4. (6 puntos) Dado el potencial magnético vectorial $\vec{A} = \frac{10}{\rho^2}\vec{a_z}\ Wb/m$, **(a)** Halle la densidad de corriente **J** para $\rho = 10m$, **(b)** Halle la expresión $\vec{B}$, dado **A**, **(c)** Calcule el flujo magnético total que cruza la superficie $\varphi = \frac{\pi}{2}, 1 \le \rho \le 2\ m, 0 \le z \le 5\ m$.

$\vec{J} = \nabla \times (\nabla \times A)$

$\vec{B} = \nabla \times A$

a) como $A = (0, 0, \frac{10}{\rho^2}) \Rightarrow \nabla \times A = -\frac{\partial A_z}{\partial \rho} a_\phi$

$$\nabla \times A = -\frac{\partial}{\partial \rho}\left(\frac{10}{\rho^2}\right) a_\phi$$

$$\nabla \times A = -\frac{\partial 10\rho^{-2}}{\partial \rho} = \frac{20}{\rho^3} a_\phi \leftarrow \vec{B}$$

$$\nabla \times (\nabla \times A) = \frac{1}{\rho} \cdot \frac{\partial (\rho A_\phi)}{\partial \rho} = \frac{1}{\rho} \frac{\partial}{\partial \rho}\left(\frac{20}{\rho^2}\right)$$

$$\nabla \times \nabla \times A = -\frac{40}{\rho^4} a_z$$

$$\vec{J} = \frac{-40}{\rho^4} a_z \qquad \vec{J}(\rho = 10) = \frac{-40}{10^4} a_z$$

$$\boxed{\therefore\ \vec{J}(\rho = 10) = -4 \times 10^{-3} a_z\ A/m}$$ R/a

$$\boxed{\vec{B} = \nabla \times A = \frac{20}{\rho^3} a_\phi}$$ R/b

c) $\psi = \int B \cdot dS \qquad d\vec{S} = d\rho\, dz\, a_\phi \qquad \vec{B} = \frac{20}{\rho^3} a_\phi$

$$\psi = \int_0^5 \int_1^2 \frac{20}{\rho^3} d\rho\, dz \Rightarrow \psi = \int_0^5 \frac{15}{2} dz = 37{,}5\ Wb$$

$$\boxed{\psi = 37{,}5\ Wb}$$ R/c