Universidad Técnica Nacional

Calculo diferencial e integral III

Tarea 1

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~~$133x + 10y + 121z = -123$~~

(1) $P(10, 12, -13)$

$Q(-11, 13, 10)$

$R(9, -11, -10)$

$\vec{u}$ $\overrightarrow{PQ} = Q - P = (-11, 13, 10) - (10, 12, -13)$

$\overrightarrow{PQ} = (-11-10, 13-12, 10--13)$

$\overrightarrow{PQ} = (-21, 1, 23)$

$\vec{v}$ $\overrightarrow{PR} = R - P = (9, -11, -10) - (10, 12, -13)$

$\overrightarrow{PR} = (9-10, -11-12, -10--13)$

$\overrightarrow{PR} = (-1, -23, 3)$

$X = t(-21, 1, 23) + s(-1, -23, 3) + (10, 12, -13)$

$X = (x, y, z)$ $\vec{u} = (u_1, u_2, u_3)$

$\vec{v} = (v_1, v_2, v_3)$ $A = (a_1, a_2, a_3)$

$t, s \in \mathbb{R}$

$x = tu_1 + sv_1 + a_1$

$y = tu_2 + sv_2 + a_2$

$z = tu_3 + sv_3 + a_3$

$X = -21t + -1s + 10$

$y = 1t + -23s + 12$

$z = 23t + 3s + -13$

$x = -21t - 1s + 10$

$y = t - 23s + 12$

$z = 23t + 3s - 13$

$\vec{u} = (-21, 1, 23)$ $\vec{v} = (-1, -23, 3)$

Producto vectorial

$\vec{u} = (u_1, u_2, u_3)$ y $\vec{v} = (V_1, V_2, V_3)$

el producto cruz

$\vec{u} \times \vec{v} = \vec{u} = (u_1, u_2, u_3) \times \vec{v} = (V_1, V_2, V_3)$

$$\vec{u} \times \vec{v} = \left(\begin{vmatrix} u_2 & u_3 \\ V_2 & V_3 \end{vmatrix}, - \begin{vmatrix} u_1 & u_3 \\ V_1 & V_3 \end{vmatrix}, \begin{vmatrix} u_1 & u_2 \\ V_1 & V_2 \end{vmatrix} \right)$$

$$\vec{u} \times \vec{v} = \left(\begin{vmatrix} 1 & 23 \\ -23 & 3 \end{vmatrix}, - \begin{vmatrix} -21 & 23 \\ -1 & 3 \end{vmatrix}, \begin{vmatrix} -21 & 1 \\ -1 & -23 \end{vmatrix} \right)$$

$$= (1 \cdot 3 - -23 \cdot 23), -(-21 \cdot 3 - -1 \cdot 23)$$

$$, (-21 \cdot -23 - -1 \cdot 1)$$

$$= (3 - -529), -(-63 - -23), (483 - -1)$$

$$= (532), -(-40), (484)$$

$$= (532, 40, 484)$$

$$\vec{h} \cdot (X - A) = 0$$

$$(h_1, h_2, h_3) \cdot ((x, y, z) - (a_1, a_2, a_3)) = 0$$

$$(h_1, h_2, h_3) \cdot (x - a_1, y - a_2, z - a_3)$$

$$h_1(x - a_1) + h_2(y - a_2) + h_3(z - a_3) = 0$$

$$h_1 x - h_1 a_1 + h_2 y - h_2 a_2 + h_3 z - h_3 a_3 = 0$$

$$h_1 x + h_2 y + h_3 z = h_1 a_1 + h_2 a_2 + h_3 a_3$$

$$532x + 40y + 484z = 532 \cdot 10 + 40 \cdot 12 + 484 \cdot -13$$

$$532x + 40y + 484z = 5320 + 480 + -6292$$

$$532x + 40y + 484z = -492$$

A)

$z = 1 + y$ $\qquad\qquad$ $z = x^2 + y^2$

$\gamma(t) = (\cos t,\ \operatorname{sen} t,\ 1 + \operatorname{sen} t)$

$0 \leq t \leq 2\pi$

B) Curva parametrica a coordenadas

$r = r(t) \qquad \theta = \theta(t) \qquad \alpha \leqq t \leqq \beta$

$x(t) = r(t)\cos\theta(t),\ y(t) = r(t)\operatorname{sen}\theta(t),\ \alpha \leqq t \leqq \beta$

$$\frac{dx}{dt} = \cos(\theta)\frac{dr}{dt} - (r\operatorname{sen}\theta)\frac{d\theta}{dt}$$

$$\frac{dy}{dt} = \operatorname{sen}(\theta)\, dr + (r\cos\theta)\frac{d\theta}{dt}$$

$$ds = \sqrt{\left(\frac{dr}{dt}\right)^2 + \left(r\frac{d\theta}{dt}\right)^2}\, dt$$

$$ds = \sqrt{\left(\frac{dr}{d\theta}\right)^2 + r^2}\, d\theta$$

3) $c(t) = (2t^{3/2}, \cos(2t), \operatorname{sen}(2t))$

$$\text{long} = \int_a^b \sqrt{x'^2 + y'^2 + z'^2}\, dt$$

$x' = (2t^{3/2})' = 3\sqrt{t}$

$y' = (\cos(2t))' = -2\operatorname{sen}(2t)$

$z' = (\operatorname{sen}(2t)) = 2\cos(2t)$

$$\text{Long} = \int_{-4/9}^{1/3} \sqrt{(3\sqrt{t})^2 + (-2\operatorname{sen}(2t))^2 + (2\cos 2t)^2}\, dt$$

$$\int_{-4/9}^{1/3} \sqrt{9t + 4\operatorname{sen}^2(2t) + 4\cos^2(2t)}$$

$$\int_{-4/9}^{1/3} \sqrt{9t + 4(\underbrace{\cos^2(2t) + \operatorname{sen}^2(2t)}_{\Rightarrow 1}}\, dt$$

$$\int_{-4/9}^{1/3} \sqrt{9t + 4}\, dt \Rightarrow 1{,}37 \,//$$

4) $R(t) = t\cos(t)\hat{i} - \frac{\operatorname{sen} t}{2}\hat{j} + \sqrt{e^t}\,\hat{k}$

$P(0,0,1)$

$t=0$

$x' = (t\cos t)' = \cos(t) - T\operatorname{sen}(t) \Rightarrow x = 1$

$y' = \left(\frac{-\operatorname{sen} t}{2}\right)' = \frac{-\cos t}{2} \Rightarrow y = -1/2$

$z' = \left(\sqrt{e^t}\right)' = \frac{\sqrt{e^t}}{2} \Rightarrow z = 1/2$

$P(0,0,1)$

$\Rightarrow x \to 0 = t\cos t \qquad t=0 \ \vee \ \cos t = 0$

$y \Rightarrow \frac{-\operatorname{sen} t}{2} = 0 \qquad t = 2\pi n \to t=0$

$z \Rightarrow \sqrt{e^t} = \underline{1}$

$e^t = 1 \qquad T=0$

$$r(t)\begin{cases} x = 0 + t\cdot 1 \\ y = 0 + t\cdot -1/2 \\ z = 1 + t\cdot 1/2 \end{cases}$$

5 $A(1,2,3)$ punto

Planos:

$x+y+2z=5$ $\}$ $3x+y+z=6$

$y=5-x-2z$ $\}$ $y=6-3x-z$

$5-x-2z=6-3x-z$

$-x-2z+3x+z=6-5$

$2x-1z=1$

$-1z=1-2x$

$z=\frac{1-2x}{-1}$

$z=-(1-2x)$

$z=-1+2x$

$x=t$

$x = t$

$y = 5 - t - 2(-1 + 2t)$

$y = 5 - t + 2 - 4t$

$y = 7 - 5t$

$z = -1 + 2t$

$(t, 7-5t, -1+2t)$

Intersección es una recta cuya parametrización

Ecuación vectorial

$X = (0, 7, -1) + \lambda(1, -5, 2)$

$\vec{u} = K \cdot \vec{v}$ Rectas paralelas

$(1, -5, 2) = K(-1, 5, -2)$

Ecuacion vectorial

$X = (1, 2, 3) + \lambda(-1, 5, -2)$

$$x = a_1 + t v_1$$
$$y = a_2 + t v_2$$
$$z = a_3 + t v_3$$

$$x = 1 + -1t$$
$$y = 2 + 5t$$
$$z = 3 + -2t$$

$$x = 1 - t$$
$$y = 2 + 5t$$
$$z = 3 - 2t$$

Parametrización de la recta paralela en el punto A

$$(1-t, 2+5t, 3-2t)$$

6.

$y = \frac{-3}{2} t^2$ $\qquad$ $y = 2x - 1$

$m = 2$

$x = -t^3 - \frac{1}{2}$

No damos $t = -\underline{1}$ por ejemplo

$x(1) = \frac{-3}{2}(-1)^2 = \frac{-3}{2}$

$y(1) = -(-1)^3 - \frac{1}{2} = \frac{1}{2}$

ec. Recta

$y - y_0 = \underbrace{m}_{\text{pendiente}}(x - x_0)$

$y - \frac{1}{2} = 2(x - (\frac{-3}{2}))$

$y - \frac{1}{2} = 2x + 3$

$\Rightarrow y = 2x + 2{,}5$