

0.5 Unitary and Hermitian Matrices

1. Let $A = \begin{bmatrix} 1 & 0 & 4 \\ 17 & 10 & 18 \end{bmatrix}$, find A^T
2. Let $A = \begin{bmatrix} 3+2i & 2e^{\pi i/3} & 4 \\ 1-7i & 10 & -3i \end{bmatrix}$, find A^* . Remember if $z = re^{i\theta}$ then $z^* = re^{-i\theta}$
3. Recall $A^\dagger = (A^*)^T = (A^T)^*$. Let $A = \begin{bmatrix} 3+2i & 2e^{\pi i/3} & 4 \\ i & 10 & -3i \end{bmatrix}$, find A^\dagger
4. A matrix U is Unitary if $U^\dagger U = I$. Is the matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$ unitary?
5. A matrix H is Hermitian if $H = H^\dagger$. Is the matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$ hermitian?

Answers

1. $\begin{bmatrix} 1 & 17 \\ 0 & 10 \\ 4 & 18 \end{bmatrix}$
2. $\begin{bmatrix} 3-2i & 2e^{-\pi i/3} & 4 \\ 1+7i & 10 & 3i \end{bmatrix}$
3. $\begin{bmatrix} 3-2i & -i \\ 2e^{-\pi i/3} & 10 \\ 4 & 3i \end{bmatrix}$
4. Yes since $AA^\dagger = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$
5. No since $A^\dagger = \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$, So $A \neq A^\dagger$