Halle la solución de la ecuación :

$$z^4-(2i+3)z^2+6i$$

10pts

$$z^4-(2i+3)z^2+6i=0$$

$$\begin{matrix} z^2 & & -3 \\ & \times & \\ z^2 & & -2i \end{matrix}$$

$$(z^2-3)(z^2-2i)=0$$

$$z^2-3=0$$

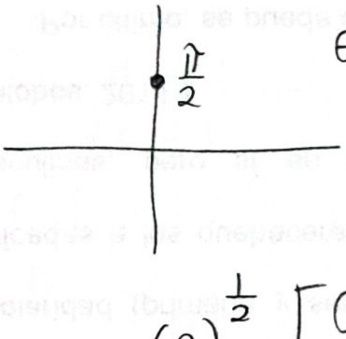
$$z=\pm\sqrt{3}$$

$$z^2-2i=0$$

$$z=\sqrt{2i}$$

$$z=\sqrt{2i}$$

$$r=\sqrt{(0)^2+(2)^2}=2$$



$$\theta=\frac{\pi}{2}$$

$$K=0 \Rightarrow (2)^{\frac{1}{2}}\left[\cos\frac{90}{2}+i\operatorname{Sen}\frac{90}{2}\right]$$

$$\Rightarrow \sqrt{2}\left[\cos 45+i\operatorname{Sen}45\right)=1+i$$

$$K=1 \Rightarrow \sqrt{2}\left[\cos 225+i\operatorname{Sen}225\right]=-1-i$$

Encontrar la serie de Laurent de la función

$$\frac{1}{z^2+9} \quad \text{en} \quad |z-4|<5$$

(10pts)

$$z^2+9 = (z-3i)(z+3i)$$

$$\frac{1}{(z-3i)(z+3i)} = \frac{A}{(z-3i)} + \frac{B}{(z+3i)} = \frac{A(z+3i)+B(z-3i)}{(z-3i)(z+3i)}$$

$$1 = \underline{\underline{Az}} + 3Ai + \underline{\underline{Bz}} - 3Bi \qquad A+B=0 \rightarrow A=-B$$

$$3Ai - 3Bi = 1$$

$$3Ai - 3Bi = 1$$

$$-3Bi - 3Bi = 1$$

$$-6Bi = 1$$

$$B = \frac{1}{-6i}\cdot\frac{i}{i} = \frac{i}{6} \qquad A = -\frac{i}{6}$$

$$f(z) = -\frac{i}{6}\cdot\frac{1}{z+3i} + \frac{i}{6}\cdot\frac{1}{(z-3i)}$$

$$\frac{1}{z-4+4+3i} = \frac{1}{(z-4)+(4+3i)} = \frac{1}{(4+3i)\left[\frac{z-4}{4+3i}+1\right]}$$

$$= \frac{1}{4+3i}\cdot\sum_{n=0}^{\infty}(-1)^n\left(\frac{z-4}{4+3i}\right)^n$$

$$= \frac{1}{4+3i}\cdot\left[1 - \frac{(z-4)}{(4+3i)} + \frac{(z-4)^2}{(4+3i)^2} - \frac{(z-4)^3}{(4+3i)^3} + \frac{(z-4)^4}{(4+3i)^4} + \cdots\right]$$

$$= \frac{1}{4+3i} - \frac{(z-4)}{(4+3i)^2} + \frac{(z-4)^2}{(4+3i)^3} - \frac{(z-4)^3}{(4+3i)^4} + \frac{(z-4)^4}{(4+3i)^5} + \cdots$$

$$\frac{1}{z-3i} = \frac{1}{z-4+4-3i} = \frac{1}{(z-4)+(4-3i)} =$$

$$\frac{1}{(4-3i)\left[\frac{z-4}{4-3i}+1\right]} = \frac{1}{(4-3i)} \cdot \sum_{n=0}^{\infty} (-1)^n \left(\frac{z-4}{4-3i}\right)^n$$

$$= \frac{1}{4-3i} \cdot \left[1 - \frac{(z-4)}{4-3i} + \frac{(z-4)^2}{(4-3i)^2} - \frac{(z-4)^3}{(4-3i)^3} + \frac{(z-4)^4}{(4-3i)^4} + \cdots\right]$$

$$= \frac{1}{4-3i} - \frac{(z-4)}{(4-3i)^2} + \frac{(z-4)^2}{(4-3i)^3} - \frac{(z-4)^3}{(4-3i)^4} + \frac{(z-4)^4}{(4-3i)^5} - \cdots$$

$$f(z) = -\frac{i}{6}\left[\frac{1}{4+3i} - \frac{(z+4)}{(4+3i)^2} + \frac{(z-4)^2}{(4+3i)^3} - \frac{(z-4)^3}{(4+3i)^4} + \frac{(z-4)^4}{(4+3i)^5} - \cdots\right] +$$

$$\frac{i}{6}\left[\frac{1}{4-3i} - \frac{(z+4)}{(4-3i)^2} + \frac{(z-4)^2}{(4-3i)^3} - \frac{(z-4)^3}{(4-3i)^4} + \frac{(z-4)^4}{(4-3i)^5} + \cdots\right]$$

$$f(z) = \frac{-i}{6(4+3i)} + \frac{i(z+4)}{6(4+3i)^2} - \frac{i(z-4)^2}{6(4+3i)^3} + \frac{i(z-4)^3}{6(4+3i)^4} + \frac{i}{6(4-3i)}$$

$$-\frac{i(z+4)}{6(4-3i)^2} + \frac{i(z-4)^2}{6(4-3i)^3} - \frac{i(z-4)^3}{6(4-3i)^4} + \frac{i(z-4)^4}{(4-3i)^5}$$

$$f(z) = \left(\frac{-i}{6(4+3i)} + \frac{i}{6(4-3i)}\right) + \left(\frac{i}{6(4+3i)^2} - \frac{i}{6(4-3i)^2}\right)(z+4) +$$

$$\left(\frac{-i}{6(4+3i)^3} + \frac{i}{6(4-3i)^3}\right)(z+4)^2 + \left(\frac{i}{6(4+3i)^4} - \frac{i}{6(4-3i)^4}\right)(z+4)^3 + \cdots$$

Calcule las singularidades de la siguiente función $f(z) = \frac{z+1}{z^3(z^2+1)}$ y clasifíquelas según corresponda. (5 pts)

Además calcule el residuo en $z = i$ $z = -i$

$z^3 = 0$ $\quad$ $z^2+1 = 0$

$z = 0$ $\quad$ $z = \pm i$

En $z = i$

$$f(z) = \frac{z+1}{z^3(z-i)(z+i)} = \frac{i+1}{0} = \infty \text{ Polo}$$

$$\lim_{z\to i} \cancel{(z-i)} \cdot \frac{z+1}{z^3\cancel{(z-i)}(z+i)} = \frac{i+1}{i^3(2i)} = \frac{i+1}{2i^4} = \frac{i+1}{2}$$

En $z = -i$

$$\lim_{z\to -i} \frac{z+1}{z^3(z-i)(z+i)} = \frac{i+1}{0} = \infty \text{ Polo}$$

$$\lim_{z\to -i} \frac{\cancel{(z+i)}(z+1)}{z^3(z-i)\cancel{(z+i)}} = \frac{-i+1}{-i^3 \cdot -2i} = \frac{-i+1}{2i^4} = \frac{-i+1}{2}$$

En $z = 0$

$$\lim_{z\to 0} \frac{z+1}{z^3(z-i)(z+i)} = \frac{1}{0} = \infty \text{ Polo}$$

b) Sabiendo que $z=0$ es un polo de orden 3. Calcule el residuo (5puntos)

$$a_{-1} = \frac{1}{(m-1)!} \lim_{z\to 0} \left\{ \frac{d^2}{dz^2}\left[\cancel{z^3} \cdot \frac{z+1}{\cancel{z^3}(z^2+1)} \right] \right\}$$

$$a_{-1} = \frac{1}{(3-1)!} \lim_{z\to 0} \left[\frac{1\cdot(z^2+1)-(z+1)\cdot 2z}{(z^2+1)^2} \right]$$

$$a_{-1} = \frac{1}{2} \lim_{z\to 0} \frac{z^2+1-2z^2-2z}{(z^2+1)^2} \Rightarrow \frac{(-z^2-2z+1)}{(z^2+1)^2}$$

$$a_{-1} = \frac{1}{2} \lim_{z\to 0} \left[\frac{(-2z-2)(z^2+1)^{\cancel{2}} - (z^2-2z+1)\cdot 2\cancel{(z^2+1)}\cdot 2z}{(z^2+1)^{\cancel{4}}} \right]$$

$$a_{-1} = \frac{1}{2} \lim_{z\to 0} \left[\frac{(-2z-2)(z^2+1)-(z^2-2z+1)\cdot 4z}{(z^2+1)^3} \right]$$

$$a_{-1} = \frac{1}{2} \left[\frac{(-2\cdot 1)-(1)\cdot 0}{(1)^3} \right]$$

$$a_{-1} = \frac{1}{2} \cdot -2 \Rightarrow \boxed{a_{-1} = -1}$$

Hallar el resultado de $\dfrac{(3-2i)(3+i)-(2i-3)^2}{i^{23}-i^{13}}$ en forma cartesiana (5pts)

$$\frac{(3-2i)(3+1)-(2i-3)^2}{i^{23}-i^{13}} = \frac{9+3i-6i+2-(-4-12i+9)}{-i-i}$$

$23 \,|\, 4$, $3 \,|\, 5$ $= -i$

$$= \frac{11-3i+4+12i-9}{-2i} = \frac{6+9i}{-2i} \cdot \frac{i}{i}$$

$13 \,|\, 4$, $1 \,|\, 3$ $= i$

$$= \frac{6i-9}{2} = 3i - \frac{9}{2}$$