Práctica Examen.

1) $$\frac{z}{9+5i} - \frac{2z+3i}{1-2i} = 4+2i$$

$$\frac{z(1-2i) - (2z+3i)(9+5i)}{(9+5i)(1-2i)} = \frac{4+2i}{1}$$

$$z(1-2i) - (2z+3i)(9+5i) = (4+2i)(9+5i)(1-2i)$$

$$z-2zi-18z-10zi-27i+15 = (36+20i+18i-10)(1-2i)$$

$$-17z-12zi-27i+15 = (26+38i)(1-2i)$$

$$-17z - 12zi - 27i + 15 = (26 + 38i)(1 - 2i)$$

$$-17z - 12zi - 27i + 15 = 26 - 52i + 38i + 76$$

$$-17z - 12zi = 102 - 14i + 27i - 15$$

$$z \cdot (-17 - 12i) = 87 + 13i$$

$$z = \frac{(87 + 3i)}{(-17 - 12i)} \cdot \frac{(-17 + 12i)}{(-17 + 12i)}$$

$$z = \frac{-1479 + 1044i - 51i - 36}{289 + 144}$$

$$z = \frac{-1515 + 993i}{433}$$

$$z = \frac{-1515}{433} + \frac{993}{433} i$$

② $\operatorname{Senh}\left(\frac{5+7i}{1-i}\right)$. Halle el principal

$$\operatorname{Senh}\left(\frac{5+7i}{1-i}\right) \qquad \operatorname{Senh}(z) = \frac{e^{z} - e^{-z}}{2}$$

$$\frac{5+7i}{1-i}\,\frac{(1+i)}{(1+i)} = \frac{5+5i+7i-7}{1+1} = \frac{-2+12i}{2} = \underbrace{\boxed{-1+6i}}^{z}$$

$$\operatorname{Senh}(-1+6i) = \frac{e^{-1+6i} - e^{1-6i}}{2}$$

$$\operatorname{Senh}(-1+6i) = \frac{e^{-1+6i} - e^{1-6i}}{2}$$

$$\boxed{e^{i\theta} = \cos\theta + i\operatorname{Sen}\theta}$$

$$\operatorname{Senh}(-1+6i) = \frac{e^{-1}\cdot e^{6i} - e^{1}\cdot e^{-6i}}{2}$$

$$\operatorname{Senh}(-1+6i) = \frac{e^{-1}\left(\cos 6 + i\operatorname{Sen} 6\right) - e\left(\cos(-6) + i\operatorname{Sen}(-6)\right)}{2}$$

$$\operatorname{Senh}(-1+6i) = \frac{1}{2}\left[\frac{1}{e}\left(\cos 6 + i\operatorname{Sen} 6\right) - e\left(\cos 6 - i\operatorname{Sen} 6\right)\right]$$

③ $z^3 - 9i = \dfrac{2-5i}{1+i}$

$$z^3 = \frac{2-5i}{1+i} + 9i$$

$$z^3 = \frac{(2-5i) + 9i(1+i)}{1+i}$$

$$z^3 = \frac{2-5i+9i-9}{1+i}$$

$$z^3 = \frac{-7+4i}{1+i}$$

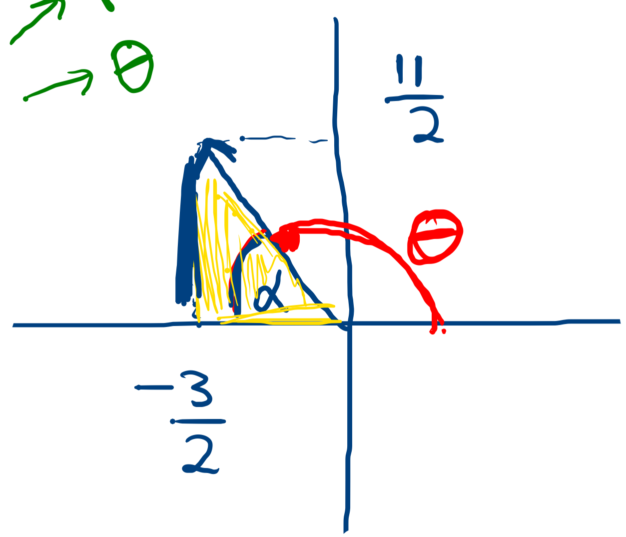
$$z^3 = \frac{(-7+4i)}{(1+i)} \frac{(1-i)}{(1-i)}$$

$$z^3 = \frac{-7+7i+4i+4}{1+1}$$

$$z^3 = \frac{-7+7i+4i+4}{1+1} \Rightarrow z^3 = -\frac{3}{2} + \frac{11}{2}i$$

$$z = \sqrt[3]{-\frac{3}{2} + \frac{11}{2}i}$$

POLAR $\to r$, $\to \theta$



$$z = -\frac{3}{2} + \frac{11}{2}i$$

$$r = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{11}{2}\right)^2}$$

$$r = \frac{\sqrt{130}}{2} \approx 5,7$$

$$\tan(\alpha) = \frac{\frac{11}{2}}{\frac{3}{2}} \to 74,74^\circ$$

$$\theta = 180 - 74,74$$

$$\theta = 105,26$$

$$z = \sqrt[3]{-\frac{3}{2} + \frac{11}{2}i}$$

$$r = \frac{\sqrt{130}}{2} = 5{,}7 \qquad \theta = 105$$

Moivre

$$z^{\frac{1}{n}} = r^{\frac{1}{n}} \left[\cos \frac{\theta + 2k\pi}{n} + i \operatorname{Sen} \frac{\theta + 2k\pi}{n} \right]$$

$$\measuredangle C = \frac{360}{3}$$

$$K = 0 \Rightarrow \left(\frac{\sqrt{130}}{2}\right)^{\frac{1}{3}} \left[\cos \frac{105}{3} + i \operatorname{Sen} \frac{105}{3} \right]$$

$$\measuredangle C = 120$$

$$K = 0 \Rightarrow \left(\frac{\sqrt{130}}{2}\right)^{\frac{1}{3}} \left[\cos 35 + i \operatorname{Sen} 35 \right]$$

$$K=0 \Rightarrow \left(\frac{\sqrt{130}}{2}\right)^{\frac{1}{3}}\left[\operatorname{Cos} 35 + i \operatorname{Sen} 35\right]$$

$$K=1 \Rightarrow \left(\frac{\sqrt{130}}{2}\right)^{1/3}\left[\operatorname{Cos} 155 + i \operatorname{Sen} 155\right]$$

$$K=2 \Rightarrow \left(\frac{\sqrt{130}}{2}\right)^{1/3}\left[\operatorname{Cos} 275 + i \operatorname{Sen} 275\right]$$

4) Desarrollar Laurent $f(z) = \dfrac{2z-3}{(z^2-9)(z+1)}$, $2 < |z+3| < 6$

$$f(z) = \frac{2z-3}{(\underline{\underline{z^2-9}})(z+1)}$$

$$f(z) = \frac{2z-3}{(z-3)(z+3)(z+1)} = \frac{A}{(z-3)} + \frac{B}{z+3} + \frac{C}{z+1}$$

$$= \frac{A(z+3)(z+1) + B(z-3)(z+1) + C(z-3)(z+3)}{(z-3)(z+3)(z+1)}$$

$$f(z) = \frac{2z-3}{\cancel{(z-3)}(z+3)(z+1)} = \frac{A}{(z-3)} + \frac{B}{z+3} + \frac{C}{z+1}$$

$$= \frac{A(z+3)(z+1) + B(z-3)(z+1) + C(z-3)(z+3)}{(z-3)\cancel{(z+3)}(z+1)}$$

$$2z-3 = Az^2 + \underline{\underline{Az}} + \underline{\underline{3zA}} + 3A + Bz^2 + Bz - 3Bz - 3B + Cz^2 + \cancel{3zC} - \cancel{3zC} - 9C$$

$$2z-3 = Az^2 + 4zA + 3A + Bz^2 - 2Bz - 3B + Cz^2 - 9C$$

$$0 = A + B + C$$

$$2 = 4A - 2B$$

$$-3 = 3A - 3B - 9C$$

Mode 5 . 2

Mode Ecuación Simul Ecuación ⇒ 3

$$A = \frac{1}{8} \qquad B = -\frac{3}{4} \qquad C = \frac{5}{8} \qquad \boxed{2 < |z+3| < 6}$$

Converje

$$f(z) = \frac{A}{z+3} + \frac{B}{z-3} + \frac{C}{z+1}$$

$$f(z) = \frac{1}{8} \cdot \frac{1}{z+3} - \frac{3}{4} \cdot \frac{1}{z-3} + \frac{5}{8} \cdot \frac{1}{z+1}$$

$$f(z) = \frac{1}{8} \cdot \frac{1}{z+3} - \frac{3}{4} \cdot \frac{1}{z-3+3-3} + \frac{5}{8} \cdot \frac{1}{z+1+3-3}$$

$$f(z)=\frac{1}{8}\cdot\frac{1}{z+3}-\frac{3}{4}\cdot\frac{1}{z-3+3-3}+\frac{5}{8}\cdot\frac{1}{z+1+3-3}$$

$$2<|z+3|<6$$

Converge

$$f(z)=\frac{1}{8}\cdot\frac{1}{z+3}\underbrace{-\frac{3}{4}\cdot\frac{1}{(z+3)-6}}\underbrace{+\frac{5}{8}\cdot\frac{1}{(z+3)-2}}$$

$$f(z)=\frac{1}{8}\cdot\frac{1}{z+3}-\frac{3}{4}\cdot\frac{1}{-6\left(1-\frac{z+3}{6}\right)}+\frac{5}{8}\cdot\frac{1}{(z+3)\left[1-\frac{2}{z+3}\right]}$$

Taylor

$$\frac{1}{1-x}=\sum_{n=0}^{\infty}x^n$$

$$\frac{1}{1+x}=\sum_{n=0}^{\infty}(-1)^n x^n$$

e^x

$\operatorname{Sen} x$

$\cos x$

$$f(z)=\frac{1}{8}\cdot\frac{1}{z+3}-\frac{3}{4}\cdot\frac{1}{-6\left(1-\frac{z+3}{6}\right)}+\frac{5}{8}\cdot\frac{1}{(z+3)\left[1-\frac{2}{z+3}\right]}$$

$$\frac{1}{1-x}=\sum_{n=0}^{\infty}x^n$$

$$f(z)=\frac{1}{8}\cdot\frac{1}{z+3}+\frac{1}{8}\cdot\sum_{n=0}^{\infty}\left(\frac{z+3}{6}\right)^n+\frac{5}{8}\cdot\frac{1}{z+3}\cdot\sum_{n=0}^{\infty}\left(\frac{2}{z+3}\right)^n$$

$$f(z)=\frac{1\cdot 1}{8(z+3)}+\frac{1}{8}\left[1+\frac{z+3}{6}+\frac{(z+3)^2}{36}+\frac{(z+3)^3}{216}+\dots\right]+$$

$$\frac{5}{8}\cdot\frac{1}{z+3}\left[1+\frac{2}{z+3}+\frac{4}{(z+3)^2}+\frac{8}{(z+3)^3}+\frac{16}{(z+3)^4}+\dots\right]$$

$$f(z) = \frac{1 \cdot 1}{8(z+3)} + \frac{1}{8}\left[1 + \frac{z+3}{6} + \frac{(z+3)^2}{36} + \frac{(z+3)^3}{216} + \dots\right] +$$

$$\frac{5}{8} \cdot \frac{1}{z+3}\left[1 + \frac{2}{z+3} + \frac{4}{(z+3)^2} + \frac{8}{(z+3)^3} + \frac{16}{(z+3)^4} + \dots\right]$$

$$f(z) = \frac{1}{8} \cdot \frac{1}{z+3} + \frac{1}{8} + \frac{z+3}{48} + \frac{(z+3)^2}{288} + \frac{(z+3)^3}{1728} + \dots +$$

$$\frac{5}{8} \cdot \frac{1}{z+3} + \frac{5}{4} \cdot \frac{1}{(z+3)^2} + \frac{5}{2} \cdot \frac{1}{(z+3)^3} + \frac{5}{(z+3)^4} + \dots$$

$$f(z) = \frac{1}{8}\cdot\frac{1}{z+3} + \frac{1}{8} + \frac{z+3}{48} + \frac{(z+3)^2}{288} + \frac{(z+3)^3}{1728} + \dots +$$

$$\frac{5}{8}\cdot\frac{1}{z+3} + \frac{5}{4}\cdot\frac{1}{(z+3)^2} + \frac{5}{2}\cdot\frac{1}{(z+3)^3} + \frac{5}{(z+3)^4} + \dots$$

Analítica

$$\boxed{a_{-1} = \frac{3}{4}}$$

$$f(z) = \frac{3}{4}\cdot\frac{1}{(z+3)} + \frac{1}{8} + \boxed{\frac{z+3}{48} + \frac{(z+3)^2}{288} + \frac{(z+3)^3}{1728} + \dots} +$$

$$\frac{5}{4}\cdot\frac{1}{(z+3)^2} + \frac{5}{2}\cdot\frac{1}{(z+3)^3} + \frac{5}{(z+3)^4} + \dots$$

Principal

⑤ $f(z) = \dfrac{\cos 3z}{2z^9}$

$2z^9 = 0 \Rightarrow z = 0$

→ polo → orden
→ esencial
→ Evitable

$$\cos z = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \frac{z^6}{6!} + \frac{z^8}{8!} - \frac{z^{10}}{10!} + \frac{z^{12}}{12!} - \dots$$

Sen z Cos z e^z

Hacer Taylor para hacer Laurent y llegar original.

$$\frac{\cos 3z}{2z^9} = \frac{1}{2z^9} - \frac{9z^2}{2!\cdot 2z^9} + \frac{81z^4}{4!\cdot 2z^9} - \frac{729z^6}{6!\cdot 2z^9} + \dots$$

$$\frac{\cos 3z}{2z^9} = \frac{1}{2z^9} - \frac{9}{4z^7} + \frac{27}{16z^5} - \frac{81}{160z^3} + \dots$$

Polo Orden 9

$z = 0 \left\{ \begin{array}{l} \text{Polo de} \\ \text{orden } 9 \end{array} \right.$

⑥ $f(z) = \dfrac{9z^2 - 2z}{\underbrace{(z^2+16)}(z+3)^2}$

$z^2 + 16 = 0$

$z^2 = -16$

$z = \sqrt{-16}$

$z = 4i$
$\quad\; -4i$

$\underbrace{z+3=0}$

$z = -3$

$f(z) = \dfrac{9z^2 - 2z}{(z-4i)(z+4i)(z+3)^2}$

$z^2 + 16.$

$z^2 - -16$

$\downarrow \qquad \downarrow$

$z \qquad 4i$

$(z-4i)(z+4i)$

Para $z = 4i$

$$\lim_{z\to 4i} \frac{9z^2 - 2z}{\underbrace{(z-4i)}_{0}(z+4i)(z+3)^2} = \frac{9(4i)^2 - 2(4i)}{0} = \frac{-144 - 8i}{0} = \infty$$

$z=4i$ Hay un polo SIMPLE

Simple

$$\lim_{z\to 4i} \cancel{(z-4i)} \frac{9z^2-2z}{\cancel{(z-4i)}(z+4i)(z+3)^2} = \frac{9(4i)^2-2(4i)}{(8i)(4i+3)^2} =$$

$$\frac{-144-8i}{8i(-16+24i+9)} = \frac{-144-8i}{8i(-7+24i)} = \frac{-144-8i}{-56i-192}$$

Residuo $= \frac{(-144-8i)}{(-56i-192)} \cdot \frac{(-192+56i)}{(-192+56i)} = \frac{27648-8064i+1536i+448}{36864+3136}$

$$a_{-1} = \frac{27648 - 8064i + 1536i + 448}{36864 + 3136} = \frac{28096 - 6528i}{40000}$$

$$a_{-1} = \frac{439}{625} - \frac{102}{625}i \quad //$$

Para $z=-4i$

$$\lim_{z\to -4i} \frac{9z^2-2z}{\underbrace{(z-4i)}_{0}(z+4i)(z+3)^2} = \frac{9(-4i)^2-2(-4i)}{0} =$$

$$\frac{-144+8i}{0} = \frac{\#}{0} = \infty.$$

Por tanto $z=-4i$ hay un <u>polo</u>.

<u>Orden</u> Simple $\lim_{z\to -4i} \cancel{(z+4i)} \cdot \frac{9z^2-2z}{(z-4i)\cancel{(z+4i)}(z+3)^2} = \#$

Para $z = -4i$ Hay un polo SIMPLE y el residuo es

$a_{-1} = $ ¿? Pendiente

Para $z=-3$

$$\lim_{z\to -3} \frac{9z^2-2z}{(z-4i)(z+4i)\underbrace{(z+3)^2}_{0}} = \frac{9(-3i)^2-2(-3i)}{0} = \frac{-81+6i}{0}$$

$$\frac{\#}{0} = \infty \Rightarrow \text{POLO}$$

$$\lim_{z\to -3} \cancel{(z+3)} \cdot \frac{(9z^2-2z)}{(z-4i)(z+4i)\underbrace{(z+3)}^{\cancel{2}}} = \infty$$

Siga

$$\lim_{z \to -3} \cancel{(z+3)^2} \cdot \frac{9z^2 - 2z}{(z-4i)(z+4i)\cancel{(z+3)^2}} = \frac{9(-3)^2 - 2(-3)}{(-3-4i)(-3+4i)} = \#$$

$$\frac{87}{9+16} = \boxed{\frac{8}{25}} \Rightarrow \text{POLO de ORDEN } 2$$

Residuo

$$a_{-1} = \frac{1}{(m-1)!} \cdot \lim_{z \to a} \left[\frac{d^{m-1}}{z^{m-1}} \left[(z-a)^m f(z) \right] \right]$$

$$a_{-1} = \underbrace{\frac{1}{(2-1)!}}_{1!} \left\{ \lim_{z \to -3} \frac{d}{dz} \left[\cancel{(z+3)^2} \cdot \frac{9z^2-2z}{(z^2+16)\cancel{(z+3)^2}} \right] \right\}$$

$$a_{-1} = \lim_{z \to -3} \frac{d}{dz} \left[\frac{9z^2-2z}{z^2+16} \right]$$

$$a_{-1} = \lim_{z \to -3} \frac{d}{dz}\left[\frac{9z^2-2z}{z^2+16}\right]$$

$$a_{-1} = \lim_{z \to -3}\left[\frac{(18z-2)(z^2+16) - (9z^2-2z)\cdot 2z}{(z^2+16)^2}\right]$$

$$\frac{(18(-3)-2)((-3)^2+16) - (9(-3)^2-2(-3))\cdot 2(-3)}{((-3)^2+16)^2}$$

$$a_{-1} = \frac{-878}{725} \quad //$$

Resumen $z=-3$

Hay un polo de orden 2

con residuo $-\frac{878}{725}$ //