2.5 Quantum Entanglement and the Bell States

1. What does it mean when qubits are maximally entangled?

2. What does it mean when qubits are paritally entangled?

3. How do we know if qubits are entangled?

4. Are the following states entangled?

(a) $|\psi\rangle = \frac{1}{\sqrt{2}}\Big(|01\rangle + |10\rangle\Big)$

(b) $|\psi\rangle = \frac{1}{\sqrt{2}}\Big(|00\rangle + |01\rangle\Big)$

(c) $|\psi\rangle = \frac{\sqrt{3}}{2\sqrt{2}}|00\rangle + \frac{1}{2\sqrt{2}}|01\rangle + \frac{\sqrt{3}}{2\sqrt{2}}|10\rangle + \frac{1}{2\sqrt{2}}|11\rangle$

Answers

1. Qubits are maximally entangled if by measuring one of the qubits we know for certain what we will measure the other qubits as

2. Qubits are partially entangled if the measurement outcome of one of the qubits affects the state the other qubits collapse into once that qubit has been measured

3. If we cannot factor the state into the tensor product of single-qubit states then the state is entangled

4. Are the following states entangled?

(a) Yes

(b) No since $\frac{1}{\sqrt{2}}\Big(|00\rangle + |01\rangle\Big) = |0\rangle \otimes \frac{1}{\sqrt{2}}\Big(|0\rangle + |1\rangle\Big)$

(c) No since $\frac{\sqrt{3}}{2\sqrt{2}}|00\rangle + \frac{1}{2\sqrt{2}}|01\rangle + \frac{\sqrt{3}}{2\sqrt{2}}|10\rangle + \frac{1}{2\sqrt{2}}|11\rangle = \Big(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\Big) \otimes \Big(\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle\Big)$