1)

Angie

$$y'' - 2y' - 3y = 4x - 5 + 6xe^{2x}$$

- $m_2 - 2m - 3 = 0$

$m = -1$

$m = 3$

$y_h = Ae^{-x} + Be^{3x}$

- Superposición

$y'' - 2y' - 3y = 4x$

$n = 1$ $\quad Y_p = x^s e^{px} (A_n x^n + A_1 x + A_0)$

$s = 0$ $\quad Y_p = x^0 e^{0x} (A + Bx)$

$Y_p = A + Bx$

$Y'_p = B$

$Y''_p = 0$

$\overbrace{y'' - 2y' - 3y}$ = 4x ← Sustituir.

$0 - 2B - 3A - 3Bx = 4x$

$-3A - 2B = 0$ $\quad Y_p = \frac{8}{9} - \frac{4}{3}x$ $\quad$ * $s = 0$, $n = 0$

$-B3 = 4$

$B = -\frac{4}{3}$ $\quad y'' - 2y' - 3y = -5$

$A = \frac{8}{9}$ $\quad Y_p = x^s e^{px} (A_n x^n + A_1 x + A_0)$

$y_p = x^0 e^{0x} \cdot A$ $\quad y'p = 0$

$y_p = A$ $\quad y''p = 0$

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Sust.

$y'' - 2y' - 3y = -5$

$0 - 0 - 3A = -5 \Rightarrow A = \frac{5}{3}$

$Y_p = \frac{5}{3}$

$y'' - 2y' - 3y = 6xe^{2x}$

$Y_p = x^s e^{\beta x}(A_n x^n + A_1 x + A_0)$

$Y_p = x^0 e^{2x}(A + Bx)$

$Y_p = e^{2x} A + Bxe^{2x}$

$Y_p = Ae^{2x} + Bxe^{2x}$

$y'_p = 2Ae^{2x} + Be^{2x} + 2Bxe^{2x}$

$y''_p = 4Ae^{2x} + 2Be^{2x} + 2Be^{2x} + 4Bxe^{2x}$

sust

$y'' - 2y' - 3y = 6xe^{2x}$

$4A\cancel{e^{2x}} + 4Be^{2x} + \cancel{4Bxe^{2x}} - \cancel{4Ae^{2x}} - 2Be^{2x} - \cancel{4Bxe^{2x}}$

$- 3Ae^{2x} - 3Bxe^{2x} = 6xe^{2x}$

$2Be^{2x} - 3Ae^{2x} - 3Bxe^{2x} = 6xe^{2x}$

$-3A + 2B = 6$

$-3B = 6$

$A = -\frac{4}{3} \quad B = -2$

$Y_p = -\frac{4}{3}e^{2x} - 2xe^{2x}$

$* \; y = Ae^{-x} + Be^{3x} + \frac{23}{9} - \frac{4}{3}x - \frac{4}{3}e^{2x} - 2xe^{2x} \cancel{*} + C_1e^{-x} + C_2e^{3x}$ //

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2) $(1-x^2)y'' - 2xy' + 2y = 0 \qquad y_1 = x$

a) $y_1' = \underline{1}$

$y_1'' = 0$

$(1-x^2)\cdot(0) - 2x(1) + 2(x) \qquad y_1$ es solución

$-2x + 2x = 0$

b) $(1-x^2)y'' - 2xy' + 2y = 0$

$$y'' - \frac{2x}{1-x^2}y' + \frac{2}{1-x^2}y = 0$$

$$y_2 = y_1 \int \frac{e^{-\int p\,dx}}{y_1^2} \rightarrow e^{-\int p\,dx} = e^{-\int \frac{2x}{1-x^2}dx} = e^{\int \frac{-2x}{1-x^2}dx}$$

$$y_2 = x \cdot \int \frac{1-x^2}{x}dx \Rightarrow e^{\ln|1-x^2|} = \underline{1-x^2} = \int \frac{-2x}{1-x^2}dx = \ln|1-x^2|$$

$$y_2 = x\left[\int \frac{1}{x}dx - \int x\,dx\right]$$

$$y_2 = x\left[\ln|x| - \frac{x^2}{2}\right]$$

$$\underline{y_2 = x\ln|x| - \frac{x^2}{2}}$$

$y_1 = x \qquad y_2 = x\ln x - \frac{x^2}{2}$

$y_1' = 1 \qquad y_2' = \ln(x) + 1 - x$

$$|w| = \begin{vmatrix} x & x\ln|x| - \frac{x^2}{2} \\ 1 & \ln|x| + 1 - x \end{vmatrix}$$

$$|w| = x(\ln|x| + 1 - x) - x\ln|x| + \frac{x^2}{2}$$

$$|w| = x\ln|x| + x - x^2 - x\ln|x| + \frac{x^2}{2}$$

$|w| = x - \frac{x^2}{2} \neq 0$ Son linealmente independientes.

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3) $\mathcal{L}[e^{2t}\operatorname{senh}3t - t^2e^{-4t} + e^t\cos 4t]$

$\mathcal{L}\{e^{2t}\operatorname{senh}3t\} - \mathcal{L}\{t^2e^{-4t}\} + \mathcal{L}\{e^t\cos 4t\}$

$\mathcal{L}\{\operatorname{senh}3t\}\Big|_{s\to s-2} - \mathcal{L}\{t^2\}\Big|_{s\to s+4} + \mathcal{L}\{\cos 4t\}\Big|_{s\to s-1}$

$$\frac{3}{s^2-9}\Big|_{s\to s-2} - \frac{2}{s^3}\Big|_{s\to s+4} + \frac{s}{s^2+16}\Big|_{s\to s-1}$$

$$\frac{3}{(s-2)^3-9} - \frac{2}{(s+4)^3} + \frac{s-1}{(s-1)^2+16}$$

$s^2-4s+13 \to s^2-4s+(2)^2-(2)^2+13$

$(s-2)^2-9$

a 4) $\mathcal{L}^{-1}\left\{\frac{2s+1}{s^2-4s+13}\right\}$

$\mathcal{L}^{-1}\left\{\frac{2s+1}{(s-2)^2-9}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{2s}{(s-2)^2-9}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{(s-2)^2-9}\right\}$

$= e^{-2t}\mathcal{L}^{-1}\left\{\frac{2(s-2)}{s^2-9}\right\} + \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{3}{(s-2)^2-9}\right\}$

$e^{-2t}\mathcal{L}^{-1}\left\{\frac{2s-4}{s^2-9}\right\} + \frac{1}{3}e^{-2t}\operatorname{sen}(3t)$

$e^{-2t}\left[2\mathcal{L}^{-1}\left\{\frac{s}{s^2-9}\right\} - 4\mathcal{L}^{-1}\left\{\frac{1}{s^2-9}\right\}\right] + \frac{1}{3}e^{-2t}\operatorname{sen}(3t)$

$= e^{-2t}\left[2\cos(3t) - \frac{4}{3}\operatorname{sen}(3t)\right] + \frac{1}{3}e^{-2t}\operatorname{sen}(3t)$

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4,B

$$\mathcal{L}^{-1}\left\{\frac{3s}{(s^2+1)(s^2+3)}\right\}$$

$$\frac{3s}{(s^2+1)(s^2+3)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+3}$$

$$3s = A\,s(s^2+3) + B(s^2+3) + Cs(s^2+1) + D(s^2+1)$$

$$3s = 3B+D+(A+C)s^3+(B+D)s^2+(3A+C)s$$

$$\begin{cases} 3B+D=0 \\ 3A+C=3 \\ B+D=0 \\ A+C=0 \end{cases}$$

Calco:

$A=\frac{3}{2}$ $\quad C=-\frac{3}{2}$

$B=0$ $\quad D=0$

$$\mathcal{L}^{-1}\left\{\frac{\frac{3}{2}s}{s^2+1} + \frac{-\frac{3}{2}s}{s^2+3}\right\}$$

$$\frac{3}{2}\mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} - \frac{3}{2}\mathcal{L}^{-1}\left\{\frac{s}{s^2+3}\right\}$$

$$\frac{3}{2}\cos t - \frac{3}{2}\cos(\sqrt{3}\,t) \;//$$

4c)

$$\mathcal{L}^{-1}\left\{\frac{e^{-8s}}{s^2} + \frac{e^{-3s}}{s^4}\right\}$$

Teorema traslación

$$\mathcal{L}^{-1}\left\{\frac{e^{-8s}}{s^2}\right\} + \mathcal{L}^{-1}\left\{\frac{e^{-3s}}{s^4}\right\}$$

$$\mu_8(t)\,\frac{t^1}{1!}\Big|_{t\to t-8} + \mu_3(t)\,\frac{t^3}{3!}\Big|_{t\to t-3}$$

$$= \mu_8(t)\,(t-8) + \mu_3(t)\,\frac{(t-3)^3}{6} \;//$$

4d)

$$\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s^2-5s+4}\right\} = \mu_2(t)\,\mathcal{L}^{-1}\left\{\frac{1}{s^2-5s+4}\right\}\Big|_{t\to t-2}$$

$$\frac{1}{s^2-5s+4} = \frac{1}{(s-1)(s-4)} = \frac{A}{s-1} + \frac{B}{s-4} = \frac{-\frac{1}{3}}{s-1} + \frac{\frac{1}{3}}{s-4}$$

$$1 = A(s-4)+B(s-1)$$

$$1 = s(A+B) - 4A - B$$

$$\begin{cases} A+B=0 \\ -4A-B=1 \end{cases}$$

$A=\frac{-1}{3}$

$B=\frac{1}{3}$

Parte (4d)

$$\Rightarrow \mu_2(t)\,\mathcal{L}^{-1}\left\{\frac{1/3}{s-4} - \frac{1/3}{s-1}\right\}\Big|_{t\to t-2}$$

$$\mu_2(t)\left[\frac{1}{3}e^{4t} - \frac{1}{3}e^{t}\right]_{t\to t-2}$$

$$\mu_2(t)\left[\frac{1}{3}e^{4(t-2)} - \frac{1}{3}e^{t-2}\right] \;//$$

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