TAREA 2

Ecuaciones diferenciales

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Tarea #2

Tarea:

1) Determine si es linealmente dependientes o independientes.

$f(x) = \cos^2 x$ $\qquad$ $g(x) = 1 + \cos 2x$

$$\begin{vmatrix} \cos^2 x & 1 + \cos 2x \\ -2\cos x \cdot \operatorname{sen} x & -2 \operatorname{sen} 2x \end{vmatrix} =$$

$\rightarrow -2\cos x \cdot \operatorname{sen} x = -\operatorname{sen} 2x \leftarrow$

$$\begin{vmatrix} \cos^2 x & 1 + \cos 2x \\ -\operatorname{sen} 2x & -2 \operatorname{sen} 2x \end{vmatrix}$$

$$\left[(\cos^2 x \cdot -2\operatorname{sen} 2x) - (-\operatorname{sen} 2x \cdot (1 + \cos 2x)) \right]$$

$$-2\operatorname{sen} 2x \cdot \cos^2 x + \operatorname{sen} 2x + \cos 2x \cdot \operatorname{sen} 2x$$

$$\operatorname{sen} 2x \left(-2\cos^2 x + 1 + \cos 2x \right)$$

$$\operatorname{sen} 2x \left[-(\cos^2 x - 1) + \cos 2x \right]$$

$$\operatorname{sen} 2x \left(-\cos 2x + \cos 2x \right)$$

$$\operatorname{sen} 2x \,(0)$$

$$0$$

$\rightarrow$ $\boxed{W = 0 \rightarrow \text{Linealmente Dependiente}}$ $\leftarrow$

Tarea #2

#2 $x^2y'' - 3xy' + 4y = 0$

$y = x^2$ $\qquad x^2(2) - 3x(2x) + 4(x^2) = 0$

$y' = 2x$ $\qquad 6x^2 - 6x^2 = 0$

$y'' = 2$ $\qquad 0 = 0$ ✓

$$y'' - \frac{3}{x}y' + 4y = 0$$

$$y_2 = x^2 \int \frac{e^{-\int -\frac{3}{x}dx}}{x^6} = x^2 \int \frac{e^{3\ln x}}{x^6} = x^2 \int \frac{\cancel{x^3}}{\cancel{x^6}} =$$

$$x^2 \int \frac{1}{x^3} = x^2 \int x^{-3} = y_2 = \frac{-1}{2}$$

$y_1 = x_2$ $\qquad y_2 = -1/2$

Fecha: / /

b) $y^{(4)} - 2y'' + y = xe^{x}$

$$y^{(4)} - 2y'' + y = 0$$
$$r^{4} - 2r^{2} + 1 = 0$$
$$(r^{2} - 1)^{2} = 0$$
$$(r-1)^{2}(r+1)^{2} = 0$$
$$y_h = C_1 e^{x} + C_2 x e^{x} + C_3 e^{-x} + C_4 x e^{-x}$$

$$y_p = (Ax^{2} + Bx^{3})e^{x}$$

$$y'_p = (2Ax + 3Bx^{2})e^{x} + (Ax^{2} + Bx^{3})e^{x}$$
$$= (2Ax + 3Bx^{2} + Ax^{2} + Bx^{3})e^{x}$$

$$y''_p = (2A + 6Bx + 2Ax + 3Bx^{2})e^{x} + (2Ax + 3Bx^{2} + Ax^{2} + Bx^{3})e^{x}$$

$$y''_p = (2A + 6Bx + 2Ax + 3Bx^{2} + 2Ax + 3Bx^{2} + Ax^{2} + Bx^{3})e^{x}$$

$$y''_p = (2A + 6Bx + 4Ax + 6Bx^{2} + Ax^{2} + Bx^{3})e^{x}$$

$$y'''_p = (6B + 4A + 12Bx + 2Ax + 3Bx^{2})e^{x} + (2A + 6Bx + 4Ax + 6Bx^{2} + Ax^{2} + Bx^{3})e^{x}$$

$$y'''_p = (6B + 6A + 18Bx + 6Ax + 9Bx^{2} + Ax^{2} + Bx^{3})e^{x}$$

$$y_p^{(4)} = (18B + 6A + 18Bx + 2Ax + 3Bx^{2})e^{x} + (6B + 6A + 18Bx + 6Ax + 9Bx^{2} + Ax^{2} + Bx^{3})e^{x}$$

$$y_p^{(4)} = (24B + 12A + 36Bx + \cancel{Ax}\, 8Ax + 12Bx^{2} + Ax^{2} + Bx^{3})e^{x}$$

$$(24B + 12A + 36Bx + 8Ax + 12Bx^{2} + Ax^{2} + Bx^{3})e^{x} - 2(2A + 6Bx + 4Ax + 6Bx^{2} + Ax^{2} + Bx^{3})e^{x} + \rightarrow$$
$$\hookrightarrow \quad (Ax^{2} + Bx^{3})e^{x} = xe^{x}$$

$$(24B + 12A + 3Bx + 8Ax + 12Bx^{2} + Ax^{2} + Bx^{3})e^{x} + (-4A - 12Bx - 8Ax - 12Bx^{2} - 2Ax^{2} - 2Bx^{3})e^{x} + \rightarrow$$
$$\hookrightarrow \quad (Ax^{2} + Bx^{3})e^{x} = xe^{x}$$

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$(24B + 8A + 24Bx)e^{x} = xe^{x}$

$(24B + 8A)e^{x} + 24Bxe^{x} = xe^{x}$

$24B + 8A = 0$

$24B = 1 \quad \rightarrow \quad 1 + 8A = 0$

$B = 1/24 \qquad A = -1/8$

$$y_p = \left(-\frac{1}{8}x^2 + \frac{1}{24}x^3\right)e^{x}$$

$$y = C_1 e^{x} + C_2 e^{x} + C_3 e^{-x} + C_4 x e^{-x} + \left(-\frac{1}{8}x^2 + \frac{1}{24}x^3\right)e^{x}$$

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$y'' - 5y' + 6y\ (x^2+3)\cos(3x)\,x e^{3x}$

$y_h = C_1 e^{2x} + C_2 e^{3x}$

$$\begin{cases} 1 & y'' - 5y + 6y = (x^2+3)\cos(3x) \\ 2 & y'' - 5y + 6y = -x e^{3x} \end{cases}$$

Para 1

$y_p = x^0\left(\cos(3x)(A_2x^2 + A_1x + A_0) + \operatorname{sen}(3x)(B_2x^2 + B_1x + B_0)\right)$

$y_p = \cos(3x)\left[\underset{A}{A_2x^2} + \underset{B}{A_1x} + \underset{C}{A_0}\right] + \operatorname{sen}(3x)\left[\underset{D}{B_2x^2} + \underset{E}{B_1x} + \underset{F}{B_0}\right]$

Para 2.

$y_p = e^{3x}(Ax^2 + Bx)$

$y_p' = e^{3x}(3Ax^2 + 2Ax + 3Bx + B)$

$y_p'' = e^{3x}(9Ax^2 + 12Ax + 2A + 9Bx + 6B)$

$y_p'' - y_p' + 6y_p = -xe^{3x}$

$e^{3x}(9Ax^2 + 12Ax + 2A + 9Bx + 6B) - 5\cdot e^{3x}(3Ax^2 + 2Ax + 3Bx + B) + 6\cdot e^{3x}(Ax^2 + Bx)$

Simpl.

$2Ae^{3x} + 2Ae^{3x}x + Be^{3x} = -xe^{3x}$

$2Ae^{3x}x + e^{3x}(2A + B) = -1\cdot e^{3x} + 0\cdot e^{3x}$

$2A + B = 0$

$2A = B$

$A = \frac{B}{2}$

$2A = -1$

$A = -\frac{1}{2}$

$B = -1$

$\Rightarrow y_p = e^{3x}\left(-\frac{1}{2}x^2 - x\right)$

Tarea #2

ver parametros

$$y'' - 5y' + 6y = (x^2+3)\cos(3x)$$

$$y_h = C_1 e^{2x} + C_2 e^{3x}$$

$$C_1 = -\int \frac{f(x) \cdot y_2}{W}\,dx$$

$$W = \begin{vmatrix} e^{2x} & e^{3x} \\ 2e^{2x} & 3e^{3x} \end{vmatrix} = e^{5x}$$

$$C_1 = -\int \frac{(x^3+3)\cos(3x) \cdot e^{3x}}{e^{5x}}\,dx \Rightarrow C_1 = -\int e^{-2x}\cos(3x)(x^2+3)\,dx$$

$$C_1 = -e^{2x}\frac{(-2\cos(3x)(169x^2-65x+461)+3(169x^2+104x+513)\cdot \operatorname{sen}(3x))}{2197}$$

$$C_2 = \int e^{-3x}\cos(3x)(x^2+3)\,dx$$

$$C_2 = \frac{1}{54}(-\cos(3x)(9x^2+26)+(9x^2+6x+28)\operatorname{sen}(3x))$$

$$y_p = \cos(3x)\left[\frac{-1}{78}x^2 - \frac{16}{169}x - \frac{3667}{59319}\right] + \operatorname{sen}(3x)\left[\frac{-5}{78}x^2 - \frac{47}{1521}x - \frac{10795}{59319}\right]$$

$$y = \cos(3x)\left[\frac{-1}{78}x^2 - \frac{16}{169}x - \frac{3667}{59319}\right] + \operatorname{sen}(3x)\left[\frac{-5}{78}x^2 - \frac{47}{1521} - \frac{10795}{59319}\right] +$$

$$C_1 e^{2x} + C_2 e^{3x} + e^{3x}\left(\frac{-1}{9}x^2 - x\right)$$