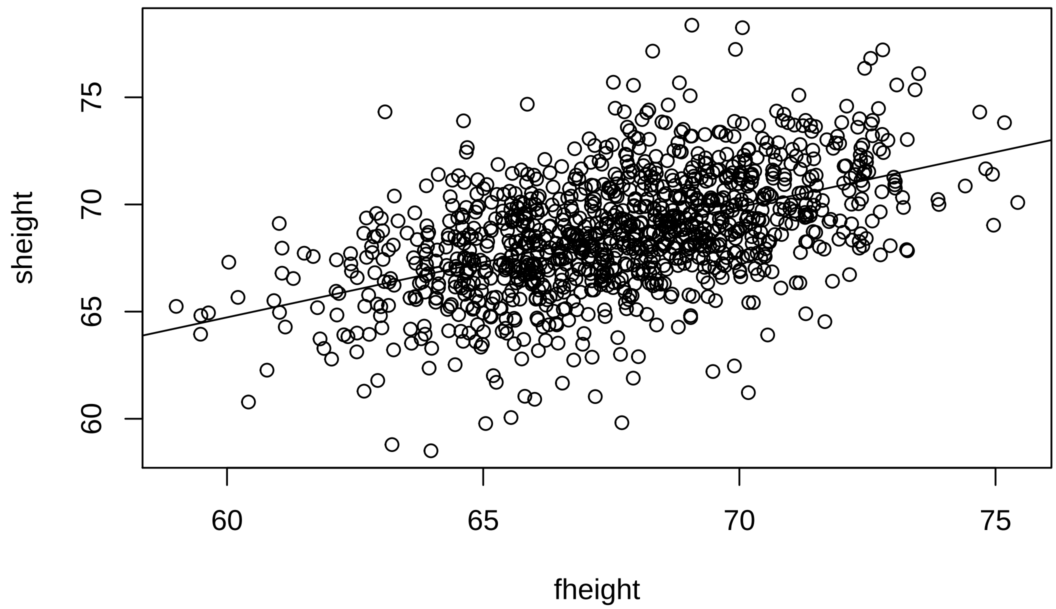
```r
sonsHeightLM
```

```
##
## Call:
## lm(formula = sheight ~ fheight)
##
## Coefficients:
## (Intercept)      fheight
##     33.8866       0.5141
```

```r
# Find the 95% confidence intervals
```

(d) Produce a visualization of the data and the least squares regression line.

```r
plot(fheight,sheight)
# visualize relationship between fheight and sheight
abline(sonsHeightLM)
```




```r
# draw the least squares regression line
```

(e) Produce a visualization of the residuals versus the fitted values. (You can inspect the elements of the linear model object in R using `names()`). Discuss what you see. Do you have any concerns about the linear model?

```r
names(sonsHeightLM)
```

```
##  [1] "coefficients"  "residuals"     "effects"       "rank"
##  [5] "fitted.values" "assign"        "qr"            "df.residual"
##  [9] "xlevels"       "call"          "terms"         "model"
```

```r
residuals(sonsHeightLM)
```

```
##            1            2            3            4            5
## -7.549320518 -3.189432294 -3.937262188 -4.897126876 -1.035698698
##            6            7            8            9           10
## -2.043843444 -3.410828758 -3.165011904 -3.236827219 -4.334628227
##           11           12           13           14           15
##  1.022303623 -0.888502884 -0.939312002 -1.389889738 -1.848607024
##           16           17           18           19           20
## -2.591991494 -2.989964076 -2.052904112 -3.438394247 -2.717010607
##           21           22           23           24           25
## -3.605699113 -4.121340976  0.546305037 -0.312543404 -0.875422298
##           26           27           28           29           30
## -1.312839748 -1.192957249 -1.230181614 -1.812453255 -1.615901663
##           31           32           33           34           35
## -2.375460072 -2.204042616 -1.619202118 -3.046443067 -2.648375866
##           36           37           38           39           40
## -2.626129125 -2.805758011 -3.212520458 -4.390581934  1.251267433
```

```
##          41          42          43          44          45
##  0.423048888 -0.045610592  0.017480915 -0.497696232 -0.474796414
##          46          47          48          49          50
##  1.651610444 -0.093493379 -6.433844280 -1.501642356 -0.690917366
##          51          52          53          54          55
##  0.167196487 -0.296185711 -0.663097904 -0.437084264  0.035412008
##          56          57          58          59          60
## -0.270065897 -0.778626280 -0.794462663 -0.920354365 -1.725711657
##          61          62          63          64          65
## -1.230643968 -0.620670541 -1.364184860 -1.572385589 -1.470870305
##          66          67          68          69          70
## -1.513833405 -1.745407337 -2.069684088 -2.846581324 -3.329580662
##          71          72          73          74          75
## -2.946462676 -3.640587809  1.860225243  2.281611743  1.304137765
##          76          77          78          79          80
##  1.278954303  1.282886041  0.483236276  0.464206224  0.684499731
##          81          82          83          84          85
##  0.541204255  0.635911455 -0.167036297  0.447075292 -0.375922595
##          86          87          88          89          90
## -0.216290319  0.116000909 -0.540760234 -0.514137299 -0.965763441
##          91          92          93          94          95
## -0.408536148 -1.177500783 -0.045185671 -1.231809237 -1.256862315
##          96          97          98          99         100
## -1.283594057 -0.807166782 -1.300360878 -0.583115367 -1.592844342
##         101         102         103         104         105
## -1.694391204 -2.726917798 -1.696968809 -2.808427293 -3.721415968
##         106         107         108         109         110
##  2.481860301  2.991852489  2.191823951  2.316841460  1.663529726
##         111         112         113         114         115
##  1.509323610  1.810568992  1.750030102  1.182320365  1.412419651
##         116         117         118         119         120
##  0.593792097  0.609404304  0.130282046  0.484382391  0.873493002
##         121         122         123         124         125
##  0.234369977 -0.507444670  0.256542544  0.113457481  0.050642153
##         126         127         128         129         130
## -0.456481594 -0.714303903 -1.006836579 -0.642387828 -0.821788545
##         131         132         133         134         135
## -0.904210026 -1.227948506 -1.903205886 -3.391252138  2.614688577
##         136         137         138         139         140
##  2.476442442  2.256267373  2.481986131  2.739712775  1.972221011
##         141         142         143         144         145
##  1.206614827  1.720636825  0.898841785  0.733899166  1.800290795
##         146         147         148         149         150
##  0.561337916  0.719309646  0.393519322  0.726053610  0.182004139
##         151         152         153         154         155
##  0.015308736  0.133562580 -0.366483521 -0.228146526 -0.531668437
##         156         157         158         159         160
## -0.322913060 -0.986682154 -2.575892643  4.288392671  2.811879391
##         161         162         163         164         165
##  3.854350538  2.964102656  3.377198232  2.462113473  2.147711164
##         166         167         168         169         170
##  2.500735498  1.712579696  1.766511791  1.668353743  1.576386544
##         171         172         173         174         175
##  1.530073205  1.281262567  1.039887763  1.046845499  1.162054402
```

```
##          176          177          178          179          180
##  0.026946632 -0.515032586 -0.680480615  3.857967348  3.386384510
##          181          182          183          184          185
##  3.613236253  2.639723729  3.303625096  2.120430141  2.598257226
##          186          187          188          189          190
##  2.241349711  1.826296299  0.819806704  1.033570797  4.187133519
##          191          192          193          194          195
##  4.073441414  3.622528817  3.353553128  2.995469909  2.616144304
##          196          197          198          199          200
##  2.324543094  8.003255538  5.442858656  3.865042378  3.616255354
##          201          202          203          204          205
##  2.560981725  6.751380784  7.093602167  5.901547869 -4.818473013
##          206          207          208          209          210
##  5.339813720  3.591276323  3.711810584 -4.456682821 -2.036181060
##          211          212          213          214          215
## -7.357182705  3.280337809 -4.094563645 -7.402425332 -4.546391439
##          216          217          218          219          220
## -3.547572138 -2.996731758 -3.058385489 -4.114599078 -5.963614403
##          221          222          223          224          225
## -1.937790543 -2.947703222 -2.850429872 -4.008435512 -3.586611265
##          226          227          228          229          230
## -4.981214007 -0.455548844 -1.843159299 -1.066749004 -2.356374230
##          231          232          233          234          235
## -2.339795194 -2.037973366 -2.089951647 -2.198188552 -3.484180192
##          236          237          238          239          240
## -3.311773147 -4.661443598 -0.009016003 -0.379773844 -0.055167765
##          241          242          243          244          245
## -0.966452762 -0.676800413 -1.302263927 -1.650971009 -1.416779793
##          246          247          248          249          250
## -1.019166136 -2.330428929 -1.957194664 -1.842430538 -3.005425792
##          251          252          253          254          255
## -2.799590649 -2.524552853 -3.378735919 -4.134563822  0.902462882
##          256          257          258          259          260
##  0.600731530  0.536295550  0.953143203 -0.014966588 -0.237475177
##          261          262          263          264          265
##  0.265131054 -0.228149267 -5.391354890 -2.849681116 -1.850511525
##          266          267          268          269          270
## -3.589695641 -0.102142686 -1.043803584 -0.666873710 -0.613530122
##          271          272          273          274          275
## -0.782423016 -0.765505802 -0.981290464 -1.070444318 -0.568468247
##          276          277          278          279          280
## -1.610169354 -1.765660462 -1.642620503 -1.486344337 -2.148516404
##          281          282          283          284          285
## -2.177641887 -1.785823380 -2.458158060 -2.459333375 -3.180891716
##          286          287          288          289          290
## -2.358938099 -3.344303343  1.737870221  1.529971642  2.126584122
##          291          292          293          294          295
##  1.837056693  1.339263165  1.180360089  0.674466681  0.463427763
##          296          297          298          299          300
## -0.123244601  0.742131310 -0.293302886 -0.188462701  0.335081797
##          301          302          303          304          305
##  0.145177756 -0.264314883 -0.427049864  0.084303897 -0.927216569
##          306          307          308          309          310
## -0.791968391 -1.187685475 -0.234938274 -1.272797267 -1.642547450
```

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##         311         312         313         314         315 
## -0.822138191 -0.732354563 -1.497519308 -1.531592328 -2.319124873 
##         316         317         318         319         320 
## -1.420630068 -1.632832247 -1.879364631 -3.047175858 -2.731998400 
##         321         322         323         324         325 
##  3.238853634  2.267149069  2.325138632  1.685806236  1.877961690 
##         326         327         328         329         330 
##  1.616795594  0.998786670  1.879586855  1.013560311  1.266195138 
##         331         332         333         334         335 
##  0.417645158  0.461415500  0.255616549  0.157141033  0.998830954 
##         336         337         338         339         340 
## -0.265556588  0.105104884 -0.294442609 -0.151975195 -0.122545542 
##         341         342         343         344         345 
##  0.047138344 -0.187101821 -0.329330033 -1.375334272 -1.103788678 
##         346         347         348         349         350 
## -0.655542638 -1.121097939 -1.017182163 -1.979390319  2.955116189 
##         351         352         353         354         355 
##  2.662128458  1.985806920  1.903415364  2.163431715  1.717846883 
##         356         357         358         359         360 
##  2.148443709  0.821629353  1.132662390  1.446649298  1.690406308 
##         361         362         363         364         365 
##  1.158488113  0.988784179  1.243774578  0.607180922  0.101055811 
##         366         367         368         369         370 
##  0.542196241  0.177075898  0.517863104 -0.564554608  0.047002321 
##         371         372         373         374         375 
## -0.653364255 -0.963612058 -1.659336497  4.138422805  3.552991910 
##         376         377         378         379         380 
##  3.349257552  2.174165598  2.560942560  2.337009107  2.182338515 
##         381         382         383         384         385 
##  1.917710543  1.828863893  1.772559456  1.904477732  1.488699510 
##         386         387         388         389         390 
##  1.486941263  1.954373022  1.111390949  0.859528685  1.197521578 
##         391         392         393         394         395 
##  0.358195739  0.240338195 -0.148113758  4.182020693  3.565863898 
##         396         397         398         399         400 
##  3.250366427  2.324158514  2.389633338  2.537895513  2.374891897 
##         401         402         403         404         405 
##  2.617766937  1.520946125  0.864434305  0.729090171  4.699917114 
##         406         407         408         409         410 
##  4.672887051  3.795810853  3.707780237  3.487485465  2.732503400 
##         411         412         413         414         415 
##  1.666374075  1.963024008  5.064569422  5.484583810  3.387810643 
##         416         417         418         419         420 
##  3.637810778  2.024863332  4.629300349  8.151747800 -4.165330595 
##         421         422         423         424         425 
## -4.397042931  0.092183980 -8.271382039 -1.875112825  0.074072255 
##         426         427         428         429         430 
## -0.294812949  2.968543040 -1.649808898 -5.637762696  1.390831221 
##         431         432         433         434         435 
##  0.549083220 -4.497157752 -3.497778992 -4.525255176 -5.520827464 
##         436         437         438         439         440 
## -1.986475485 -2.387577959 -3.302808121 -3.025704983 -3.618434519 
##         441         442         443         444         445 
## -4.864695754  0.821873405 -1.462888129 -0.933447502 -2.631248222 
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##         446         447         448         449         450 
## -2.581049069 -2.088575488 -2.392377869 -2.632835083 -2.985479647 
##         451         452         453         454         455 
## -3.746318585 -4.570049223  0.123878967  0.017222335 -0.636215736 
##         456         457         458         459         460 
## -0.343478636 -1.308177593 -0.996858657 -1.356835908 -1.482939709 
##         461         462         463         464         465 
## -1.045068074 -2.380873782 -2.035909345 -2.405678827 -2.577727027 
##         466         467         468         469         470 
## -2.282927149 -2.269134787 -2.804976822 -3.263977852  1.507905714 
##         471         472         473         474         475 
##  1.401090723  0.220064646 -0.064974348  0.629756131 -0.350102478 
##         476         477         478         479         480 
## -0.248751352 -0.711485837 -3.486091137 -4.676935637 -1.760437753 
##         481         482         483         484         485 
## -2.470469682 -0.068363565 -0.003059954 -0.667482049 -0.834641792 
##         486         487         488         489         490 
## -1.015693781 -0.740362645 -0.875387261 -1.605599758 -1.349426932 
##         491         492         493         494         495 
## -0.864106255 -0.919440855 -0.822352004 -1.658792381 -1.976064268 
##         496         497         498         499         500 
## -1.658146023 -1.891878029 -2.229115654 -1.760182271 -2.455244350 
##         501         502         503         504         505 
## -3.012410151 -2.582018386  2.218731817  2.286712411  1.032497054 
##         506         507         508         509         510 
##  1.134678004  0.918782672  1.073320623  0.796418419 -0.170741632 
##         511         512         513         514         515 
##  0.287647361 -0.118998035  0.014531984 -0.341609600 -0.512497595 
##         516         517         518         519         520 
## -0.166179153  0.058683229  0.157404465 -0.070758990 -0.434102977 
##         521         522         523         524         525 
## -0.193019062 -0.485859072 -0.864082612 -1.463250029 -0.804839027 
##         526         527         528         529         530 
## -1.232652814 -1.401964163 -1.186450226 -1.064415076 -1.499661599 
##         531         532         533         534         535 
## -1.203505578 -2.081020787 -1.941188075 -2.456274391 -2.494140757 
##         536         537         538         539         540 
##  3.857451958  3.073832008  2.427127077  1.603469118  2.417727796 
##         541         542         543         544         545 
##  1.823501510  1.593041702  0.940236448  0.577220003  0.910136172 
##         546         547         548         549         550 
##  0.515233980  0.703753170 -0.233272042  0.196287715  0.100233396 
##         551         552         553         554         555 
##  0.476631073 -0.467795173 -0.484116104 -0.394556034  0.026199662 
##         556         557         558         559         560 
##  0.121192064 -0.615982756 -0.349044921 -0.111379734 -0.520127446 
##         561         562         563         564         565 
## -0.956518584 -0.900222426 -1.879284146 -1.571540721  3.390140321 
##         566         567         568         569         570 
##  2.714972594  2.529337171  2.331660578  2.185673182  2.205944965 
##         571         572         573         574         575 
##  1.617506217  1.841601958  0.994719840  1.925553736  1.311455510 
##         576         577         578         579         580 
##  0.206828320  1.042816425  0.505300846  0.523866950  0.632331719
```

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##          581          582          583          584          585 
##  0.122432249  0.605779235  0.420151602 -0.498952678 -0.565600690 
##          586          587          588          589          590 
## -0.006776941 -1.034715921 -0.712179528  4.139316630  3.906986317 
##          591          592          593          594          595 
##  3.293088672  2.426219526  2.224091815  2.731735177  2.764149030 
##          596          597          598          599          600 
##  1.733662782  2.318791395  2.003560969  1.544138142  1.888508198 
##          601          602          603          604          605 
##  1.384333399  1.614683392  1.222460389  1.958252376  0.723632483 
##          606          607          608          609          610 
##  0.246105987 -0.211022623  0.359560060  5.307896020  4.380018759 
##          611          612          613          614          615 
##  3.032073323  3.081352240  2.559210884  3.158959859  1.998324951 
##          616          617          618          619          620 
##  2.155955791  2.167794730  1.924072275  0.860063659  1.295236203 
##          621          622          623          624          625 
##  4.209837674  3.804104926  4.104828835  3.690333035  3.165760595 
##          626          627          628          629          630 
##  1.968800825  1.632180526  6.933304176  4.754464919  4.110180686 
##          631          632          633          634          635 
##  2.919973210  2.715892034  6.406283771  4.432464139  8.343688769 
##          636          637          638          639          640 
## -6.912291784  2.729159120 -2.864912195 -1.004644842  1.402358037 
##          641          642          643          644          645 
##  0.310794534  4.414340456  0.593138224 -6.197892353 -0.863888343 
##          646          647          648          649          650 
## -0.960672325 -3.003959566 -7.392165742 -2.912550245 -3.631370065 
##          651          652          653          654          655 
## -4.822340049 -2.113980471 -2.389505185 -2.863227011 -3.241792826 
##          656          657          658          659          660 
## -3.660770957 -3.557003416  0.347453597 -1.046381008 -1.210875925 
##          661          662          663          664          665 
## -1.199720032 -2.198016722 -1.753317097 -2.283677937 -2.250404110 
##          666          667          668          669          670 
## -3.154485655 -4.073990691 -3.496240977  1.147346305 -0.176037520 
##          671          672          673          674          675 
## -0.807078631 -0.557475725 -0.869483985 -0.886144096 -1.789389167 
##          676          677          678          679          680 
## -1.879991423 -2.216156355 -1.873581809 -2.065795216 -2.224427754 
##          681          682          683          684          685 
## -2.153025312 -2.756852481 -2.821668994 -2.891438726 -3.107287648 
##          686          687          688          689          690 
##  2.556559092  1.585146235  0.912853044  0.243258742  0.084565160 
##          691          692          693          694          695 
## -0.176879207 -0.582579831  0.728382928 -2.031688861 -5.673090644 
##          696          697          698          699          700 
##  0.128901089 -0.848534235 -0.425037232 -0.026043450 -0.461419334 
##          701          702          703          704          705 
## -0.666059743 -0.583233298 -0.576804909 -1.025709317 -1.727829408 
##          706          707          708          709          710 
## -0.656146609 -1.173511313 -1.483330169 -1.102655255 -1.984596098 
##          711          712          713          714          715 
## -1.925230237 -2.075224673 -2.241378133 -2.105931208 -2.162257874 
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##          716          717          718          719          720 
## -2.301354229 -2.591485584 -2.352828690  2.679176774  2.552197385 
##          721          722          723          724          725 
##  1.475345605  1.081885037  1.336060999  1.320918248 -0.163937907 
##          726          727          728          729          730 
##  0.226689115  0.155647222  0.615597172  0.841266298  0.306471871 
##          731          732          733          734          735 
## -0.177464407  0.347112755  0.422040392 -0.615294736 -0.348105883 
##          736          737          738          739          740 
## -0.791977664 -0.518978094 -0.636249443 -0.960427270 -0.107662718 
##          741          742          743          744          745 
## -1.446767273 -0.847153161 -0.853009530 -1.087654005 -1.037981022 
##          746          747          748          749          750 
## -1.341329846 -1.545713529 -2.050437241 -1.931290112 -3.112732942 
##          751          752          753          754          755 
## -2.649142210 -3.309949996  2.785279007  2.263076122  1.531179258 
##          756          757          758          759          760 
##  1.905201983  1.764683451  1.213794428  1.666860943  1.826353542 
##          761          762          763          764          765 
##  1.094292047  0.642080206  0.786003233  0.293665575  0.561125983 
##          766          767          768          769          770 
##  0.183822293 -0.296645354  0.459255598  0.257705551 -0.463951074 
##          771          772          773          774          775 
## -0.687939067  0.165302297 -0.485975879 -0.702747144 -0.516705767 
##          776          777          778          779          780 
## -0.798849741 -1.189519749 -1.284811307 -1.137687029 -2.215477427 
##          781          782          783          784          785 
##  3.981979061  3.257037431  2.441659396  1.865190889  2.184233319 
##          786          787          788          789          790 
##  2.438748847  1.523302204  1.744129527  1.792821336  0.977441966 
##          791          792          793          794          795 
##  0.761255989  0.987603262  0.845677064  0.751871029  1.311344445 
##          796          797          798          799          800 
##  1.121127694  0.939099389 -0.076258444  0.328537243  0.054671476 
##          801          802          803          804          805 
##  0.280665660  0.367567953 -0.843409666 -1.183476486  4.547311529 
##          806          807          808          809          810 
##  3.923177995  3.928424900  2.511007480  3.132594693  2.421288524 
##          811          812          813          814          815 
##  2.210515640  2.619886458  2.615086752  2.144988651  1.925027600 
##          816          817          818          819          820 
##  2.140899245  0.995929277  1.676481318  0.899878283  1.421671863 
##          821          822          823          824          825 
##  0.706562463  1.010632168  0.330221258 -0.676492952  5.512939661 
##          826          827          828          829          830 
##  3.582462901  3.565717639  3.207917457  2.921717005  2.891910573 
##          831          832          833          834          835 
##  1.970314406  2.583362960  2.434336198  1.514810192  1.805870562 
##          836          837          838          839          840 
##  1.103459451  4.316685112  4.858847022  3.737191773  2.971788177 
##          841          842          843          844          845 
##  3.110645457  2.544724923  2.029458434  6.796174721  5.605938444 
##          846          847          848          849          850 
##  4.054330034  3.889141502  3.209280692  5.693750433  5.221715677 
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```
##         851         852         853         854         855
##  8.968478813 -8.743284614  1.540916577  0.396943250  1.471849840
##         856         857         858         859         860
##  2.905875337 -1.061552721 -6.910148818  1.510890334  2.068747073
##         861         862         863         864         865
## -5.731984395 -2.572537507 -4.664720036 -2.422006258 -3.815460108
##         866         867         868         869         870
## -4.266914447 -0.520498395 -2.221230289 -2.771972252 -2.997079420
##         871         872         873         874         875
## -2.925450034 -3.579032884 -6.245864307 -0.992296736 -1.651270933
##         876         877         878         879         880
## -1.731973307 -1.784466334 -2.309649546 -2.035579488 -2.274169372
##         881         882         883         884         885
## -3.139890119 -3.757720851 -3.203750953 -4.590130726  0.172744471
##         886         887         888         889         890
## -0.916724729 -0.730549555 -0.887759564 -0.822638186 -1.364119639
##         891         892         893         894         895
## -1.144799831 -1.514609136 -2.067856275 -1.915276036 -1.980686728
##         896         897         898         899         900
## -1.697328359 -2.642619860 -2.485659262 -3.179389457 -3.698535260
##         901         902         903         904         905
## -4.187737492  1.975566233  1.138536012  0.039401362  0.476744056
##         906         907         908         909         910
## -0.358221327  0.233489628  0.326412660 -0.385024862 -5.470119405
##         911         912         913         914         915
## -0.943585721 -3.858235770  1.721985741  0.411202107 -0.570615118
##         916         917         918         919         920
## -0.584183699 -0.618681331 -1.021739968  0.031520090 -1.290869634
##         921         922         923         924         925
## -1.464087582 -1.339519526 -0.922204781 -1.427366675 -1.125437078
##         926         927         928         929         930
## -1.689962615 -1.566518148 -1.994245576 -2.162774545 -1.965427571
##         931         932         933         934         935
## -1.932382933 -2.225195673 -2.552031012 -2.562924943  1.567945618
##         936         937         938         939         940
##  1.705210854  1.478785397  1.632961910  0.689264831  1.447444579
##         941         942         943         944         945
##  0.386428459  0.627950374  0.422352636  0.435988172  0.030293202
##         946         947         948         949         950
##  0.403164614  0.131251766  0.216064953  0.141792061 -0.575767023
##         951         952         953         954         955
## -0.894873844 -1.086366203 -0.876240723 -0.332976163 -0.722298637
##         956         957         958         959         960
## -0.940250308 -0.954278596 -1.291021065 -1.371546810 -1.005424910
##         961         962         963         964         965
## -1.193308727 -1.825408209 -1.966702622 -2.205402332 -2.280651354
##         966         967         968         969         970
## -2.668852466 -2.801679762  3.350174082  1.944998698  1.834501349
##         971         972         973         974         975
##  1.240292894  1.318071750  1.018360789  1.293638770  0.673796039
##         976         977         978         979         980
##  0.978309682  0.547953795  0.711440073  0.545759462  0.662990964
##         981         982         983         984         985
##  0.273431755  0.180373017  0.208792946 -0.016661794  0.164318768
```

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##         986         987         988         989         990
##  0.179359096  0.199053067  0.037893666 -0.341811880 -1.120156569
##         991         992         993         994         995
## -0.837729072 -1.109617521 -0.933549710 -1.109290052 -0.735894733
##         996         997         998         999        1000
## -1.633203894  3.028308450  2.181361796  3.050111495  2.889867743
##        1001        1002        1003        1004        1005
##  2.195399495  1.535078644  2.191354836  1.104142120  1.014290782
##        1006        1007        1008        1009        1010
##  1.997966028  1.384245909  1.004036442  0.598681741  1.378897978
##        1011        1012        1013        1014        1015
##  1.086436763  0.119568553  0.163910582  0.322355679  0.554619095
##        1016        1017        1018        1019        1020
## -0.414866878  0.171365277 -0.236351870 -0.520593564 -1.634873942
##        1021        1022        1023        1024        1025
##  3.436372382  3.506197296  3.668358411  2.939776083  2.466993091
##        1026        1027        1028        1029        1030
##  2.385146036  2.790695191  2.584203554  1.583647664  1.912701017
##        1031        1032        1033        1034        1035
##  1.586134683  1.580070675  1.332671761  1.946137415  1.254843310
##        1036        1037        1038        1039        1040
##  1.309027961  0.602998826  0.414716046 -0.364875489 -1.278200379
##        1041        1042        1043        1044        1045
##  4.136000253  3.682608593  3.774236726  3.211133930  2.378770554
##        1046        1047        1048        1049        1050
##  2.131856040  2.588984847  2.049679070  1.836506945  1.220304149
##        1051        1052        1053        1054        1055
##  0.962337404  4.331060985  4.028872564  4.195720592  3.619775234
##        1056        1057        1058        1059        1060
##  2.313494450  2.811195962  2.270234930  1.997450895  5.865147140
##        1061        1062        1063        1064        1065
##  4.680763131  3.452156878  2.926232447  1.284000836  4.123250188
##        1066        1067        1068        1069        1070
##  7.400658027 -7.523332626  5.628732981 -4.251839278 -7.593037101
##        1071        1072        1073        1074        1075
## -3.658603782 -6.671128941 -8.877150660  2.423122015 -2.290051307
##        1076        1077        1078
## -1.483926919 -0.950717935 -3.015475796
```
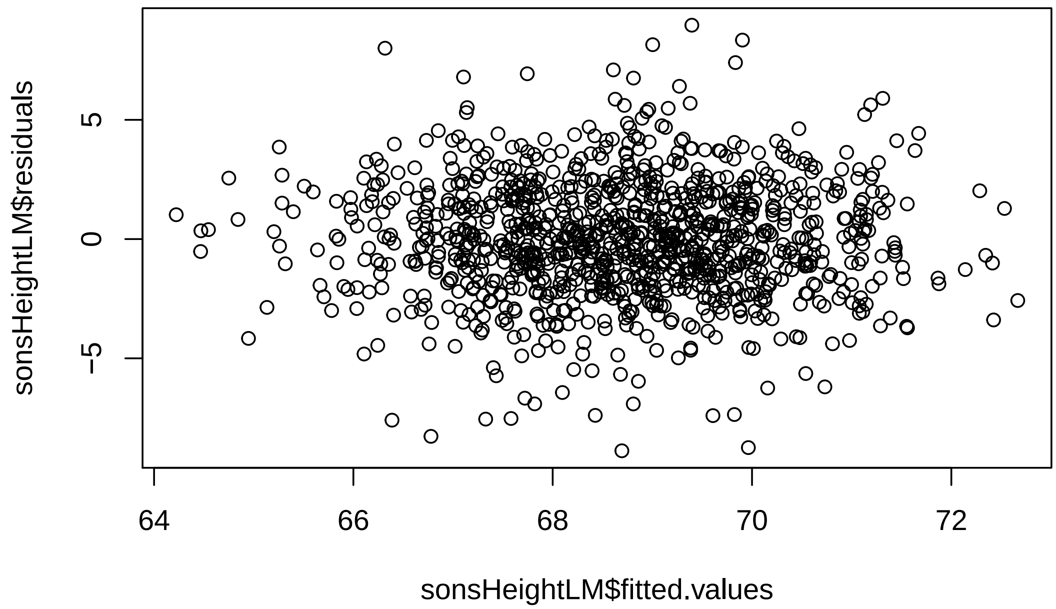

```r
plot(sonsHeightLM$fitted.values, sonsHeightLM$residuals)
```




```r
# visualization of the residuals
```

As you can see in the visualization, it is a random distribution and there are few outliers. This model might be a good represenation for this data is what my concern since there is no definite pattern.

(f) Using the model you fit in part (b) predict the height was 5 males whose father are 50, 55, 70, 75, and 90 inches respectively. You may find the `predict()` function helpful.

```r
predictH = data.frame(fheight = c(50, 55, 70, 75, 90))
# set heights
predict(sonsHeightLM, predictH, interval = "predict")
```

```
##        fit      lwr      upr
## 1 59.59126 54.71685 64.46566
## 2 62.16172 57.33140 66.99204
## 3 69.87312 65.08839 74.65785
## 4 72.44358 67.64470 77.24246
## 5 80.15498 75.22740 85.08255
```

```r
# predict son's heights
```

(a) Son's height is 59.59126 inches when Father's height is 50
(b) Son's height is 62.16172 inches when Father's height is 55
(c) Son's height is 69.87312 inches when Father's height is 70
(d) Son's height is 72.44358 inches when Father's height is 75
(e) Son's height is 80.15498 inches when Father's height is 90

3. **Extra Credit:**

(a) What assumptions are made about the distribution of the explanatory variable in the normal simple linear regression model?

Linear regression makes several key assumptions about the distribution of the explanatory variable:
1. Distribution is normal

    (a) No or little multicollinearity i.e. explanatory variables are independent of each other

(b) Why can an R^2 close to one not be used as evidence that the simple linear regression model is appropriate?

R^2 is used to measure the relationship between the predictors and the response. Even if the R^2 value is close to 1, it doesn't imply the bias in the coefficient values and the predictors and requires that the residual plots to be checked. It is also possible that high R^2 value my not fit the data well and if the sample is biased, R^2 will be biased as well.

(c) Consider a regression of weight on height for a sample of adult males. Suppose the intercept is 5 kg. Does this imply that males of height 0 weigh 5 kg, on average? Would this imply that the simple linear regression model is meaningless?

It is incorrect to intrepret that males of height 0 weigh 5 kg on average. And also, it doesn't mean that the regression model is meaningless.
The Males of height 0 would weigh 5 kg only if x approaches to 0. We know that it is rare to see because model is not valid as x=0.

(d) Suppose you had data on pairs (X, Y) which gave the scatterplot been below. How would you approach the analysis?

Based on the scatterplot visualizations, we can see two patterns in the form a linear regression lines. By introducing the dummy variables, we can create a model to fit two different regression lines and analyze the data accordingly.