Computational Finance - Modelling Stock prices

Problem 5

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Following piece of code download the prices of TCS since 2007

```
library(quantmod)
getSymbols('TCS.NS')
```

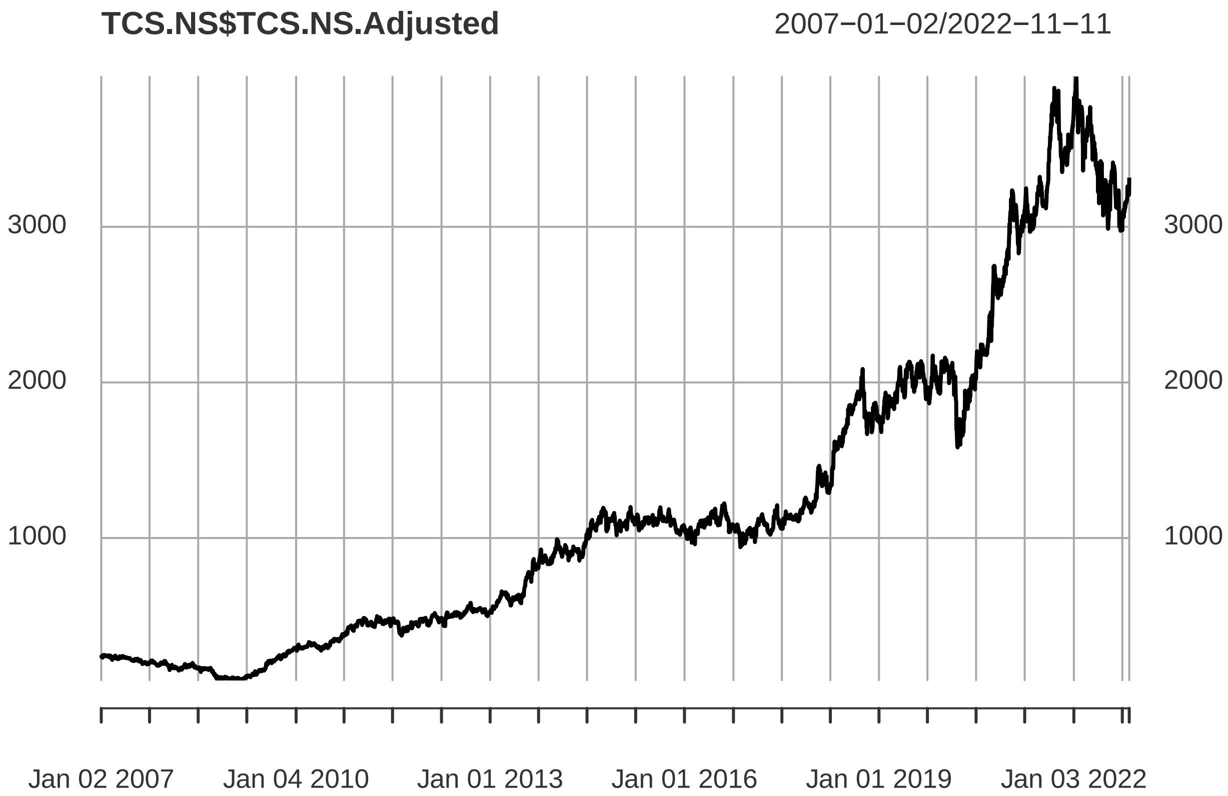
```
## [1] "TCS.NS"
```

```
tail(TCS.NS)
```

```
##            TCS.NS.Open TCS.NS.High TCS.NS.Low TCS.NS.Close TCS.NS.Volume
## 2022-11-03     3228.05     3228.05    3195.00      3206.75       1422652
## 2022-11-04     3217.00     3220.05    3166.15      3217.40       1464013
## 2022-11-07     3229.00     3242.80    3195.10      3233.70       1474498
## 2022-11-09     3249.80     3249.80    3201.65      3216.05       1162267
## 2022-11-10     3170.00     3225.00    3170.00      3205.65       1573092
## 2022-11-11     3269.60     3341.60    3255.05      3315.95       3265394
##            TCS.NS.Adjusted
## 2022-11-03         3206.75
## 2022-11-04         3217.40
## 2022-11-07         3233.70
## 2022-11-09         3216.05
## 2022-11-10         3205.65
## 2022-11-11         3315.95
```

Plot the adjusted close prices of TCS

```
## [1] 3918
```



Download the data of market index Nifty50. The Nifty 50 index indicates how the over all market has done over the similar period.

```
getSymbols('^NSEI')
```

```
## [1] "^NSEI"
```

```
tail(NSEI)
```

```
##            NSEI.Open NSEI.High NSEI.Low NSEI.Close NSEI.Volume NSEI.Adjusted
## 2022-11-03  17968.35   18106.3 17959.20   18052.70      213000      18052.70
## 2022-11-04  18053.40   18135.1 18017.15   18117.15      267900      18117.15
## 2022-11-07  18211.75   18255.5 18064.75   18202.80      314800      18202.80
## 2022-11-09  18288.25   18296.4 18117.50   18157.00      307200      18157.00
## 2022-11-10  18044.35   18103.1 17969.40   18028.20      256500      18028.20
## 2022-11-11  18272.35   18362.3 18259.35   18349.70      378500      18349.70
```

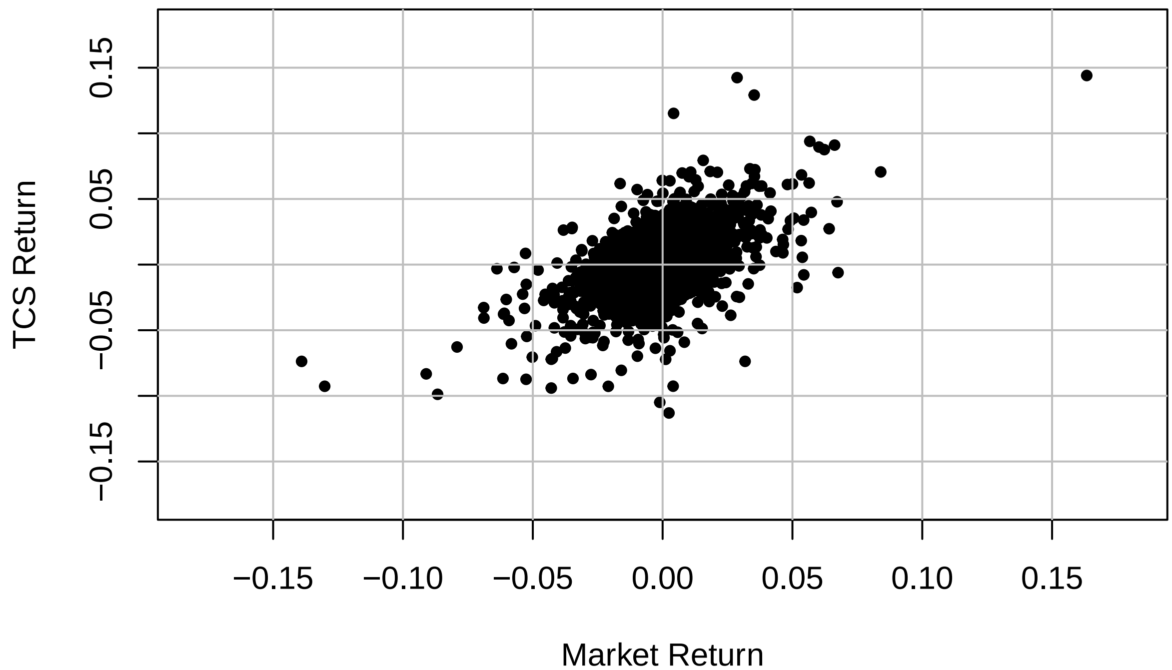
Plot the adjusted close value of Nifty50

Log-Return

We calculate the daily log-return, where log-return is defined as

$$r_t = \log(P_t) - \log(P_{t-1}) = \Delta \log(P_t),$$

where P_t is the closing price of the stock on t^{th} day.



- Consider the following model:

$$r_t^{TCS} = \alpha + \beta r_t^{Nifty} + \varepsilon$$

where $\mathbb{E}(\varepsilon) = 0$ and $\mathbb{V}ar(\varepsilon) = \sigma^2$.

Sub-problem 1

Estimate the parameters of the models $\theta = (\alpha, \beta, \sigma)$ using the method of moments type plug-in estimator discussed in the class.

Solution:

Brief Introduction on Method of Moments Method of Moments is a method of finding point estimators by equating sample moments with the corresponding population moments.

We know, j^{th} Moment of variable X,

$$M_j = \frac{1}{n} \sum_{i=1}^{n} X_i^j \tag{1}$$

Calculating Expectation by Method of Moments:

$$\mathbb{E}(X) = M_1 = m_1 = \frac{1}{n} \sum_{i=1}^{n} X_i \tag{2}$$

After simplifying, we get `equation (1)`

$$\mathbb{E}(X) = M_1 = \overline{X} \tag{3}$$

Calculating Variance by Method of Moments:

$$\begin{aligned}
\mathbb{E}(X^2) = M_2 = m_2 = \frac{1}{n}\sum_{i=1}^{n} X_i^2 && (4)\\
or, \mathbb{V}ar(X) + (\mathbb{E}(X))^2 = M_2 = \overline{X^2} && (5)\\
or, \mathbb{V}ar(X) = M_2 + (\mathbb{E}(X))^2 = \overline{X^2} + (\mathbb{E}(X))^2 && (6)
\end{aligned}$$

After replacing from `equation (1)`, we get `equation (2)`,

$$\mathbb{V}ar(X) = M_2 + M_1^2 = \overline{X^2} + \overline{X}^2 \tag{7}$$

Solving Approach

First Moments: We have already seen (`equation (1)`) that the Expectation and First moment of a random variable is same.

So, $\mathbb{E}(r^{Nifty}) = 3.5020103 \times 10^{-4}$ and $\mathbb{E}(r^{TCS}) = 7.2156422 \times 10^{-4}$

Second Moments: $\mathbb{E}((r^{Nifty})^2) = 1.9392909 \times 10^{-4}$ and $\mathbb{E}((r^{TCS})^2) = 3.6964472 \times 10^{-4}$

Variance calculation: As we already have calculated First and Second moment for both the variable, now Variance can be calculated using `equation (2)`.

$\mathbb{V}ar(r^{TCS}) = \mathbb{E}((r^{TCS})^2) + (\mathbb{E}(r^{TCS}))^2 = 3.7016538 \times 10^{-4}$

and

$\mathbb{V}ar(r^{Nifty}) = \mathbb{E}((r^{Nifty})^2) + (\mathbb{E}(r^{Nifty}))^2 = 1.9405173 \times 10^{-4}$

Forming equations between parameters: Applying property of Expectation on the given model, we get `equation (3)`

$$\begin{aligned}
\mathbb{E}(r^{TCS}) &= \mathbb{E}(\alpha + \beta r^{Nifty} + \varepsilon) && (8)\\
\Rightarrow \mathbb{E}(r^{TCS}) &= \alpha + \beta * \mathbb{E}(r^{Nifty}) + \mathbb{E}(\varepsilon) && (9)\\
\Rightarrow \mathbb{E}(r^{TCS}) &= \alpha + \beta * \mathbb{E}(r^{Nifty}) \qquad [as, \mathbb{E}(\varepsilon) = 0] && (10)
\end{aligned}$$

Therefore putting values of $\mathbb{E}(r^{TCS})$ and $\mathbb{E}(r^{Nifty})$ in `equation (3)` we get `equation (4)`

$\alpha +$ 3.5020103×10^{-4} $\beta = 7.2156422 \times 10^{-4}$

Multiplying explanatory variable r^{Nifty} on both sides of the model, we get,

$$r^{TCS} r^{Nifty} = \alpha r^{Nifty} + \beta (r^{Nifty})^2 + \varepsilon r^{Nifty} \tag{11}$$

Now, applying property of Expectation on the above equation `equation (5)` is formed,

$$\mathbb{E}(r^{TCS}r^{Nifty}) = \mathbb{E}(\alpha r^{Nifty} + \beta(r^{Nifty})^2 + \varepsilon r^{Nifty}) \tag{12}$$
$$\Rightarrow \mathbb{E}(\alpha r^{Nifty}) + \mathbb{E}(\beta(r^{Nifty})^2) = \mathbb{E}(r^{TCS}r^{Nifty}) - \mathbb{E}(\varepsilon r^{Nifty}) \tag{13}$$
$$\Rightarrow \alpha\mathbb{E}(r^{Nifty}) + \beta\mathbb{E}((r^{Nifty})^2) - \mathbb{E}(r^{TCS}r^{Nifty}) = \mathbb{E}(r^{TCS}r^{Nifty}) \tag{14}$$

Replacing, expected values in the above equation we get, `equation (6)`

$3.5020103 \times 10^{-4}\ \alpha + 1.9392909 \times 10^{-4}\ \beta = 1.443859 \times 10^{-4}$

Applying property of Variance on the given model, we get `equation (7)`

$$\mathbb{V}ar(r^{TCS}) = \mathbb{V}ar(\alpha + \beta r^{Nifty} + \varepsilon) \tag{15}$$
$$\Rightarrow \mathbb{V}ar(r^{TCS}) = \mathbb{V}ar(\beta * r^{Nifty}) + \mathbb{V}ar(\varepsilon) \tag{16}$$
$$\Rightarrow \mathbb{V}ar(r^{TCS}) = \beta^2 * \mathbb{V}ar(r^{Nifty}) + \sigma^2 \qquad [as, \mathbb{V}ar(\varepsilon) = \sigma^2] \tag{17}$$

Therefore putting values of $\mathbb{V}ar(r^{TCS})$ and $\mathbb{V}ar(r^{Nifty})$ in `equation (7)`, we get `equation (8)`

$1.9405173 \times 10^{-4}\ \beta^2 + \sigma^2 = 3.7016538 \times 10^{-4}$

Solving for values of the parameters `equation (4)`

$\alpha + 3.5020103 \times 10^{-4}\ \beta = 7.2156422 \times 10^{-4}$

`equation (6)`

$3.5020103 \times 10^{-4}\ \alpha + 1.9392909 \times 10^{-4}\ \beta = 1.443859 \times 10^{-4}$

`equation (8)`

$1.9405173 \times 10^{-4}\ \beta^2 + \sigma^2 = 3.7016538 \times 10^{-4}$

- Solving `equation (4)` & `(6)` we get the values of parameters α & β
- Now putting the solved value of β into `equation (8)` we get the value of σ

Estimated values of the parameters are in the following table of Problem 3

Sub-problem 2

Estimate the parameters using the `lm` built-in function of `R`. Note that `lm` using the OLS method.

Solution:

Given Model,

$$r_t^{TCS} = \alpha + \beta r_t^{Nifty} + \varepsilon$$

Here, in the model r_t^{TCS} is the target or dependent variable and r_t^{Nifty} is the explanatory variable. Also, α is the co-efficient and β is slope of the linear model.

- From the **Model Coefficients** of the `model` object derived from `lm` function, we get the values of intercept(α_{lm}) and the slope(β_{lm}). After that, we can predict the values of the target variable with the help of derived parameters.

where,

- Predicted value of the TCS stock, $r_{pred}^{TCS} = \alpha_{lm} + \beta_{lm} * r_{actual}^{Nifty}$
- Error in prediction, $\varepsilon = r_{pred}^{TCS} - r_{actual}^{TCS}$

Estimated values of the parameters are in the following table of Problem 3

Sub-problem 3

Fill-up the following table

Parameters	Method of Moments	OLS
α	4.6112088×10^{-4}	4.6112088×10^{-4}
β	0.7436967	0.7436967
σ	0.0162123	0.0161865

Sub-problem 4

If the current value of Nifty is 18000 and it goes up to 18200. The current value of TCS is Rs. 3200/-. How much you can expect TCS price to go up?

Solution:

```
nif_current = 18000
nif_future = 18200
tcs_current = 3200

nif_return = log(nif_future) - log(nif_current)
tcs_return_pred = predict(model, data.frame(NSEI.Adjusted = c(nif_return)))

tcs_forecast = round(exp(tcs_return_pred) * tcs_current)
```

After prediction by the model, we can say TCS stock price would go up to 3228