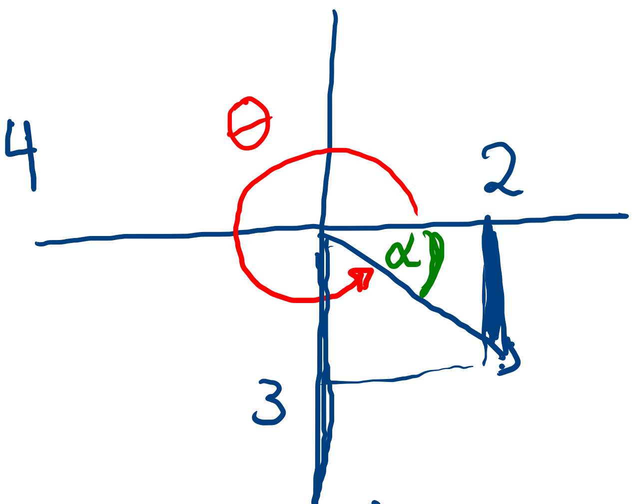
Potencias POLAR

Teorema Moivre : $\underset{\#\text{ real}}{}\; z^{n} = \left(r(\cos\theta + i\,\text{Sen}\,\theta)\right)^{n}$ ← # real

$= r^{n}(\cos n\theta + i\,\text{Sen}\,n\theta)$

1) $(\underbrace{2-3i}_{z})^{4}$ ← # real



$r = \sqrt{(2)^2 + (3)^2}$

$r = \sqrt{14}$

$\text{Tan}\,\alpha = \dfrac{3}{2}$

$\alpha = 56^\circ$

$\theta = 360 - 56$

$\theta = 304^\circ$

$\left[\left(\sqrt{14}(\cos 304 + i\,\text{Sen}\,304)\right)\right]^{4}$

$\underbrace{(\sqrt{14})^{4}}(\cos(4\cdot 304) + i\,\text{Sen}(4\cdot 304))$

$196(\cos 1016 + i\,\text{Sen}\,1016)$

Raíces : POLAR

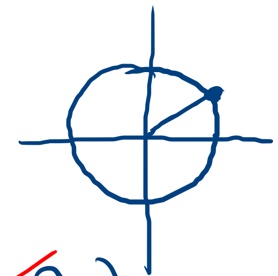
$$z^{\frac{1}{n}} = \left\{r\left(\cos\theta + i\,\mathrm{Sen}\,\theta\right)\right\}^{\frac{1}{n}} \Rightarrow r^{\frac{1}{n}}\left(\cos\left(\frac{\theta+2k\pi}{n}\right) + i\,\mathrm{Sen}\left(\frac{\theta+2k\pi}{n}\right)\right)$$

Calcular: $\sqrt[3]{2\left(\cos 60 + i\,\mathrm{Sen}\,60\right)}$

$K=0$

$K=1$

$K=2$



$\measuredangle C = \dfrac{360}{\text{índice}}$

$$K=0 \Rightarrow (2)^{\frac{1}{3}}\left(\cos\left(\frac{60+\cancel{2\cdot 0\cdot \pi}}{3}\right) + i\,\mathrm{Sen}\left(\frac{60+\cancel{2\cdot 0\cdot \pi}}{3}\right)\right)$$

$$K=0 \Rightarrow (2)^{1/3}\left(\cos 20 + i\,\mathrm{Sen}\,20\right) \checkmark$$

$\measuredangle C = \dfrac{360}{3}$

$$K=1 \Rightarrow (2)^{1/3}\left(\cos\left(\frac{60+2\cdot 1\cdot \pi}{3}\right) + i\,\mathrm{Sen}\left(\frac{60+2\cdot 1\cdot \pi}{3}\right)\right)$$

$\boxed{\measuredangle C = 120}$

$$K=1 \Rightarrow (2)^{1/3}\left(\cos 140 + i\,\mathrm{Sen}\,140\right) \checkmark$$

$$K=2 \Rightarrow (2)^{\frac{1}{3}}\left(\cos\left(\frac{60+2\cdot 2\cdot \pi}{3}\right)+\operatorname{Sen}\left(\frac{60+2\cdot 2\cdot \pi}{3}\right)\right)$$

$$K=2 \Rightarrow (2)^{1/3}\left(\cos 260+\operatorname{Sen} 260\right) \checkmark$$

Ejemplo 1 Sea $z = 1 + \sqrt{3}\, i$ $\Rightarrow z = 2\left(\text{Cos}\, 30 + i\, \text{Sen}\, 30\right)$

a) z^4

b) $\sqrt{z}$

a) z^4

$$\left(2\left(\text{Cos}\, 30 + i\, \text{Sen}\, 30\right)\right)^4 = (2)^4\left(\text{Cos}\,(30 \cdot 4) + i\, \text{Sen}\,(30 \cdot 4)\right)$$

$$= 16\left(\text{Cos}\, 120 + i\, \text{Sen}\, 120\right)$$

b) $\sqrt{z} = \sqrt{2(\cos 30 + i \operatorname{Sen} 30)}$

$K=0$

$K=1$

$\measuredangle c = \frac{360}{2}$

$\measuredangle c = 180$

$$\left[2(\cos 30 + i \operatorname{Sen} 30)\right]^{\frac{1}{2}}$$

$$K=0 \Rightarrow \left[(2)^{\frac{1}{2}}\left(\cos\frac{30}{2} + i \operatorname{Sen}\frac{30}{2}\right)\right] \Rightarrow (2)^{1/2}\left[\cos 15 + i \operatorname{Sen} 15\right]$$

$$\Rightarrow (\sqrt{2})_{15^\circ}$$

$$K=1 \Rightarrow (2)^{1/2}\left[\cos 195 + i \operatorname{Sen} 195\right] \Rightarrow (\sqrt{2})_{195^\circ}$$

c) Calcule $\sqrt[3]{\dfrac{1+i}{\sqrt{3}+i}}$

$$K\,C = \frac{360}{3} \Rightarrow 120$$

$$\frac{(1+i)}{(\sqrt{3}+i)} \cdot \frac{(\sqrt{3}-i)}{(\sqrt{3}-i)} = \frac{(1+i)(\sqrt{3}-i)}{3+1} = \frac{\sqrt{3}-i+\sqrt{3}i+1}{4}$$

$$\left(\frac{\sqrt{3}+1}{4}\right) + \left(\frac{\sqrt{3}-1}{4}\right)i$$

Pasado a polaR

$$Z = \frac{\sqrt{2}}{2}\left(\text{Cos } 15 + i \text{ Sen } 15\right)$$

$$K=0 \Rightarrow \left(\frac{\sqrt{2}}{2}\right)^{\frac{1}{3}}\left(\text{Cos } \frac{15}{3} + i \text{ Sen } \frac{15}{3}\right) = \left(\frac{\sqrt{2}}{2}\right)^{\frac{1}{3}}\left(\text{Cos } 5 + i \text{ Sen } 5\right)$$

$$K=1 \Rightarrow \left(\frac{\sqrt{2}}{2}\right)^{1/3}\left(\text{Cos } 125 + i \text{ Sen } 125\right)$$

$$K=2 \Rightarrow \left(\frac{\sqrt{2}}{2}\right)^{1/3}\left(\text{Cos } 245 + i \text{ Sen } 245\right)$$

Pág 12 Calcule "m" "n"

c) $(3m+2i)-(5-2ni)=2-6i$

$3m+2i-5+2ni=2-6i$

$(3m-5)+(2+2n)i=2-6i$

parte reales =

partes imaginarias =

$3m-5=2$

$\boxed{m=\frac{7}{3}}$

$2+2n=-6$

$\boxed{n=-4}$

$2+6i=2+6i$

ECUACIONES

a) $4x - 2(3-2i) = 6yi$

$4x - 6 + 4i = 6yi$

$4x - 6 = 0$ $\qquad$ $4 = 6y$

$x = \frac{3}{2}$ $\qquad$ $y = \frac{2}{3}$

b) $z^4 - 5 + 5i = 0$.

$\sphericalangle$ central $\frac{360}{\text{índice}}$

$\sphericalangle = \frac{360}{4} = \underline{\underline{90}}$

$z^4 = 5 - 5i$

POLAR

$z = \sqrt[4]{5-5i}$

$z = 5\sqrt{2}\,(\cos 315 + i \operatorname{Sen} 315)$

$K=0 \Rightarrow (5\sqrt{2})^{1/4}\left(\cos \frac{315}{4} + i \operatorname{Sen} \frac{315}{4}\right) \Rightarrow (5\sqrt{2})^{\frac{1}{4}}(\cos \underline{\underline{79}} + i \operatorname{Sen} \underline{\underline{79}})$

$K=1 \Rightarrow (5\sqrt{2})^{1/4}(\cos 169 + i \operatorname{Sen} 169)$

$K=2 \Rightarrow (5\sqrt{2})^{1/4}(\cos 259 + i \operatorname{Sen} 259)$

$K=3 \Rightarrow (5\sqrt{2})^{1/4}(\cos 349 + i \operatorname{Sen} 349) \Rightarrow (5\sqrt{2})^{1/4}_{349}$.

c) $\dfrac{z-1}{1+iz\sqrt{3}} = \dfrac{3i\sqrt{3}}{1}$

$3i\sqrt{3}\,(1+iz\sqrt{3})$

$3i\sqrt{3}-9z$

$z-1 = 3i\sqrt{3}-9z$

$z+9z = 3i\sqrt{3}+1$

$10z = 3i\sqrt{3}+1$

$z = \dfrac{1}{10} + \dfrac{3\sqrt{3}}{10}i$

g) $z = 3 - 4i$. Encuentre w tq $\bar{z} \cdot \bar{w} = 2i - 1$

$$\bar{z} = 3 + 4i$$

$$(3+4i) \cdot \bar{w} = 2i - 1$$

$$\bar{w} = \frac{(2i-1)}{(3+4i)} \cdot \frac{(3-4i)}{(3-4i)} = \frac{6i + 8 - 3 + 4i}{9 + 16} = \boxed{\frac{5}{25} + \frac{10i}{25}}$$

R/ $w = \dfrac{5}{25} - \dfrac{10i}{25}$ //

6 $\underbrace{(\sqrt{2})_{45}}$ $(\sqrt{2})_{315}$

$$x_1 = \sqrt{2}\,(\cos 45 + i \operatorname{Sen} 45) \Rightarrow 1 + i$$

$$x_2 = \sqrt{2}\,(\cos 315 + i \operatorname{Sen} 315) \Rightarrow 1 - i$$

$$(x-1-i)(x-1+i) = 0$$

$$x^2 - x + \cancel{xi} - x + 1 - \cancel{i} - \cancel{ix} + \cancel{i} + 1 = 0$$

$$x^2 - 2x + 2 = 0 \quad //$$

$x = 3$ $x = -2$

$$(x-3)(x+2) = 0$$

$$x^2 + 2x - 3x - 6 = 0$$

$$x^2 - x - 6 = 0$$

mode 5-3

mate 1

Pág 15.

Círculo $(x-h)^2+(y-k)^2=r^2$ $C(h,k)$

Elipse $\dfrac{(x-h)^2}{a^2}+\dfrac{(y-k)^2}{b^2}=1$ $C(h,k)$

Hipérbola $\dfrac{(x-h)^2}{a^2}-\dfrac{(y-k)^2}{b^2}=1$ $C(h,k)$

C $x^2+y^2=r^2$

H $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$

E $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$

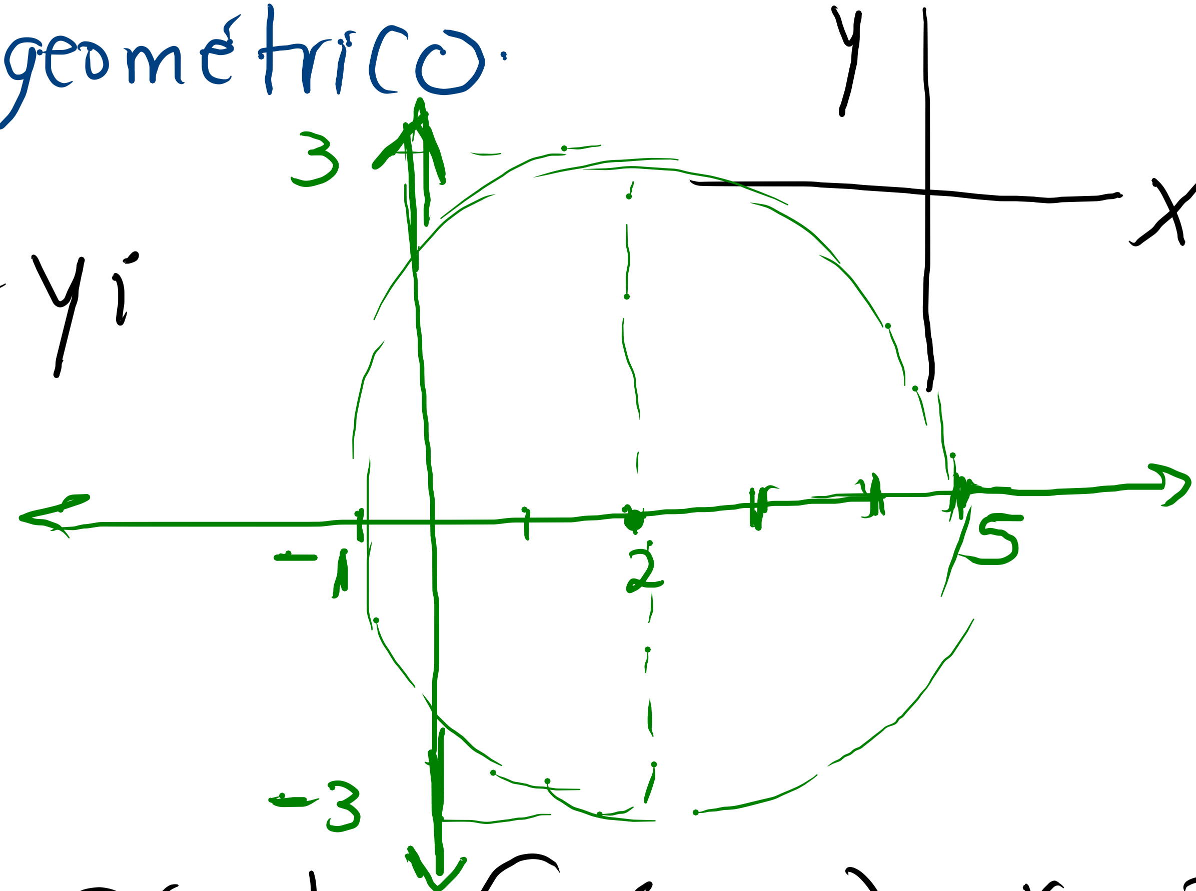
Pág 15: Determine el lugar geométrico.

$$|z-2| = 3 \qquad z = x + yi$$

$$|x+yi-2| = 3$$

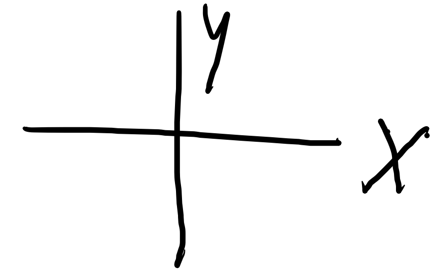
$$\sqrt{(x-2)^2 + (y)^2} = 3$$

$$(x-2)^2 + y^2 = 9$$



Círculo $C(2,0)$ $r = 3$

$$|z-2| = |z+4|$$



$$|x+yi-2| = |x+yi+4|$$

$$\left(\cancel{\sqrt{}}\,\sqrt{(x-2)^2+(y)^2}\right)^{\cancel{2}} = \left(\cancel{\sqrt{}}\,\sqrt{(x+4)^2+y^2}\right)^{\cancel{2}}$$

$$(x-2)^2 + y^2 = (x+4)^2 + y^2$$

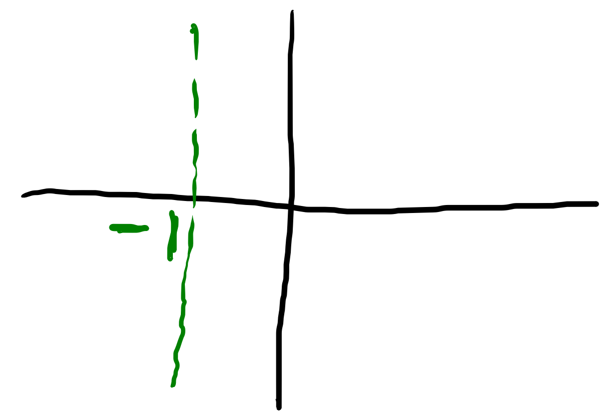
$$\cancel{x^2} - 4x + 4 + \cancel{y^2} = \cancel{x^2} + 8x + 16 + \cancel{y^2}$$

$$-4x + 4 = 8x + 16$$

$$-4x - 8x = 16 - 4$$

$$-12x = 12$$

$$x = -1$$



$$|z-3| + |z+3| = 10. \qquad z = x+yi$$

$$|x+yi-3| + |x+yi+3| = 10$$

$$\underline{\sqrt{(x-3)^2+y^2}} + \sqrt{(x+3)^2+y^2} = 10$$

$$\left[\cancel{\sqrt{(x-3)^2+y^2}}\right]^{\cancel{2}} = \left[10 - \sqrt{(x+3)^2+y^2}\right]^2$$

$$(x-3)^2+y^2 = 100 - 20\sqrt{(x+3)^2+y^2} + (x+3)^2+y^2$$

$$(x-3)^2+y^2=100-20\sqrt{(x+3)^2+y^2}+(x+3)^2+y^2$$

$$\cancel{x^2}-6x+\cancel{9}+\cancel{y^2}=100-20\sqrt{(x+3)^2+y^2}+\cancel{x^2}+6x+\cancel{9}+\cancel{y^2}$$

$$-6x=100-20\sqrt{(x+3)^2+y^2}+6x$$

¿?

$$-12x-100=-20\sqrt{(x+3)^2+y^2}$$

$$(12x+100)^2=\left[20\sqrt{(x+3)^2+y^2}\right]^2$$

$$144x^2+2400x+10000=400\left((x+3)^2+y^2\right)$$

$$144x^2+2400x+10\,000=400\left((x+3)^2+y^2\right)$$

$$144x^2+2400x+10000=400(x^2+6x+9+y^2)$$

$$144x^2+\cancel{2400x}+10\,000=400x^2+\cancel{2400x}+3600+400y^2$$

$$144x^2-400x^2-400y^2+10000-3600=0$$

$$-256x^2-400y^2+6400=0$$

Cálculo II

$$\frac{6400}{6400}=\frac{256x^2}{6400}+\frac{400y^2}{6400}$$

$$1=\frac{x^2}{25}+\frac{y^2}{16}$$

Elipse C (0, 0)