1- $\vec{A} = 4\vec{a_r} + 3\vec{a_\phi} + 5\vec{a_z}$ $\qquad P = (6, 45°, 2)$

$\vec{B} = 7\vec{a_r} + 2\vec{a_\phi} + 1\vec{a_z}$

a) Pasamos $\vec{A}$ $(r, \phi, z) \rightarrow (x, y, z)$

$\vec{A} = A_x a_x + A_y a_y + A_z a_z$

$A_x = A_r \cos\phi - A_\phi \operatorname{sen}\phi$

$A_x = 4 \cdot \cos(45) - 3 \operatorname{sen}(45)$

$A_x = \frac{\sqrt{2}}{2}$

$A_y = A_r \operatorname{sen}\phi + A_\phi \cos\phi$

$A_y = 4 \operatorname{sen}(45) + 3 \cos(45)$

$A_y = \frac{7\sqrt{2}}{2}$

$A_z = A_z$

$A_z = 5$

$\vec{A} = \left(\frac{\sqrt{2}}{2}, \frac{7\sqrt{2}}{2}, 5\right)$

$\vec{B} = B_x a_x + B_y a_y + B_z a_z$

$B_x = B_r \cdot \cos\phi - B_\phi \operatorname{sen}\phi$

$B_x = 7 \cdot \cos(45) - 2 \operatorname{sen}(45)$

$B_x = \frac{5\sqrt{2}}{2}$

$B_y = B_r \operatorname{sen}\phi + B_\phi \cos\phi$

$B_y = 7 \operatorname{sen}(45) + 2 \cos(45)$

$B_y = \frac{9\sqrt{2}}{2}$

$B_z = B_z$

$B_z = 1$

$\vec{B} = \left(\frac{5\sqrt{2}}{2}, \frac{9\sqrt{2}}{2}, 1\right)$

a) $A + B = (A_x + B_x, A_y + B_y, A_z + B_z)$

$A + B = \left(\frac{\sqrt{2}}{2} + \frac{5\sqrt{2}}{2}, \frac{7\sqrt{2}}{2} + \frac{9\sqrt{2}}{2}, 5 + 1\right)$

$A + B = (3\sqrt{2}, 8\sqrt{2}, 6)$

$\boxed{A + B = 3\sqrt{2}\,\vec{a_x} + 8\sqrt{2}\,\vec{a_y} + 6\,a_z}$ R/a

$\vec{A} = \left(\frac{\sqrt{2}}{2}, \frac{7\sqrt{2}}{2}, 5\right) \qquad \vec{B} = \left(\frac{5\sqrt{2}}{2}, \frac{9\sqrt{2}}{2}, 1\right)$

b) $A \cdot B = A_x \cdot B_x + A_y \cdot B_y + A_z \cdot B_z$

$A \cdot B = \frac{\sqrt{2}}{2} \cdot \frac{5\sqrt{2}}{2} + \frac{7\sqrt{2}}{2} \cdot \frac{9\sqrt{2}}{2} + 5 \cdot 1$

$\boxed{A \cdot B = 39}$ R/b

c) $A \times B =$

$$\begin{array}{c} \begin{array}{ccc} a_x & a_y & a_z \end{array} \\ \begin{vmatrix} \frac{\sqrt{2}}{2} & \frac{7\sqrt{2}}{2} & 5 \\ \frac{5\sqrt{2}}{2} & \frac{9\sqrt{2}}{2} & 1 \end{vmatrix} \end{array} = \begin{vmatrix} \frac{7\sqrt{2}}{2} & 5 \\ \frac{9\sqrt{2}}{2} & 1 \end{vmatrix} \vec{a_x} - \begin{vmatrix} \frac{\sqrt{2}}{2} & 5 \\ \frac{5\sqrt{2}}{2} & 1 \end{vmatrix} \vec{a_y} + \begin{vmatrix} \frac{\sqrt{2}}{2} & \frac{7\sqrt{2}}{2} \\ \frac{5\sqrt{2}}{2} & \frac{9\sqrt{2}}{2} \end{vmatrix} \vec{a_z}$$

$$= \left(\frac{7\sqrt{2}}{2} \cdot 1 - 5 \cdot \frac{9\sqrt{2}}{2}\right) \vec{a_x} - \left(\frac{\sqrt{2}}{2} \cdot 1 - 5 \cdot \frac{5\sqrt{2}}{2}\right) \vec{a_y} + \left(\frac{\sqrt{2}}{2} \cdot \frac{9\sqrt{2}}{2} - \frac{7\sqrt{2}}{2} \cdot \frac{5\sqrt{2}}{2}\right) \vec{a_z}$$

$$= -19\sqrt{2}\, \vec{a_x} + 12\sqrt{2}\, \vec{a_y} - 13\, \vec{a_z}$$

$\Rightarrow \boxed{A \times B = -19\sqrt{2}\, \vec{a_x} + 12\sqrt{2}\, \vec{a_y} - 13\, \vec{a_z}}$ R/c

d)

$|A| = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{7\sqrt{2}}{2}\right)^2 + 5^2} = 5\sqrt{2}$

$|B| = \sqrt{\left(\frac{5\sqrt{2}}{2}\right)^2 + \left(\frac{9\sqrt{2}}{2}\right)^2 + 1^2} = 3\sqrt{6}$

$\cos \alpha = \frac{\vec{A} \cdot \vec{B}}{|A| \cdot |B|} = \frac{39}{5\sqrt{2} \cdot 3\sqrt{6}}$

$\cos \alpha = \frac{13\sqrt{3}}{30}$

$\alpha = \cos^{-1}\left(\frac{13\sqrt{3}}{30}\right)$

$\boxed{\alpha = 41,36}$ R/d

e) $\vec{A} = \left(\frac{\sqrt{2}}{2}, \frac{7\sqrt{2}}{2}, 5\right)$ de $(x, y, z) \to (r, \theta, \phi)$

$A = A_r \vec{a}_r + A_\theta \vec{a}_\theta + A_\phi \vec{a}_\phi$ $P(6, 45; 2)$

Primero pasamos $P(6, 45, 2) \to P(x, y, z)$
$P(x, y, z) \to P(r, \theta, \phi)$

$P = (6, 45, 2)$

$6 = \sqrt{x^2 + y^2}$ $\tan(45) = \frac{y}{x}$

$6 = \sqrt{x^2 + x^2}$

$6 = \sqrt{2x^2}$ $1 = \frac{y}{x} \Rightarrow \underline{x = y}$

$6 = \sqrt{2}\, x$

$\frac{6}{\sqrt{2}} = x = 3\sqrt{2} = y$ $z = 2$

$P(\overset{r}{6}, \overset{\phi}{45}, \overset{z}{2}) = P(\overset{x}{3\sqrt{2}}, \overset{y}{3\sqrt{2}}, \overset{z}{2})$

$P(3\sqrt{2}, 3\sqrt{2}, 2) \to P(r, \theta, \phi)$

$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{(3\sqrt{2})^2 + (3\sqrt{2})^2 + 2^2}$

$r = 2\sqrt{10}$

$\theta = \tan^{-1}\left(\frac{\sqrt{(3\sqrt{2})^2 + (3\sqrt{2})^2}}{2}\right)$ $\varphi = \tan^{-1}\left(\frac{3\sqrt{2}}{3\sqrt{2}}\right)$

$\theta = 71,56°$ $\varphi = 45°$

$P(r, \theta, \varphi) = (2\sqrt{10}, 71,56°, 45°)$

$\vec{A} = 4\,\vec{a_r} + 3\,\vec{a_\phi} + 5\,\vec{a_z}$

$\vec{A} = \left(\frac{\sqrt{2}}{2}, \frac{7\sqrt{2}}{2}, 5\right)$

$\vec{A} = A_r a_r + A_\theta a_\theta + A_\phi a_\phi \qquad P(r, \theta, \varphi) = (2\sqrt{10}, 71{,}56°, 45°)$

$\vec{a_x} = \operatorname{sen}\theta\cos\phi\,\vec{a_r} + \cos\theta\cos\phi\,\vec{a_\theta} - \operatorname{sen}\phi\,\vec{a_\phi}$

$\vec{a_x} = \operatorname{sen}(71{,}56)\cos(45)\,\vec{a_r} + \cos(71{,}56)\cos(45)\,\vec{a_\theta} - \operatorname{sen}(45)\,\vec{a_\phi}$

$\Rightarrow \vec{a_x} = 0{,}67\,\vec{a_r} + 0{,}22\,\vec{a_\theta} - 0{,}70\,\vec{a_\phi}$

$\vec{a_y} = \operatorname{sen}\theta\,\operatorname{sen}\phi\,\vec{a_r} + \cos\theta\,\operatorname{sen}\phi\,\vec{a_\theta} + \cos\phi\,\vec{a_\phi}$

$\vec{a_y} = \operatorname{sen}(71{,}56)\operatorname{sen}(45)\,\vec{a_r} + \cos(71{,}56)\operatorname{sen}(45)\,\vec{a_\theta} + \cos(45)\,\vec{a_\phi}$

$\Rightarrow a_y = 0{,}67\,\vec{a_r} + 0{,}22\,\vec{a_\theta} + 0{,}70\,\vec{a_\phi}$

$a_z = \cos\theta\,\vec{a_r} - \operatorname{sen}\theta\,\vec{a_\phi}$

$a_z = \cos(71{,}56)\,\vec{a_r} - \operatorname{sen}(71{,}56)\,\vec{a_\phi}$

$\Rightarrow a_z = 0{,}31\,\vec{a_r} - 0{,}94\,\vec{a_\phi}$

$A = \frac{\sqrt{2}}{2}\vec{a_x} + \frac{7\sqrt{2}}{2}\vec{a_y} + 5\,a_z$

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