② $f(x)=\begin{cases}-x, & -1\le x\le 0\\ 0, & 0\le x\le 1\end{cases} \Rightarrow f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)+\sum_{n=1}^{\infty} b_n \operatorname{Sen}\left(\frac{n\pi x}{L}\right)$

$\Rightarrow a_0=\frac{1}{L}\int_{-L}^{L} f(x)\,dx=\frac{1}{1}\left[\int_{-1}^{0}(-x)\,dx+\int_{0}^{1}0\,dx\right]=\int_{-1}^{0}(-x)\,dx+\underbrace{\int_0^1 0\,dx}_{\text{"0"}}$

$a_0=-\frac{x^2}{2}\Big|_{-1}^{0}=-\frac{0^2}{2}+\frac{(-1)^2}{2}=\frac{1}{2}.$

$a_n=\frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx=\frac{1}{1}\left[\int_{-1}^{0}-x\cos(n\pi x)\,dx+\underbrace{\int_0^1 0\cdot\cos(n\pi x)\,dx}_{\text{"0"}}\right]$

$a_n=-\int_{-1}^{0}x\cos(n\pi x)\,dx \Rightarrow x=u \Rightarrow du=dx,\ \cos(n\pi x)\,dx=dv \Rightarrow v=\frac{1}{n\pi}\operatorname{Sen}(n\pi x)$

$a_n=-\left(\frac{x}{n\pi}\operatorname{Sen}(n\pi x)\Big|_{-1}^{0}-\int_{-1}^{0}\frac{1}{n\pi}\operatorname{Sen}(n\pi x)\,dx\right)=\frac{-x}{n\pi}\operatorname{Sen}(n\pi x)\Big|_{-1}^{0}-\frac{1}{n^2\pi^2}\cos(n\pi x)\Big|_{-1}^{0}$

$a_n=\underbrace{\frac{-0}{n\pi}\operatorname{Sen}(n\pi\cdot 0)+\frac{1}{n\pi}\operatorname{Sen}(\pi n)}_{\text{"0"}}-\frac{1}{n^2\pi^2}\cos(n\pi\cdot 0)+\frac{1}{n^2\pi^2}\cos(n\pi)$

$a_n=\frac{-1}{n^2\pi^2}+\frac{1}{n^2\pi^2}(-1)^n \quad \left\{\cos(n\pi)\begin{cases}-1 \Rightarrow n=\text{Impar}\\ 1 \Rightarrow n=\text{Par}\end{cases}\right.$

$b_n=\frac{1}{L}\int_{-L}^{L} f(x)\operatorname{Sen}\left(\frac{n\pi x}{L}\right)dx=\frac{1}{1}\left[\int_{-1}^{0}-x\operatorname{Sen}(n\pi x)\,dx+\underbrace{\int_0^1 0\cdot\operatorname{Sen}(n\pi x)\,dx}_{\text{"0"}}\right]$

$b_n=-\int_{-1}^{0}x\operatorname{Sen}(n\pi x)\,dx \Rightarrow x=u \Rightarrow dx=du,\ \operatorname{Sen}(n\pi x)\,dx=dv \Rightarrow v=\frac{-1}{n\pi}\cos(n\pi x)$

$b_n=-\left(\frac{-x}{n\pi}\cos(n\pi x)\Big|_{-1}^{0}-\int_{-1}^{0}\frac{-1}{n\pi}\cos(n\pi x)\,dx\right)=\frac{x}{n\pi}\cos(n\pi x)\Big|_{-1}^{0}-\frac{1}{n^2\pi^2}\operatorname{Sen}(n\pi x)\Big|_{-1}^{0}$

$b_n=\underbrace{\frac{0}{n\pi}\cos(n\pi\cdot 0)}_{\text{"0"}}-\frac{(-1)}{n\pi}\cos(-n\pi)-\underbrace{\frac{1}{n^2\pi^2}\operatorname{Sen}(n\pi\cdot 0)+\frac{1}{n^2\pi^2}\operatorname{Sen}(n\pi\cdot -1)}_{\text{"0"}}$

$b_n=\frac{1}{n\pi}(-1)^n$ porque $\cos(-n\pi)\begin{cases}-1, \text{"n" impar}\\ 1, \text{"n" par.}\end{cases}$

$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n\cos\left(\frac{n\pi x}{L}\right)+\underbrace{\sum_{n=1}^{\infty} b_n\operatorname{Sen}\left(\frac{n\pi x}{L}\right)}_{\text{"0"}}$

$f(x)=\frac{1}{4}+\frac{-2}{\pi}\cos(\pi x)+\frac{-2}{\pi}\cos(3\pi x)+\frac{-2}{\pi}\cos(5\pi x)+\ldots+\underbrace{\frac{-1}{\pi}\operatorname{Sen}(\pi x)+\frac{1}{\pi}\operatorname{Sen}(2\pi x)+\ldots}_{\text{"0"}}$

R/ $f(x)=\frac{1}{4}-\frac{2}{\pi}\cos(\pi x)-\frac{2}{\pi}\cos(3\pi x)-\frac{2}{\pi}\cos(5\pi x)+\ldots$

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