Pregunta 4

$x^2 + y^2 + z^2 = 80$ $z = \frac{1}{2}(x^2 + y^2)$ Calculamos la intersección
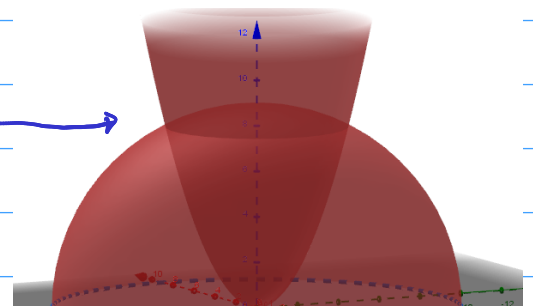
$2z = x^2 + y^2$

$2z + z^2 = 80$ $z = 8 \leftarrow$ Si $\longrightarrow$

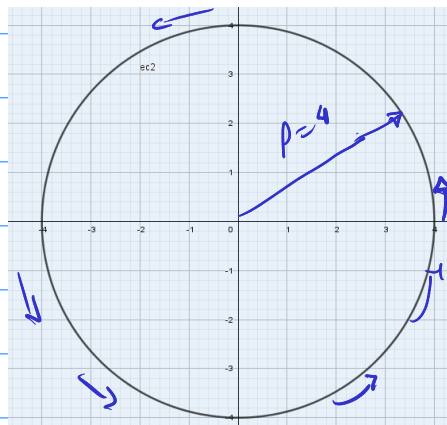$z^2 + 2z - 80 = 0$ $z = -10 \leftarrow$ No

realizamos proyección en el plano XY

$x^2 + y^2 + (8)^2 = 80$

$x^2 + y^2 = 16$

$\rho = 4$

$\theta = 0$

$\theta = 2\pi$

$x^2 + y^2 + z^2 = 80$ $z = \frac{1}{2}(x^2 + y^2)$ $\boxed{\text{Limites}}$

$z = \sqrt{80 - (x^2 + y^2)}$ $\qquad 0 \leq \theta \leq 2\pi$

$z = \sqrt{80 - \rho^2}$ $z = \frac{1}{2}\rho^2$ $0 \leq \rho \leq 4$

$\frac{1}{2}\rho^2 \leq z \leq \sqrt{80 - \rho^2}$

construimos la integral.

$$V = \int_0^{2\pi} \int_0^4 \int_{\frac{1}{2}\rho^2}^{\sqrt{80 - \rho^2}} \rho \, dz \, d\rho \, d\theta$$

$$V = \int_0^{2\pi} \int_0^4 \left[\rho z \right]_{\frac{1}{2}\rho^2}^{\sqrt{80 - \rho^2}} d\rho \, d\theta$$

$$V = \int_0^{2\pi} \int_0^4 \left(\rho\sqrt{80-\rho^2} - \frac{\rho^3}{2} \right) d\rho \, d\theta$$

$$V = \int_0^{2\pi} \left[-\frac{1}{3}(80-\rho^2)^{\frac{3}{2}} - \frac{\rho^4}{8} \right]_0^4 d\theta$$

$$V = \int_0^{2\pi} 35.847 \, d\theta = 35.847 \, \theta \Big|_0^{2\pi}$$

$$V = 35.847 \cdot 2\pi = \underline{225.23 \ ul^3}$$