Práctica Laplace

① $\mathcal{L}\left\{4 + t\cos(4t) - 3te^{5t}\operatorname{Sen}t\right\}$

$$4\mathcal{L}[1] + \underbrace{\mathcal{L}[\overset{\text{derive}}{t}\cos 4t]} - 3\mathcal{L}[te^{5t}\operatorname{Sen}t]$$

$$\mathcal{L}[\overset{\text{derive}}{t}\cos 4t] = \frac{s}{s^2+16} = \frac{1\cdot(s^2+16) - s\cdot 2s}{(s^2+16)^2} = \frac{s^2+16-2s^2}{(s^2+16)^2}$$

$$= \frac{-s^2+16}{(s^2+16)^2} = \boxed{\frac{s^2-16}{(s^2+16)^2}} \; //$$

$$\mathcal{L}\left[t\, e^{5t}\, \text{Sen}t\right] = \frac{1}{s^2+1} = \frac{\cancel{0\cdot(s^2+1)} - 1\cdot 2s}{(s^2+1)^2} = \frac{-2s}{(s^2+1)^2}$$

derivar

e^{5t}

$$= \frac{2s}{(s^2+1)^2} = \frac{2(s-5)}{((s-5)^2+1)^2}$$

Retomando

$$4\mathcal{L}[1] + \mathcal{L}[t\cos 4t] - 3\,\mathcal{L}\left[t\, e^{5t}\, \text{Sen}t\right]$$

$$4\cdot\frac{1}{s} + \frac{s^2-16}{(s^2+16)^2} - 3\cdot\frac{2(s-5)}{((s-5)^2+1)^2} = \boxed{\frac{4}{s} + \frac{s^2-16}{(s^2+16)^2} - \frac{6(s-5)}{((s-5)^2+1)^2}}$$

② $\mathcal{L}^{-1}\left\{\frac{6}{3s+2}+\frac{9s}{3s^2+12}\right\}$

$$6\,\mathcal{L}^{-1}\left\{\frac{1}{3s+2}\right\}+9\,\mathcal{L}^{-1}\left\{\frac{s}{3s^2+12}\right\}$$

$$6\,\mathcal{L}^{-1}\left\{\frac{1}{3\left(s+2/3\right)}\right\}+9\,\mathcal{L}^{-1}\left\{\frac{s}{3\left(s^2+4\right)}\right\}$$

$$6\cdot\frac{1}{3}\,\mathcal{L}^{-1}\left\{\frac{1}{s+2/3}\right\}+\frac{9}{3}\,\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\}$$

$$2e^{-2/3t}+3\cos 2t \quad //$$

③ $\mathcal{L}^{-1}\left\{\frac{4s}{(s+2)(s^2+4)}\right\}$

$$\frac{4s}{(s+2)\cancel{(s^2+4)}} = \frac{A}{(s+2)} + \frac{Bs+C}{(s^2+4)} = \frac{A(s^2+4)+(s+2)(Bs+C)}{(s+2)\cancel{(s^2+4)}}$$

$$4s = As^2 + 4A + Bs^2 + Cs + 2Bs + 2C$$

$0 = A + B$

$4 = C + 2B$

$0 = 4A + 2C$

mode 5.2

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 4 \\ 4 & 0 & 2 & 0 \end{array}\right]$$

$A = -1$

$B = 1$

$C = 2$

$$\mathcal{L}^{-1}\left[\frac{A}{s+2}\right] + \mathcal{L}^{-1}\left[\frac{Bs+C}{s^2+4}\right]$$

$A = -1 \quad C = 2$

$B = 1$

$$\mathcal{L}[\operatorname{Sen} at] = \frac{a}{s^2+a^2}$$

$$-1\,\underbrace{\mathcal{L}^{-1}\left[\frac{1}{s+2}\right]} + \underbrace{\mathcal{L}^{-1}\left[\frac{s+2}{s^2+4}\right]}$$

$$-e^{-2t} + \mathcal{L}^{-1}\left[\frac{s}{s^2+4}\right] + \frac{2}{2}\mathcal{L}^{-1}\left[\frac{1\cdot 2}{s^2+4}\right]$$

$$-e^{-2t} + \cos 2t + \operatorname{Sen} 2t \;\; //$$

(4) $\mathcal{L}^{-1}\left\{\dfrac{3s-4}{s^2-4s+2}\right\}$

s^2-4s+2

$s^2-4s+2+4-4$

$(s^2-4s+4)-2$

$(s-2)^2-2$

$\dfrac{4}{2} = (2)^2 = 4$

$\mathcal{L}^{-1}\left\{\dfrac{3s-4}{(s-2)^2-2}\right\}$

$\mathcal{L}[\cosh at] = \dfrac{s}{s^2-a^2}$

$3\mathcal{L}^{-1}\left\{\dfrac{s-2+2}{(s-2)^2-2}\right\} - 4\mathcal{L}^{-1}\left[\dfrac{1}{(s-2)^2-2}\right]$

$$3\mathcal{L}^{-1}\left\{\frac{s-2+2}{(s-2)^2-2}\right\}-4\mathcal{L}^{-1}\left[\frac{1}{(s-2)^2-2}\right]$$

$$3\left[\mathcal{L}^{-1}\left\{\underset{\dfrac{s}{s^2-a^2}}{\frac{s-2}{(s-2)^2-2}}\right\}+\frac{2}{\sqrt{2}}\mathcal{L}^{-1}\left\{\underset{\dfrac{1}{s^2-2}}{\frac{1\cdot\sqrt{2}}{(s-2)^2-2}}\right\}\right]-\frac{4}{\sqrt{2}}\mathcal{L}^{-1}\left[\frac{1\cdot\sqrt{2}}{(s-2)^2-2}\right]$$

$$3\left[e^{2t}\cosh(\sqrt{2}t)+\frac{2}{\sqrt{2}}\operatorname{Senh}(\sqrt{2}t)\cdot e^{2t}\right]-\frac{4}{\sqrt{2}}\operatorname{Senh}(\sqrt{2}t)\,e^{2t}$$

$$3\left[e^{2t}\cosh(\sqrt{2}t)+\frac{2}{\sqrt{2}}\operatorname{Senh}(\sqrt{2}t)\cdot e^{2t}\right]-\frac{4}{\sqrt{2}}\operatorname{Senh}(\sqrt{2}t)\,e^{2t}$$

$$3e^{2t}\cosh(\sqrt{2}t)+\frac{6}{\sqrt{2}}e^{2t}\operatorname{Senh}(\sqrt{2}t)-\frac{4}{\sqrt{2}}e^{2t}\operatorname{Senh}(\sqrt{2}t)$$

$$3e^{2t}\cosh(\sqrt{2}t)+\sqrt{2}\,e^{2t}\operatorname{Senh}(\sqrt{2}t)\;//$$

⑤ $\mathcal{L}\{(t+1)\cdot u(t-2)\}$

$\mathcal{L}\{(t+1)\cdot u_{\underline{\underline{2}}}(t)\}$

$\mathcal{L}\{(t+\underline{\underline{1}}-2+\underline{\underline{2}})\cdot u_2(t)\}$

$\mathcal{L}\{(\cancel{t-2})\cdot u_2(t)\}+3\mathcal{L}\{u_2(t)\}$

$\frac{1}{s^2}\cdot e^{-2s} + \frac{3e^{-2s}}{s}$

(5) Otra forma:

$$\mathcal{L}\{(t+1)\cdot \mu_2(t)\}$$

$$\mathcal{L}\{t\cdot \mu_2(t)\} + \underbrace{\mathcal{L}\{\mu_2(t)\}}$$

$$\frac{2e^{-2s}}{s} + \frac{e^{-2s}}{s^2} + \frac{e^{-2s}}{s} = \frac{3e^{-2s}}{s} + \frac{e^{-2s}}{s^2} \quad //$$

$$\mathcal{L}\{t\cdot \mu_2(t)\} = \frac{e^{-2s}}{s} = \frac{-2e^{-2s}\cdot s - e^{-2s}\cdot 1}{s^2} = \frac{-2e^{-2s}\cdot \cancel{s}}{s^{\cancel{2}}} - \frac{e^{-2s}}{s^2} =$$

derivada

$$\frac{-2e^{-2s}}{s} - \frac{e^{-2s}}{s^2} = \frac{2e^{-2s}}{s} + \frac{e^{-2s}}{s^2}$$

(6) $\mathcal{L}\{(2t-7)\cdot u(t-3)\}$

$$\mathcal{L}\{(2t-7)\cdot u_3(t)\} \Rightarrow \mathcal{L}\left\{2\left(t-\frac{7}{2}-3+3\right)\cdot u_3(t)\right\}$$

$$2\left\{\mathcal{L}\,(t-3)\cdot u_3(t) - \frac{1}{2}\,\mathcal{L}\left[u_3(t)\right]\right\}$$

$$2\,\underbrace{\mathcal{L}\,(t\cancel{-3})\cdot u_3(t)} - \mathcal{L}\left[u_3(t)\right]$$

$$2\cdot\frac{1}{s^2}e^{-3s} - \frac{e^{-3s}}{s} \Rightarrow \frac{2}{s^2}e^{-3s} - \frac{e^{-3s}}{s} \quad //$$

⑥ Otra forma

$$\mathcal{L}\{(2t-7)\cdot u_3(t)\}$$

$$\mathcal{L}\{2t\cdot u_3(t)\} - 7\mathcal{L}\{u_3(t)\}$$

$$2\mathcal{L}\{t\cdot u_3(t)\} - 7\mathcal{L}\{u_3(t)\}$$

Calcular $u_3(t)$

Derivar una vez

⑦ $\mathcal{L}\{t^2\, \mathcal{U}_1(t)\}$

derivo 2 veces

$$\mathcal{L}[\mathcal{U}_a(t)] = \frac{e^{-as}}{s}$$

$$\mathcal{L}\{t^2 \cdot \mathcal{U}_1(t)\} = \frac{e^{-s}}{s} = -\frac{e^{-s}\cdot s - e^{-s}\cdot 1}{s^2} = \frac{se^{-s}}{s^2} + \frac{e^{-s}}{s^2} = \underbrace{\frac{e^{-s}}{s} + \frac{e^{-s}}{s^2}}_{\text{deriva 1 vez}}$$

$$-\frac{e^{-s}\cdot s - e^{-s}\cdot 1}{s^2} + \frac{-e^{-s}\cdot s^2 - e^{-s}\cdot 2s}{s^4} = \frac{se^{-s}}{s^2} + \frac{e^{-s}}{s^2} + \frac{s^2e^{-s}}{s^4} + \frac{2se^{-s}}{s^4}$$

$$= \frac{e^{-s}}{s} + \frac{e^{-s}}{s^2} + \frac{e^{-s}}{s^2} + \frac{2e^{-s}}{s^3}$$

$$= \frac{e^{-s}}{s} + \frac{e^{-s}}{s^2} + \frac{e^{-s}}{s^2} + \frac{2e^{-s}}{s^3}$$

$$= \frac{e^{-s}}{s} + \frac{2e^{-s}}{s^2} + \frac{2e^{-s}}{s^3} \quad //$$

⑧ $\mathcal{L}^{-1}\left\{\dfrac{4+5e^{-2s}}{s+4}\right\}$

$$\mathcal{L}\left[e^{at}\right] = \frac{1}{s-a}$$

$$\underbrace{4\,\mathcal{L}^{-1}\left[\frac{1}{s+4}\right]}_{} + 5\,\mathcal{L}^{-1}\left[\frac{e^{-2s}}{s+4}\right]$$

$$4e^{-4t} + 5\,e^{-4(t-2)}\cdot u_2(t) \;\;//$$

$$\mathcal{L}^{-1}\left[\frac{e^{-2s}}{s+4}\right] \Rightarrow \mathcal{L}^{-1}\left[\frac{1}{s+4}\right] = e^{-4t} = e^{-4(t-2)} \cdot u_2(t)$$

$\mathcal{L}$

→ Tabla

→ Desplazamiento

$\mathcal{L}[t \cdot \mathcal{U}_2(t)]$

$\mathcal{L}[e^{at} F(t)] = $ calculo Laplace, Corro "a" unidades, cambio la "s" por "s-a"

→ $\mathcal{L}[\mathcal{U}_a(t)] = \dfrac{e^{-as}}{s}$

→ $\mathcal{L}[\mathcal{U}_a(t) F(t-a)] = $ Calculo Laplace sin correr

- e^{-as}

→ $\mathcal{L}[t^n F(t)]$.

$\Downarrow$

- Calcule Laplace
- Derive las veces q' el exponente le diga
- Cambie el signo

* Por cada derivada se cambio el signo, antes de volver a derivar

$\mathcal{L}^{-1}$

$$\mathcal{L}^{-1}\left[\frac{e^{-2s}}{s+4}\right]$$

- Tabla
- Fracciones Parciales
- Completar cuadrados
- $\mathcal{L}^{-1}\left[e^{-as}F(s)\right] = \begin{cases} \text{Calcule } \mathcal{L}^{-1} \\ \text{Desplazar "a" unidades (Cambio "t" por "t-a")} \\ \cdot\ \mathcal{U}_a(t) \end{cases}$