Exercises 2 : Interaction and Concurrency

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Exercise I.1

Suppose a labelled transition system is given by the following transition relation:

$\{\langle 1,a,2\rangle, \langle 1,a,3\rangle, \langle 2,a,3\rangle, \langle 2,b,1\rangle, \langle 3,a,3\rangle, \langle 3,b,1\rangle, \langle 4,a,5\rangle, \langle 5,a,5\rangle, \langle 5,b,6\rangle, \langle 6,a,5\rangle, \langle 7,a,8\rangle, \langle 8,a,8\rangle, \langle 8,b,7\rangle\}$

Prove or refute $1 \sim 4 \sim 6 \sim 7$.

Exercise I.2

Given two labelled transition systems $\langle S_A, \mathcal{N}, \longrightarrow_{\mathcal{A}}\rangle$ and $\langle S_B, \mathcal{N}, \longrightarrow_{\mathcal{B}}\rangle$, two states p and q are *equisimilar* iff

$$p \doteqdot q \;\equiv\; p \lesssim q \wedge q \lesssim p$$

1. Show that $\doteqdot$is an equivalence relation.
2. Compare this equivalence with bisimilarity $\sim$.

Exercise I.3

Suppose that the existential quantifiers in the definition of bisimulation were replaced by universal quantifiers. Characterise the resulting bisimilarity relation.

Exercise I.4

Show that bisimilarity is strictly included in equisimilarity, and that the latter is also strictly included on trace equivalence.

Exercise I.5

Discuss whether bisimilarity $\sim$

- is closed for union
- is closed for intersection

Exercise I.6

A relation R over the state space of a labelled transition system is a *word bisimulation* if, whenever $\langle p, q\rangle \in R$ and $s \in \mathcal{N}^*$, we have

$$p \xrightarrow{s} p' \;\Rightarrow\; \langle \exists\, q' \;:\; q' \in S_2 :\; q \xrightarrow{s} q' \wedge \langle p', q'\rangle \in R\rangle$$
$$q \xrightarrow{s} q' \;\Rightarrow\; \langle \exists\, p' \;:\; p' \in S_1 :\; p \xrightarrow{s} p' \wedge \langle p', q'\rangle \in R\rangle$$

1. Define formally relation $\xrightarrow{s}$, for $s \in \mathcal{N}^*$
2. Two states are *word bisimilar* iff they belong to a word bisimulation. Show that two states p and q are word bisimilar iff $p \sim q$.
