UNIVERSIDAD TECNICA NACIONAL
INGENIERIA ELECTRONICA

Tarea 3

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Teoría electromagnética

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1- $\vec{D} = (3yz)^2 \vec{a_x} + 2y \vec{a_y} + (x-y)^3 \vec{a_z}$ $\quad \begin{cases} 1 \le x \le 2 \\ 0 \le y \le 3 \\ 2 \le z \le 4 \end{cases}$

a) $Q = \oint \rho_v \, dV$

b) $Q = \oint D \cdot dS$

$\rho_v = \nabla \cdot \vec{D}$

$$\rho_v = \frac{\partial}{\partial x} D_x + \frac{\partial}{\partial y} D_y + \frac{\partial}{\partial z} D_z$$

$$\rho_v = \cancel{\frac{\partial}{\partial x}(3yz)^2}^{\,0} + \frac{\partial}{\partial y} 2y + \cancel{\frac{\partial}{\partial z}(x-y)^3}^{\,0}$$

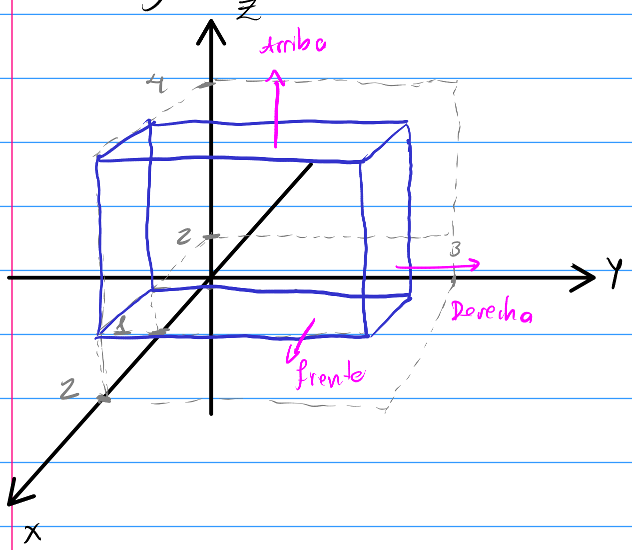
$$\rho_v = \frac{\partial}{\partial y} 2y = \underline{2} \ \frac{C}{m^3}$$

$\Rightarrow Q = \iiint \rho_v \, dV$

$$Q = \int_2^4 \int_0^3 \int_1^2 2 \, dx\, dy\, dz$$

$$Q = 12 \ C$$ Ra

b) $Q = \oint D \, dS$



frente $\Rightarrow d\vec{S} = dy\,dz \; \vec{a}_x$

derecha $\Rightarrow d\vec{S} = dx\,dz \; \vec{a}_y$

arriba $\Rightarrow d\vec{S} = dx\,dy \; \vec{a}_z$

1- $\vec{D} = (3yz)^2 \vec{a}_x + 2y \; \vec{a}_y + (x-y)^3 \; \vec{a}_z$ $\quad \begin{cases} 1 \leq x \leq 2 \\ 0 \leq y \leq 3 \\ 2 \leq z \leq 4 \end{cases}$

a) $Q = \oint \rho_v \, dV$

frente $\Rightarrow d\vec{S} = dy\,dz \; \vec{a}_x$

derecha $\Rightarrow d\vec{S} = dx\,dz \; \vec{a}_y$

arriba $\Rightarrow d\vec{S} = dx\,dy \; \vec{a}_z$

frente

$$Q_1 = \int\int \underset{x=2}{D_x} \cdot dy\,dz$$

atras

$$Q_2 = -\int\int \underset{x=1}{D_x} \, dy\,dz$$

$$Q_1 = \int_2^4 \int_0^3 (3yz)^2 \, dy\,dz$$

No tienen x

$$Q_2 = \int_2^4 \int_0^3 (3yz)^2 \, dy\,dz$$

no tienen x

Q_1 y Q_2 son iguales pero signo contrario...

Derecha

$$Q_3 = \iint D_y \, dx\,dz \quad y=3$$

$$Q_3 = \int_2^4 \int_1^2 2y \, dx\,dz \quad y=3$$

$$Q_3 \int_2^4 \int_1^2 2\cdot 3 \, dx\,dz$$

$$Q_3 = 12\,C$$

izquierda

$$Q_4 = -\iint D_y \, dx\,dz \quad y=0$$

$$Q_4 = -\int_2^4 \int_1^2 2y \, dx\,dz \quad y=0$$

$$Q_4 = -\int_2^4 \int_1^2 2\cdot 0 \, dx\,dz$$

$$Q_4 = 0$$

Arriba

$$Q_5 = \iint D_z \, dx\,dy$$

$$Q_5 = \iint (x-y)^3 \, dx\,dy$$

No tienen z

Abajo

$$Q_6 = -\iint D_z \, dx\,dy$$

$$Q_6 = \int_2^4 \int_0^3 (x-y)^3 \, dx\,dy$$

no tienen z

Q_5 y Q_6 son iguales pero signo contrario...

$$Q_{total} = Q_4 + Q_5 = 12\,C$$

R b

2.- $\vec{D} = r^{-2}\vec{a_r} + sen\phi\,\vec{a_\phi} + z^{-1/3}\,\vec{a_z}\ \frac{C}{m^3}$ $\quad P(1, 15°, 3)$

$\nabla D = \rho$

$\Rightarrow \nabla D = \frac{1}{r}\frac{\partial}{\partial r}(r \cdot r^{-2}) + \frac{1}{r}\frac{\partial}{\partial \phi}(sen\phi) + \frac{\partial}{\partial z}(z^{-1/3})$

$\nabla D = \frac{-1}{r^3} - \frac{1}{3z^{4/3}}$

$\nabla D(P) = \frac{-1}{(1)^3} - \frac{1}{3\cdot(3)^{4/3}}$

$\nabla D(P) = -1{,}07\ \frac{C}{m^2}$

$\boxed{\text{Densidad en el punto } P = -1{,}07\ \frac{C}{m^2}}$

3- $\vec{D} = r^{-1}\,\vec{a_r} + \cos\theta\,\vec{a_\theta} + \varphi\cos\theta\,\text{sen}\varphi\,\vec{a_\varphi}\ \frac{C}{m^2}$ $\qquad P = (5, 25°, 15°)$

a) Gauss $\qquad \oint E\,ds = \frac{q_{en}}{\epsilon_0} \qquad \rho_s = \nabla D_0$

$$q_{enc} = \int \nabla\cdot D\,dV$$

$$\nabla D = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \cdot r^{-1}) + \frac{1}{r\,\text{sen}\theta}\frac{\partial}{\partial\theta}(\text{sen}\theta\cdot\cos\theta) + \frac{1}{r\,\text{sen}\theta}\frac{\partial}{\partial\phi}(\varphi\cos\theta\,\text{sen}\varphi)$$

$$\nabla D = \frac{r\cos(2\theta)\csc\theta + 1}{r^2}$$

$$\nabla D(P) = \frac{5\cos(2\cdot 25)\csc(25)+1}{5^2}$$

$$\nabla D(P) = 2{,}15\ \frac{C}{m^2} = \rho$$

$$\boxed{\text{Densidad de carga} \Rightarrow 2{,}15\ \frac{C}{m^2}}$$

4. $\nabla \cdot D = \frac{dD_x}{dx} = 4 pC/m^3$

$D = \int 4\ pC/m^3\ dx$

$D = 4x\ pC/m^3\ dx$

$D_{x=5cm} = 4 \cdot (0{,}05\,m)\ pC/m^3$

$D_{x=5cm} = 200\ \frac{fC}{m^2}$

$Q = \int \rho_v\, dV$

$Q = \int_0^{12}\int_5^{15}\int_0^{10} 40 p\frac{C}{m^3}\, dx\, dy\, dz$

$\underline{Q = 48 nC}$ R/