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1.

$$f(t)=\begin{cases} 0, & 0\leq t<4 \\ 1-t^2, & 4\leq t<2\pi \\ \cos t, & t\geq 2\pi \end{cases}$$

$$f(t)= 0\cdot\mu_0(t)+(1-t^2-0)\,\mu_4(t)+[\cos t-(1-t^2)]\,\mu_{2\pi}(t)$$

$$f(t)= (1-t^2)\,\mu_4(t)+(\cos t-1+t^2)\,\mu_{2\pi}(t)$$

$$\mathcal{L}\{f(t)\}=\mathcal{L}\{(1-t^2)\,\mu_4(t)\}+\mathcal{L}\{(\cos t-1+t^2)\,\mu_{2\pi}(t)\}$$

$$=e^{-4s}\mathcal{L}\{1-(t+4)^2\}+e^{-2\pi s}\mathcal{L}\{\cos(t+2\pi)-1+(t+2\pi)^2\}$$

$$=e^{-4s}\left[\mathcal{L}\{1\}-\mathcal{L}\{t^2+8t+16\}\right]+e^{-2\pi s}\left[\mathcal{L}\{\cos(t)\}-\mathcal{L}\{1\}+\mathcal{L}\{t^2+2\pi t+4\pi^2\}\right]$$

$$=e^{-4s}\left[\frac{1}{s}-\frac{2}{s^3}\quad\frac{8}{s^2}\quad\frac{16}{s}\right]+e^{-2\pi s}\left[\frac{s}{s^2+1}-\frac{1}{s}+\frac{2}{s^3}+\frac{2\pi}{s^2}+\frac{4\pi^2}{s}\right]$$

$$\boxed{R/\quad \mathcal{L}\{f(t)\}=e^{-4s}\left[\frac{1}{s}-\frac{2}{s^3}-\frac{8}{s^2}-\frac{16}{s}\right]+e^{-2\pi s}\left[\frac{s}{s^2+1}-\frac{1}{s}+\frac{2}{s^3}+\frac{2\pi}{s^2}+\frac{4\pi^2}{s}\right]}$$

2-

$$\begin{cases} 2y_1' + y_2' - 3y_2 = 0 \\ y_1' + y_2' = t \end{cases} \qquad y_1(0) = y_2(0) = 0$$

$$\begin{cases} 2\mathcal{L}[y_1'] + \mathcal{L}[y_2'] - 3\mathcal{L}[y_2] = 0 \\ \mathcal{L}[y_1'] + \mathcal{L}[y_2'] = \mathcal{L}[t] \end{cases}$$

$$\begin{cases} 2\cdot[s\mathcal{L}[y_1] - \cancel{y_1(0)}] + s\mathcal{L}[y_2] - \cancel{y_2(0)} - 3\mathcal{L}[y_2] = 0 \\ s\mathcal{L}[y_1] - \cancel{y_1(0)} + s\mathcal{L}[y_2] - \cancel{y_2(0)} = \frac{1}{s^2} \end{cases}$$

$$\begin{cases} 2s\mathcal{L}[y_1] + s\mathcal{L}[y_2] - 3\mathcal{L}[y_2] = 0 \\ s\mathcal{L}[y_1] + s\mathcal{L}[y_2] = \frac{1}{s^2} \end{cases}$$

$$\begin{cases} 2s\mathcal{L}[y_1] + (s-3)\mathcal{L}[y_2] = 0 \\ \cdot -2 \rightarrow s\mathcal{L}[y_1] + s\mathcal{L}[y_2] = \frac{1}{s^2} \end{cases}$$

$$\begin{cases} \cancel{2s\mathcal{L}[y_1]} + (s-3)\mathcal{L}[y_2] = 0 \\ \cancel{-2s\mathcal{L}[y_1]} - 2s\mathcal{L}[y_2] = \frac{-2}{s^2} \end{cases}$$

$+$

$$(s-3-2s)\mathcal{L}[y_2] = -\frac{2}{s^2}$$

$$(-s-3)\mathcal{L}[y_2] = -\frac{2}{s^2}$$

$$\mathcal{L}[y_2] = \frac{-2}{s^2(-s-3)}$$

$$\mathcal{L}[Y_2] = \frac{2}{s^2(s+3)}$$

$$Y_2 = \mathcal{L}^{-1}\left\{ \frac{2}{s^2(s+3)} \right\}$$

$$Y_2 = 2\,\mathcal{L}^{-1}\left\{ \frac{1}{s^2(s+3)} \right\}$$

$$\frac{1}{s^2(s+3)} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+3}$$

$$1 = A(s+3) + B(s+3)s + Cs^2$$

$s = -3 \rightarrow \quad 1 = A(0) + B(0) + 9C$

$* \quad \frac{1}{9} = C$

$s \rightarrow 0 \quad 1 = 3A + B(0) + C(0)$

$* \quad \frac{1}{3} = A$

$s = 1 \rightarrow 1 = \frac{1}{3}(4) + B(4) + \frac{1}{9}$

$\frac{-4}{9} = 4B$

$* \quad \frac{-1}{9} = B$

$$\frac{1}{s^2(s+3)} = \frac{1}{3s^2} - \frac{1}{9s} + \frac{1}{9(s+3)}$$

$$Y_2 = 2\,\mathcal{L}^{-1}\left\{ \frac{1}{3s^2} - \frac{1}{9s} + \frac{1}{9(s+3)} \right\}$$

$$Y_2 = 2 \cdot \frac{1}{3} t - 2 \cdot \frac{1}{9} + 2 \cdot \frac{1}{9} e^{-3t}$$

$$\boxed{Y_2 = \frac{2}{3} t - \frac{2}{9} + \frac{2}{9} e^{-3t}}$$

$$\mathcal{L}[y_2] = \frac{2}{s^2(s+3)}$$

$$2s\,\mathcal{L}[y_1] + (s-3)\,\mathcal{L}[y_2] = 0$$

$$2s\,\mathcal{L}[y_1] + \frac{(s-3)\cdot 2}{s^2(s+3)} = 0$$

$$\cancel{2}s\,\mathcal{L}[y_1] = \frac{-\cancel{2}(s-3)}{s^2(s+3)}$$

$$\mathcal{L}[y_1] = \frac{3-s}{s^3(s+3)}$$

$$\frac{3-s}{s^3(s+3)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s+3}$$

$$3-s = Ds^3 + A(s+3)s^2 + B(s+3)s + C(s+3)$$

$s=-3 \rightarrow \quad 6 = -27D + A(0) + B(0) + C(0)$

$\ast \quad -\frac{2}{9} = D$

$s=0 \rightarrow \quad 3 = D(0) + A(0) + B(0) + 3C$

$\ast \quad 1 = C$

$$\begin{array}{ll} s=1 \rightarrow & \begin{cases} 2 = A + 4B + 4 + \frac{2}{9} \\ 1 = 20A + 10B + 5 + \frac{16}{9} \end{cases} \\ s=2 & \end{array} \qquad \begin{array}{l} A = \frac{2}{9} \\ B = \frac{-2}{3} \end{array}$$

$$\frac{3-s}{s^3(s+3)} = \frac{2}{9s} - \frac{2}{3s^2} + \frac{1}{s^3} - \frac{2}{9(s+3)}$$

$$\mathcal{L}^{-1}\left\{\mathcal{L}[y_1]\right\} = \mathcal{L}^{-1}\left\{\frac{2}{9s} - \frac{2}{3s^2} + \frac{1}{s^3} - \frac{2}{9(s+3)}\right\}$$

$$\boxed{y_1 = \frac{2}{9} - \frac{2}{3}t + \frac{1}{2}t^2 - \frac{2}{9}e^{-3t}}$$

$$\therefore \begin{cases} y_1 = \frac{2}{9} - \frac{2}{3}t + \frac{1}{2}t^2 - \frac{2}{9}e^{-3t} \\ y_2 = \frac{2}{3}t - \frac{2}{9} + \frac{2}{9}e^{-3t} \end{cases}$$

3- $y'' + \int_0^t y'(u)\, e^{2(t-u)}\, du = e^{2t} \qquad y(0)=0 \qquad y'(0)=1$

$$\mathcal{L}\{y''\} + \mathcal{L}\{\int_0^t y'(u) e^{2(t-u)}\} du = \mathcal{L}\{e^{2t}\}$$

$$s^2 \mathcal{L}[y] - s\,\cancel{y(0)} - y'(0) + \mathcal{L}\{y'\} \cdot \mathcal{L}\{e^{2t}\} = \frac{1}{s-2}$$

$$s^2 \mathcal{L}[y] - 1 + (s\mathcal{L}[y] - \cancel{y(0)})\left(\frac{1}{s-2}\right) = \frac{1}{s-2}$$

$$s^2 Y(s) - 1 + \frac{s}{s-2} Y(s) = \frac{1}{s-2}$$

$$Y(s)\left[s^2 + \frac{s}{s-2}\right] = \frac{1}{s-2} + 1$$

$$Y(s)\left[\frac{s^3 - 2s^2 + s}{s-2}\right] = \frac{1}{s-2} + 1$$

$$Y(s) = \frac{\frac{1}{\cancel{s-2}}}{\frac{s^3-2s^2+s}{\cancel{s-2}}} + \frac{1}{\frac{s^3-2s^2+s}{s-2}}$$

$$Y(s) = \frac{1}{s^3-2s^2+s} + \frac{s-2}{s^3-2s^2+s}$$

$$Y(s) = \frac{s-1}{s^3-2s^2+s}$$

$$Y(s) = \frac{\cancel{(s-1)}}{s(s-1)^{\cancel{2}}} \Rightarrow Y(s) = \frac{1}{s(s-1)}$$

$$Y(s) = \frac{1}{s(s-1)}$$

$$\frac{1}{s(s-1)} = \frac{A}{s} + \frac{B}{s-1}$$

$$1 = A(s-1) + Bs$$

$$s=1 \rightarrow 1 = A(0) + B$$

$$\underline{1 = B}$$

$$s=0 \rightarrow 1 = -A + B(0)$$

$$\underline{-1 = A}$$

$$Y(s) = \frac{1}{s-1} - \frac{1}{s}$$

$$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}\left\{\frac{1}{s-1}\right\} - \mathcal{L}\left\{\frac{1}{s}\right\}$$

$$\boxed{y(t) = e^{t} - 1}$$ R/

$$4 - \quad \mathcal{L}^{-1}\left\{ \frac{e^{-\pi s}(s+1)}{s^2+2s+5} + \ln\left(\frac{s-3}{s+1}\right) - \operatorname{arcot}\left(\frac{4}{s}\right)\right\}$$

$$= s^2+2s+\left(\tfrac{2}{2}\right)^2-\left(\tfrac{2}{2}\right)^2+5$$
$$=(s+1)^2+4$$

$$\mu_{\pi}(t)\,\mathcal{L}\left\{\frac{s+1}{s^2+2s+5}\right\}_{t=t-\pi} + \mathcal{L}^{-1}\left\{\ln\left(\frac{s-3}{s+1}\right)\right\} - \mathcal{L}^{-1}\left\{\operatorname{arcot}\left(\frac{4}{s}\right)\right\}$$

$$\mu_{\pi}(t)\,\mathcal{L}\left\{\frac{s}{(s+1)^2+4} + \frac{1}{(s+1)^2+4}\right\}_{t=t-\pi} - \frac{1}{t}\mathcal{L}^{-1}\left\{\frac{d}{ds}\left[\ln\left(\frac{s-3}{s+1}\right)\right]\right\} + \frac{1}{t}\mathcal{L}^{-1}\left\{\frac{d}{ds}\operatorname{arcot}\left(\frac{4}{s}\right)\right\}$$

$$\mu_{\pi}(t)\left[e^{-t}\mathcal{L}^{-1}\left\{\frac{s-1}{s^2+4}\right\} + e^{-t}\mathcal{L}^{-1}\left\{\frac{1}{s^2+4}\right\}\right]_{t=t-\pi} - \frac{1}{t}\mathcal{L}^{-1}\left\{\frac{1}{s-3} + \frac{1}{s+1}\right\} + \frac{1}{t}\mathcal{L}^{-1}\left\{\frac{4}{s^2+16}\right\}$$

$$\mu_{\pi}(t)\left[e^{-t}\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\} - \cancel{e^{-t}\mathcal{L}^{-1}\left\{\frac{1}{s^2+4}\right\}} + \cancel{e^{-t}\mathcal{L}^{-1}\left\{\frac{1}{s^2+4}\right\}}\right]_{t=t-\pi} - \frac{1}{t}\left[e^{3t} + e^{-t}\right] + \frac{1}{t}\operatorname{sen}4t$$

$$\mu_{\pi}(t)\left[e^{-t}\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\}\right]_{t=t-\pi} - \frac{1}{t}\left[e^{3t} + e^{-t}\right] + \frac{1}{t}\operatorname{sen}4t$$

$$\boxed{\mu_{\pi}(t)\,e^{-(t-\pi)}\cos(2(t-\pi)) - \frac{1}{t}\left[e^{3t}+e^{-t}\right] + \frac{1}{t}\operatorname{sen}4t}$$

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5- $\mathcal{L}\left\{ t^2 \int_0^t \operatorname{sen} u \, e^{3t-3u} \, du + e^{-7t} t^2 \mu_4(t) \right\}$

$$\frac{d^2}{ds^2}\left[\mathcal{L}\left\{\int_0^t \underbrace{\operatorname{sen} u \, e^{3(t-u)}} \, du\right\}\right] + \left[\mathcal{L}\left\{t^2 \mu_4(t)\right\}\right]_{s \to s+7}$$

$f(t) = \operatorname{sen} t \qquad g(u) = e^{3t}$

$$\frac{d^2}{ds^2}\left[\mathcal{L}\{\operatorname{sen} t\} \cdot \mathcal{L}\{e^{3t}\}\right] + \frac{d^2}{ds^2}\mathcal{L}\{\mu_4(t)\}\Big|_{s \to s+7}$$

$$\frac{d^2}{ds^2}\left[\frac{1}{s^2+1} \cdot \frac{1}{s-1}\right] + \frac{d^2}{ds^2} \cdot \frac{e^{-4s}}{s}\Big|_{s \to s+7}$$

$$\frac{d^2}{ds^2}\left(\frac{1}{(s^2+1)(s-1)}\right) + \frac{2e^{-4s}(4s^2+4s+1)}{s^3}\Big|_{s \to s+7}$$

$$\boxed{\frac{d^2}{ds^2}\left(\frac{1}{(s^2+1)(s-1)}\right) + \frac{2e^{-4(s+7)}\left(4(s+7)^2+4(s+7)+1\right)}{(s+7)^3}}$$

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