

Linear Algebra and its Applications

Assignment 4

Due: 17 February 2023

Write a your own LU solver, based on Gaussian elimination with partial pivoting, in Python. The input is a square invertible matrix A and a right hand side vector b (both randomly generated). The output should be L, U and the (row) permutation matrix and the solution to the problem $Ax = b$. Once you are sure that the code is working properly you should experiment with various sizes and note the time taken. Prepare comparison tables (one for factorization and the other for the solution, i.e., forward and backward substitution) that shows the time needed for your algorithm, for the SciPy's LU solver. For each system you should also note down how far were you from the actual solution. [20 points]

To be precise, here are some guidelines.

- There is a built-in LU solver in SciPy. However, you should write your own without calling any readily available routine from any of the Python libraries.
- Use the NumPy random library to generate an $n \times n$ real matrix for $n = 10, 100, 1000, \dots$ etc (more the better).
- Implement the Gaussian elimination with partial pivoting algorithm. Do add an array that keeps track of pivots. In case you reach a situation where there is no (nonzero) pivot available in that column then you should exit the program, saying “the matrix is singular and the rank is at least $-$ ”.
- The ideal output of the program should be the permutation matrix P , the unit lower triangular L and the upper triangular U . Of course, you don't want to print these matrices individually.
- Measure the time taken only for the factorization (don't count matrix initialization, verification etc.) and substitutions.
- Also prepare the following tables: matrix norm of $PA - LU$ and the vector norm of $Ax_0 - b$ for both the solvers. If the norm is zero then you are sure that there were no precision errors.

Note: Your submission should be a PDF output of your Jupyter Notebook. Make sure that unnecessary things like, error messages, trial runs etc. are not included in the submission.