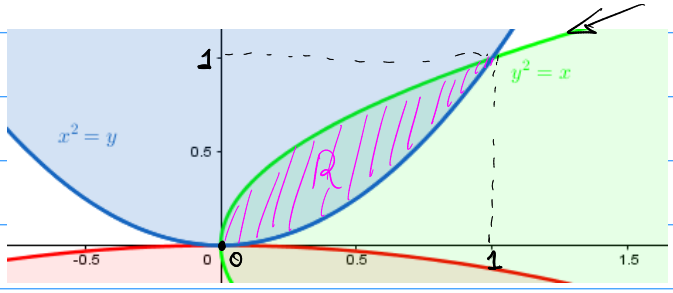
Problema:

Determine el volumen del sólido limitado por el paraboloide $x^2 + 4y^2 = z$, el plano $z = 0$ y los cilindros $y^2 = x$ e $x^2 = y$.

$$V = \iint_R f(x,y)\, dA$$

Graficamos la proyeccion en el plano XY

$y = x^2$



$y^2 = x$

$y = +\sqrt{x}$

Determinamos los limites de la region R en XY

$$\begin{cases} 0 \le x \le 1 \\ x^2 \le y \le \sqrt{x} \end{cases}$$

Volumen $\iint_R f_{techo} - f_{piso}\, dA$

$f_{Techo} \Rightarrow z = x^2 + 4y^2$

$f_{piso} \Rightarrow z = 0$

$$V = \int_0^1 \int_{x^2}^{\sqrt{x}} x^2 + 4y^2 \, dy\, dx = \int_0^1 \left[x^2 y + \frac{4}{3} y^3 \right]_{x^2}^{\sqrt{x}} dx$$

$$V = \int_0^1 \left[x^2\sqrt{x} + \frac{4}{3}(\sqrt{x})^3 - x^2 \cdot x^2 - \frac{4}{3}(x^2)^3 \right] dx$$

$$V = \int_0^1 \left(x^{\frac{5}{2}} + \frac{4}{3}x^{\frac{3}{2}} - x^4 - \frac{4}{3}x^6 \right) dx = 0{,}4286\ u^3 \quad (\text{unidades} = u)$$