Angie Marchena Mondell

(5 Puntos) Sea la densidad de flujo $\vec{D} = \frac{8}{r}\cos(2\theta)\,\vec{a}_\theta \ \frac{C}{m^2}$, utilizar dos métodos diferentes para encontrar la **carga total** dentro de la región $1 < r < 3$ m, $1 < \theta < 2$ rad, $1 < \varphi < 2$ rad

$$\nabla\cdot D = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\cdot 0) + \frac{1}{r\,sen\theta}\cdot\frac{\partial}{\partial\theta}\left(sen\theta\cdot\frac{8}{r}\cos(2\theta)\right) + \frac{1}{r\,sen\theta}\cdot\frac{\partial}{\partial\phi}(0)$$

$$\nabla\cdot D = \frac{1}{r\,sen\theta}\cdot\frac{\partial}{\partial\theta}\left(sen\theta\cdot\frac{8}{r}\cos(2\theta)\right)$$

$$\nabla\cdot D = \frac{8}{r^2}\cdot\frac{1}{sen\theta}\,\frac{\partial}{\partial\theta}(sen\theta\cos(2\theta))$$

$$dV = r^2\,sen\theta\,dr\,d\theta\,d\phi$$

$$\nabla\cdot D = \frac{8}{r^2}\left[\frac{\cos(2\theta)\cos\theta}{sen\theta} - 2sen(2\theta)\right]$$

$$Q_{enc} = \int_V \nabla\cdot D\,dV = \int_1^2\int_1^2\int_1^3 \frac{8}{\cancel{r^2}}\left[\frac{\cos(2\theta)\cos\theta}{sen\theta} - 2sen(2\theta)\right]\cancel{r^2}\,sen\theta\,dr\,d\theta\,d\phi$$

$$Q_{enc} = \int_1^2\int_1^2\int_1^3 8\left[\cos(2\theta)\cos\theta - 2sen(2\theta)\,sen\theta\right]dr\,d\theta\,d\phi$$

$$Q_{enc} = 8\underbrace{\int_1^2\int_1^2\int_1^3\left[\cos(2\theta)\cos\theta - 2sen(2\theta)\,sen\theta\right]dr\,d\theta\,d\phi}$$

$$Q_{enc} = 8\cdot(-0{,}488362)$$

$$\boxed{Q_{enc} = -3{,}906\ C}$$

$\vec{D} = \frac{8}{r^2} \cos(2\theta)\, a_\theta \ \frac{C}{m^2}$ como $D_r = 0$ y $D_\phi = 0$

Solo se usa D_θ en $\theta = 1$ y $\theta = 2$

$dS_1 = -r\, sen\,\theta\, dr\, d\phi$ $dS_2 = r\, sen\,\theta\, dr\, d\phi$

$$\Rightarrow Q_{enc} = \int D_{\theta=1}\, dS_1 + \int D_{\theta=2}\, dS_2$$

$$Q_{enc} = \int_1^2 \int_1^3 \frac{8}{\cancel{r}} \cos(2\cdot 1) \cdot -\cancel{r}\, sen(1)\, dr\, d\phi + \int_1^2 \int_1^3 \frac{8}{\cancel{r}} \cos(2\cdot 2) \cdot -\cancel{r}\, sen(2)\, dr\, d\phi$$

$$Q_{enc} = -8\cos(2)\,sen(1) \underbrace{\int_1^2 \int_1^3 dr\, d\phi}_{2} + 8\cos(4)\,sen(2) \underbrace{\int_1^2 \int_1^3 dr\, d\phi}_{2}$$

$$Q_{enc} = -8\cos(2)\,sen(1) \cdot 2 + 8\cos(4)\,sen(2) \cdot 2$$

$$\boxed{Q_{enc} = -3{,}906 \ C}$$