Soluções da Folha Prática 6

1. (a) 0.050 (b) 0.647 (c) 0.185 (d) 0.998 (e) $K = 4$

2. Valor exato/Aproximação:
   i) 0.03782949/0.03783327 *ii*) 0.9972315/0.9972306 *iii*) 0.9329235/0.932914

3. (a) 0.099 (b) 0.014 (c) $f(k) = \begin{cases} e^{-0.6p}\frac{(0.6p)^k}{k!} & se \quad k \in \mathbb{N}_0 \\ 0 & se \quad c.c. \end{cases}$ (d) $Poisson(\lambda p)$

4. (a) 0.081 (b) 0.271 (c) 0.531 (d) 0.81

5. (a) $F(c) = \begin{cases} 0 & se \quad c < -1 \\ c + \frac{1}{2}c^2 + \frac{1}{2} & se \quad -1 \leq c < 0 \\ c - \frac{1}{2}c^2 + \frac{1}{2} & se \quad 0 \leq c < 1 \\ 1 & se \quad c \geq 1 \end{cases}$ (b) $0; \frac{7}{8}; \frac{3}{8}; \frac{1}{8}; \frac{5}{9}$

6. (a) $F(c) = \begin{cases} \frac{1}{2}e^c & se \quad c < 0 \\ 1 - \frac{1}{2}e^{-c} & se \quad c \geq 0 \end{cases}$ (b) $\frac{1}{2}; \frac{1}{2}; \frac{1}{2}(1 - e^{-1}); \frac{1}{2}(1 - e^{-1}); 1 - e^{-1}$ (c) $Exp(1)$

7. (a) $F(c) = \begin{cases} 0 & se \quad c < 0 \\ 1 - e^{-\lambda c} & se \quad c \geq 0 \end{cases}$; $e^{-\lambda c}$ (b) — (c) $\frac{3e^{-4}}{3e^{-4}+e^{-2}}$

8. (a) $a = 0; b = 1; k = 2; f(x) = \begin{cases} 0 & se \quad x \leq 0 \vee x \geq 2 \\ 1 - \frac{x}{2} & se \quad 0 < x < 2 \end{cases}$ (b) $\frac{1}{16}$ (c) 0.034 (d) 20kg

9. $Y \sim Exp(1)$

Soluções da Folha Prática 7

1. —

2. (a) $0.472; \frac{1}{2}; 0.056; 0.115$ (b) 0.145

3. (a) 0.683 (b) 0.954 (c) 0.997

4. (d)

5. 0.046

6. (a) $F(c) = \begin{cases} 0 & se \quad c < 2 \\ \frac{c-2}{10} & se \quad 2 \leq c \leq 12 \\ 1 & se \quad c > 12 \end{cases}$ (b) $U([2, 12])$ (c) 0.6; 0.121

7. (a) 0.25 (b) 0.25 (c) 0.6 (d) 359ml

8. (a) 0.393 (b) 0.368 (c) 0.393

9. $P(Y = 0) = 1 - e^{-\lambda a}$, $F_Y(c) = \begin{cases} 0 & se \quad c < 0 \\ 1 - e^{-\lambda(c+a)} & se \quad c \geq 0 \end{cases}$

10. (a) — (b) $H_1(t) = \begin{cases} 0 & se \quad t < -1 \\ 1 & se \quad t \geq -1 \end{cases}$ e $H_2(t) = \begin{cases} 0 & se \quad t < \frac{1}{2} \\ \frac{2t-1}{3} & se \quad \frac{1}{2} \leq t \leq 2 \\ 1 & se \quad t > 2 \end{cases}$

Soluções da Folha Prática 8

1. —
2. (a)

	$E[X]$	$Var[X]$	σ_X	$\chi_{0.25}$	$\chi_{0.5}$	$\chi_{0.75}$
5. i.	$\frac{70}{36}$	$\frac{2660}{1296}$	$\sqrt{\frac{2660}{1296}}$	1	2	3
5. ii.	$\frac{161}{36}$	$\frac{2555}{1296}$	$\sqrt{\frac{2555}{1296}}$	3	5	5
5. iii.	$\frac{91}{36}$	$\frac{2555}{1296}$	$\sqrt{\frac{2555}{1296}}$	1	2	3
5. iv.	1	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	0	1	1
5. v.	1	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	0	1	1
5. vi.	7	$\frac{210}{36}$	$\sqrt{\frac{210}{36}}$	5	7	9

(b)

	$E[X]$	$Var[X]$	σ_X	$\chi_{0.25}$	$\chi_{0.5}$	$\chi_{0.75}$
5.	0	$\frac{1}{6}$	$\frac{1}{\sqrt{6}}$	$-1+\frac{\sqrt{2}}{2}$	0	$1-\frac{\sqrt{2}}{2}$
6.	0	2	$\sqrt{2}$	$-\log(2)$	0	$\log(2)$
8.	$\frac{2}{3}$	$\frac{2}{9}$	$\frac{\sqrt{2}}{3}$	$2-\sqrt{3}$	$2-\sqrt{2}$	1

3. $E[Y]$ existe e $E[Y] = \frac{1-e^{-\lambda}}{\lambda}$
4. (a) $\frac{1}{2}$ (b) $Y \sim U([0,1])$, $E[Y] = \frac{1}{2}$ e $Var[Y] = \frac{1}{12}$
5. (a) $F(c) = \begin{cases} 0 & se \quad c < 0 \\ 4c^3 - 3c^4 & se \quad 0 \leq c \leq 1 \\ 1 & se \quad c > 1 \end{cases}$; $E[X] = \frac{3}{5}$ e $Var[X] = \frac{1}{25}$

   (b) $E[L] = \frac{8v_1+40v_2+33v_3}{81} - k$; $\sigma_L = \sqrt{\frac{8v_1^2+40v_2^2+33v_3^3}{81} - \left(\frac{8v_1+40v_2+33v_3}{81}\right)^2}$
6. —
7. —

Soluções da Folha Prática 9

1. (a) $f((a_1,a_2)) = \begin{cases} 4/36 & se \quad (a_1,a_2) \in \{(0,0),(1,0)\} \\ 12/36 & se \quad (a_1,a_2) \in \{(0,1)\} \\ 9/36 & se \quad (a_1,a_2) \in \{(0,2)\} \\ 6/36 & se \quad (a_1,a_2) \in \{(1,1)\} \\ 1/36 & se \quad (a_1,a_2) \in \{(2,0)\} \\ 0 & se \quad c.c. \end{cases}$ (b) $\frac{5}{36}$

   (c) $f_X(a_1) = \begin{cases} 25/36 & se \quad a_1 = 0 \\ 10/36 & se \quad a_1 = 1 \\ 1/36 & se \quad a_1 = 2 \\ 0 & se \quad c.c. \end{cases}$, $f_Y(a_2) = \begin{cases} 9/36 & se \quad a_2 \in \{0,2\} \\ 18/36 & se \quad a_2 = 1 \\ 0 & se \quad c.c. \end{cases}$

   (d) Não (e) $Cov(X,Y) = -\frac{1}{6}$, $\rho(X,Y) = -0.447$

2. (a) $f((a_1,a_2)) = \begin{cases} 1/16 & se \quad (a_1,a_2) \in \{(0,0),(0,2),(1,0),(1,3),(2,1),(2,3)\} \\ 2/16 & se \quad (a_1,a_2) \in \{(0,1),(2,2)\} \\ 3/16 & se \quad (a_1,a_2) \in \{(1,1),(1,2)\} \\ 0 & se \quad c.c. \end{cases}$

   (b) $\frac{11}{16}$

   (c) $f_{X_1}(a_1) = \begin{cases} 4/16 & se \quad a_1 \in \{0,2\} \\ 8/16 & se \quad a_1 = 1 \\ 0 & se \quad c.c. \end{cases}$; $f_{X_2}(a_2) = \begin{cases} 2/16 & se \quad a_2 \in \{0,3\} \\ 6/16 & se \quad a_2 \in \{1,2\} \\ 0 & se \quad c.c. \end{cases}$; Não

   (d) $Cov(X_1,X_2) = \frac{1}{4}$, $\rho(X_1,X_2) = \frac{1}{\sqrt{6}}$

3. (a) $C_{(X,Y)} = \{(0,1),(1,0),(1,1),(2,0),(2,1),(3,0),(3,1)\}$;

   $f_X(x) = \begin{cases} 1/32 & se \quad x = 0 \\ 3/32 & se \quad x = 1 \\ 9/32 & se \quad x = 2 \\ 19/32 & se \quad x = 3 \\ 0 & se \quad c.c. \end{cases}$; $f_Y(y) = \begin{cases} 14/32 & se \quad y = 0 \\ 18/32 & se \quad y = 1 \\ 0 & se \quad c.c. \end{cases}$

   (b) Não (c) $\frac{78}{32}$; $\frac{18}{32}$; 0.621; 0.246; -0.059; -0.151

4. (a) $F_M(c) = [F(c)]^n$; $F_N(c) = 1 - [1 - F(c)]^n$ (b) $Exp(n\lambda)$

5. São independentes.

6. $X_1 + X_2 + \ldots + X_n \sim Bin(n,p)$

7. (a) $f_X(x) = \begin{cases} 0 & se \quad x \leq 0 \\ e^{-x} & se \quad x > 0 \end{cases}$; $f_Y(y) = \begin{cases} 0 & se \quad y \leq 0 \\ e^{-y} & se \quad y > 0 \end{cases}$ (b) $\frac{1}{2}$

   (c) $P(X+Y \leq u) = \begin{cases} 0 & se \quad u \leq 0 \\ 1 - e^{-u}[1+u] & se \quad u > 0 \end{cases}$; $2e^{-1} - 3e^{-2}$ (d) São independentes

   (e) $E[X] = E[Y] = 1$, $Var[X] = Var[Y] = 1$, $Cov(X,Y) = \rho(X,Y) = 0$

8. $Cov(X,X^2) = \rho(X,X^2) = 0$

9. (a) $k = 1/8$;

   $f_X(x) = \begin{cases} \frac{1}{4}x^3 & se \quad 0 < x < 2 \\ 0 & se \quad c.c. \end{cases}$; $f_Y(y) = \begin{cases} \frac{1}{8}[\frac{8}{3} - 2y + \frac{5}{6}y^3] & se \quad -2 < y \leq 0 \\ \frac{1}{8}[\frac{8}{3} - 2y + \frac{1}{6}y^3] & se \quad 0 < y < 2 \\ 0 & se \quad c.c. \end{cases}$

   (b) Não (c) $\frac{8}{5}$; $-\frac{8}{15}$; 0.107; 0.604; -0.036; -0.142

10. (a) $f_X(x) = \begin{cases} 0 & se \quad x < 0 \vee x > 1 \\ \frac{2}{5}(x+2) & se \quad 0 \leq x \leq 1 \end{cases}$; $f_Y(y) = \begin{cases} 0 & se \quad y < 0 \vee y > 1 \\ \frac{1}{5}(1+8y) & se \quad 0 \leq y \leq 1 \end{cases}$; Não são independentes

    (b) $\frac{8}{15}$ (c) $\frac{2}{5}$ (d) $\frac{8}{15}$; $\frac{19}{30}$; $\frac{37}{450}$; $\frac{59}{900}$; $-\frac{1}{225}$; -0.061

11. 0.12

12. 0.0034

13. $(X,Y) \sim M(2; \frac{1}{6}, \frac{1}{2})$; Não

14. —

15. —

16. (a) —
    (b) $Cov(X_1, X_2) = \rho(X_1, X_2) = \frac{2}{\pi}$
    (c) —

17. (a) $\chi_{0.25} = 1.75; \chi_{0.5} = 2.5; \chi_{0.75} = 3.25$ (b) 0.0003
    (c) $f((x,y)) = \begin{cases} \frac{1}{9} & se \quad x \in [1,4], y \in [1,4] \\ 0 & se \quad c.c. \end{cases}$ (d) $\frac{1}{2}$ (e) $\frac{4}{9}$

Soluções da Folha Prática 10

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1. —

2. —

3. —

4. (a) $N(270, 67)$ (b) 0.033

5. 0.028

6. $n = 62$