

$$1- \quad f(x) = x^2 - x + 3 \quad , \quad -2 \leq x \leq 2$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right] \quad L=2$$

$$a_0 = \frac{1}{2} \int_{-2}^2 (x^2 - x + 3) dx = \frac{1}{2} \left(\frac{x^3}{3} - \frac{x^2}{2} + 3x \right) \Big|_{-2}^2 = \frac{26}{3}$$

$$a_n = \frac{1}{2} \int_{-2}^2 (x^2 - x + 3) \cos\left(\frac{n\pi x}{2}\right) dx$$

$$\begin{array}{rcl} x^2 - x + 3 & \begin{array}{c} | \\ + \\ | \\ + \\ + \\ | \end{array} & \cos\left(\frac{n\pi x}{2}\right) \\ 2x - 1 & \begin{array}{c} | \\ - \\ | \\ - \\ - \\ | \end{array} & \frac{2}{n\pi} \sin\left(\frac{n\pi x}{2}\right) \\ 2 & \begin{array}{c} | \\ + \\ | \\ + \\ + \\ | \end{array} & -\frac{4}{n^2\pi^2} \cos\left(\frac{n\pi x}{2}\right) \\ 0 & \begin{array}{c} | \\ - \\ | \\ - \\ - \\ | \end{array} & \frac{-8}{n^3\pi^3} \sin\left(\frac{n\pi x}{2}\right) \end{array}$$

$$a_n = \frac{1}{2} (2x-1) \left(\frac{4}{n^2\pi^2} \cos\left(\frac{n\pi x}{2}\right) \right) \Big|_{-2}^2 = \frac{2}{n^2\pi^2} (2x-1) \left[\cos\left(\frac{n\pi x}{2}\right) \right]_{-2}^2$$

$$a_n = \frac{2}{n^2\pi^2} \left[3(\cos(n\pi)) - (-5)\cos(-n\pi) \right]$$

$$a_n = \frac{2}{n^2\pi^2} \left[3(-1)^n + 5(-1)^n \right] = \frac{16(-1)^n}{n^2\pi^2}$$

$$b_n = \frac{1}{2} \int_{-2}^2 (x^2 - x + 3) \sin\left(\frac{n\pi x}{2}\right) dx$$

$$\begin{array}{rcl} x^2 - x + 3 & \begin{array}{c} | \\ + \\ | \\ + \\ | \\ + \\ | \end{array} & \sin\left(\frac{n\pi x}{2}\right) \\ 2x - 1 & \begin{array}{c} | \\ - \\ | \\ - \\ | \\ - \\ | \end{array} & \frac{-2}{n\pi} \cos\left(\frac{n\pi x}{2}\right) \\ 2 & \begin{array}{c} | \\ + \\ | \\ + \\ | \\ + \\ | \end{array} & \frac{-4}{n^2\pi^2} \sin\left(\frac{n\pi x}{2}\right) \\ 0 & \begin{array}{c} | \\ - \\ | \\ - \\ | \\ - \\ | \end{array} & \frac{8}{n^3\pi^3} \cos\left(\frac{n\pi x}{2}\right) \end{array}$$

$$b_n = \frac{1}{2} \left[-\frac{2}{n\pi} (x^2 - x + 3) \cos\left(\frac{n\pi x}{2}\right) + \frac{16}{n^3\pi^3} \cos\left(\frac{n\pi x}{2}\right) \right]_{-2}^2$$

$$b_n = \frac{1}{2} \left[-\frac{2}{n\pi} (9) \cos(n\pi) + \frac{16}{n^3\pi^3} \cos(n\pi) \right] - \frac{1}{2} \left[-\frac{2}{n\pi} \cdot 9 \cos(-n\pi) + \frac{16}{n^3\pi^3} \cos(-n\pi) \right]$$

$$b_n = -\frac{9}{n\pi} (-1)^n + \frac{8}{\pi^3 n^3} (-1)^n + \frac{9}{n\pi} (-1)^n - \frac{8}{n^3\pi^3} (-1)^n$$

$$b_n = \frac{4}{n\pi} (-1)^n$$

$$f(x) = \frac{B}{3} + \sum_{n=1}^{\infty} \left[\frac{16(-1)^n}{n^2\pi^2} \cos\left(\frac{n\pi x}{2}\right) + \frac{4(-1)^n}{n\pi} \sin\left(\frac{n\pi x}{2}\right) \right]$$