Tarea #1

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① $3x(y^2+1)dx + y(x^2+2)dy = 0$

$\Rightarrow 3x(y^2+1)dx = -y(x^2+2)dy$

$\Rightarrow \int \frac{3x}{(x^2+2)}dx = \int \frac{-y}{(y^2+1)}dy$

$\Rightarrow 3\int \frac{x}{(x^2+2)}dx = -1\int \frac{y}{(y^2+1)}dy$

$\Rightarrow$ Sustitución:

$u = (x^2+2)$

$\frac{du}{2} = x\,dx$

$\Rightarrow$ Sustitución:

$v = (y^2+1)$

$\frac{dv}{2} = y\,dy$

$\Rightarrow 3\int \frac{1}{2u}du = -1\int \frac{1}{2v}dv$

$\Rightarrow \frac{3}{2}\int \frac{1}{u}du = -\frac{1}{2}\int \frac{1}{v}dv$

$\Rightarrow \frac{3}{2}\ln|u| = -\frac{1}{2}\ln|v|$

$\Rightarrow \frac{3}{2}\ln|x^2+2| = -\frac{1}{2}\ln|y^2+1| + C$

2 = Ejercicio # 2.

$$u = \frac{y}{x}$$

$$xy\,dx - (x^2 + 4xy + 4y^2)\,dy = 0$$

$$xy\,dx = (x^2 + 4xy + 4y^2)\,dy$$

$$* \quad \frac{xy}{x^2 + 4xy + 4y^2} = \frac{dy}{dx}$$

$$* \quad \frac{\frac{xy}{x^2}}{\frac{\cancel{x^2} + 4xy + 4y^2}{\cancel{x^2}}} = \frac{dy}{dx}$$

$$* \quad \frac{\frac{y}{x}}{1 + \frac{4y}{x} + \frac{4y^2}{x^2}} = \frac{dy}{dx}$$

$$* F(u) = \frac{u}{1 + 4u + 4u^2}$$

$$\rightarrow \quad \frac{1}{x} = \frac{1}{F(u) - u}$$

$$\frac{1}{x} = \frac{1}{\frac{u}{1 + 4u + 4u^2} - \frac{u}{1}}$$

$$\frac{1}{x} = \frac{\frac{1}{1}}{\frac{\cancel{u} - \cancel{u} - 4u^2 - 4u^3}{1 + 4u + 4u^2}}$$

$$\frac{1}{x} = \frac{1 + 4u + 4u^2}{-4u^2 - 4u^3}$$

$$\frac{1}{x} = \frac{1 + 4u + 4u^2}{-4u^2 - 4u^3}$$

$$\frac{1}{x} = \frac{-1 - 4u - 4u^2}{4u^2(1 + u)}$$

$$\frac{-1 - 4u - 4u^2}{4u^2(1 + u)} = \frac{Au + B}{4u^2} + \frac{C}{1 + u}$$

$$\frac{-1 - 4u - 4u^2}{4u^2(1 + u)} = \frac{(1 + u)(Au + B) + 4Cu^2}{\cancel{4u^2(1 + u)}}$$

$$-1 - 4u - 4u^2 = Au + B + Au^2 + Bu + 4Cu^2$$

* sistema de ecuación

$$A + 4C = -4$$
$$A + B = -4$$
$$B = -1$$

$$A + B = -4 \rightarrow -1 = B$$
$$A + 1 = -4$$
$$A = -4 + 1$$
$$\boxed{A = -3}$$

$$A + 4C = -4$$
$$-3 + 4C = -4$$
$$4C = -4 + 3$$
$$4C = -1$$
$$\boxed{C = -\frac{1}{4}}$$

$$\frac{1}{x} = \frac{Au+B}{4u^2} + \frac{C}{1+u}$$

$$\frac{1}{x} = \frac{3u-1}{4u^2} - \frac{\frac{1}{4}}{1+u}$$

$$\Rightarrow \int \frac{1}{x}dx = \int \frac{-3u}{4u^2}du - \int \frac{1}{4u^2}du - \frac{1}{4}\int \frac{1}{1+u}du$$

$$= \ln|x| + C = -\frac{3}{4}\int \frac{1}{u}du - \frac{1}{4}\int u^{-2}du - \frac{1}{4}\int \frac{1}{1+u}du$$

$$= \ln|x| + C = -\frac{3}{4}\ln|u| + \frac{1}{4}u^{-1} - \frac{1}{4}\ln|1+u|$$

$$= \ln|x| + C = \frac{-3}{4}\ln\left(\frac{y}{x}\right) + \frac{1}{4}\left(\frac{x}{y}\right) - \frac{1}{4}\ln\left|1 + \frac{y}{x}\right|$$

Tarea #1

Pregunta 3:

$(3x^2y + e^y)\,dx + (x^3 + xe^y - 2y)\,dy = 0$

$M = 3x^2y + e^y$

$N = x^3 + xe^y - 2y$

$\frac{\partial M}{\partial y} = 3x^2 + e^y$

$\frac{\partial N}{\partial x} = 3x^2 + e^y$

$\frac{\partial f}{\partial x} = 3x^2y + e^y$

$\frac{\partial f}{\partial y} = x^3 + xe^y - 2y$

$f = \int 3x^2y + e^y\,dx$

$f = 3y\int x^2\,dx + \int e^y\,dx$

$f = \cancel{3}y \cdot \frac{x^3}{\cancel{3}} + e^y \cdot x$

$f = yx^3 + e^yx$

$f = yx^3 + e^yx + g(y)$

$\frac{\partial f}{\partial y} = x^3 + xe^y + g'(y)$

$\cancel{x^3} + \cancel{xe^y} + g'(y) = \cancel{x^3} + \cancel{xe^y} - 2y$

$g'(y) = -2y$

$g(y) = \int -2y\,dy$

$g(y) = -2\int y\,dy$

$g(y) = -\cancel{2} \cdot \frac{y^2}{\cancel{2}}$

$g(y) = -y^2$

$f = yx^3 + e^yx + g(y)$

$f = yx^3 + e^yx - y^2$

$\boxed{yx^3 + e^yx - y^2 = C}$

4) $y' = \dfrac{2x - \operatorname{sen} y}{x\cos y}$

$\dfrac{dy}{dx} = \dfrac{2x - \operatorname{sen} y}{x\cos y}$

$(x\cos y)dy = (2x - \operatorname{sen} y)\,dx$

$x\cos y\,dy + (\operatorname{sen} y - 2x)\,dx = 0$

$N dy + M dy = 0$

$\left.\begin{array}{l} M_y = \cos y \\ N_x = \cos y \end{array}\right\}$ sí es exacta

$\int x\cos y\,dy = x\int \cos y = x\operatorname{sen} y$

$f(x,y) = x\operatorname{sen} y + g(x)$

$f_x = \cancel{\operatorname{sen} y} + g'(x) = \cancel{\operatorname{sen} y} - 2x$

$g'(x) = -2x$

$g(x) = \int -2x\,dx = -x^2$

$f(x,y) = x\operatorname{sen} y - x^2$

$x\operatorname{sen} y - x^2 = C$

5. $(y \ln y - 2xy)\,dx + (x + y)\,dy = 0$

$$\frac{\partial M}{\partial y} = 1 \ln y + \cancel{y} \frac{1}{\cancel{y}} + 1 - 2x = \ln y + 1 - 2x$$

$$\frac{\partial N}{\partial x} = 1$$

$$\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = g(y) = \frac{1}{y \ln y - 2xy}\,(1 - \ln y - 1 + 2x)$$

$$\frac{-\ln y + 2x}{y(\ln y - 2x)} \Rightarrow \frac{-(\ln y - 2x)}{y(\ln y - 2x)} = \frac{-1}{y}$$

$$g(y) = \frac{-1}{y} \qquad \mu = e^{\int \frac{-1}{y}} = \cancel{e}^{\ln y^{-1}} = y^{-1} = \frac{1}{y}$$

$$\frac{1}{y}(y \ln y - 2xy)\,dx + \frac{1}{y}(x + y)\,dy = 0$$

$$\left(\ln y - \frac{2xy}{y}\right)dx + \left(\frac{x}{y} + \frac{y}{y}\right)dy = 0$$

$$(\ln y - 2x)\,dx + \left(\frac{x}{y} + 1\right)dy = 0$$

$$\boxed{\frac{\partial M}{\partial y} = \frac{1}{y} \qquad \frac{\partial N}{\partial x} = \frac{1}{x} \qquad \rightarrow \text{son Exactos}}$$

$$\frac{\partial f}{\partial x} = (\ln y - 2x)\,dx$$

$$f = \int \ln y - 2x \;dx$$

$$f = x \ln y - 2\frac{x^2}{2}$$

$$f = x \ln y - x^2 + g(y)$$

$$\frac{\partial f}{\partial y} = \frac{x}{y} + g'(y) \qquad \frac{\partial f}{\partial y} = \frac{x}{y} + 1$$

$$\cancel{\frac{x}{y}} + g'(y) = \cancel{\frac{x}{y}} + 1$$

$$g'(y) = 1$$

$$g(y) = \int 1 = y$$

$$\boxed{f = x \ln y - x^2 + y + C}$$