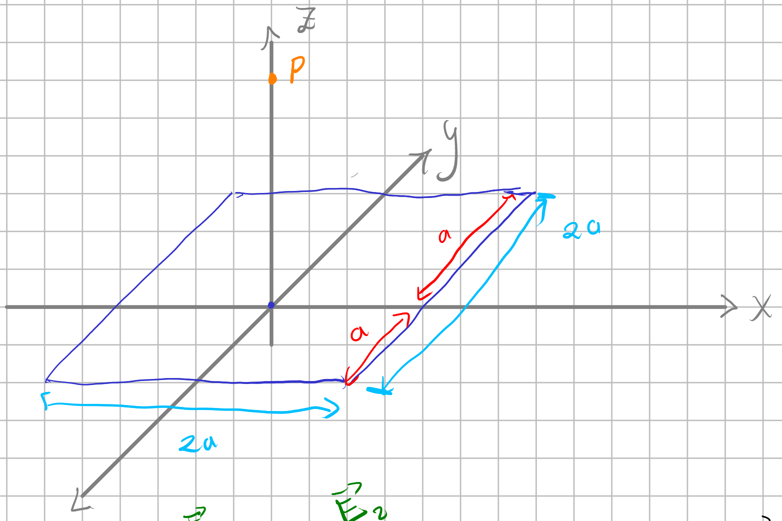
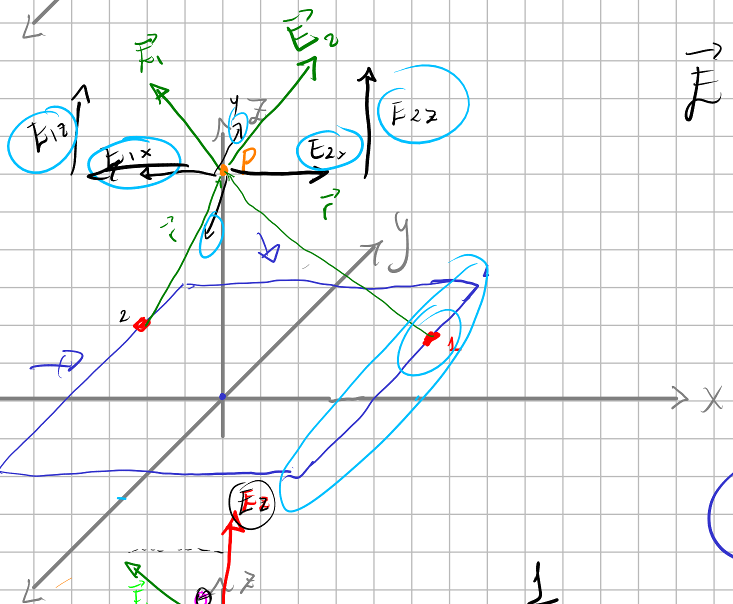
1. (4pts.) Una espira cuadrada de lado 2a se encuentra en z = 0 centrada en el origen.
Cada lado de la espira tiene una distribución de carga ρL uniforme.
Determine la expresión del campo eléctrico para un punto cualquiera en el eje z mayor que cero.

$\rho L =$ línea

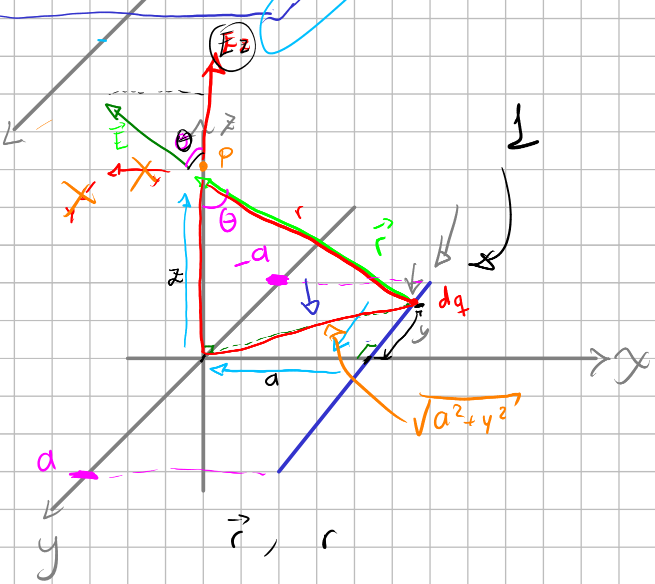




$\vec{E} \Rightarrow E_x = 0 \; , \; E_y = 0$

$\underline{E_z \neq 0}$

$$\boxed{E = K\frac{Q}{r^2}\hat{r} \rightsquigarrow K\frac{Q}{r^3}\vec{r}}$$



$\rho_L = \lambda$

$$d\vec{E} = K\frac{dq}{r^2}\, a_n$$

$$d\vec{E} = K\frac{dq}{r^2}\, a_z$$

$$\vec{r} = -a(a_x) - y(a_y) + z(a_z)$$

$$r = \sqrt{a^2 + y^2 + z^2}$$

$$r^2 = a^2 + y^2 + z^2$$

$\underline{dq = \rho_L\, dy}$

$$\boxed{E_{tot} = 4E_{uno}}$$

$$E_z = E\cos\theta\,(a_z)$$

$$dE_z = dE\cos\theta\,(a_z)$$

$$dE_z = dE \cos\theta \quad (a_z)$$

$$dq = \rho_L\, dy$$

$$\vec{r} = -a\,(a_x) - y\,(a_y) + z\,(a_z)$$

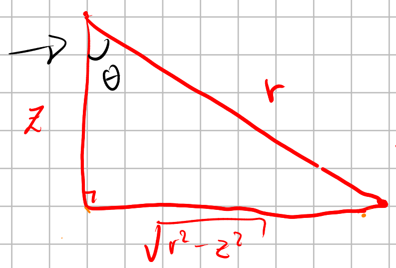
$$r = \sqrt{a^2 + y^2 + z^2}$$

$$r^2 = a^2 + y^2 + z^2$$

$$dE = K \frac{dq}{r^2} \cos\theta \quad (a_z)$$

$$E_z = E \cos\theta \quad (a_z)$$

$$dE_z = dE \cos\theta \quad (a_z)$$



$$\cos\theta = \frac{z}{r}$$

$$d\vec{E} = K \frac{dq}{r^2} \cdot \frac{z}{r} \quad (a_z)$$

$$d\vec{E} = K \frac{z\, dq}{r^3} \quad (a_z)$$

$$\hookrightarrow d\vec{E} = K \frac{z \cdot \rho_L\, dy}{\left(\sqrt{a^2 + z^2 + y^2}\right)^3} \quad (a_z)$$

$$\vec{E} = \int_{-a}^{a} K \frac{z \cdot \rho_L\, dy}{\left(\sqrt{a^2 + z^2 + y^2}\right)^3} \quad (a_z)$$

$$\left(\sqrt{a^2 + z^2 + y^2}\right)^3 = (a^2 + z^2 + y^2)^{3/2}$$

$$\vec{E} = K z \rho_L \int_{-a}^{a} \frac{dy}{(\underbrace{a^2 + z^2}_{\omega^2} + y^2)^{3/2}} \quad (a_z)$$

$$\omega^2 = a^2 + z^2$$

$$\vec{E} = K z \rho_L \int_{-a}^{a} \frac{dy}{(\omega^2 + y^2)^{3/2}} \quad (a_z)$$

$$\int \frac{dy}{(\omega^2 + y^2)^{3/2}} = \frac{y}{\omega^2 \sqrt{\omega^2 + y^2}}$$

$$\vec{E} = 2 K z \rho_L \int_{0}^{a} \frac{dy}{(\omega^2 + y^2)^{3/2}} \quad (a_z)$$

$$\vec{E} = 2 K z \rho_L \left[\frac{y}{\omega^2 \sqrt{\omega^2 + y^2}} \right]_0^a \quad (a_z)$$

$$\vec{E} = 2 K z \rho_L \left[\frac{a}{\omega^2 \sqrt{\omega^2 + a^2}} - \cancel{\frac{0}{\omega^2 \sqrt{\omega^2 + 0^2}}} \right]$$

$$\vec{E} = 2 K z \rho_L \cdot \frac{a}{\omega^2 \sqrt{\omega^2 + a^2}} \quad (a_z)$$

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$$\vec{E} = 2K z \rho_L \cdot \frac{a}{w^2\sqrt{w^2+a^2}} \, (a_z)$$

$w^2 = a^2 + z^2$

$$\vec{E} = 2K \cdot z \, \rho_L \cdot \frac{a}{(a^2+z^2)\sqrt{a^2+z^2+a^2}} \, (a_z)$$

$$E = 2K \cdot z \, \rho_L \cdot \frac{a}{(a^2+z^2)\sqrt{2a^2+z^2}} \, (a_z)$$

$$k = \frac{1}{4\pi\epsilon_0}$$

$$\rho_L = \frac{Q}{L} = \frac{Q}{2a}$$

$$E_{total} = 4 \cdot E$$

$$E_{total} = 4 \cdot 2K \cdot z \, \rho_L \cdot \frac{a}{(a^2+z^2)\sqrt{2a^2+z^2}} \, (a_z)$$

$$E_{tot} = \cancel{4} \cdot 2 \cdot z \cdot \left(\frac{1}{\cancel{4}\pi\epsilon_0}\right) \rho_L \frac{a}{(a^2+z^2)\sqrt{2a^2+z^2}} \, (a_z)$$

$$\boxed{E_{tot} = \frac{2z \, \rho_L}{\pi\epsilon_0} \cdot \frac{a}{(a^2+z^2)\sqrt{2a^2+z^2}} \, (a_z)}$$

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