UNIVERSIDAD TECNICA NACIONAL
INGENIERIA ELECTRONICA

Tarea 5

Angie Marchena Mondell

Teoría electromagnética

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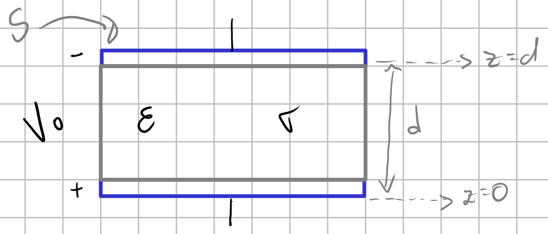
1. El espacio entre dos superficies de conductor perfecto está relleno de un material dieléctrico de permitividad $\boldsymbol{\varepsilon}$ y conductividad $\boldsymbol{\sigma}$, y tienen una diferencia de potencia $\boldsymbol{V_0}$ entre ellas. a) Demuestre que el producto [resistencia x capacitancia] entre los conductores es un valor constante. b) Demuestre la ecuación de resistencia, c) demuestra la ecuación de capacitancia para el problema anterior.

$$\boxed{R = \frac{L}{\sigma S}}$$

$$\boxed{C = \frac{Q}{V_0} = \frac{\epsilon S}{d}}$$



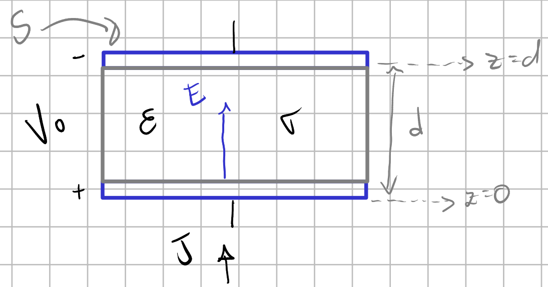
$$R = \frac{L}{\sigma S} = \frac{d}{\sigma S}$$

$$C = \frac{\epsilon S}{d}$$

$$R \cdot C = \frac{\cancel{d}}{\sigma \cancel{S}} \cdot \frac{\epsilon \cancel{S}}{\cancel{d}} = \frac{\epsilon}{\sigma}$$

$$\Rightarrow \boxed{R \cdot C = \frac{\epsilon}{\sigma}} \quad R/a$$

b)



$$J = \frac{I}{S} \qquad E = \frac{V}{d}$$

$$J = \sigma E \qquad V = Ed$$

Ley de Ohm

$$\boxed{R = \frac{V}{I}}$$

$$\Rightarrow \quad J = \sigma E = \frac{I}{S}$$

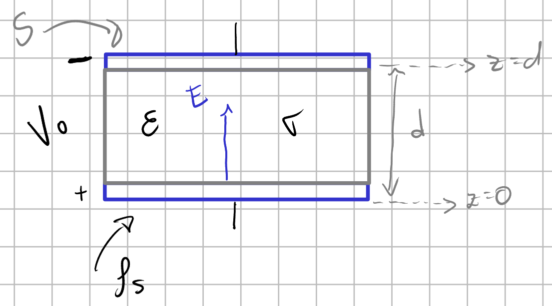
$$J = \sigma \cdot \frac{V}{d} = \frac{I}{S}$$

$$\sigma \frac{V}{d} = \frac{I}{S}$$

$$V = \frac{I \cdot d}{\sigma S}$$

$$\frac{V}{I} = \frac{d}{\sigma S}$$

$$\Rightarrow \boxed{R = \frac{d}{\sigma S}} \quad R/b$$



$$Q = \rho_s \cdot S$$

$$C = \frac{Q}{V_0} = \frac{\cancel{\rho_s}\; S}{\frac{\cancel{\rho_s} \cdot d}{\epsilon}}$$

$$\boxed{C = \frac{\epsilon S}{d}}$$

R/c

$$\vec{E} = \frac{\rho_s}{\epsilon} a_z$$

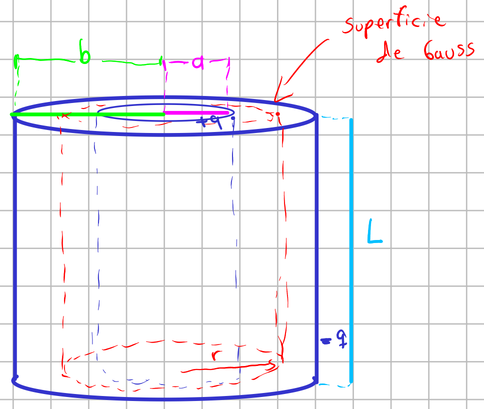
$$V_0 = -\int \vec{E} \cdot d\vec{L}$$

$$V_0 = -\int_d^0 E\, dL \cos\theta$$

$$V_0 = \int_0^d E\, dL$$

$$V_0 = \int_0^d \frac{\rho_s}{\epsilon} dz = \frac{\rho_s}{\epsilon} z \Big|_0^z = \frac{\rho_s d}{\epsilon}$$

2.



$+q$ cilindro $r = a$

$-q$ cilindro $r = b$

$$\Phi = \int_S \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0}$$

Campo Electrico en dirección radial

$$\int E\, dS \cos\theta = \frac{q}{\epsilon_0}$$

$$E \cdot S = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{\epsilon_0 S}$$

$$E = \frac{q}{2\pi r L \epsilon_0}$$

$$S = 2\pi r L$$

$$\Delta V = -\int \vec{E}\, d\vec{r}$$

$$\Delta V = -\int \frac{q}{2\pi L \epsilon_0} \cdot \frac{dr}{r}$$

$$\Delta V = -\frac{q}{2\pi L \epsilon_0} \int_a^b \frac{dr}{r}$$

$$\Delta V = \frac{-q}{2\pi L \epsilon_0} \ln\left(\frac{b}{a}\right)$$

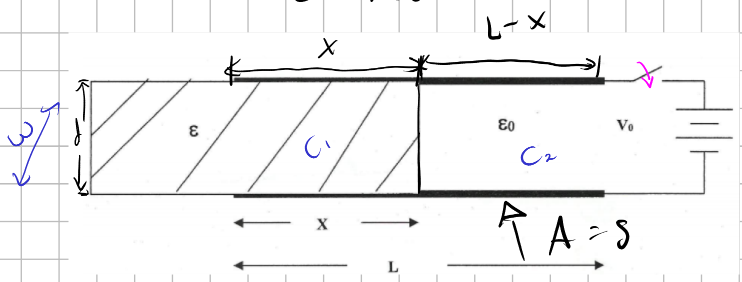
$$C = \frac{Q}{\Delta V} \rightarrow C = \frac{\cancel{q}}{\frac{\cancel{q}}{2\pi \epsilon_0 L} \cdot \ln\left(\frac{b}{a}\right)}$$

$$\boxed{C = \frac{2\pi \epsilon_0 L}{\ln\left(\frac{b}{a}\right)}}$$

3-

$$C = K\epsilon_0 \frac{A}{d}$$

$\varepsilon = K\epsilon_0$



$$C_1 = \epsilon \frac{A_1}{d} \qquad C_2 = \epsilon_0 \frac{A_2}{d}$$

$$C_1 = \frac{\epsilon \cdot x \cdot \omega}{d} \qquad C_2 = \frac{\epsilon_0 \omega (L-x)}{d}$$

$$C_1 = \frac{K\epsilon_0 x \cdot \omega}{d}$$

$$C_{tot} = \frac{K\epsilon_0 x \omega}{d} + \frac{\epsilon_0 \omega (L-x)}{d}$$

$$F_x = \left(\frac{\partial U}{\partial x}\right)_V = \frac{1}{2} V_0^2 \left[\frac{\partial}{\partial x}\left(\frac{K\epsilon_0 x \omega}{d}\right) + \frac{\partial}{\partial x}\left(\frac{\epsilon_0 \omega (L-x)}{d}\right)\right]$$

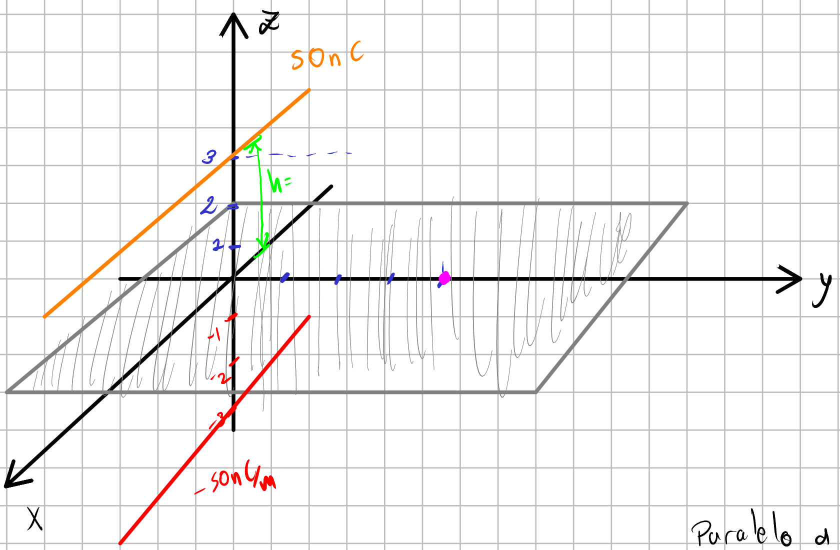
$$F_x = \frac{1}{2} V_0^2 \cdot \frac{\epsilon_0 (K-1)\omega}{2d}$$

$$\epsilon = \epsilon_0 K$$

$$F_x = \frac{1}{2} V_0^2 \frac{(\epsilon_0 K - \epsilon_0)\omega}{2d}$$

$$F_x = \frac{1}{2} V_0^2 \frac{(\epsilon - \epsilon_0)\omega}{2d}$$

4. Una línea de carga infinita de 50nC/m está a 3m sobre el suelo, el cual está a potencial cero. Elija el plano xy como el de tierra y la línea de carga paralela al eje x. Use el método de imágenes para determinar lo siguiente:
   a) Campo eléctrico E en (0, 4, 3).
   b) Campo eléctrico y ρ_s en (0, 4, 0).



$$\mathbf{E} = \frac{\rho_L}{2\pi\varepsilon_o}\left[\frac{x\mathbf{a}_x + (z-h)\mathbf{a}_z}{x^2+(z-h)^2} - \frac{x\mathbf{a}_x + (z+h)\mathbf{a}_z}{x^2+(z+h)^2}\right]$$

$$\rho_S = D_n = \varepsilon_o E_z\bigg|_{z=0} = \frac{-\rho_L h}{\pi(x^2+h^2)}$$

Sadiku
3ra Ed.

Paralelo a "y"

$$\mathbf{E} = \frac{\rho_L}{2\pi\varepsilon_o}\left[\frac{y\mathbf{a}_y + (z-h)\mathbf{a}_z}{y^2+(z-h)^2} - \frac{y\mathbf{a}_y + (z+h)\mathbf{a}_z}{y^2+(z+h)^2}\right]$$

$$\rho_S = D_n = \varepsilon_o E_z\bigg|_{z=0} = \frac{-\rho_L h}{\pi(y^2+h^2)}$$

Paralelo a "x"

a) $P(\overset{x}{0}, \overset{y}{4}, \overset{z}{3})$

$$E = \frac{50nC/m}{2\pi\epsilon_0}\left[\frac{4\,a_y + \cancel{(3-3)\,a_z}^{\,0}}{4^2 + \cancel{(3-3)^2}_{\,0}} - \frac{4\,a_y + (3+3)a_z}{4^2 + (3+3)^2}\right]$$

$$E = \frac{50nC/m}{2\pi\epsilon_0}\left[4\,a_y - \frac{4a_y + 9a_z}{97}\right]$$

$$E = \frac{50nC/m}{2\pi\epsilon_0}\left[3{,}96\,a_y - 0{,}93\,a_z\right]$$

$$\boxed{E = 3559{,}1\,a_y - 835{,}84\,a_z}$$

b) $\overset{x\ y\ z}{P(0,4,0)}$

$$E = \frac{50nC/m}{2\pi\epsilon_0}\left[\frac{4a_y + (0-3)a_z}{4^2 + (0-3)^2} - \frac{4a_y + (0+3)a_z}{4^2 + (0+3)^2}\right]$$

$$E = \frac{50nC/m}{2\pi\epsilon_0}\left[\frac{4a_y - 3a_z}{25} - \frac{4a_y + 3a_z}{25}\right]$$

$$E = \frac{50nC/m}{2\pi\epsilon_0}\left[\cancel{\frac{4a_y}{25}} - \frac{3a_z}{25} - \cancel{\frac{4a_y}{25}} - \frac{3a_z}{25}\right]$$

$$E = \frac{50nC/m}{\cancel{2}\pi\epsilon_0}\cdot\left[-\cancel{2}\cdot\frac{3a_z}{25}\right]$$

$$E = \frac{50nC/m}{\pi\epsilon_0}\cdot\frac{3a_z}{25}$$

$$\boxed{E = \frac{150nC/m}{\pi\epsilon_0}\ a_z}$$

$$\rho_s = \frac{-\rho_L\cdot h}{\pi(y^2+h^2)} = \frac{-50nC/m\cdot 3}{\pi\ (4^2+3^2)}$$

$$\boxed{\rho_s = 1{,}91\ nC/m2}$$