Pregunta 1

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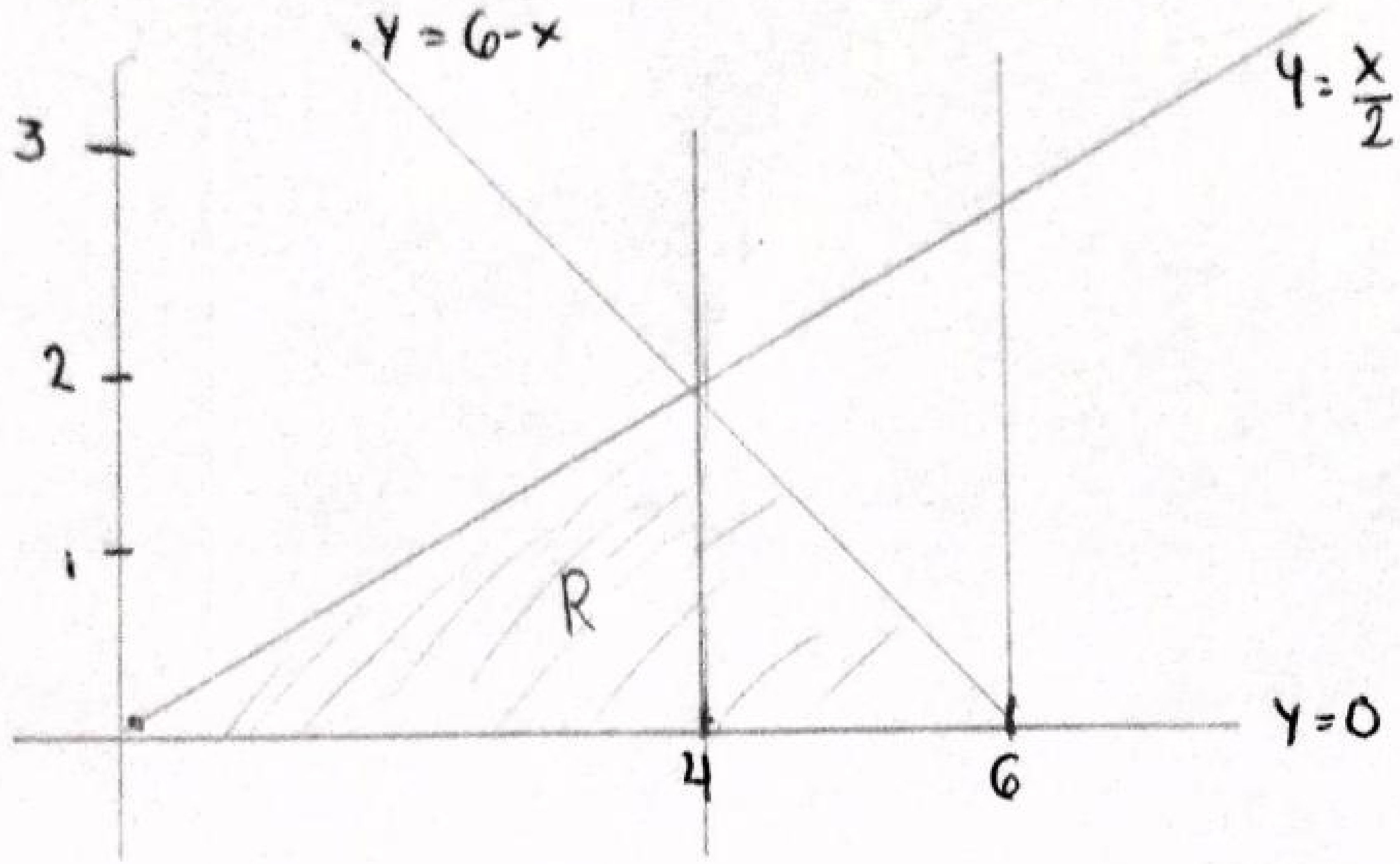
Examen 2.

a)



b) Cambiar el orden despejamos las X

$y = 6-x$
$\Rightarrow x = 6-y$

$y = \frac{x}{2}$
$\Rightarrow 2y = x$

Intersección
$2y = 6-y$
$2y+y = 6$
$3y = 6$
$y = 2$

Integral $\int_0^2 \int_{2y}^{6-y} dx\,dy$

$= \int_0^2 (6-y-2y)\,dy$

$= \int_0^2 (6-3y)\,dy = 6y - \frac{3y^2}{2} \Big|_0^2$

$= 6\cdot 2 - \frac{3-(2)^2}{2} - \left(6\cdot 0 - \frac{3-0^2}{2}\right)$

$= 6\ ul^2$

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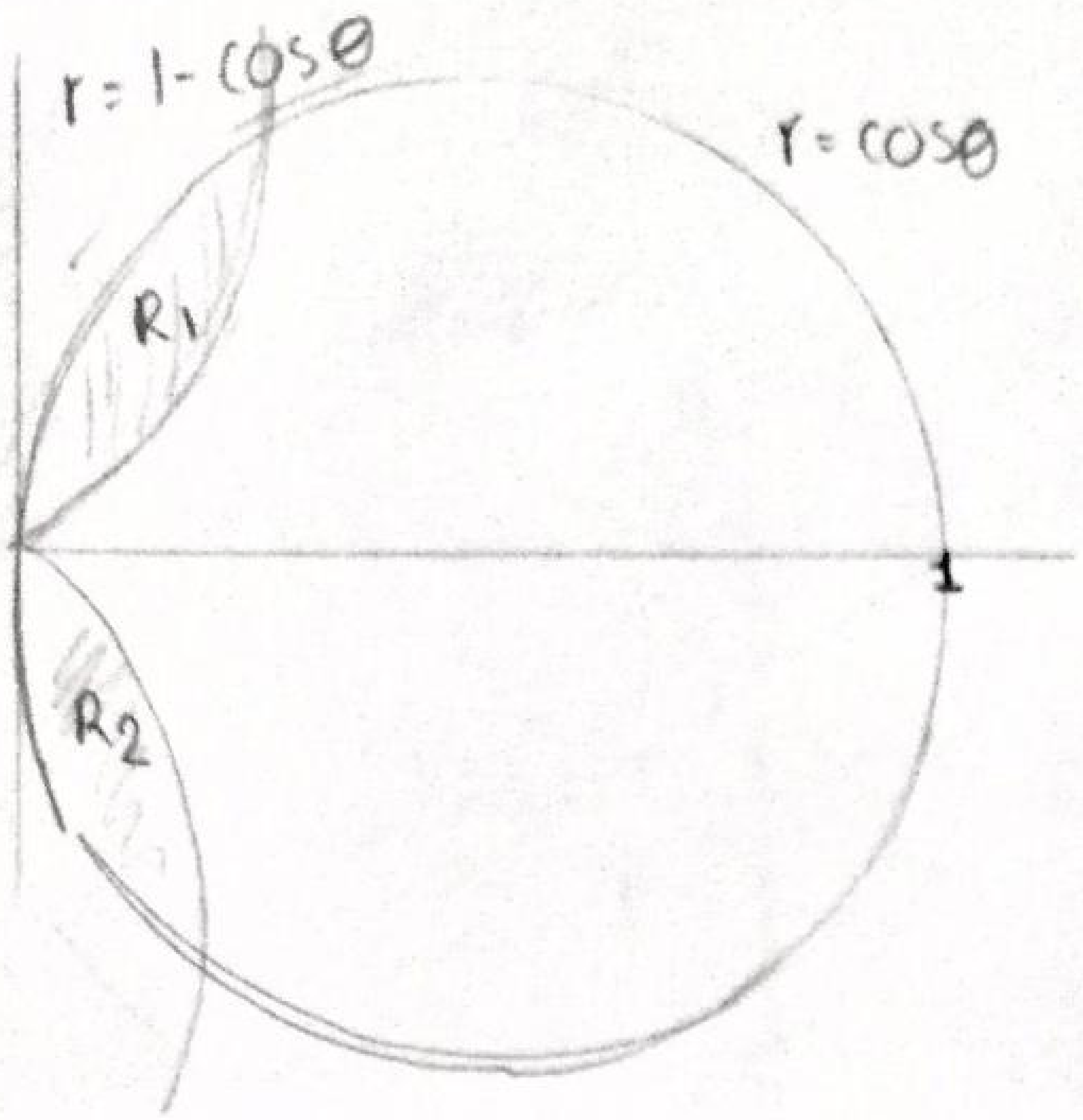
Pregunta 2



$R = R_1 \perp R_2$
Son Regiones iguales

Calculamos el valor del angulo de intersección

$1 - \cos\theta = \cos\theta$

$1 = 2\cos\theta$

$\frac{1}{2} = \cos\theta \rightarrow \theta = 60^{\circ} = \frac{\pi}{3}$ rad

=> el área total es 2· Integral en una región

$$\Rightarrow A = 2\cdot \int_{R_1}\int r\,dr\,d\theta = 2\cdot\left[\int_0^{\pi/3}\int_0^{1-\cos\theta} r\,dr\,d\theta + \int_{\pi/3}^{\pi/2}\int_0^{\cos\theta} r\,dr\,d\theta\right.$$

$$A = 2\cdot\left[\int_0^{\pi/3} \frac{r^2}{2}\Big|_0^{1-\cos\theta} d\theta + \int_{\pi/3}^{\pi/2}\frac{r^2}{2}\Big|_0^{\cos\theta} d\theta\right.$$

$$A = \cancel{2}\cdot\frac{1}{\cancel{2}}\left[\int_0^{\pi/3}(1-\cos\theta)^2 d\theta + \int_{\pi/3}^{\pi/2}\cos^2\theta\, d\theta\right]$$

" $\cos^2\theta = \frac{1+\cos(2\theta)}{2}$ "

$$A = \int_0^{\pi/3}(1 - 2\cos\theta + \cos^2\theta)\,d\theta + \int_{\pi/3}^{\pi/2}\frac{1+\cos(2\theta)}{2}\,d\theta$$

$$A = \theta\Big|_0^{\pi/3} - 2\cdot \operatorname{sen}\theta\Big|_0^{\pi/3} + \int_0^{\pi/3}\frac{1+\cos(2\theta)}{2}\,d\theta + \frac{\theta}{2}\Big|_{\pi/3}^{\pi/2} + \frac{1}{2}\int_{\pi/3}^{\pi/2}\cos(2\theta)\,d\theta$$

$$A = \frac{\pi}{3} - \sqrt{3} + 0{,}74 + \frac{\pi}{6} + \frac{\pi}{12} + \left(-\frac{\sqrt{3}}{8}\right) = 0{,}1005\ u^2 \;//$$

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Pregunta 3

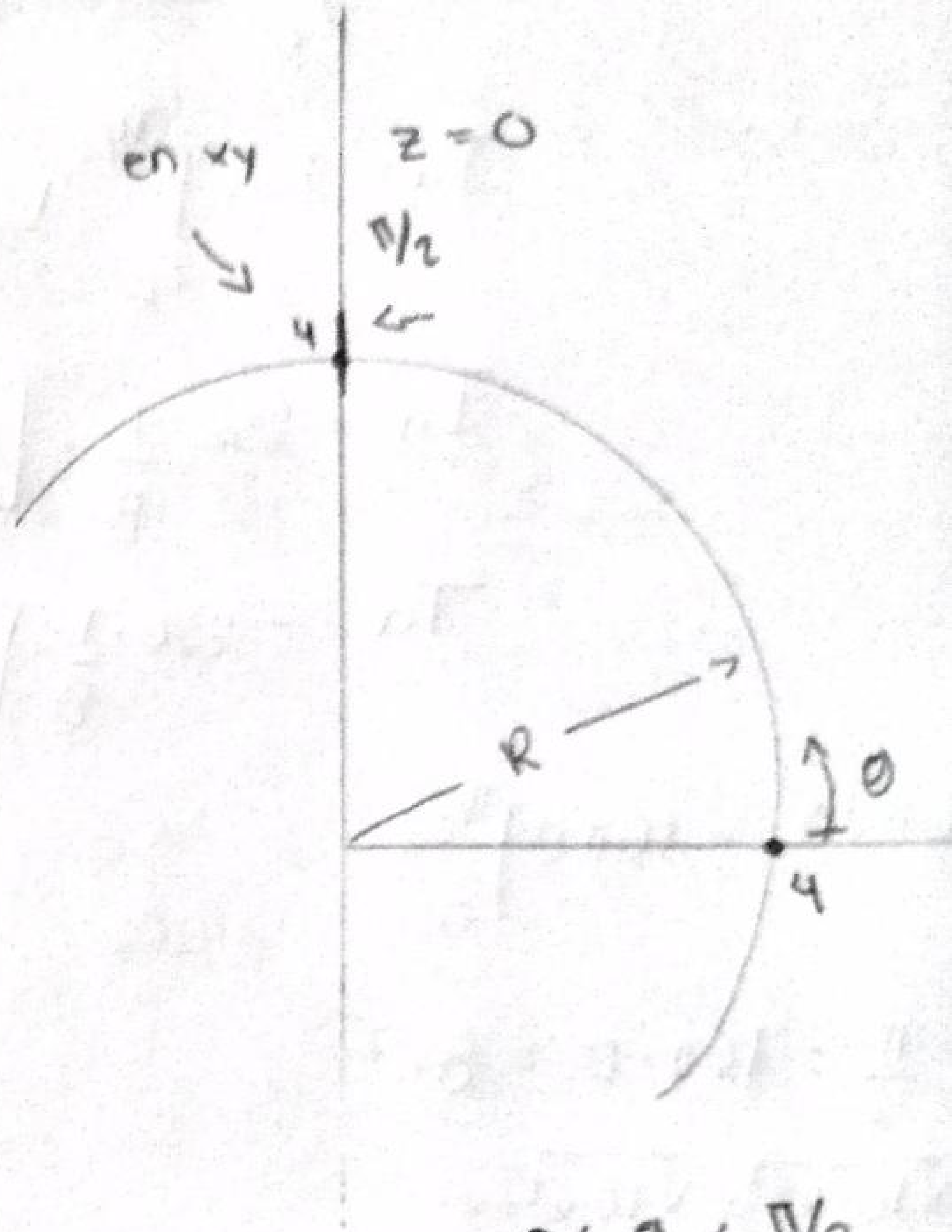
$$I = \int_0^4 \int_0^{\sqrt{16-x^2}} \int_0^{\sqrt{16-x^2-y^2}} \sqrt{x^2+y^2}\; dz\,dy\,dx$$

Para facilitar pasamos a coordenadas cilindricas

$0 \leq x \leq 4$

$0 \leq y \leq \sqrt{16-x^2}$ ← Cilindro de radio 4

$0 \leq z \leq \sqrt{16-x^2-y^2}$ ← esfera de radio 4.



$0 \leq \theta \leq \pi/2$

$0 \leq p \leq 4$

el radio es constante

=> Si realizamos el cambio a cilindricas

$$z = \sqrt{16-x^2-y^2}$$

$$z = \sqrt{16-(x^2+y^2)} = \sqrt{16-p^2}$$

$$\Rightarrow \int_0^{\pi/2} \int_0^4 \int_0^{\sqrt{16-p^2}} \sqrt{p^2}\; p\,dz\,dp\,d\theta$$

$$= \int_0^{\pi/2} \int_0^4 p^2 z \Big|_0^{\sqrt{16-p^2}} dp\,d\theta$$

$$= \int_0^{\pi/2} \underbrace{\int_0^4 p^2\sqrt{16-p^2}\; dp}_{Ia}\, d\theta$$

$p = 4\,\text{sen}(a)$ $\quad p=0 \quad u=0$

$dp = 4\cos(a)\,du$ $\quad p=(4) \quad u=\frac{\pi}{2}$

$$= \int_0^{\pi/2} Ia\, d\theta$$

$$I_a = \int_0^4 p^2\sqrt{16-p^2}\,dp = \int_0^{\pi/2} (4\,sen(u))^2 \sqrt{16-(4\,sen\,u)^2}\cdot 4\cos u\,du$$

$$I_a = \int_0^{\pi/2} 256\,sen^2 u\,\cos^2 u\,du$$

$$I_a = 256\int_0^{\pi/2} \frac{1-\cos(4u)}{8}\,du$$

$$I_a = 256\cdot\frac{1}{8}\cdot\left[\int_0^{\pi/2} 1\,du - \int_0^{\pi/2}\cos(4u)\,du\right.$$

$$I_a = 256\cdot\frac{1}{8}\cdot\left[\frac{\pi}{2}-0\right] = 16\pi$$

$$\Rightarrow \int_0^{\pi/2} 16\pi\,d\theta = 16\pi\theta\Big|_0^{\pi/2}$$

$$= 16\pi\cdot\frac{\pi}{2} - 16\pi\cdot 0 = 8\pi^2$$

$$\int_0^4\int_0^{\sqrt{16-x^2}}\int_0^{\sqrt{16-x^2-y^2}} \sqrt{x^2+y^2}\,dz\,dy\,dx = 8\pi^2$$

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Pregunta 4

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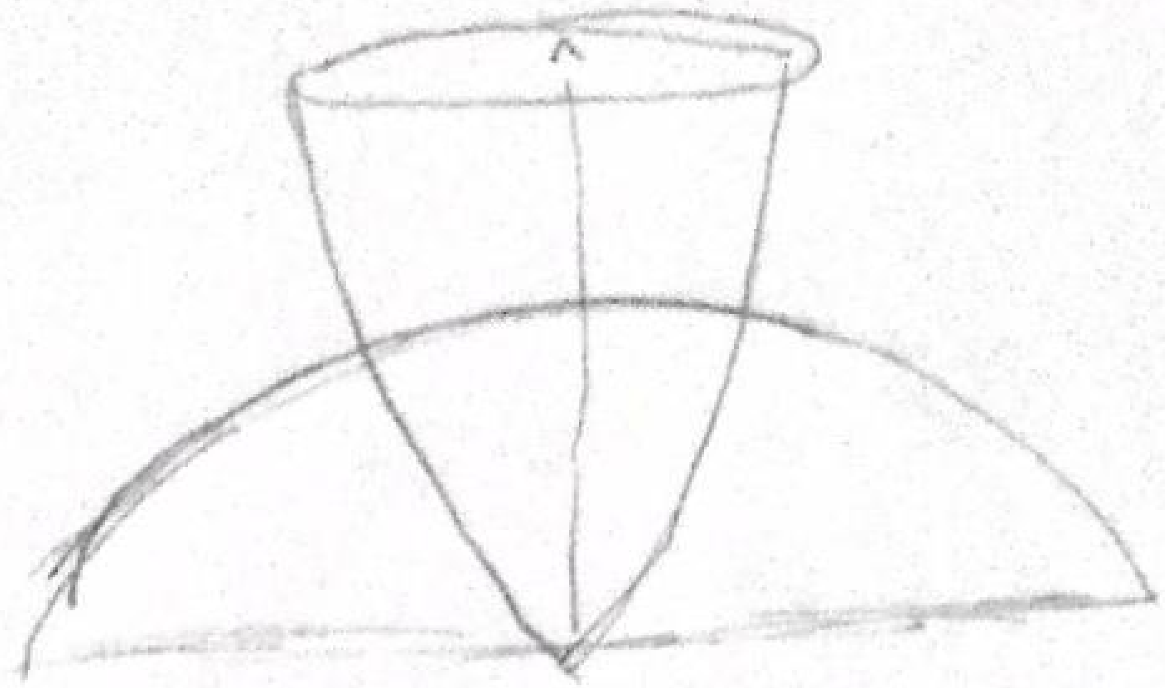
$x^2+y^2+z^2=80 \qquad z=\frac{1}{2}(x^2+y^2)$

$2z = x^2+y^2$

$2z+z^2=80 \qquad z=8 \leftarrow$ Si

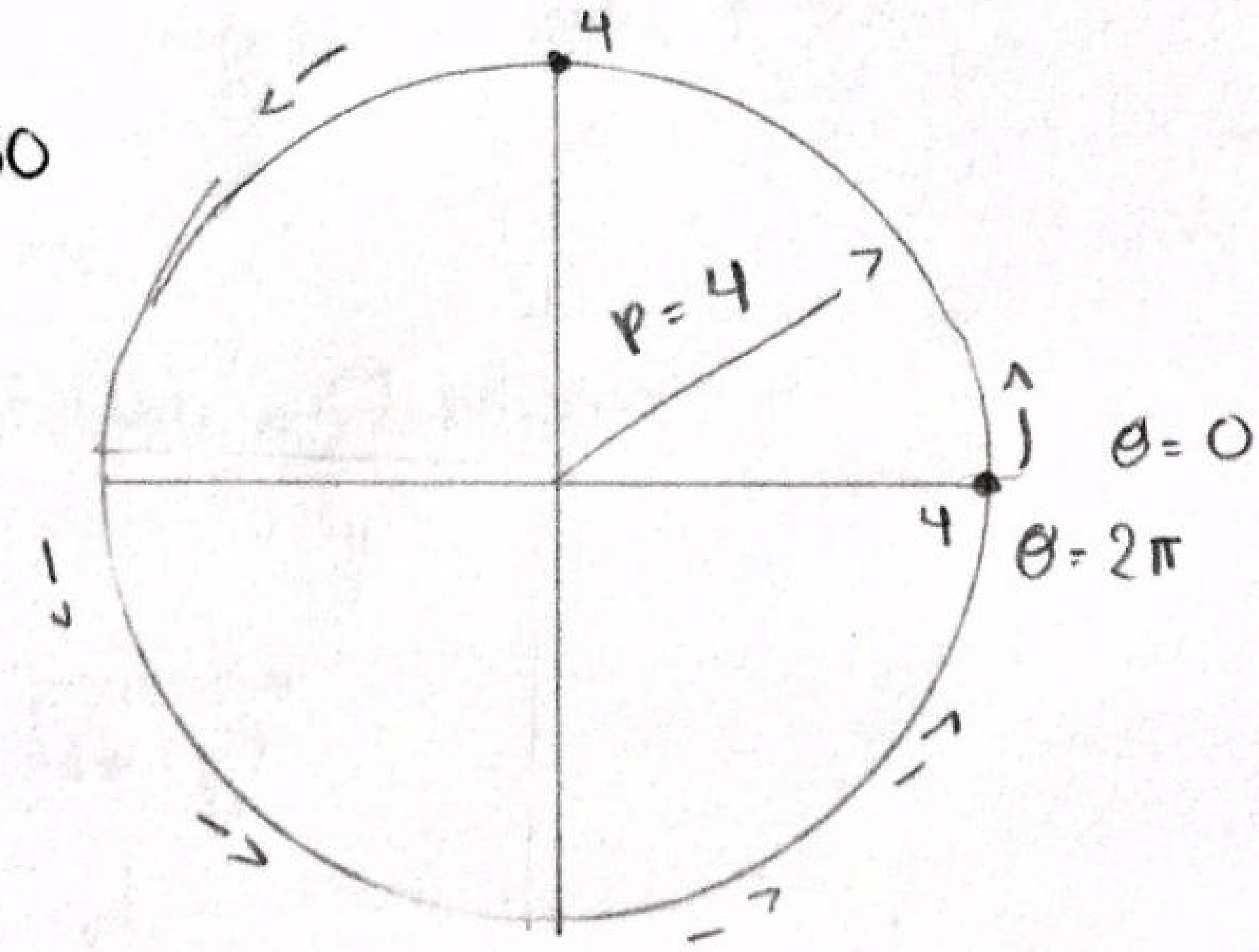
$z^2+2z-80=0 \qquad z=-10 \leftarrow$ NO



realizamos proyección en el plano xy

$x^2+y^2+(8)^2=80$

$x^2+y^2=16$



$x^2+y^2+z^2=80$

$z=\sqrt{80-(x^2+y^2)} \qquad z=\frac{1}{2}(x^2+y^2)$

$z=\sqrt{80-\rho^2} \qquad z=\frac{1}{2}\rho^2$

limites

$0 \leq \theta \leq 2\pi$

$0 \leq \rho \leq 4$

$\frac{1}{2}\rho^2 \leq z \leq \sqrt{80-\rho^2}$

construimos la integral

$$V=\int_0^{2\pi}\int_0^4\int_{\frac{1}{2}\rho^2}^{\sqrt{80-\rho^2}} \rho\, dz\, d\rho\, d\theta$$

$$V=\int_0^{2\pi}\int_0^4 \left[\rho z\right]_{\frac{1}{2}\rho^2}^{\sqrt{80-\rho^2}} d\rho\, d\theta \Rightarrow V=\int_0^{2\pi}\int_0^4 \left(\rho\sqrt{80-\rho^2}-\frac{\rho^3}{3}\right)d\rho\, d\theta$$

$$V=\int_0^{2\pi}\left[-\frac{1}{3}(80-\rho^2)^{3/2}-\frac{\rho^4}{8}\right]_0^4 d\theta$$

$$V=\int_0^{2\pi} 35{,}847\, d\theta = 35{,}847\,\theta\Big|_0^{2\pi}$$

$$V=35{,}847\cdot 2\pi = 225{,}23\ ul^3$$

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Pregunta 5

$2y^2 = x \quad , z = 0 \quad , \quad x + 2y + z = 4$

Necesitamos los cortes cuando $z = 0$

$x + 2y = 4 \quad ; \quad 2x^2 = x$

$2y^2 + 2y - 4 = 0 \Rightarrow y_1 = 1 \quad y_2 = -2$

despejamos la $z \to \begin{cases} z = 4 - x - 2y \\ z = 0 \end{cases}$

la region en el plano xy

$2y^2 \leq x \leq 4 - 2y$

$-2 \leq y \leq 1$

$0 \leq z \leq 4 - x - 2y$

$-6 \quad V = \int_{-2}^{1} \int_{2y^2}^{4-2y} \int_{0}^{4-x-2y} dz\,dx\,dy \Rightarrow V = \int_{2}^{1} \int_{2y^2}^{4-2y} (4 - x - 2y)\,dx\,dy$

$V = \int_{2}^{1} \left(4x - \frac{x^2}{2} - 2yx\right)\Big|_{2y^2}^{4-2y} dy$

$V = \int_{2}^{1} (4(4-2y) - (\frac{4-2y^2}{2} - 2y(4-2)) - (4 \cdot 2y^2 - \frac{(2y^2)^2}{2} - 2y(2y^2))\,dy$

$V = \int_{-2}^{1} (2y^4 + 4y^3 - 4y^2 - 16y - \frac{(-2y+4)^2}{2} + 16)\,dy$

$V = \frac{81}{5} = 16{,}2\ ul^3$ //

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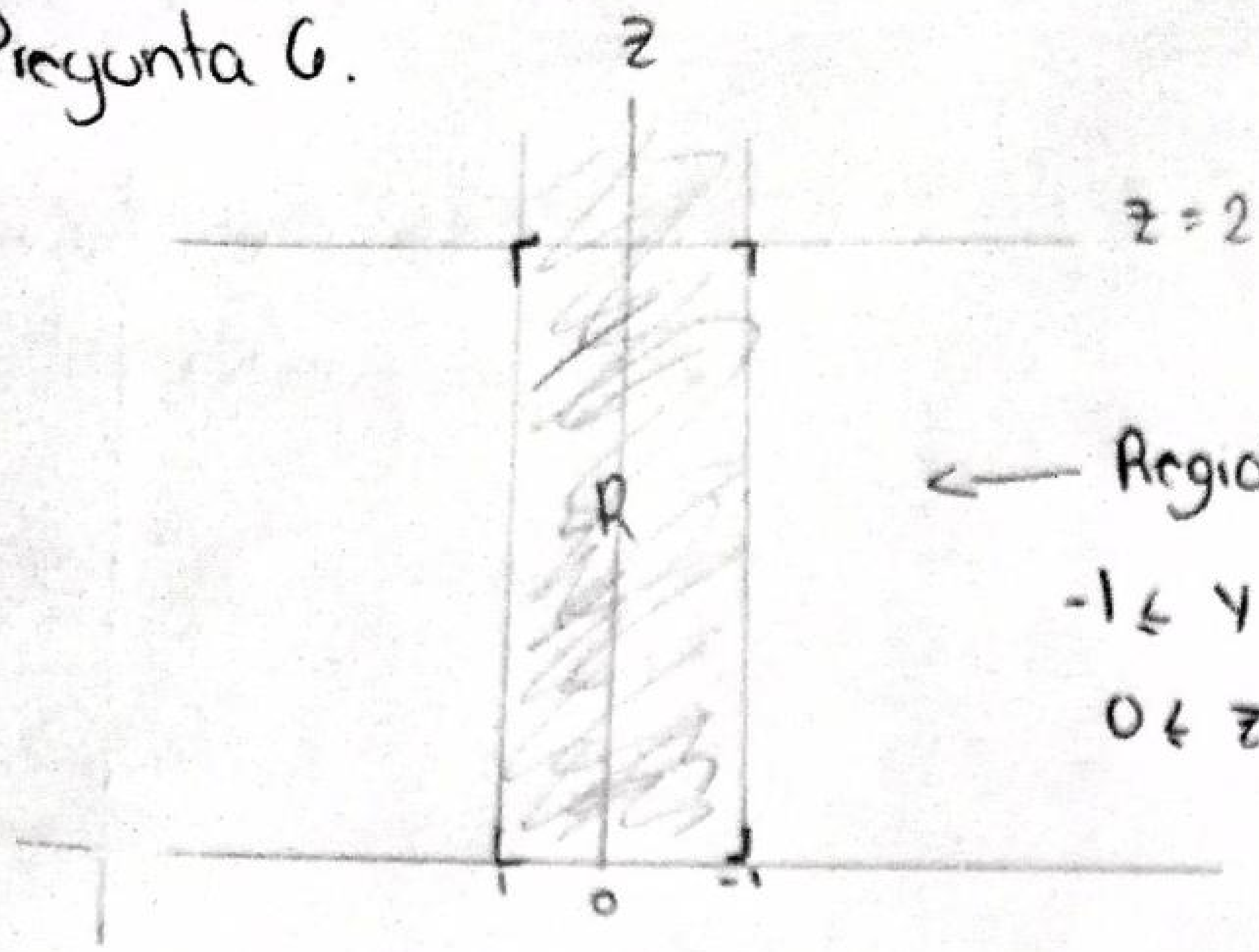
Pregunta 6.



← Región de integración

$-1 \leq y \leq 1$

$0 \leq z \leq 2$

=> Despejamos x

$x^2 + y^2 = 1$ $\qquad$ $x + y = 3$

$x = \pm\sqrt{1-y^2}$ $\qquad$ $x = 3 - y$

$$V = \int_{-1}^{0}\int_{0}^{2}\int_{-\sqrt{1-y^2}}^{2-y} dx\,dz\,dy + \int_{0}^{1}\int_{0}^{2}\int_{\sqrt{1-y^2}}^{3-x} dx\,dz\,dy$$

$$V = \int_{-1}^{0}\int_{0}^{2} 3 - y + \sqrt{1-y^2}\,dz\,dy + \int_{0}^{1}\int_{0}^{2} 3 - y - \sqrt{1-y^2}\,dz\,dy$$

$$V = \int_{-1}^{0} (3 - y + \sqrt{1-y^2})z\Big|_0^2\,dy + \int_{0}^{1} 3 - y - \sqrt{1-y^2}\Big|z\Big|_0^2\,dy$$

$$V = \int_{-1}^{0} 2(3 - y + \sqrt{1-y^2})\,dy + \int_{0}^{1} 2(3 - y - \sqrt{1-y^2})\,dy$$

$$V = 2\left[\int_{-1}^{0} (3 - y + \sqrt{1-y^2})\,dy + \int_{0}^{1} (3 - y - \sqrt{1-y^2})\,dy\right]$$

$V = 2 \cdot 4,28 + 2 \cdot 1,71$

$\Rightarrow V = 11,98\,ul^3$