



### Algumas propriedades das funções trigonométricas

1.  $\forall a \in \mathbb{R} \quad \sin^2 a + \cos^2 a = 1$
2.  $\forall a \in \mathbb{R} \setminus \{\frac{\pi}{2} + k\pi : k \in \mathbb{Z}\} \quad 1 + \operatorname{tg}^2 a = \sec^2 a$
3.  $\forall a \in \mathbb{R} \setminus \{k\pi : k \in \mathbb{Z}\} \quad 1 + \operatorname{cotg}^2 a = \operatorname{cosec}^2 a$
4.  $\forall a \in \mathbb{R} \quad \sin(-a) = -\sin a \quad (\sin \text{ é ímpar})$
5.  $\forall a \in \mathbb{R} \quad \cos(-a) = \cos a \quad (\cos \text{ é par})$
6.  $\forall a \in \mathbb{R} \quad \cos(\frac{\pi}{2} - a) = \sin a \quad \text{e} \quad \sin(\frac{\pi}{2} - a) = \cos a$
7.  $\forall a \in \mathbb{R} \quad \sin(a + 2\pi) = \sin a \quad (\sin \text{ tem período } 2\pi)$
8.  $\forall a \in \mathbb{R} \quad \cos(a + 2\pi) = \cos a \quad (\cos \text{ tem período } 2\pi)$
9.  $\forall a, b \in \mathbb{R} \quad \sin(a + b) = \sin a \cos b + \sin b \cos a$
10.  $\forall a, b \in \mathbb{R} \quad \cos(a + b) = \cos a \cos b - \sin b \sin a$
11.  $\forall a, b \in \mathbb{R} \quad \cos a - \cos b = -2 \sin \frac{a-b}{2} \sin \frac{a+b}{2}$
12.  $\forall a, b \in \mathbb{R} \quad \sin a - \sin b = 2 \sin \frac{a-b}{2} \cos \frac{a+b}{2}$

### Algumas propriedades das funções hiperbólicas

$$\begin{array}{ccc} \operatorname{sh} : \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \longmapsto & \frac{e^x - e^{-x}}{2} \end{array} \qquad \begin{array}{ccc} \operatorname{ch} : \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \longmapsto & \frac{e^x + e^{-x}}{2} \end{array}$$

1.  $\forall a \in \mathbb{R} \quad \operatorname{ch}^2 a - \operatorname{sh}^2 a = 1$
2.  $\forall a \in \mathbb{R} \quad \operatorname{th}^2 a + \operatorname{sech}^2 a = 1$
3.  $\forall a \in \mathbb{R} \setminus \{0\} \quad \operatorname{coth}^2 a - \operatorname{cosech}^2 a = 1$
4.  $\forall a \in \mathbb{R} \quad \operatorname{sh}(-a) = -\operatorname{sh} a \quad (\text{a função sh é ímpar})$
5.  $\forall a \in \mathbb{R} \quad \operatorname{ch}(-a) = \operatorname{ch} a \quad (\text{a função ch é par})$
6.  $\forall a, b \in \mathbb{R} \quad \operatorname{sh}(a + b) = \operatorname{sh} a \operatorname{ch} b + \operatorname{sh} b \operatorname{ch} a$
7.  $\forall a, b \in \mathbb{R} \quad \operatorname{ch}(a + b) = \operatorname{ch} a \operatorname{ch} b + \operatorname{sh} b \operatorname{sh} a$
8.  $\forall n \in \mathbb{N} \quad \forall a \in \mathbb{R} \quad (\operatorname{ch} a + \operatorname{sh} a)^n = \operatorname{ch}(na) + \operatorname{sh}(na)$