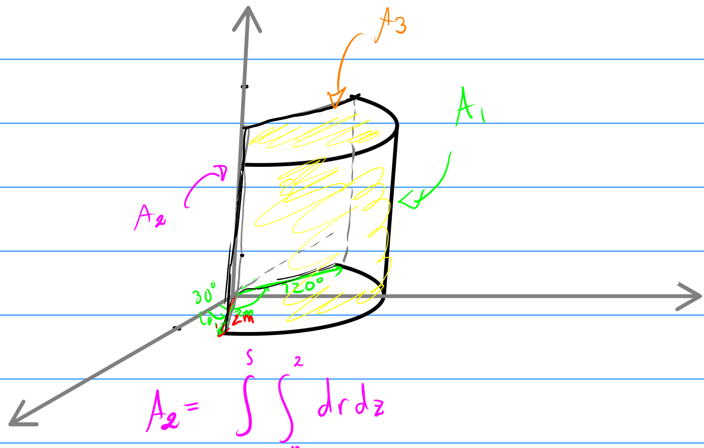
3- $r = 2$ $\quad 30 \leq \varphi \leq 120$

$h = 5$



$$A_2 = \int_0^5 \int_0^2 dr\,dz$$

$$A_2 = \int_0^5 2\,dz = 2 \cdot 5 = 10$$

$$A_3 = \int_{\pi/3}^{2\pi/3} \int_0^2 r\,dr\,d\phi = \int_{\pi/3}^{2\pi/3} \frac{r^2}{2}\Big|_0^2 d\phi$$

$$A_3 = \int_{\pi/3}^{2\pi/3} 2\,d\phi = 2\left(\frac{2\pi}{3} - \frac{\pi}{3}\right) = \frac{2}{3}\pi$$

$$A_1 = \int_0^5 \int_{\pi/3}^{2\pi/3} r\,d\phi\,dz$$

$$A_1 = r \int_0^5 \left(\frac{2\pi}{3} - \frac{\pi}{3}\right) dz$$

$$A_1 = r \int_0^5 \frac{\pi}{3}\,dz$$

$$A_1 = r \cdot \frac{5\pi}{3} = \frac{2 \cdot 5\pi}{3} = \frac{10\pi}{3}$$

$$A_{tot} = A_1 + 2A_2 + 2A_3$$

$$A_{tot} = \frac{10\pi}{3} + 2 \cdot 10 + 2 \cdot \frac{2}{3}\pi$$

$$A_{tot} = 20 + 4\pi$$

1- $A = (5, 3, 2)$

$B = (7, 30, 70) = \left(7, \frac{\pi}{6}, \frac{7\pi}{18}\right)$

$x = r \operatorname{sen}\phi \cos\theta$

$y = r \operatorname{sen}\phi \operatorname{sen}\theta$

$z = r\cos\phi$

B: $x = 7 \operatorname{sen}\left(\frac{7\pi}{18}\right)\cos\left(\frac{\pi}{6}\right) = \frac{7}{2}\sqrt{3}\cos\left(\frac{\pi}{9}\right) \approx 5{,}70$

$y = 7 \operatorname{sen}\left(\frac{7\pi}{18}\right)\operatorname{sen}\left(\frac{\pi}{6}\right) = \frac{7}{2}\cos\left(\frac{\pi}{9}\right) \approx 3{,}29$

$z = 7\cos\left(\frac{7\pi}{18}\right) \approx 2{,}39$

$B = (5{,}70,\ 3{,}29,\ 2{,}39)$

$A + B = (5 + 5{,}70,\ 3 + 3{,}29,\ 2 + 2{,}39)$

$= (10{,}70,\ 6{,}29,\ 4{,}39)$

$A \cdot B = 5 \cdot 5{,}70 + 3 \cdot 3{,}29 + 2 \cdot 2{,}39 = 43{,}15$

$|A| = \sqrt{5^2 + 3^2 + 2^2} = \sqrt{38}$

$|B| = \sqrt{5{,}7^2 + 3{,}29^2 + 2{,}39^2} = 7{,}00$

$\cos\theta = \frac{43{,}15}{\sqrt{38} \cdot 7{,}00} \rightsquigarrow$

$\theta = \cos^{-1}\left(\frac{43{,}15}{\sqrt{38} \cdot 7}\right)$

$\theta = 0{,}36°$

$$\vec{A} \times \vec{B} = \begin{array}{c} \begin{matrix} \hat{i} & \hat{j} & k \end{matrix} \\ \begin{vmatrix} 5 & 3 & 2 \\ 5{,}7 & 3{,}29 & 2{,}39 \end{vmatrix} \end{array}$$

$$= \begin{vmatrix} 3 & 2 \\ 3{,}29 & 2{,}39 \end{vmatrix} \vec{i} - \begin{vmatrix} 5 & 2 \\ 5{,}7 & 2{,}39 \end{vmatrix} \vec{j} + \begin{vmatrix} 5 & 3 \\ 5{,}7 & 3{,}29 \end{vmatrix} \vec{k}$$

$$= (3 \cdot 2{,}39 - 2 \cdot 3{,}29)\,\hat{i} - (5 \cdot 2{,}39 - 2 \cdot 5{,}7)\,\hat{j} + (5 \cdot 3{,}29 - 3 \cdot 5{,}7)\,\hat{k}$$

$$\vec{A} \times \vec{B} = (0{,}59,\ 0{,}55,\ -0{,}65)$$

Area del paralelogramo $\quad |A \times B| = \sqrt{0{,}59^2 + 0{,}55^2 + (-0{,}65)^2}$

$$= 1{,}03\,u^2$$

2) Punto$_c$ $(4, 45°, 2) \rightarrow (4, \frac{\pi}{4}, 2)$

Cartesianas

$x = r\cos\theta = 4\cos(\frac{\pi}{4}) = 2\sqrt{2}$

$y = r\,\text{sen}\,\theta = 4\,\text{sen}(\frac{\pi}{4}) = 2\sqrt{2}$

$z = z = 2$

$(2\sqrt{2}, 2\sqrt{2}, 2)$

Polares

$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{(2\sqrt{2})^2 + (2\sqrt{2})^2 + 2^2}$

$= 2\sqrt{5}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{2\sqrt{2}}{2\sqrt{2}}\right) = 45°$

$\phi = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{2}\right) = 63{,}44°$

$(2\sqrt{5}, 63{,}44°, 45°)$

4) $F = 2\cos\theta \, a_r + \operatorname{sen}\theta \, a_\theta$

$$\begin{bmatrix} A_x \\ A_\varphi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\varphi & \operatorname{sen}\varphi & 0 \\ -\operatorname{sen}\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2\cos\theta \\ \operatorname{sen}\theta \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\varphi \cdot 2\cos\theta + \operatorname{sen}\varphi \operatorname{sen}\theta + 0 \\ -\operatorname{sen}\varphi \cdot 2\cos\theta + \cos\varphi \operatorname{sen}\theta + 0 \\ 0 + 0 + 0 \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} 2\cos\varphi\cos\theta + \operatorname{sen}\varphi \operatorname{sen}\theta \\ -2\operatorname{sen}\varphi \cos\theta + \cos\varphi \operatorname{sen}\theta \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad F = (2\cos\varphi\cos\theta + \operatorname{sen}\varphi\operatorname{sen}\theta)\,a_x + (-2\operatorname{sen}\varphi\cos\theta + \cos\varphi\operatorname{sen}\theta)\,a_y$$