1)

$$\sum_{i=2}^{\infty}\left[\frac{(-2)^{3i}+4^{i}}{7^{i+3}}\right]$$

$$=\sum_{i=2}^{\infty}\frac{(-2)^{3i}}{7^{i+3}}+\sum_{i=2}^{\infty}\frac{4^{i}}{7^{i+3}}$$

$$=\sum_{i=2}^{\infty}\frac{(-8)^{i}}{7^{i}\cdot 7^{3}}+\sum_{i=2}^{\infty}\frac{4^{i}}{7^{i}\cdot 7^{3}}$$

$$=\frac{1}{7^{3}}\sum_{i=2}^{\infty}\left(\frac{-8}{7}\right)^{i}+\frac{1}{7^{3}}\sum_{i=2}^{\infty}\left(\frac{4}{7}\right)^{i}$$

$$\# \quad \left|\frac{-8}{7}\right| \geq 1 \quad y \quad \left|\frac{4}{7}\right| < 1$$

$\therefore$ Diverge en $\sum_{i=2}^{\infty}\left(\frac{-8}{7}\right)^{i}$

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2

$$\sum_{n=2}^{\infty} \left[\frac{2}{4n^2+12n+5} \right]$$

$$= \frac{2}{(2n+1)(2n+5)} = \frac{A}{2n+1} + \frac{B}{2n+5}$$

$$\frac{2}{\cancel{(2n+1)(2n+5)}} = \frac{A(2n+5)+B(2n+1)}{\cancel{(2n+1)(2n+5)}}$$

$$2 = A(2n+5) + B(2n+1)$$

* $n = -\frac{1}{2}$

$$2 = A(2(-\tfrac{1}{2})+5) + \cancel{B(2(-\tfrac{1}{2})+1)}^{0}$$

$$2 = A4$$

$$\frac{2}{4} = A \Rightarrow \frac{1}{2}$$

* $n = -\frac{5}{2}$

$$2 = \cancel{A(2(-\tfrac{5}{2})+5)}^{0} + B(2(-\tfrac{5}{2})+1)$$

$$2 = -4B$$

$$-\frac{2}{4} = B \Rightarrow \frac{-1}{2}$$

$$\sum_{n=2}^{\infty}\left(\frac{1/2}{2n+1}-\frac{1/2}{2n+5}\right)$$

$$\frac{1}{2}\sum_{n=2}^{\infty}\left(\frac{1}{2n+1}-\frac{1}{2n+5}\right) \Rightarrow \frac{1}{2(n+2)+1} \Rightarrow \frac{1}{2n+5}$$

$$\frac{1}{2}\sum_{n=2}^{\infty}\left(\frac{1}{2n+1}-\frac{1}{2n+5}\right)\frac{1}{-2}\left(a_1+a_2-\cancelto{0}{\lim_{n=\infty} a_{n+2}}\right)$$

$$\Rightarrow \text{Suma} = \frac{1}{2}\left(\frac{1}{2\cdot2+1}+\frac{1}{2\cdot3+1}-0\right)$$

$$\text{Suma} = \frac{1}{2}\left(\frac{12}{35}\right)$$

$$\text{suma} = \frac{6}{35} \quad \text{converge}$$

Parte 3

a) $\sum_{n=1}^{\infty} \frac{3}{n^2+10n+25}$

$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2+10n+25} = 3\sum_{i=1}^{\infty} \frac{1}{(n+5)^2}$

=> Esta serie se puede comparar

$\sum^{\infty} 1/n^2$ ya que $n >> 5$

si hacemos comparacion en el limite

$\lim_{n=\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{1}{n^2+10n+25}}{1/n^2}$

$= \lim_{n\to\infty} \frac{n^2}{n^2+10n+25}$

$= \lim_{n\to\infty} \frac{n^2}{n^2(1+10/n+25/n^2)} = \frac{1}{1+10/\infty+25/\infty^2} = 1$

como el limite da $1>0$

por comparacion en el limite

$\therefore \sum_{i=1}^{\infty} \frac{3}{n^2+10n+25}$ converge

Parte b

$$\sum_{i=1}^{\infty} \frac{7^n + 5\cos^2(n) + 1}{1+3^{2n}}$$

$$0 \leq \cos^2(n) \leq 1$$

$$0 \leq 5\cos^2(n) \leq 5$$

$$7^n + 1 \leq 5\cos^2 n + 7^n + 1 \leq 6 + 7^n$$

$$\frac{7^n+1}{1+3^n} \leq \frac{5\cos^2 n + 7^n + 1}{1+3^n} \leq \frac{6+7^n}{1+3^{2n}}$$

$$\sum_{i=1}^{\infty} \frac{6+7^n}{1+3^{2n}} = \sum_{i=1}^{\infty} \frac{6}{1+3^{2n}} + \sum_{i=1}^{\infty} \frac{7^n}{1+3^{2n}}$$

como $(3^2)^n >> 1$

$9^n >> 1$

se reescribe

$$6 \cdot \sum_{i=1}^{\infty} \frac{1}{9^n} + \sum_{i=1}^{\infty} \left(\frac{7^n}{9^n}\right)$$

$$6 \cdot \sum_{i=1}^{\infty} \left(\frac{1}{9}\right)^n + \sum_{i=1}^{\infty}$$

como $\left|\frac{1}{9}\right| < 1$ y $\left|\frac{7}{9}\right| < 1$ Ambas convergen y como $\sum_{i=1}^{\infty} b_n$ converge

- Por comparación directa

$$\therefore \sum_{i=1}^{\infty} \frac{7^n + 5\cos^2(2) + 1}{1+3^{2n}} \quad \text{converge}$$

d) $\displaystyle\sum_{k=1}^{\infty}\left(\frac{k^2+1}{2k^2+k}\right)^{2k+1}$

• Aplicando criterio de la Raiz

$$\lim_{k\to\infty} \sqrt[k]{\left|\left(\frac{k^2+1}{2k^2+k}\right)^{2k+1}\right|}$$

$$\lim_{k\to\infty} \sqrt[k]{\left(\frac{k^2+1}{2k^2+k}\right)^{2k}\cdot\left(\frac{k^2+1}{2k^2+k}\right)^{1}} \Rightarrow \lim_{k\to\infty} \sqrt[k]{\left(\frac{k^2+1}{2k^2+k}\right)^{2k}} \cdot \sqrt[k]{\left(\frac{k^2+1}{2k^2+k}\right)}$$

$$\lim_{k\to\infty} \left(\frac{k^2+1}{2k^2+k}\right)^{2}\cdot\left(\frac{k^2+1}{2k^2+k}\right)^{1/k}$$

$$\lim_{k\to\infty} \left(\frac{\cancel{k^2}(1+1/k^2)}{\cancel{k^2}(2+1/k)}\right)^{2}\cdot\left(\frac{k^2(1+1/k^2)}{k^2(2+1/k)}\right)^{1/k}$$

$$=\left(\frac{1}{2}\right)^{2}\cdot\left(\frac{1}{2}\right)^{0}=\frac{1}{4}$$

como $\lim_{n\to\infty} \sqrt[k]{|a_k|} < 1$

la serie

$\displaystyle\sum_{k=1}^{\infty}\left(\frac{k^2+1}{2k^2+k}\right)^{2k+1}$ converge

d) $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot n!}{4 \cdot 7 \cdot 10 \ldots (3n+1)}$

• Aplicamos criterio del cociente

$$\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n = \infty} \left| \frac{\dfrac{(-1)^{n+1} \cdot (n+1)!}{4 \cdot 7 \cdot 10 \ldots (3n+1)(3(n+1)+1)}}{\dfrac{(-1)^n \cdot n!}{4 \cdot 7 \cdot 10 \ldots (3n+1)}} \right|$$

$$\lim_{n = \infty} \left| \frac{\dfrac{\cancel{(-1)^n} \cdot (-1) \cdot (n+1) \cdot \cancel{n!}}{\cancel{4 \cdot 7 \cdot 10 (3n+1)}(3n+4)}}{\dfrac{\cancel{(-1)^n} \cdot \cancel{n!}}{\cancel{4 \cdot 7 \cdot 10 \ldots (3n+1)}}} \right|$$

$$= \lim_{n \to \infty} \frac{1 + 1 \cdot (n+1)}{3n+4}$$

$$= \lim_{n \to \infty} \frac{n+1}{3n+4} = \lim_{n \to \infty} \frac{n(1 + 1/n)}{n(3 + 4/n)} = \frac{1+0}{3+0} = \frac{1}{3}$$

Como $\lim_{n \infty} \left| \frac{a_{n+1}}{a_n} \right| \to \frac{1}{3} < 1$ la serie $\sum_{\lambda=1}^{\infty} \frac{(-1)^n \cdot n!}{4 \cdot 7 \cdot 10 \cdots (3n+1)}$

converge

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