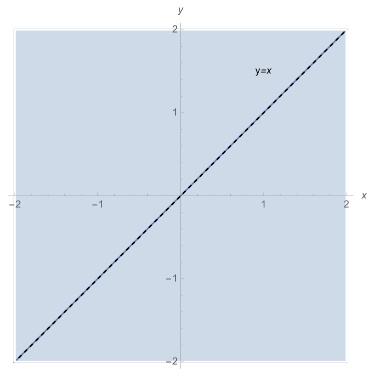
Universidade do Minho
Departamento de Matemática

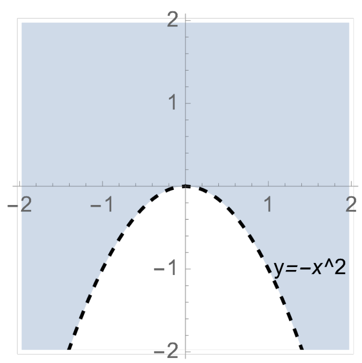
Licenciatura em Ciências da Computação
Análise
2021/22

Funções reais de n variáveis reais: limite e continuidade

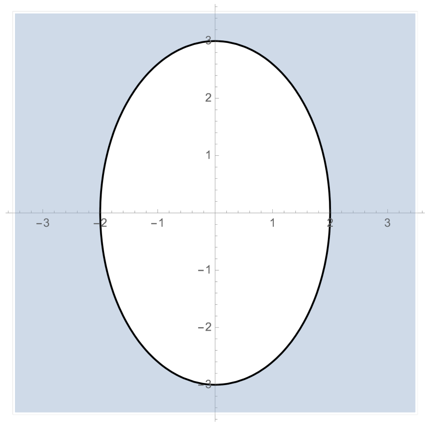
1. (a) $\mathcal{D}_f = \{(x, y) \in \mathbb{R}^2 : \, y \neq x\}$



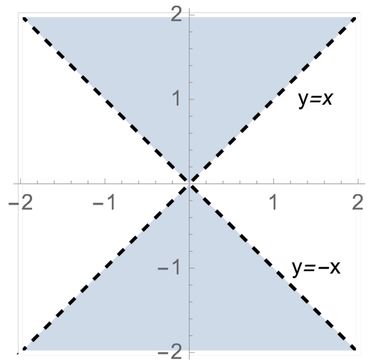
(b) $\mathcal{D}_f = \{(x, y) \in \mathbb{R}^2 : \, x^2 + y > 0\} = \{(x, y) \in \mathbb{R}^2 : \, y > -x^2\}$



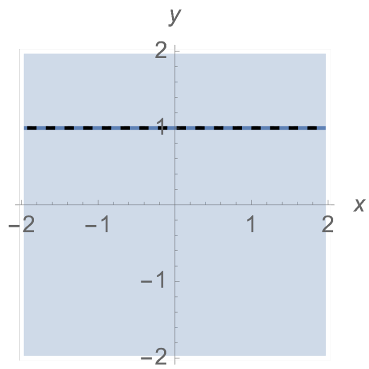
(c) $\mathcal{D}_f = \{(x, y) \in \mathbb{R}^2 : \, 9x^2 + 4y^2 - 36 \geq 0\} = \{(x, y) \in \mathbb{R}^2 : \, \frac{x^2}{4} + \frac{y^2}{9} \geq 1\}$



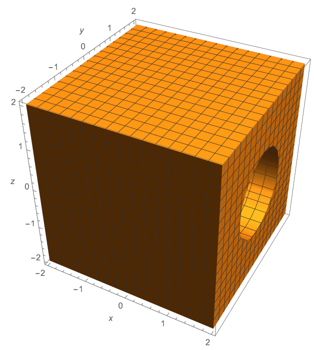
(d) $\mathcal{D}_f = \{(x, y) \in \mathbb{R}^2 : \, y^2 - x^2 > 0\} = \{(x, y) \in \mathbb{R}^2 : \, (y - x)(y + x) > 0\}$



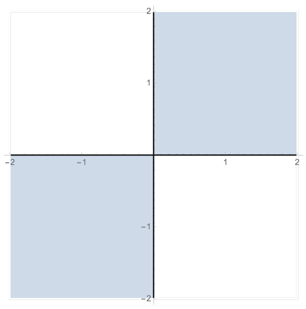
2. (a) $\mathcal{D}_f = \{(x, y) \in \mathbb{R}^2 : y \neq 1\}$.



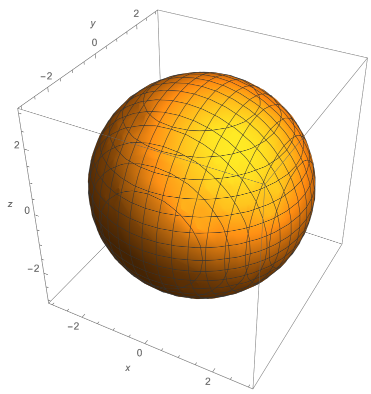
(b) $\mathcal{D}_f = \{(x, y, z) \in \mathbb{R}^3 : y^2 + z^2 - 1 > 0\} = \{(x, y, z) \in \mathbb{R}^3 : y^2 + z^2 > 1\}$.



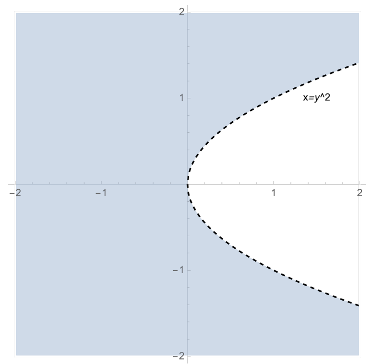
(c) $\mathcal{D}_f = \{(x, y) \in \mathbb{R}^2 : xy \geq 0\}$.



(d) $\mathcal{D}_f = \{(x,y,z) \in \mathbb{R}^3 : 9 - x^2 - y^2 - z^2 \geq 0\} = \{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 9\}$.

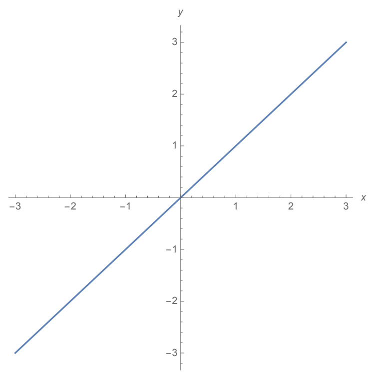


(e) $\mathcal{D}_f = \{(x,y) \in \mathbb{R}^2 : y^2 - x > 0\} = \{(x,y) \in \mathbb{R}^2 : x < y^2\}$.

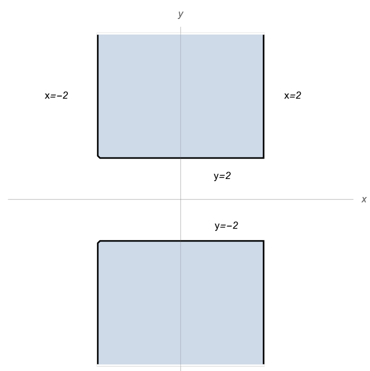


(f) $\mathcal{D}_f = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 - 2xy \geq 0\} = \{(x,y) \in \mathbb{R}^2 : (x-y)^2 \geq 0\} = \mathbb{R}^2$.

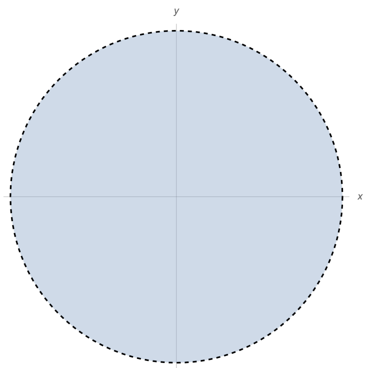
(g) $\mathcal{D}_f = \{(x,y) \in \mathbb{R}^2 : -(x-y)^2 \geq 0\} = \{(x,y) \in \mathbb{R}^2 : x = y\}$.



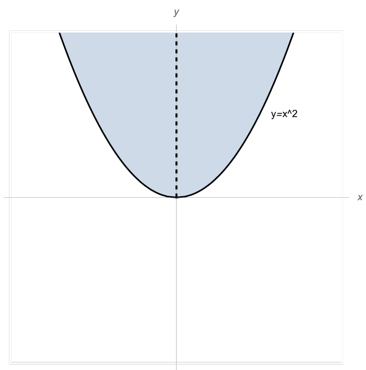
(h) $\mathcal{D}_f = \{(x,y) \in \mathbb{R}^2 : 4 - x^2 \geq 0 \wedge y^2 - 4 \geq 0\} = \{(x,y) \in \mathbb{R}^2 : x^2 \leq 4 \wedge y^2 \geq 4\}$.



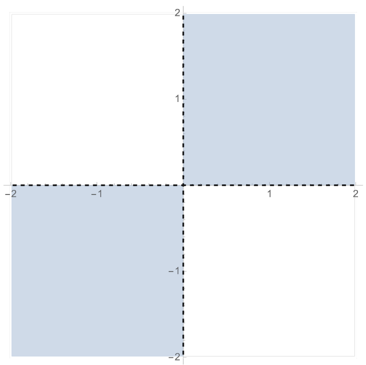
(i) $\mathcal{D}_f = \{(x,y) \in \mathbb{R}^2 :\, 1 - x^2 - y^2 > 0\} = \{(x,y) \in \mathbb{R}^2 :\, x^2 + y^2 < 1\}$.



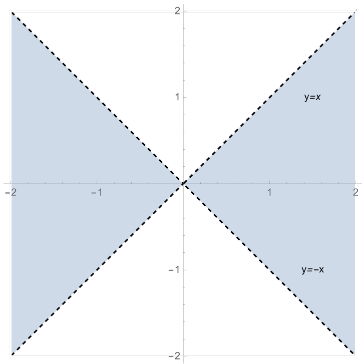
(j) $\mathcal{D}_f = \{(x,y) \in \mathbb{R}^2 :\, x \neq 0 \,\wedge\, y - x^2 \geq 0\} = \{(x,y) \in \mathbb{R}^2 :\, x \neq 0 \,\wedge\, y \geq x^2\}$.



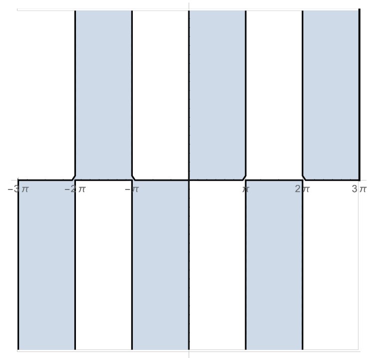
(k) $\mathcal{D}_f = \{(x,y) \in \mathbb{R}^2 :\, xy > 0\}$



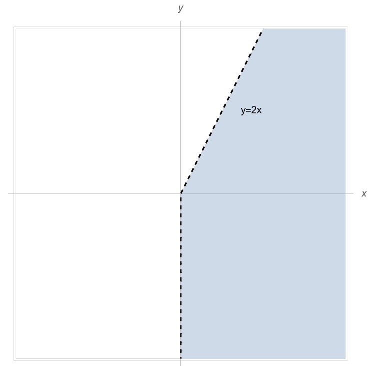
(l) $\mathcal{D} = \{(x,y) \in \mathbb{R}^2 :\, x^2 - y^2 > 0\} = \{(x,y) \in \mathbb{R}^2 :\, (x-y)(x+y) > 0\}$



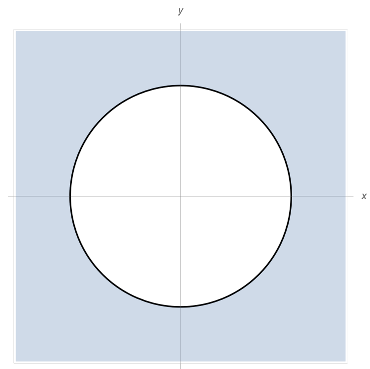
(m) $\mathcal{D}_f = \{(x,y) \in \mathbb{R}^2 : \, y \operatorname{sen} x \geq 0\}$



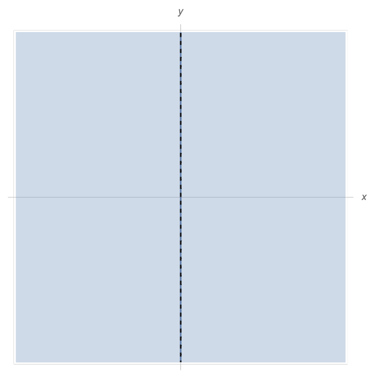
(n) $\mathcal{D}_f = \{(x,y) \in \mathbb{R}^2 : \, x > 0 \,\wedge\, 2x - y > 0\} = \{(x,y) \in \mathbb{R}^2 : \; x > 0 \,\wedge\, y < 2x\}$



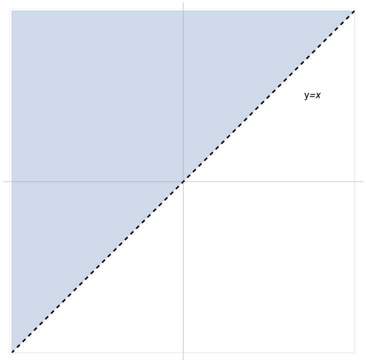
(o) $\mathcal{D}_f = \{(x,y) \in \mathbb{R}^2 : \, x^2 + y^2 - 4 \geq 0\} = \{(x,y) \in \mathbb{R}^2 : \; x^2 + y^2 \geq 4\}$



(p) $\mathcal{D}_f = \{(x,y) \in \mathbb{R}^2 : \, x \neq 0\}$



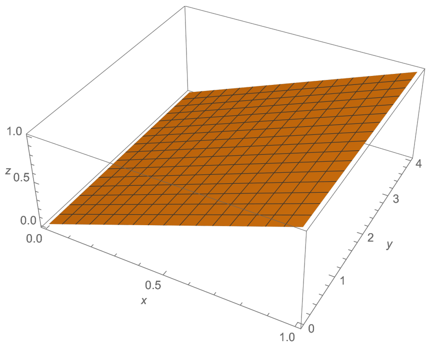
(q) $\mathcal{D}_f = \{(x,y) \in \mathbb{R}^2 : y > x\}$



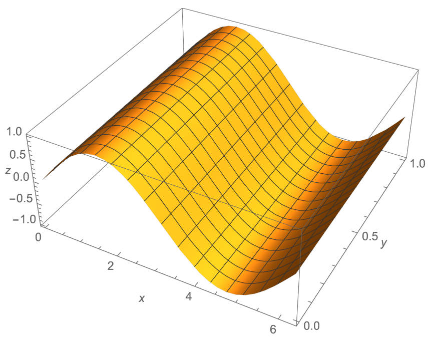
(r) $\mathcal{D}_f = \{(x,y) \in \mathbb{R}^2 : y \geq x \wedge x \leq 1\}$

3. Esboce o gráfico das seguintes funções:

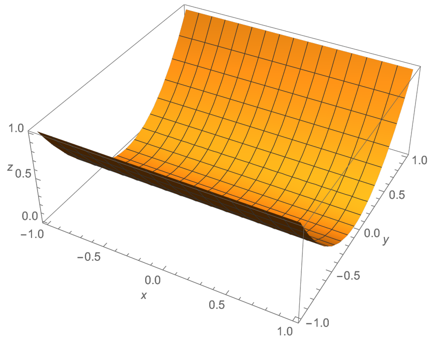
(a) $f : [0,1] \times [0,4] \longrightarrow \mathbb{R}$ tal que $f(x,y) = x$



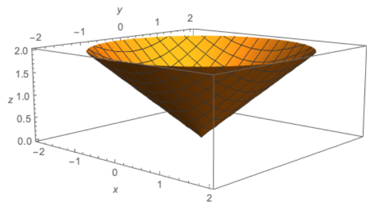
(b) $f : [0,2\pi] \times [0,1] \longrightarrow \mathbb{R}$ tal que $f(x,y) = \operatorname{sen} x$



(c) $f : [-1,1] \times [-1,1] \longrightarrow \mathbb{R}$ tal que $f(x,y) = y^2$

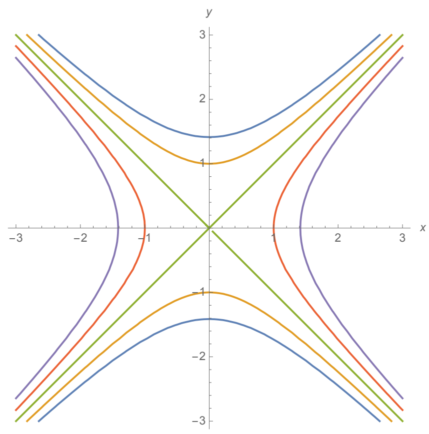


(d) $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ tal que $f(x,y) = \sqrt{x^2 + y^2}$.

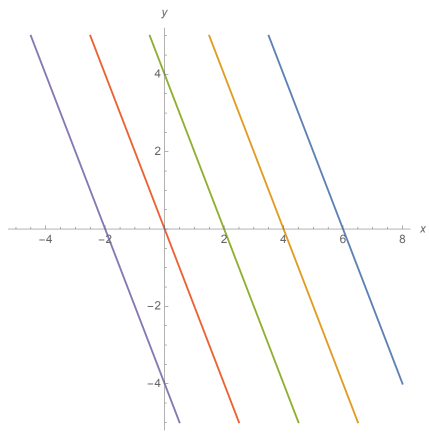


4. Determine e esboce algumas curvas de nível das seguintes funções:

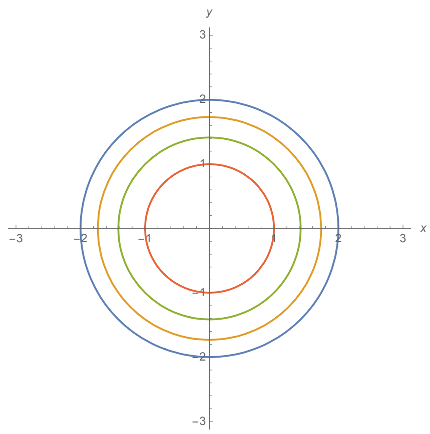
(a) $\mathcal{C}_k = \{(x,y) \in \mathbb{R}^2 : x^2 - y^2 = k\}$: por exemplo, $k = -2, -1, 0, 1, 2$



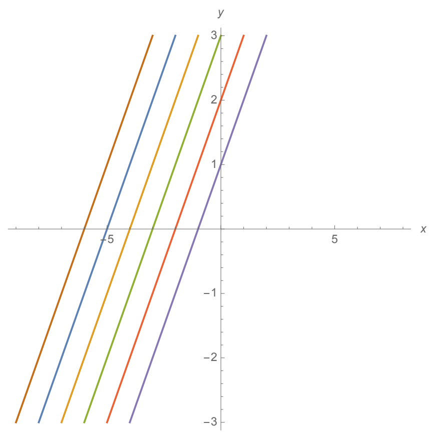
(b) $\mathcal{C}_k = \{(x,y) \in \mathbb{R}^2 : 1 - \frac{x}{2} - \frac{y}{4} = k\}$: por exemplo, $k = -2, -1, 0, 1, 2$



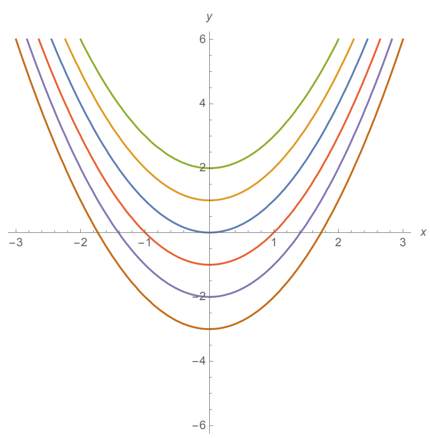
(c) $\mathcal{C}_k = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 4 - k^2,\ \ 0 \leq k \leq 2\}$: por exemplo, $k = 0, 1, \sqrt{2}, \sqrt{3}, 2$



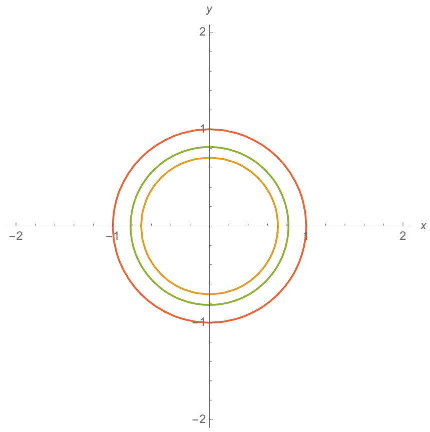
(d) $\mathcal{C}_k = \{(x,y) \in \mathbb{R}^2 : y = x + 5 - k\ \}$: por exemplo, $k = -1, 0, 1, 2, 3, 4$



(e) $\mathcal{C}_k = \{(x,y) \in \mathbb{R}^2 : y = x^2 + k\ \}$: por exemplo, $k = -3, -2, -1, 0, 1, 2$



(f) $\mathcal{C}_k = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 = 1-k^2,\ -1 \leq k \leq 1\}$: por exemplo, $k = 0, \sqrt{2}/2, \sqrt{3}/3, 1$



5. (a) Resolvido na aula.

   (b) Resolvido na aula.

   (c) $\mathcal{S}_k = \{(x,y,z) \in \mathbb{R}^3 : x+2y+3z = k,\ k \in \mathbb{R}\}$:
   Plano ortogonal ao vetor $u = (1,2,3)$ e que passa no ponto $(0,0,\frac{k}{3})$.

   (d) $\mathcal{S}_k = \{(x,y,z) \in \mathbb{R}^3 : z = x^2+y^2+k,\ k \in \mathbb{R}\}$:
   Paraboloide de vértice $(0,0,k)$.

6. (a) Seja $\mathcal{P} = \{(x,y) \in \mathbb{R}^2 : y = x^2\}$ e $\mathcal{D} = \mathbb{R}^2 \backslash \mathcal{P}$. Temos que:

$$\lim_{\substack{(x,y) \to (0,0) \\ (x,y) \in \mathcal{P}}} f(x,y) = 1 \quad \text{e} \quad \lim_{\substack{(x,y) \to (0,0) \\ (x,y) \in \mathcal{D}}} f(x,y) = 0\,.$$

   Então $\nexists \lim\limits_{(x,y)\to(0,0)} f(x,y)$.

   (b) $\lim\limits_{(x,y)\to(0,0)} g(x,y) = 1$.

   (c) Seja $\mathcal{R} = \{(x,y) \in \mathbb{R}^2 : y = x\}$ e $\mathcal{D} = \mathbb{R}^2 \backslash \mathcal{R}$. Temos que:

$$\lim_{\substack{(x,y) \to (0,0) \\ (x,y) \in \mathcal{R}}} h(x,y) = \lim_{(x,y)\to(0,0)} x^2 = 0 \quad \text{e} \quad \lim_{\substack{(x,y) \to (0,0) \\ (x,y) \in \mathcal{D}}} h(x,y) = \lim_{(x,y)\to(0,0)} \operatorname{sen}(xy) = 0\,.$$

   Então $\lim\limits_{(x,y)\to(0,0)} h(x,y) = 0$.

   (d) Sejam
   $\mathcal{D}_1 = \{(x,y) \in \mathbb{R}^2 : x \geq 0 \wedge y \geq 0\}$
   $\mathcal{D}_2 = \{(x,y) \in \mathbb{R}^2 : x \geq 0 \wedge y < 0\}$
   $\mathcal{D}_3 = \{(x,y) \in \mathbb{R}^2 : y < 0\}$.

Temos que:

$$\lim_{\substack{(x,y)\to(0,0)\\(x,y)\in\mathcal{D}_1}} k(x,y) = \lim_{(x,y)\to(0,0)} 0 = 0, \quad \lim_{\substack{(x,y)\to(0,0)\\(x,y)\in\mathcal{D}_2}} k(x,y) = \lim_{(x,y)\to(0,0)} x = 0, \quad \text{e}$$

$$\lim_{\substack{(x,y)\to(0,0)\\(x,y)\in\mathcal{D}_3}} k(x,y) = \lim_{(x,y)\to(0,0)} y = 0\,.$$

Então, $\displaystyle\lim_{(x,y)\to(0,0)} k(x,y) = 0$.

7. (a) $\displaystyle\lim_{(x,y)\to(1,1)} f(x,y) = 0$

Em relação ao limite $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y)$ temos que:

$$\lim_{\substack{(x,y)\to(0,0)\\x=0}} f(x,y) = \lim_{y\to 0}\frac{-2y^2}{y} = -2 \quad \text{e} \quad \lim_{\substack{(x,y)\to(0,0)\\y=0}} f(x,y) = \lim_{x\to 0}\frac{0}{x^2} = 0\,.$$

Então $\nexists \displaystyle\lim_{(x,y)\to(0,0)} f(x,y)$.

(b) $\displaystyle\lim_{(x,y)\to(0,2)} g(x,y) = 0$

Em relação ao limite $\displaystyle\lim_{(x,y)\to(0,0)} g(x,y)$ temos que:

$$\lim_{\substack{(x,y)\to(0,0)\\x=0}} f(x,y) = \lim_{y\to 0}\frac{y^2-2y}{y^2} = \lim_{y\to 0}\left(1-\frac{2}{y}\right) \quad \text{que não existe}\,.$$

Então $\nexists \displaystyle\lim_{(x,y)\to(0,0)} g(x,y)$.

(c) $\displaystyle\lim_{(x,y)\to(0,0)} h(x,y) = 0$

Em relação ao limite $\displaystyle\lim_{(x,y)\to(0,-1)} h(x,y)$ temos que:

$$\lim_{\substack{(x,y)\to(0,-1)\\x=0}} h(x,y) = \lim_{y\to -1} h(x,y) = 0 \quad \text{e} \quad \lim_{\substack{(x,y)\to(0,-1)\\y=x^2-1}} h(x,y) = \lim_{x\to 0}\frac{x^2}{x^2} = 1\,.$$

Então $\nexists \displaystyle\lim_{(x,y)\to(0,-1)} h(x,y)$.

8. (a) $\displaystyle\lim_{(x,y)\to(0,0)} (x^2+y^2)\,\text{sen}\frac{1}{\sqrt{x^2+y^2}} = 0$

porque $\displaystyle\lim_{(x,y)\to(0,0)} (x^2+y^2) = 0$ e $\left|\text{sen}\frac{1}{\sqrt{x^2+y^2}}\right| \leq 1,\ \forall (x,y) \neq (0,0)$.

(b) Considerando as retas não verticais que passam pela origem de equação $y = mx$, $m \in \mathbb{R}$, vem

$$\lim_{\substack{(x,\,y)\,\to\,(0,\,0)\\ y\,=\,mx}} \frac{xy}{x^2+y^2} = \lim_{x\to 0} \frac{mx^2}{x^2+m^2x^2} = \frac{m}{1+m^2}$$

e este limite existe para todo $m \in \mathbb{R}$ mas o seu valor depende de m.

Então, $\nexists \lim\limits_{(x,y)\to(0,0)} \dfrac{xy}{x^2+y^2}$.

(c) Temos que: $\lim\limits_{\substack{(x,\,y)\,\to\,(0,\,0)\\ x\,=\,0}} \dfrac{x+y}{x^2+y^2} = \lim\limits_{y\to 0} \dfrac{1}{y}$ e este limite não existe.

Então, $\nexists \lim\limits_{(x,y)\to(0,0)} \dfrac{x+y}{x^2+y^2}$.

(d) Considerando as curvas que passam pela origem de equação $x = ky^3$, $k \in \mathbb{R}$, vem

$$\lim_{\substack{(x,\,y)\,\to\,(0,\,0)\\ x\,=\,ky^3}} \frac{xy^3}{x^2+y^6} = \lim_{y\to 0} \frac{ky^6}{k^2y^6+y^6} = \frac{k}{k^2+1}$$

e este limite existe para todo $k \in \mathbb{R}$ mas o seu valor depende de k.

Então, $\nexists \lim\limits_{(x,y)\to(0,0)} \dfrac{xy^3}{x^2+y^6}$.

(e) Temos que:

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} = \lim_{(x,y)\to(0,0)} \frac{x}{\sqrt{x^2+y^2}}\, y\,.$$

Observemos que a função $h(x,y) = \frac{x}{\sqrt{x^2+y^2}}$, $(x,y) \in \mathbb{R}^2\backslash\{(0,0)\}$, é limitada. Com efeito,

$$|x| = \sqrt{x^2} \leq \sqrt{x^2+y^2} \text{ e, portanto, } \left|\frac{x}{\sqrt{x^2+y^2}}\right| \leq 1,\ \forall (x,y) \in \mathbb{R}^2\backslash\{(0,0)\}\,.$$

Então, $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy}{\sqrt{x^2+y^2}} = \lim\limits_{(x,y)\to(0,0)} \dfrac{x}{\sqrt{x^2+y^2}}\, y = 0$ porque $\lim\limits_{(x,y)\to(0,0)} y = 0$ e $\left|\frac{x}{\sqrt{x^2+y^2}}\right| \leq 1,\ \forall (x,y) \in \mathbb{R}^2\backslash\{(0,0)\}$.

(f) Temos que:

$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^2} = \lim_{(x,y)\to(0,0)} \frac{y^2}{x^2+y^2}\, x\,.$$

Observemos que a função $h(x,y) = \frac{y^2}{x^2+y^2}$, $(x,y) \in \mathbb{R}^2\backslash\{(0,0)\}$, é limitada. Com efeito,

$$\left|\frac{y^2}{x^2+y^2}\right| \leq 1,\ \forall (x,y) \in \mathbb{R}^2\backslash\{(0,0)\}\,.$$

Então, $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy^2}{x^2+y^2} = \lim\limits_{(x,y)\to(0,0)} \dfrac{x}{x^2+y^2}\, x = 0$ porque $\lim\limits_{(x,y)\to(0,0)} x = 0$ e $\left|\frac{y^2}{x^2+y^2}\right| \leq 1,\ \forall (x,y) \in \mathbb{R}^2\backslash\{(0,0)\}$.

(g) $\lim\limits_{(x,y)\to(0,0)} \dfrac{\operatorname{sen}(x-y)}{\cos(x-y)} = 0$

(h) Considerando as curvas que passam pela origem de equação $x = ky^3$, $k \in \mathbb{R}$, vem

$$\lim_{\substack{(x,\,y)\,\to\,(0,\,0)\\ x\,=\,kx^3}} \frac{2xy^3}{3x^2+4y^6} = \lim_{y\to 0} \frac{2ky^6}{3k^2y^6+4y^6} = \frac{2k}{3k^2+4}$$

e este limite existe para todo $k \in \mathbb{R}$ mas o seu valor depende de k.

Então, $\nexists \lim_{(x,y)\to(0,0)} \dfrac{2xy^3}{3x^2+4y^6}$.

(i) Temos que:

$$\lim_{(x,y)\to(0,1)} \frac{x^2(y-1)^2}{x^2+(y-1)^2} = \lim_{(x,y)\to(0,1)} \frac{x^2}{x^2+(y-1)^2}\,(y-1)^2\,.$$

Observemos que a função $h(x,y) = \frac{x^2}{x^2+(y-1)^2}$, $(x,y) \in \mathbb{R}^2\backslash\{(0,1)\}$, é limitada. Com efeito,

$$\left|\frac{x^2}{x^2+(y-1)^2}\right| \le 1,\ \forall (x,y) \in \mathbb{R}^2\backslash\{(0,1)\}\,.$$

Então, $\lim_{(x,y)\to(0,1)} \dfrac{x^2(y-1)^2}{x^2+(y-1)^2} = \lim_{(x,y)\to(0,1)} \dfrac{x^2}{x^2+(y-1)^2}\,(y-1)^2 = 0$ porque $\lim_{(x,y)\to(0,1)} (y-1)^2 = 0$ e $\left|\frac{x^2}{x^2+(y-1)^2}\right| \le 1,\ \forall (x,y) \in \mathbb{R}^2\backslash\{(0,1)\}$.

(j) Temos que: $\lim_{\substack{(x,\,y)\,\to\,(0,\,0)\\ x\,=\,0}} \dfrac{-2x^2+3y}{x^2+y^2} = \lim_{y\to} \dfrac{3}{y}$ e este limite não existe.

Então, $\nexists \lim_{(x,y)\to(0,0)} \dfrac{-2x^2+3y}{x^2+y^2}$.

(k) Temos que:

$$\lim_{(x,y,z)\to(0,0,0)} \frac{y^2z}{x^2+y^2+z^2} = \lim_{(x,y,z)\to(0,0,0)} \frac{y^2}{x^2+y^2+z^2}\,z\,.$$

Observemos que a função $h(x,y,z) = \frac{y^2}{x^2+y^2+z^2}$, $(x,y,z) \in \mathbb{R}^3\backslash\{(0,0,0)\}$, é limitada. Com efeito,

$$\left|\frac{y^2}{x^2+y^2+z^2}\right| \le 1,\ \forall (x,y,z) \in \mathbb{R}^3\backslash\{(0,0,0)\}\,.$$

Então, $\lim_{(x,y)\to(0,0,0)} \dfrac{y^2z}{x^2+y^2+z^2} = \lim_{(x,y,z)\to(0,0,0)} \dfrac{y^2}{x^2+y^2+z^2}\,z = 0$ porque $\lim_{(x,y)\to(0,0,0)} z = 0$ e $\left|\frac{y^2}{x^2+y^2+z^2}\right| \le 1,\ \forall (x,y,z) \in \mathbb{R}^3\backslash\{(0,0,0)\}$.

(l) Considerando as curvas que passam pela origem de equação $x = 0 \wedge y = kz^2$, $k \in \mathbb{R}$, vem

$$\lim_{\substack{(x,\,y,\,z)\,\to\,(0,\,0,\,0)\\ x\,=\,0\\ y\,=\,kz^2}} \frac{x^3+yz^2}{x^4+y^2+z^4} = \lim_{z\to 0} \frac{kz^4}{k^2z^4+z^4} = \frac{k}{k^2+1}$$

e este limite existe para todo $k \in \mathbb{R}$ mas o seu valor depende de k.

Então, $\nexists \lim_{(x,y,z)\to(0,0,0)} \dfrac{x^3+yz^2}{x^4+y^2+z^4}$.

(m) $\displaystyle\lim_{(x,y)\to(2,0)} \frac{(x-2)^2y^2}{(x-2)^2+y^2} = 0.$

(n) $\displaystyle\nexists \lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}.$

(o) $\displaystyle\nexists \lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$

(p) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y}{(x^2+y^2)^2}\operatorname{sen}(x^2+y^2) = 0.$

9. (a) Considerando as curvas que passam pela origem de equação $y = kx^2$, $k \in \mathbb{R}$, vem

$$\lim_{\substack{(x,\,y)\,\to\,(0,\,0)\\ y\,=\,kx^2}} \frac{x^2y}{x^4+y^2} = \lim_{x\to 0} \frac{kx^4}{x^4+k^2x^4} = \frac{k}{1+k^2}$$

e este limite existe para todo $k \in \mathbb{R}$ mas o seu valor depende de k.

Então, $\displaystyle\nexists \lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}.$

Consequentemente, f não é contínua em $(0,0)$.

(b) Queremos mostrar que $\displaystyle\lim_{\substack{(x,\,y)\,\to\,(0,\,0)\\ y\,=\,mx}} f(x,y) = f(0,0) = 0.$

Temos que:

$$\lim_{\substack{(x,\,y)\,\to\,(0,\,0)\\ y\,=\,mx}} \frac{x^2y}{x^4+y^2} = \lim_{x\to 0} \frac{mx^3}{x^4+m^2x^2} = \lim_{x\to 0} \frac{mx}{x^2+m^2} = 0.$$

10. Estude a continuidade das seguintes funções:

(a) f é contínua em $\mathbb{R}^2$.

(b) (i) Para $\underline{(x,y) \neq (0,0)}$: f é contínua porque é uma função racional cujo denominador não se anula.

(ii) Para $\underline{(x,y) = (0,0)}$:

Temos que:

$$\lim_{(x,y)\to(0,0)} \frac{x^2}{\sqrt{x^2+y^2}} = \lim_{(x,y)\to(0,0)} \frac{x}{\sqrt{x^2+y^2}}\, x\,.$$

Observemos que a função $h(x,y) = \frac{x}{\sqrt{x^2+y^2}}$, $(x,y) \in \mathbb{R}^2\backslash\{(0,0)\}$, é limitada. Com efeito,

$$|x| = \sqrt{x^2} \leq \sqrt{x^2+y^2} \text{ e, portanto, } \left|\frac{x}{\sqrt{x^2+y^2}}\right| \leq 1,\ \forall (x,y) \in \mathbb{R}^2\backslash\{(0,0)\}\,.$$

Então, $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2}{\sqrt{x^2+y^2}} = \lim_{(x,y)\to(0,0)} \frac{x}{\sqrt{x^2+y^2}}\, x = 0$ porque $\displaystyle\lim_{(x,y)\to(0,0)} x = 0$ e $\left|\frac{x}{\sqrt{x^2+y^2}}\right| \leq 1,\ \forall (x,y) \in \mathbb{R}^2\backslash\{(0,0)\}\,.$

Como $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y) = f(0,0) = 0$, f é contínua em $(0,0)$.

Concluímos por (i) e (ii) que f é contínua em $\mathbb{R}^2$.

(c) f é contínua em $\mathbb{R}^2$.

(d) f é contínua em $\mathbb{R}^2\backslash\{(0,0)\}$.

(e) f é contínua em $\mathbb{R}^2\backslash\{(0,0)\}$.

(f) f é contínua em $\mathbb{R}^2$.

(g) f é contínua em $\mathbb{R}^2\backslash\{(x,y)\in\mathbb{R}^2 : y=0\}$.

(h) f é contínua em $\mathbb{R}^2$.

(i) f é contínua em $\mathbb{R}^2$.

(j) f é contínua em $\mathbb{R}^2\backslash\{(0,0)\}$.

(k) f é contínua em $\mathbb{R}^2\backslash\{(x,y)\in\mathbb{R}^2 : x\neq 0\}$.

(l) f é contínua em $\mathbb{R}^2\backslash(\{(x,y)\in\mathbb{R}^2 : y=0\}\cup\{(x,y)\in\mathbb{R}^2 : y=x^2\})$.

(m) f é contínua em $\mathbb{R}^2$.

(n) f é contínua em $\mathbb{R}^2\backslash\{(x,y)\in\mathbb{R}^2 : x^2+y^2=1\}$ (Justifique).

(o) f é contínua em $\mathbb{R}^2\backslash\{(x,y)\in\mathbb{R}^2 : y=2\}$.

(p) f é contínua em $\mathbb{R}^2$.

11. As funções f e h admitem prolongamentos contínuos a $\mathbb{R}^2$ e a função g não admite prolongamento contínuo a $\mathbb{R}^2$. Justifique.