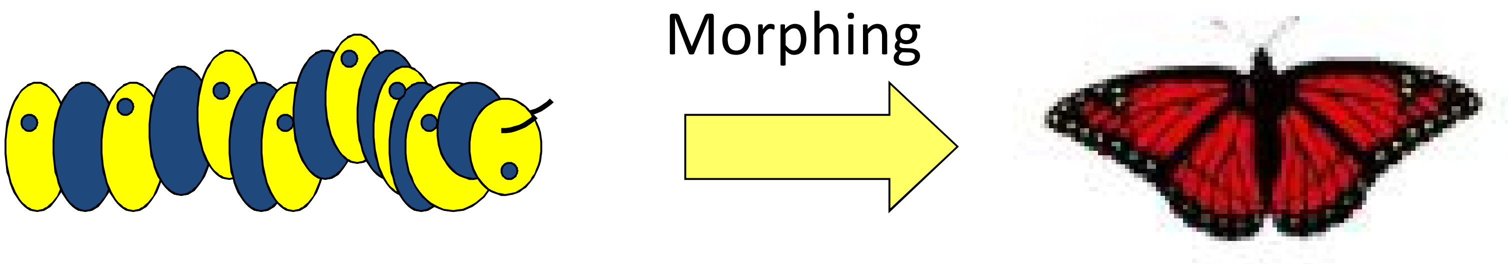
Mahidol University *Wisdom of the Land*

Chapter 10

Morphological Image Processing

Morphological Image Processing

- The word morphology refers to the scientific branch that deals the forms and structures of animals/plants.
- Morphology in image processing is a tool for extracting image components that are useful in the representation and description of region shape, such as boundaries and skeletons.
- Furthermore, the morphological operations can be used for filtering, thinning and pruning.
- The language of the Morphology comes from the set theory, where image objects can be represented by sets.



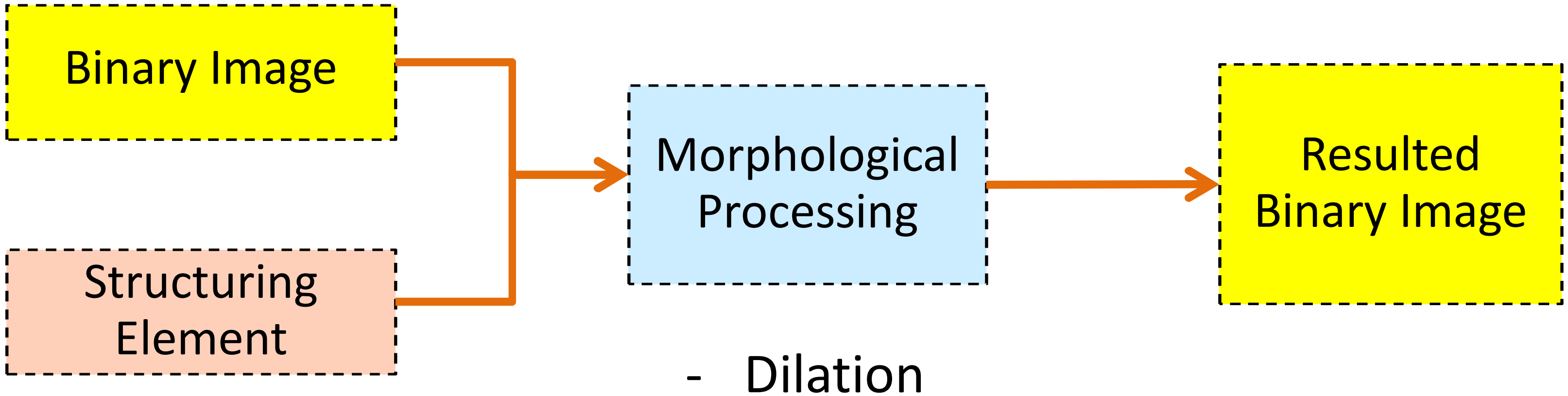
Morphological Image Processing

- Pre-processing used to prepare binary (thresholded) images for object segmentation/recognition.
- Extracting image components that are useful in the representation and description of region shape.

Image after segmentation

Image after segmentation and morphological processing

Basic Elements

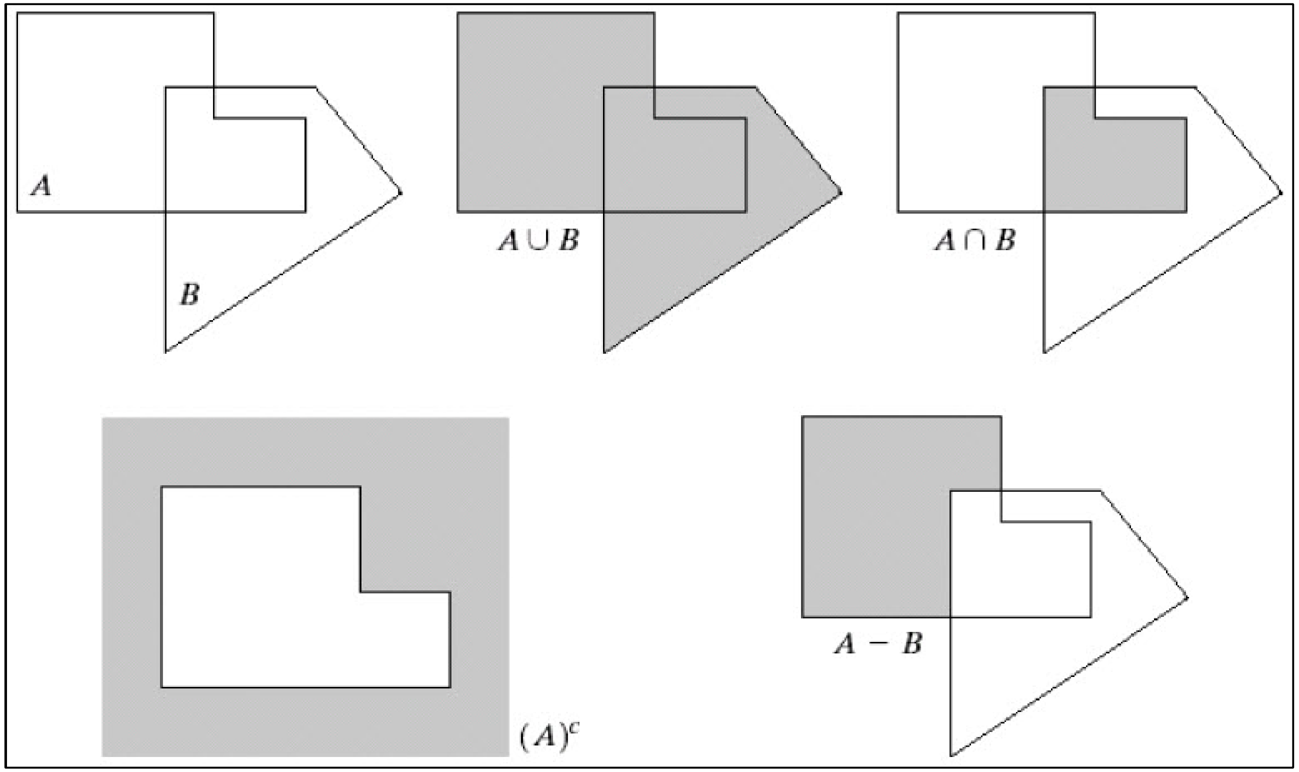


- Dilation
- Erosion
- Opening
- Closing

- A shape mask with any shapes or sizes
  - Square (Box)
  - Circle (Disk)
  - Cross
  - Hexagon
  - Any shape

Set Theory Fundamentals

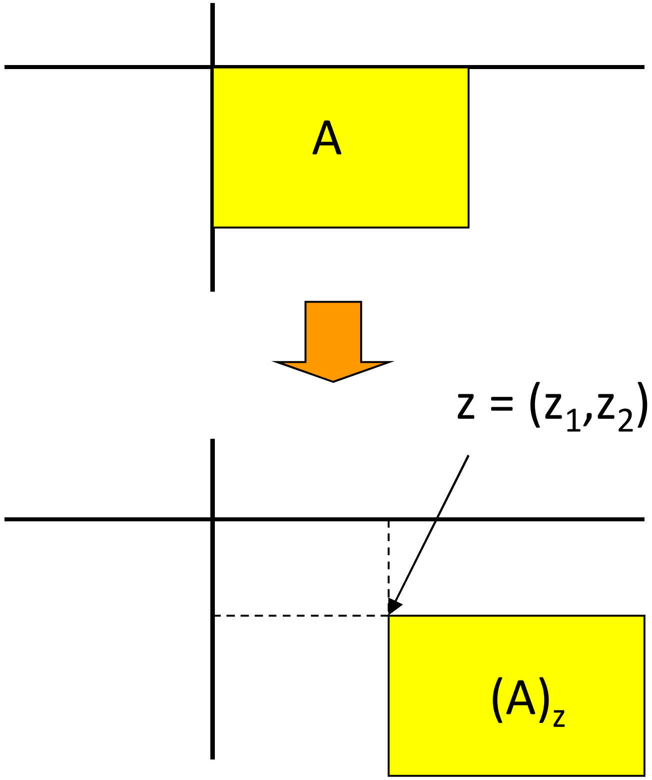
- Concept of a set in binary image morphology : Each set may represent one object.
- Each pixel (x, y) has its status : belong to a set or not belong to a set.
- Given 2 sets A and B



Set Theory Fundamentals

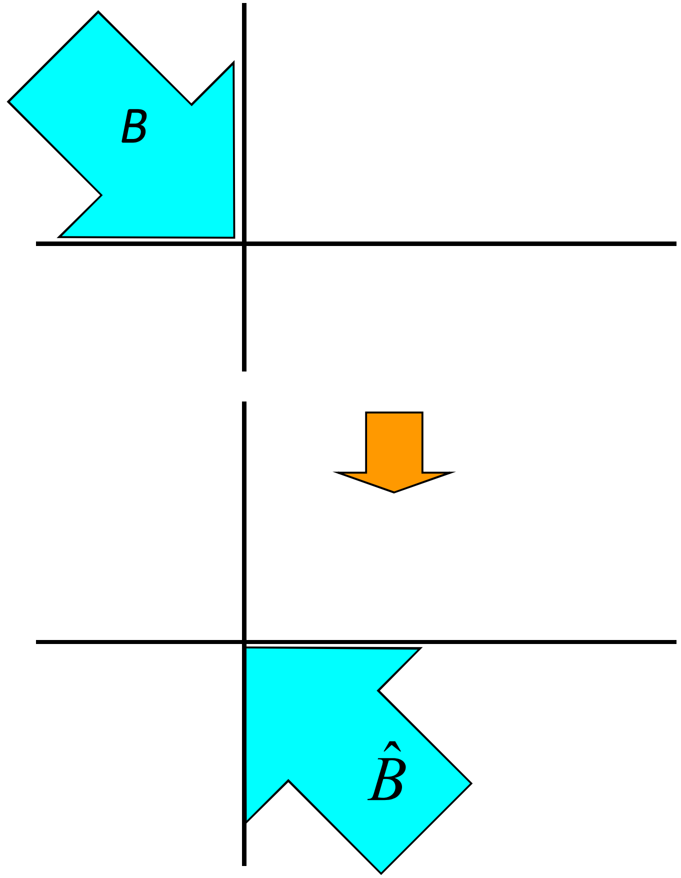
Translation

$$(A)_z = \{c \mid c = a + z, \quad \text{for } a \in A\}$$



Reflection

$$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$$



Logic Operations Involving Binary Images

- The three basic logical operations

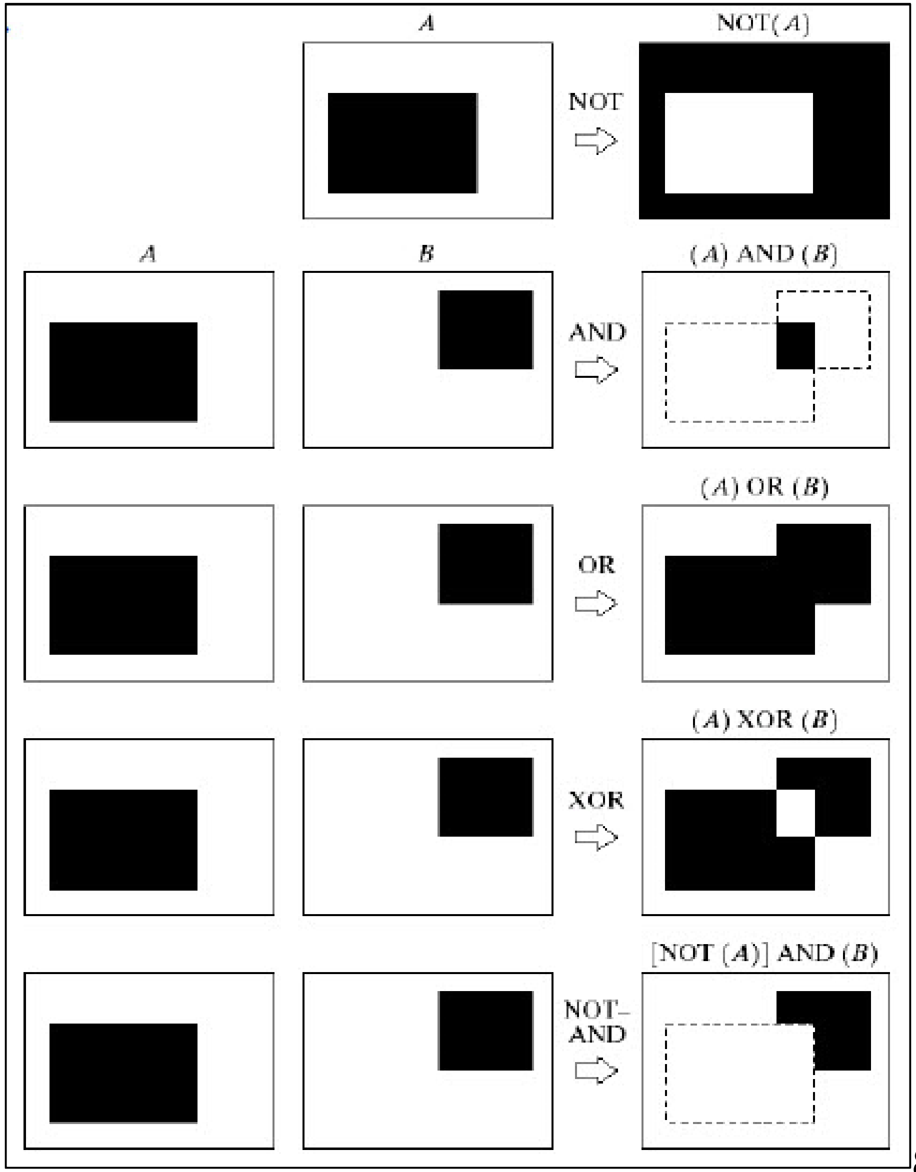
p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

- Which are defined and illustrated in the discussion that follows.

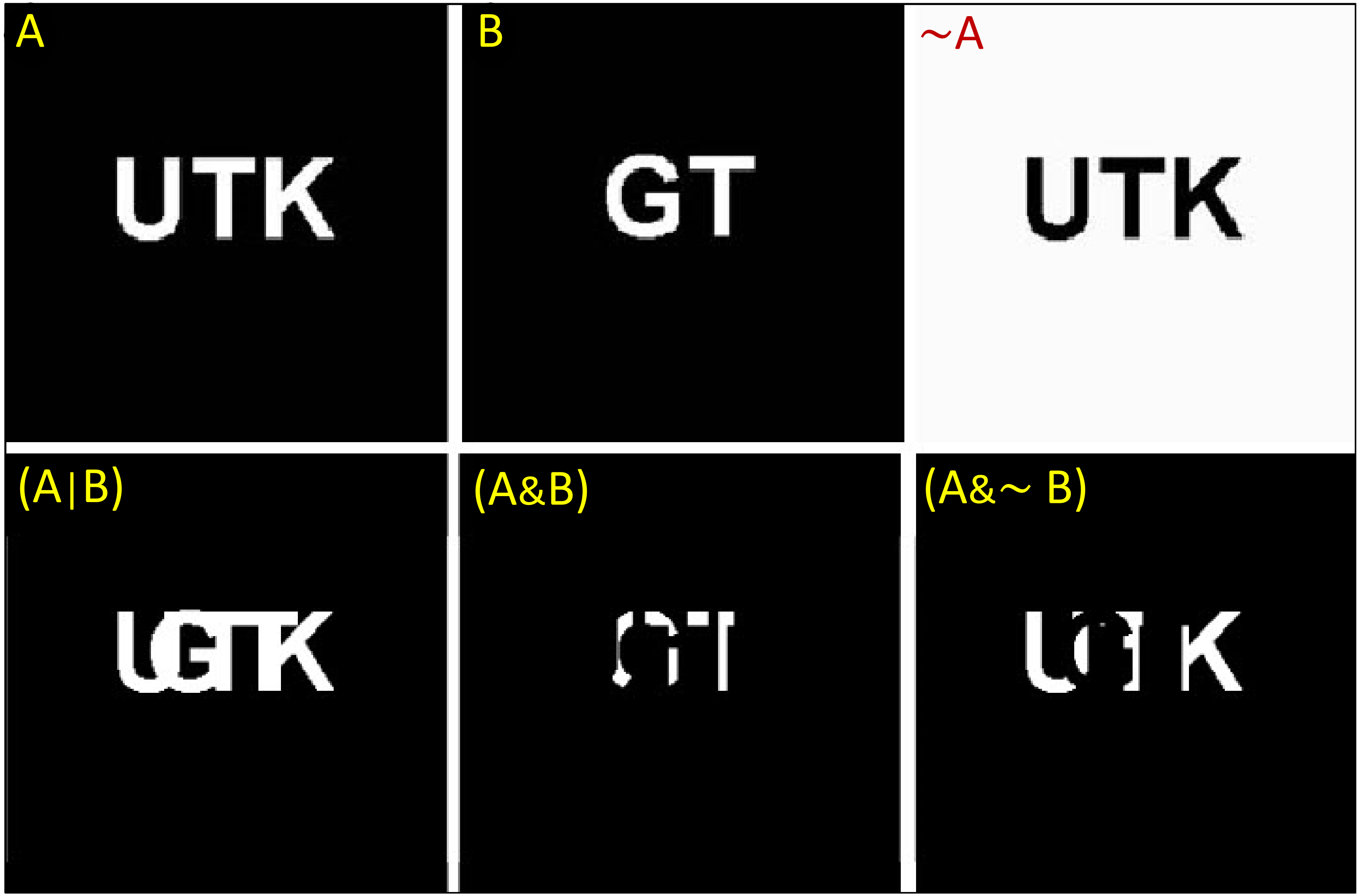
Set Operation	MATLAB Expression	Name
$A \cap B$	A & B	AND
$A \cup B$	A | B	OR
A^c	~A	NOT
$A - B$	A & ~B	DIFFERENCE

Example : Logic Operations

- Given 1-bit binary images, A and B, the basic logical operations are illustrated :
- Note that the black indicates binary 1 and white indicates binary 0 here.
- Logical Operations : For binary images only.



Example : Logic Operations



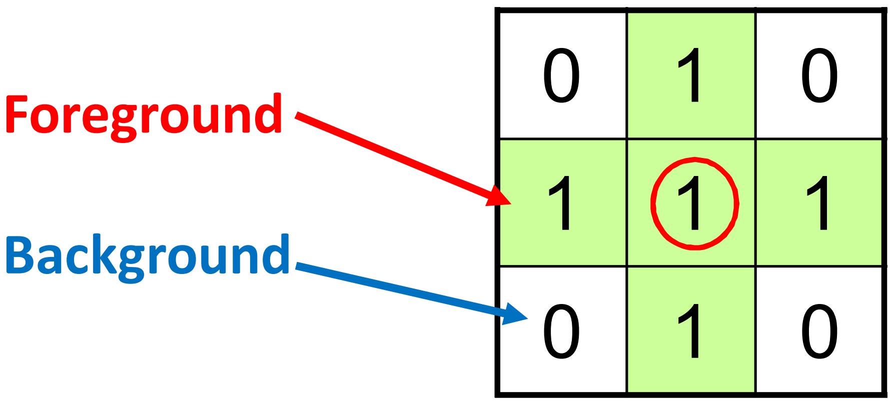
Structuring Element (SE)

- A morphing operation is based on the use of a filter-like binary pattern called the structuring element of the operation.
- Structuring Elements (SEs) : small sets of sub-images used to probe an analyzed image for properties of interest.
- The structuring element is sometimes called the kernel, but we reserve that term for the similar objects used in convolutions.
- Structuring Element is represented by a matrix of 0s and 1s; for simplicity, the zero entries are often omitted.
- Structuring elements can be any size and make any shape.
- However, for simplicity we will use rectangular structuring elements with their origin at the middle pixel.

Structuring Element (SE)

- White pixels
  - Usually represent foreground regions (binary representation ‘1’)
- Black pixels
  - Represent background (binary representation ‘0’)

- The origins of SEs are marked by a circle border.



Structuring Element

Structuring Element (SE)

- Structuring elements have varying shapes and sizes.

Square (Box)

1	1	1
1	1	1
1	1	1

Direct Line (Box)

1
1
1

Circle (Disk)

0	1	0
1	1	1
0	1	0

Circle (Disk)

0	0	1	0	0
0	1	1	1	0
1	1	1	1	1
0	1	1	1	0
0	0	1	0	0

Dilation and Erosion Operations

- Two fundamental operations used in morphological image processing.
- Almost all morphological algorithms depend on these two operations.
  - Dilation : adds pixels to the boundaries of objects in an image.
  - Erosion : removes pixels on object boundaries.
- The number of pixels added or removed from the objects in an image depends on the size and shape of the structuring element used to process the image.

Dilation Operations

- Dilation : to gradually enlarge the boundaries of regions of foreground pixels.
- Given A and B sets in Z^2, the dilation of A by B, is defined by :

$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \phi\}$$

- The dilation of A and B is a set of all displacements, z , such that B and A overlap by at least one element. The definition can also be written as :

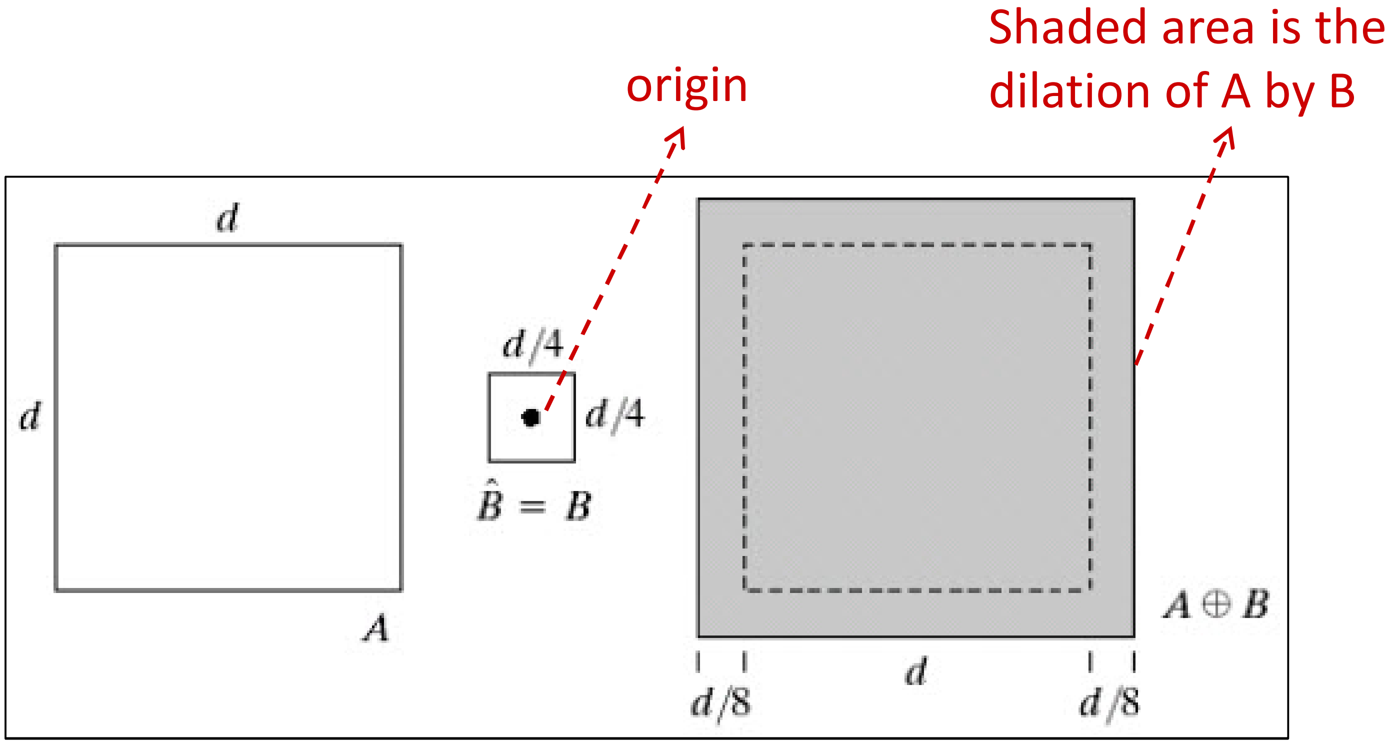
$$A \oplus B = \{z \mid [(\hat{B})_z \cap A] \subseteq A\}$$

- Note : Set B is referred to as the structuring element and used in dilation as well as in other morphological operations.

Dilation Operations

- Dilation : “grows” or “thickens” an object in a binary image.
- Extent of thickening controlled by a structuring element.
- Dilation enlarges foreground, shrinks background.
- Dilation expands an image.
- While holes within those regions become smaller.

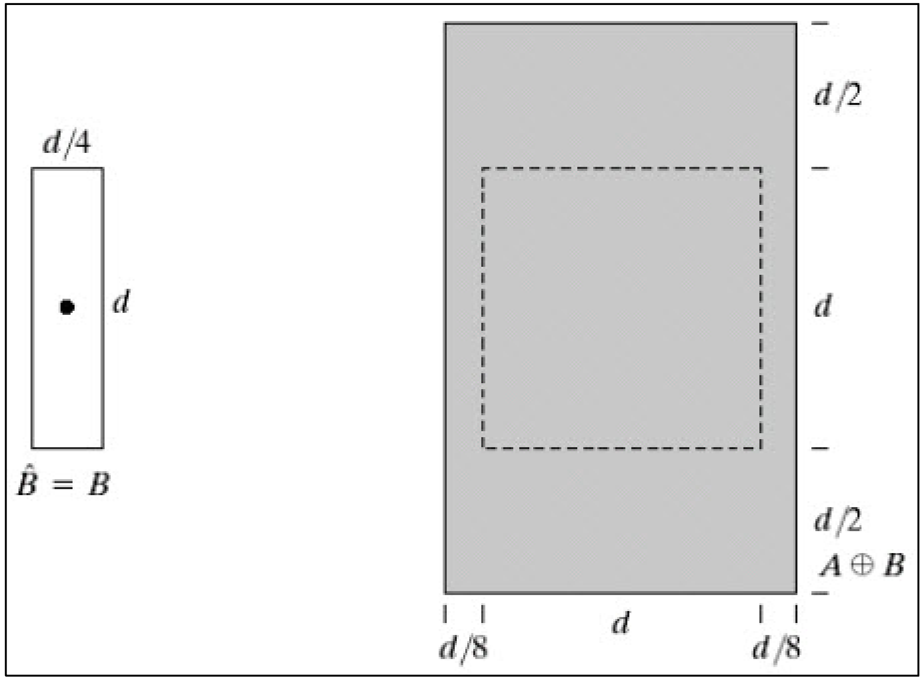
- Given the structuring element B and set A.



- The structuring element B enlarges the size of A at its boundaries. Dilation simply expands a given image.

Dilation Operations

- Given the structuring element B and set A.

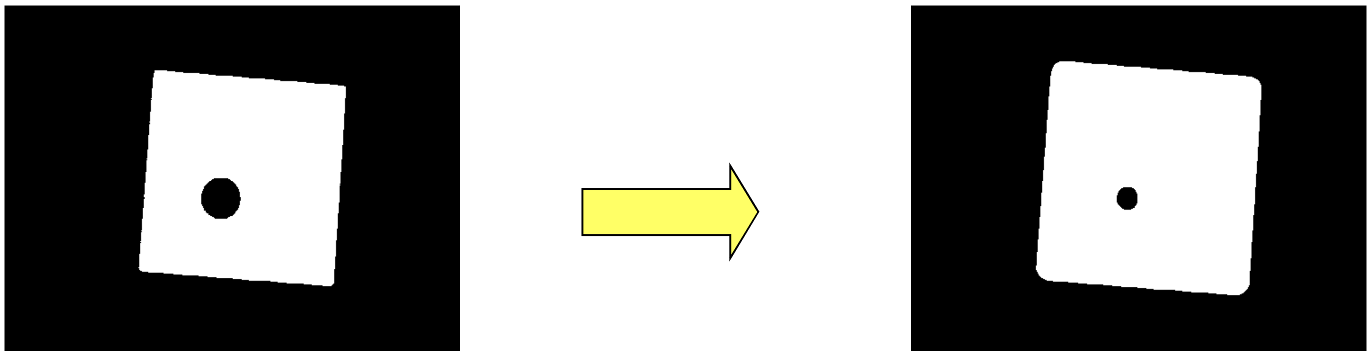


- The structuring element B enlarges the size of A at its boundaries, in relation to the distance from the origin of the structuring element.

Dilation Operations

- consider each of the foreground pixels.
- superimpose the structuring element on top.
- Consider each foreground pixel in the input image.
  - If any the structuring element touches in foreground pixels,

  ➔ write a “1” at the origin of the structuring element!



The pixels on the image using the set theory fundamentals

- We are set to pixels that have a value of 1 in black and white as an element in set A.

(Column)

(Row)

	1	2	3	4	5
1	0	0	0	0	0
2	0	1	1	0	0
3	0	0	1	1	0
4	0	1	1	0	0
5	0	0	0	0	0

Black and white
image size 5x5 pixels

- The elements in set A are all 6 different pixels have a value of 1 (the shadows).

$p(2,2)$	$p(2,3)$	$p(3,3)$
$p(3,4)$	$p(4,2)$	$P(4,3)$

Example : Dilation Operations (1/8)

0	0	0	0	0
0	1	1	0	0
0	0	1	1	0
0	1	1	0	0
0	0	0	0	0

Black and white image size 5x5 pixels

Set A

0	1	0
1	1	1
0	1	0

Structuring Element size 3x3

Set B

- To lay the structuring element B on the image A and sliding it across the image that are an element of image A. Which pixels from the Union between B and A, the output pixel is set to 1.

Example : Dilation Operations (2/8)

1st points

- Slide B for 6 points because A has pixel that is 1 for 6 points.

B

0	1	0
1	1	1
0	1	0

A

0	1	0	0	0
1	1	1	0	0
0	1	1	1	0
0	1	1	0	0
0	0	0	0	0

Input Image

0	1	0	0	0
1	1	1	0	0
0	1	1	0	0
0	0	0	0	0
0	0	0	0	0

Output Image

Example : Dilation Operations (3/8)

2nd points

- Slide B for 6 points because A has pixel that is 1 for 6 points.

B

0	1	0
1	1	1
0	1	0

A

0	0	1	0	0
0	1	1	1	0
0	0	1	1	0
0	1	1	0	0
0	0	0	0	0

Input Image

0	1	1	0	0
1	1	1	1	0
0	1	1	1	0
0	0	0	0	0
0	0	0	0	0

Output Image

Example : Dilation Operations (4/8)

3rd points

B

0	1	0
1	1	1
0	1	0

- Slide B for 6 points because A has pixel that is 1 for 6 points.

A

0	0	0	0	0
0	1	1	0	0
0	1	1	1	0
0	1	1	0	0
0	0	0	0	0

Input Image

0	1	1	0	0
1	1	1	1	0
0	1	1	1	0
0	1	1	0	0
0	0	0	0	0

Output Image

Example : Dilation Operations (5/8)

4th points

B

0	1	0
1	1	1
0	1	0

- Slide B for 6 points because A has pixel that is 1 for 6 points.

A

0	0	0	0	0
0	1	1	1	0
0	0	1	1	1
0	1	1	1	0
0	0	0	0	0

Input Image

0	1	1	0	0
1	1	1	1	0
0	1	1	1	1
0	1	1	1	0
0	0	0	0	0

Output Image

Example : Dilation Operations (6/8)

5th points

B

0	1	0
1	1	1
0	1	0

- Slide B for 6 points because A has pixel that is 1 for 6 points.

A

0	0	0	0	0
0	1	1	0	0
0	1	1	1	0
1	1	1	0	0
0	1	0	0	0

Input Image

0	1	1	0	0
1	1	1	1	0
0	1	1	1	1
1	1	1	1	0
0	1	0	0	0

Output Image

Example : Dilation Operations (7/8)

6th points (the last point)

B

0	1	0
1	1	1
0	1	0

- Slide B for 6 points because A has pixel that is 1 for 6 points.

A

0	0	0	0	0
0	1	1	0	0
0	0	1	1	0
0	1	1	1	0
0	0	1	0	0

Input Image

0	1	1	0	0
1	1	1	1	0
0	1	1	1	1
1	1	1	1	0
0	1	1	0	0

Output Image

Example : Dilation Operations (8/8)

- Shows the result output image obtained when B complete sliding for every 6 points of A.

B

0	1	0
1	1	1
0	1	0

The object is larger because Dilation expands an image.

A

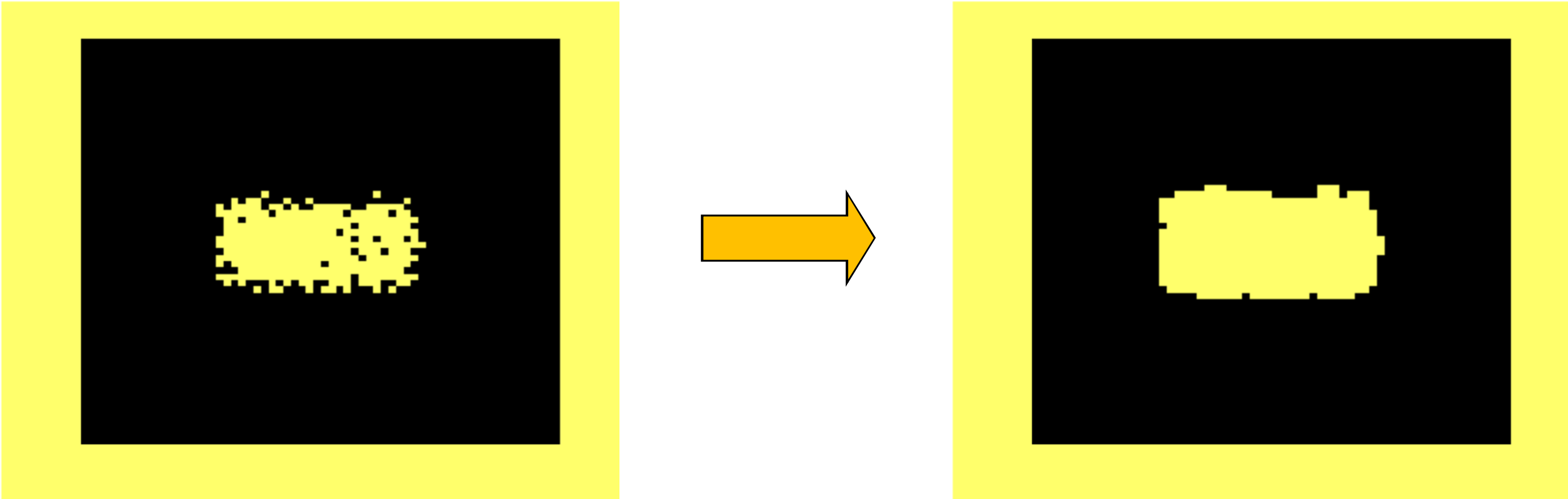
0	0	0	0	0
0	1	1	0	0
0	0	1	1	0
0	1	1	0	0
0	0	0	0	0

Input Image

0	1	1	0	0
1	1	1	1	0
0	1	1	1	1
1	1	1	1	0
0	1	1	0	0

Output Image

Example : Dilation Applications



- Fill holes in objects.
- Smooth object boundaries.
- Adds an extra outer ring of pixels onto object boundary, ie, object becomes slightly larger.

Example : Dilation Applications

Original image

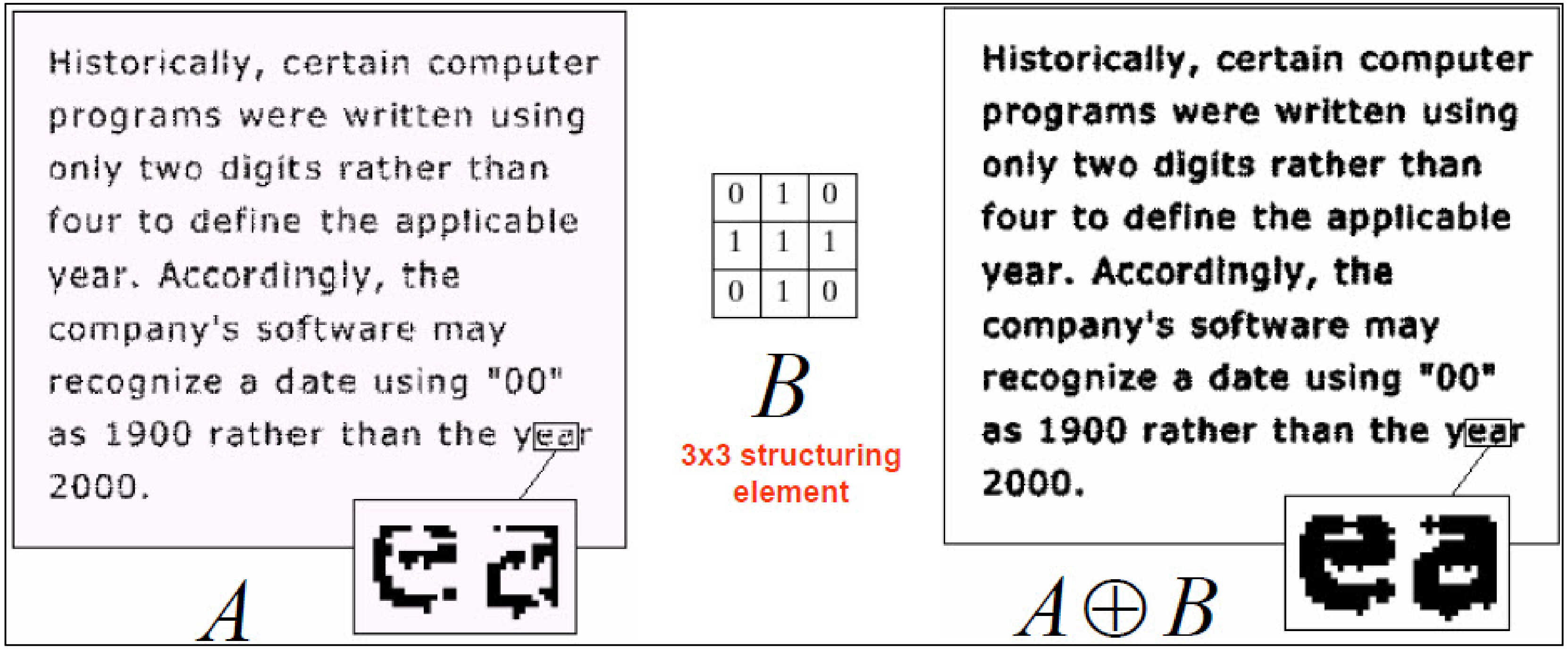
Dilation by 3*3 square structuring element

Dilation by 5*5 square structuring element

Watch out: In these examples a 1 refers to a black pixel!

Example : Dilation Applications

- Given the following distorted text image where the maximum length of the broken characters are 2 pixels. The image can be enhanced by bridging the gaps by using the structuring element given below :



Note that the broken characters are joined.

What Is Dilation For?

- Dilation can repair breaks

- Dilation can repair intrusions

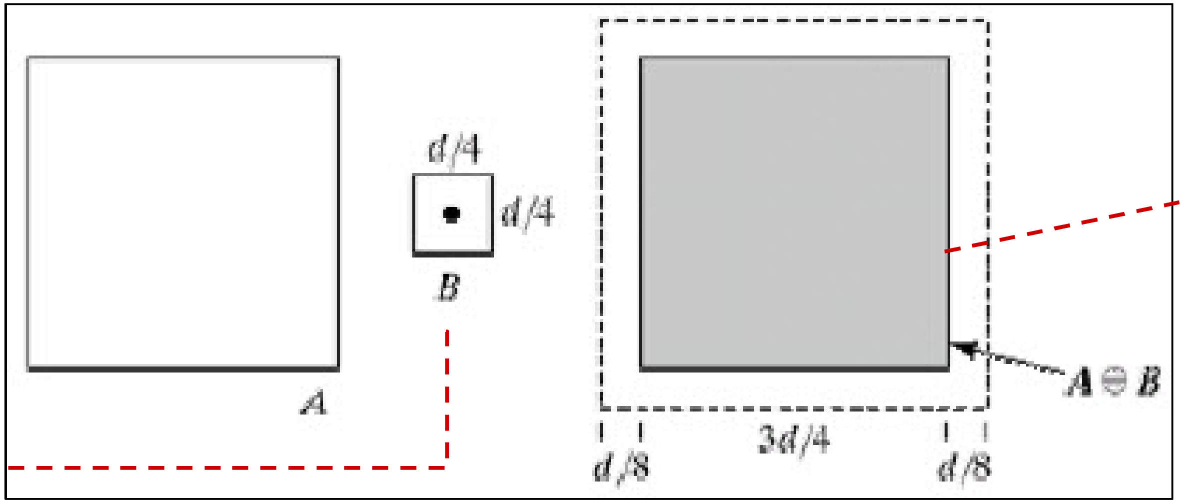
- **Watch out:** Dilation enlarges objects

- Given A and B sets in Z^2, the erosion of A by structuring element B, is defined by :

$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

- The erosion of A by structuring element B is the set of all points z, such that B, translated by z, is contained in A.

structuring element

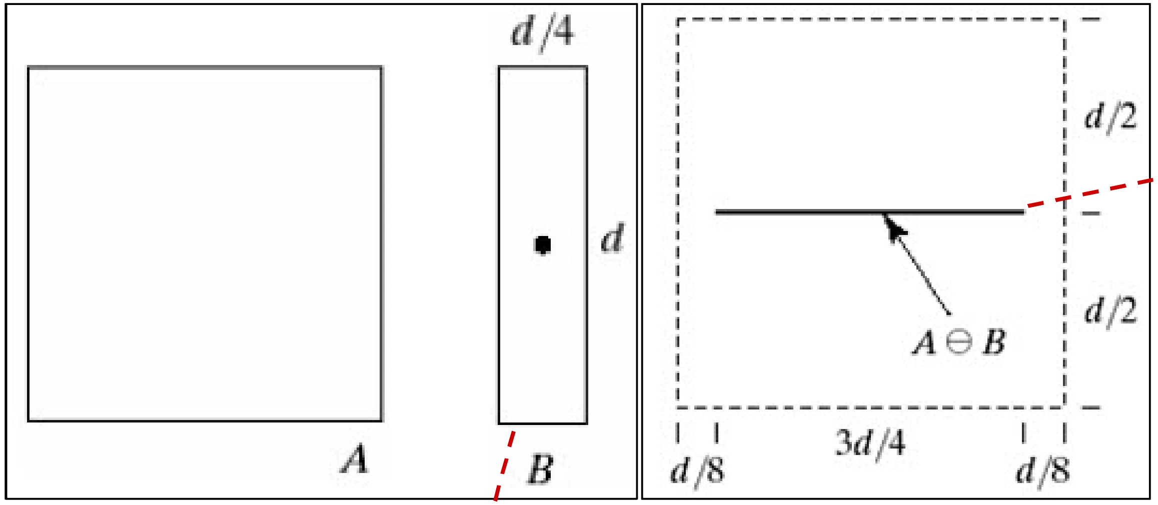


Shaded area is the erosion of A by B

- Note that in erosion the structuring element B erodes the input image A at its boundaries. Erosion shrinks a given image.

Erosion Operations

- Given the structuring element B and set A.



Shaded line is what is left from the erosion of A by B

structuring element

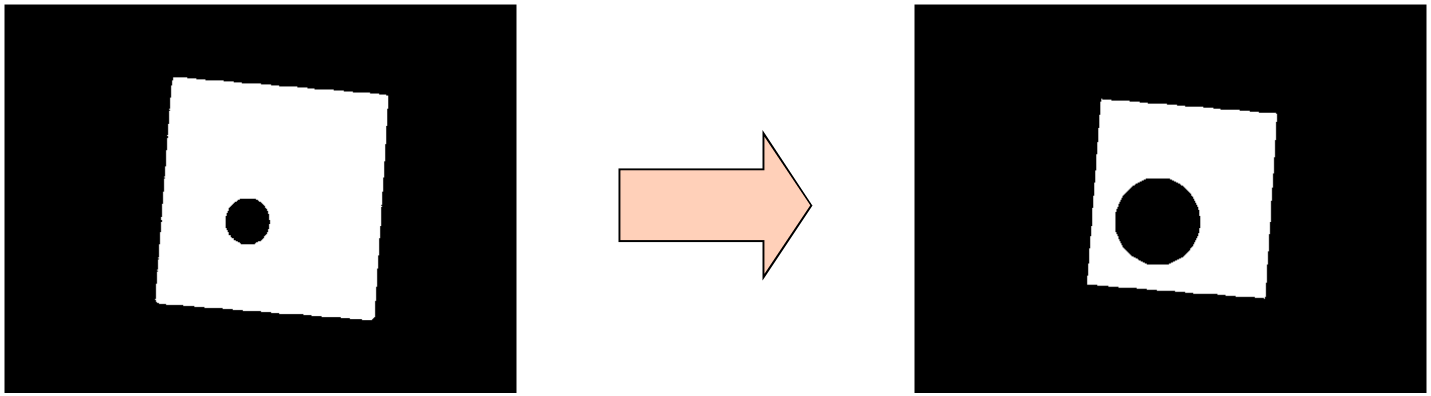
Erosion Operations

- Erosion : to erode away the boundaries of regions of foreground pixels.
- Erosion : “shrink” or “thin” an object in a binary image.
- Extent of shrinking controlled by a structuring element.
- Erosion shrinks foreground, enlarges background.
- Erosion shrinks an image.
- While holes within those areas become larger.

Erosion Operations

- consider each of the foreground pixels.
- superimpose the structuring element on top.
- Consider each foreground pixel in the input image.
    - If all the structuring element fits in foreground pixels,

  → write a “1” at the origin of the structuring element!



Example : Erosion Operations (1/9)

0	0	0	0	0
0	1	1	0	0
0	1	1	1	0
0	0	1	1	0
0	0	0	0	0

Black and white image size 5x5 pixels

Set A

0	0	0
1	1	0
0	1	0

Structuring Element size 3x3

Set B

- To lay the structuring element B on the image A and sliding it across the image that are an element of image A. The origin B is at its center, for each pixel in A superimpose the origin of B, if B is completely contained by A the pixel is retained, else deleted.

Example : Erosion Operations (2/9)

1st points

B

0	0	0
1	1	0
0	1	0

A

0	0	0	0	0
1	1	1	0	0
0	1	1	1	0
0	0	1	1	0
0	0	0	0	0

Input Image

- Slide B for 7 points because A has pixel that is 1 for 7 points.

B is not contained inside A. Therefore, the 1st points of A gets deleted or eroded.

0	0	0	0	0
0	~~0~~	1	0	0
0	1	1	1	0
0	0	1	1	0
0	0	0	0	0

Output Image

Example : Erosion Operations (3/9)

2nd points

B

0	0	0
1	1	0
0	1	0

- Slide B for 7 points because A has pixel that is 1 for 7 points.

B is contained inside A. Therefore, the 2nd points of A is retained.

A

0	0	0	0	0
0	1	1	0	0
0	1	1	1	0
0	0	1	1	0
0	0	0	0	0

Input Image

0	0	0	0	0
0	~~0~~	1	0	0
0	1	1	1	0
0	0	1	1	0
0	0	0	0	0

Output Image

Example : Erosion Operations (4/9)

3rd points

B

0	0	0
1	1	0
0	1	0

- Slide B for 7 points because A has pixel that is 1 for 7 points.

B is not contained inside A. Therefore, the 3rd points of A gets deleted or eroded.

A

0	0	0	0	0
0	1	1	0	0
1	1	1	1	0
0	1	1	1	0
0	0	0	0	0

Input Image

0	0	0	0	0
0	~~0~~	1	0	0
0	~~0~~	1	1	0
0	0	1	1	0
0	0	0	0	0

Output Image

Example : Erosion Operations (5/9)

4th points

B

0	0	0
1	1	0
0	1	0

- Slide B for 7 points because A has pixel that is 1 for 7 points.

B is contained inside A. Therefore, the 4th points of A is retained.

A

0	0	0	0	0
0	1	1	0	0
0	1	1	1	0
0	0	1	1	0
0	0	0	0	0

Input Image

0	0	0	0	0
0	~~0~~	1	0	0
0	~~0~~	1	1	0
0	0	1	1	0
0	0	0	0	0

Output Image

5th points

B

0	0	0
1	1	0
0	1	0

- Slide B for 7 points because A has pixel that is 1 for 7 points.

B is contained inside A. Therefore, the 5th points of A is retained.

A

0	0	0	0	0
0	1	1	0	0
0	1	1	1	0
0	0	1	1	0
0	0	0	0	0

Input Image

0	0	0	0	0
0	~~0~~	1	0	0
0	~~0~~	1	1	0
0	0	1	1	0
0	0	0	0	0

Output Image

Example : Erosion Operations (7/9)

6th points

B

0	0	0
1	1	0
0	1	0

- Slide B for 7 points because A has pixel that is 1 for 7 points.

B is not contained inside A. Therefore, the 6th points of A gets deleted or eroded.

A

0	0	0	0	0
0	1	1	0	0
0	1	1	1	0
0	1	1	1	0
0	0	1	0	0

Input Image

0	0	0	0	0
0	~~0~~	1	0	0
0	~~0~~	1	1	0
0	0	~~0~~	1	0
0	0	0	0	0

Output Image

Example : Erosion Operations (8/9)

7th points

B

0	0	0
1	1	0
0	1	0

- Slide B for 7 points because A has pixel that is 1 for 7 points.

B is not contained inside A. Therefore, the 7th points of A gets deleted or eroded.

A

0	0	0	0	0
0	1	1	0	0
0	1	1	1	0
0	0	1	1	0
0	0	0	1	0

Input Image

0	0	0	0	0
0	~~0~~	1	0	0
0	~~0~~	1	1	0
0	0	~~0~~	~~0~~	0
0	0	0	0	0

Output Image

Example : Erosion Operations (9/9)

- Shows the result output image obtained when B complete sliding for every 7 points of A.

B

0	0	0
1	1	0
0	1	0

The object is smaller because Erosion shrinks an image.

A

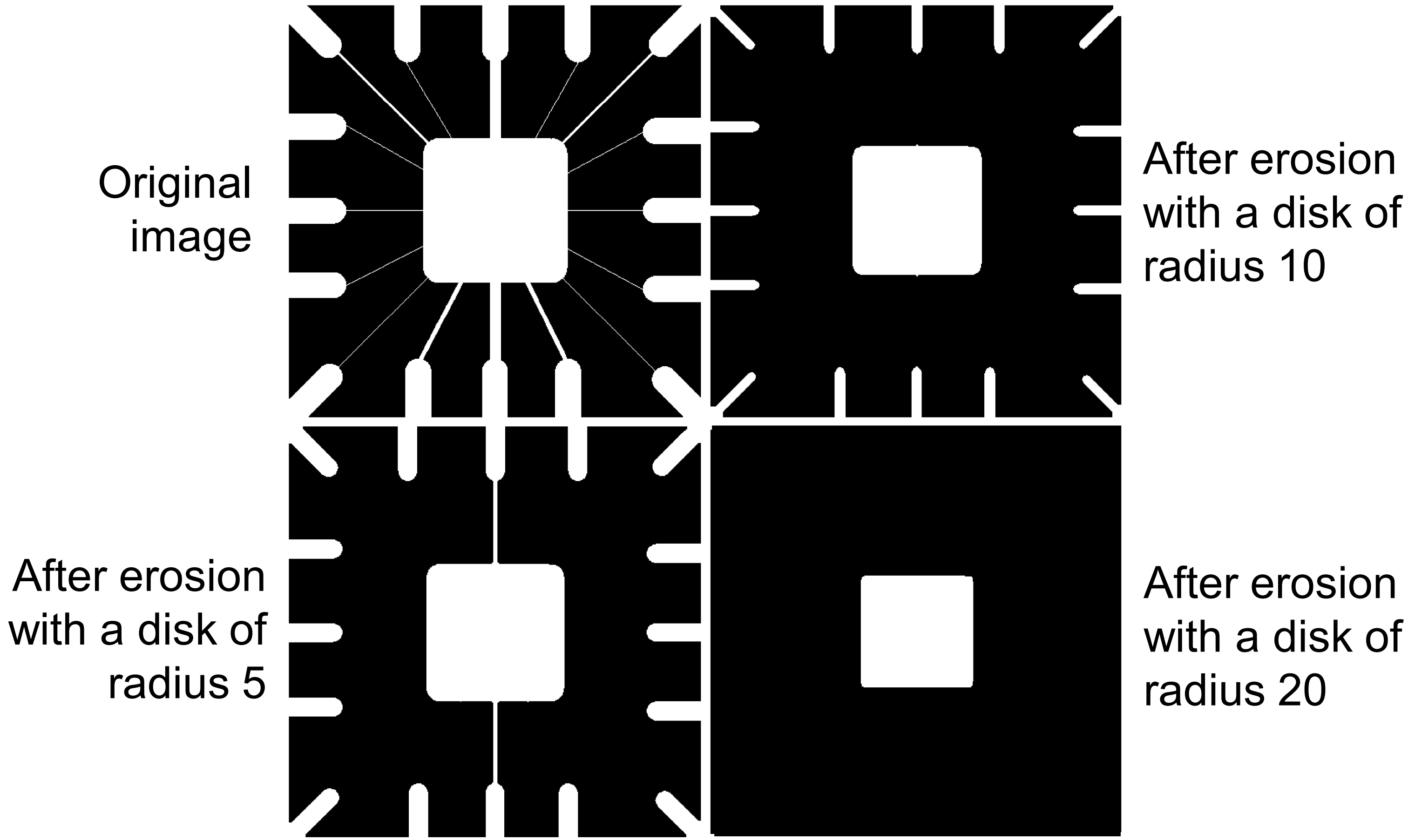
0	0	0	0	0
0	1	1	0	0
0	1	1	1	0
0	0	1	1	0
0	0	0	0	0

Input Image

0	0	0	0	0
0	0	1	0	0
0	0	1	1	0
0	0	0	0	0
0	0	0	0	0

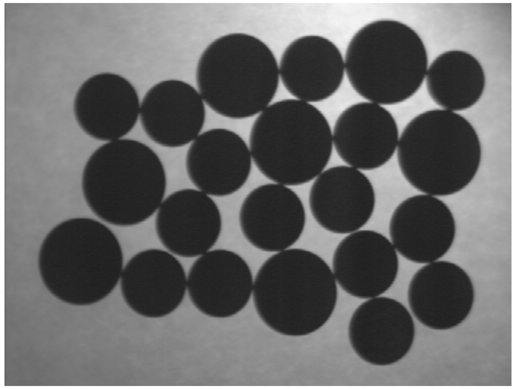
Output Image

Example : Erosion applications

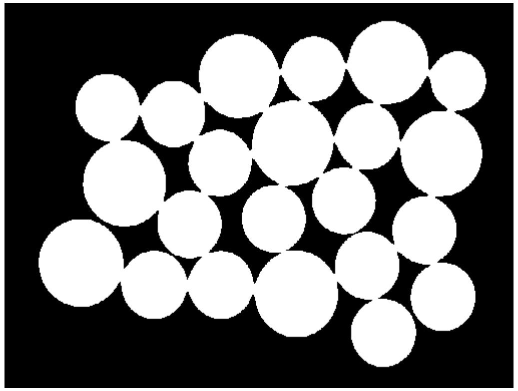


Example : Erosion Applications

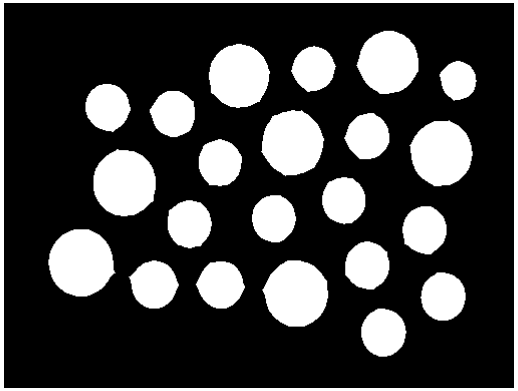
- Simple application of pattern matching.



Original Image

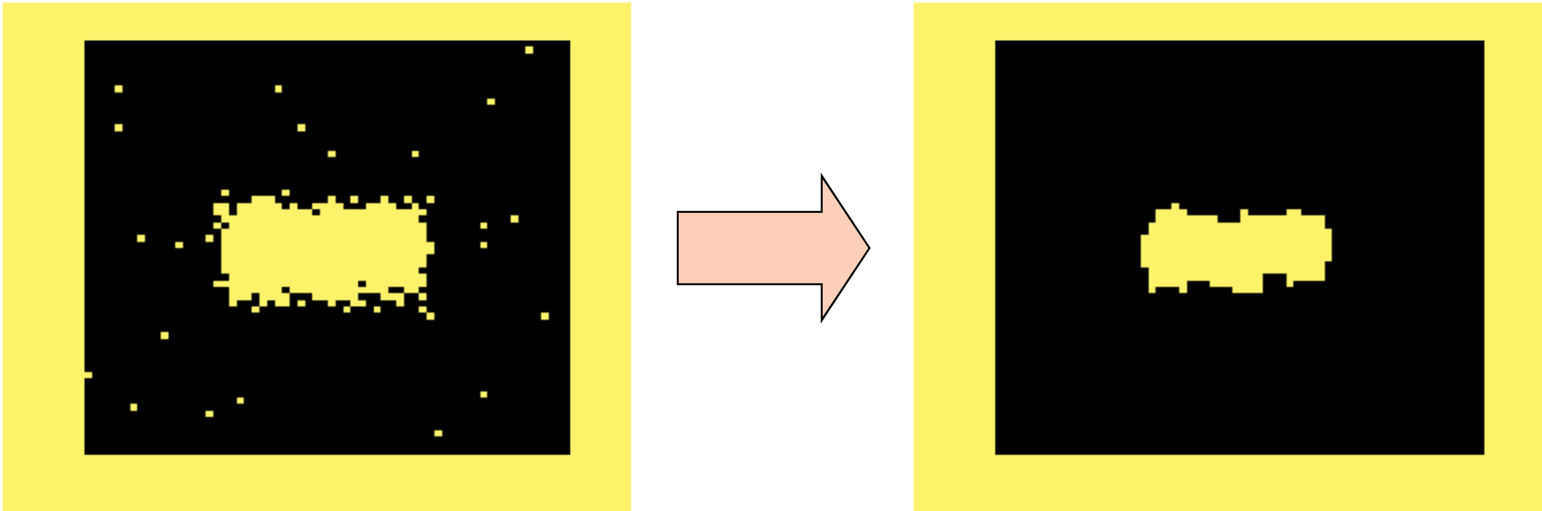


Segmented Image



Erosion Result

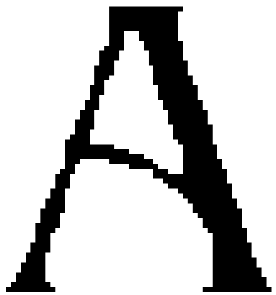
Example : Erosion Applications



- Removing noise.
- Removes the outer layer of object pixels.

Example : Erosion Applications

Original image



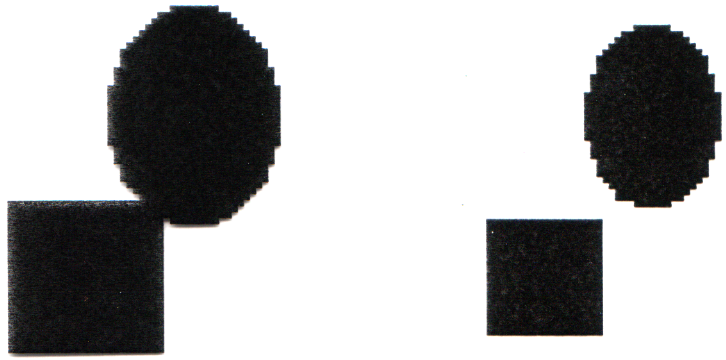
Erosion by 3*3 square structuring element

Erosion by 5*5 square structuring element

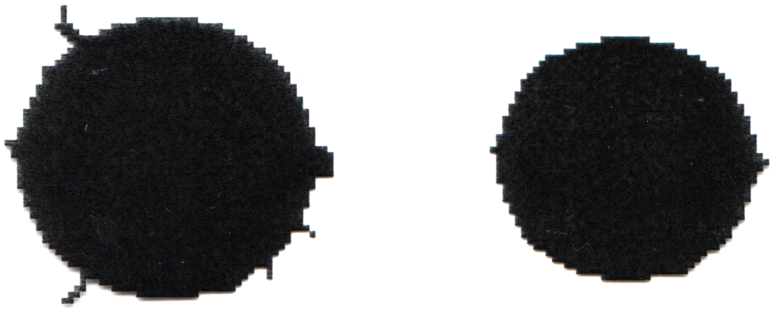
Watch out: In these examples a 1 refers to a black pixel!

What Is Erosion For?

- Erosion can split apart joined objects



- Erosion can strip away extrusions



- **Watch out:** Watch out: Erosion shrinks objects

Dilation and Erosion Summary

- Useful
  - Dilation : filling of holes of certain shape and size, given by SE
  - Erosion : removal of structures of certain shape and size, given by SE
- Combining Dilation and Erosion
  - WANTED : remove structures / fill holes, without affecting remaining parts
  - SOLUTION : combine erosion and dilation, (using same SE)

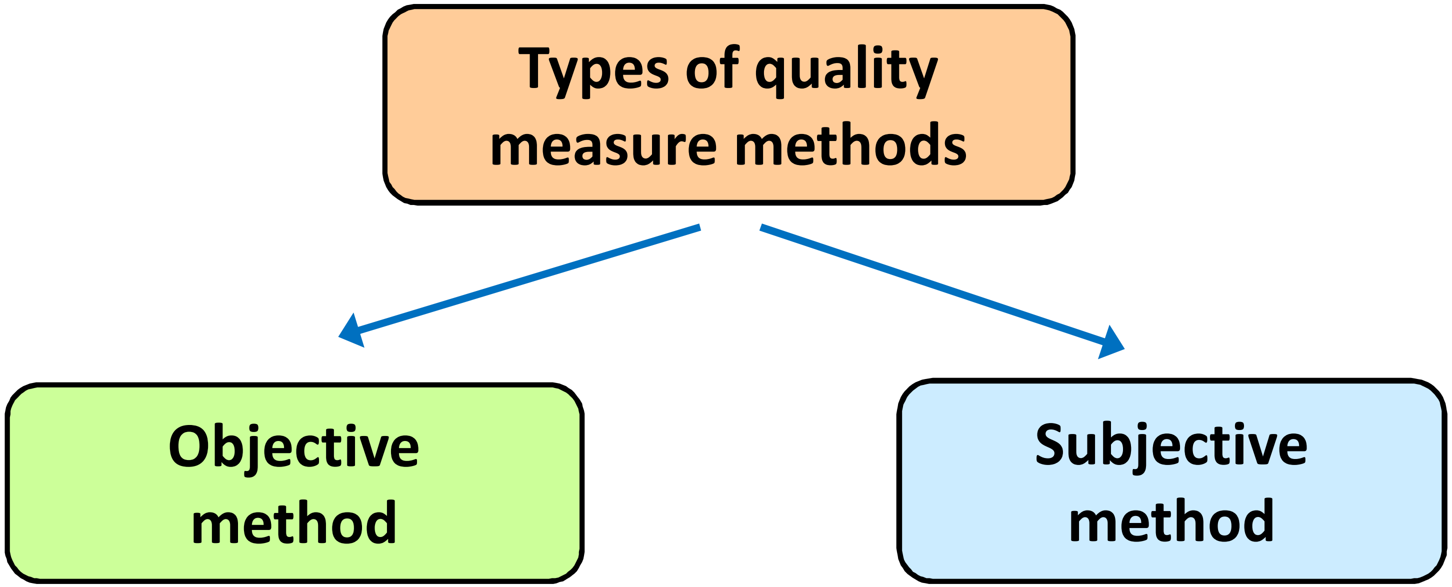
Opening and Closing Operations

- Opening and Closing are two important operators from mathematical morphology.
- They are both derived from the fundamental operations of erosion and dilation.
- They are normally applied to binary images, but gray value images are also possible.
- These operations are dual to each other.

Opening and Closing Operations

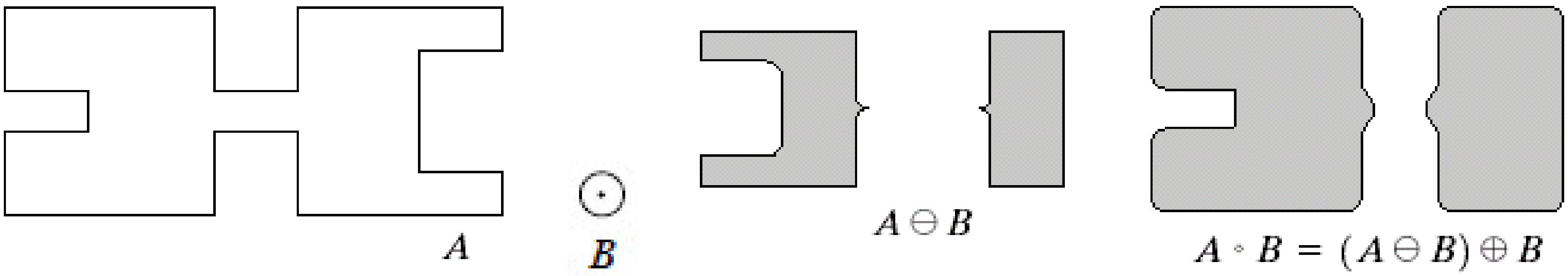
- Opening : generally smoothes the contour of an object, breaks narrow isthmuses, and eliminates thin protrusions.
- Closing : also tends to smooth sections of contours but, as opposed to opening, it generally fuses narrow breaks and long thin gulfs, eliminates small holes, and fills gaps in the contour.
- Opening => Erosion + Dilation
- Closing => Dilation + Erosion

Quality Measure of a Compressed Image



- Opening : The process of erosion followed by dilation is called opening. It has the effect of eliminating small and thin objects, breaking the objects at thin points and smoothing the boundaries/contours of the objects.
- Given set A and the structuring element B. Opening of A by structuring element B is defined by :

$$A \circ B = (A \ominus B) \oplus B \qquad A \circ B = \cup\{(B)_z \mid (B)_z \subseteq A\}$$



Original shape SE After erosion After dilation

Opening Operations

- The basic effect of an opening is similar to erosion.
  - Tends to remove some of the foreground pixels from the edges of regions of foreground pixels.
  - Less destructive than erosion.
- The exact operation is determined by a structuring element.
- Erosion can be used to eliminate small clumps of undesirable foreground pixels, e.g. “salt noise”.
- However, it affects all regions of foreground pixels indiscriminately.
- Opening gets around this by performing both an erosion and a dilation on the image.

Example : Opening Applications

Original Image

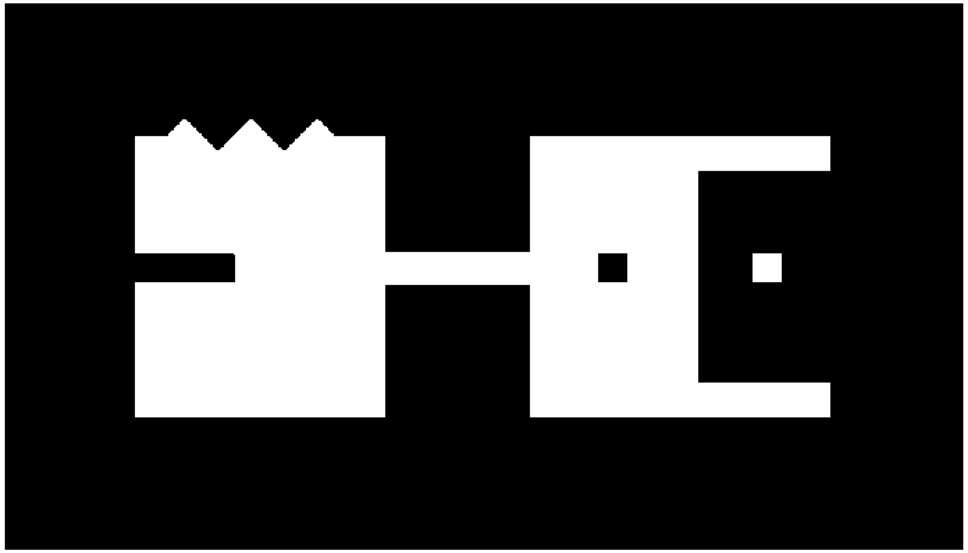
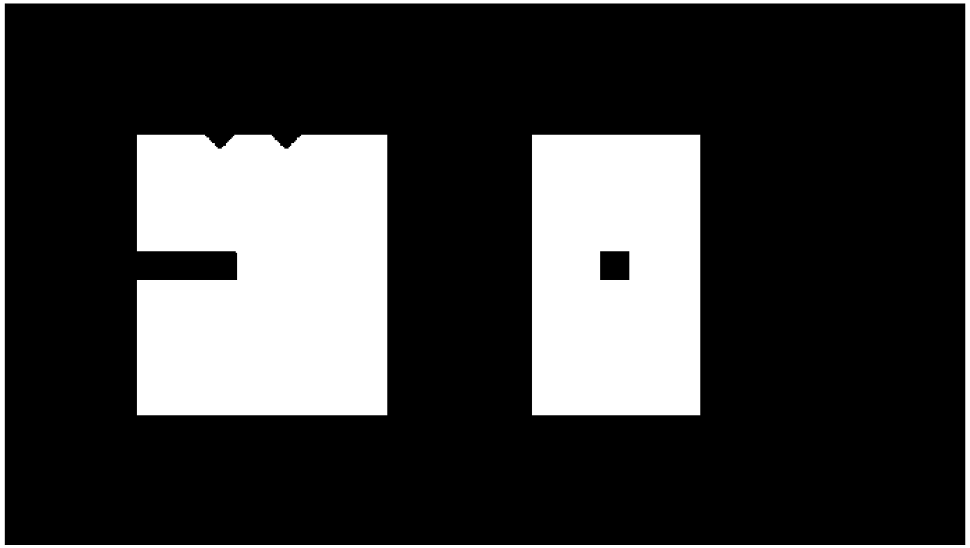


Image After Opening



Example : Opening Applications

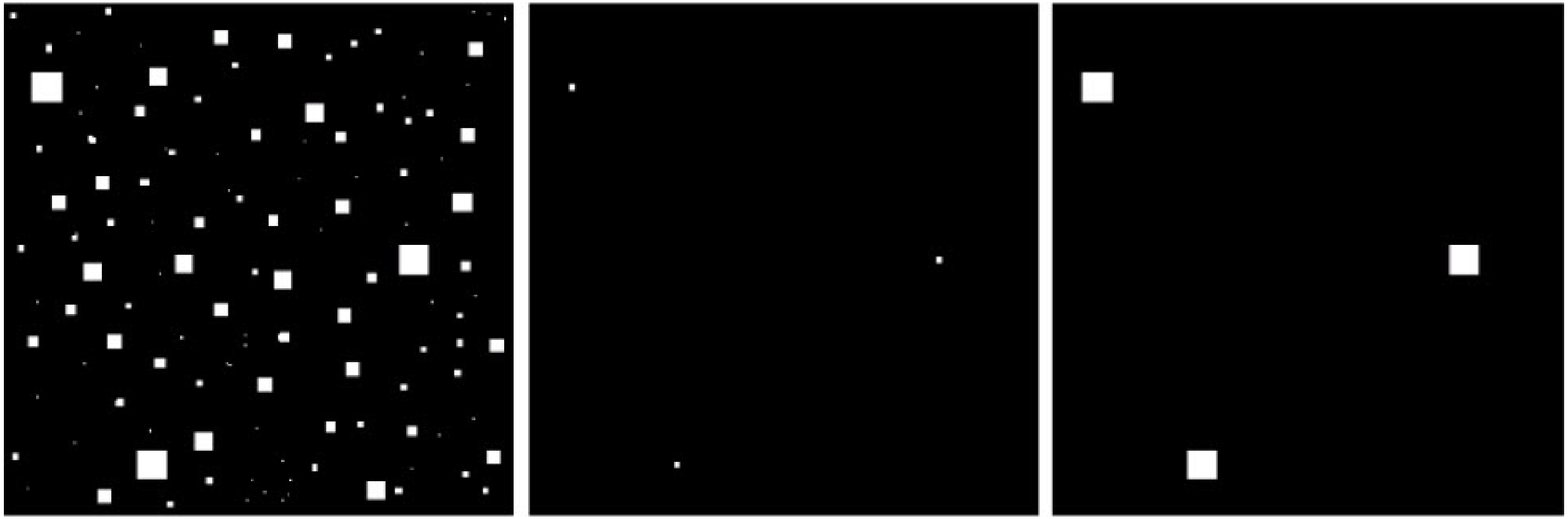


Image of squares of size 1,3,5,7,9 and 15 pixels on the side

Erosion of (a) with a square structuring element of 1's, 13 pixels on the side

Dilation of (b) with a same structuring element

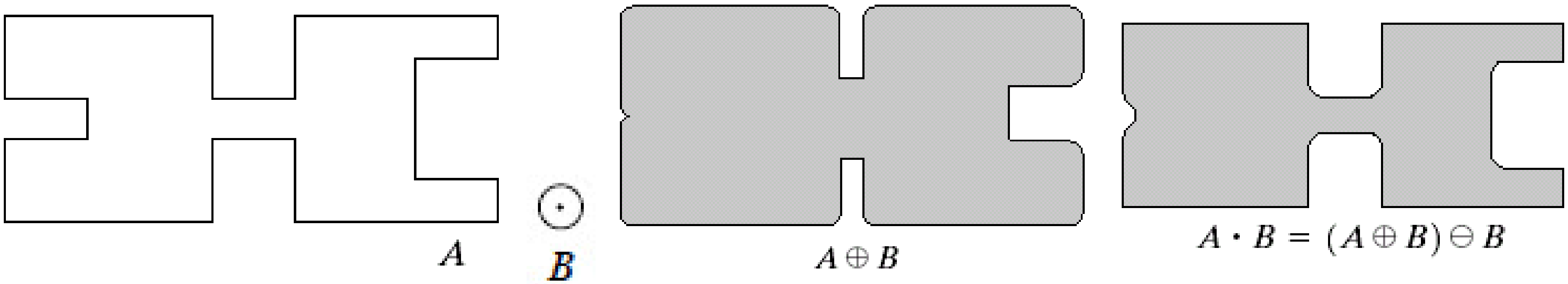
Remove small objects such as noise

Remember! Erosion "shrinks" or "thins" objects in binary image.

Closing Operations

- Closing : The process of dilation followed by erosion is called closing. It has the effect of filling small and thin holes, connecting nearby objects and smoothing the boundaries/contours of the objects.
- Given set A and the structuring element B. Closing of A by structuring element B is defined by :

$$A \bullet B = (A \oplus B) \ominus B \qquad A \bullet B = \cup\{(B)_z \mid (B)_z \cap A \neq \phi\}$$



Original shape SE After dilation After erosion

Closing Operations

- The basic effect of an closing is similar to dilation.
  - Tends to enlarge the boundaries of foreground regions.
  - Less destructive of the original boundary shape.
- The exact operation is determined by a structuring element.
- Closing is opening performed in reverse. It is defined simply as a dilation followed by an erosion using the same.
- fuse narrow breaks and long thin gulfs.
- fill gaps in the contour.

Example : Closing Applications

Original Image

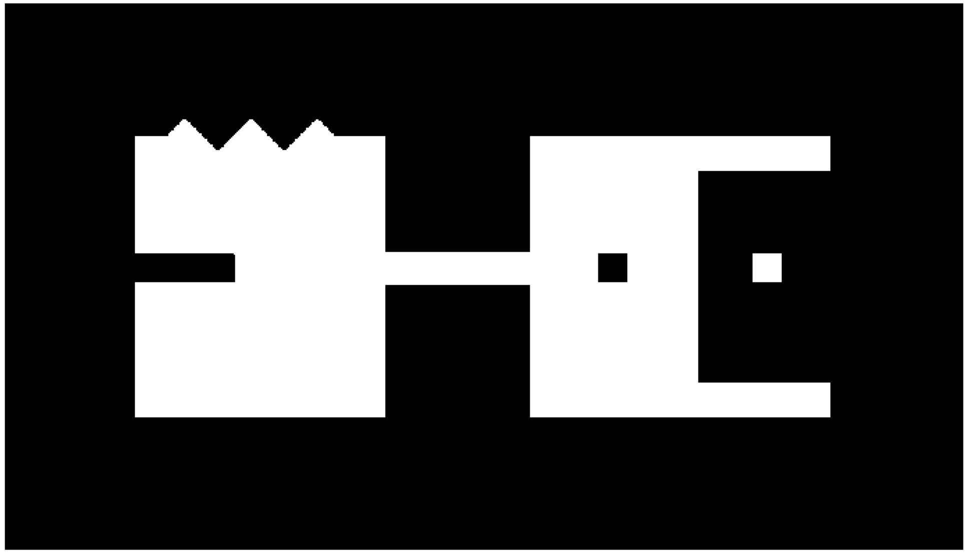
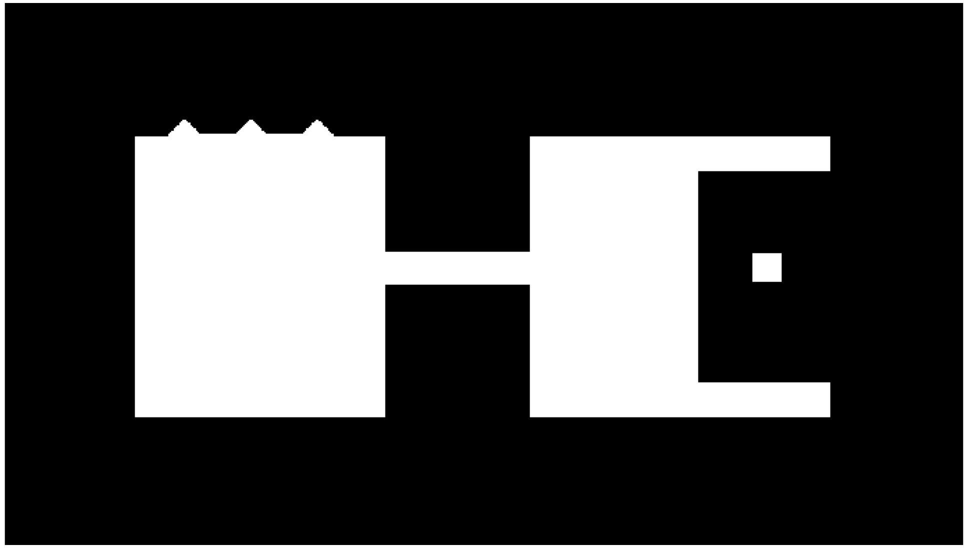
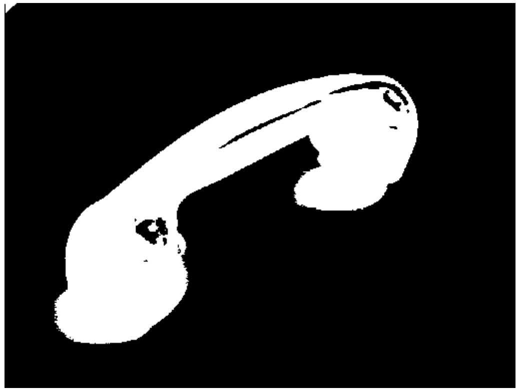


Image After Closing

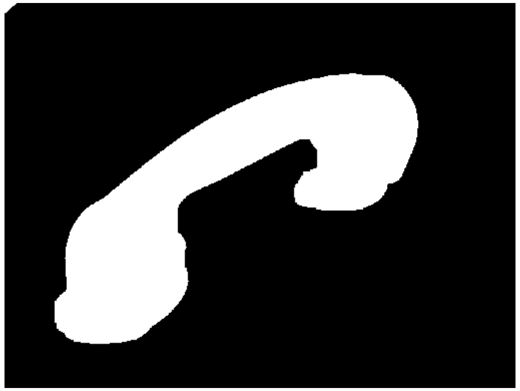


Example : Closing Operations

Original Image



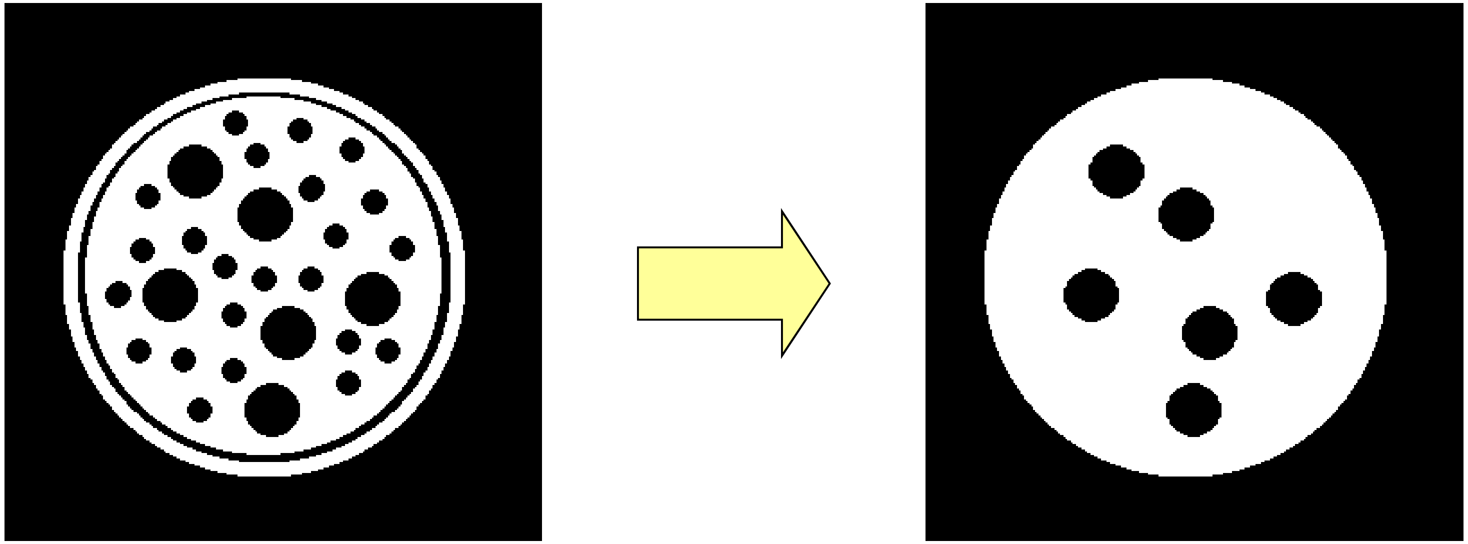
Segmented Image



Closing Result

- Closing with disc of size 20

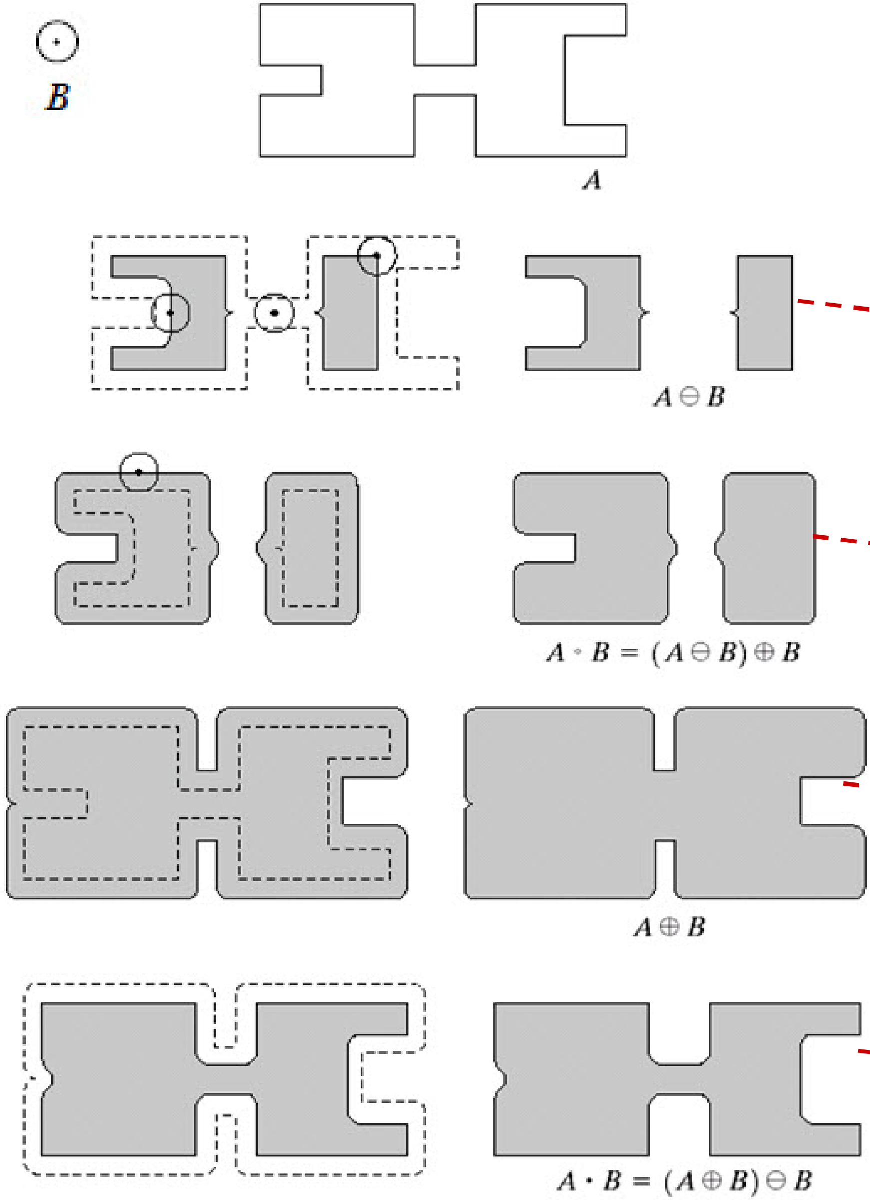
Example : Closing Operations



- Closing operation with a 22 pixel disc.
- Closes small holes in the foreground.

Example : Opening and Closing Operations

circular structuring element



result of erosion of A by B

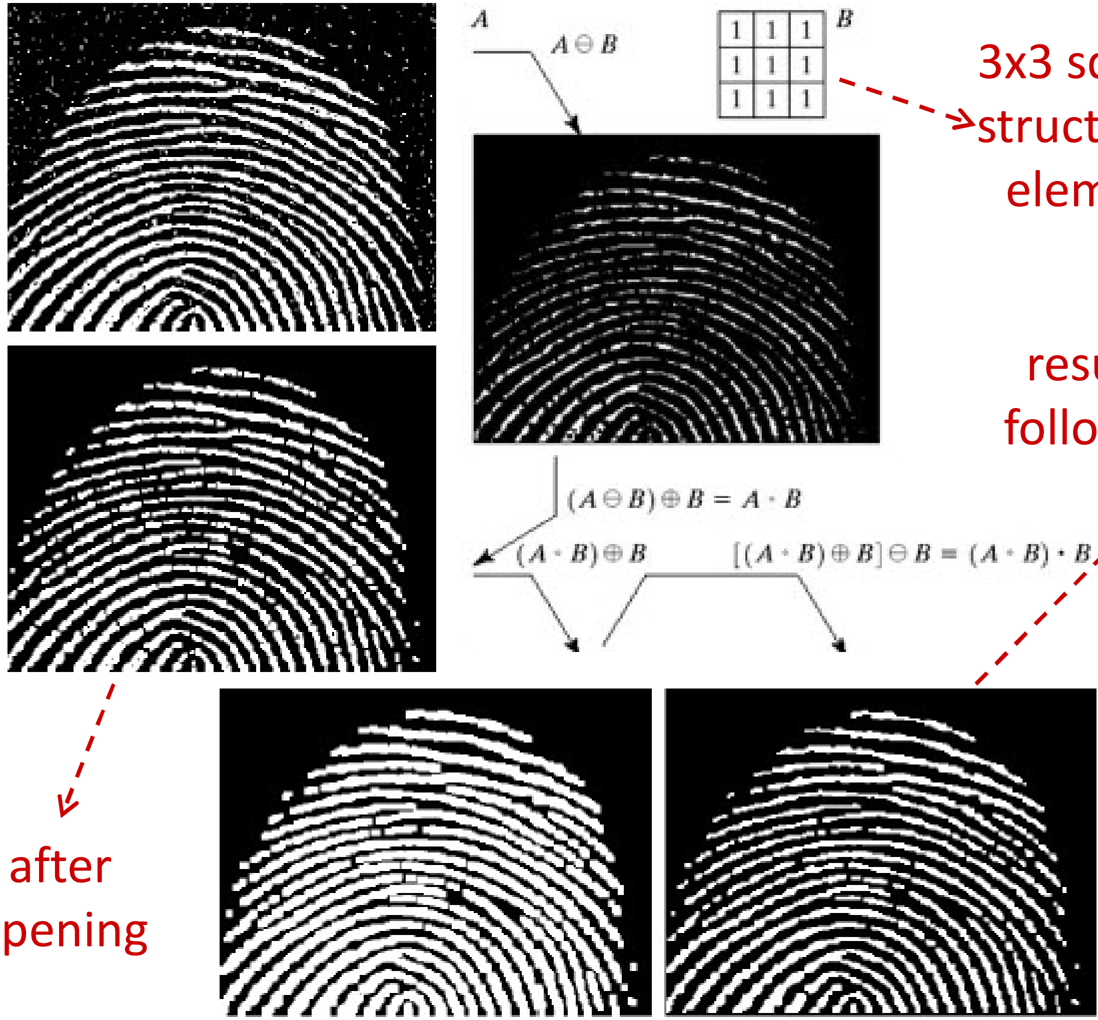
result of opening of A by B

result of dilation of A by B

result of closing of A by B

Example : Opening and Closing Operations

- Noise Filtering : The morphological operations can be used to remove the noise as in the following example :



3x3 square structuring element

result of opening followed by closing

after opening

Note that impulsive noise within the background and the fingerprints is removed.

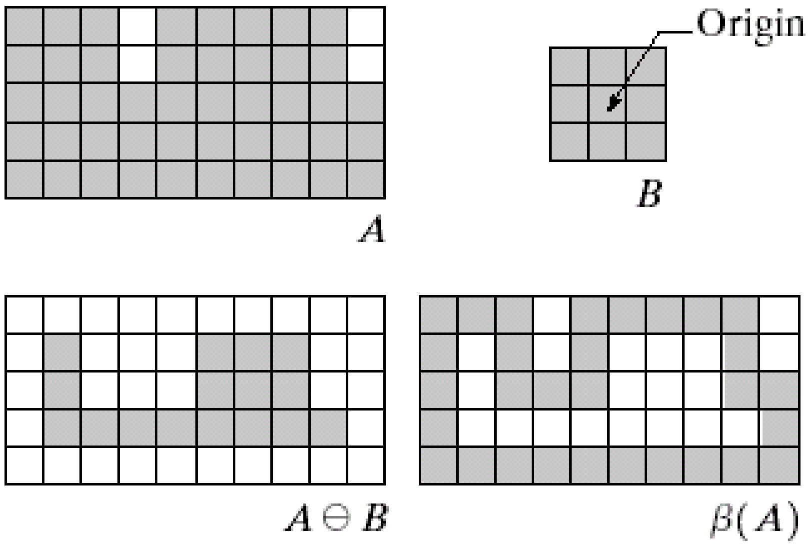
Some Basic Morphological Algorithms

- We are now ready to consider some practical uses of morphology.
- In particular, we consider morphological algorithms for extracting boundaries, connected components.
- We make extensive use in this section of "mini-images" designed to clarify the mechanics of each morphological process as we introduce it.
- The images are binary, with 1 's shown shaded and 0 's shown in white.

Boundary Extraction

- **Boundary Extraction** : The boundaries/edges of a region/shape can be extracted by first applying erosion on A by B and subtracting the eroded A from A.

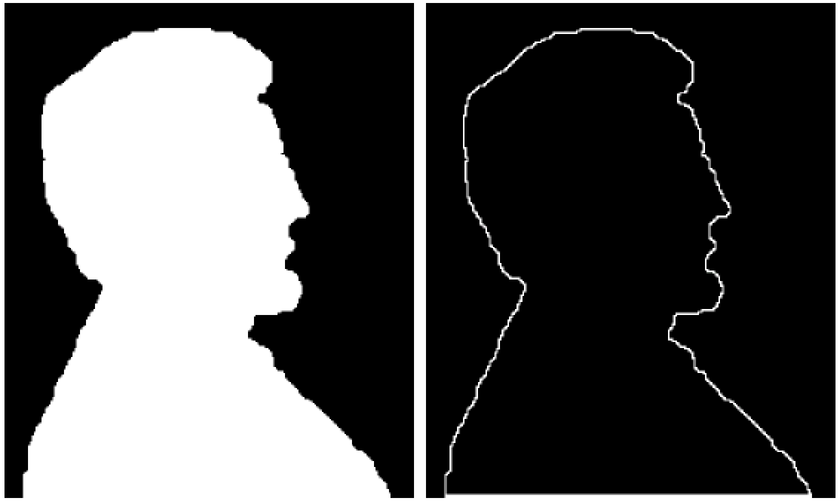
$$\beta(A) = A - (A \ominus B)$$



Ex 1 : 3x3 Square structuring element is used for boundary extraction

Original image

Boundary



Ex 2 : The same structuring element in Ex 1 is used

Note that thicker boundaries can be obtained by increasing the size of structuring element.

Region Filling

- **Region Filling** : Region filling can be performed by using the following definition.
- Given a symmetric structuring element B, one of the non-boundary pixels (X_k) is consecutively diluted and its intersection with the complement of A is taken as follows :

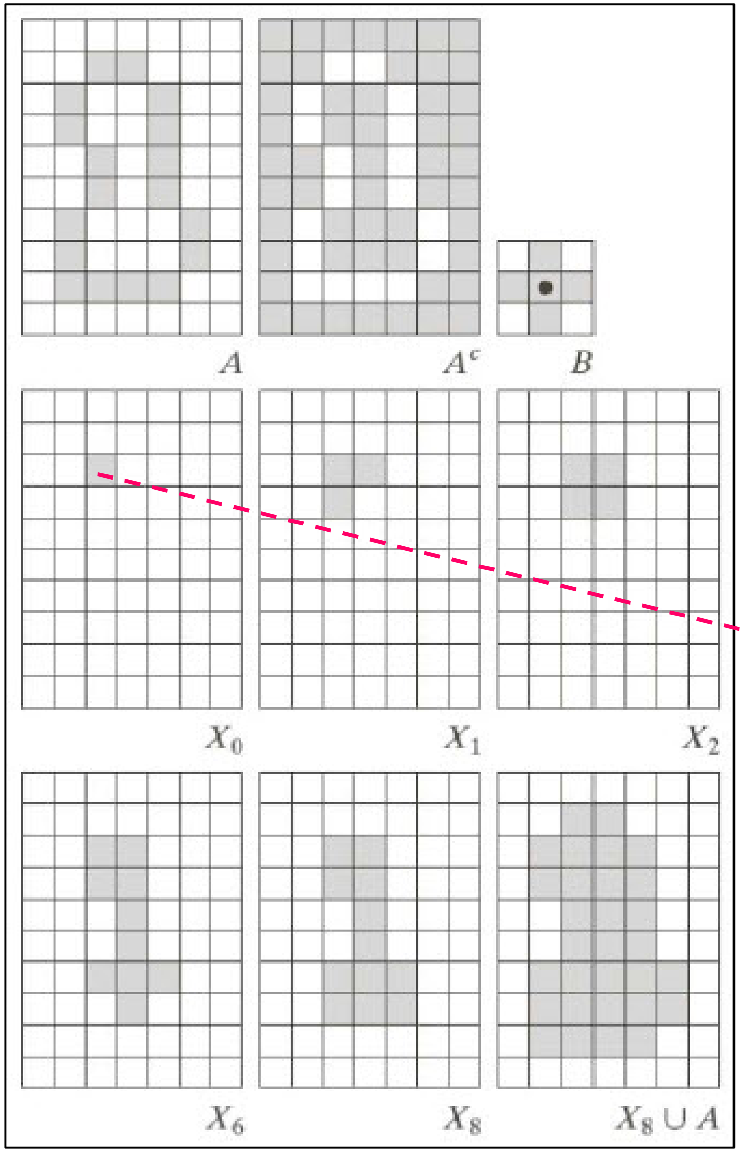
$$X_k = \left(X_{k-1} \oplus B\right) \cap A^c$$

k=1,2,3,...
terminates when $X_k = X_{k-1}$
$X_0 = 1$ (inner pixel)

- Following consecutive dilations and their intersection with the complement of A, finally resulting set is the filled inner boundary region and its union with A gives the filled region $\mathrm{F}(A)$.

$$F(A) = X_k \cup A$$

Example : Region Filling

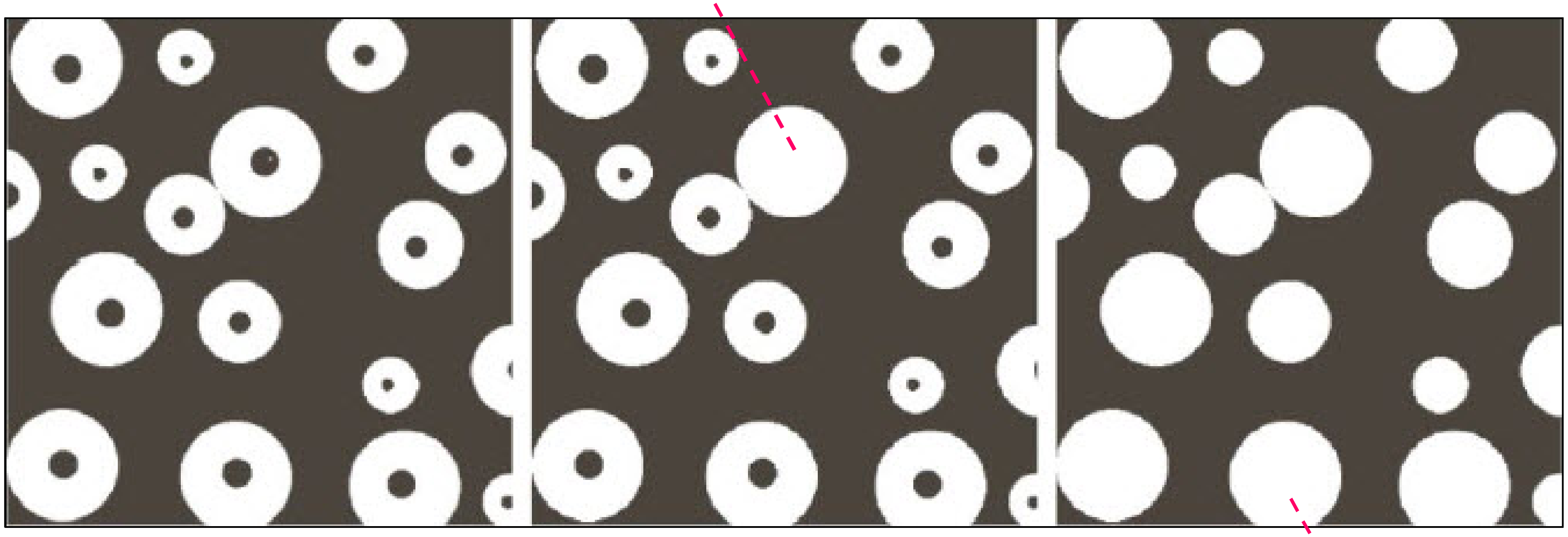


- Ex 1: $X_0 = 1$ (Assume that the shaded boundary points are 1 and the white pixels are 0)

A non-boundary pixel

Example : Region Filling

This region is filled first



Original image

Results of region filling

Filling of all the other regions

Thanks for your attention