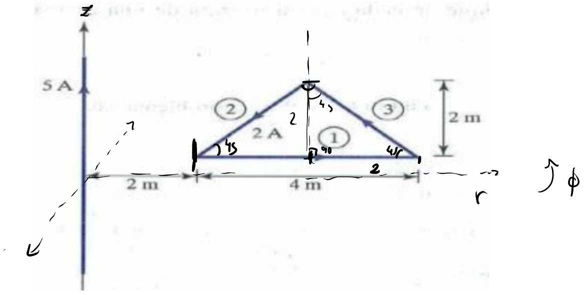
5. (8 puntos) Una espira conductora triangular portadora de una corriente de 2 A se sitúa cerca de un conductor recto de longitud infinita con una corriente de 5 A, como se muestra en la figura.

$$B = \frac{\mu_0 I}{2\pi r} \, (a_\phi)$$



$r \quad \phi \quad z$

Calcule **(a)** (2 pts.) la fuerza sobre el lado 1 de la espira triangular, **(b)** (3 pts.) la fuerza total sobre la espira, **(c)** (3 pts.) Determine la inductancia mutua entre un alambre recto muy largo y la espira conductora con forma de triángulo.

$$F = I(L \times B)$$

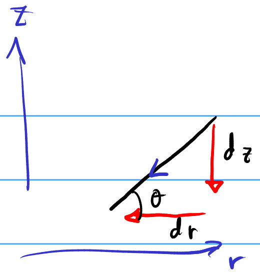
$$dF = I \, dl \times B \qquad dl = dr \, (a_r)$$

$$F = I_{2A} \int dr \; a_r \times \left(\frac{\mu_0 I_{5A}}{2\pi r} \right) (a_\phi) \qquad a_r \times a_\phi = a_z$$

$$F = \frac{2A \cdot 5A \cdot \mu_0}{2\pi} \int_2^6 \frac{1}{r} dr \; a_z$$

$$F = \frac{10A \cdot}{2\pi} \mu_0 \ln\left(\frac{6}{2}\right) \rightsquigarrow \boxed{F = 2,20 \times 10^{-6} \, N \; a_z} \quad R/a$$

b) $dF_2 = I \cdot dl_2 \times B$



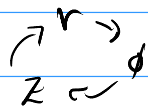
$d\vec{l}_2 = dz\,(-a_z) + dr\,(-a_r)$

$\tan\theta = \frac{dz}{dr}$

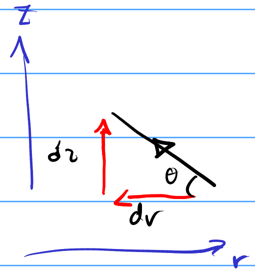
$$dF_2 = 2A \cdot (dz(-a_z) + dr(-a_r)) \times \frac{\mu_0 S A}{2\pi r}(a_\phi)$$

$\vec{B} = \left(\frac{\mu_0 I_{SA}}{2\pi r}\right)(a_\phi)$

$$dF_2 = \frac{2 \cdot dz\, \mu_0 \cdot S}{2\pi r}\, a_r + \cancel{\frac{2 \cdot dr \cdot \mu_0 S}{2\pi r}(-a_z)}$$



$$dF_3 = 2 \cdot dl_3 \times B$$



$dl = dr\,(-a_r) + dz\,(a_z)$

$\tan\theta = \frac{dz}{dr}$

$dz = \tan\theta \cdot dr$

$$dF_3 = \frac{2 \cdot dz\, \mu_0 S}{2\pi r}\, a_r + \cancel{\frac{2dr\,\mu_0 S}{2\pi r}(a_z)}$$

$F_2 = F_3 = 0$

$$\boxed{F_{Total} = 2{,}77 \times 10^{-6}\,(a_r) + 2{,}20 \times 10^{-6}\,(a_z)\ N}$$

R/b

$$dF_{23} = \frac{\cancel{2} \cdot 2 \cdot S \cdot \mu_0}{\cancel{2}\pi r} \cdot dz\ (a_r)$$

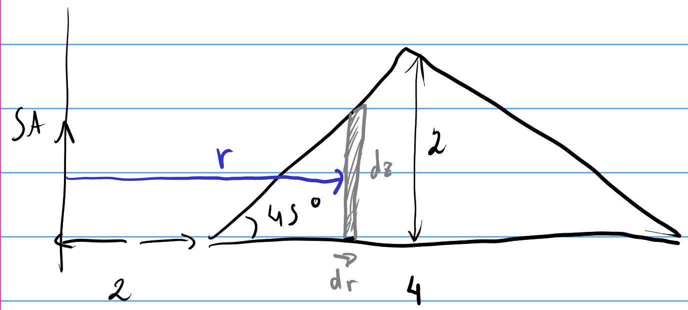
$$dF_{23} = \frac{10\,\mu_0}{\pi r} \tan\theta\, dr\ (a_r)$$

$$F_{23} = \frac{10\,\mu_0}{\pi} \tan\theta \int_2^4 \frac{dr}{r}\ (a_r)$$

$$F_{23} = \frac{10}{\pi}\mu_0 \tan\theta \ln\left(\frac{4}{2}\right)\ N\ (a_r)$$

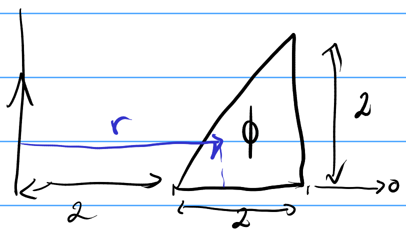
$$F_{23} = 2{,}77 \times 10^{-6} \tan\theta\ N\ (a_r$$

$$F_{23} = 2{,}77 \times 10^{-6}\ N\ (a_r)$$



$$\phi = \int B\,dA \qquad dA = dr\,dz$$

$$B = \frac{\mu_0 S}{2\pi r}$$



$$2\phi = \phi_{Total}$$

$$\phi = \iint \frac{\mu_0 S}{2\pi r}\cdot dr\,dz$$

$$\phi = \int_0^2 \int_2^4 \frac{\mu_0 S}{2\pi r}\, dr\,dz$$

$$\phi = \int_0^2 \frac{S\mu_0}{2\pi} \ln\left(\frac{4}{2}\right) dz$$

$$\phi = \frac{S}{2}\frac{\mu_0}{\pi} \ln(2)\cdot 2 \qquad \phi_{Total} = S\mu_0 \ln(2)\ 2$$

$$\phi_{tot} = 8{,}71 \times 10^{-6}\ \text{Wb}$$

$$\boxed{M = \frac{\phi_{Total}}{I_{SA}} = 1{,}74 \times 10^{-6}\ \text{H}}$$

R/c