UNIVERSIDAD TECNICA NACIONAL
INGENIERIA ELECTRONICA

Tarea 2

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Teoría electromagnética

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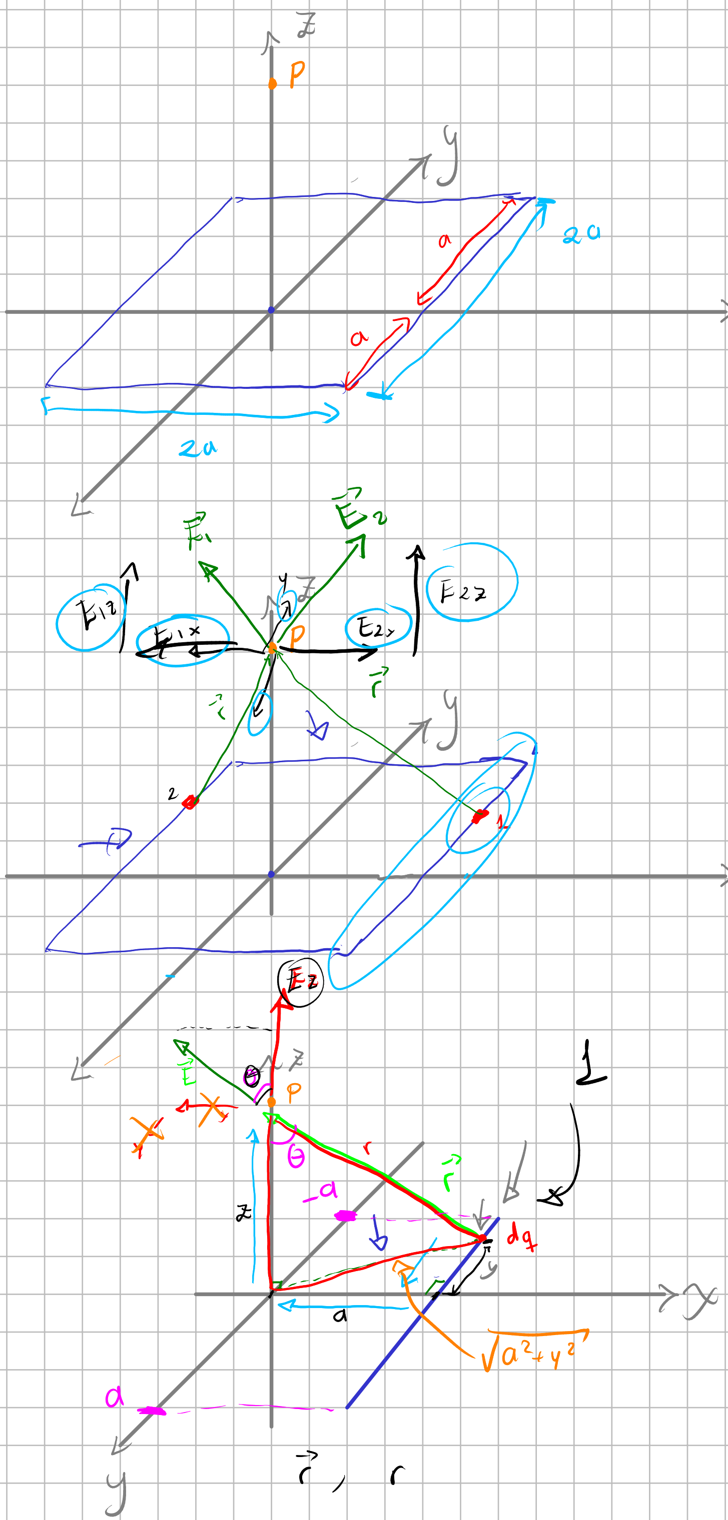
1. (4pts.) Una espira cuadrada de lado 2a se encuentra en z = 0 centrada en el origen.
Cada lado de la espira tiene una distribución de carga ρL uniforme.
Determine la expresión del campo eléctrico para un punto cualquiera en el eje z mayor que cero.

$\rho L =$ línea



$\vec{E} \Rightarrow E_x = 0 \,,\; E_y = 0$

$\underbrace{E_z \neq 0}$

$$\boxed{E = k \frac{Q}{r^2} \hat{r} \rightsquigarrow k \frac{q}{r^3} \vec{r}}$$

$\rho_L = \lambda$

$$d\vec{E} = k \frac{dq}{r^2} a_n$$

$$d\vec{E} = k \frac{dq}{r^2} a_z$$

$$\vec{r} = -a\,(a_x) - y\,(a_y) + z\,(a_z)$$

$$r = \sqrt{a^2 + y^2 + z^2}$$

$$r^2 = a^2 + y^2 + z^2$$

$$\underline{dq = \rho_L \, dy}$$

$$\boxed{E_{tot} = 4 E_{uno}}$$

$$E_z = E \cos\theta \; (a_z)$$

$$dE_z = dE \cos\theta \; (a_z)$$

$$dE_z = dE \cos\theta \ (a_z)$$

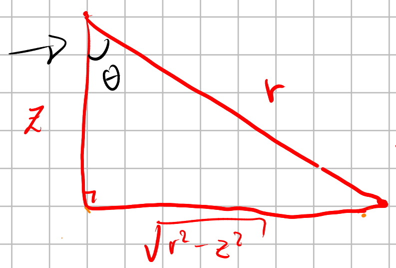
$$dq = \rho_L\, dy$$

$$\vec{r} = -a\,(a_x) - y\,(a_y) + z\,(a_z)$$

$$r = \sqrt{a^2 + y^2 + z^2}$$

$$r^2 = a^2 + y^2 + z^2$$

$$dE = K \frac{dq}{r^2} \cos\theta \ (a_z)$$



$$\cos\theta = \frac{z}{r}$$

$$E_z = E\cos\theta \ (a_z)$$

$$dE_z = dE \cos\theta \ (a_z)$$

$$d\vec{E} = K \frac{dq}{r^2} \cdot \frac{z}{r} \ (a_z)$$

$$d\vec{E} = K \frac{z\, dq}{r^3} \ (a_z)$$

$$d\vec{E} = K \frac{z \cdot \rho_L\, dy}{\left(\sqrt{a^2+z^2+y^2}\right)^3} \ (a_z)$$

$$\vec{E} = \int_{-a}^{a} K \frac{z \cdot \rho_L\, dy}{\left(\sqrt{a^2+z^2+y^2}\right)^3} \ (a_z)$$

$$\left(\sqrt{a^2+z^2+y^2}\right)^3 = (a^2+z^2+y^2)^{3/2}$$

$$\vec{E} = K z \rho_L \int_{-a}^{a} \frac{dy}{(\underbrace{a^2+z^2}_{\omega^2}+y^2)^{3/2}} \ (a_z)$$

$$\omega^2 = a^2 + z^2$$

$$\vec{E} = K z \rho_L \int_{-a}^{a} \frac{dy}{(\omega^2+y^2)^{3/2}} \ (a_z)$$

$$\int \frac{dy}{(\omega^2+y^2)^{3/2}} = \frac{y}{\omega^2\sqrt{\omega^2+y^2}}$$

$$\vec{E} = 2K z \rho_L \int_{0}^{a} \frac{dy}{(\omega^2+y^2)^{3/2}} \ (a_z)$$

$$\vec{E} = 2K z \rho_L \left[\frac{y}{\omega^2\sqrt{\omega^2+y^2}} \right]_0^a \ (a_z)$$

$$\vec{E} = 2K z \rho_L \left[\frac{a}{\omega^2\sqrt{\omega^2+a^2}} - \cancel{\frac{0}{\omega^2\sqrt{\omega^2+0^2}}} \right]$$

$$\vec{E} = 2K z \rho_L \cdot \frac{a}{\omega^2\sqrt{\omega^2+a^2}} \ (a_z)$$

una varilla

$$\vec{E} = 2K z \rho_L \cdot \frac{a}{w^2\sqrt{w^2+a^2}} (a_z)$$

$w^2 = a^2 + z^2$

$$\vec{E} = 2K \cdot z \rho_L \cdot \frac{a}{(a^2+z^2)\sqrt{a^2+z^2+a^2}} (a_z)$$

$$E = 2K \cdot z \rho_L \cdot \frac{a}{(a^2+z^2)\sqrt{2a^2+z^2}} (a_z)$$

$k = \frac{1}{4\pi\epsilon_0}$

$\rho_L = \frac{Q}{L} = \frac{Q}{2a}$

$$E_{total} = 4 \cdot E$$

$$E_{total} = 4 \cdot 2K \cdot z \rho_L \cdot \frac{a}{(a^2+z^2)\sqrt{2a^2+z^2}} (a_z)$$

$$E_{tot} = \cancel{4} \cdot 2 \cdot z \cdot \left(\frac{1}{\cancel{4}\pi\epsilon_0}\right) \rho_L \frac{a}{(a^2+z^2)\sqrt{2a^2+z^2}} (a_z)$$

$$\boxed{E_{tot} = \frac{2z\rho_L}{\pi\epsilon_0} \cdot \frac{a}{(a^2+z^2)\sqrt{2a^2+z^2}} (a_z)}$$

R/

2. Una carga 5nC en $(r, \varphi, z) = (2, 115, 4)$

a) $\vec{E}$ en $(4, -2, 4)$ en rectangulares

$r = \sqrt{x^2 + y^2}$ $\quad z = z$

$\tan\theta = \dfrac{y}{x}$

$(r, \varphi, z) = (2, 115, 4)$

$\tan(115) = \dfrac{y}{x}$

$-2{,}14 = \dfrac{y}{x} \rightarrow \underbrace{-2{,}14\,x = y}$

$2^2 = x^2 + y^2$

$4 = x^2 + (-2{,}14\,x)^2$

$4 = x^2 + 4{,}6x^2$

$4 = 5{,}6\,x^2$

$\dfrac{4}{5{,}6} = x^2$

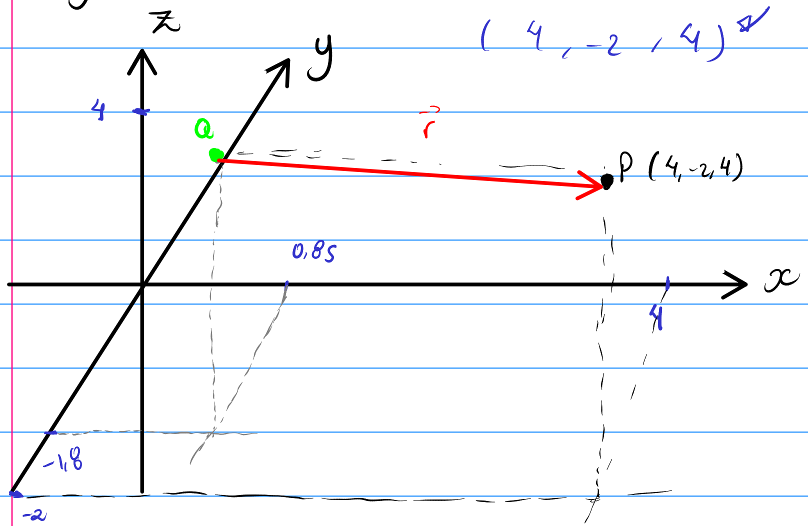
$\sqrt{\dfrac{4}{5{,}6}} = x = 0{,}85$

$-2{,}14 \cdot 0{,}85 = y$

$-1{,}8 = y$

Carga esta $(x, y, z) = (0.85, -1.8, 4)$

$(4, -2, 4)$



$\vec{E} = K \cdot \dfrac{Q}{r^3}\vec{r} \rightarrow K\dfrac{Q}{r^2}\hat{r}$

$\vec{r} = (4 - 0{,}85, -2 - (-1{,}8), 4 - 4)$

$\vec{r} = (3{,}15, -0{,}2, 0)$

$\vec{r} = 3{,}15\,a_x - 0{,}2\,a_y + 0\,a_z$

$r = \sqrt{(3{,}15)^2 + (-0{,}2)^2}$

$r = 3{,}16$

$Q =$

$\vec{E} = K \cdot \dfrac{Q}{r^3}\vec{r}$

$\vec{E} = K \cdot \dfrac{5nC}{(3{,}16)^3} \cdot (3{,}15\,a_x - 0{,}2\,a_y)$

$\vec{E} = 1{,}43 \times 10^{-18}(3{,}15a_x - 0{,}2a_y)$

$\boxed{\vec{E} = 4{,}49 \times 10^{-18}\,a_x - 285 \times 10^{-21}\,a_y}$

b) Determine la Fuerza de una carga $-15nC$ en $(4,-2,4)$ en cilíndricas

$F = ?$

$F = K \frac{Q_1 Q_2}{r^3} \vec{r}$

$\Rightarrow F = q \cdot \vec{E}$

$\vec{F} = (-15nC) \cdot (4{,}49 \times 10^{-18} a_x - 285 \times 10^{-21} a_y)$

$\vec{F} = \underbrace{-67{,}4 \times 10^{-27}}_{A_x} a_x + \underbrace{4{,}28 \times 10^{-27}}_{A_y} a_y$

$(x, y, z) \rightarrow (r, \phi, z)$

$\rightarrow (A_x \cos\phi + A_y \operatorname{sen}\phi) a_r$

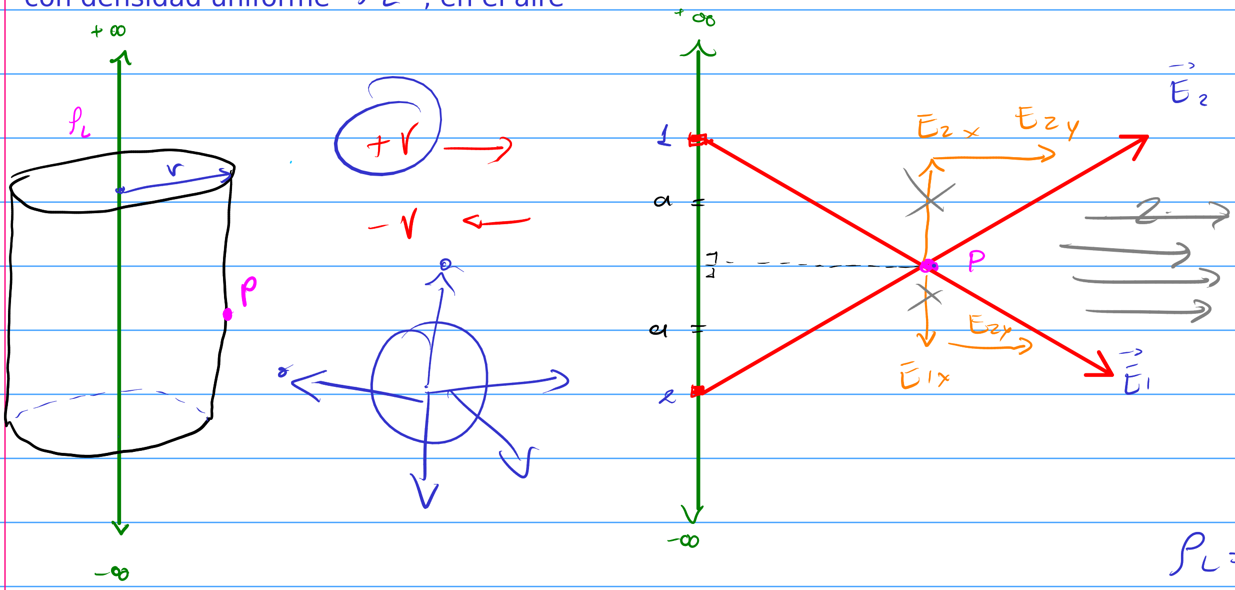
$\rightarrow (A_y \cos\phi - A_x \operatorname{sen}\phi) a_\phi$

$\rightarrow A_z a_z = 0$

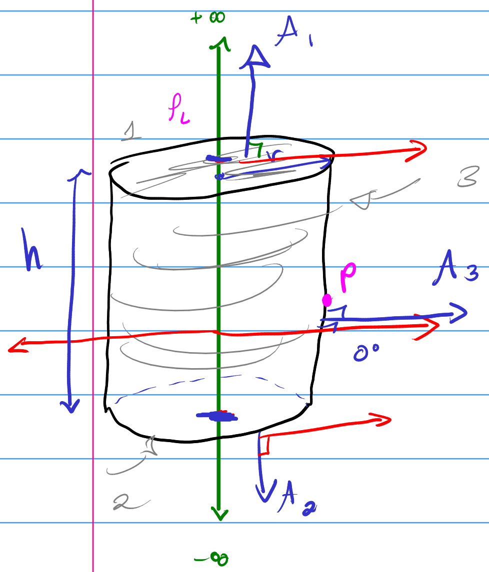
$\vec{F}_\theta = (-67{,}4 \times 10^{-27} \cos(115) + 4{,}28 \times 10^{-27} \operatorname{sen}(115)) a_r + (4{,}28 \times 10^{-27} \cos(115) + 67{,}4 \times 10^{-27} \operatorname{sen}(115)) a_\phi$

$$\boxed{\vec{F} = 3{,}23 \times 10^{-26} a_r + 59{,}3 \times 10^{-27} a_\phi}$$

Determine la intensidad de campo eléctrico de una línea de carga recta, infinitamente larga, con densidad uniforme ρ_L, en el aire



$$\rho_L = \frac{Q}{l}$$



$$\Phi_E = \int \vec{E}\, d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$Q = \rho_L \cdot l$$

$$\cancel{\Phi_1} + \cancel{\Phi_2} + \Phi_3$$

$$\Phi_E = \int E\, dA \cancel{\cos\theta} = \frac{Q_{en}}{\epsilon_0}$$

$$E\int dA = \frac{Q_{en}}{\epsilon_0}$$

?

$$E A = \frac{Q_{enc}}{\epsilon_0}$$

$Q_{enc} = \rho_L\, l$

$Q_{enc} = \rho_L \cdot h$

$$E(2\pi r h) = \frac{Q_{enc}}{\epsilon_0}$$

$$E = \frac{Q_{enc}}{2\pi r \epsilon_0 h}$$

$$E = \frac{\rho_L \cancel{h}}{2\pi r \epsilon_0 \cancel{h}}$$

$$E = \frac{\rho_L}{2\pi r \epsilon_0} a_r$$

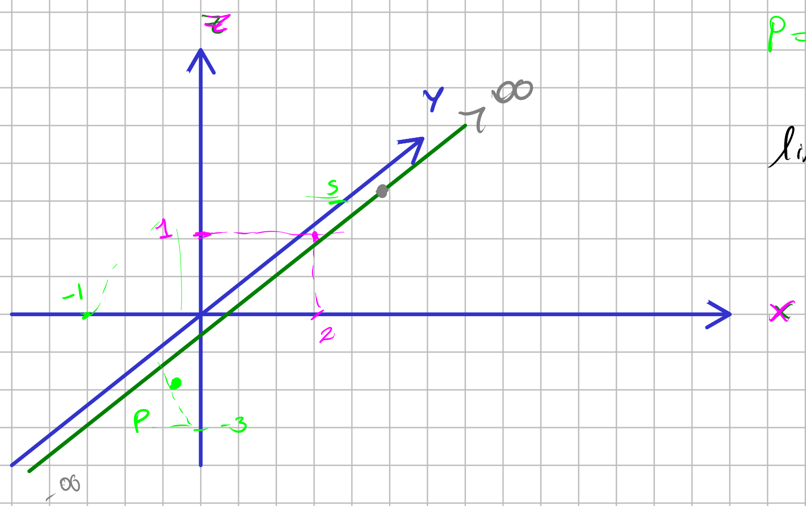
$$E = \frac{\rho_L}{2\pi r \epsilon_0} a_r$$

$\rho_L = 50\,nC/m$

línea esta

$x = 2 \quad -\infty < y < \infty$

$z = 1$



$P = (-1, 5, 3)$

línea $(2, 5, 1)$

$$\vec{r} = P - \text{línea}$$
$$= (-1-2, 5-5, 3-1)$$
$$= (-3, 0, 2)$$
$$r = \sqrt{(-3)^2 + (0)^2 + (2)^2}$$
$$r = \sqrt{13}$$

$$\vec{E} = \frac{\rho_L}{2\pi r \epsilon_0} a_r$$

$$\vec{E} = \frac{50 \frac{nC}{m}}{2\pi(\sqrt{13}) \cdot \epsilon_0} a_r$$

$$\vec{E} = 2{,}21 \frac{nC}{m^2} \cdot \frac{1}{\epsilon_0} a_r$$

$$\boxed{\vec{E} = 249 \frac{nC}{Fm} a_r} \quad R/a$$

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b) calcular D en $(-1, 5, 3)$

$$D = \frac{\rho_L}{2\pi r} a_r$$

$$D = \frac{50\,nC/m}{2\pi(\sqrt{13}\,m)} a_r$$

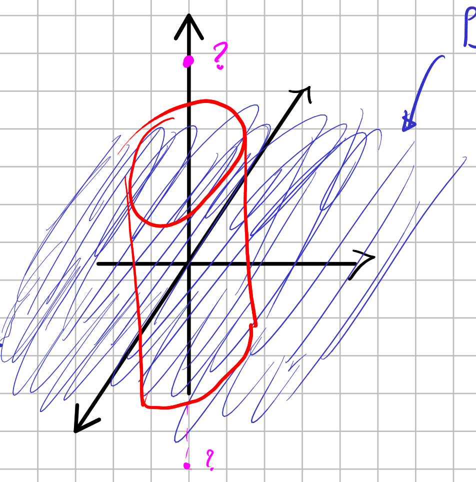
$$\boxed{D = 2{,}21 \frac{nC}{m^2} a_r} \quad R/b$$

4. (6 pts.) Determine la intensidad de campo eléctrico de un plano de carga infinito con densidad superficial de carga uniforme ρ_S , para z>0 y z<0.

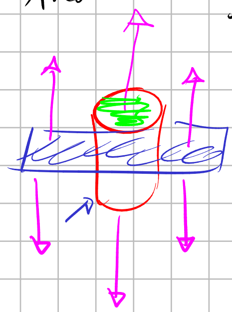
$\rho_L \to$ línea

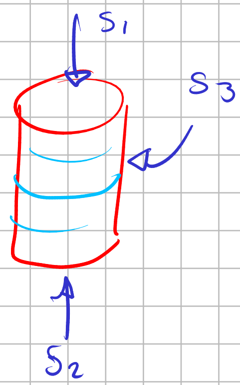
$\rho_S \to$ Área

$\rho_V \to$ Volumen



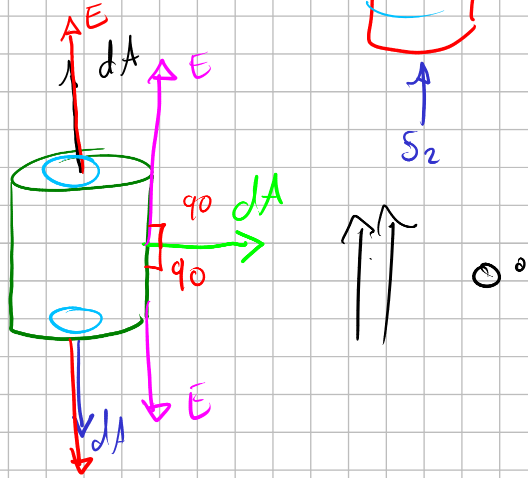
Ley de Gauss





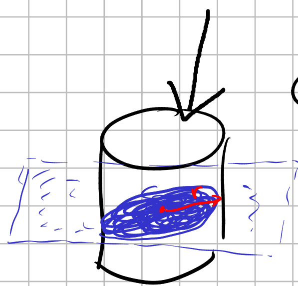
$$\Phi_E = \Phi_{S_1} + \Phi_{S_2} + \Phi_{S_3}$$

$$\Phi_E = \int_S \vec{E} \cdot d\vec{A}$$



$$\begin{cases} \Phi_{S_1} = \int_{S_1} E\, dA \cancel{\cos\theta} & \theta = 0,\ \cos\theta = 1 \\ \Phi_{S_2} = \int_{S_2} E\, dA \cancel{\cos\theta} & \theta = 0,\ \cos\theta = 1 \end{cases}$$

$$\cancel{\Phi_{S_3}} = \int_{S_3} E\, dA \underbrace{\cos\theta}_{0} \quad \theta = 90 \quad = 0$$



$Q_S = \rho_S \cdot A$

$Q_{enc} = \rho_S \pi r^2$

Ley de Gauss

$$\int \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$2 \cdot \int E\, dA \cancel{\cos\theta} = \frac{Q_{enc}}{\epsilon_0}$$

$$2 \cdot E \int dA = \frac{Q_{enc}}{\epsilon_0}$$

$$2 E A = \frac{Q_{enc}}{\epsilon_0}$$

$$2 E \cdot \cancel{\pi r^2} = \frac{\rho_S \cancel{\pi r^2}}{\epsilon_0}$$

$$\boxed{E = \frac{\rho_S}{2 \epsilon_0}}$$

$\rho_L = \frac{Q}{L}$

$\Rightarrow \rho_S = \frac{Q}{A}$

$\rho_V = \frac{Q}{V}$

Para $z > 0$

$$E = \frac{\rho_S}{2 \epsilon_0} (a_z)$$

Para $z < 0$

$$E = -\frac{\rho_S}{2 \epsilon_0} (a_z)$$