



Exercises 4 : Interaction and Concurrency

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Exercise I.1

Suppose two variants of parallel composition have been added to the process language \mathbb{P} and defined through the following rules:

$$\begin{array}{c} \frac{E \xrightarrow{a} E'}{E \otimes F \xrightarrow{a} E' \otimes F} (O_1) \qquad \frac{F \xrightarrow{a} F'}{E \otimes F \xrightarrow{a} E \otimes F'} (O_2) \\[10pt] \frac{E \xrightarrow{a} E' \quad \wedge \quad \bar{a} \notin \mathcal{L}(F)}{E \parallel F \xrightarrow{a} E' \parallel F} (P_1) \qquad \frac{F \xrightarrow{a} F' \quad \wedge \quad \bar{a} \notin \mathcal{L}(E)}{E \parallel F \xrightarrow{a} E \parallel F'} (P_2) \\[10pt] \frac{E \xrightarrow{a} E' \quad F \xrightarrow{\bar{a}} F'}{E \parallel F \xrightarrow{\tau} E' \parallel F'} (P_3) \end{array}$$

1. Explain, in your own words, the meaning of \otimes e \parallel .
2. Guided by the semantic rules given, show how the synchronisation diagrams for $E \otimes F$ and $E \parallel F$ can be built from the corresponding diagrams for E and F .
3. Is \parallel associative with respect to \sim ?

Exercise I.2

Identify, in the list of process pairs below, which of them can be related by \approx . And by $=$?

1. $a.\tau.b.0 \text{ e } a.b.0$
2. $a.(b.0 + \tau.c.0) \text{ e } a.(b.0 + c.0)$
3. $a.(b.0 + \tau.c.0) \text{ e } a.(b.0 + c.0) + a.c.0$
4. $a.0 + b.0 + \tau.b.0 \text{ e } a.0 + \tau.b.0$
5. $a.0 + b.0 + \tau.b.0 \text{ e } a.0 + b.0$
6. $a.(b.0 + (\tau.(c.0 + \tau.d.0))) \text{ e } a.(b.0 + (\tau.(c.0 + \tau.d.0))) + a.(c.0 + \tau.d.0)$
7. $a.(b.0 + (\tau.(c.0 + \tau.d.0))) \text{ e } a.(b.0 + c.0 + d.0) + a.(c.0 + d.0) + a.d.0$
8. $\tau.(a.b.0 + a.c.0) \text{ e } \tau.a.b.0 + \tau.a.c.0$
9. $\tau.(a.\tau.b.0 + a.b.\tau.0) \text{ e } a.b.0$
10. $\tau.(\tau.a.0 + \tau.b.0) \text{ e } \tau.a.0 + \tau.b.0$
11. $A \triangleq a.\tau.A \text{ e } B \triangleq a.B$
12. $A \triangleq \tau.A + a.0 \text{ e } a.0$
13. $A \triangleq \tau.A \text{ e } 0$


Exercise I.3

Suppose processes R and T have transitions $R \xrightarrow{\tau} T$ and $T \xrightarrow{\tau} R$, among others. Show that, under this condition, $R = T$.

Exercise I.4

Consider the following statements about a binary relation S on \mathbb{P} . Discuss whether you may conclude from each of them whether S is (or is not) a weak bisimulation.

observacional:

- | | |
|---|--|
| 1. S is the identity in \mathbb{P} . | sim |
| 2. S is a subset of the identity in \mathbb{P} . | nao |
| 3. S is a strict bisimulation up to  | sim (corrido a vermelho para -> equivalencia estrutural (simbolo mod)) |
| 4. S is the empty relation. | sim |
| 5. $S = \{(a.E, a.F) \mid E \approx F\}$. | nao |
| 6. $S = \{(a.E, a.F) \mid E \approx F\} \cup \approx$. | sim |
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Exercise I.5

Show that

- $E + \tau.(E + F) = \tau.(E + F)$
 - $a.(E + \tau.\tau.E) = a.E$
 - $\tau.(G + a.(E + \tau.F)) = \tau.(G + a.(E + \tau.F)) + a.F$
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Exercise I.6

Show that any process $\tau.(\tau.P + a.0)$ is a solution to equation $X = a.0 + \tau.X$.

Exercise I.7

Let E be a process such that $\text{fn}(E) = \emptyset$. Prove or refute the following statements:

- $E \mid Q \approx Q$.
 - $E \mid Q = Q$.
 - $E \mid Q = \tau.Q$.
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Exercise I.8

Although concurrent systems usually deal with components exhibiting non terminating behaviour, it is sometimes useful also to consider terminating processes and their composition. Let T be a class of terminating processes which perform a special action \dagger to announce completion of all their tasks and evolve to 0 after that. In this class it is possible to define a combinator for *sequential* composition $P ; Q$, whose behaviour is informally explained as *once P terminates, $P ; Q$ behaves like Q* . Formally,

$$P ; Q \triangleq \text{new } \{m\} (\{m/\dagger\} P \mid \overline{m} \cdot Q)$$

where m is fresh identifier, not occurring neither in P nor Q .

1. Define a process $U \in T$ such that $U ; P \approx P$. Justify your proposal.
2. Prove or refute that, for any $P, Q, R \in T$,

$$(P + Q) ; R \approx (P ; R) + (Q ; R)$$
3. As sequential composition is a particular case of parallel composition, the law above could be regarded as a particular case of

$$(P + Q) | R \approx (P | R) + (Q | R)$$
 This equation, however, is false. Confirm this by providing a suitable counter-example..

Exercise I.9

Consider the following specification of a *pipe*, as supported e.g. in UNIX:

$$U \triangleright V \stackrel{\text{abv}}{=} \text{new } \{c\} (\{c/out\}U \mid \{c/in\}V)$$

under the assumption that, in both processes, actions \overline{out} e in stand for, respectively, the output and input ports.

1. Consider now the following processes only partially defined:

$$\begin{aligned} U_1 &\triangleq \overline{out}.T \\ V_1 &\triangleq in.R \\ U_2 &\triangleq \overline{out}.\overline{out}.\overline{out}.T \\ V_2 &\triangleq in.in.in.R \end{aligned}$$

Prove, by equational reasoning, or refute the following properties:

- (a) $U_1 \triangleright V_1 \sim T \triangleright R$
 - (b) $U_2 \triangleright V_2 = U_1 \triangleright V_1$
2. Show or refute the associativity of \triangleright wrt process equality, i.e., for all $P, T, V \in \mathbb{P}$,

$$(U \triangleright V) \triangleright T = U \triangleright (V \triangleright T)$$
 3. Show that $\mathbf{0} \triangleright \mathbf{0} = \mathbf{0}$.

Exercise I.10

Consider a combinator \circ_n whose operational semantics is given by following rule

$$\frac{E \xrightarrow{a} E'}{\circ_0 E \xrightarrow{a} E'} \quad \frac{E \xrightarrow{a} E'}{\circ_n E \xrightarrow{a} \circ_{n-1} E'} \quad \text{for } n > 0$$

1. Explain its purpose.
2. Discuss whether, and for which values of m and n , one may have $\circ_n (\circ_m E) \sim \circ_n E$.
3. Show that $E \sim F$ implies $\circ_n E \sim \circ_n F$.
4. Show, by a counter-example, that, whenever \sim is replaced by \approx , the implication above fails.
5. How could the operational semantics of this new combinator be changed so that the implication mentioned above holds? I.e. so that $E \approx F \Rightarrow \circ_n E \approx \circ_n F$?

Exercise I.11

Consider a combinator whose operational semantics is given by following rule

$$\frac{E \xrightarrow{x} E'}{E \downarrow a \xrightarrow{x} E'} \quad \text{if } x \neq a, x \neq \bar{a}$$

1. Explain its purpose.
2. Show that $P \downarrow a \sim Q \downarrow a$ if $P \sim Q$.
3. Define two processes E and F such that $E \approx F$ but $E \downarrow a \not\approx F \downarrow a$.
4. Prove or refute that if $P = Q$ then $P \downarrow a = Q \downarrow a$.

Exercise I.12

Consider a new process combinator, called an *action duplicator*, and defined by the following rule:

$$\frac{E \xrightarrow{a} E'}{\odot(E) \xrightarrow{a} E}$$

Note that the derivative in the rule's conclusion is E (and not E'). For example, $\odot(a.\mathbf{0}) \xrightarrow{a} a.\mathbf{0}$. Prove or refute that

1. $E \sim F$ implies $\odot(E) \sim \odot(F)$.
2. $E \approx F$ implies $\odot(E) \approx \odot(F)$.
3. $\odot(E + F) \sim \odot(E) + \odot(F)$.