$$\sum_{i=1}^{\infty} \frac{(-1)^n \cdot n!}{4 \cdot 7 \cdot 10 \cdots (3n+1)}$$

Aplicamos criterio del cociente

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{\dfrac{(-1)^{n+1} \cdot (n+1)!}{4 \cdot 7 \cdot 10 \cdots (3n+1)(3(n+1)+1)}}{\dfrac{(-1)^n \cdot n!}{4 \cdot 7 \cdot 10 \cdots (3n+1)}} \right|$$

$$\lim_{n\to\infty} \left| \frac{\dfrac{\cancel{(-1)^n} \cdot (-1) \cdot (n+1) \cdot \cancel{n!}}{\cancel{4 \cdot 7 \cdot 10 \cdot (3n+1)}(3n+4)}}{\dfrac{\cancel{(-1)^n} \cdot \cancel{n!}}{\cancel{4 \cdot 7 \cdot 10 \cdots (3n+1)}}} \right|$$

$$\lim_{n\to\infty} \frac{|-1|(n+1)}{3n+4}$$

$$\lim_{n\to\infty} \frac{n+1}{3n+4} = \lim_{n\to\infty} \frac{\cancel{n}(1+1/n)}{\cancel{n}(3+4/n)} = \frac{1+0}{3+0} = \frac{1}{3}$$

como

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \frac{1}{3}$$

$$\frac{1}{3} < 1$$

La serie

$$\sum_{i=1}^{\infty} \frac{(-1)^n \cdot n!}{4 \cdot 7 \cdot 10 \cdots (3n+1)}$$ ~~converge~~