1) Angie Marchena Mondell

C_1 = Recta que va de $(2,0,0) \to (2,5,0)$

$$\left\{ \begin{aligned} \vec{v} &= (2,5,0) - (2,0,0) \\ \vec{v} &= (0,5,0) \end{aligned} \right.$$

$$\left\{ \begin{aligned} r &= A + t\vec{v} \\ r &= (2,0,0) + t(0,5,0) \end{aligned} \right.$$

$$\left\{ \begin{aligned} x &= 2 \\ y &= 5t \\ z &= 0 \end{aligned} \right. \qquad 0 \leq t < 1$$

C_2: $x = t$ $\quad y = 5 \quad z = 4 - t^2$

$$\left\{ \begin{aligned} x &= t \\ y &= 5 \\ z &= 4 - t^2 \end{aligned} \right. \qquad -2 \leq t \leq 2$$

C_3: recta de $(-2,5,0) \to (-2,0,0)$

$\vec{v}\ (0,-5,0)$

$r = (-2,5,0) + t(0,-5,0)$

$$\left\{ \begin{aligned} x &= -2 \\ y &= 5 - 5t \\ z &= 0 \end{aligned} \right. \qquad 0 \leq t \leq 1$$

C_4: $x = t \quad y = 0 \quad z = 4 - t^2$

$$\left\{ \begin{aligned} x &= t \\ y &= 0 \\ z &= 4 - t^2 \end{aligned} \right. \qquad -2 \leq t \leq 2$$

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2) $x^2+y^2+z^2=9$ $\qquad x+y=1/8$

$x^2+\left(\frac{1}{8}-x\right)^2+z^2=9$, $y=\frac{1}{8}-x$

$x^2+\frac{1}{64}-\frac{x}{4}+x^2+z^2=9$

$2x^2+\frac{1}{64}-\frac{x}{4}+z^2=9$

$2x^2-\frac{x}{4}+z^2=\frac{575}{64}$

$2x^2-\frac{x}{4}+\left(\frac{1}{16}\right)^2-\left(\frac{1}{16}\right)^2+z^2=9-\frac{1}{64}$

$2\left(x-\frac{1}{16}\right)^2-\frac{1}{128}+z^2=9-\frac{1}{64}$

$2\left(x-\frac{1}{16}\right)^2+z^2=9-\frac{1}{64}+\frac{1}{128}$

$\dfrac{\left(x-\frac{1}{16}\right)^2}{1/2}+z^2=\frac{1151}{128}$ elipse

$\dfrac{(x-1/16)^2}{\frac{64}{1151}}+\dfrac{z^2}{\frac{128}{1151}}=1$

$x=1/16+\sqrt{\frac{64}{1151}}\cos t$, $y=\frac{1}{8}-\frac{1}{16}-\sqrt{\frac{64}{1151}}\cos t$, $z=\sqrt{\frac{128}{1151}}\operatorname{sen} t$

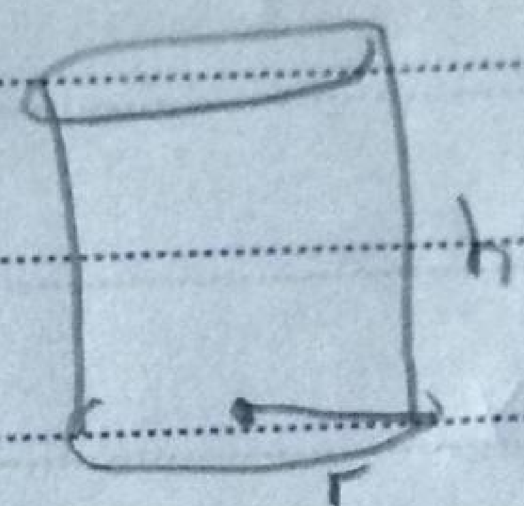
$$*\begin{cases} x=\frac{1}{16}+0{,}23\cos t \\ y=\frac{1}{16}-0{,}23\cos t \\ z=0{,}33\operatorname{sen} t \end{cases}$$

$t\in[0,2\pi]$

3)

$V = \pi r^2 h$

$$\frac{dv}{dt} = \frac{\partial v}{\partial r} \cdot \frac{dr}{dt} + \frac{\partial v}{\partial h} \cdot \frac{dh}{dt}$$



$$\frac{dr}{dt} = -5 \text{ cm/min}$$

$$\frac{dh}{dt} = +12 \text{ cm/min}$$

$$\frac{dv}{dt} = ? \quad \begin{cases} r = 20 \text{cm} \\ V = 1600\pi \text{cm}^2 \\ h = 4 \text{ cm} \end{cases}$$

$$\frac{\partial v}{\partial r} = 2r\pi h$$

$$\frac{\partial v}{\partial h} = \pi r^2$$

$$\frac{dv}{dt} = 2\pi \cdot r \cdot h \cdot \frac{dr}{dt} + \pi r^2 \cdot \frac{dh}{dt}$$

$$\frac{dv}{dt} = 2\pi \cdot 20 \cdot 4 \cdot -5 + \pi \cdot (20)^2 \cdot 12$$

$$\frac{dv}{dt} = 4000\pi \text{ cm}^3/\text{min}$$

☐ volumen aumenta a razón de $4000\pi \text{ cm}^3/\text{min}$

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Angie Marchena Mondell

4) Sea "c" la curva descrita por $R(t) = (3t - t^3, 3t^2, 3t + t^3)$

a)

$R'(t) = (3 - 2t, 6t, 3 + 3t^2)$

$\| R' \| = \sqrt{(3-2t)^2 + (6t)^2 + (3+3t^2)^2}$

$\| R' \| = \sqrt{36t^2 + 9 - 12t + 4t^2 + 9 + 18t^2 + 9t^4}$

$\| R' \| = \sqrt{9t^4 + 58t^2 - 12t + 18}$

$T(t) = \dfrac{R'(t)}{\| R' \|}$

6t

$T(t) = \left(\dfrac{3-2t}{\sqrt{9t^4+58t^2-12t+18}}, \dfrac{6t}{\sqrt{9t^4+58t^2-12t+8}}, \dfrac{3+3t^2}{\sqrt{9t^4+58t^2-12t+18}} \right)$

b) $R'(t) = (3-2t, 6t, 3+3t^2)$

$R(1/2) = (3-2t, 6t, 3+3t^2)$

$R(1/2) = \left(\dfrac{5}{4}, \dfrac{3}{4}, \dfrac{13}{8} \right)$

$R'(1/2) = \left(2, 3, \dfrac{15}{4} \right)$

Recta T = $x(t) = \left(\dfrac{5}{4}, \dfrac{3}{4}, \dfrac{13}{8} \right) + \left(2, 3, \dfrac{15}{4} \right) t$

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5) $f(x,y) = x^3 + 12xy^2 - 15x - 24y$

$f_x = 3x^2 + 12y^2 - 15$ $\quad$ $f_y = 24xy - 24$

$f_{xx} = 6x$ $\quad$ $f_{yy} = 24x$

$f_{xy} = 24y$ $\leftrightarrow$ $f_{yx} = 24y$

$3x^2 + 12y^2 - 15 = 0$ $\quad$ $24xy - 24 = 0$

$3x^2 + 12\left(\frac{1}{x}\right)^2 - 15 = 0$ $\quad$ $24xy = 24$

$3x^2 + \frac{12}{x^2} = 15$ $\quad$ $xy = 1$

$y = \frac{1}{x}$

$\frac{3x^4 + 12}{x^2} = 15$

$3(x^4 + 4) = 15x^2$ $\quad$ $x = \pm 1, \pm 2$

$x^4 + 4 = 5x^2$ $\quad$ $y = \pm 1, \pm 1/2$

$x^4 - 5x^2 + 4 = 0$

$(1,1)\ (-1,-1),\ (2, 1/2),\ (-2, -1/2)$ = puntos criticos

Punto $(1,1)$ $\quad$ $d = F_{xx} \cdot F_{xy} - F_{xy}^2$

$F_{xy}(1,1) = 6$ $\quad$ $d = \underline{-432}$ | silla

$F_{yy}(1,1) = 24$

$F_{xy}(1,1) = 24$

Para $(-1,-1)$

$f_{xx}(-1,-1) = -6$ $\quad d = f_{xy} \cdot f_{yy} - f_{xy}^2$

$f_{xy}(-1,-1) = -24$ $\quad d = -432 \Rightarrow$ silla

$f_{xy}(-1,-1) = 24$

Para $(2, \frac{1}{2})$

$f_{xx}(2, \frac{1}{2}) = 12$ $\quad d = f_{xy} \cdot f_{yy} - f_{xy}^2$

$f_{yy}(2, \frac{1}{2}) = 48$ $\quad d = 432$ minimo

$f_{xy}(2, \frac{1}{2}) = 12$

Para $(-2, -\frac{1}{2})$

$f_{xy}(-2, -\frac{1}{2}) = -12$ $\quad d = f_{xx} - f_{yy} - f_{xy}^2$

$f_{yy}(-2, -\frac{1}{2}) = 48$ $\quad d = 432$ Minimo

$f_{xy}(-2, -\frac{1}{2}) = -12$

$\Rightarrow \begin{cases} (1,1) \text{ y } (-1,-1) & \text{puntos silla} \\ (2, \frac{1}{2}) \text{ y } (-2, -\frac{1}{2}) & \text{puntos minimos} \end{cases}$

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6) $r(t) = (4\cos 2t, 4\operatorname{sen} 2t, 4t)$

$x = 4\cos(2t)$

$x' = -8\operatorname{sen}(2t) \qquad y = 4\operatorname{sen}(2t) \qquad z = 4t$

$T(t) = \dfrac{R'(t)}{\|R'\|} \qquad y' = 8\cos(2t) \qquad z' = 4$

$R'(t) = (-8\operatorname{sen}(2t), 8\cos(2t), 4)$

$\|R'\| = \sqrt{(-8\operatorname{sen}(2t))^2 + (8\operatorname{sen}(2t))^2 + 4^2}$

$\|R'\| = \sqrt{64(\operatorname{sen}^2 2t + \cos^2 2t) + 16}$

$\|R'\| = \sqrt{64+16} = 4\sqrt{5}$

$T(t) = \left(\dfrac{-8\operatorname{sen}(2t)}{4\sqrt{5}}, \dfrac{8\cos(2t)}{4\sqrt{5}}, \dfrac{4}{4\sqrt{5}}\right)$

$T'(t) = \left(\dfrac{-2\operatorname{sen}(2t)}{\sqrt{5}}, \dfrac{2\cos(2t)}{\sqrt{3}}, \dfrac{1}{\sqrt{5}}\right)$

$T'(t) \left(\dfrac{-4\cos(2t)}{\sqrt{5}}, \dfrac{4\operatorname{sen}(2t)}{\sqrt{5}}, 0\right)$

$\dfrac{T'(t)}{\|R'\|} = \left(\dfrac{-1}{5}\cos(2t), \dfrac{-1}{5}\operatorname{sen}(2t), 0\right) \Rightarrow K(t)$

$\|K(t)\| \sqrt{\left(\dfrac{-1}{5}\cos(2t)\right)^2 + \left(\dfrac{-1}{5}\operatorname{sen}(2t)\right)^2}$

$\|K(t)\| = \sqrt{\dfrac{1}{25}} = \dfrac{1}{5}$ curvatura $= \dfrac{1}{5}$

7) $f(x,y)\ ye^{2xy} - z \qquad P(0,0,2) \leftarrow A$

$\nabla f = (f_x, f_y, f_z)$

$\nabla f = (2y^2e^{2xy}, e^{2xy}(2xy+1), -1)$

$\nabla f(p) = (0,1,-1) \leftarrow \vec{n}$

Plano T.

$\vec{n}\,(x-A) = 0$

$(0,1,-1)((x,y,z)-(0,0,2)) = 0$

$(0,1,-1)(x,y,z-2)$

$y - z + 2 = 0$

$y - z = -2$ + Tangente

Normal

$r(t) = A + t\vec{n}$

$$\begin{cases} x = A_x + t\vec{n}_x \\ y = A_y + t\vec{n}_x \\ z = A_z + t\vec{n}_z \end{cases} = \begin{cases} x = 0 + 0t \\ x\ 0 + 1t \\ z = 2t + -1t \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = t \\ z = 2 - t \end{cases}$$

8) $x = \sqrt{1+t} \qquad y = 2 + \frac{1}{3}t$

$T_x(2,3) = 4 \qquad T_y(2,3) = 3$

$$\frac{\partial T}{\partial t} = \frac{\partial T}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial T}{\partial y} \cdot \frac{\partial y}{\partial t}$$

$$\frac{\partial x}{\partial t} = \frac{1}{2}(1+t)^{1/2-1} = \frac{1}{2\sqrt{1+t}}$$

$$\frac{\partial y}{\partial t} = \frac{1}{3}$$

- Necesitamos $\frac{\partial T}{\partial t}$ en $t = 3$

$$\frac{\partial T}{\partial t} = T_x \cdot \frac{\partial x}{\partial t} + T_y \cdot \frac{\partial y}{\partial t}$$

$$\frac{\partial T}{\partial t} = \frac{2}{\sqrt{1+t}} + 1 = \frac{2}{\sqrt{1+3}} + 1 = \boxed{2}$$

la temperatura aumenta con una velocidad de 2