Problema1

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$x^2y'' + 5xy' + 4y = 16x^2 - 8\ln x$

$(y'' - y') + 5(y') + 4y = 16e^{2t} - 8t$

$y'' - y' + 5y' + 4y = 16e^{2t} - 8t$

$y'' + 4y' + 4y = 16e^{2t} - 8t$

homogenea

$y'' + 4y' + 4y = 0$

$m^2 + 4m + 4 = 0$

$m_1 = -2$

$y_h = c_1 e^{-2t} + c_2 t e^{-2t}$

$-8t \rightarrow y_{p_1} = A + Bt$

$y_{p_1}' = B$

$y_{p_1}'' = 0$

$16e^{2t} \rightarrow y_{p_2} = Ae^{2t}$

$y_{p_2}' = 2Ae^{2x}$

$y_{p_2}'' = 4Ae^{2x}$

$e^t = x \rightarrow \ln x = t$

$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} = \frac{dy}{dx}e^t$

$\rightarrow \frac{dy}{dt} = x\frac{dy}{dx}$ ← $y' = xy'$

$\frac{d^2y}{dt^2} = \frac{d}{dt}\left(\frac{dy}{dt}\right) = \frac{d}{dt}\left(\frac{dy}{dx}e^t\right)$

$= \left[\frac{d}{dt}\left(\frac{dy}{dx}\right)\right]e^t + \frac{dy}{dx}\left(\frac{d}{dt}e^t\right)$

$= \left[\frac{d}{dx}\left(\frac{dy}{dx}\right)\frac{dx}{dt}\right]e^t + \frac{dy}{dx}e^t$

$= \left(\frac{d^2y}{dx^2}e^t\right)e^t + \frac{dy}{dx}e^t$

$\rightarrow \frac{d^2y}{dt^2} = \frac{d^2y}{dx^2}e^{2t} + \frac{dy}{dx}e^t = \frac{dy}{dx^2}x^2 + \frac{dy}{dx}x$

$\rightarrow \frac{d^2y}{dt^2} - \frac{dy}{dt} = x^2\frac{dy}{dx}$

⇒ Para y_{p_1}

$0 + 4\cdot(B) + 4\cdot(A + Bt) = -8t$

$4B + 4A + 4Bt = -8t$

$4B + 4A = 0$

$A + B = 0$

$A - 2 = 0$

$A = 2$

$4B = -8$

$B = -8/4 = -2$

$y_{p_1} = 2 - 2t$

→ Para y_{p2}:

$$4Ae^{2t} + 4(2Ae^{2t}) + 4(Ae^{2t}) = 16e^{2x}$$

$$16Ae^{2t} = 16e^{2t}$$

$$A = 1$$

$$\Rightarrow y_{p2} = e^{2t}$$

$$\therefore y = C_1 e^{-2t} + C_2 t e^{-2t} + e^{2t} + 2 - 2t$$

$$y = C_1 \cdot (e^t)^{-2} + C_2 t \cdot (e^t)^{-2} + (e^t)^2 + 2 - 2t$$

$t = \ln x$

$e^t = x$

$$y = C_1 x^{-2} + C_2 \ln(x) \cdot x^{-2} + x^2 + 2 - 2\ln(x)$$