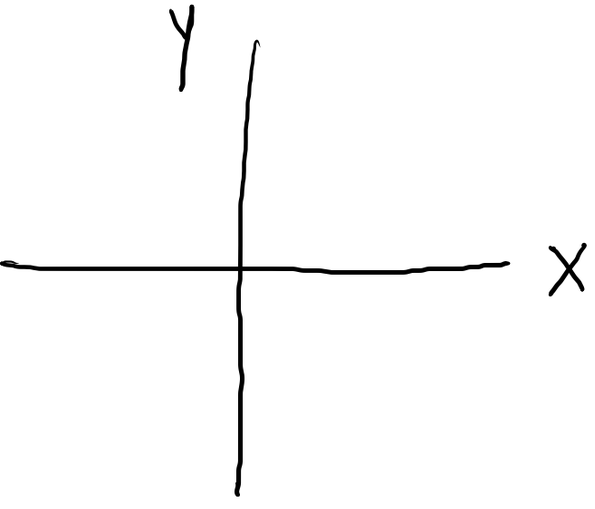
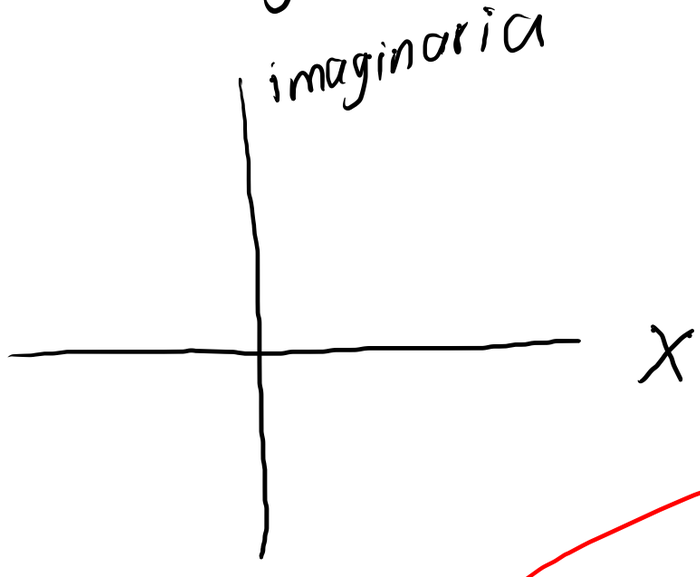
Folleto 1 : Números Complejos





$$\sqrt{-1} = i$$

$$\sqrt{-1} \cdot \sqrt{-1} = \left(\sqrt{-1}\right)^2 = -1$$

$$i \cdot i = -1$$

$$\boxed{i^2 = -1}$$

Número complejo: $X = a + bi$

a → real

bi → imaginaria.

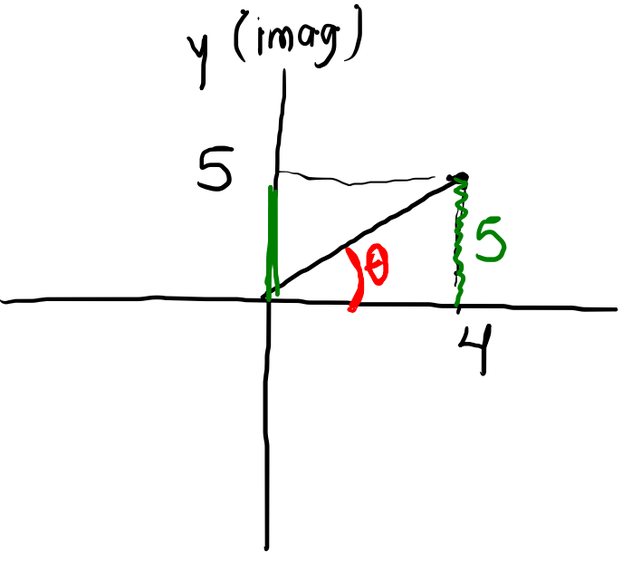
→ módulo

→ Argumento

ángulo → + / −

Se toma con el eje "X"

$X = 4 + 5i$



X real

$\|X\|$

$\|X\| = \sqrt{(a)^2 + (b)^2}$

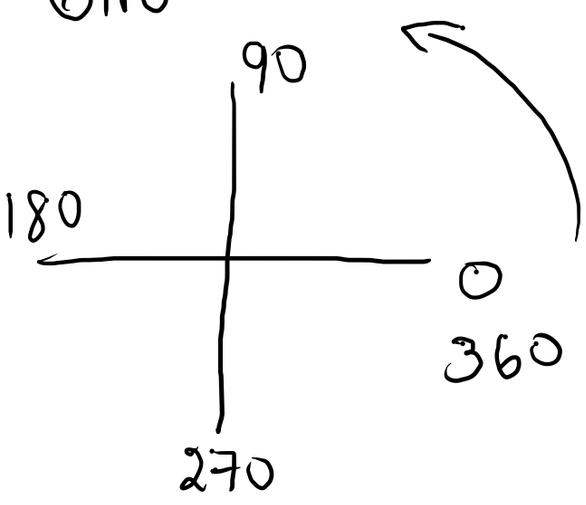
$\|X\| = \sqrt{(4)^2 + (5)^2}$

$\|X\| = \sqrt{41} \approx 6,40$

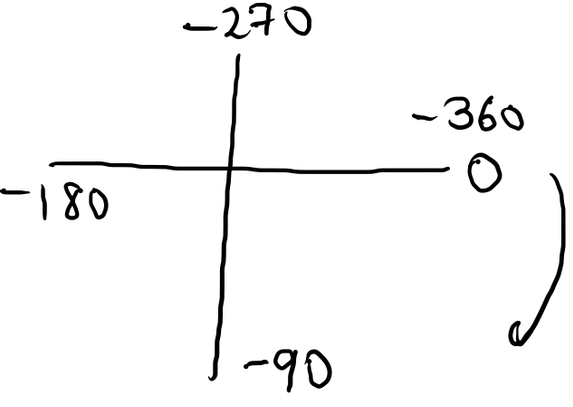
$\text{Tan}\theta = \frac{5}{4}$

$\theta = 51,34$

Giro +

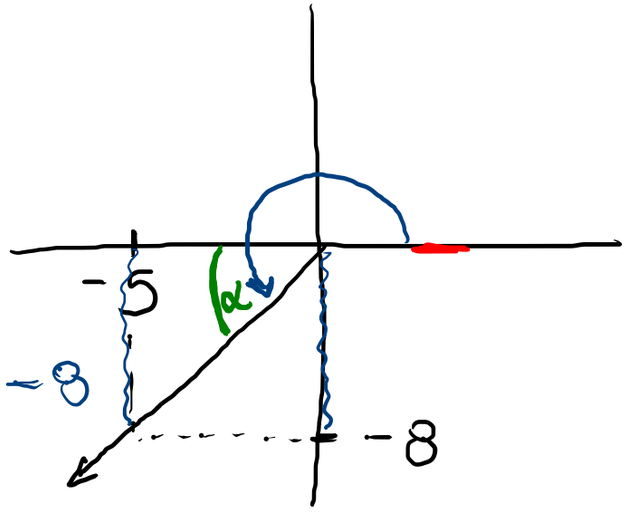


Giro −



(2) $X = -5 - 8i$

Cartesiano



$$\|X\| = \sqrt{(5)^2 + (8)^2}$$

$$\|X\| = \sqrt{89} \approx 9,43$$

$$\text{Tan}\,\alpha = \frac{8}{5} \Rightarrow \alpha = 58^\circ$$

$$\theta = 180 + 58$$

$$\theta = 238^\circ \Rightarrow \frac{238 \cdot \pi}{180}$$

$$\frac{119\,\pi}{90}$$

- Igualdad de # complejos

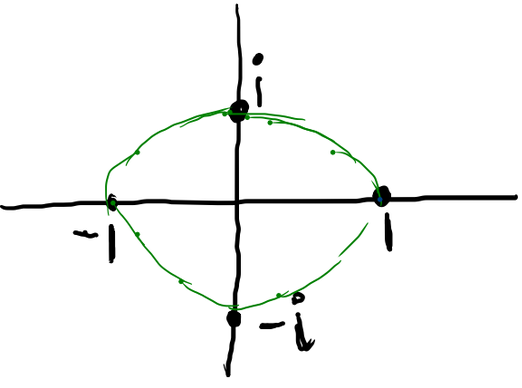
$x_1 = a + bi \qquad x_2 = c + di \quad \Rightarrow \quad \begin{matrix} a=c \\ b=d \end{matrix}$

- Opuesto $\quad x = -3+2i \quad \Rightarrow \quad$ opuesto $= 3-2i$

- Conjugado: Solo le cambio el signo a la parte imaginaria

$x = -3+2i \qquad \overline{x} = -3-2i$

Operación



$$i(2+i) = 2i + i^2 = 2i - 1$$

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = i^2 \cdot i = -1 \cdot i = -i$

$i^4 = i^3 \cdot i = -i \cdot i = 1$

$i^5 = i^4 \cdot i = 1 \cdot i = i$

$i^6 = i^5 \cdot i = i \cdot i = -1$

$i^7 = i^6 \cdot i = -1 \cdot i = -i$

$i^8 = i^7 \cdot i = -i \cdot i = 1$

$i^9 = i^8 \cdot i = 1 \cdot i = i$

Potencias de i

Residuo es cero = 1

Residuo es uno = i

Residuo es dos = -1

Residuo es tres = $-i$

Calculemos

$i^{256} = 1$

$i^{307} = -i$

$i^{37} = i$

$i^{50} = -1$

$i^{104} = 1$

$i^{219} = -i$

$i^{500} = 1$

256	4
16	64
0	

307	4
27	76
3	

37	4
1	9

104	4
24	26
0	

i
1
$-i$
-1

Ejemplo adicional

$z_1 = -4 + 5i$

$z_2 = 8 - 9i$

a) $z_1 \cdot z_2$

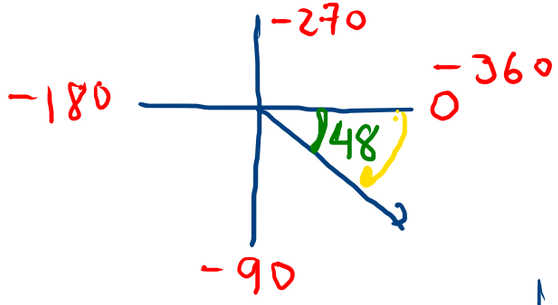
$$(-4+5i)(8-9i) = -32 + 36i + 40i - 45i^2$$

$$= -32 + 36i + 40i + 45$$

$$= 13 + 76i$$

$z_1 = -4 + 5i$

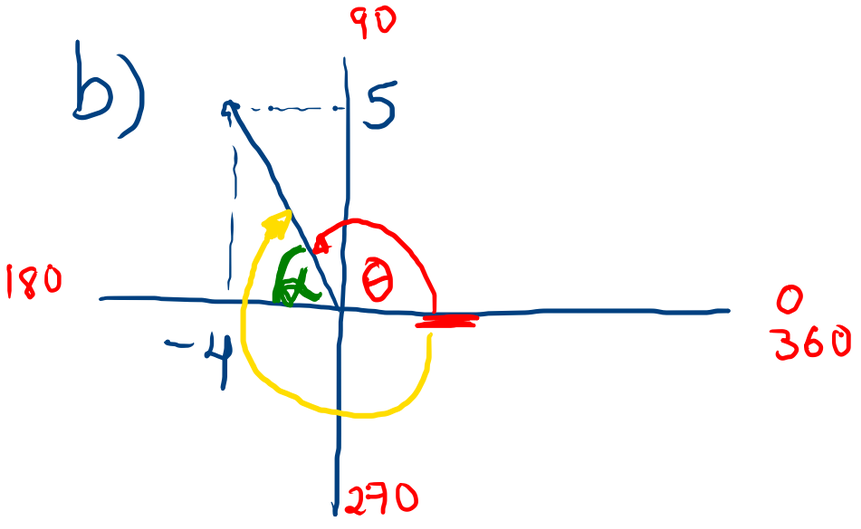
$z_2 = 8 - 9i$

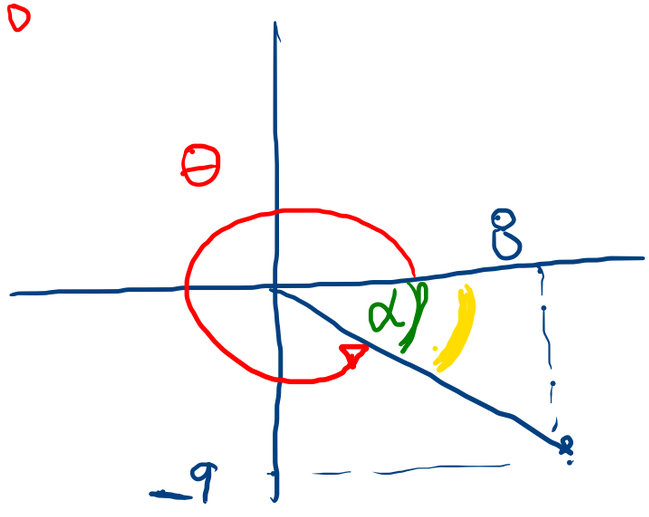


$\tan \alpha = \frac{9}{8}$

$\alpha = 48°$

b)





$\|z_1\| = \sqrt{(-4)^2 + (5)^2}$

$\|z_1\| = \sqrt{41}$ Giro +

$\tan \alpha = \frac{5}{4} \Rightarrow \alpha = 51°$

$\theta = 129$

Giro −

$\theta = -231$

$\|z_2\| = \sqrt{(8)^2 + (9)^2} = \sqrt{145}$

Giro (+) $\theta = 312$

Giro (−) $\theta = -48$

Ejemplo 2

$z = 5 + 7i$ $\qquad z' = -\sqrt{3} + 2i$

c) $z \cdot z' \cdot z = z^2 \cdot z'$

$(5+7i)^2 \cdot (-\sqrt{3} + 2i)$

$(25 + 70i - 49) \cdot (-\sqrt{3} + 2i)$ $\quad -1$

$(-24 + 70i)(-\sqrt{3} + 2i) + 140 i^2$

$24\sqrt{3} - 48i - 70\sqrt{3}\, i - 140$

$(24\sqrt{3} - 140) - (48 + 70\sqrt{3})\, i$

$-98.43 - 169.24\, i$

e) $2z - 5\overline{z'}$

$z = 5 + 7i \qquad z' = -\sqrt{3} + 2i$

$2(5 + 7i) - 5(-\sqrt{3} - 2i)$

$10 + 14i + 5\sqrt{3} + 10i$

$(10 + 5\sqrt{3}) + 24i$ //

Representaciones de un complejo.

1) Cartesiana
2) Gráfica
3) Polar
4) Exponencial

Polar

$z = x + yi$

$z = r\cos\theta + r\,\text{Sen}\,\theta\, i$

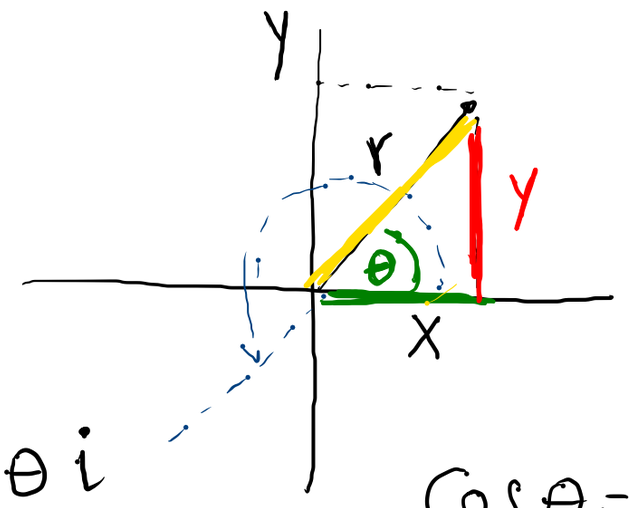
$z = r(\cos\theta + i\,\text{Sen}\,\theta)$

Euler

$e^{i\theta} = \cos\theta + i\,\text{Sen}\,\theta$

Exponencial

$z = r\,e^{i\theta}$



radio
ángulo

$\cos\theta = \frac{x}{r}$

$r\cos\theta = x$

$\text{Sen}\,\theta = \frac{y}{r}$

$r\,\text{Sen}\,\theta = y$

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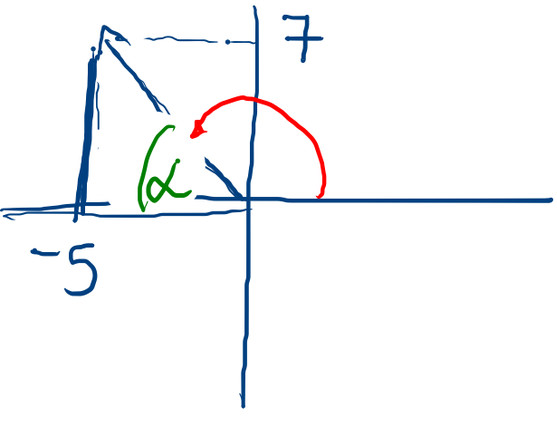
a) $-5+7i$



$\|r\| = \sqrt{(7)^2+(5)^2}$

$r = \sqrt{74}$

Polar

$z = r(\cos\theta + i\,\mathrm{Sen}\,\theta)$

$\boxed{z = \sqrt{74}(\cos 126 + i\,\mathrm{Sen}\,126)}$ manera larga.

$z = (\sqrt{74})_{126^\circ}$ → corta

$(\sqrt{74})\ \frac{7}{10}\pi$

radio ↓ módulo

$\mathrm{Tan}\,\alpha = \frac{7}{5}$

$\alpha = 54^\circ$

$\theta = 180 - 54$

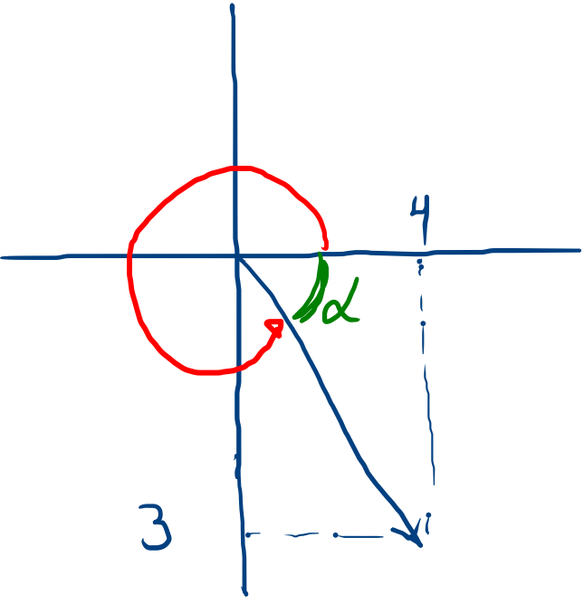
$\theta = 126^\circ \rightarrow \frac{126}{180}\pi \Rightarrow \frac{7}{10}\pi$

Exponencial

$z = re^{i\theta}$

$z = \sqrt{74}\,e^{i\cdot\frac{7}{10}\pi}$

b) $4-3i$



$$\|r\| = \sqrt{(4)^2+(3)^2}$$

$$\|r\| = 5$$

$$\text{Tan}\,\alpha = \frac{3}{4}$$

$$\alpha = 36{,}87$$

$$\alpha = 37°$$

$$\theta = 360-37$$

$$\theta = 323$$

POLAR

$$z = r(\cos\theta + i\,\text{Sen}\,\theta)$$

$$z = 5(\cos 323 + i\,\text{Sen}\,323)$$

$$z = (5)_{323°}$$

EXPONENCIAL

$$z = re^{i\theta}$$

$$z = 5e^{i.323}$$

Multiplicación

a) $(-2+2i)(3+i)$

$-6-2i+6i-2$

$-8+4i$

b) $(a+bi)(a-bi)$

$(a)^2-(bi)^2$

a^2+b^2

Division

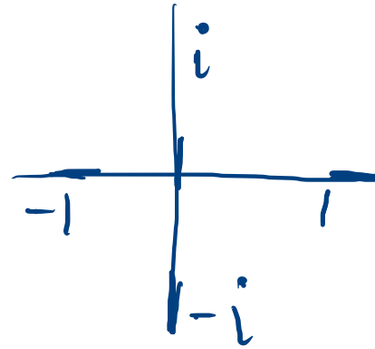
a) $\frac{(3+i)}{(1+2i)} \cdot \frac{(1-2i)}{(1-2i)} = \frac{3-6i+i+2}{1+4} = \frac{5-5i}{5}$

$= 1 - i$ //

b) $\frac{1}{i} \cdot \frac{i}{i} = \frac{i}{-1} = -i$ //

Ejemplo Calcule $\dfrac{i^7 - i^{-7}}{2i}$

$$7 \mid 4$$
$$3 \mid 1$$



$$\frac{i^7 - \frac{1}{i^7}}{2i} = \frac{-i - \frac{1}{-i}}{2i} = \frac{-i + \frac{1}{i}}{2i}$$

$$\frac{\frac{1+1}{i}}{\frac{2i}{1}} = \frac{2}{-2} = -1 \;//$$

Potencias. Teorema de Moivre. POLAR

$$z^n = \{ r(\cos\theta + i\,\text{sen}\,\theta) \}^n = r^n(\cos n\theta + i\,\text{sen}\,n\theta)$$

POLAR

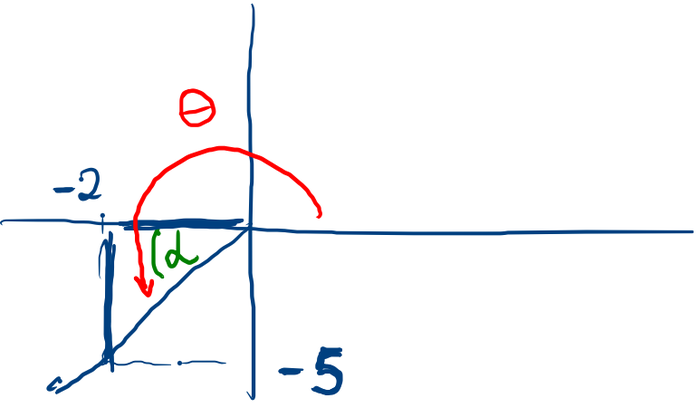
Ejemplo adicional

$z = -2 - 5i$. Halle z^5

$$\left(\sqrt{29}(\cos 248 + i\,\text{sen}\,248\right)^5$$

$$(\sqrt{29})^5(\cos 248\cdot 5 + i\,\text{sen}\,248\cdot 5)$$

$$4529(\cos 1240 + i\,\text{sen}\,1240)$$



$$r = \sqrt{(2)^2 + (5)^2}$$

$$r = \sqrt{29}$$

$$\text{Tan}\,\alpha = \frac{5}{2} \Rightarrow \alpha = 68$$

$$\theta = 180 + 68 \Rightarrow 248^\circ$$