2.4 Measuring Singular Qubits

1. Consider the state $|\psi\rangle = \frac{1}{\sqrt{3}}\Big(|001\rangle + |011\rangle + |101\rangle\Big)$

(a) What is the probability of measuring the left qubit as 0

(b) What would the state collapse to if we were to measure the left qubit as 0

2. Consider the state $|\psi\rangle = \frac{\sqrt{2}}{2}|000\rangle + \frac{\sqrt{2}}{4}|001\rangle + \frac{\sqrt{5}}{4}|110\rangle + \frac{1}{4}|111\rangle$

(a) What is the probability of measuring the middle qubit as 1

(b) What would the state collapse to if we were to measure the middle qubit as 1

3. Consider the state $|\psi\rangle = \frac{1}{2}|01\rangle + \frac{\sqrt{3}}{2}|11\rangle$, what is the probability of measuring the right qubit as 0, explain your answer

4. Consider the state $|\psi\rangle = \frac{1}{2}|00\rangle + \frac{\sqrt{3}}{2}|11\rangle$

(a) What would the state collapse to if we measure one of the qubits as 0

(b) What would the state collapse to if we measure one of the qubits as 1

Answers

1.

(a) $\left|\frac{1}{\sqrt{3}}\right|^2 + \left|\frac{1}{\sqrt{3}}\right|^2 = \frac{2}{3}$

(b) We know that $\left|\frac{A}{\sqrt{3}}\right|^2 + \left|\frac{A}{\sqrt{3}}\right|^2 = 1$, giving us $A = \sqrt{\frac{3}{2}}$,

Therefore state collapses to $\sqrt{\frac{3}{2}}\frac{1}{\sqrt{3}}\Big(|001\rangle + |011\rangle\Big) = \frac{1}{\sqrt{2}}\Big(|001\rangle + |101\rangle\Big)$

2.

(a) $\left|\frac{\sqrt{5}}{4}\right|^2 + \left|\frac{1}{4}\right|^2 = \frac{3}{8}$

(b) We know that $\left|\frac{A\sqrt{5}}{4}\right|^2 + \left|\frac{A}{4}\right|^2 = 1$, giving us $A = \frac{2\sqrt{2}}{\sqrt{3}}$,

Therefore state collapses to $\frac{2\sqrt{2}}{\sqrt{3}}\left(\frac{\sqrt{5}}{4}|110\rangle + \frac{1}{4}|111\rangle\right) = \frac{\sqrt{10}}{2\sqrt{3}}|110\rangle + \frac{\sqrt{2}}{2\sqrt{3}}|111\rangle$

3. The probability of measuring 0 is 0 since there are no superpostion states in the state where the right qubit is a 0.

4. Consider the state $|\psi\rangle = \frac{1}{2}|00\rangle + \frac{\sqrt{3}}{2}|11\rangle$

(a) The state would collapse to $|00\rangle$

(b) The state would collapse to $|11\rangle$