Mahidol University *Wisdom of the Land*

Chapter 3

Image Enhancement in the Spatial Domain

Sharpening Spatial Filters

Blurred image
***f*(x,y)**

→

Sharper Image
***g*(x,y)**

Cause:

a) **Error while taking image**

b) **Either in error or as natural effect of a particular method of image acquisition.**

Differentiation:

a) **First-order derivative**

$$\frac{\partial}{\partial x} \quad \frac{\partial}{\partial y}$$

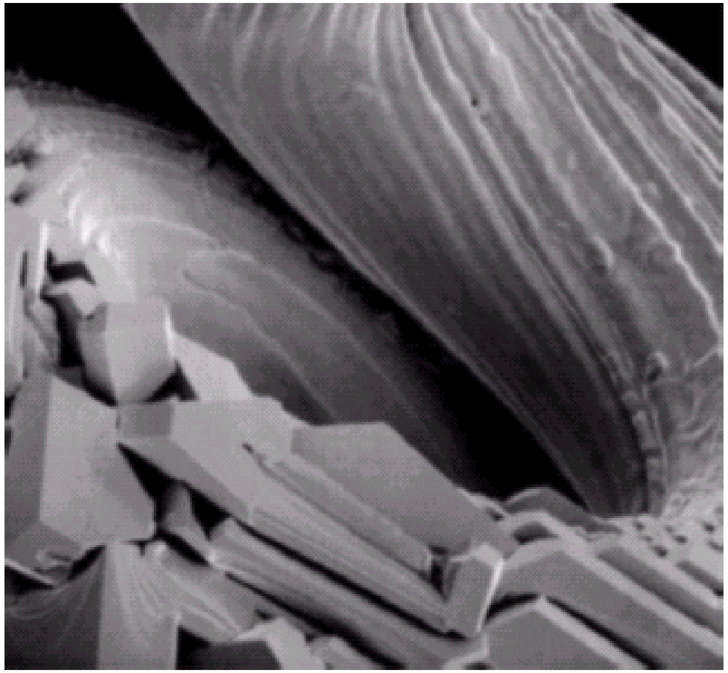
b) **Second-order derivative**

$$\frac{\partial^2}{\partial x^2} \quad \frac{\partial^2}{\partial y^2}$$

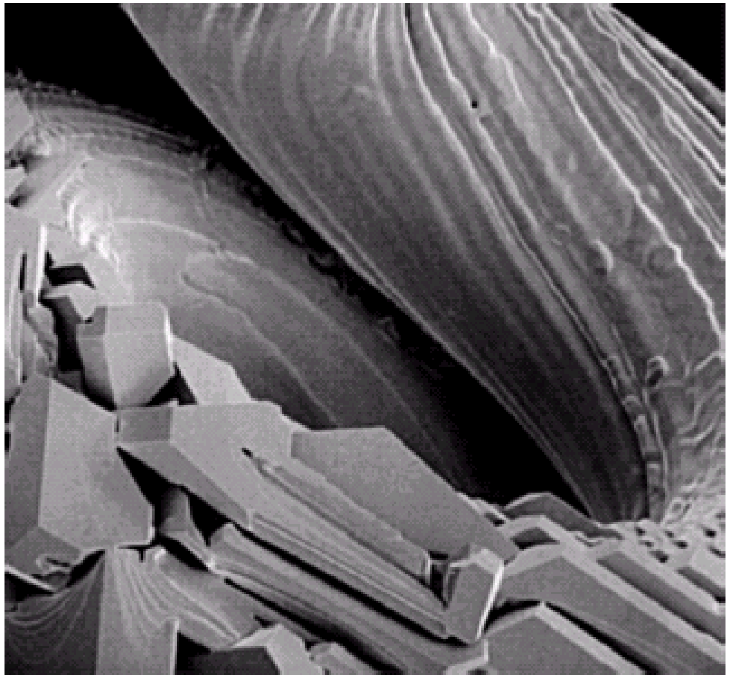
Sharpening Spatial Filters

Blurred image
***f*(x,y)**

Sharper Image
***g*(x,y)**



A given image



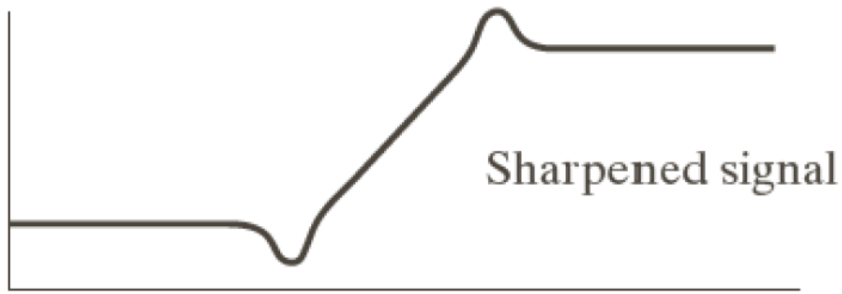
Result of a given image

Sharpening Spatial Filters

Blurred image
***f*(x,y)**

Sharper Image
***g*(x,y)**

A blurred signal



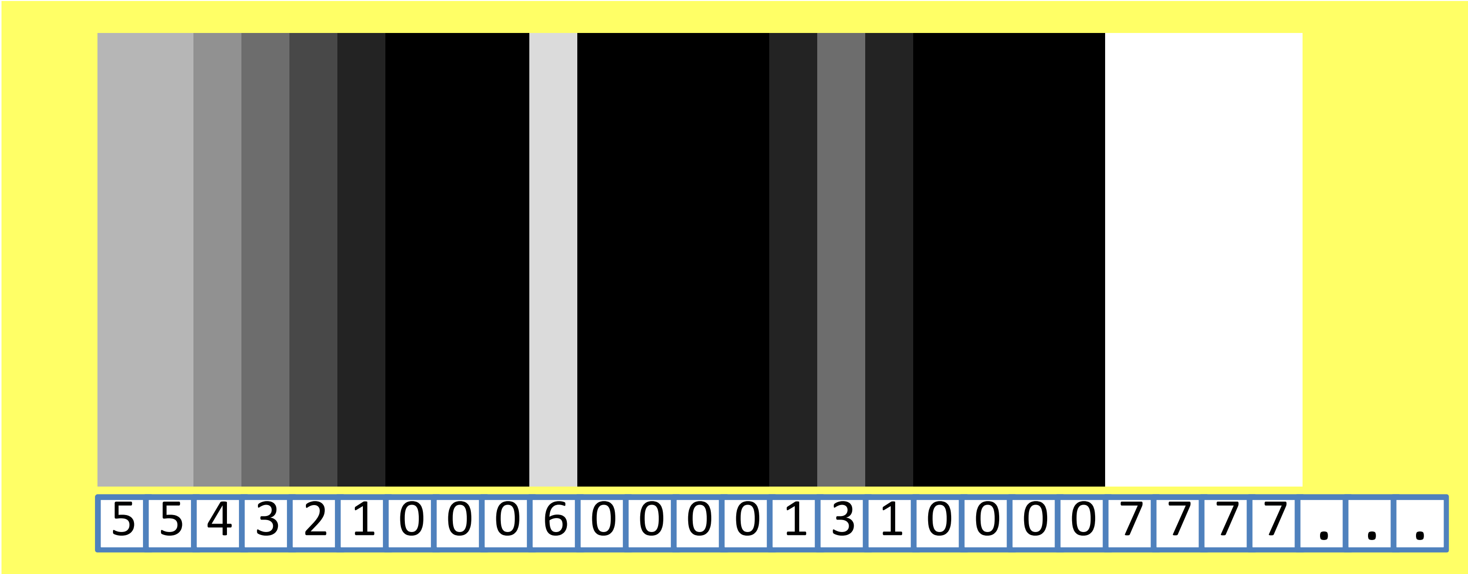
Result of a blurred signal

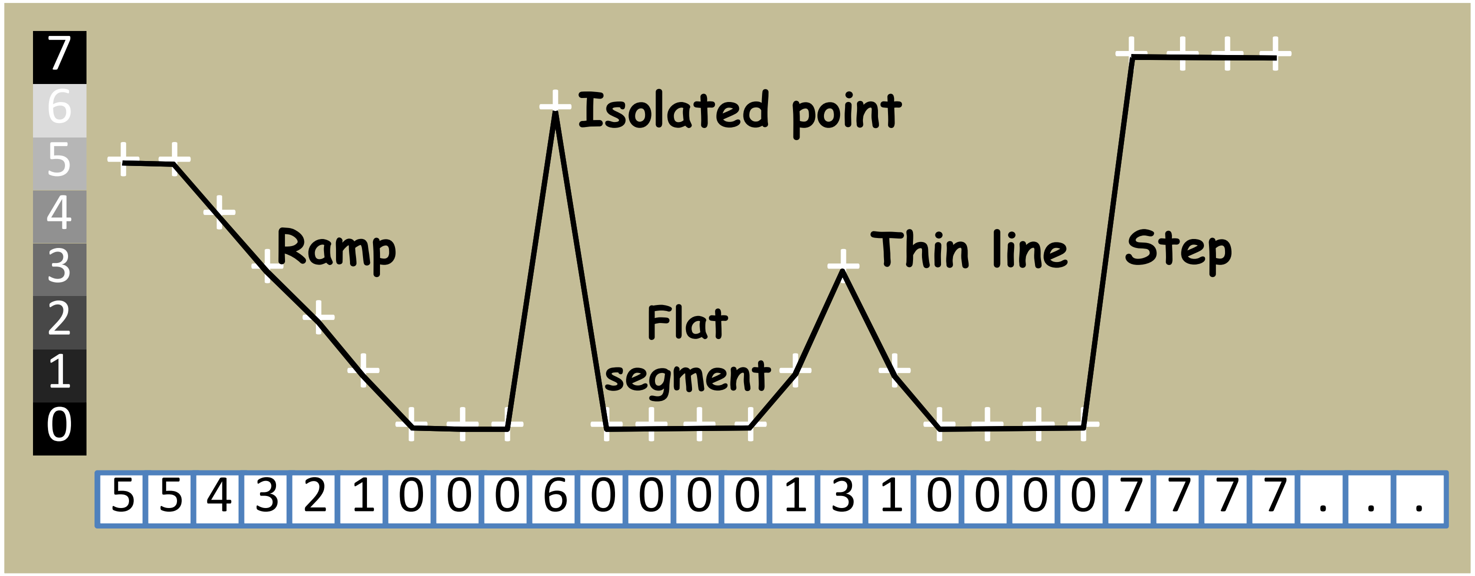
Sharpening Spatial Filters

- Sharpening is the operation to highlight fine details or enhance the details that has been blurred.
- Image blurring can be achieved using averaging filters, and hence sharpening can be achieved by operators that invert averaging operators.
- In mathematics, averaging is equivalent to the concept of integration, and differentiation inverts integration.
- Therefore the sharpening could be accomplished by differentiation.

averaging <--> integration
sharpening <--> differentiation

Sharpening Spatial Filters





Sharpening Spatial Filters

We are interested in the behavior of the derivatives

- in areas of constant gray level (flat segments)
- at the onset and end of discontinuities (step and ramp discontinuities)
- along gray-level ramps
  - These types of discontinuities can be used to model noise points, lines, and edges in an image.

- There are two main types of sharpening spatial filters :
  - Use of First Derivatives for Enhancement—The Gradient.
  - Use of Second Derivatives for Enhancement–The Laplacian.

First Derivatives-The Gradient

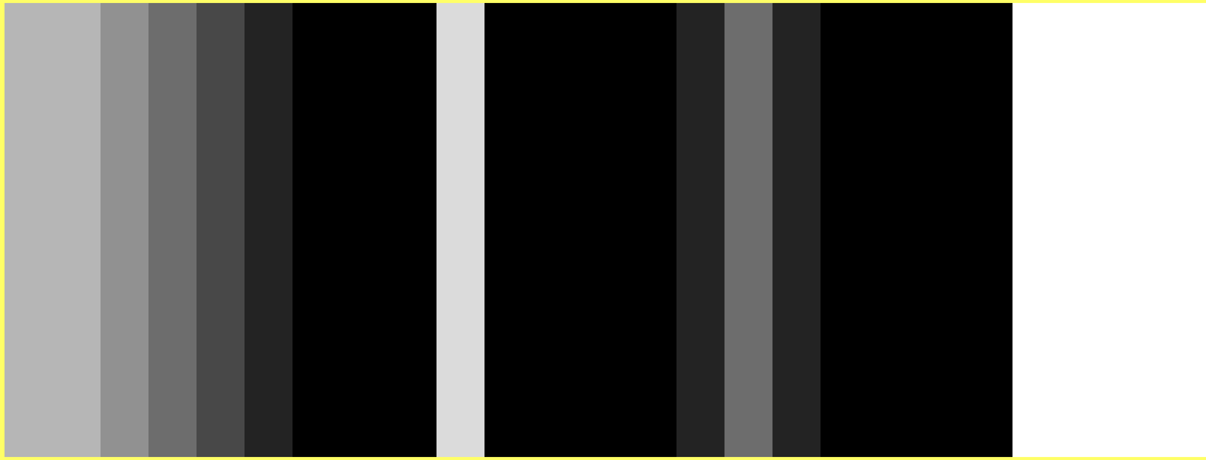
Use of First Derivatives for Enhancement—The Gradient

- First-order derivatives generally produce "thicker" edges in an image.

The first derivative must be :

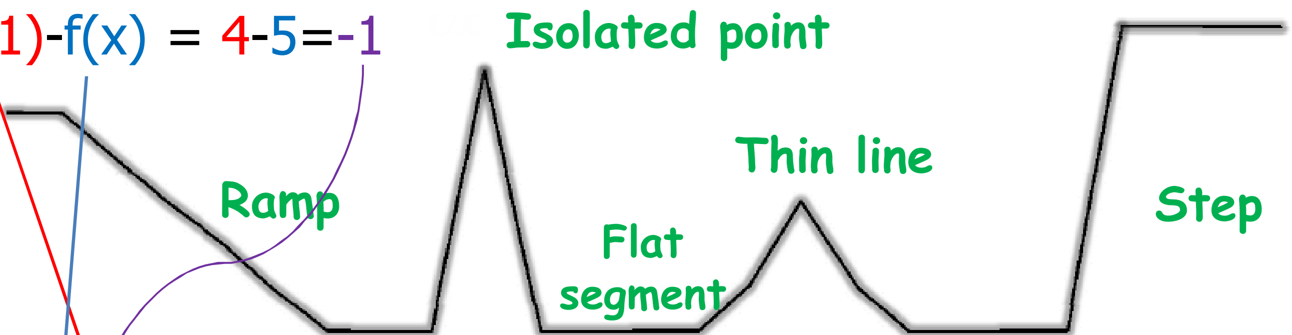
- Zero in areas of constant intensity;
- Non-zero at the onset and end of an intensity step or ramp;
- Non-zero along ramps of constant slope.

First Derivatives-The Gradient



5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7	.	.	.

1st $= f(x+1) - f(x) = 4 - 5 = -1$



5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7	.	.	.
	-1	-1	-1	-1	-1	0	0	6	-6	0	0	0	1	2	-2	-1	0	0	0	7	0	0	0		.	.	.

First Derivatives-The Gradient

- **First Derivatives**

$$\frac{\partial I}{\partial x} = I(x+1) - I(x)$$

- **Gradient vector**

$$\nabla I = \begin{bmatrix} I_x \\ I_y \end{bmatrix} = \begin{bmatrix} \frac{\partial I}{\partial x} \\ \\ \frac{\partial I}{\partial y} \end{bmatrix}$$

- **Magnitude of this vector is given by:**

$$|\nabla I| = mag(\nabla I) = \sqrt{I_x^2 + I_y^2} = \sqrt{\left(\frac{\partial I}{\partial x}\right)^2 + \left(\frac{\partial I}{\partial y}\right)^2}$$

- **Angle of change**

$$\theta = \tan^{-1}\left(\frac{I_y}{I_x}\right)$$

First Derivatives-The Gradient

- The derivative of a digital function is defined in terms of differences, where a first order derivative of a one dimensional function is :

$$G(x, y) = \nabla I(x, y)$$

- where : $$\nabla = \frac{\partial}{\partial x} + \frac{\partial}{\partial y}$$

$$G(x, y) = \sqrt{\left(\frac{\partial I}{\partial x}\right)^2 + \left(\frac{\partial I}{\partial y}\right)^2}$$

$$G(x, y) = \left|\frac{\partial I}{\partial x}\right| + \left|\frac{\partial I}{\partial y}\right| = I_x + I_y$$

First Derivatives-The Gradient

$$G(x,y) = \nabla F(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} H(s,t) f(x-s, y-t)$$

$$G(x,y) = I(x,y) + H(x,y)$$

$$G_R(x,y) = \left| I(x,y) * H_R(x,y) \right|$$

$$G_C(x,y) = \left| I(x,y) * H_c(x,y) \right|$$

$$\boxed{G_R(x,y) = \left| I(x,y+1) - I(x,y) \right|}$$

$$\boxed{G_C(x,y) = \left| I(x+1,y) - I(x,y) \right|}$$

$$\boxed{G(x,y) = G_R(x,y) + G_c(x,y)}$$

First Derivatives-The Gradient

- The Gradient can be implemented by these filter masks:

-1	-1	-1
0	0	0
1	1	1

-1	0	1
-1	0	1
-1	0	1

Prewitt

-1	-2	-1
0	0	0
1	2	1

-1	0	1
-2	0	2
-1	0	1

Sobel

First Derivatives-The Gradient

- The Gradient can be implemented by these filter masks:

1	0
0	-1

0	1
-1	0

Roberts

First Derivatives-The Gradient

How to use a 2x2 Roberts filter mask to process an image?

1	0
0	-1

0	1
-1	0

Roberts

0	0	0
0	1	0
0	0	-1

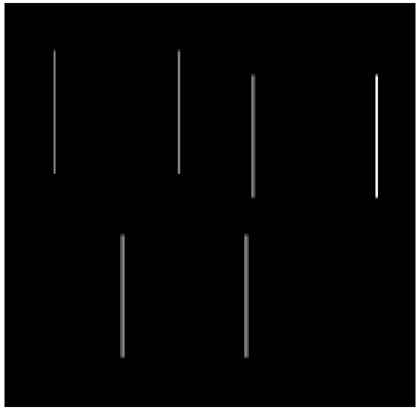
0	0	0
0	0	1
0	-1	0

3x3 Roberts

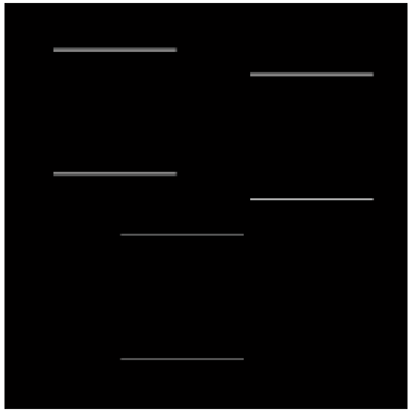
First Derivatives-The Gradient

$\mathrm{I}(x, y)$

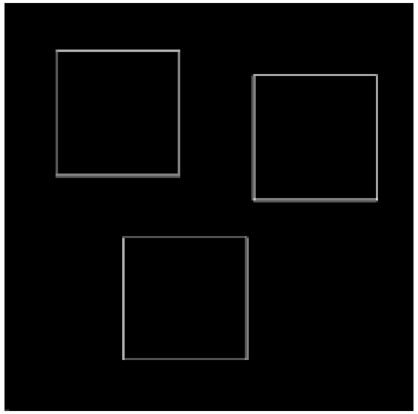
$G_C(x, y)$ Gradient column



$G_R(x, y)$ Gradient row



$G(x, y) = G_R(x, y) + G_C(x, y)$



Example : First Derivatives-The Gradient

Input image

2	3	2	1	3
1	1	1	2	2
2	3	2	1	3
2	3	2	1	3
1	1	1	2	2

$I(x, y)$

Prewitt Mask

-1	-1	-1
0	0	0
1	1	1

$H_R(x, y)$

-1	0	1
-1	0	1
-1	0	1

$H_C(x, y)$

Example : First Derivatives-The Gradient

Step 1 : The size of the image is extended by padding with zeros.

Zero padding

0	0	0	0	0	0	0
0	2	3	2	1	3	0
0	1	1	1	2	2	0
0	2	3	2	1	3	0
0	2	3	2	1	3	0
0	1	1	1	2	2	0
0	0	0	0	0	0	0

$I(x, y)$

Example : First Derivatives-The Gradient

Step 2 : evaluate $G_R(x,y) = \left| I(x,y) * H_R(x,y) \right|$

$$G_R(1,1) = \left| (-1) + (-1) \right| = 2$$

-1	-1	-1
0	0	0
1	1	1

$H_R(x,y)$

0	0	0	0	0	0	0
0	2	3	2	1	3	0
0	1	1	1	2	2	0
0	2	3	2	1	3	0
0	2	3	2	1	3	0
0	1	1	1	2	2	0
0	0	0	0	0	0	0

$I(x,y)$

2				

$G_R(x,y)$

Example : First Derivatives-The Gradient

Step 2 : evaluate $G_R(x,y) = \left| I(x,y) * H_R(x,y) \right|$

$$G_R(5,5) = \left| (1) + (3) \right| = 4$$

-1	-1	-1
0	0	0
1	1	1

$H_R(x,y)$

0	0	0	0	0	0	0
0	2	3	2	1	3	0
0	1	1	1	2	2	0
0	2	3	2	1	3	0
0	2	3	2	1	3	0
0	1	1	1	2	2	0
0	0	0	0	0	0	0

$I(x,y)$

2	3	4	5	4
0	0	0	0	0
3	4	2	1	0
3	4	2	1	0
5	7	6	6	4

$G_R(x,y)$

Example : First Derivatives-The Gradient

Step 2 : evaluate $G_C(x,y) = \left| I(x,y) * H_C(x,y) \right|$

-1	0	1
-1	0	1
-1	0	1

$H_C(x,y)$

$G_C(1,1) = \left|(-3)+(-1)\right| = 4$

0	0	0	0	0	0	0
0	2	3	2	1	3	0
0	1	1	1	2	2	0
0	2	3	2	1	3	0
0	2	3	2	1	3	0
0	1	1	1	2	2	0
0	0	0	0	0	0	0

$I(x,y)$

4				

$G_C(x,y)$

Example : First Derivatives-The Gradient

Step 2 : evaluate $G_C(x,y) = \left| I(x,y) * H_C(x,y) \right|$

-1	0	1
-1	0	1
-1	0	1

$H_C(x,y)$

$G_C(5,5) = \left|(1)+(2)\right| = 3$

0	0	0	0	0	0	0
0	2	3	2	1	3	0
0	1	1	1	2	2	0
0	2	3	2	1	3	0
0	2	3	2	1	3	0
0	1	1	1	2	2	0
0	0	0	0	0	0	0

$I(x,y)$

4	0	1	2	3
7	0	3	3	4
7	0	3	3	4
7	0	3	3	4
4	0	1	2	3

$G_C(x,y)$

Example : First Derivatives-The Gradient

Step 4 : evaluate $G(x,y) = G_R(x,y) + G_c(x,y)$

$G_R(x,y)$

2	3	4	5	4
0	0	0	0	0
3	4	2	1	0
3	4	2	1	0
5	7	6	6	4

$G_C(x,y)$

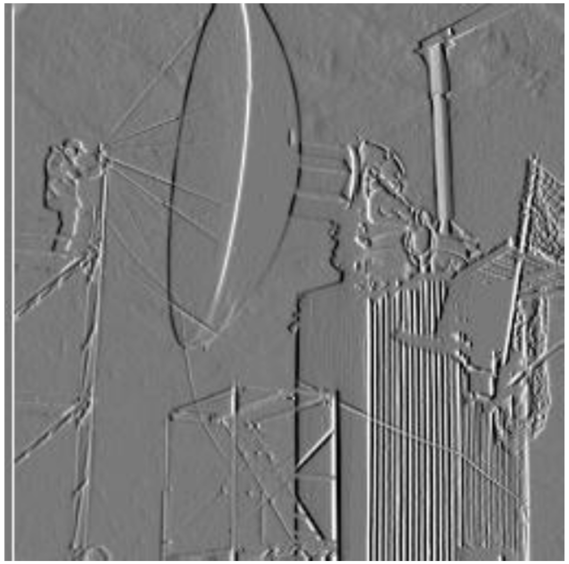
4	0	1	2	3
7	0	3	3	4
7	0	3	3	4
7	0	3	3	4
4	0	1	2	3

6	3	5	7	7
7	0	3	3	4
10	4	5	4	4
10	4	5	4	4
9	7	7	8	7

$G(x,y) = G_R(x,y) + G_c(x,y)$

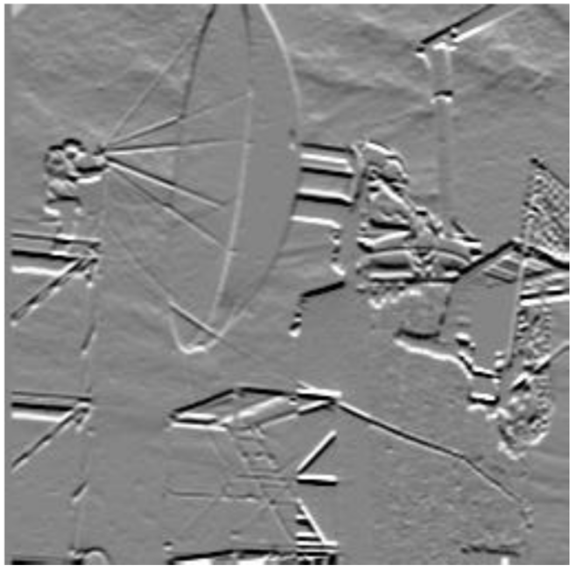
Example : First Derivatives-The Gradient

Input Image



Gradient Column

Gradient Row



Gradient Magnitude

Second Derivatives-The Laplacian

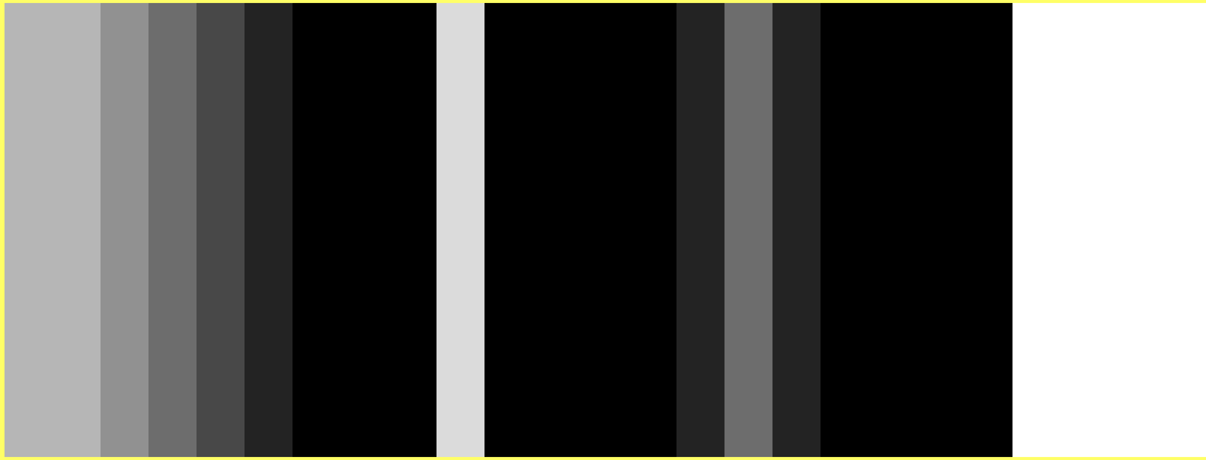
Use of Second Derivatives for Enhancement–The Laplacian

- Second-order derivatives produce a double response at step changes in gray-level.
- Second-order derivatives have a stronger response to finer detail, such as thin lines and isolated points.

The second derivative must be :

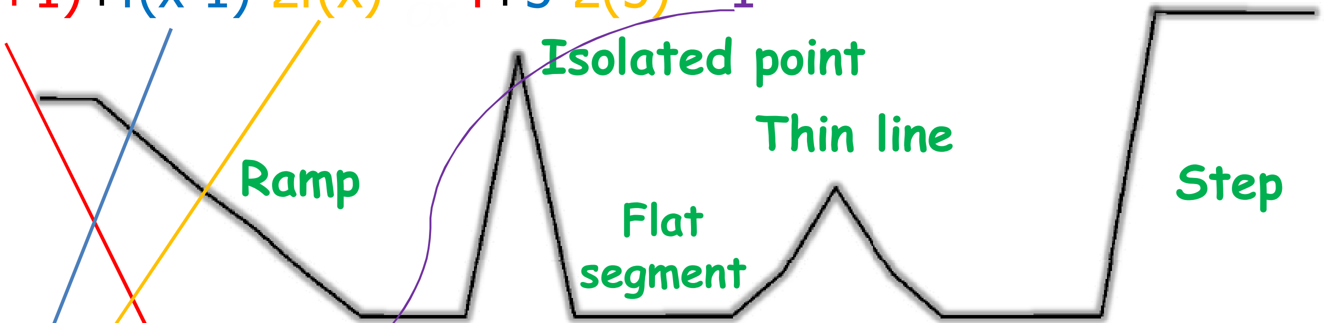
- Zero in areas of constant intensity;
- Non-zero at the onset and end of an intensity step or ramp;
- Zero along ramps of constant slope

Second Derivatives-The Laplacian



5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7	.	.	.

2nd $= f(x+1) + f(x-1) - 2f(x) = 4 + 5 - 2(5) = -1$



5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7	.	.	.
	-1	0	0	0	0	1	0	6	-12	6	0	0	1	1	-4	1	1	0	0	7	-7	0	0		.	.	.

Second Derivatives-The Laplacian

- **First Derivatives**

$$\frac{\partial I}{\partial x} = I(x+1) - I(x)$$

- **Second Derivatives**

$$\frac{\partial^2 I}{\partial^2 x} = I(x+1) - I(x) - (I(x) - I(x-1))$$

$$= I(x+1) + I(x-1) - 2I(x)$$

- **Laplacian vector**

$$\nabla^2 I = \begin{bmatrix} I_x \\ I_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial^2 I}{\partial x^2} \\ \\ \dfrac{\partial^2 I}{\partial y^2} \end{bmatrix}$$

Second Derivatives-The Laplacian

- The second derivative of a digital function is defined in terms of differences, where a second order derivative of a one dimensional function is :

$$G(x,y) = \nabla^2 I(x,y) = \frac{\partial^2 I(x,y)}{\partial x^2} + \frac{\partial^2 I(x,y)}{\partial y^2}$$

$$\frac{\partial^2 I(x,y)}{\partial x^2} = \frac{\partial I(x+1,y) - I(x,y)}{\partial x}$$

$$\frac{\partial^2 I(x,y)}{\partial x^2} = I(x+2,y) - I(x+1,y) + I(x+1,y) + I(x,y)$$

$$\frac{\partial^2 I(x,y)}{\partial x^2} = I(x+2,y) - 2I(x+1,y) + I(x,y)$$

Second Derivatives-The Laplacian

$$\frac{\partial^2 I(x,y)}{\partial x^2} = I(x+1,y) + I(x-1,y) - 2I(x,y)$$

1	-2	1

$$\frac{\partial^2 I(x,y)}{\partial y^2} = I(x,y+1) + I(x,y-1) - 2I(x,y)$$

1
-2
1

$$G(x,y) = \frac{\partial^2 I(x,y)}{\partial x^2} + \frac{\partial^2 I(x,y)}{\partial y^2}$$

$$= I(x+1,y) + I(x-1,y) + I(x,y+1) + I(x,y-1) - 4I(x,y)$$

0	1	0
1	-4	1
0	1	0

Second Derivatives-The Laplacian

- The Laplacian can be implemented by these filter masks:

0	1	0
1	-4	1
0	1	0

→ Laplacian filter invariant to 90◦ rotations.

1	1	1
1	-8	1
1	1	1

→ Laplacian filter invariant to 45◦ rotations.

Example : Second Derivatives-The Laplacian

Input image

16	81	34	120	255
120	255	29	50	105
220	45	57	68	18
25	14	12	21	8
9	55	6	4	36

$I(x, y)$

Mask

0	1	0
1	-4	1
0	1	0

$H(x, y)$

Example : Second Derivatives-The Laplacian

Step 1 : The size of the image is extended by padding with zeros.

$I(x, y)$

0	0	0	0	0	0	0
0	16	81	34	120	255	0
0	120	255	29	50	105	0
0	220	45	57	68	18	0
0	25	14	12	21	8	0
0	9	55	6	4	36	0
0	0	0	0	0	0	0

Zero padding

Example : Second Derivatives-The Laplacian

Step 2 : evaluate $G(x,y) = \left| I(x,y) * H(x,y) \right|$

$$G(1,1) = \left| (0\times0)+(0\times1)+(0\times0)+(0\times1)+(16\times(-4))+(81\times1)+(0\times0)+(120\times1)+(255\times0) \right|$$
$$= 137$$

0	1	0
1	-4	1
0	1	0

$H(x,y)$

0	0	0	0	0	0	0
0	16	81	34	120	255	0
0	120	255	29	50	105	0
0	220	45	57	68	18	0
0	25	14	12	21	8	0
0	9	55	6	4	36	0
0	0	0	0	0	0	0

$I(x,y)$

137				

$G(x,y)$

Example : Second Derivatives-The Laplacian

Step 3 : evaluate $G(x,y) = \left| I(x,y) * H(x,y) \right|$

0	1	0
1	-4	1
0	1	0

$H(x,y)$

$$G(5,5) = \left| (21\times0)+(8\times1)+(0\times0)+(4\times1)+(36\times(-4))+(0\times1)+(0\times0)+(0\times1)+(0\times0) \right|$$
$$=132$$

0	0	0	0	0	0	0
0	16	81	34	120	255	0
0	120	255	29	50	105	0
0	220	45	57	68	18	0
0	25	14	12	21	8	0
0	9	55	6	4	36	0
0	0	0	0	0	0	0

$I(x,y)$

137	19	94	141	795
11	745	280	122	97
690	366	74	126	109
143	81	50	8	43
44	191	47	47	132

$G(x,y)$

Example : Second Derivatives-The Laplacian

Input image

16	81	34	120	255
120	255	29	50	105
220	45	57	68	18
25	14	12	21	8
9	55	6	4	36

$$I(x,y)$$

Output image

137	19	94	141	795
11	745	280	122	97
690	366	74	126	109
143	81	50	8	43
44	191	47	47	132

$$G(x,y) = \left| I(x,y) * H(x,y) \right|$$

Example : Second Derivatives-The Laplacian

Input Image

Laplacian Filtered Image

Laplacian Image Enhancement

- The Laplacian operator highlights gray level discontinuities and de-emphasizes the slowly varying gray-levels.

- Depending on the choice of the Laplacian coefficients the following criteria is used for enhancement:

$$G(x,y) = I(x,y) \pm \nabla^2 I$$

$$G(x,y) = I(x,y) - \nabla^2 I$$

If the center coefficient of the Laplacian mask is negative

$$G(x,y) = I(x,y) + \nabla^2 I$$

If the center coefficient of the Laplacian mask is positive

Laplacian Image Enhancement

- The Laplacian can be implemented by these filter masks:

1	1	1
1	-8	1
1	1	1

0	1	0
1	-4	1
0	1	0

The center of the Laplacian mask is negative.

or

-1	-1	-1
-1	8	-1
-1	-1	-1

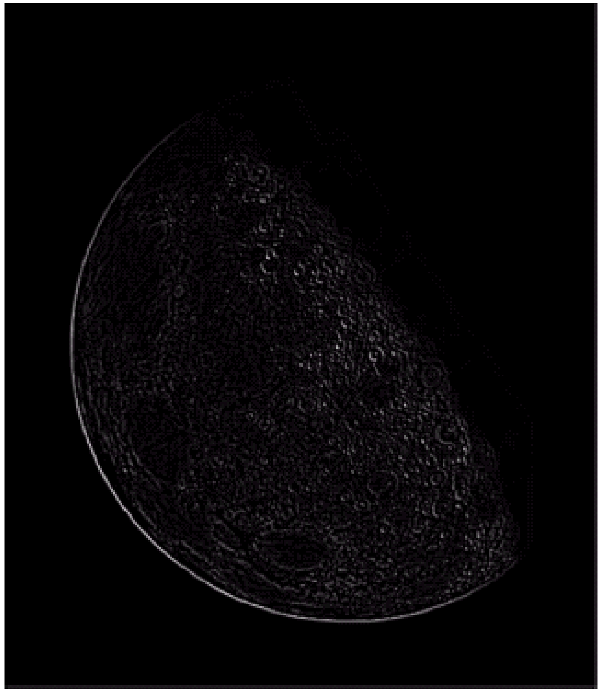
0	-1	0
-1	4	-1
0	-1	0

The center of the Laplacian mask is positive.

Laplacian Image Enhancement

- Applying the Laplacian to an image we get a new image that highlights edges and other discontinuities.

Input Image

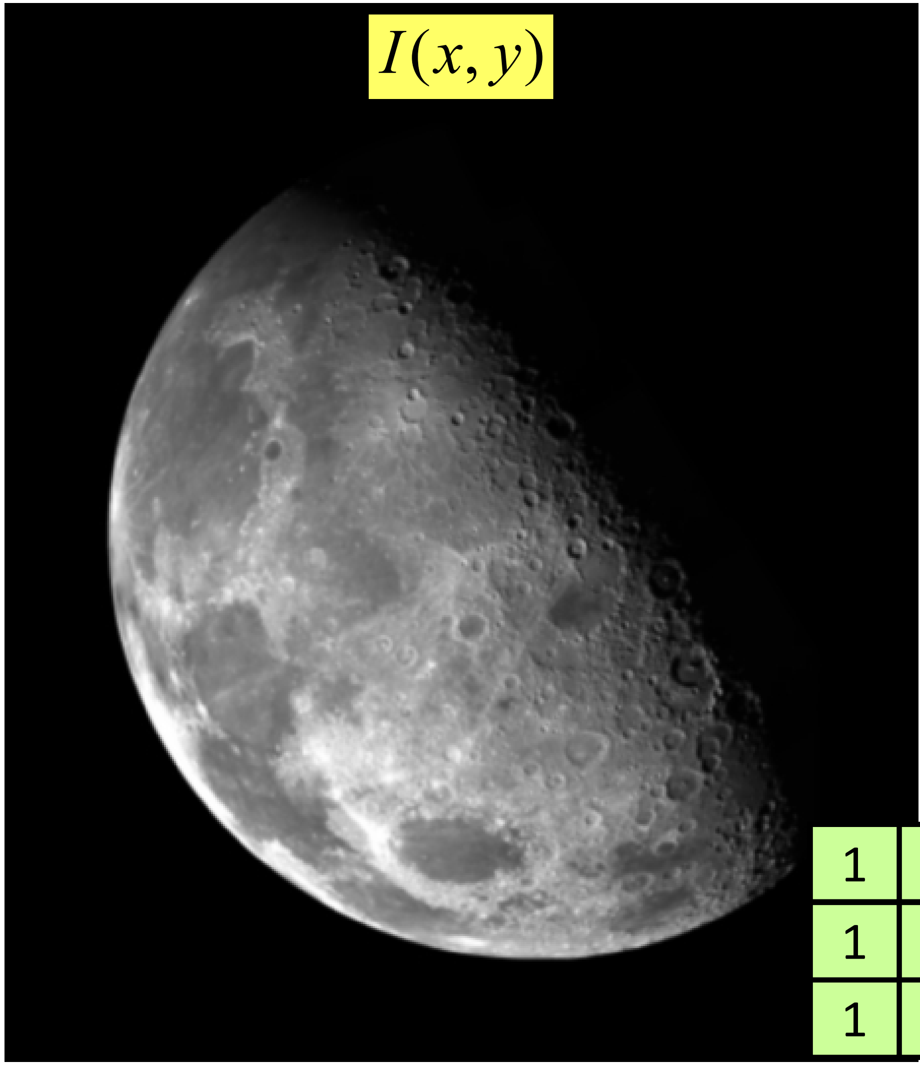


Laplacian Filtered Image

Sharpened Image

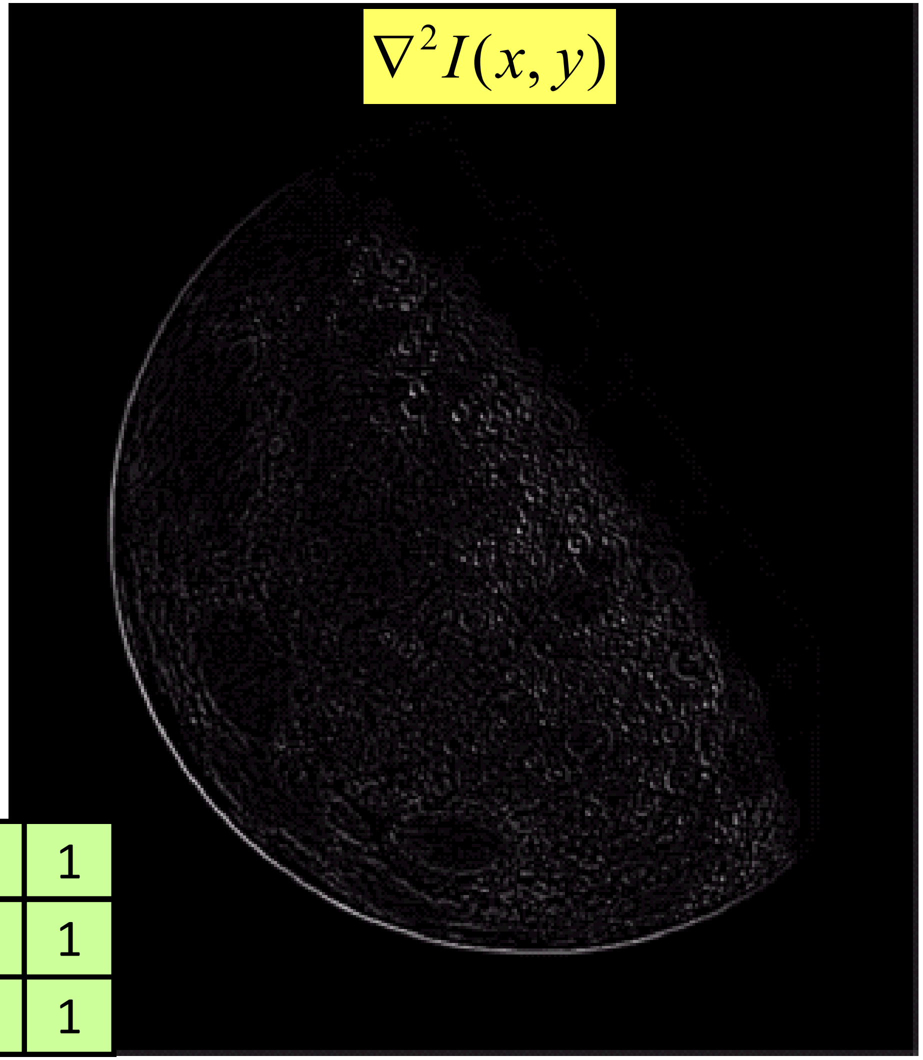
- In the final sharpened image edges and fine detail are much more obvious.

Example : Laplacian Image Enhancement



Input Image

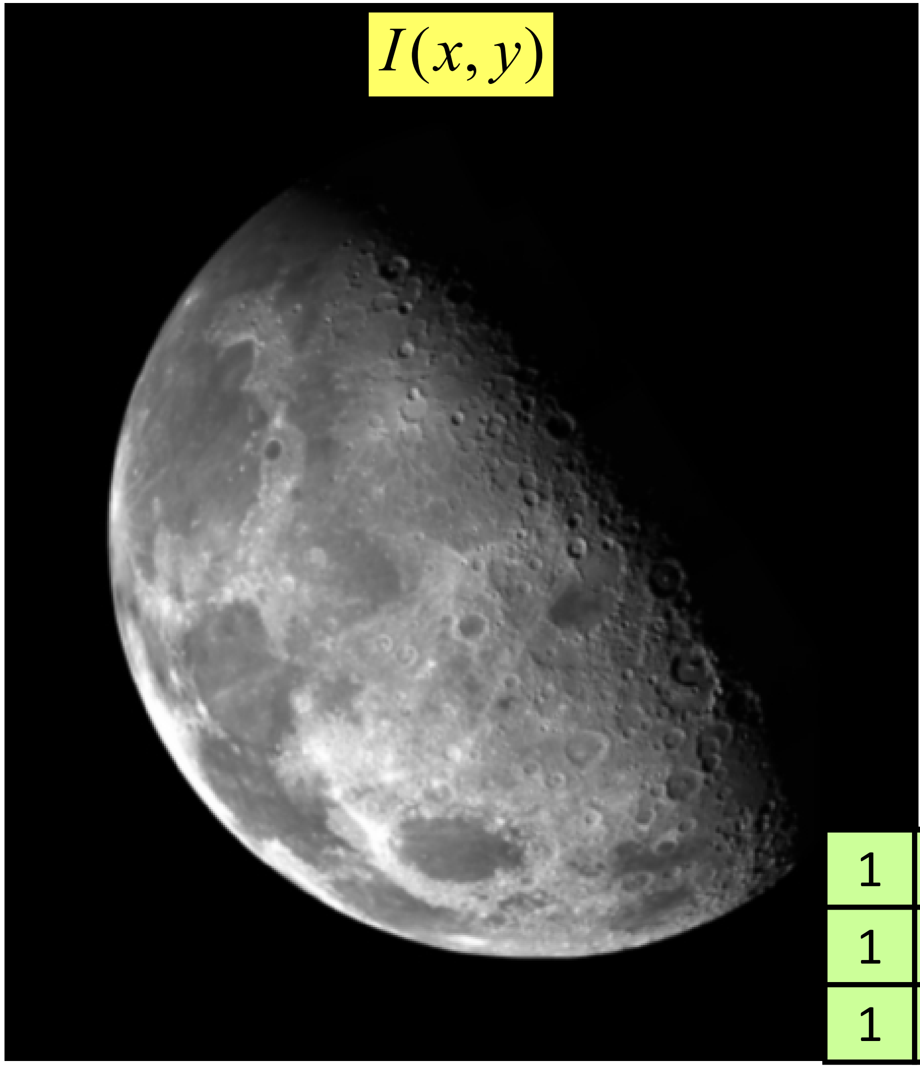
1	1	1
1	-8	1
1	1	1



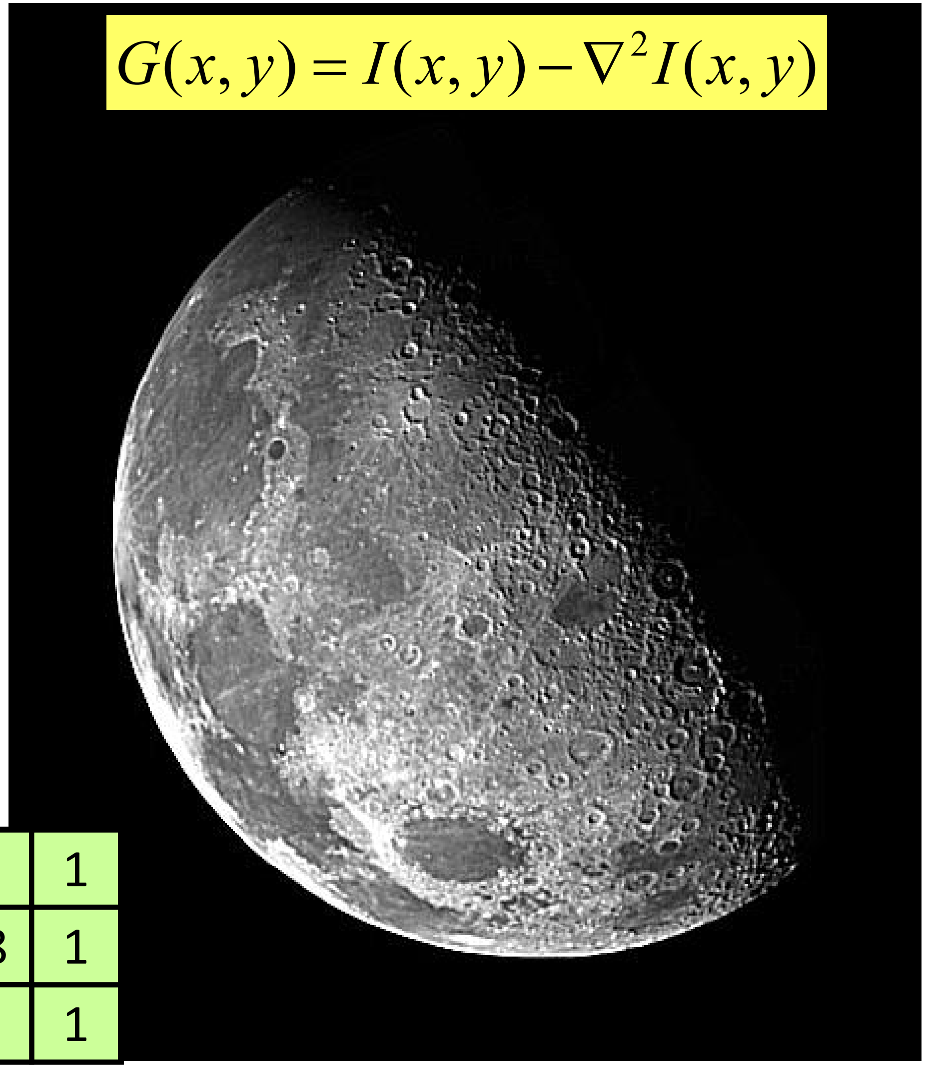
Laplacian Filtered Image

Example : Laplacian Image Enhancement

$I(x,y)$



$G(x,y) = I(x,y) - \nabla^2 I(x,y)$



1	1	1
1	-8	1
1	1	1

Input Image

Sharpened Image

Simplified Image Enhancement

- The entire enhancement can be combined into a single filtering operation

$$H = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\begin{aligned} G(x,y) &= I(x,y) - \nabla^2 I(x,y) \\ &= I(x,y) - H(x,y) * I(x,y) \end{aligned}$$

$$G(x,y) = I(x,y) - \begin{bmatrix} I(x-1,y) + I(x,y-1) - 4I(x,y) \\ + I(x,y+1) + I(x+1,y) \end{bmatrix}$$

$$= -\begin{bmatrix} I(x-1,y) + I(x,y-1) - 5I(x,y) \\ + I(x,y+1) + I(x+1,y) \end{bmatrix}$$

$$H = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

Simplified Image Enhancement

$$H=\begin{bmatrix}0 & 1 & 0\\ 1 & -4 & 1\\ 0 & 1 & 0\end{bmatrix} \quad H=\begin{bmatrix}1 & 1 & 1\\ 1 & -8 & 1\\ 1 & 1 & 1\end{bmatrix}$$

$$H=\begin{bmatrix}0 & -1 & 0\\ -1 & 4 & -1\\ 0 & -1 & 0\end{bmatrix} \quad H=\begin{bmatrix}-1 & -1 & -1\\ -1 & 8 & -1\\ -1 & -1 & -1\end{bmatrix}$$

$$G(x,y)=I(x,y)-\nabla^2 I$$

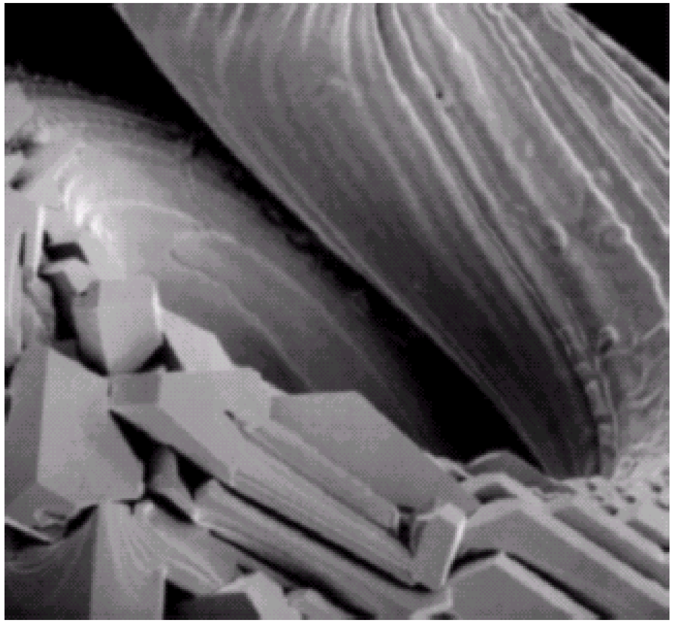
$$G(x,y)=I(x,y)+\nabla^2 I$$

$$H=\begin{bmatrix}0 & -1 & 0\\ -1 & 5 & -1\\ 0 & -1 & 0\end{bmatrix} \quad H=\begin{bmatrix}-1 & -1 & -1\\ -1 & 9 & -1\\ -1 & -1 & -1\end{bmatrix}$$

$$H=\begin{bmatrix}0 & 1 & 0\\ 1 & -5 & 1\\ 0 & 1 & 0\end{bmatrix} \quad H=\begin{bmatrix}1 & 1 & 1\\ 1 & -9 & 1\\ 1 & 1 & 1\end{bmatrix}$$

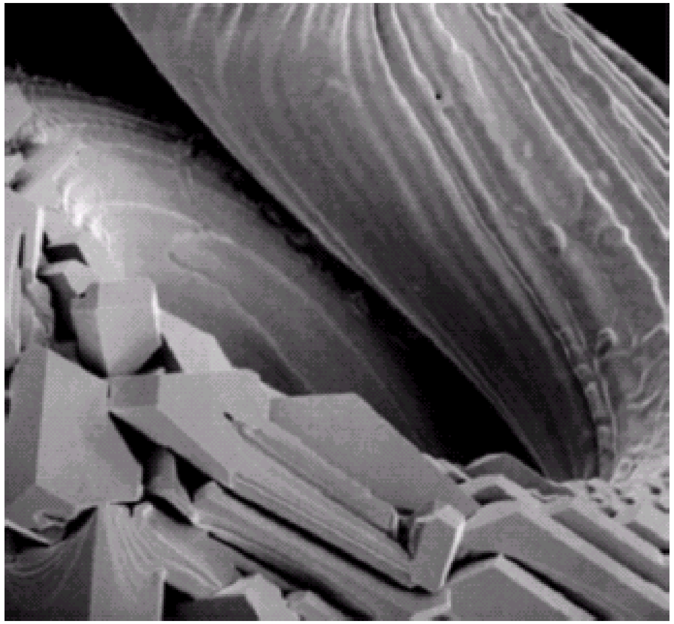
Example : Simplified Image Enhancement

- This gives us a new filter which does the whole job for us in one step.



Input Image

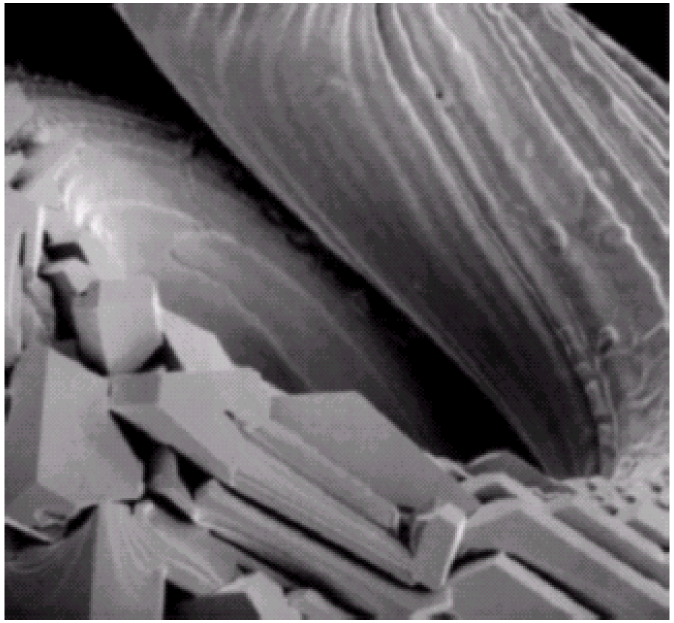
0	-1	0
-1	5	-1
0	-1	0



Sharpened Image

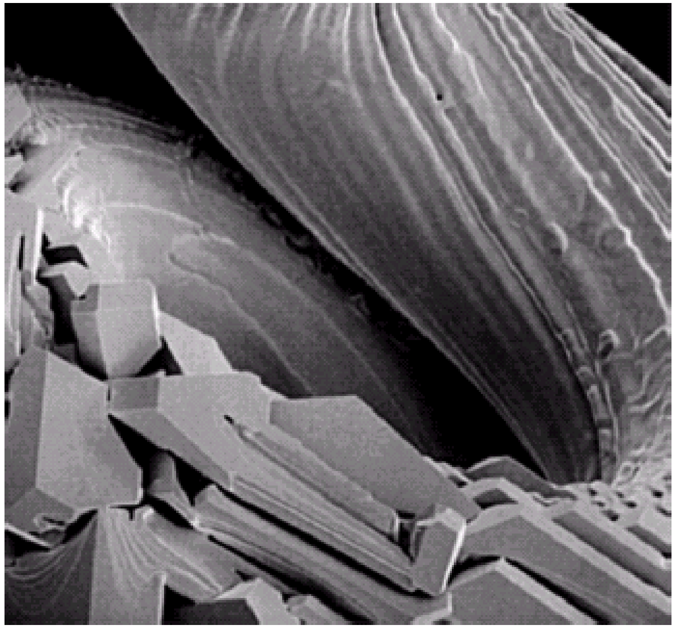
Example : Simplified Image Enhancement

- There are lots of slightly different versions of the Laplacian that can be used:



Input Image

-1	-1	-1
-1	9	-1
-1	-1	-1



Sharpened Image

Thanks for your attention