Respoestas Portafolio 1 Angie Marchena M.

1 $A = (5,3,2)$

$B = (7,30,70) = (7, \frac{\pi}{6}, \frac{7\pi}{18})$

$x = r \operatorname{sen} \phi \cos\theta$

$y = r \operatorname{sen} \phi \operatorname{sen}\theta$

$z = r \cos\phi$

B: $x = 7\operatorname{sen}\left(\frac{7\pi}{18}\right)\cos\left(\frac{\pi}{6}\right) = \frac{7}{2}\sqrt{3}\cos\left(\frac{\pi}{9}\right) \approx 5{,}70$

$y = 7\operatorname{sen}\left(\frac{7\pi}{18}\right)\operatorname{sen}\left(\frac{\pi}{6}\right) = \frac{7}{2}\cos\left(\frac{\pi}{9}\right) \approx 3{,}29$

$z \quad 7\cos\left(\frac{7\pi}{18}\right) \approx 2{,}39$

$B = (5{,}70 , 3{,}29 , 2{,}39)$

$=$

$A+B = (5+5{,}70 , 3+3{,}29 , 2+2{,}39)$

$= (10{,}70 , 6{,}29 , 4{,}39)$

$A \cdot B$

$5 \cdot 5{,}70 + 3 \cdot 3{,}29 + 2 \cdot 2{,}39 = 43{,}15$

$|A| = \sqrt{5^2 + 3^2 + 2^2} = \sqrt{38}$

$|B| = \sqrt{5{,}70^2 + 3{,}29^2 + 2{,}39} = 7{,}00$

$\cos\theta = \frac{43{,}15}{\sqrt{38} \cdot 7{,}00} \rightsquigarrow \theta = \cos^{-1}\left(\frac{43{,}15}{\sqrt{38} \cdot 7}\right)$

$\theta = 0{,}36°$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 3 & 2 \\ 5{,}7 & 3{,}29 & 2{,}39 \end{vmatrix}$$

$$= \begin{vmatrix} 3 & 2 \\ 3{,}29 & 2{,}39 \end{vmatrix} \hat{i} - \begin{vmatrix} 5 & 2 \\ 5{,}7 & 2{,}39 \end{vmatrix} \hat{j} + \begin{vmatrix} 5 & 3 \\ 5{,}7 & 3{,}29 \end{vmatrix} \hat{k}$$

$$= (3 \cdot 2{,}39 - 2 \cdot 3{,}29)\hat{i} - (5 \cdot 2{,}39 - 2 \cdot 5{,}7)\hat{j} + (5 \cdot 3{,}29 - 3 \cdot 5{,}7)\hat{k}$$

$$\vec{A} \times B = (0{,}59 \, , \; 0{,}55 \, , \; -0{,}65)$$

Área del paralelogramo $|A \times B| = \sqrt{0{,}59^2 + 0{,}55^2 + (-0{,}65)^2}$

$= 1{,}03 u^2$

2) Punto$_C$ $(4, 45°, 2) \rightarrow (4, \frac{\pi}{4}, 2)$

Cartesianas

$X = r\cos\theta = 4\cos(\frac{\pi}{4}) = 2\sqrt{2}$

$Y = r\,sen\,\theta = 4\,sen(\frac{\pi}{4}) = 2\sqrt{2}$

$Z = Z = 2$

$(2\sqrt{2}, 2\sqrt{2}, 2)$

Polares

$r = \sqrt{x^2+y^2+z^2} = \sqrt{(2\sqrt{2})^2+(2\sqrt{2})^2+2^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{2\sqrt{2}}{2\sqrt{2}}\right) = 45°$

$\phi \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{2}\right) = 63,44°$

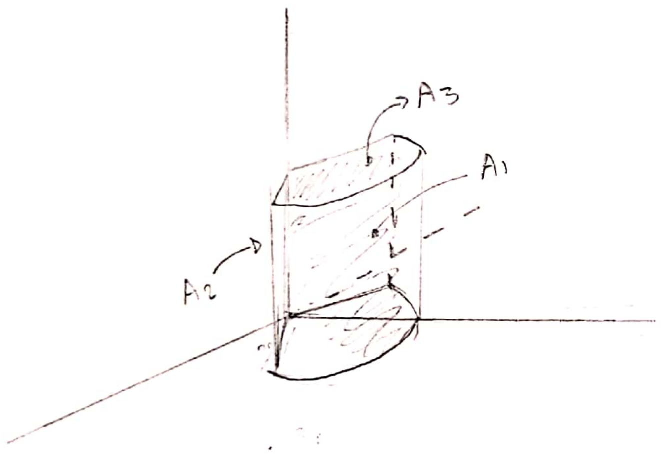
$(2\sqrt{5}, 63,44°, 45°)$

3)

$r = 2$

$h = 5$

$30 \leq \phi \leq 120$



$A_2 = \int_0^5 \int_0^2 dr\,dz$

$A_2 = \int_0^5 2dz = 2 \cdot 5 = 10$

$A_3 = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \int_0^2 r\,dr\,d\phi = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{r^2}{2}\Big|_0^2 d\phi$

$A_3 = \int_{\pi/3}^{\frac{2\pi}{3}} 2d\phi = 2\left(\frac{2\pi}{3} - \frac{\pi}{3}\right) = \frac{2}{3}\pi$

$A_1 = \int_0^5 \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} r\,d\phi\,dz$

$A_1 = r\int_0^5 \left(\frac{2\pi}{3} - \frac{\pi}{3}\right) dz$

$A_1 = r\int_0^5 \frac{\pi}{3} dz$

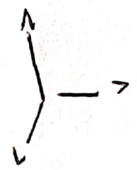
$A_1 = r \cdot \frac{5\pi}{3} = \frac{2 \cdot 5\pi}{3} = \frac{10\pi}{3}$

$A_{Tot} = A_1 + 2A_2 + 2A_3$

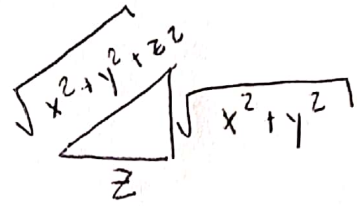
$A_{Tot} = \frac{10\pi}{3} + 2 \cdot 10 + 2 \cdot \frac{2}{3}\pi$

$A_{tot} = 20 + 4\pi$

4)



$\theta \tan^{-1} \sqrt{\frac{x^2+y^2}{z}}$ $\qquad r = $



$\tan\theta \sqrt{\frac{x^2+y^2}{z}}$

$\operatorname{sen}\theta \; \frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}, \quad \cos\theta = \frac{z}{\sqrt{x^2+y^2+z^2}}$

$$\begin{vmatrix} A_x \\ A_y \\ A_z \end{vmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \frac{2z}{r} \\ \frac{\sqrt{x^2+y^2}}{r} \\ 0 \end{bmatrix}$$

$$A_x = \frac{x}{\sqrt{x^2+y^2+z^2}} \cdot \frac{2z}{\sqrt{x^2+y^2+z^2}} + \frac{xz}{\sqrt{x^2+y^2}\,z} \cdot \frac{\sqrt{x^2+y^2}}{(\sqrt{x^2+y^2+z^2})^2} + 0$$

$$A_y = \boxed{\frac{xz}{(x^2+y^2+z^2)} \cdot (2+1)\,\vec{a}_x}$$

$$A_y = \frac{y}{\sqrt{x^2}} \cdot \frac{2z}{\sqrt{x^2+y^2+z^2}} + \frac{yz \cdot \sqrt{x^2+y^2}}{\sqrt{x^2+y^2} \cdot (x^2+y^2+z^2)} + 0$$

$$A_y = \boxed{\frac{3yz}{x^2+y^2+z^2}\,\vec{a}_y}$$

$$A_z = \frac{z}{(x^2+y^2+z^2)} \cdot 2z - \frac{\sqrt{x^2+y^2} \cdot \sqrt{x^2+y^2}}{(x^2+y^2+z^2)}$$

$$A_z = \boxed{\frac{1}{x^2+y^2+z^2}\,(2z^2 - x^2 - y^2)\,\vec{a}_z}$$

$$\vec{F} = \frac{3xz}{(x^2+y^2+z^2)}\vec{a}_x + \frac{3y^2}{(x^2+y^2+z^2)}\vec{a}_y + \left(\frac{2z^2-x^2-y^2}{x^2+y^2+z^2}\right)\vec{a}_z$$