1.7 The Phase Gates (S and T Gates)

1. Apply an S-gate to a qubit in the state $|\psi\rangle = \alpha|0\rangle + e^{i\theta}\beta|1\rangle$. What happened to the relative phase of the qubit?

1. Apply an $S^\dagger$-gate to a qubit in the state $|\psi\rangle = \alpha|0\rangle + e^{i\theta}\beta|1\rangle$. What happened to the relative phase of the qubit?

1. Apply an T-gate to a qubit in the state $|\psi\rangle = \alpha|0\rangle + e^{i\theta}\beta|1\rangle$. What happened to the relative phase of the qubit?

1. Apply an $T^\dagger$-gate to a qubit in the state $|\psi\rangle = \alpha|0\rangle + e^{i\theta}\beta|1\rangle$. What happened to the relative phase of the qubit?

Answers

1. $S|\psi\rangle = \alpha|0\rangle + e^{i(\theta+\pi/2)}\beta|1\rangle$, a relative phase of $e^{i\pi/2}$ was added to the qubit

1. $S^\dagger|\psi\rangle = \alpha|0\rangle + e^{i(\theta-\pi/2)}\beta|1\rangle$, a relative phase of $e^{-i\pi/2}$ was added to the qubit

1. $T|\psi\rangle = \alpha|0\rangle + e^{i(\theta+\pi/4)}\beta|1\rangle$, a relative phase of $e^{i\pi/4}$ was added to the qubit

1. $T^\dagger|\psi\rangle = \alpha|0\rangle + e^{i(\theta-\pi/4)}\beta|1\rangle$, a relative phase of $e^{-i\pi/4}$ was added to the qubit