1- $f(x) = x^2 - x + 3 \quad , \quad -2 \leq x \leq 2$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \operatorname{sen}\left(\frac{n\pi x}{L}\right) \right] \qquad L = 2$$

$$a_0 = \frac{1}{2}\int_{-2}^{2} (x^2 - x + 3)\,dx = \frac{1}{2}\left(\frac{x^3}{3} - \frac{x^2}{2} + 3x\right)\Bigg|_{-2}^{2} = \frac{26}{3}$$

$$a_n = \frac{1}{2}\int_{-2}^{2} (x^2 - x + 3)\cos\left(\frac{n\pi x}{2}\right) dx$$

$x^2 - x + 3$	$+$	$\cos\left(\frac{n\pi x}{2}\right)$
$2x - 1$	$-$	$\frac{2}{n\pi}\operatorname{sen}\left(\frac{n\pi x}{2}\right)$
2	$+$	$-\frac{4}{n^2\pi^2}\cos\left(\frac{n\pi x}{2}\right)$
0		$-\frac{8}{n^3\pi^3}\operatorname{sen}\left(\frac{n\pi x}{2}\right)$

$$a_n = \frac{1}{2}(2x-1)\left(\frac{4}{n^2\pi^2}\cos\left(\frac{n\pi x}{2}\right)\right)\Bigg|_{-2}^{2} = \frac{2}{n^2\pi^2}(2x-1)\left[\cos\left(\frac{n\pi x}{2}\right)\right]_{-2}^{2}$$

$$a_n = \frac{2}{n^2\pi^2}\left[3(\cos(n\pi)) - (-5)\cos(-n\pi)\right]$$

$$a_n = \frac{2}{n^2\pi^2}\left[3(-1)^n + 5(-1)^n\right] = \frac{16(-1)^n}{n^2\pi^2}$$

$$b_n = \frac{1}{2}\int_{-2}^{2} (x^2 - x + 3)\,\text{sen}\left(\frac{n\pi x}{2}\right) dx$$

$$\begin{array}{c|c|l}
x^2 - x + 3 & & \text{sen}\left(\frac{n\pi x}{2}\right) \\
2x - 1 & + & -\frac{2}{n\pi}\cos\left(\frac{n\pi x}{2}\right) \\
2 & - & -\frac{4}{n^2\pi^2}\,\text{sen}\left(\frac{n\pi x}{2}\right) \\
0 & + & \frac{8}{n^3\pi^3}\left(\frac{n\pi x}{2}\right)
\end{array}$$

$$b_n = \frac{1}{2}\left[-\frac{2}{n\pi}(x^2 - x + 3)\cos\left(\frac{n\pi x}{2}\right) + \frac{16}{n^3\pi^3}\cos\left(\frac{n\pi x}{2}\right)\right]_{-2}^{2}$$

$$b_n = \frac{1}{2}\left[-\frac{2}{n\pi}(5)\cos(n\pi) + \frac{16}{n^3\pi^3}\cos(n\pi)\right] - \frac{1}{2}\left[-\frac{2}{n\pi}\,9\cos(-n\pi) + \frac{16}{n^3\pi^3}\cos(-n\pi)\right]$$

$$b_n = -\frac{5}{n\pi}(-1)^n + \frac{8}{\pi^3 n^3}(-1)^n + \frac{9}{n\pi}(-1)^n - \frac{8}{n^3\pi^3}(-1)^n$$

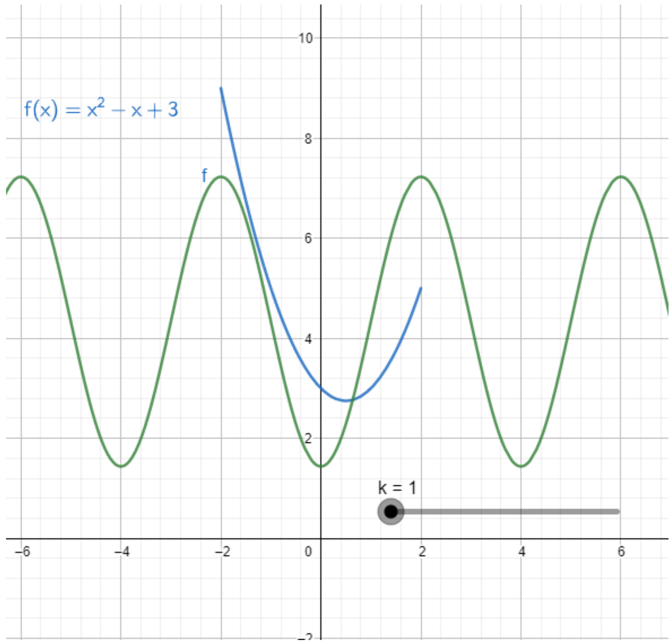
$$b_n = \frac{4}{n\pi}(-1)^n$$

$$\boxed{f(x) = \frac{13}{3} + \sum_{n=1}^{\infty}\left[\frac{16(-1)^n}{n^2\pi^2}\cos\left(\frac{n\pi x}{2}\right) + \frac{4(-1)^n}{n\pi}\,\text{sen}\left(\frac{n\pi x}{2}\right)\right]}$$

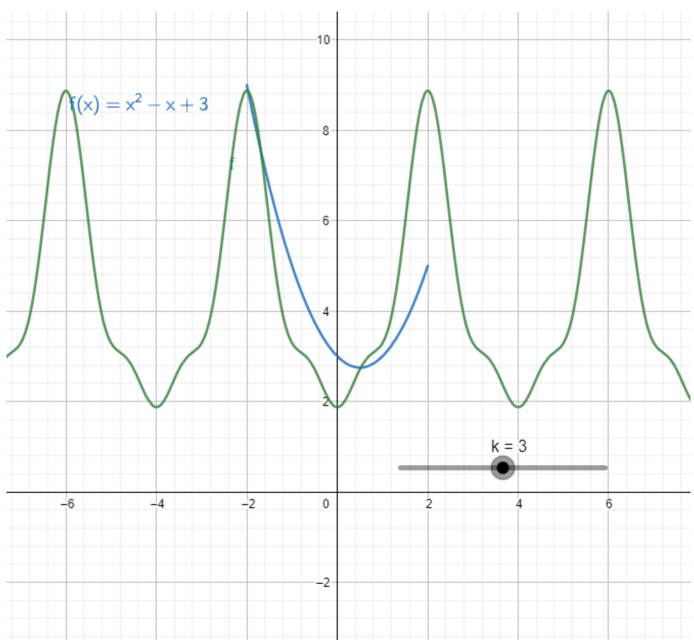
Grafica de la función en GeoGebra.

$$f(x) = x^2 - x + 3; \qquad -2 \leqslant x \leqslant 2$$

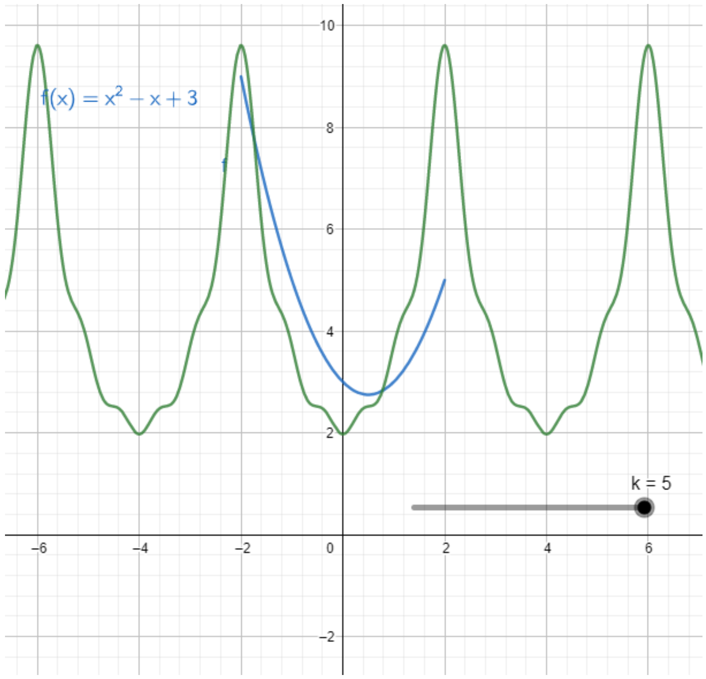
Para $n = 1$



Para $n = 3$



Para $n = 5$



② $f(x)=\begin{cases}-x, & -1\le x\le 0\\ 0, & 0\le x\le 1\end{cases} \Rightarrow f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)+\sum_{n=1}^{\infty} b_n \operatorname{Sen}\left(\frac{n\pi x}{L}\right)$

$\Rightarrow a_0=\frac{1}{L}\int_{-L}^{L} f(x)\,dx=\frac{1}{1}\left[\int_{-1}^{0}(-x)\,dx+\int_0^1 0\,dx\right]=\int_{-1}^{0}(-x)\,dx+\underbrace{\int_0^1 0\,dx}_{\text{"0"}}$

$a_0=-\frac{x^2}{2}\Big|_{-1}^{0}=-\frac{0^2}{2}+\frac{(-1)^2}{2}=\frac{1}{2}.$

$a_n=\frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx=\frac{1}{1}\left[\int_{-1}^{0}-x\cos(n\pi x)\,dx+\underbrace{\int_0^1 0\cdot\cos(n\pi x)\,dx}_{\text{"0"}}\right]$

$a_n=-\int_{-1}^{0} x\cos(n\pi x)\,dx \Rightarrow x=u \Rightarrow du=dx,\ \cos(n\pi x)\,dx=dv \Rightarrow v=\frac{1}{n\pi}\operatorname{Sen}(n\pi x)$

$a_n=-\left(\frac{x}{n\pi}\operatorname{Sen}(n\pi x)\Big|_{-1}^{0}-\int_{-1}^{0}\frac{1}{n\pi}\operatorname{Sen}(n\pi x)\,dx\right)=\frac{-x}{n\pi}\operatorname{Sen}(n\pi x)\Big|_{-1}^{0}-\frac{1}{n^2\pi^2}\cos(n\pi x)\Big|_{-1}^{0}$

$a_n=\underbrace{\frac{-0}{n\pi}\operatorname{Sen}(n\pi\cdot 0)+\frac{1}{n\pi}\operatorname{Sen}(\pi n)}_{\text{"0"}}-\frac{1}{n^2\pi^2}\cos(n\pi\cdot 0)+\frac{1}{n^2\pi^2}\cos(n\pi)$

$a_n=\frac{-1}{n^2\pi^2}+\frac{1}{n^2\pi^2}(-1)^n \quad \left\{\cos(n\pi) \begin{array}{l}\to -1 \Rightarrow n=\text{Impar}\\ \to 1 \Rightarrow n=\text{Par}\end{array}\right.$

$b_n=\frac{1}{L}\int_{-L}^{L} f(x)\operatorname{Sen}\left(\frac{n\pi x}{L}\right)dx=\frac{1}{1}\left[\int_{-1}^{0}-x\operatorname{Sen}(n\pi x)\,dx+\underbrace{\int_0^1 0\cdot\operatorname{Sen}(n\pi x)\,dx}_{\text{"0"}}\right]$

$b_n=-\int_{-1}^{0} x\operatorname{Sen}(n\pi x)\,dx \Rightarrow x=u \Rightarrow dx=du,\ \operatorname{Sen}(n\pi x)\,dx=dv \Rightarrow v=\frac{-1}{n\pi}\cos(n\pi x)$

$b_n=-\left(\frac{-x}{n\pi}\cos(n\pi x)\Big|_{-1}^{0}-\int_{-1}^{0}\frac{-1}{n\pi}\cos(n\pi x)\,dx\right)=\frac{x}{n\pi}\cos(n\pi x)\Big|_{-1}^{0}-\frac{1}{n^2\pi^2}\operatorname{Sen}(n\pi x)\Big|_{-1}^{0}$

$b_n=\underbrace{\frac{0}{n\pi}\cos(n\pi\cdot 0)}_{\text{"0"}}-\frac{(-1)}{n\pi}\cos(-n\pi)-\underbrace{\frac{1}{n^2\pi^2}\operatorname{Sen}(n\pi\cdot 0)+\frac{1}{n^2\pi^2}\operatorname{Sen}(n\pi\cdot -1)}_{\text{"0"}}$

$b_n=\frac{1}{n\pi}(-1)^n$ porque $\cos(-n\pi) \begin{array}{l}\to -1,\ \text{"n" impar}\\ \to 1,\ \text{"n" par.}\end{array}$

$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n\cos\left(\frac{n\pi x}{L}\right)+\sum_{n=1}^{\infty} b_n \operatorname{Sen}\left(\frac{n\pi x}{L}\right)$

$f(x)=\frac{1}{4}+\frac{-2}{\pi}\cos(\pi x)+\frac{-2}{\pi}\cos(3\pi x)+\frac{-2}{\pi}\cos(5\pi x)+\ldots+\underbrace{\frac{-1}{\pi}\operatorname{Sen}(\pi x)+\frac{1}{\pi}\operatorname{Sen}(2\pi x)+\ldots}_{\text{"0"}}$

R/ $f(x)=\frac{1}{4}-\frac{2}{\pi}\cos(\pi x)-\frac{2}{\pi}\cos(3\pi x)-\frac{2}{\pi}\cos(5\pi x)+\ldots$

Escaneado con CamScanner