# Folleto 3: Derivadas con complejos

$f(z) = u(x,y) + i\,v(x,y)$

Analítica $\longrightarrow$ I. $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}$ II. $\dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x}$

$\downarrow$

Deriva normal

**Ejemplo 3**: Muestre que $f(z) = \text{Sen } 4z$ es analítica y derive

$$\text{Sen } z = \frac{e^{iz} - e^{-iz}}{2i}$$

$$f(z) = \text{Sen } 4z$$

$$f(z) = \frac{e^{i4z} - e^{-i\cdot 4z}}{2i} \Rightarrow f(z) = \frac{e^{i4(x+yi)} - e^{-i4(x+yi)}}{2i} \Rightarrow$$

$$f(z) = \frac{e^{i\cdot 4x - 4y} - e^{-4ix + 4y}}{2i} \Rightarrow f(z) = \frac{e^{i\cdot 4x} \cdot e^{-4y} - e^{-4xi} \cdot e^{4y}}{2i}$$

$$f(z) = \frac{e^{i\cdot 4x} \cdot e^{-4y} - e^{-4xi} \cdot e^{4y}}{2i} \Rightarrow$$

Euler $e^{i\theta} = \cos\theta + i\,\text{Sen}\,\theta$

$$f(z) = \frac{e^{-4y}\left[\cos 4x + i\,\text{Sen}\,4x\right] - e^{4y}\left[\overbrace{\cos(-4x)}^{\text{PAR}} + \overbrace{i\,\text{Sen}(-4x)}^{\text{iMPAR.}}\right]}{2i}$$

$$\cos(-x) = \cos x \qquad \text{Sen}(-x) = -\text{Sen}(x)$$

$$f(z) = \frac{e^{-4y}\cos 4x + i e^{-4y}\,\text{Sen}\,4x - e^{4y}\cos 4x + e^{4y}\, i\,\text{Sen}\,4x}{2i}$$

$$f(z)=\left(\frac{e^{-4y}\cos 4x+ie^{-4y}\operatorname{Sen}4x-e^{4y}\cos 4x+e^{4y}i\operatorname{Sen}4x}{2i}\right)\frac{i}{i}$$

$$f(z)=\frac{ie^{-4y}\cos 4x-e^{-4y}\operatorname{Sen}4x-ie^{4y}\cos 4x-e^{4y}\operatorname{Sen}4x}{-2}$$

$$u=\frac{-e^{-4y}\operatorname{Sen}4x-e^{4y}\operatorname{Sen}4x}{-2} \qquad v=\frac{ie^{-4y}\cos 4x-ie^{4y}\cos 4x}{-2}$$

$$u=\frac{e^{-4y}\operatorname{Sen}4x+e^{4y}\operatorname{Sen}4x}{2} \qquad v=\frac{ie^{4y}\cos 4x-ie^{-4y}\cos 4x}{2}$$

$$u = \frac{e^{-4y}\,\text{Sen}\,4x + e^{4y}\,\text{Sen}\,4x}{2} \qquad v = \left(\frac{e^{4y}\,\text{Cos}\,4x - e^{-4y}\,\text{Cos}\,4x}{2}\right) i$$

I. $$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\frac{4e^{-4y}\,\text{Cos}\,4x + 4e^{4y}\,\text{Cos}\,4x}{2} \quad \Big| \quad \frac{4e^{4y}\,\text{Cos}\,4x + 4e^{-4y}\,\text{Cos}\,4x}{2}$$

$$u = \frac{e^{-4y}\,\text{Sen}\,4x + e^{4y}\,\text{Sen}\,4x}{2} \qquad v = \left(\frac{e^{4y}\,\text{Cos}\,4x - e^{-4y}\,\text{Cos}\,4x}{2}\right) i$$

II. $\dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x}$

$$\frac{-4e^{-4y}\,\text{Sen}\,4x + 4e^{4y}\,\text{Sen}\,4X}{2} \;\Bigg|\; -\left(\frac{-4e^{4y}\,\text{Sen}\,4x + 4e^{-4y}\,\text{Sen}\,4x}{2}\right)$$

$$\frac{4e^{4y}\,\text{Sen}\,4x - 4e^{-4y}\,\text{Sen}\,4x}{2} \checkmark$$

$\Rightarrow$ Analítica

Derive $f(z) = \operatorname{Sen} 4z \Rightarrow f'(z) = 4\cos 4z$

OTRA FORMA

$f(z) = \operatorname{Sen} 4z$

$f(z) = \operatorname{Sen} 4(x+yi) \Rightarrow f(z) = \operatorname{Sen}(4x+4yi)$

6 Folleto 2 Pág 4 j) $\operatorname{Sen}(x+yi) = \operatorname{Sen} x \cosh y + i \cos x \operatorname{Senh} y$

$$f(z) = \underbrace{\operatorname{Sen} 4x \cosh 4y}_{u} + i\,\underbrace{\cos 4x \operatorname{Senh} 4y}_{v}$$

$$f(z) = \underbrace{\operatorname{Sen} 4x \operatorname{Cosh} 4y}_{u} + i\,\underbrace{\operatorname{Cos} 4x \operatorname{Senh} 4y}_{v}$$

$$(\operatorname{Senh} z)' = \operatorname{Cosh} z$$

$$(\operatorname{Cosh} z)' = \operatorname{Senh} z$$

I. $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}$

$$4 \operatorname{Cos} 4x \operatorname{Cosh} 4y = 4 \operatorname{Cos} 4x \operatorname{Cosh} 4y \quad \checkmark$$

II. $\dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x}$

$$4 \operatorname{Sen} 4x \operatorname{Senh} 4y = 4 \operatorname{Sen} 4x \operatorname{Senh} 4y \quad \checkmark$$

$\Rightarrow$ Analítica

Ejemplo 5

$$\left(\tanh^{-1} z\right)' = \frac{1}{1-z^2} \cdot 1$$

a) $\left\{\tanh^{-1}(iz+2)\right\}^{-1}$

$$-\left\{\tanh^{-1}(iz+2)\right\}^{-2} \cdot \frac{1}{1-(iz+2)^2} \cdot i$$

$$-\frac{i}{1-(iz+2)^2}\left\{\tanh^{-1}(iz+2)\right\}^{-2}$$

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b) $(z-3i)^{4z+2} \longrightarrow [a]^b = e^{b\ln(a)}$

Potencia con variable compleja

$(e^x)' = e^x \cdot$ deriv exp

$(\ln x)' = \frac{1}{x} \cdot$ derivada argumen

$$e^{(4z+2)\ln(z-3i)} =$$

$$e^{\overbrace{(4z+2)\ln(z-3i)}^{\text{Producto.}}} \cdot \left(4 \cdot \ln(z-3i) + (4z+2) \cdot \frac{1}{z-3i}\right)$$

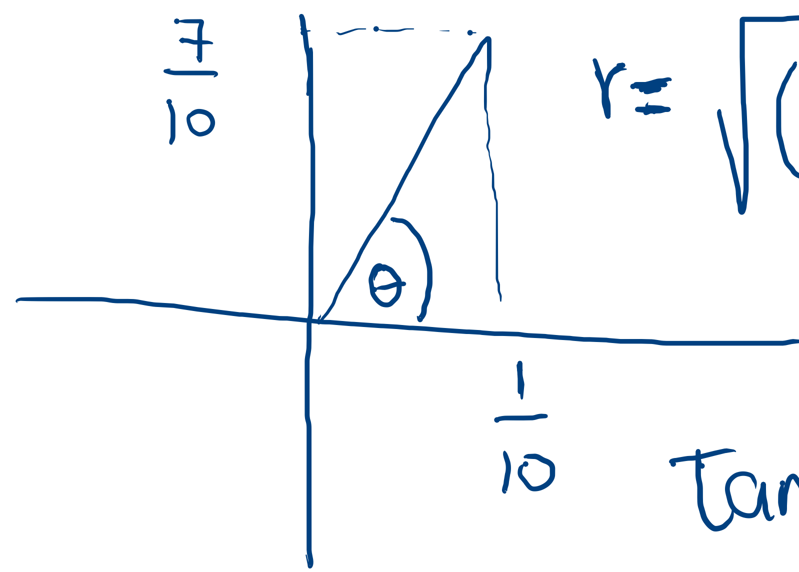
# PRACTIQUEMOS

1) Calcule $\sqrt[3]{\frac{1+2i}{3-i}}$

$$\frac{1+2i}{(3-i)}\cdot\frac{(3+i)}{(3+i)} = \frac{(1+2i)(3+i)}{9+1}$$

$$\frac{3+i+6i-2}{10} = \frac{1}{10}+\frac{7i}{10}$$

$$\sqrt[3]{\underbrace{\frac{1}{10}+\frac{7}{10}i}_{\text{POLAR}}}$$




$$r=\sqrt{\left(\frac{1}{10}\right)^2+\left(\frac{7}{10}\right)^2}$$

$$r=\frac{\sqrt{2}}{2}$$

$$\tan\theta = \frac{\frac{7}{10}}{\frac{1}{10}} \rightarrow 82$$

$$\sqrt[3]{\underbrace{\frac{1}{10}+\frac{7}{10}i}_{\text{POLAR}}}$$

$r=\dfrac{\sqrt{2}}{2}$ $\theta=82°$

$$\left(\frac{\sqrt{2}}{2}\left(\cos 82+i\operatorname{Sen} 82\right)\right)$$

$\measuredangle C=\dfrac{360}{3}\Rightarrow \underline{\underline{120}}$

$$K=0 \Rightarrow \left(\frac{\sqrt{2}}{2}\right)^{\frac{1}{3}}\cdot\left[\cos\frac{82}{3}+i\operatorname{Sen}\frac{82}{3}\right]$$

$$K=1 \Rightarrow \left(\frac{\sqrt{2}}{2}\right)^{\frac{1}{3}}\left[\cos\frac{442}{3}+i\operatorname{Sen}\frac{442}{3}\right]$$

$$K=2 \Rightarrow \left(\frac{\sqrt{2}}{2}\right)^{\frac{1}{3}}\left[\cos\frac{562}{3}+i\operatorname{Sen}\frac{562}{3}\right]$$

② Resuelva la siguiente ecuación $3i^7x^2+(2-4i)x+4i-8=0$

$a \rightarrow 3i^7$

$b \rightarrow 2-4i$

$c \rightarrow 4i-8$

$$\Delta = (b)^2-4\cdot a\cdot c$$

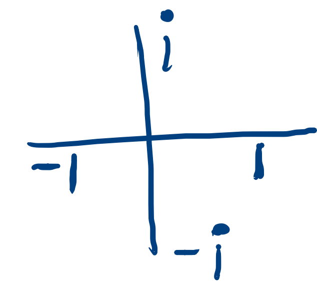
$$\Delta = (2-4i)^2-4\cdot 3i^7(4i-8)$$

$$\Delta = 4-16i-16-48i^8+96i^7$$

$$\Delta = 4-16i-16-48-96i$$

$$\Delta = -60-112i$$

$$\begin{array}{r|l} 7 & 4 \\ \hline 3 & 1 \end{array}$$




$$x_1 = \frac{-(2-4i)+\sqrt{-60-112i}}{2\cdot 3i^7}$$

$$x_2 = \frac{-(2-4i)-\sqrt{-60-112i}}{2\cdot 3i^7}$$

$$x_1 = \frac{-(2-4i)+\sqrt{-60-112i}}{2\cdot 3\underline{i}^7}$$

$x_2 = -(2-4i)$ [redacted] $\sqrt{-60-112i}$

$$\overline{\quad 2\cdot 3\underline{i}^7 \quad}$$

$$x_1 = \frac{-2+4i+\sqrt{-60-112i}}{-6i}\cdot\frac{i}{i}$$

$$x_2 = \frac{-2+4i-\sqrt{-60-112i}}{-6i}\cdot\frac{i}{i}$$

$$x = \frac{-2i-4+i\sqrt{-60-112i}}{6}$$

$$x_2 = \frac{-2i-4-i\sqrt{-60-112i}}{6}$$

(3) Determinar el valor de "a" para que el módulo de $\dfrac{a-i}{1-i}$ sea 5

$$\frac{a-i}{1-i}\cdot\frac{(1+i)}{(1+i)}=\frac{(a-i)(1+i)}{2}=\frac{a+ai-i+1}{2}$$

$$\frac{a+1}{2}+\frac{(a-1)i}{2} \qquad \sqrt{\left(\frac{a+1}{2}\right)^2+\left(\frac{a-1}{2}\right)^2}=5$$

$$\sqrt{\left(\frac{a+1}{2}\right)^2 + \left(\frac{a-1}{2}\right)^2} = 5$$

$$\begin{array}{r|l} 24 & 2 \\ 12 & 2 \\ 6 & 2 \\ 3 & 3 \end{array}$$

$$\left(\frac{a+1}{2}\right)^2 + \left(\frac{a-1}{2}\right)^2 = 25$$

$$\frac{a^2}{2} + \cancel{\frac{2a}{2}} + \frac{1}{2} + \frac{a^2}{2} - \cancel{\frac{2a}{2}} + \frac{1}{2} = 25$$

$$a^2 + 1 = 25$$

$$a = \sqrt{24}$$

$$+\,2\sqrt{6}$$

$$-\,2\sqrt{6}$$

Resuelva la ecuación $\dfrac{z}{1+\overline{z}} = 3+4i$

$z = x+yi$

$\dfrac{x+yi}{1+x-yi} = \dfrac{3+4i}{1}$

$x+yi = (3+4i)(1-x-yi)$

$x+yi = 3-3x-3yi+4i-4xi+4y$

$$\begin{cases} 3-3x+4y = x \\ y = -3y+4-4x \end{cases}$$

$$\begin{cases} \cancel{-4x}+4y = -3 \\ \cancel{4x}+4y = 4 \end{cases}$$

$8y = 1$

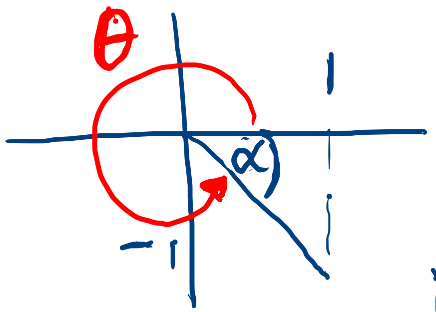
$y = \dfrac{1}{8}$

$x = \dfrac{4}{3}$

Calcular $(1-i)^{4i}$

$$e^{4i\left(\ln(1-i)\right)}$$

$$e^{4i\left(\ln\sqrt{2} + i\cdot 315^\circ\right)}$$




$r=\sqrt{(1)^2+(1)^2}$

$r=\sqrt{2}$

$\tan\theta = 1$

$\alpha = 45^\circ$

$\theta = 315^\circ$

Derive: $\underbrace{\cosh^{-1}(2z+3)}\cdot\underbrace{e^{z-3}}$

$$\left(\cosh^{-1} z\right)' = \frac{1}{\sqrt{z^2-1}}$$

$$\frac{1}{\sqrt{(2z+3)^2-1}}\cdot 2\cdot e^{z-3} + \cosh^{-1}(2z+3)\cdot e^{z-3}\cdot 1$$

$$\frac{2e^{z-3}}{\sqrt{(2z+3)^2-1}} + e^{z-3}\cdot\cosh^{-1}(2z+3)$$

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Muestre que es analítica y derive $f(z) = \cos 2z$

$f(z) = \cos 2z$

$\cos(x+yi) = \cos x \cdot \cosh y + i \operatorname{Sen} x \operatorname{Sen} h y$

$f(z) = \cos 2(x+yi) \Rightarrow f(z) = \cos(2x+2yi)$

$f(z) = \underbrace{\cos 2x \cdot \cosh 2y}_{u} + \underbrace{i \operatorname{Sen} 2x \operatorname{Sen} h 2y}_{v}$

I. $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}$

$-2 \operatorname{Sen} 2x \cdot \cosh 2y = 2 \operatorname{Sen} 2x \cosh 2y$

No es analítico