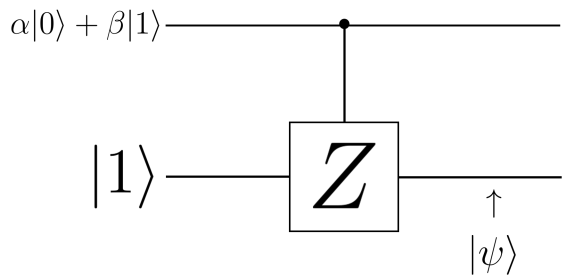
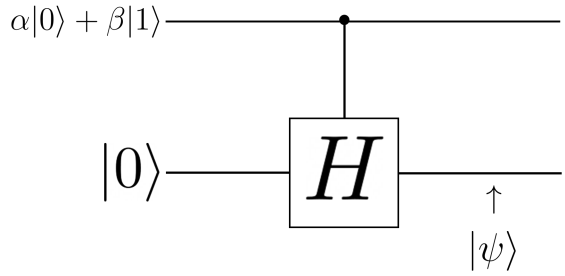
2.6 Phase Kickback

1. We know that $|1\rangle$ is an eigenvector of the Z-gate since $Z|1\rangle = -|1\rangle$. Consider the circuit below,



Prove that phase kickback occurs by showing a relative phase of -1 is applied to the control qubit

2. Does phase kickback occur in the following circuit. Why/why not?



Answers

1. $|\psi\rangle = CZ\Big(\alpha|0\rangle + \beta|1\rangle\Big)|1\rangle$

$= CZ\Big(\alpha|01\rangle + \beta|11\rangle\Big)$

$= CZ\alpha|01\rangle + CZ\beta|11\rangle$

$= \alpha|01\rangle - \beta|11\rangle$

$= \Big(\alpha|0\rangle - \beta|1\rangle\Big)|1\rangle$

Therefore a relative phase of -1 was added to the control qubit

2. Phase kickback does not occur since the state $|0\rangle$ is not an eigenvector of the Hadamard Gate