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$2 < |z+5| < 5$

$z^2+4z+1 = (z+3)(z+1)$

$$f(z) = \frac{3z+1}{z^2+4z+1}$$

$$f(z) = \frac{3z+1}{(z+3)(z+1)}$$

$$\frac{3z+1}{(z+3)(z+1)} = \frac{A}{z+3} + \frac{B}{z+1}$$

$$\frac{3z+1}{\cancel{(z+3)(z+1)}} = \frac{A(z+1)+B(z+3)}{\cancel{(z+3)(z+1)}}$$

$3z+1 = A(z+1)+B(z+3)$ $\quad z=-3$

$3(-3)+1 = A(-3+1)+\cancel{B(-3+3)}$

$-8 = A(-2)$

$\boxed{4 = A}$

$3z+1 = A(z+1)+B(z+3)$ $\quad z=-1$

$3(-1)+1 = \cancel{A(-1+1)}+B(-1+3)$

$-2 = B(2)$

$\boxed{-1 = B}$

$$f(z) = \frac{3z+1}{(z+3)(z+1)} = \frac{A}{z+3} + \frac{B}{z+1}$$

$$f(z) = \frac{4}{z+3} - \frac{1}{z+1}$$

$$f(z) = 4\cdot\frac{1}{z+3} - 1\,\frac{1}{z+1}$$

$$f(z) = 4\cdot\frac{1}{z+3+2-2} - 1\cdot\frac{1}{z+1+4-4}$$

$$f(z) = 4\cdot\frac{1}{(z+5)-2} - 1\cdot\frac{1}{(z+5)-4}$$

$$f(z) = \frac{4}{(z+5)-2} - \frac{1}{(z+5)-4}$$

$$f(z) = \frac{4}{(z+5)\left[1 - \frac{2}{z+5}\right]} - \frac{1}{-4\left[1 - \frac{(z+5)}{4}\right]}$$

$$f(z) = \frac{4}{z+5} \cdot \sum_{n=0}^{\infty} \left(\frac{2}{z+5}\right)^n - \frac{-1}{4} \sum_{n=0}^{\infty} \left(\frac{z+5}{4}\right)^n$$

$$f(z) = \underbrace{\frac{4}{z+5} \cdot \sum_{n=0}^{\infty} \left(\frac{2}{z+5}\right)^n}_{\text{Parte Principal}} + \underbrace{\frac{1}{4} \sum_{n=0}^{\infty} \left(\frac{z+5}{4}\right)^n}_{\text{Parte Analítica}}$$

residuo

$a_{-1} = 4$

$$\frac{4}{z+5} \cdot \cancel{\left(\frac{2}{z+5}\right)^0} + \frac{4}{z+5} \left(\frac{2}{z+5}\right)^1 \ldots$$

$$a_{-1} \rightarrow \frac{4}{(z+5)^1} + \frac{8}{(z+5)^2} \ldots +$$