$$\sum_{i=1}^{\infty}\left(\frac{k^2+1}{2k^2+k}\right)^{2k+1}$$

Aplicamos criterio de la Raíz

$$=\lim_{k\to\infty}\sqrt[k]{\left|\left(\frac{k^2+1}{2k^2+k}\right)^{2k+1}\right|}$$

$$\lim_{k\to\infty}\sqrt[k]{\left(\frac{k^2+1}{2k^2+k}\right)^{2k}\cdot\left(\frac{k^2+1}{2k^2+k}\right)^{1}}=\lim_{k\to\infty}\sqrt[k]{\left(\frac{k^2+1}{2k^2+k}\right)^{2k}}\cdot\sqrt[k]{\left(\frac{k^2+1}{2k^2+k}\right)}$$

$$=\lim_{k\to\infty}\left(\frac{k^2+1}{2k^2+k}\right)^{2}\cdot\left(\frac{k^2+1}{2k^2+k}\right)^{1/k}$$

$$=\lim_{k\to\infty}\left(\frac{\cancel{k^2}(1+1/k^2)}{\cancel{k^2}(2+1/k)}\right)^{2}\cdot\left(\frac{\cancel{k^2}(1+1/k^2)}{k^2(2+1/k)}\right)^{1/k}$$

$$=\left(\frac{1}{2}\right)^{2}\cdot\left(\frac{1}{2}\right)^{0}=\boxed{\frac{1}{4}}$$

Como

$$\lim_{n\to\infty}\sqrt[k]{|a_k|}<1$$

la serie

$$\sum_{i=1}^{\infty}\left(\frac{k^2+1}{2k^2+k}\right)^{2k+1}$$

converge