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1-

$$f(t) = \begin{cases} 0, & 0 < t < 2 \\ t^2, & 2 < t \leq 4 \\ e^{2t}, & t \geq 4 \end{cases}$$

$$f(t) = 0 \cdot \mu_0(t) + (t^2 - 0)\mu_2(t) + (e^{2t} - t^2)\mu_4(t)$$

$$f(t) = t^2 \mu_2(t) + (e^{2t} - t^2)\mu_4(t)$$

$$\Rightarrow \mathcal{L}\{f(t)\} = \mathcal{L}\{t^2\mu_2(t) + (e^{2t} - t^2)\mu_4(t)\}$$

$$= \mathcal{L}\{t^2\mu_2(t)\} + \mathcal{L}\{(e^{2t} - t^2)\mu_4(t)\}$$

$$= \mathcal{L}\{t^2\mu_2(t)\} + \mathcal{L}\{e^{2t}\mu_4(t)\} - \mathcal{L}\{t^2\mu_4(t)\}$$

$$= e^{-2s}\mathcal{L}\{(t+2)^2\} + e^{-4s}\mathcal{L}\{e^{2(t+4)}\} - e^{-4s}\mathcal{L}\{(t+4)^2\}$$

$$= e^{-2s}\mathcal{L}\{t^2 + 4t + 4\} + e^{-4s}\mathcal{L}\{e^{2t} \cdot e^{8}\} - e^{-4s}\mathcal{L}\{t^2 + 8t + 16\}$$

$$= e^{-2s}\left[\frac{2}{s^3} + \frac{4}{s^2} + \frac{4}{s}\right] + e^{-4s} \cdot e^{8}\left[\frac{1}{s-2}\right] - e^{4s}\left[\frac{2}{s^3} + \frac{8}{s^2} + \frac{16}{s}\right]$$

$$= e^{-2s}\left[\frac{2}{s^3} + \frac{4}{s^2} + \frac{4}{s}\right] + e^{-4s+8}\left[\frac{1}{s-2}\right] - e^{s}\left[\frac{2}{s^3} + \frac{8}{s^2} + \frac{16}{s}\right]$$

$$\boxed{\mathcal{L}\{f(t)\} = e^{-2s}\left[\frac{2}{s^3} + \frac{4}{s^2} + \frac{4}{s}\right] + e^{-4s+8}\left[\frac{1}{s-2}\right] - e^{s}\left[\frac{2}{s^3} + \frac{8}{s^2} + \frac{16}{s}\right]}$$ R/

2)

a) $\mathcal{L}\left[e^{-3t}\int_0^t \cos(2t-2u)\,u^2 e^u\,du\right]$

$$\mathcal{L}\left[\int_0^t \cos(2(t-u))\,u^2 e^u\,du\right]\Bigg|_{s\to s+3}$$

$$\mathcal{L}\left[\int_0^t u^2 e^u \cdot \cos(2(t-u))\,du\right]\Bigg|_{s\to s+3}$$

$$f(t) = t^2 e^t \qquad g(t) = \cos(2t)$$

$$\mathcal{L}\{t^2 e^t\} \cdot \mathcal{L}\{\cos(2t)\}\Bigg|_{s\to s+3}$$

$$\mathcal{L}\{t^2\}\Big|_{s\to s-1} \cdot \mathcal{L}\{\cos(2t)\}\Bigg|_{s\to s+3}$$

$$\frac{2}{s^3}\Big|_{s\to s-1} \cdot \frac{s}{s^2+2^2}\Bigg|_{s\to s+3}$$

$$\frac{2}{(s-1)^3} \cdot \frac{s}{s^2+4}\Bigg|_{s\to s+3}$$

$$\boxed{\frac{2}{(s+3-1)^3} \cdot \frac{s+3}{(s+3)^2+4}}$$ R/

b) $\mathcal{L}\left\{ t^2 \int_0^t \operatorname{sen}(3u)\,du \right\}$

$$\frac{d^2}{ds^2}\mathcal{L}\left\{ \int_0^t \operatorname{sen}(3u)\,du \right\}$$

$f(t) = \operatorname{sen}(3t) \qquad g(t) = 1$

$$\frac{d^2}{ds^2}\left(\mathcal{L}\{\operatorname{sen}(3t)\}\cdot\mathcal{L}\{1\}\right)$$

$$\frac{d^2}{ds^2}\left(\frac{3}{s^2+3^2}\cdot\frac{1}{s}\right)$$

$$\frac{d^2}{ds^2}\left(\frac{3}{s(s^2+9)}\right)$$

$$\boxed{\mathcal{L}\left\{ t^2 \int_0^t \operatorname{sen}(3u)\,du \right\} = \frac{d^2}{ds^2}\left(\frac{3}{s(s^2+9)}\right)}$$ R(

3-

$$\mathcal{L}^{-1}\left\{\frac{e^{-3s}\cdot s}{s^2+6s+13}+\ln\left(\frac{s^2+4}{s+1}\right)\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{e^{-3s}\cdot s}{s^2+6s+13}\right\}+\mathcal{L}^{-1}\left\{\ln\left(\frac{s^2+4}{s+1}\right)\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s+3-3}{(s+3)^2+4}\right\}+\frac{1}{s}\mathcal{L}^{-1}\left\{-\frac{d}{ds}\left(\ln\left(\frac{s^2+4}{s+1}\right)\right)\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s+3}{(s+3)^2+4}\right\}+\frac{1}{s}\mathcal{L}^{-1}\left\{-\frac{d}{ds}\left[\ln(s^2+4)-\ln(s+1)\right]\right\}$$

$$e^{-3t}\cos(2t)+\frac{1}{s}\mathcal{L}^{-1}\left\{-\left(\frac{2s}{s^2+4}-\frac{1}{s+1}\right)\right\}$$

$$e^{-3t}\cos(2t)+\frac{1}{s}\mathcal{L}^{-1}\left\{-2\cdot\frac{s}{s^2+4}+\frac{1}{s+1}\right\}$$

$$\boxed{e^{-3t}\cos(2t)+\frac{1}{s}\left[-2\cdot\cos(2t)+e^{-t}\right]}\ \mathbb{R}$$