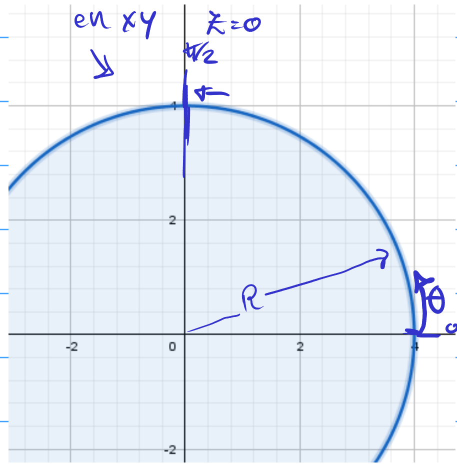
Pregunta 3

$$I=\int_0^4\int_0^{\sqrt{16-x^2}}\int_0^{\sqrt{16-x^2-y^2}}\sqrt{x^2+y^2}\,dz\,dy\,dx$$

Para facilitar pasamos a coordenadas cilindricas



$0\le x\le 4$

$0\le y\le\sqrt{16-x^2}$ ← cilindro de radio 4.

$0\le z\le\sqrt{16-x^2-y^2}$ ← esfera de radio 4.

el radio es constante.

si realizamos el cambio a cilindricas

$0\le\theta\le\pi/2$

$0\le p\le 4$

$z=\sqrt{16-x^2-y^2}$

$z=\sqrt{16-(x^2+y^2)}=\sqrt{16-p^2}$

$$\Rightarrow\int_0^{\pi/2}\int_0^4\int_0^{\sqrt{16-p^2}}\sqrt{p^2}\,p\,dz\,dp\,d\theta$$

$$=\int_0^{\pi/2}\int_0^4 p^2 z\Big|_0^{\sqrt{16-p^2}}dp\,d\theta$$

$$=\int_0^{\pi/2}\underbrace{\int_0^4 p^2\sqrt{16-p^2}\,dp}_{Ia}\,d\theta$$

$p=4\,\text{sen}(u)$ $\quad$ $p=0$ $\quad u=0$

$dp=4\cos(u)\,du$ $\quad$ $p=(4)$ $\quad u=\pi/2$

$$=\int_0^{\pi/2}Ia\,d\theta$$

$$I_a = \int_0^4 p^2\sqrt{16-p^2}\,dp = \int_0^{\frac{\pi}{2}} (4\,\text{sen}(u))^2\sqrt{16-(4\,\text{sen}\,u)^2}\cdot 4\cos u\,du$$

$$I_a = \int_0^{\frac{\pi}{2}} 256\,\text{sen}^2 u\cos^2 u\,du$$

$$I_a = 256\int_0^{\pi/2} \frac{1-\cos(4u)}{8}\,du$$

$$I_a = 256\cdot\frac{1}{8}\cdot\left[\int_0^{\frac{\pi}{2}} 1\,du - \int_0^{\pi/2}\cos(4u)\,du\right]$$

$$I_a = 256\cdot\frac{1}{8}\cdot\left[\frac{\pi}{2}-0\right] = 16\pi$$

$$\Rightarrow \int_0^{\frac{\pi}{2}} 16\pi\,d\theta = 16\pi\,\theta\Big|_0^{\frac{\pi}{2}}$$

$$= 16\pi\cdot\frac{\pi}{2} - 16\pi\cdot 0 = \underline{8\pi^2}$$

$$\therefore \int_0^4\int_0^{\sqrt{16-x^2}}\int_0^{\sqrt{16-x^2-y^2}} \sqrt{x^2+y^2}\,dz\,dy\,dx = \underline{8\pi^2}$$