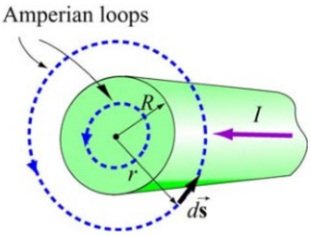
Problema 2. (20 puntos) La ley de Ampere es aplicable a las siguientes configuraciones actuales: Conductor coaxial infinito, lámina infinita, solenoide infinito y toroide.

a) **Calcule el campo dentro y fuera de un cable portador de corriente.** Considere un cable recto largo de radio R que lleva una corriente I de densidad de corriente uniforme, como se muestra en la Figura.
1. Encuentra el campo magnético en todas partes.
2. Calcule la inductancia de un conductor coaxial infinito.



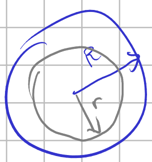
$$\oint \vec{B}\, d\vec{l} = \mu_0 I_{enc}$$

$I_{total} = I$

$A_{total} = \pi R^2$

$S = 2\pi r$

1- Para $r < R$



$$\oint \vec{B} \cdot d\vec{S} = \mu_0 I_{enc}$$

$$J = \frac{I_{total}}{A_{total}} \rightsquigarrow \frac{I}{\cancel{\pi} R^2} = \frac{I_{enc}}{\cancel{\pi} r^2}$$

$$I_{enc} = \frac{I r^2}{R^2}$$

$$B \cdot S = \mu_0 I_{en}$$

$$B = \frac{\mu_0 I_{enc}}{S}$$

$$B = \frac{\mu_0 I_{enc}}{2\pi r} \longrightarrow B = \frac{\mu_0}{2\pi \cancel{r}}\left(\frac{I r^{\cancel{2}}}{R^2}\right)$$

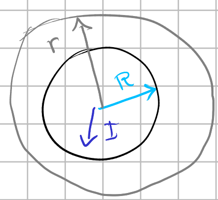
$$\boxed{B = \frac{\mu_0 I r}{2\pi R^2} \quad \text{para} \quad r < R}$$

Para $r > R$

$$\oint \vec{B} \cdot d\vec{S} = \mu_0 I_{enc}$$

$$J = \frac{I_{total}}{A_{total}}$$

$I_{encerrada} = I$

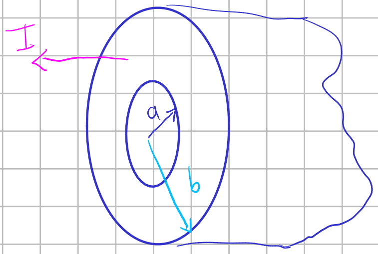


$$B \cdot S = \mu_0 I_{en}$$

$$B = \frac{\mu_0 I_{enc}}{S}$$

$$B = \frac{\mu_0 I_{enc}}{2\pi r} \longrightarrow \boxed{B = \frac{\mu_0 I}{2\pi r} \quad \text{Para} \quad r > R}$$

2- Conductor coaxial infinito



$$\phi = \int B \cdot dr$$

$$\phi = \int_a^b \frac{\mu_0 I}{2\pi r} dr$$

$$\phi = \frac{\mu_0 I}{2\pi} \int_a^b \frac{dr}{r}$$

$$\phi = \frac{\mu_0 I}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$L = \frac{N\phi}{I} \qquad N=1$$

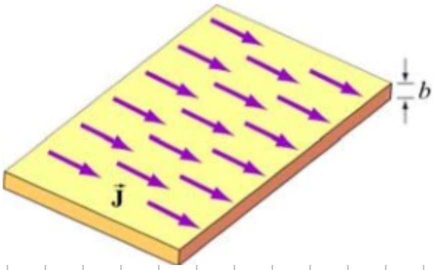
$$L = \frac{\phi}{I}$$

$$B = \frac{\mu_0 I}{2\pi r}$$

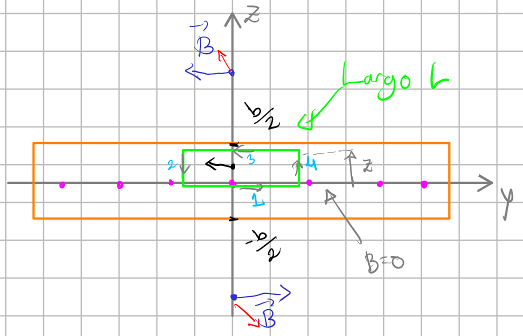
$$L = \frac{\mu_0 \cancel{I}}{2\pi} \ln\left(\frac{b}{a}\right) / \cancel{I}$$

$$\boxed{L = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right) [H/m]} \quad R/$$

b) **Campo magnético debido a una hoja de corriente infinita.** Considere una hoja infinitamente grande de espesor b que se encuentra en el plano xy con una uniforme densidad de corriente $\vec{J} = J_0\vec{a_x}$. 1. Encuentra el campo magnético en todas partes: $para\ z > \frac{b}{2}$ $,para\ -\frac{b}{2} < z < \frac{b}{2},\ para\ z < -\frac{b}{2}$.



$\vec{J} = J_0 \vec{a_x}$



$$\boxed{\text{Para el plano XY } \vec{B} = 0}$$

$\oint B\,ds = \mu_0 I_{enc}$

$$\oint_C \vec{B}\,d\vec{s} = \underbrace{\cancel{\oint_1 \vec{B}\,d\vec{s}}}_{0} + \underbrace{\cancel{\oint_2 \vec{B}\,d\vec{s}}}_{0} + \oint_3 \vec{B}\,d\vec{s} + \underbrace{\cancel{\oint_4 \vec{B}\,d\vec{s}}}_{0}$$

$$\oint_C \vec{B}\,d\vec{s} = \oint_3 \vec{B}\cdot d\vec{s}$$
$$= \oint B\,ds \cdot \cos 0°$$
$$= B\,s$$
$$= BL$$

$$\oint_C \vec{B}\cdot d\vec{s} = \mu_0 I_{enc}$$
$$B\cdot L = \mu_0 I_{enc}$$
$$\boxed{B = \frac{\mu_0 I_{enc}}{L}}$$

$I_{enc} = J\cdot A$ $\qquad$ $A = z\cdot L$

$I_{enc} = J_0 \cdot zL$

$$\vec{B} = \frac{\mu_0 J z L}{L}\,(a_y)$$

$$\vec{B} = \mu_0 J_0 z\ (a_y)$$

* Para $-\frac{b}{2} < z < \frac{b}{2}$

$$\vec{B} = -\mu_0 J_0 z\ (a_y)$$

* Para $z > \frac{b}{2}$

$$\vec{B} = -\mu_0 J_0 \frac{b}{2}\ (a_y)$$

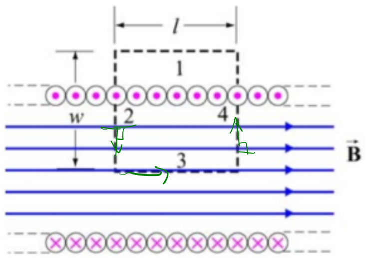
* Para $z < -\frac{b}{2}$

$$\vec{B} = \mu_0 J_0 \frac{b}{2}\ (a_y)$$

R/

c) **1. Calcule el campo magnético de un solenoide ideal de N vueltas y**
2. Calcule la inductancia de un solenoide ideal.

$$\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = \int_1 \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} + \int_2 \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} + \int_3 \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} + \int_4 \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}}$$
$$= 0 + 0 + Bl + 0$$



$$\oint \vec{B}\cdot d\vec{s} = \oint_3 \vec{B}\, d\vec{s}$$
$$= \oint_3 B\, ds \cdot \cos 0°$$
$$= \int_l B\, ds$$
$$= \underline{B\cdot l}$$

Por ley de Amperè

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{enc}$$
$$B\cdot l = \mu_0\, n\, l\, I$$
$$B = \frac{\mu_0\, n\, \cancel{l}\, I}{\cancel{l}}$$

$$n = \frac{\#\text{Vuetas}}{\text{Longitud}} = \frac{N}{l} \rightsquigarrow N = n\,l$$

$$I_{enc} = \#\text{Vuetas}\cdot I \qquad n = \frac{N}{l}$$
$$I_{enc} = N\cdot I$$
$$I_{enc} = n\, l\, I$$

$$B = \mu_0\, n\, I$$

$$\boxed{B = \frac{\mu_0 N I}{l}\ [T]}\ R/_1$$

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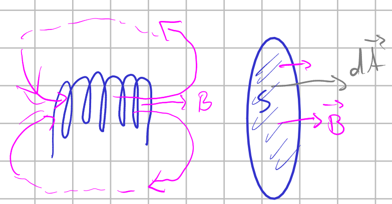
2 - Inductancia

$$\Phi_M = L\cdot I$$
$$L = \frac{\Phi_M}{I}$$
$$\Phi_M = \oint_S \vec{B}\, d\vec{A}$$



$$\Phi_{M_1} = \oint_S B\, dA \cos 0°$$
$$\Phi_{M_1} = B\cdot A$$
$$\Phi_{M\,Total} = N\cdot B\cdot A$$
$$\Phi_M = N\cdot\left(\frac{\mu_0 N I}{l}\right)\cdot A$$
$$\Phi_M = \frac{N^2 \mu_0 I A}{l}$$

A = area del solenoide

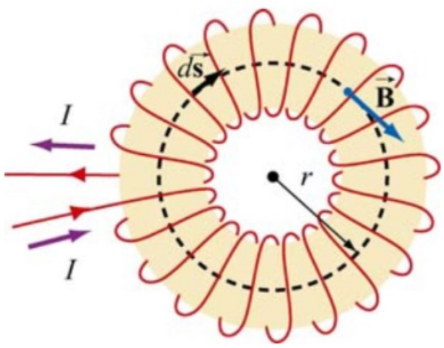
$$L = \frac{\Phi_M}{I}$$
$$L = \frac{\left[\frac{N^2 \mu_0 I A}{l}\right]}{I}$$

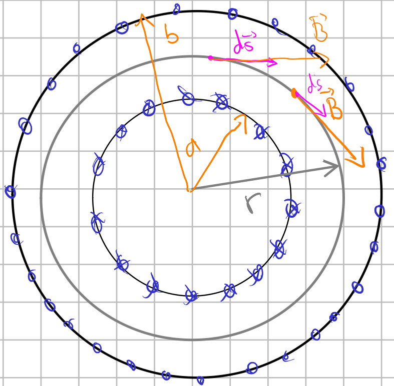
$$\boxed{L = \frac{\mu_0 N^2 A}{l}}\ R$$

d) **Considere un toroide que consta de N vueltas**, como se muestra en la figura siguiente.

1. **Encuentra el magnético campo en todas partes y la inductancia.**
2. **Calcule la inductancia de un ~~solenoide~~ ideal.**

Toroide





$$\oint \vec{B}\, d\vec{s} = \oint_c B\, ds \cos 0^\circ$$

$$= \oint_c B\, ds$$

$$= B \int_c ds$$

$$= B \cdot C$$

$$C = 2\pi r$$

$$\oint \vec{B}\, d\vec{s} = B \cdot 2\pi r$$

$$I_{enc} = N \cdot I$$

$$\oint_c \vec{B}\, d\vec{s} = \mu_0 I_{enc}$$

$$B \cdot 2\pi r = \mu_0 N I$$

$$B = \frac{\mu_0 N I}{2\pi r} \quad [T]$$

* Para $a < r < b$ [dentro]

$$B = \frac{\mu_0 N I}{2\pi r}$$

* Para $r > b$ ó $r < a$ [fuera]

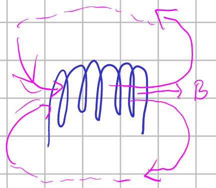
$$B = 0$$

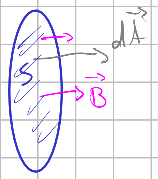
R/1

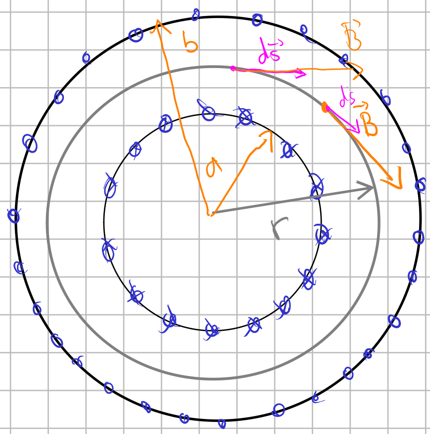
$\phi_M = L \cdot I$

$L = \frac{\phi_M}{I}$

$\phi_M = \oint_S \vec{B} \cdot d\vec{A}$







$\phi_M = \int B \, dA \cos 0^\circ$

$\phi_{M_1} = \int B \cdot dA$

$\phi_{M_1} \approx B A$

$\phi_M \approx N \cdot B \cdot A$

$\phi_M \approx N \cdot \left(\frac{\mu_0 N I}{2\pi r}\right) \cdot A$

$\phi_M \approx \frac{\mu_0 N^2 I A}{2\pi r}$

$B = \frac{\mu_0 N I}{2\pi r}$

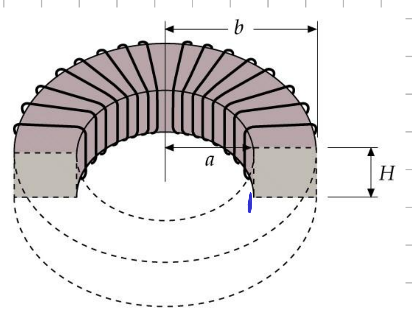
$L \approx \phi_M / I$

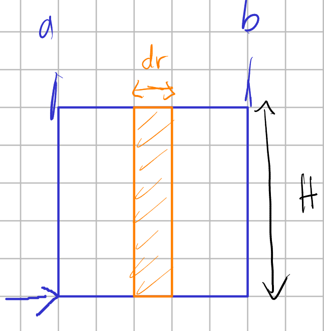
$L \approx \frac{\mu_0 N^2 \cancel{I} A}{2\pi r} / \cancel{I}$

$$\boxed{L \approx \frac{\mu_0 N^2 A}{2\pi r}}$$ R/.

Por Aproximació

Respuesta exacta





$dA = N \cdot H \cdot dr$

$\phi_M = \int B \cdot dA$

$\phi_M = \int \frac{\mu_0 N I}{2\pi r} N H \cdot dr$

$\phi_M = \frac{\mu_0 N^2 I H}{2\pi} \int_a^b \frac{dr}{r}$

$\phi_M = \frac{\mu_0 N^2 I H}{2\pi} \cdot \ln\left(\frac{b}{a}\right)$

$\Rightarrow L = \frac{\phi_M}{I} = \left[\frac{\mu_0 N^2 \cancel{I} H}{2\pi} \cdot \ln\left(\frac{b}{a}\right)\right] / \cancel{I}$

$$\boxed{L = \frac{\mu_0 N^2 H}{2\pi} \ln\left(\frac{b}{a}\right)}$$ R/ exacta

Asumiendo A rectangular //