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1. $f(z) = \dfrac{4z-9}{(z+3)^2(z-1)}$ $\qquad 2 < |z+5| < 6$

$$\boxed{\frac{-5}{16} = A \qquad \frac{5}{16} = B \qquad \frac{21}{4} = C}$$

$$\frac{4z-9}{(z+3)^2(z-1)} = \frac{A}{z-1} + \frac{B}{z+3} + \frac{C}{(z+3)^2}$$

$$4z-9 = A(z+3)^2 + B(z-1)(z+3) + C(z-1)$$

$$4\cdot 1 - 9 = A(1+3)^2 + B\cancel{(1-1)}(1+3) + C\cancel{(1-1)} \qquad z = 1$$

$$-5 = A(4)^2 \rightsquigarrow A = \frac{-5}{16}$$

$$4z-9 = A(z+3)^2 + B(z-1)(z+3) + C(z-1)$$

$$4\cdot -3 - 9 = A\cancel{(-3+3)}^2 + B(-3-1)\cancel{(-3+3)} + C\cdot(-3-1) \qquad z = -3$$

$$-21 = C(-4) \rightsquigarrow C = \frac{-21}{-4}$$

$$4z-9 = A(z+3)^2 + B(z-1)(z+3) + C(z-1)$$

$$z = 2$$

$$4\cdot 2 - 9 = -\frac{5}{16}(2+3)^2 + B(2-1)(2+3) + \frac{21}{4}(2-1)$$

$$-1 = -\frac{125}{16} + B\cdot(5) + \frac{21}{4}$$

$$-1 + \frac{125}{16} - \frac{21}{4} = 5B$$

$$\frac{25}{16} = 5B \rightsquigarrow \frac{5}{16} = B$$

$$f(z) = \frac{4z-9}{(z-1)(z+3)^2}$$

$$f(z) = \frac{-5}{16(z-1)} + \frac{5}{16(z+3)} + \frac{21}{4(z+3)^2} \qquad 2 < |z+5| < 6$$

$$f(z) = \frac{-5}{16}\,\frac{1}{(z-1)} + \frac{5}{16}\cdot\frac{1}{(z+3)} + \frac{21}{4}\cdot\frac{1}{(z+3)^2}$$

$$f(z) = \frac{-5}{16}\,\frac{1}{z-1+5-5} + \frac{5}{16}\cdot\frac{1}{z+3+5-5} + \frac{21}{4}\,\frac{1}{(z+3+5-5)^2}$$

$$f(z) = \frac{-5}{16}\cdot\frac{1}{(z+5)-6} + \frac{5}{16}\cdot\frac{1}{(z+5)-2} + \frac{21}{4}\,\frac{1}{[(z+5)-2]^2}$$

$$f(z) = -\frac{5}{16} \cdot \frac{1}{(z+5)-6} + \frac{5}{16} \cdot \frac{1}{(z+5)-2} + \frac{21}{4} \frac{1}{[(z+5)-2]^2} \qquad 2 < |z+5| < 6$$

$$\frac{1}{(z+5)-2} = \frac{1}{(z+5)\left[1 - \frac{2}{(z+5)}\right]} = \frac{1}{z+5} \sum_{n=0}^{\infty} \left(\frac{2}{z+5}\right)^n$$

$$\frac{1}{[(z+5)-2]^2} = \frac{1}{(z+5)^2\left[1 - \frac{2}{(z+5)}\right]^2} = \frac{1}{(z+5)^2} \sum_{n=0}^{\infty} (n+1) \left(\frac{2}{z+5}\right)^n$$

$$\frac{1}{(z+5)-6} = \frac{1}{-6\left(1 - \frac{z+5}{6}\right)} = \frac{-1}{6} \sum_{n=0}^{\infty} \left(\frac{z+5}{6}\right)^n$$

$$f(z) = \frac{5}{16} \cdot \frac{1}{z+5} \sum_{n=0}^{\infty} \left(\frac{2}{z+5}\right)^n + \frac{21}{4} \frac{1}{(z+5)^2} \cdot \sum_{n=0}^{\infty} (n+1) \left(\frac{2}{z+5}\right)^n + \frac{5}{96} \sum_{n=0}^{\infty} \left(\frac{z+5}{6}\right)^n$$

$$f(z) = \underbrace{\frac{5}{16} \cdot \frac{1}{z+5}}_{a_{-1}} + \underbrace{\frac{5}{16} \cdot \frac{2}{(z+5)^2} + \frac{5}{16} \cdot \frac{4}{(z+5)^3} + \cdots + \frac{21}{4} \frac{1}{(z+5)^2} + \frac{21}{4} \cdot \frac{2}{(z+5)^3}}_{\text{Parte principal}} + \cdots + \frac{5}{96} + \underbrace{\frac{5}{96} \cdot \frac{(z+5)}{6}}_{\text{Parte Analítica}} + \cdots$$

$$a_{-1} = \frac{5}{16}$$ Residuo.

2- $|z+2i| + |z-2i| = 6$

$$z = x + yi$$

$$|x+yi+2i| + |x+yi-2i| = 6$$

$$|x+(y+2)i| + |x+(y-2)i| = 6$$

$$\sqrt{x^2+(y+2)^2} + \sqrt{x^2+(y-2)^2} = 6$$

$$\left(\sqrt{x^2+(y-2)^2}\right)^2 = \left(6-\sqrt{x^2+(y+2)^2}\right)^2$$

$$x^2+(y-2)^2 = 36 - 12\sqrt{x^2+(y+2)^2} + x^2 + (y+2)^2$$

$$\cancel{x^2} + \cancel{y^2} - 4y + \cancel{4} = 36 - 12\sqrt{x^2+(y+2)^2} + \cancel{x^2} + \cancel{y^2} + 4y + \cancel{4}$$

$$-36 - 8y = -12\sqrt{x^2+(y+2)^2}$$

$$(8y+36)^2 = \left(12\sqrt{x^2+(y+2)^2}\right)^2$$

$$1296 + 576y + 64y^2 = 144(x^2+(y+2)^2)$$

$$1296 + 576y + 64y^2 = 144(x^2+y^2+4y+4)$$

$$1296 + \cancel{576y} + 64y^2 = 144x^2 + 144y^2 + \cancel{576y} + 576$$

$$1296 - 576 = 144x^2 + 80y^2$$

$$720 = 144x^2 + 80y^2 \qquad (/720)$$

$$\boxed{\Rightarrow \frac{x^2}{5} + \frac{y^2}{9} = 1}$$

Elipse centrada en el origen

ejes $\sqrt{5}$ y 3 ✓

3- $f(z) = \dfrac{\operatorname{sen}(5z)}{2z^4}$

$2z^4 = 0$

$z = 0$

$\operatorname{Sen} z = z - \dfrac{z^3}{3!} + \dfrac{z^5}{5!} - \dfrac{z^7}{7!} + \ldots$

$$\frac{\operatorname{Sen} 5z}{2z^4} = \frac{5z}{2z^4} - \frac{125 z^3}{3!\, 2z^4} + \frac{3125 z^5}{3!\, 2z^4} - \frac{78125 z^7}{7!\, 2z^4}$$

$$\frac{\operatorname{sen}(5z)}{2z^4} = \frac{5}{2z^3} - \frac{125}{12z} + \frac{625 z}{48} - 7{,}75 z^3 + \ldots$$

$$\boxed{\text{Polo en } z=0 \text{ de orden } 3 \quad \text{y} \quad \text{residuo} \quad a_{-1} = \frac{-125}{12}}$$

4- $f(z) = \dfrac{4z^2-5z}{(z+4)^2(z^2+25)}$

se indefine en:

$z+4=0 \qquad z^2+25=0$

$z=-4 \qquad z = 5i, -5i$

Para $z=-4$

$$\lim_{z\to -4} \frac{4z^2-5z}{(z+4)^2(z^2+25)} = \lim_{z\to -4} \frac{64+20}{0} = \infty$$

Polo

$$\lim_{z\to -4} \cancel{(z+4)^2} \cdot \frac{4z^2-5z}{\cancel{(z+4)^2}(z^2+25)} = \frac{64+20}{16+25} = \frac{84}{41}$$

polo de orden 2

Residuo

$$a_{-1} = \frac{1}{(2-1)!}\left[\lim_{z\to -4} \frac{d}{dz}\left[\cancel{(z+4)^2} \cdot \frac{4z^2-5z}{\cancel{(z+4)^2}(z^2+25)}\right]\right]$$

$$a_{-1} = \lim_{z\to -4} \frac{d}{dz}\left[\frac{4z^2-5z}{z^2+25}\right] = \lim_{z\to -4} \frac{(8z-5)(z^2+25)-(4z^2-5z)(2z)}{(z^2+25)^2}$$

$$a_{-1} = -\frac{845}{1681}$$

$$\boxed{\text{Polo en } z=-4 \text{ orden } 2, \quad a_{-1} = \frac{-845}{1681}}$$

Para $z=5i$

$$\lim_{z\to 5i} \frac{4z^2-5z}{(z+4)^2\underbrace{(z^2+25)}_{0}} = \frac{\#}{0}$$

Polo simple

$$\lim_{z\to 5i} \cancel{(z^2+25)} \frac{4z^2-5z}{(z+4)^2\cancel{(z^2+25)}} = \lim_{z\to 5i} \frac{4z^2-5z}{(z+4)^2} = \frac{4(5i)^2-5(5i)}{(5i+4)^2} = \frac{-100-25i}{-25+40i+16}$$

$$= \frac{-100-25i}{9+40i} \cdot \frac{(9-40i)}{(9-40i)}$$

$$= \frac{(-100-25i)(9-40i)}{81+1600} = \frac{-1900}{1681} + \frac{3775}{1681}i$$

$$\boxed{\begin{array}{ll} z=5i & \text{Polo simple} \\ \text{Residuo} & \\ \dfrac{-1900}{1681} + \dfrac{3775}{1681}i & \end{array}}$$

Para $z=-5i$

$$\lim_{z \to 5i} \frac{4z^2-5z}{(z+4)^2\underbrace{(z^2+25)}_{0}} = \frac{\#}{0}$$ Polo simple

$$\lim_{z\to -5i} \cancel{(z^2+25)} \frac{4z^2-5z}{(z+4)^2\cancel{(z^2+25)}} = \lim_{z\to -5i} \frac{4z^2-5z}{(z+4)^2} = \frac{4(-5i)^2-5(-5i)}{(-5i+4)^2} = \frac{-100+25i}{-25-40i+16}$$

$$= \frac{-100+25i}{-9-40i} \cdot \frac{(-9+40i)}{(-9+40i)}$$

$$= \frac{(-100+25i)(-9+40i)}{81+1600} = -\frac{100}{1681} + \frac{4225}{1681} i$$

$z=-5i$ Polo simple

Residuo

$$-\frac{100}{1681} + \frac{4225}{1681} i$$

5 - $\left(\frac{3-7i}{4-i}\right)^{2\pi i}$

$\ln z = \ln(r) + i\theta$

$(a)^b = e^{b(\ln a + 2k\pi i)}$

$\frac{3-7i}{4-i} \cdot \frac{(4+i)}{(4+i)} = \frac{19}{17} - \frac{25}{17}i$

$= \frac{\sqrt{986}}{17}$, $\theta = -53 = 307$

$$\left(\frac{3-7i}{4-i}\right)^{2\pi i} = e^{2\pi i\left[\ln\left(\frac{3-7i}{4-i}\right)\right]}$$

$$\left(\frac{3-7i}{4-i}\right)^{2\pi i} = e^{2\pi i\left[\ln\left(\frac{\sqrt{986}}{17}\right) + 307i\right]}$$

$$\boxed{e^{2\pi i\left[\ln\left(\frac{\sqrt{986}}{17}\right) + 307i\right]}}$$

6- $\mathcal{L}\{te^{4t}\cos(5t)+t^2\mu_3(t)-2t^5e^{-2t}\}$

$$\mathcal{L}\{te^{4t}\cos(5t)+t^2\mu_3(t)-2t^5e^{-2t}\}$$

$$\frac{d}{ds}\left(\frac{s-4}{(s-4)^2+s^2}\right)+e^{-3s}\mathcal{L}\{(t+3)^2\}-2\mathcal{L}\{t^5\}\Big|_{s\to s+2}$$

$$\frac{(1)((s-4)^2+25)-(s-4)((s-4)^2+25)'}{((s-4)^2+25)^2}+e^{-3s}\mathcal{L}\{t^2+6t+9\}-2\left.\frac{5!}{s^{5+1}}\right|_{s\to s+2}$$

$$\frac{[(s-4)^2+25]-(s-4)(2s-8)}{(s^2-8s+41)^2}+e^{-3s}\left(\frac{2}{s^3}+\frac{6}{s^2}+\frac{9}{s}\right)-\frac{240}{(s+2)^6}$$

$$\boxed{\frac{s^2-8s-9}{(s^2-8s+41)^2}+e^{-3s}\left(\frac{2}{s^3}+\frac{6}{s^2}+\frac{9}{s}\right)-\frac{240}{(s+2)^2}}$$

7)

a) $\mathcal{L}^{-1}\left\{ \dfrac{e^{-3s} \cdot s}{s^2+6s+13} + \dfrac{s-3}{(s+7)^2} \right\}$

$s+7 \to s$

$s \to s-7$

$$\mu_3(t)\mathcal{L}^{-1}\left\{ \frac{s}{s^2+6s+13} \right\} + e^{-7t}\mathcal{L}^{-1}\left\{ \frac{s-7-3}{s^2} \right\}$$

$$\mu_3(t)\mathcal{L}^{-1}\left\{ \frac{s}{s^2+6s+13} \right\} + e^{-7t}\mathcal{L}^{-1}\left\{ \frac{s-10}{s^2} \right\}$$

$s^2+6s+\left(\frac{6}{2}\right)^2-\left(\frac{6}{2}\right)^2+13$

$(s+3)^2-(3)^2+13$

$(s+3)^2+4$

$$\mu_3(t)\mathcal{L}^{-1}\left\{ \frac{s}{(s+3)^2+4} \right\} + e^{-7t}\mathcal{L}^{-1}\left\{ \frac{1}{s} - \frac{10}{s^2} \right\}$$

$$\mu_3(t)e^{-3t}\mathcal{L}^{-1}\left\{ \frac{s-3}{s^2+4} \right\} + e^{-7t}\left[1-10t\right]$$

$$\mu_3(t)e^{-3t}\left[\mathcal{L}^{-1}\left\{ \frac{s}{s^2+4} \right\} - 3\mathcal{L}^{-1}\left\{ \frac{1}{s^2+4} \right\}\right] + e^{-7t}\left[1-10t\right]$$

$$\mu_3(t)e^{-3t}\left[\cos(2t) - \frac{3}{2}\mathcal{L}^{-1}\left\{ \frac{2}{s^2+4} \right\}\right] + e^{-7t}\left[1-10t\right]$$

$$\boxed{\mu_3(t)e^{-3t}\left[\cos(2t) - \frac{3}{2}\,\mathrm{sen}(2t)\right] + e^{-7t}\left[1-10t\right]}$$

b) $\mathcal{L}^{-1}\left\{\frac{5s^2-10s+4}{s^3-4s^2+4s}+\frac{3}{s^4}\right\}$

$$\frac{5s^2-10s+4}{s^3-4s^2+4s}=\frac{5s^2-10s+4}{s(s-2)^2}=\frac{A}{s-2}+\frac{B}{(s-2)^2}+\frac{C}{s}$$

$$5s^2-10s+4=As(s-2)+Bs+C(s-2)^2$$

$$5s^2-10s+4=A(s^2-2s)+Bs+C(s^2-4s+4)$$

$$5s^2-10s+4=s^2(A+C)+s(-2A+B-4C)+4C$$

$$\begin{cases}5=A+C\\-10=-2A+B-4C\\4=4C\end{cases}$$

$A=4$

$B=2$

$C=1$

$$\Rightarrow\frac{5s^2-10s+4}{s^3-4s^2+4s}=\frac{1}{s}+\frac{4}{s-2}+\frac{2}{(s-2)^2}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s}+\frac{4}{s-2}+\frac{2}{(s-2)^2}+\frac{3}{s^4}\right\}$$

$$1+4e^{2t}+2e^{2t}t+3\mathcal{L}^{-1}\left\{\frac{1}{3!}\cdot\frac{3!}{s^4}\right\}$$

$$1+4e^{2t}+2e^{2t}t+\frac{3}{3!}\mathcal{L}^{-1}\left\{\frac{3!}{s^4}\right\}$$

$$\boxed{1+4e^{2t}+2e^{2t}t+\frac{1}{2}t^3}$$