$$\sum_{i=1}^{\infty} \frac{7^n + 5\cos^2(n) + 1}{1+3^{2n}}$$

$$0 \leq \cos^2(n) \leq 1$$

$$0 \leq 5\cos^2 n \leq 5$$

$$7^n + 1 \leq 5\cos^2 n + 7^n + 1 \leq 6 + 7^n$$

$$\frac{7^n+1}{1+3^n} \leq \underbrace{\frac{5\cos^2 n + 7^n + 1}{1+3^{2n}}}_{a_n} \leq \underbrace{\frac{6+7^n}{1+3^{2n}}}_{b_n}$$

$$\sum_{i=1}^{\infty} \frac{6+7^n}{1+3^{2n}} = \sum_{i=1}^{\infty} \frac{6}{1+3^{2n}} + \sum_{i=1}^{\infty} \frac{7^n}{1+3^{2n}}$$

Como $(3^2)^n >> 1$

$$q^n >> 1$$

Se reescribe

$$6\sum_{i=1}^{\infty} \frac{1}{q^n} + \sum_{i=1}^{\infty} \left(\frac{7}{q^n}\right)$$

$$6 \cdot \sum_{i=1}^{\infty} \left(\frac{1}{q}\right)^n + \sum_{i=2}^{\infty} \left(\frac{7}{q}\right)^n$$

como $\left|\frac{1}{q}\right| < 1$ y $\left|\frac{7}{q}\right| < 1$:)

Ambas convergen

Y como $\sum_{i=1}^{\infty} b_n$ converge

Por comparacion directa

$$\therefore \sum_{i=1}^{\infty} \frac{7^n + 5\cos^2(n) + 1}{1+3^{2n}}$$ converge