1.1 Introduction to the Qubit and Superposition

1. An arbitrary qubit state can be represented as $|\psi\rangle = \begin{bmatrix}\alpha \\ \beta\end{bmatrix}$. The probability of measuring the $|0\rangle$ state is $|\alpha|^2$ and the probability of measuring the $|1\rangle$ state is $|\beta|^2$

(a) What is the probability of measuring $|0\rangle$ with a qubit in the state $|\psi\rangle = \begin{bmatrix}\sqrt{3}/2 \\ 1/2\end{bmatrix}$

(b) What is the probability of measuring $|1\rangle$ with a qubit in the state $|\psi\rangle = \begin{bmatrix}\sqrt{3}/2 \\ 1/2\end{bmatrix}$

(c) What is the probability of measuring $|0\rangle$ when a qubit in the state $|\psi\rangle = \begin{bmatrix}1/\sqrt{2} \\ 1/\sqrt{2}\end{bmatrix}$

(d) What is the probability of measuring $|1\rangle$ when a qubit in the state $|\psi\rangle = \begin{bmatrix}1/\sqrt{2} \\ 1/\sqrt{2}\end{bmatrix}$

2. For an arbitrary qubit state $|\psi\rangle = \begin{bmatrix}\alpha \\ \beta\end{bmatrix}$, the probability of measuring $|0\rangle$ plus the probability of measuring $|1\rangle$ must equal 1 since when we measure a qubit we will get one of those outcomes. This gives us the equation $|\alpha|^2 + |\beta|^2 = 1$. Are these qubit states valid qubit states? Why?

(a) $|\psi\rangle = \begin{bmatrix}\sqrt{3}/2 \\ 1/2\end{bmatrix}$ (b) $|\psi\rangle = \begin{bmatrix}1 \\ 0\end{bmatrix}$ (c) $|\psi\rangle = \begin{bmatrix}\sqrt{2}/5 \\ 3/5\end{bmatrix}$

Answers

1.

(a) 3/4 (b) 1/4 (c) 1/2 (d) 1/2

2.

(a) Yes, it is a valid qubit state since $|\sqrt{3}/2|^2 + |1/2|^2 = 1$

(b) Yes, it is a valid qubit state since $|1|^2 + |0|^2 = 1$

(c) No, it is not a valid qubit state since $|\sqrt{2}/5|^2 + |3/5|^2 = 11/25 \neq 1$