3.2.B Functions on Quantum Computers

The notation $f : \{0,1\}^n \to \{0,1\}^m$ means the function f takes in a bit string of length n and returns a bit string of length m.

1. Apply U_f to the state $|\psi\rangle = \frac{1}{\sqrt{3}}\Big(|001\rangle|0\rangle + |010\rangle|0\rangle + |111\rangle|0\rangle\Big)$, where the function $f : \{0,1\}^3 \to \{0,1\}$. Let the first register of qubits be the input to the function and second register be the output register.

2. Apply U_f to the state $|\psi\rangle = \frac{1}{\sqrt{3}}\Big(|001\rangle|-\rangle + |010\rangle|-\rangle + |111\rangle|-\rangle\Big)$, where the function $f : \{0,1\}^3 \to \{0,1\}$. Let the first register of qubits be the input to the function and second register be the output register. Recall the phase oracle formula: $U_f|x\rangle|-\rangle = (-1)^{f(x)}|x\rangle|-\rangle$

Answers

1. $U_f|\psi\rangle = U_f\frac{1}{\sqrt{3}}\Big(|001\rangle|0\rangle + |010\rangle|0\rangle + |111\rangle|0\rangle\Big)$

$= \frac{1}{\sqrt{3}}\Big(U_f|001\rangle|0\rangle + U_f|010\rangle|0\rangle + U_f|111\rangle|0\rangle\Big)$

$= \frac{1}{\sqrt{3}}\Big(|001\rangle|f(001)\rangle + |010\rangle|f(010)\rangle + |111\rangle|f(111)\rangle\Big)$

2. $U_f|\psi\rangle = U_f\frac{1}{\sqrt{3}}\Big(|001\rangle|-\rangle + |010\rangle|-\rangle + |111\rangle|-\rangle\Big)$

$= \frac{1}{\sqrt{3}}\Big(U_f|001\rangle|-\rangle + U_f|010\rangle|-\rangle + U_f|111\rangle|-\rangle\Big)$

$= \frac{1}{\sqrt{3}}\Big((-1)^{f(001)}|001\rangle|-\rangle + (-1)^{f(010)}|010\rangle|-\rangle + (-1)^{f(111)}|111\rangle|-\rangle\Big)$