Práctica

$$\mathcal{L}\{t\cos wt\} = \frac{s^2 - w^2}{(s^2+w^2)^2}$$

① $\mathcal{L}\{4 + t\cos 4t - 3te^{5t}\operatorname{Sen} t\}$

$$4\mathcal{L}\{1\} + \mathcal{L}\{t\cos 4t\} - 3\,\mathcal{L}\{te^{5t}\operatorname{Sen} t\}$$

$$4 \cdot \frac{1}{s}$$

$$\mathcal{L}\{t\cos 4t\} = \frac{s^2 - 16}{(s^2+16)^2} \checkmark$$

derivar

$$\mathcal{L}\{t\cos 4t\} = \frac{s}{s^2+16} = \frac{1\cdot(s^2+16) - s\cdot 2s}{(s^2+16)^2} = \frac{s^2+16-2s^2}{(s^2+16)^2}$$

$$\frac{-s^2+16}{(s^2+16)^2}$$

$$\frac{s^2-16}{(s^2+16)^2}$$

$$\mathcal{L}\{t\, e^{5t}\, \text{Sen}\, t\} = \frac{2\cdot 1\cdot s}{(s^2+1)^2}$$

$$\mathcal{L}\{t\, \text{Sen}\, wt\} = \frac{2ws}{(s^2+w^2)^2}$$

$$= \frac{2s}{(s^2+1)^2} = \frac{2(s-5)}{((s-5)^2+1)^2}$$

$$\Rightarrow \frac{4}{s} + \frac{s^2-16}{(s^2+16)^2} - \frac{3\cdot 2(s-5)}{((s-5)^2+1)^2}$$

$$\frac{4}{s} + \frac{s^2-16}{(s^2+16)^2} + \frac{-6s+30}{((s-5)^2+1)^2} \quad //$$

(2) $\mathcal{L}^{-1}\left\{\dfrac{6}{3s+2}+\dfrac{9s}{3s^2+12}\right\}$

$\mathcal{L}\{e^{at}\}=\dfrac{1}{s-a}$

$6\,\mathcal{L}^{-1}\left\{\dfrac{1}{3s+2}\right\}+9\,\mathcal{L}^{-1}\left\{\dfrac{s}{3s^2+12}\right\}$

$6\,\mathcal{L}^{-1}\left\{\dfrac{1}{3\left(s+\frac{2}{3}\right)}\right\}+9\,\mathcal{L}^{-1}\left\{\dfrac{s}{3(s^2+4)}\right\}$

$\dfrac{6}{3}\,\mathcal{L}^{-1}\left\{\dfrac{1}{s+2/3}\right\}+\dfrac{9}{3}\,\mathcal{L}^{-1}\left\{\dfrac{s}{s^2+4}\right\}$

$2e^{-2/3t}+3\cos 2t$

3 $\mathcal{L}^{-1}\left\{\frac{4s}{(s+2)(s^2+4)}\right\}$

$$4\mathcal{L}^{-1}\left\{\frac{s}{(s+2)(s^2+4)}\right\}$$

$$\frac{s}{\cancel{(s+2)(s^2+4)}} = \frac{A}{s+2} + \frac{Bs+C}{s^2+4}$$

$$= \frac{A(s^2+4) + (Bs+C)(s+2)}{\cancel{(s+2)(s^2+4)}}$$

$$= 1s = As^2 + 4A + Bs^2 + 2Bs + Cs + 2C$$

$$(s+4)^2 = \frac{A}{s+4} + \frac{B}{(s+4)^2}$$

$0 = A + B$

$1 = 2B + C$

$0 = 4A + 2C$

$$\begin{array}{ccc|c} A & B & C & = \\ 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 1 \\ 4 & 0 & 2 & 0 \end{array}$$

$A = -1/4$

$B = 1/4$

$C = 1/2$

$$\mathcal{L}^{-1}\left\{\frac{A}{s+2}\right\} + \mathcal{L}^{-1}\left\{\frac{Bs+C}{s^2+4}\right\} \qquad A=-1/4 \quad B=1/4 \quad C=1/2$$

$$-\frac{1}{4}\underbrace{\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\}} + \mathcal{L}^{-1}\left\{\frac{\frac{1}{4}s+\frac{1}{2}}{s^2+4}\right\}$$

$$-\frac{1}{4}e^{-2t} + \mathcal{L}^{-1}\left\{\frac{\frac{1}{4}s}{s^2+4}\right\} + \mathcal{L}^{-1}\left\{\frac{\frac{1}{2}}{s^2+4}\right\}$$

$$-\frac{1}{4}e^{-2t} + \underbrace{\frac{1}{4}\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\}} + \underbrace{\frac{1}{2}\cdot\frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1\cdot 2}{s^2+4}\right\}}$$

$$-\frac{1}{4}e^{-2t} + \frac{1}{4}\cos 2t + \frac{1}{4}\operatorname{Sen} 2t \quad //$$

4 $\mathcal{L}^{-1}\left\{\dfrac{3s-4}{s^2-4s+2}\right\}$

s^2-4s+2

$x=2-\sqrt{2}$

$x=2+\sqrt{2}$

$s^2-4s+2+4-4$

$\frac{4}{2}=(2)^2=4$

$(s^2-4s+4)-2$

$(s-2)^2-2$

$\mathcal{L}^{-1}\left\{\dfrac{3s-4}{(s-2)^2-2}\right\}$

$\underbrace{3\mathcal{L}^{-1}\left\{\dfrac{s}{(s-2)^2-2}\right\}}-4\mathcal{L}^{-1}\left\{\dfrac{1}{(s-2)^2-2}\right\}$

$$3\,\underbrace{\mathcal{L}^{-1}\left\{\frac{s-2+2}{(s-2)^2-2}\right\}}\;-\;4\,\mathcal{L}^{-1}\left\{\frac{1}{(s-2)^2-2}\right\}$$

$$3\left[\mathcal{L}^{-1}\left\{\underset{\dfrac{s}{s^2-2}}{\frac{s-2}{(s-2)^2-2}}\right\}+\frac{2}{\sqrt{2}}\,\mathcal{L}^{-1}\left\{\underset{\dfrac{1\cdot\sqrt{2}}{s^2-2}}{\frac{1}{(s-2)^2-2}}\right\}\right]-\frac{4}{\sqrt{2}}\,\mathcal{L}^{-1}\left\{\frac{1\cdot\sqrt{2}}{(s-2)^2-2}\right\}$$

$$3\left[\cosh(\sqrt{2}\,t)\,e^{2t}+\frac{2}{\sqrt{2}}\,\operatorname{Senh}(\sqrt{2}\,t)\,e^{2t}\right]-\frac{4}{\sqrt{2}}\,\operatorname{Senh}(\sqrt{2}\,t)\,e^{2t}$$

$$3\left[\cosh(\sqrt{2}t)e^{2t}+\frac{2}{\sqrt{2}}\operatorname{Senh}(\sqrt{2}t)e^{2t}\right]-\frac{4}{\sqrt{2}}\operatorname{Senh}(\sqrt{2}t)e^{2t}$$

$$3\cosh(\sqrt{2}t)e^{2t}+3\sqrt{2}\,e^{2t}\operatorname{Senh}(\sqrt{2}t)-\frac{4}{\sqrt{2}}\operatorname{Senh}(\sqrt{2}t)e^{2t}$$

$$3\cosh(\sqrt{2}t)e^{2t}+\sqrt{2}\,e^{2t}\operatorname{Senh}(\sqrt{2}t) \quad //$$

⑤ $\mathcal{L}\{(t+1)\cdot u(t-2)\}$

-$\mathcal{L}$ sin correr

- $\cdot\, e^{-as}$

$\mathcal{L}\{u_a(t)\} = \dfrac{e^{-as}}{s}$

$\mathcal{L}\{(t+1)\cdot u_2(t)\}$

$$\mathcal{L}\{(t+1+2-2)\cdot u_2(t)\} = \underbrace{\mathcal{L}\{(t\cancel{-2})\cdot u_2(t)\}}_{\frac{1}{s^2}\cdot e^{-2s}} + 3\,\mathcal{L}\{u_2(t)\}$$

$$\frac{1}{s^2}\cdot e^{-2s} + \frac{3\cdot e^{-2s}}{s} \quad //$$

Otra forma:

$$\mathcal{L}\{(t+1)\cdot \mathcal{U}_2(t)\} = \underbrace{\mathcal{L}\{t\cdot \mathcal{U}_2(t)\}}_{\frac{2}{s}e^{-2s}+\frac{e^{-2s}}{s^2}} + \underbrace{\mathcal{L}\{\mathcal{U}_2(t)\}}_{\frac{e^{-2s}}{s}}$$

$\Rightarrow$ R/ $\frac{3}{s}e^{-2s} + \frac{e^{-2s}}{s^2}$ //

derivada

$$\mathcal{L}\{t\cdot \mathcal{U}_2(t)\} = \frac{e^{-2s}}{s} = \frac{-2e^{-2s}\cdot s - e^{-2s}\cdot 1}{s^2} = \frac{-2se^{-2s}}{s^2} - \frac{e^{-2s}}{s^2}$$

cambio signo

$$= \frac{2e^{-2s}}{s} + \frac{e^{-2s}}{s^2}$$

⑥ $\mathcal{L}\{e^{-3t}\cdot u_4(t) + t^2\cdot u_6(t) + (t-3)\cdot u_2(t)\}$

$$\mathcal{L}\{e^{-3t}\cdot u_4(t)\} + \mathcal{L}\{t^2\cdot u_6(t)\} + \mathcal{L}\{(t-3)\cdot u_2(t)\}$$

$$\mathcal{L}\{e^{-3t}\cdot u_4(t)\} = \frac{e^{-4s}}{s} = \frac{e^{-4(s+3)}}{(s+3)}$$

$$\underbrace{\mathcal{L}\{t^2 \cdot \mathcal{U}_6(t)\}} = \frac{e^{-6s}}{s} \overset{1ra}{=} \frac{-6e^{-6s}\cdot s - e^{-6s}\cdot 1}{s^2} = \frac{6e^{-6s}}{s} + \frac{e^{-6s}}{s^2}$$

derivo 2veces

2davez

$$\frac{-36e^{-6s}\cdot s - 6e^{-6s}\cdot 1}{s^2} + \frac{-6e^{-6s}\cdot s^2 - e^{-6s}\cdot 2s}{s^4}$$

$$\frac{36e^{-6s}}{s} + \frac{6e^{-6s}}{s^2} + \frac{6e^{-6s}}{s^2} + \frac{2e^{-6s}}{s^3}$$

$$\frac{36}{s}e^{-6s} + \frac{12}{s^2}e^{-6s} + \frac{2e^{-6s}}{s^3}$$

$$\mathcal{L}\{(t-3)\cdot u_2(t)\}$$

$$\mathcal{L}\{(\underset{=}{t}\underbrace{-3+2}-\underset{=}{2})\cdot u_2(t)\} = \mathcal{L}\{(t\cancel{-2})\cdot u_2(t)\} - \mathcal{L}\{u_2(t)\}$$

$$= \frac{1}{s^2}e^{-2s} - \frac{e^{-2s}}{s}$$

R/ $$\frac{e^{-4(s+3)}}{(s+3)} + \frac{36}{s}e^{-6s} + \frac{12}{s^2}e^{-6s} + \frac{2e^{-6s}}{s^3} + \frac{1}{s^2}e^{-2s} - \frac{e^{-2s}}{s}$$

7) $\mathcal{L}\{\operatorname{Senh}(t-\cancel{3}) \cdot \mathcal{U}_3(t)\}$

$\mathcal{L}$ sin correr $\cdot\, e^{-as}$

indicador

$$\frac{1}{s^2-1} \cdot e^{-3s} \quad //$$

8 $\mathcal{L}^{-1}\left\{\frac{e^{-2s}(s+5)}{(s-7)^2}+\frac{2s^2+15s+7}{(s+1)^2(s-2)}\right\}$

$$\mathcal{L}^{-1}\left\{\frac{e^{-2s}(s+5)}{(s-7)^2}\right\}+\underbrace{\mathcal{L}^{-1}\left\{\frac{2s^2+15s+7}{(s+1)^2(s-2)}\right\}}$$

$$\mathcal{L}^{-1}\left\{\frac{2s^2+15s+7}{\underline{\underline{(s+1)^2}}\ \underline{\underline{(s-2)}}}\right\}=\frac{A}{s+1}+\frac{B}{(s+1)^2}+\frac{C}{s-2}$$

$$2s^2+15s+7=\frac{A(s+1)(s-2)+B(s-2)+C(s+1)^2}{(s+1)^2(s-2)}$$

$$\frac{2s^2+15s+7}{\cancel{(s+1)^2(s-2)}} = \frac{A(s+1)(s-2)+B(s-2)+C(s+1)^2}{\cancel{(s+1)^2(s-2)}}$$

$C(s^2+2s+1)$

$$2s^2+15s+7 = \underline{\underline{As^2}}\underbrace{-2sA+sA}-2A+\underbrace{Bs}-2B+\underline{\underline{Cs^2}}+\underbrace{2Cs}+C$$

$2 = A + C$

$15 = -A + B + 2C$

$7 = -2A - 2B + C$

A	B	C	
1	0	1	2
-1	1	2	15
-2	-2	1	7

$A = -3$

$B = 2$

$C = 5$

$$\mathcal{L}^{-1}\left\{\frac{A}{s+1}\right\} + \mathcal{L}^{-1}\left\{\frac{B}{(s+1)^2}\right\} + \mathcal{L}^{-1}\left\{\frac{C}{(s-2)}\right\}$$

$A = -3$
$B = 2$
$C = 5$

$$\underbrace{-3\,\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}} + \underbrace{2\,\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\}}_{\frac{1}{s^2}} + 5\,\mathcal{L}^{-1}\left\{\frac{1}{(s-2)}\right\}$$

$$-3e^{-t} + 2te^{-t} + 5e^{2t} \quad //$$

$$\mathcal{L}^{-1}\left\{\frac{e^{-2s}(s+5)}{(s-7)^2}\right\}$$

- Calculamos $\mathcal{L}^{-1}$
- Desplazarlo
- $\mathcal{U}_a(t)$

$$\mathcal{L}^{-1}\left\{\frac{s+5}{(s-7)^2}\right\} = \underbrace{\mathcal{L}^{-1}\left\{\frac{s+7-7}{(s-7)^2}\right\}} + 5\mathcal{L}^{-1}\left\{\frac{1}{(s-7)^2}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{\cancel{s-7}}{(s-7)^{\cancel{2}}}\right\} + 7\mathcal{L}^{-1}\left\{\underset{\frac{1}{s^2}}{\frac{1}{(s-7)^2}}\right\} + 5\mathcal{L}^{-1}\left\{\underset{\frac{1}{s^2}}{\frac{1}{(s-7)^2}}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s-7}\right\}$$

$$e^{7t} + 7te^{7t} + 5te^{7t} \Rightarrow e^{7t} + 12te^{7t} \;//$$

R/ $e^{7t} + 12te^{7t} - 3e^{-t} + 2te^{-t} + 5e^{2t}$

9) Calcule el principal $\left(\dfrac{3i+4}{2i+3}\right)^{4i}$

potencia variable compleja.

$$[a]^b = e^{b[\log a + \cancel{2k\pi i}]}$$

$$\ln z = \ln \underline{r} + i\underline{\theta}$$

$$e^{4i\left[\log\left(\frac{3i+4}{2i+3}\right)\right]} \Rightarrow \overset{R/}{e^{4i\left[\ln \frac{5\sqrt{13}}{13} + i\cdot 3^\circ\right]}}$$

POLAR

$r \quad \theta$

$$\frac{(3i+4)}{2i+3} \cdot \frac{(3-2i)}{(3-2i)} = \frac{9i+6+12-8i}{9+4} \Rightarrow \frac{i}{13} + \frac{18}{13}$$

(10) Determine el lugar geométrico de $|z-1|+|z+3|=10$.

$$z=x+yi$$

$$|x+yi-1|+|x+yi+3|=10$$

$$\sqrt{(x-1)^2+y^2}+\sqrt{(x+3)^2+y^2}=10$$

$$\left(\cancel{\sqrt{(x-1)^2+y^2}}\right)^{\cancel{2}}=\left(10-\sqrt{(x+3)^2+y^2}\right)^2$$

$$(x-1)^2+y^2=100-20\sqrt{(x+3)^2+y^2}+(x+3)^2+y^2$$

$$(x-1)^2+y^2=100-20\sqrt{(x+3)^2+y^2}+(x+3)^2+y^2$$

$$\cancel{x^2}-2x+1+\cancel{y^2}=100-20\sqrt{(x+3)^2+y^2}+\cancel{x^2}+6x+9+\cancel{y^2}$$

$$-2x+1-100-6x-9=-20\sqrt{(x+3)^2+y^2}$$

$$(-8x-108)=-20\sqrt{(x+3)^2+y^2}$$

$$(8x+108)^2=\left(20\sqrt{(x+3)^2+y^2}\right)^2$$

$$64x^2+1728x+11664=400\left[(x+3)^2+y^2\right]$$

$$64x^2 + 1728x + 11664 = 400\left[(x+3)^2 + y^2\right]$$

$$64x^2 + 1728x + 11664 = 400(x^2 + 6x + 9 + y^2)$$

$$64x^2 + 1728x + 11664 = 400x^2 + 2400x + 3600 + 400y^2$$

$$64x^2 + 1728x - 400x^2 - 2400x - 400y^2 = \underbrace{3600 - 11664}$$

$$-336x^2 - 672x - 400y^2 = -8064$$

$$336x^2 + 672x + 400y^2 = 8064.$$

$$\frac{336x^2}{8064} + \frac{672x}{8064} + \frac{400y^2}{8064} = \frac{8064}{8064}.$$

$$\frac{x^2}{24} + \frac{1}{12}x + \frac{25}{504}y^2 = 1$$

$$\frac{1}{24}\left(x^2 + 2x + 1 - 1\right) + \frac{25}{504}y^2 = 1$$

$$\frac{1}{24}(x+1)^2 - \frac{1}{24} + \frac{25}{504}y^2 = 1$$

Recta $y = mx + b.$

Círculo $(x-h)^2 + (y-k)^2 = r^2$

Elipse $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

Hipérbola $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$$\frac{1}{24}(x+1)^2 - \frac{1}{24} + \frac{25}{504}y^2 = 1$$

$$\frac{(x+1)^2}{24} + \frac{y^2}{\frac{504}{25}} = 1 + \frac{1}{24}$$

$$\frac{(x+1)^2}{24 \cdot \frac{25}{24}} + \frac{y^2}{\frac{504}{25} \cdot \frac{25}{24}} = \frac{\frac{25}{24}}{\frac{25}{24}}$$

$$\frac{(x+1)^2}{25} + \frac{y^2}{21} = 1$$

Elipse

$C(-1, 0)$

Folleto 1

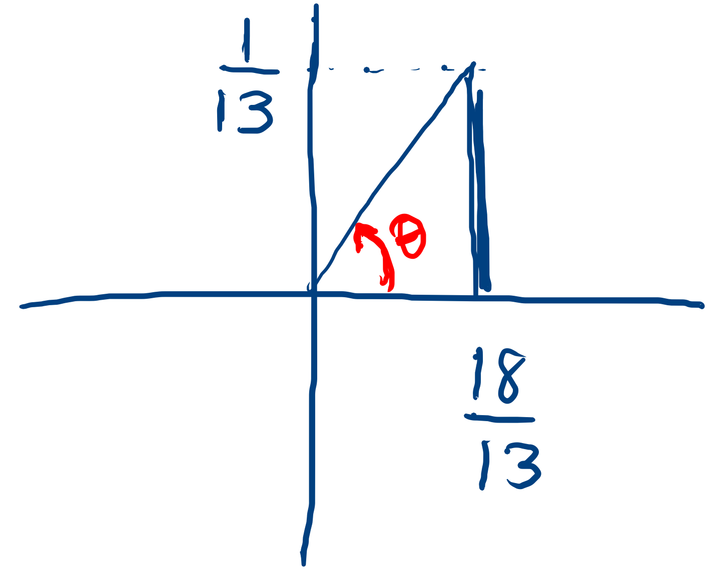
Folleto 2

Folleto 4

Folleto 5

Folleto 6

$$z = \frac{i}{13} + \frac{18}{13}$$



$$\tan\theta = \frac{\frac{1}{13}}{\frac{18}{13}}$$

$$\theta = 3^\circ$$

$$r = \sqrt{\left(\frac{18}{13}\right)^2 + \left(\frac{1}{13}\right)^2}$$

$$r = \frac{5\sqrt{13}}{13}$$