

1.6 The Hadamard Gate and the $|+\rangle, |-\rangle, |i\rangle$ and $|-i\rangle$ states

We usually factor out the $\frac{1}{\sqrt{2}}$ out of the $|+\rangle, |i\rangle, |-\rangle, |-i\rangle$ states.

For example $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, $|-i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$

1.

(a) Represent i in exponential form ($e^{i\theta}$)

(b) The $|i\rangle$ state is defined by $|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$. Substitute in i in exponential form, how many radians has the qubit been rotated around the Bloch Sphere and does it line up with the $|i\rangle$ state on the Bloch Sphere?

(c) Represent -1 in exponential form ($e^{i\theta}$)

(d) The $|-\rangle$ state is defined by $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$. Substitute in -1 in exponential form, how many radians has the qubit been rotated around the Bloch Sphere and does it line up with the $|-\rangle$ state on the Bloch Sphere?

(e) Represent $-i$ in exponential form ($e^{i\theta}$)

(f) The $|-i\rangle$ state is defined by $|-i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$. Substitute in $-i$ in exponential form, how many radians has the qubit been rotated around the Bloch Sphere and does it line up with the $|-i\rangle$ state on the Bloch Sphere?

2. Find

(a) $H|0\rangle$

(b) $H|1\rangle$

(c) $H|+\rangle$

(d) $H|-\rangle$

Answers

1.

(a) $i = e^{i\pi/2}$

(b) $|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\pi/2}|1\rangle)$

(c) $-1 = e^{i\pi}$

(d) $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\pi}|1\rangle)$

(e) $-i = e^{3\pi i/2}$

(f) $| - i \rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{3\pi i/2}|1\rangle)$

2. Find

(a) $H|0\rangle = |+\rangle$

(b) $H|1\rangle = |-\rangle$

(c) $H|+\rangle = |0\rangle$

(d) $H|-\rangle = |1\rangle$