

0.2 Complex Numbers on the Number Plane Problem Set

Summary of lesson: We can represent a complex number $a+ib$ as $r(\cos(\theta)+isin(\theta)) = re^{i\theta}$ where r is the distance from the origin of the complex number and θ is the angle it makes with the positive x-axis.

1. Plot each of these complex numbers on the complex plane

(a) $2 + 3i$ (b) $2(\cos(\pi/4) + isin(\pi/4))$ (c) $3e^{5\pi i/4}$

2. Plot each of these complex numbers on the complex plane. What happens to the angle they make with the positive x-axis when multiplied together?

(a) $e^{7\pi i/8}$ (b) $e^{\pi i/4}$ (c) $e^{7\pi i/8}e^{\pi i/4}$

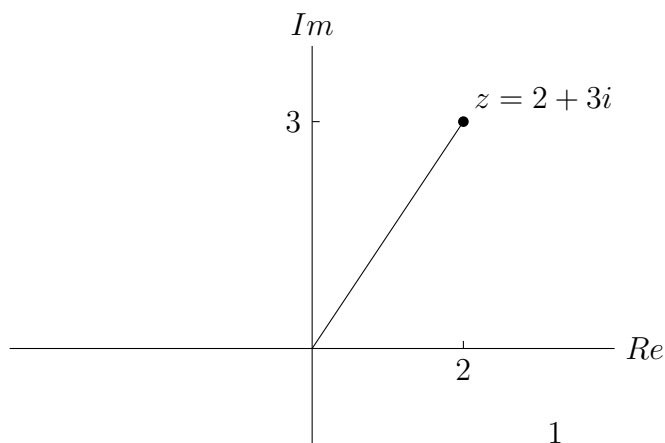
3. Represent each of these numbers in polar and exponential form

(a) $1 + i$ (b) i (c) -1

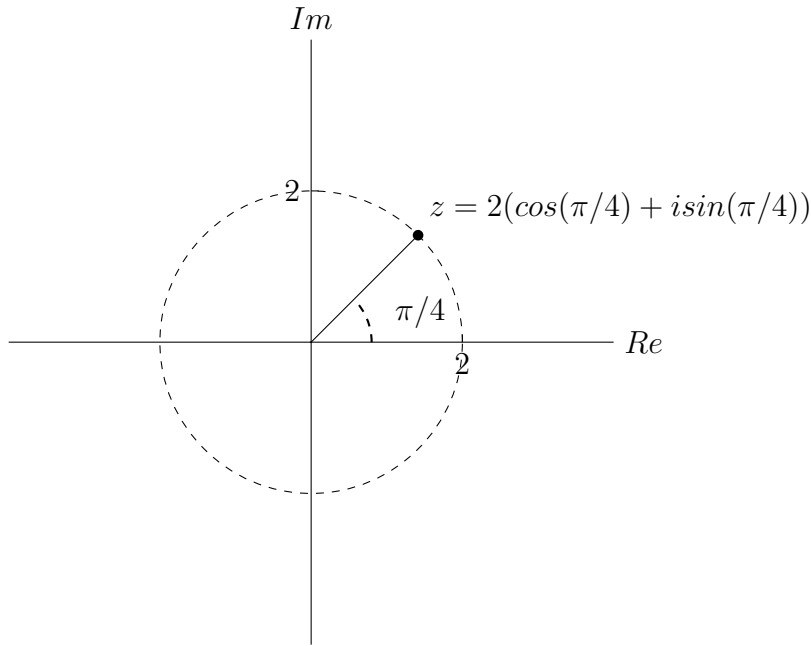
4. Plot $1 + i$ on the complex plane then plot $i(1 + i) = -1 + i$ on the complex plane. In Q3 we found $i = e^{i\pi/2}$, what happens when we multiply a complex number by i ?

Answers

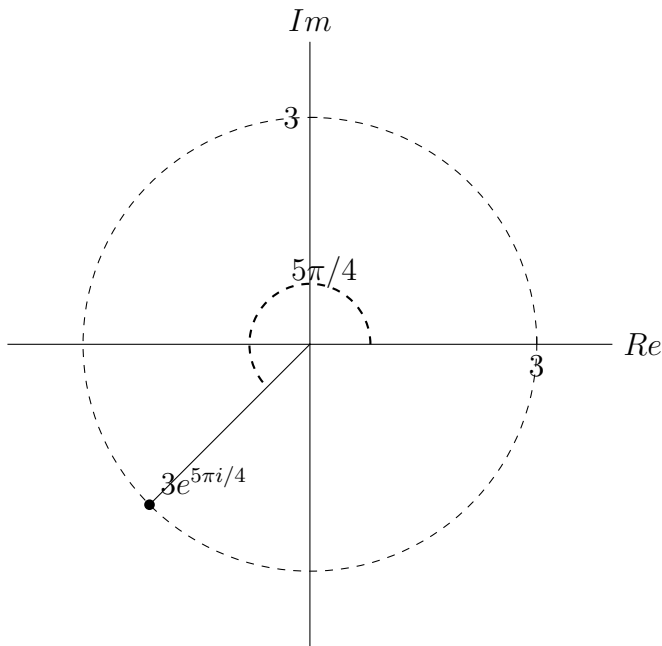
1.



(a)



(b)



(c)

2. When multiplying two complex numbers the angle adds together, $e^{i\theta}e^{i\phi} = e^{i(\theta+\phi)}$

3.

(a) $1 + i = \sqrt{2}(\cos(\pi/4) + i\sin(\pi/4)) = \sqrt{2}e^{i\pi/4}$

(b) $i = \cos(\pi/2) + i\sin(\pi/2) = e^{i\pi/2}$

(c) $-1 = \cos(\pi) + i\sin(\pi) = e^{i\pi}$

4. When we multiply a complex number by i , geometrically we are rotating that complex number anti-clockwise by $\pi/2$ radians.