1.5 Introduction to Phase

1. If we have a qubit in superposition that has a relative phase of $e^{5\pi i/4}$, on the Bloch Sphere how many radians has the qubit 'spun' around the z-axis

2. Simplify the qubit state $|\psi\rangle = \alpha e^{i\phi}|0\rangle + \beta e^{i\phi}|1\rangle = e^{i\phi}(\alpha|0\rangle + \beta|1\rangle)$ by omitting the global phase

3. Simplify the following qubit states by first factoring out the phase from the 0 state, creating a global and relative phase, then omitting the global phase. (HINT for (c): represent -1 as a complex number in exponential form, $-1 = e^{i\pi}$)

(a) $e^{i\theta}\alpha|0\rangle + e^{i\phi}\beta|1\rangle$

(b) $e^{\pi i/2}\alpha|0\rangle + e^{3\pi i/4}\beta|1\rangle$

(c) $e^{3\pi i/2}\alpha|0\rangle - \beta|1\rangle$

Answers

1. $5\pi/4$ radians

2. $e^{i\phi}(\alpha|0\rangle + \beta|1\rangle) \equiv \alpha|0\rangle + \beta|1\rangle$

3.

(a) $e^{i\theta}\alpha|0\rangle + e^{i\phi}\beta|1\rangle = e^{i\theta}(\alpha|0\rangle + e^{-i\theta}e^{i\phi}\beta|1\rangle) = e^{i\theta}(\alpha|0\rangle + e^{i(\phi-\theta)}\beta|1\rangle) \equiv \alpha|0\rangle + e^{i(\phi-\theta)}\beta|1\rangle$

(b) $e^{\pi i/2}\alpha|0\rangle + e^{3\pi i/4}\beta|1\rangle \equiv \alpha|0\rangle + e^{\pi i/4}\beta|1\rangle$

(c) $e^{3\pi i/2}\alpha|0\rangle - \beta|1\rangle = e^{3\pi i/2}\alpha|0\rangle + e^{\pi i}\beta|1\rangle \equiv \alpha|0\rangle + e^{-\pi i/2}\beta|1\rangle \equiv \alpha|0\rangle + e^{3\pi i/2}\beta|1\rangle$ (Since rotating $-\pi/2$ radians is the same as rotating $3\pi/2$ radians)