1.4 Manipulating a Qubit with Single Qubit Gates

$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$, $Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

1. Apply an X-gate to the state $|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$ by using matrix multiplication

2. Apply an Y-gate to the state $|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$ by using matrix multiplication

3. Apply an Z-gate to the state $|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$ by using matrix multiplication

4. Apply an X-gate to the state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ in Dirac Notation (by looking at the columns of the X-gate matrix)

5. Apply an Y-gate to the state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ in Dirac Notation (by looking at the columns of the Y-gate matrix)

6. Apply an Z-gate to the state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ in Dirac Notation (by looking at the columns of the Z-gate matrix)

Answers

1. $X|\psi\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \beta \\ \alpha \end{bmatrix}$
2. $Y|\psi\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} -i\beta \\ i\alpha \end{bmatrix}$
3. $Z|\psi\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \alpha \\ -\beta \end{bmatrix}$
4. $X|\psi\rangle = X(\alpha|0\rangle + \beta|1\rangle) = \alpha X|0\rangle + \beta X|1\rangle = \alpha \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \alpha|1\rangle + \beta|0\rangle = \beta|0\rangle + \alpha|1\rangle$
5. $Y|\psi\rangle = Y(\alpha|0\rangle + \beta|1\rangle) = \alpha Y|0\rangle + \beta Y|1\rangle = \alpha \begin{bmatrix} 0 \\ i \end{bmatrix} + \beta \begin{bmatrix} -i \\ 0 \end{bmatrix} = i\alpha|1\rangle - i\beta|0\rangle = -i\beta|0\rangle + i\alpha|1\rangle$
6. $Z|\psi\rangle = Z(\alpha|0\rangle + \beta|1\rangle) = \alpha Z|0\rangle + \beta Z|1\rangle = \alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \alpha|0\rangle - \beta|1\rangle$