Series de Laurent QUIZ #2

① $f(z) = \dfrac{3z+1}{z^2+4z+3}$ $\qquad 2 < |z+5| < 4$

z^2+4z+3

$z \quad 3$

$z \quad 1$

$(z+3)(z+1)$

$$f(z) = \frac{3z+1}{(z+3)(z+1)} = \frac{A}{z+3} + \frac{B}{z+1}$$

$$\frac{3z+1}{\cancel{(z+3)(z+1)}} = \frac{A(z+1)+B(z+3)}{\cancel{(z+3)(z+1)}}$$

$$3z+1 = Az + A + Bz + 3B$$

$A+B=3 \qquad A=4$

$A+3B=1 \qquad B=-1$

$$f(z) = \frac{4}{z+3} - \frac{1}{z+1}$$

$$\frac{4}{z+3} = \frac{4}{z+5-5+3} = \frac{4}{(z+5)-2} = \frac{4}{(z+5)\left[1-\frac{2}{z+5}\right]}$$

$$= \frac{4}{z+5}\sum_{n=0}^{\infty}\left(\frac{2}{(z+5)}\right)^n$$

$$\frac{4}{z+5}\left[1 + \frac{2}{z+5} + \frac{4}{(z+5)^2} + \frac{8}{(z+5)^3} + \frac{16}{(z+5)^4} + \cdots\right]$$

$$\frac{4}{z+5} + \frac{8}{(z+5)^2} + \frac{16}{(z+5)^3} + \frac{32}{(z+5)^4} + \frac{64}{(z+5)^5} + \cdots$$

$$\frac{1}{z+1} = \frac{1}{z+5-5+1} = \frac{1}{(z+5)-4} = \frac{1}{4\left[\frac{z+5}{4}-1\right]} =$$

$$-\frac{1}{4} \cdot \frac{1}{\left[1-\left(\frac{z+5}{4}\right)\right]} = -\frac{1}{4} \cdot \sum_{n=0}^{\infty} \left(\frac{z+5}{4}\right)^n$$

$$-\frac{1}{4}\left[1 + \frac{z+5}{4} + \frac{(z+5)^2}{16} + \frac{(z+5)^3}{64} + \frac{(z+5)^4}{256} + \cdots\right]$$

$$-\frac{1}{4} - \frac{z+5}{16} - \frac{(z+5)^2}{64} - \frac{(z+5)^3}{256} - \frac{(z+5)^4}{1024} + \cdots$$

$$f(z) = \frac{4}{z+3} - \frac{1}{z+1}$$

le cambia signo

$$f(z) = \underbrace{\frac{4}{z+5} + \frac{8}{(z+5)^2} + \frac{16}{(z+5)^3} + \frac{32}{(z+5)^4}}_{\text{Parte principal}} + \cdots + \frac{1}{4} + \frac{(z+5)}{16}$$

$$\underbrace{+ \frac{(z+5)^2}{16} + \frac{(z+5)^3}{256} + \frac{(z+5)^4}{1024} + \cdots}_{\text{Parte analítica}}$$

$$\boxed{a_{-1} = 4}$$

② $f(z) = \dfrac{z+5}{z^2+z-2}$ $\qquad 1 < |z+3| < 4$

z^2+z-2

$$\begin{array}{ccc} z & & 2 \\ & \times & \\ z & & -1 \end{array}$$

$(z+2)(z-1)$

$$f(z) = \frac{z+5}{(z+2)(z-1)} = \frac{A}{(z+2)} + \frac{B}{(z-1)}$$

$$\frac{z+5}{\cancel{(z+2)(z-1)}} = \frac{A(z-1)+B(z+2)}{\cancel{(z+2)(z-1)}}$$

$$z+5 = Az - A + Bz + 2B$$

$A+B=1 \qquad A=-1$

$-A+2B=5 \qquad B=2$

$$f(z) = \frac{-1}{(z+2)} + \frac{2}{(z-1)}$$

$$\frac{-1}{(z+2)} = -\frac{1}{(z+3-3+2)} = \frac{-1}{(z+3-1)} = \frac{-1}{(z+3)\left[1-\frac{1}{z+3}\right]}$$

$$= \frac{-1}{z+3}\sum_{n=0}^{\infty}\left(\frac{1}{z+3}\right)^n$$

$$= \frac{-1}{z+3}\left[1+\frac{1}{z+3}+\frac{1}{(z+3)^2}+\frac{1}{(z+3)^3}+\frac{1}{(z+3)^4}+\cdots\right]$$

$$= \frac{-1}{z+3}+\frac{1}{(z+3)^2}+\frac{1}{(z+3)^3}+\frac{1}{(z+3)^4}+\frac{1}{(z+3)^5}+\cdots$$

$$\frac{2}{z-1} = \frac{2}{z+3-3-1} = \frac{2}{(z+3)-4} = \frac{2}{4\left[\frac{(z+3)}{4} - 1\right]} =$$

$$-\frac{1}{2} \cdot \frac{1}{\left[1 - \frac{z+3}{4}\right]} = -\frac{1}{2} \cdot \sum_{n=0}^{\infty} \left(\frac{z+3}{4}\right)^n$$

$$-\frac{1}{2}\left[1 + \frac{z+3}{4} + \frac{(z+3)^2}{16} + \frac{(z+3)^3}{64} + \frac{(z+3)^4}{256} + \dots\right]$$

$$-\frac{1}{2} + \frac{z+3}{8} + \frac{(z+3)^2}{32} - \frac{(z+3)^3}{128} - \frac{(z+3)^4}{512} + \dots$$

$$f(z) = -\frac{1}{z+2} + \frac{2}{z-1}$$

$$f(z) = \underbrace{-\frac{1}{z+3} - \frac{1}{(z+3)^2} - \frac{1}{(z+3)^3} - \frac{1}{(z+3)^4}}_{\text{Parte principal}} + \dots -\frac{1}{2} - \frac{z+3}{8} - \frac{(z+3)^2}{32}$$

$$+ \frac{(z+3)^3}{128} - \frac{(z+3)^4}{512} + \dots$$

Parte analítica

$$\boxed{a_{-1} = -1}$$

(3) $f(z) = \dfrac{z^2+2z-1}{z^3-z}$ $\qquad 1 < |z+1| < 2$

$z^3 - z$

$z(z^2-1) = z(z-1)(z+1)$

$$\frac{z^2+2z-1}{z(z-1)(z+1)} = \frac{A}{z} + \frac{B}{z-1} + \frac{C}{z+1}$$

$$= \frac{A(z^2-1) + Bz(z+1) + Cz(z-1)}{z(z-1)(z+1)}$$

$$z^2+2z-1 = Az^2 - A + Bz^2 + Bz + Cz^2 - Cz$$

$A+B+C=1$ $\qquad A=1$

$B-C=2$ $\qquad B=1$

$-A=-1$ $\qquad C=-1$

$$f(z) = \frac{1}{z} + \frac{1}{z-1} - \frac{1}{z+1}$$

ya está en potencias de $(z+1)$ solo se escribe en la serie

$$\frac{1}{z} = \frac{1}{z+1-1} = \frac{1}{(z+1)-1} = \frac{1}{(z+1)\left[1-\frac{1}{z+1}\right]} = \frac{1}{z+1}\sum_{n=0}^{\infty}\left(\frac{1}{z+1}\right)^n$$

$$\frac{1}{z+1}\left[1 + \frac{1}{z+1} + \frac{1}{(z+1)^2} + \frac{1}{(z+1)^3} + \frac{1}{(z+1)^4} + \dots\right]$$

$$\frac{1}{z+1} + \frac{1}{(z+1)^2} + \frac{1}{(z+1)^3} + \frac{1}{(z+1)^4} + \frac{1}{(z+1)^5} + \dots$$

$$\frac{1}{z-1} = \frac{1}{z+1-1-1} = \frac{1}{(z+1)-2} = \frac{1}{2\left[\frac{z+1}{2}-1\right]} =$$

$$-\frac{1}{2}\cdot\frac{1}{1-\frac{z+1}{2}} = -\frac{1}{2}\cdot\sum_{n=0}^{\infty}\left(\frac{z+1}{2}\right)^n$$

$$-\frac{1}{2}\left[1+\frac{z+1}{2}+\frac{(z+1)^2}{4}+\frac{(z+1)^3}{8}+\frac{(z+1)^4}{16}+\dots\right]$$

$$-\frac{1}{2}-\frac{z+1}{4}-\frac{(z+1)^2}{8}-\frac{(z+1)^3}{16}-\frac{(z+1)^4}{32}+\dots$$

$$f(z)=\frac{1}{z}+\frac{1}{z-1}-\frac{1}{z+1}$$

$$f(z)=\cancel{\frac{1}{z+1}}+\frac{1}{(z+1)^2}+\frac{1}{(z+1)^3}+\frac{1}{(z+1)^4}+\dots-\frac{1}{2}-\frac{(z+1)}{4}-\frac{(z+1)^2}{8}-\frac{(z+1)^3}{16}+\dots\cancel{\frac{-1}{z+1}}$$

$$f(z)=\underbrace{\frac{1}{(z+1)^2}+\frac{1}{(z+1)^3}+\frac{1}{(z+1)^4}+\dots}_{\text{Parte principal}}\underbrace{-\frac{1}{2}-\frac{(z+1)}{4}-\frac{(z+1)^2}{8}-\frac{(z+1)^3}{16}+\dots}_{\text{Parte analítica}}$$

$$a_{-1}=0$$