Universidade do Minho
Departamento de Matemática

Primitivas

1. Sendo $f : \mathbb{R} \longrightarrow \mathbb{R}$ definida por $f(x) = x^2 \operatorname{sen} x$, calcule a primitiva de f cujo gráfico passa pelo ponto $(\frac{\pi}{2}, \pi)$.

2. Em cada alínea, determine a única função $f : \mathbb{R} \longrightarrow \mathbb{R}$ duas vezes derivável tal que

   (a) $f''(x) = 4x - 1, \quad x \in \mathbb{R}, \quad f(1) = 3 \quad \text{e} \quad f'(2) = -2$
   (b) $f''(x) = \operatorname{sen} x \cos x, \quad x \in \mathbb{R}, \quad f(0) = 0 \quad \text{e} \quad f'(0) = 1.$

3. [Primitivas imediatas] Calcule:

   (1) $\displaystyle\int (\sqrt{x} + 2)^2 \, dx$
   (2) $\displaystyle\int (3x^2 - 2x^5) \, dx$
   (3) $\displaystyle\int (2x + 10)^{20} \, dx$
   (4) $\displaystyle\int x^2 e^{x^3} \, dx$
   (5) $\displaystyle\int x^4 (x^5 + 10)^9 \, dx$
   (6) $\displaystyle\int \frac{2x + 1}{x^2 + x + 3} \, dx$
   (7) $\displaystyle\int \sqrt{2x + 1} \, dx$
   (8) $\displaystyle\int \frac{x}{3 - x^2} \, dx$
   (9) $\displaystyle\int \frac{1}{4 - 3\,x} \, dx$
   (10) $\displaystyle\int \frac{1}{e^{3x}} \, dx$
   (11) $\displaystyle\int \frac{-7}{\sqrt{1 - 5x}} \, dx$
   (12) $\displaystyle\int \frac{\sqrt{1 + 3 \log x}}{x} \, dx$
   (13) $\displaystyle\int x \operatorname{sen}(x^2) \, dx$
   (14) $\displaystyle\int \frac{1}{x(\log^2 x + 1)} \, dx$
   (15) $\displaystyle\int \left(\frac{2}{x} - 3\right)^2 \frac{1}{x^2} \, dx$
   (16) $\displaystyle\int \operatorname{sen}\,(\pi - 2\,x) \, dx$
   (17) $\displaystyle\int \operatorname{th} x \, dx$
   (18) $\displaystyle\int \operatorname{sen}\, x \cos\, x \, dx$
   (19) $\displaystyle\int \operatorname{sen}\,(2x) \cos\, x \, dx$
   (20) $\displaystyle\int \operatorname{sen}^2\left(\frac{x}{2}\right) \cos^2\left(\frac{x}{2}\right) \, dx$

(21) $\int \operatorname{sen}^2 x\, dx$

(22) $\int \cos^3 x\, dx$

(23) $\int \dfrac{x}{x^2-1}\, dx$

(24) $\int \dfrac{x}{\sqrt{x^2-1}}\, dx$

(25) $\int \dfrac{1}{x}\operatorname{sen}\,(\log x)\, dx$

(26) $\int \dfrac{-3}{x\,(\log x)^3}\, dx$

(27) $\int \dfrac{e^x}{1+e^{2x}}\, dx$

(28) $\int \dfrac{e^x}{1-2e^x}\, dx$

(29) $\int \dfrac{1}{\cos^2(7x)}\, dx$

(30) $\int \left(\sqrt{2x-1}-\sqrt{1+3x}\right)\, dx$

(31) $\int \dfrac{1}{x}\,\left[1+(\log\, x)^2\right]\, dx$

(32) $\int \dfrac{2\,+\,\sqrt{\operatorname{arctg}\,(2x)}}{1+4x^2}\, dx$

(33) $\int \dfrac{e^{\operatorname{arctg} x}}{1+x^2}\, dx$

(34) $\int \dfrac{\operatorname{sen}\, x}{\sqrt{1+\cos\, x}}\, dx$

4. [Primitivação por partes] Calcule:

(1) $\int \log\, x\, dx$

(2) $\int x \operatorname{sen}\,(2x)\, dx$

(3) $\int \operatorname{arctg}\, x\, dx$

(4) $\int x \cos\, x\, dx$

(5) $\int \log(1-x)\, dx$

(6) $\int x \log\, x\, dx$

(7) $\int x^2 \operatorname{sen} x\, dx$

(8) $\int x \operatorname{sen} x \cos x\, dx$

(9) $\int \log^2 x\, dx$

(10) $\int e^x \cos\, x\, dx$

(11) $\int \operatorname{arcsen}\, x\, dx$

(12) $\int e^{\operatorname{sen} x} \operatorname{sen}\, x \cos\, x\, dx$

(13) $\int \dfrac{\operatorname{arcsen}\sqrt{x}}{\sqrt{x}}\, dx$

(14) $\int x \operatorname{arctg} x\, dx$

(15) $\int x^2 \log x\, dx$

(16) $\int \operatorname{sen}\,(\log\, x)\, dx$

(17) $\int \operatorname{ch} x \operatorname{sen}\,(3\,x)\, dx$

(18) $\int x^3\, e^{x^2}\, dx$

5. [Primitivação por substituição] Efetuando a substituição sugerida, calcule:

(1) $\int x\sqrt{x-1}\,dx$, sugestão $x = 1 + t^2$, $t \geq 0$;

(2) $\int \sqrt{1-x^2}\,dx$, sugestão $x = \operatorname{sen} t$, $t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$;

(3) $\int \sqrt{4+x^2}\,dx$, sugestão $x = 2\operatorname{sh} t$, $t \geq 0$;

(4) $\int x\,(x+3)^{1/3}\,dx$, sugestão $t = x + 3$;

(5) $\int \frac{x}{\sqrt{2-3x}}\,dx$, sugestão $\sqrt{2-3x} = t$;

(6) $\int \frac{e^{2x}}{3+e^x}\,dx$, sugestão $e^x = t$, $t > 0$.

6. [Primitivação de funções racionais] Calcule:

(1) $\int \frac{2x^2+x+1}{(x-1)(x+1)^2}\,dx$

(2) $\int \frac{3x^2-4x-1}{(x^2-1)(x-2)}\,dx$

(3) $\int \frac{2x^2-x-2}{x^2(x-2)}\,dx$

(4) $\int \frac{2x^3+5x^2+6x+2}{x(x+1)^3}\,dx$

(5) $\int \frac{x^2-x+2}{x(x^2-1)}\,dx$

(6) $\int \frac{27}{x^4-3\,x^3}\,dx$

(7) $\int \frac{x+3}{(x-2)\,(x^2-2\,x+5)}\,dx$

(8) $\int \frac{x+1}{x(x^2+1)^2}\,dx$

7. Calcule:

(1) $\int \frac{1}{(2+\sqrt{x})^7\,\sqrt{x}}dx$

(2) $\int \operatorname{tg}^2 x\,dx$

(3) $\int \frac{x+(\operatorname{arcsen}(3x))^2}{\sqrt{1-9\,x^2}}dx$

(4) $\int \frac{1}{1+e^{\,x}}\,dx$

(5) $\int \frac{1}{\cos^2 x\,\operatorname{sen}^2 x}\,dx$

(6) $\int \frac{1}{x^2\sqrt{4-x^2}}\,dx$