ÉCOLE DES PONTS ET CHAUSSÉES

SUPPLY FUNCTION EQUILIBRIA ON THE ELECTRICITY MARKET

THÈSE

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Chapter 1

Dynamics of the Electricity Day-Ahead Market : Supply Function Equilibria and Ramping Costs

1.1 Introduction

One traditional understanding for the role of markets is that a market allows to aggregate information that would be otherwise dispersed. When applying this view to the electricity markets, the litterature has regarded the existence of the capacity market as a way to aggregate information about sunk costs when building a plant, the existence of the forward market as a way to aggregate information about the fixed costs, and the existence of the spot market as a way to aggregate information about marginal costs. However, the spot market is actually divided into two markets : the day-ahead market and the intraday or balancing market, that the litterature does not differentiate clearly. At the same time, the litterature as well as the industry is well aware of the existence of ramping costs, that is costs that depend on the variation of production. We think that the two spot markets can be understood as aggregating information about marginal costs and ramping costs. In this paper we model the day-ahead market using the supply function equilibria framework and considering that there exists ramping costs. This allows us to obtain unique solutions, in contrast to the usual continuum, and to describe the dynamic evolution of the optimal bid in a symmetric oligopoly. It also opens the possibility to distinguish between day-ahead and intraday markets.

The supply function equilibria litterature has been active since the seminal paper by Klemperer and Meyer in 1989 [Klemperer and Meyer, 1989] (henceforth known as KM). In the wake of the electricity industry liberalisation, authors, notably Green and Newbery in 1992 [Green and Newbery, 1992], have used this framework to evaluate the expected level of competition and argue for certain regulatory paths.

Groundbreaking and fertile, the original model by KM studied how demand uncertainty collapses dramatically the set of available supply function equilibria to a well defined continuum. This continuum collapses further to a single Nash equilibria by considering an infinite support of demand shocks. All of these equilibria are ex-post optimal, meaning that changes in anticipated demand shocks do not impact the actual solutions, but only the parts of the solutions that are actually explored as shocks realize. In this setup markets are always efficient, a very strong result.

On the electricity day ahead markets, producers are generally required to submit supply schedules once a day for all the auctions taking place during the next day. The APX (England) and the EPEX (Austria, France, Germany and Switzerland) markets allow hourly auctions [APX, 2017, EPEX, 2015], and EPEX allows for bids comprising up to 256 price quantity combinations, effectively approximating smooth supply functions. Producers can submit different supply schedules for each individual auction, but every bid must be placed at the same time one day in advance for each block of 24 hours. Customers go through the same process and submit their demand schedules, then the market operator matches supply and demand for each auction. Producers thus have to submit schedules facing uncertain demand, this is the reason for the popularity of supply function equilibria approaches to the electricity market.

However, on this market, bids change from auction to auction. From the point of view of KM's model, this should happen only through a coordination of agents agreeing to collectively swap from one Nash equilibrium to another in the available continuum. Describing these dynamics, however, is increasingly important as the energetic mix is bound to include an increasing fraction of renewables. Power production can be separated in two classes: dispatchable and non-dispatchable technologies. Nuclear, coal, land-fill gas or hydro power generation are mainly dispatchable as one can actually choose their level of production whereas the two rising stars of renewable energy, namely wind and solar, are non-dispatchable: they react to weather conditions. Having these technologies in the mix introduces uncertainty on the production side, which comes down to dispatchable units facing a more uncertain residual demand [Boyle, 2007]. In this paper we want to explore how to model these dynamics.

Electricity production faces very specific technological constraints. These constraints, generally labelled as ramping costs, vary across production technologies, and have yet to be captured in a model. We propose to do so by introducing a multivariate cost function, depending as always on the quantity produced, but also on the rate at which production varies: $C(S, \frac{dS}{dt})$. We call this class of cost functions dynamic cost functions.

All power plants face maintenance costs. However part of these maintenance costs are induced by the dynamics of production, and can be seen as ramping costs. More precisely, whatever the production technology, fluctuations in production are costly. Indeed, they imply fluctuations in the temperature of the core of the power plant, thus dilation and contraction cycles of the different parts, which cause wear and tear. The industry is aware of these effects [GE, 2015], some B2B companies even specialize in minimizing the related long term costs. For example, W‰rtsil‰ Power Plants, a supplier of power plants and tools to forecast long term costs, explains in a white paper [Arima, 2012]:

> Increased variability in net load demand means that dispatchable generating units have to ramp considerably more steeply and deeper than traditionally, thus increasing wear and tear to components.

We are going to model these ramping costs through a dynamic cost function, increasing in the absolute value of its second argument : any change in production is costly. This paper will focus on the implications of considering this type of ramping costs. Other types of ramping costs exist, for example startup costs, but they will not be studied in this paper.

These effects cannot be captured by traditionnal cost functions depending on the level of production alone. One needs to take into account the actual path leading to a given quantity produced. This implies that we need to impose structure on the dynamics of the system while retaining uncertainty, the key ingredient of KM's paper. To do so, we use stochastic dynamics.

This seemingly small addition to KM's framework has a lot of implications on the results obtained. The solutions are not ex-post optimal anymore, allowing to account more satisfactorily for the dynamics of optimal supply schedules, and our solutions are unique, even for bounded demand shocks. We also define a novel selection rule to choose from KM's continuum of equilibria. Finally these results open the possibility to distinguish intraday and day-ahead markets.

In section 1.2 we will present a heuristic approach to get the intuition of the model. Then, in section 1.3 we will introduce the mathematical tools needed to use stochastic dynamics in this context, in section 1.4 and section 1.5 we will solve the monopoly and the symmetric oligopoly cases while considering that producers have information about the overall distribution of shocks during the day, but do not have information about differences in the shocks at different dates. Finally in section 1.6 we will discuss the dynamic variation of the optimal bids, while sections 1.7 and 1.8 will respectively cover a few implications of these results and conclude the paper.

1.2 Heuristic Description of the Model

In this section the essence of the model is presented before introducing the proper mathematical tools needed to treat this problem rigorously in the next section.

As in KM's setup, the producer faces uncertain demand, $D(\theta, p)$, with θ a stochastic shock to the demand and p the price, to which we add ramping costs and uncertain dynamics of demand: $\theta(t)$ is the time trajectory of a stochastic shock with stochastic dynamics. In the real market, bidders submit a finite number of bids once a day, and face the ramping costs inter-period, that is, when production has to be adjusted to reach the subsequent market outcome. Here we assume that time is continuous, that ramping costs are incurred continuously, and that bidders are allowed to submit a different supply schedule for every point in time. This amounts to being asked to submit a surface of strategy in the price-quantity-time space.

The producer maximises her expected profits, and we consider here the simplest case in which the distribution of shocks is static and the producer is asked by the market operator to submit a single supply schedule a day in advance. In an oligopoly the program maximised by producer i is therefore:

$$\max_{S_i(p)} \mathbb{E}\left[\int_0^T \left(p(\theta(t))S_i(p(\theta(t))) - C\left(S_i(p(\theta(t))), \frac{dS_i(p(\theta(t)))}{dt}\right)\right) dt\right] \tag{1.1}$$

with $p(\theta(t))$ the price given the demand shock $\theta(t)$ at date t, $S_i(\cdot)$ the supply schedule of producer i and $C(\cdot,\cdot)$ the dynamic cost function.

The goal of this section is to provide the intuition of the model, therefore we will not describe here the conditions that must be verified by the different terms of the model. We will simply assume that the dynamic cost function is additively separable between a static and a ramping term, $C(S_i, \frac{dS_i}{dt}) = C_s(S_i) + C_r(\frac{dS_i}{dt})$, and that the demand shocks θ are bounded in $[\underline{\theta}, \overline{\theta}]$. Lastly we require the ramping term $C_d(\cdot) = \frac{\gamma}{2}(\cdot)^k$ for clarity, and $k \geq 2$ an integer. We distribute the expectation operator and write that $\frac{dS_i}{dt} = \frac{dS_i}{dp}\frac{dp}{d\theta}\frac{d\theta}{dt} = S_i' \cdot \dot{p} \cdot \frac{d\theta}{dt}$, with X' the derivative of univariate function X with respect to its argument, $\dot{X} = \frac{dX}{d\theta}$. The maximisation program can then be written as follows :

$$\max_{S_i(p)} \int_{\underline{\theta}}^{\overline{\theta}} f(\theta)\left(p(\theta)S_i(p(\theta(t))) - C_s(S_i(p(\theta(t)))) - \frac{\gamma}{2}\left(S_i' \cdot \dot{p}\right)^k \mathbb{E}\left[\left(\frac{d\theta}{dt}\right)^k \middle| \theta\right]\right) d\theta \tag{1.2}$$

with $f(\theta)$ the distribution of shocks, and γ the ramping cost parameter capturing the magnitude of the ramping costs.

Note that $\mathbb{E}\left[\left(\frac{d\theta}{dt}\right)^k \middle| \theta\right]$ depends only on θ, and that producer i faces a residual demand so that $S_i(p(\theta(t))) = D(\theta, p(\theta(t))) - S_{-i}(p(\theta(t)))$ which depends only on θ and p, with S_{-i} the aggregate supply schedule of all the other producers, taken as given by producer i. This implies that the integrand in eq. 1.2 depends only on three variables: θ, p and $\dot{p}$. The maximisation pogram is therefore equivalent to an Euler-Lagrange problem, a very well described mathematical object: $\max_p \int \mathcal{L}(\theta, p, \dot{p}) d\theta$.

The information obtained from taking the first-order condition of an Euler-Lagrange problem yields a second order differential equation as well as two boundary conditions: $\frac{\partial \mathcal{L}}{\partial p} = \frac{d}{d\theta}\frac{\partial \mathcal{L}}{\partial \dot{p}}$ and $\frac{\partial \mathcal{L}}{\partial \dot{p}}\Big|_{\underline{\theta}} = \frac{\partial \mathcal{L}}{\partial \dot{p}}\Big|_{\overline{\theta}} = 0$. This is why we obtain unique solutions: if the boundary conditions are not verified there exists profitable deviations. In less mathematical terms, taking ramping costs into account as specified above means that for a given level of shock, one not only cares about the optimal level of production for this shock, but also about the optimal slope of the supply schedule at this level of production. Effectively, this means that optimal levels of production cannot be chosen independently for different level of shocks anymore, thus shrinking the continuum of equilibria. The boundary conditions' argument explains why the continuum not only shrinks, but collapses to a unique equilibrium.

Note that if the ramping cost parameter γ is taken equal to 0 we are back to KM's model: one doesn't care about the slope of the supply schedule anymore, and the problem comes down to a pointwise maximisation which therefore yields ex-post optimal equilibria. We want to stress that this means that it is not sufficient to specify the dynamics of the shocks to obtain a dynamic model, one needs to take into account ramping costs.

The maximisation program 1.2 is a heuristic description of the situation. We want to model the stochastic nature of demand and of its dynamics. We do this by using Itō processes, a class of stochastic processes built through brownians, to describe the stochastic trajectory of the demand shocks with respect to time. The difficulty is that brownians are everywhere continuous but nowhere differentiable, therefore the way program 1.2 is written, with a term in $\frac{d\theta}{dt}$, is incorrect.

In the next section we introduce the stochastic dynamics properly without using the concept of derivative.

1.3 Stochastic Dynamics

As described in the previous section, we consider that bidders submit surfaces, that is supply schedules for every point in time. The reason to describe a discrete dynamic market as a continuous one is that although discrete time is conceptually more easily understood, continuous time allows to use much more powerful mathematical tools and to obtain closed form solutions, which we think are crucial in gaining intuitive insights about these dynamics. Therefore we consider that demand fluctuates continuously and that ramping costs are incurred instantaneously. This approximation would need to be tested, although it should be noted that day ahead markets operate with hourly or half-hourly periods and are therefore facing a reasonnable amount of periods each day.

Our strategy to describe a discrete process as a continuous one is as follows. We approximate the stochastic process (continuous in time) that we want to use to describe the market by a random walk (discrete in time), and then decrease its timescale towards 0 to converge towards the stochastic process. We are first going to describe the target stochastic process that we will be using throughout this paper, before illustrating how we go from a discrete description to a continuous one.

In the electricity market, demand shocks, denoted $\theta(t)$, are bounded: there are no days for which demand is null nor are there days for which demand tends towards infinity. The structure to be imposed on the dynamics of the shocks has to imply bounded shocks.

1.3.1 The stochastic process

A simple Itō process one can consider that leads to bounded shocks is defined by the following stochastic differential equation (SDE):

$$d\theta(t) = -2\theta(t)dt + \sqrt{1-\theta(t)^2}dB_t \tag{1.3}$$

with B_t a brownian and dX an infinitesimal variation of quantity X.

Observe that this SDE is formed by a deterministic mean-returning term $-2\theta(t)dt$ and a bounded stochastic one $\sqrt{1-\theta(t)^2}dB_t$. As $\theta(t)$ approaches -1 or 1 the stochastic term goes to 0, thus $\theta(t) \in [-1, 1]$. Without loss of generality we can restrain ourselves to this special case. Other bounded supports, $\theta \in [\underline{\theta}, \overline{\theta}]$, can be captured through renormalisations of θ.

Such a stochastic process has a distribution of probability $f(\theta)$ given by Fokker-Planck's equation, easily solved here. In the general case of an Itō process given by SDE 1.4, one obtains in 1.5 the generic Fokker-Planck equation for its distribution of probability $f(\theta, t)$:

$$d\theta = \mu(\theta, t)dt + \sigma(\theta, t)dB_t \tag{1.4}$$

$$\frac{\partial}{\partial t}f(\theta, t) = \frac{\partial}{\partial \theta}(\mu(\theta, t)f(\theta, t)) + \frac{1}{2}\frac{\partial^2}{\partial \theta^2}(\sigma(\theta, t)^2 f(\theta, t)) \tag{1.5}$$

Here, for SDE 1.3, this yields that $f(\theta) = \frac{3}{4}(1-\theta^2)$ on $[-1, 1]$ and 0 elsewhere.

1.3.2 The ramping costs

In the rest of the paper we are going to consider quadratic ramping costs. More precisely we consider the costs induced by fluctuations in the production level. As described in the introduction, fluctuations imply increased wear and tear, whether the production is increasing or decreasing. In addition, these ramping costs are null in the absence of fluctuations. This means that they can be captured by a function $C_r(\cdot)$ verifying $C_r(0) = 0$, $C_r(\cdot) \geq 0$ and increasing in the absolute value of its argument. In the abscence of more detailed knowledge about the actual shape of these ramping costs, it seems reasonable to consider a quadratic cost function, that is the first term in a Taylor expansion of the actual real ramping cost function.

We cannot compute $\frac{d\theta}{dt}$ as it appears in Eq. 1.2, as a stochastic process, although everywhere continuous, is nowhere differentiable. We are therefore going to consider a random walk of timestep Δt which converges towards the Itō process 1.4, using the Euler-Maruyama approximation, a generalisation of the Euler method to stochastic differential equations. We consider a Markov chain Y defined as follows :

$$\Delta Y_n = Y_{n+1} - Y_n = \mu(Y_n, n\Delta t)\Delta t + \sigma(Y_n, n\Delta t)\Delta B_n \tag{1.6}$$

where $\Delta B_n = B_{(n+1)\Delta t} - B_{n\Delta t}$. These ΔB_n are *i.i.d.* normal random variables of mean 0 and variance Δt. Note that as Δt is taken towards 0, this Markov chain converges towards its underlying stochastic process defined by eq.(1.4).

The ramping costs are taken as quadratic in the variation of the production, and also depend on a ramping cost parameter $\Gamma(\Delta t)$, that is the cost per unit of quadratic variation at horizon Δt, so we compute the following quantity :

$$\mathbb{E}\left[\frac{\Gamma(\Delta t)}{2} \cdot \left(\frac{Y_{n+1} - Y_n}{\Delta t}\right)^2 \middle| Y_n\right] = \frac{\Gamma(\Delta t)}{2} \cdot \frac{\sigma(Y_n, n\Delta t)^2}{\Delta t} \tag{1.7}$$

For this quantity to converge to a finite value when the Markov chain is taken towards its underlying stochastic process we have to consider that for small enough timescales, the ramping cost parameter $\Gamma(\Delta t)$ is linear in Δt, i.e. $\Gamma(\Delta t) = \gamma\Delta t + o(\Delta t)$. Mathematically, if $\Gamma(\Delta t)$ had a slower than linear relationship at small timescales, the ramping costs would diverge, and if it was faster they would converge to 0. A physical constraint, namely thermal inertia, ensures that the ramping cost parameter does actually behave in this way[1].

Consider for now that the mean function μ and the variance function σ from eq. 1.4 do not depend on time explicitly and are therefore written $\mu(\theta)$ and $\sigma(\theta)$. Consider now

[1]Ramping costs come from thermal fluctuations in the core of the plant. Therefore we have to describe how temperature responds to fluctuations in production. Thermal inertia acts as a low pass filter, meaning that it smoothes out fluctuations on short timescales. Think about heating a saucepan full of water: although lighting the stove is almost instantaneous, the temperature of the water being heated increases only progressively, in an exponential fashion that is therefore linear in time for short timescales.

a transformation $T(\cdot)$ that we apply to the Markov chain Y. Then:

$$\mathbb{E}\left[\frac{\Gamma(\Delta t)}{2}\cdot\left(\frac{T(Y_{n+1})-T(Y_n)}{\Delta t}\right)^2\middle|Y_n\right]=\mathbb{E}\left[\frac{\Gamma(\Delta t)}{2}\cdot\left(\frac{T(Y_{n+1})-T(Y_n)}{Y_{n+1}-Y_n}\cdot\frac{Y_{n+1}-Y_n}{\Delta t}\right)^2\middle|Y_n\right] \tag{1.8}$$

And in the limit where the markov process Y converges towards the Itō process θ of equation 1.4:

$$\lim_{\Delta t\to 0}\mathbb{E}\left[\frac{\Gamma(\Delta t)}{2}\cdot\left(\frac{T(Y_{n+1})-T(Y_n)}{\Delta t}\right)^2\middle|Y_n\right]=\frac{\gamma}{2}\cdot T'(\theta(t))^2\cdot\sigma(\theta)^2 \tag{1.9}$$

We apply this result to the problem at hand, that is that we evaluate the ramping costs in the case where the demand shocks are given by eq. 1.3:

$$\lim_{\Delta t\to 0}\mathbb{E}\left[\frac{\Gamma(\Delta t)}{2}\cdot\left(\frac{\Delta S_i(p(\theta(t)))}{\Delta t}\right)^2\middle|\theta(t)\right]=\frac{\gamma}{2}\cdot S_i'(p(\theta(t)))^2\dot{p}(\theta(t))^2(1-\theta^2) \tag{1.10}$$

with X' the derivative of quantity X with respect to its argument and $\dot{X}$ its derivative with respect to θ. Note that we considered here that the variance term $\sigma(\theta)=1-\theta^2$ depends only on θ and not explicitly on t, which in turn implies that the strategy S_i does not depend explicitly on t either.

Let us consider the case where the strategy and the variance depend explicitly on time, and are thus written $S_i(p(\theta(t),t),t)$ and $\sigma(\theta,t)$ respectively. By using a first order expansion as before, the ramping cost function can be approximated as follows:

$$\begin{aligned}\lim_{\Delta t\to 0}\mathbb{E}\left[\frac{\Gamma(\Delta t)}{2}\left(\frac{\Delta S_i(p(\theta(t),t),t)}{\Delta t_c}\right)^2\middle|\theta(t)\right] &= \lim_{\Delta t\to 0}\mathbb{E}\left[\frac{\gamma}{2}(\partial_1 S(p(\theta(t),t),t)\partial_1 p(\theta(t),t))^2\frac{\Delta\theta^2}{\Delta t}+\mathcal{O}(\Delta t)\right]\\ &= \frac{\gamma}{2}(\partial_1 S(p(\theta(t),t),t)\partial_1 p(\theta(t),t))^2\sigma(\theta,t)^2\end{aligned} \tag{1.11}$$

with $\partial_i X$ the partial derivative of quantity X with respect to its i^{th} argument.

Now, we can write down the instantaneous expected value of the profit of producer i if the demand shock is $\theta(t)$, $\pi_i^e(t)$, that is the profit that one expects to obtain when demand is at $\theta(t)$ given the expected value of the ramping costs:

$$\pi_i^e(t)=p(\theta(t),t)S_i(p(\theta(t),t),t)-C_s(S_i(p(\theta(t),t),t))-\frac{\gamma}{2}\partial_1 S_i(p(\theta(t),t),t)^2\partial_1 p(\theta(t),t)^2\sigma(\theta,t)^2 \tag{1.12}$$

Lastly we have to write down the expected profit for a day's worth of submitted strategies. Let us consider that the chosen unit of time is the day. Therefore, the total expected profit Π_i^e writes:

$$\begin{aligned}
\Pi_i^e &= \int_0^1 \mathbb{E}[\pi_i^e(t)]dt \\
&= \int_0^1 \int_{\underline{\theta}}^{\overline{\theta}} f(\theta,t)\Big[p(\theta,t)S_i(p(\theta,t),t) - C_s(S_i(p(\theta),t)) \\
&\quad -\frac{\gamma}{2}\partial_1 S_i(p(\theta,t),t)^2 \partial_1 p(\theta,t)^2 \sigma(\theta,t)^2\Big]\, d\theta dt
\end{aligned} \tag{1.13}$$

1.3.3 Discussion of the approximations

We want a tractable mathematical formulation of the dynamic problem faced by producers on the electricity market. To achieve this we seek to describe the discrete real life problem by an approximated continuous one. We first use two technological facts: fluctuations in production are costly and these costs decrease linearly in time for short timescales. We then rely heavily on first order expansions of the different terms we have to compute.

1.3.4 The maximisation program

Here, we consider that the dynamics of demand shocks are given by eq.(1.3), and that therefore $\sigma(\theta,t)^2 = \sigma(\theta)^2 = (1-\theta^2)$.

We now introduce the different conditions that have to be satisfied by the various terms in this problem. First, on most electricity markets, schedules must be increasing, therefore here we take $S_i'(\cdot) \geq 0$. Second, the aggregate demand is non negative as consumers do not have production facilities at their disposal: $D(\theta(t), p(\theta(t))) = \sum_i S_i(p(\theta(t))) \geq 0$. Last, we consider that the shocks θ are ordered so that the demand is increasing in θ, i.e. $\frac{\partial D}{\partial \theta} \geq 0$, and that the price has to weakly increase with the shocks, i.e. $\dot{p} \geq 0$. Our initial stochastic maximisation program can thus be rewritten as a regular optimal control problem:

$$\max_{S_i(p)} \int_{-1}^{1} f(\theta)\left(p(\theta)S_i(p(\theta)) - C_s(S_i(p(\theta))) - \frac{\gamma}{2}(1-\theta^2)\left(S_i'(p(\theta))\dot{p}(\theta)\right)^2\right) d\theta \tag{1.14}$$

$$\begin{aligned}
s.t. \qquad\qquad & S_i'(\cdot) \geq 0 \\
& \dot{p} \geq 0 \qquad\qquad (1.15) \\
& D(\cdot,\cdot) \geq 0
\end{aligned}$$

$$\tag{1.16}$$

The next section solves this problem for a monopoly.

1.4 The Monopoly

Let us consider that the aggregate demand is linear, that is:

$$D(\theta(t), p(\theta(t))) = a\theta(t) + b - p(\theta(t))$$

with a and b parameters taken to describe any bounded support of shocks given the stochastic dynamics introduced in the previous section for which $\theta \in [-1, 1]$. Here $(a\theta + b) \in [b - a, b + a]$.

In a monopoly situation we have $S = D(\theta(t), p(\theta(t)))$, therefore the constraints reduce to:

$$\dot{p}(\theta) \in [0, a], \text{ and } p(\theta) \leq a\theta + b$$

where $\dot{X}$ corresponds to $\frac{dX}{d\theta}$.

Consider in addition that the static cost function is also quadratic: $C_s(S_i) = \frac{\lambda}{2}S_i^2$. The maximisation program is rewritten as:

$$\max_{p(\cdot)} \int_{-1}^{1} f(\theta) \left(p(\theta)(a\theta + b - p(\theta)) - \frac{\lambda}{2}(a\theta + b - p(\theta))^2 - \frac{\gamma}{2}(1 - \theta^2)\left(a - \dot{p}(\theta)\right)^2 \right) d\theta \tag{1.17}$$

$$\begin{aligned} s.t. \qquad\qquad & \dot{p}(\theta) \in [0, a] \\ & p(\theta) \leq a\theta + b \end{aligned}$$

Results

Proposition 1.4.1 *The solution exists, is unique, and has the following form:*

$$\forall \theta \in [-1, 1] \; p^*(\theta) = a\frac{4\gamma + 1 + \lambda}{4\gamma + 2 + \lambda}\theta + b\frac{1 + \lambda}{2 + \lambda} \tag{1.18}$$

The optimal schedule is parametrised by θ so that $S(p(\theta))$ is formed by the points of coordinate $(a\theta + b - p(\theta), p(\theta))$. Its equation is given by:

$$S^*(p) = \frac{1}{4\gamma + 1 + \lambda}\left(p + \frac{4\gamma}{2 + \lambda}b\right) \tag{1.19}$$

Proof See annex 1.A. ■

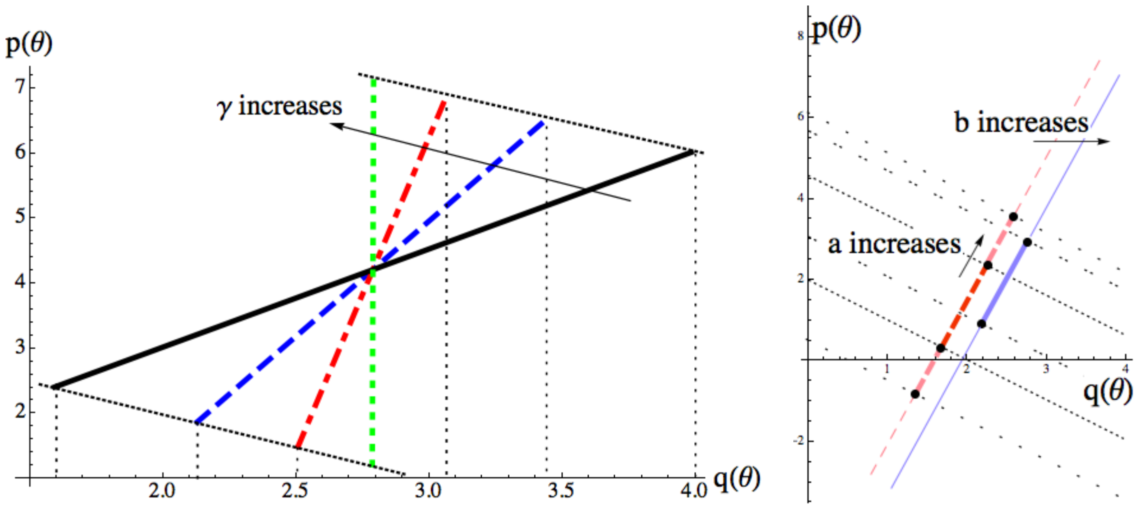
We present in Fig. 1.1(a) the results obtained for increasing values of the ramping cost parameter γ, starting at $\gamma = 0$ in black and moving progressively from black to blue to red to green. The supply schedules are obtained by noting that the optimal solution

As expected, adding these inertial costs narrows down the domain of attainable quantities produced, as a larger quantity domain implies larger incurred dynamic costs.

More interesting is the way the quantity domain is narrowed down. The domain of prices increases conversely, so that the solutions are steeper than the traditional monopoly situation, bringing the schedules ever closer to a Cournot-like situation. In addition, the optimal supply schedules do not depend on a, the parameter determining the width of the possible shocks, but only on b which defines the average value of the shocks.

One can then study the comparative statics when the values for a and b are varied, as illustrated in Fig. 1.1(b). In particular, if we consider an increase in a without changing b, the solution is represented by the same "master" function, but the explored region expands. On the other hand, if we consider a fixed a but an increasing b, the explored length is kept constant, but the optimal schedule is translated towards the north-east region of the plane as expected intuitively: more demand implies a given mix between higher quantities and prices, which is given by the direction of the vector of translation. Note that the independence of the solution on variations of a comes from the fact that we are considering comparative statics, which is very different from dynamically evolving values of a and b, case which will be treated in detail in section 1.6.

Lastly, note that all schedules cross at a single point. These quadratic ramping costs imply a symmetric deformation of the solution without ramping costs. The limit of extremely high ramping costs is a Cournot-like schedule, i.e. a vertical one, taken at this crossing point.



(a) $S^{-1\,*}(p)$ (b) $S^{-1\,*}(p)$

Figure 1.1: **(a)** Four optimal supply schedules are plotted. In black (full line) $\gamma = 0$. As γ increases we transition from the black curve to the blue curve (large dashes), then the red curve (mixed dashes) and then finally for $\gamma \to \infty$ to the green one (small dashes). The range of production is highlighted for each curve through the thin vertical dotted lines.

(b) The thin black dotted lines represent the extremal demand functions given a and b, i.e. $D(\underline{\theta}, p)$ and $D(\overline{\theta}, p)$. From . . . to . . . b is kept fixed while a is increased, and from . . . to . . . a is kept constant while b is increased. In red (dashed) the solution for a given value of b. As a increases, the solution widens from the thick deep red region to the thick light red one. In the case for which a is kept constant and b is increased the solution shifts from the dashed deep red region to the full thick blue one.

1.5 The Symmetric Oligopoly

We keep the same linear demand specification as in the monopoly, therefore, with n competitors one has to consider the residual demand faced by each producer:

$$S(p(\theta)) = a\theta + b - (n-1)S(p(\theta)) - p \tag{1.20}$$

$$S(p(\theta)) = \frac{a\theta + b - p}{n} \tag{1.21}$$

$$S'(p(\theta)) = \frac{a - \dot{p}}{n\dot{p}} \tag{1.22}$$

$$S''(p(\theta)) = -\frac{a\ddot{p}}{n\dot{p}^3} \tag{1.23}$$

For concision, we drop the explicit dependencies of the different functions on their arguments in the following equations; $f(\theta)$, $p(\theta)$ and $S(p(\theta))$ will be noted f, p and S respectively. The maximisation program now writes:

$$\max_{p(\cdot)} \int_{-1}^{1} f\left(p(a\theta+b-p-(n-1)S) - \frac{\lambda}{2}(a\theta+b-p-(n-1)S)^2 - \frac{\gamma}{2}(1-\theta^2)\left(a - \dot{p}(1+(n-1)S')\right)^2\right) d\theta \tag{1.24}$$

$$
\begin{aligned}
s.t. \qquad & \dot{p} \in [0, a] \\
& p \leq a\theta + b
\end{aligned}
$$

with, as before, $\dot{X} = \frac{dX}{d\theta}$ and X' is the derivative of function X with respect to its argument.

Results

Proposition 1.5.1 *The solution exists, is unique, and has the following form:*

$$\forall \theta \in [-1, 1], \; p^*(\theta) = aK_1\theta + bK_2 \tag{1.25}$$

with

$$K_1 = \frac{n\sqrt{(4\gamma + \lambda + n)^2 - 4n + 4} - (4\gamma + \lambda + n)(n-2)}{2(4\gamma + \lambda + 2n)} \tag{1.26}$$

$$K_2 = \frac{\lambda(n-1) + K_1(\lambda + n)}{(\lambda + n)(n-1) + K_1(\lambda + 2n)} \tag{1.27}$$

and the supply schedule has the following expression:

$$S^*(p) = \frac{1}{n}\left(p\left(\frac{1}{K_1} - 1\right) + b\left(1 - \frac{K_2}{K_1}\right)\right) \tag{1.28}$$

Proof See Annex 1.B. ■

We are now going to focus on the graphical representation of these solutions. As in the monopoly case we obtain unique solutions of increasing steepness in the ramping cost parameter γ. When the ramping costs increase, it becomes more and more costly to allow for a large domain of potential quantities to be produced.

The black curve in Fig. 1.2 corresponds to the limit solution when $\gamma \to 0$, for which the problem gets closer to that of KM, i.e. no ramping costs. Note that as long as $\gamma \neq 0$ the solutions are unique, thus our framework does not converge to that of KM.

Proposition 1.5.2 *When $\gamma \to 0$, the solution remains unique and converges towards the linear schedule available in KM's set of solutions, that is the same schedule selected with KM's selection rule obtained when considering an infinite support for the shocks.*

Proof It is straightforward to check that K_1 and K_2 have the same values as KM for $\gamma \to 0$.
More intuitively the argument is as follows. When $\gamma \to 0$, with $\gamma > 0$, we retain a unique

solution although the problem itself converges towards that of KM. We should select an equilibrium present in KM's continuum. When KM take the limiting case of an infinite support of shocks they select a unique equilibrium. In our case we can do the same thing by taking $a \to \infty$. In the limit, our solution being in their set which converges to a unique equilibrium, those two selected equilibria should be equal. Now note that our solution does not depend explicitly on a so that when the support is finite, we still select the same equilibria out of what is now a continuum of equilibria in KM's framework. ■

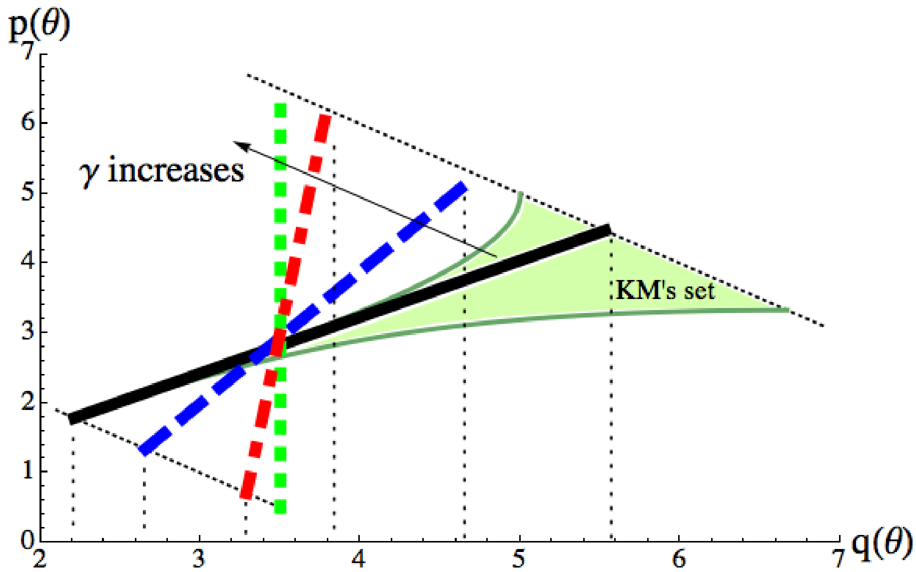


Figure 1.2: This graph plots $S^*(p)$ for different values of the ramping cost parameter, and compares them to the set of equilibria obtained in KM's framework. Four optimal supply schedules are plotted. The black curve (full line) corresponds to the case where $\gamma \to 0$. As before, as γ increases the optimal schedules get steeper and steeper until in the limit of $\gamma \to \infty$, the optimal schedule attains a vertical slope. In addition, we show the set of available equilibria in KM's model in light green, and the extremal demand schedules in dashed black.

Intuitively, as we take γ to 0 we come closer to the situation captured in KM, but as long as $\gamma > 0$, the producer still faces dynamic costs, and therefore converges towards the only linear schedule available in KM's set, as shown in Fig. 1.2, in which we plot our solutions on top of KM's solution set in order to clarify the comparison.

Note that it isn't possible to transition smoothly from our model to that of KM, although they are obviously closely related. Indeed, $\forall \gamma > 0$, our model yields unique solutions, but for $\gamma = 0$ we return to KM's model for which there is a continuum of equilibria. There is an intrinsic discontinuity between these two models, namely, the correspondence $\Gamma(\gamma)$ associating the set of equilibria to the symmetric oligopoly problem obtained for a given value of the ramping cost parameter γ is not lower hemicontinuous at $\gamma = 0$.

In addition to proposing a way to take into account dynamic technological constraints, our model provides a selection rule to choose from the continuum of equilibria described in KM's seminal work, i.e. the solutions' stability to ramping costs.

We have here a model which solutions depend on the distribution of shocks, therefore we are able to capture the interday variation of bids by assuming that the distribution of shocks varies from day to day. In this case, there exists only one symmetric equilibria each day, function of the distribution of shocks.

In the next section we are going to present how to capture richer dynamics, and especially how the surface of bids should evolve with time when the producers have information about the anticipated variation of shocks during the day.

1.6 Dynamic behavior of the bids

On the day-ahead market, bids, although made once per day for each period included in the next 24 hours, vary from one another. This is because demand is expected to vary according to a daily cycle with, roughly speaking, low demand during the night and higher demand during the day. The model described above doesn't account for these hourly dynamics. Here we present a way to capture these intraday variations, by considering bids that depend continuously on the date t.

Previously, the SDE defining the dynamics of the problem was written as:

$$d\theta(t) = -2\theta(t)dt + \sqrt{1-\theta(t)^2}dB_t$$

This specification implies a stochastic trajectory for the shocks, bounded by a constant envelope.

To account for these intraday variations we define the envelope by two functions, $(\underline{\theta}(t), \overline{\theta}(t))$, respectively the lower and upper bounds of the shocks. These two functions, although very easy to comprehend, are not the most useful way to define the boundary. Instead we are going to use the average value of the shocks, and the half width of the envelope, $(\hat{\theta}(t), \omega(t))$. This means that $\underline{\theta}(t) = \hat{\theta}(t) - \omega(t)$ and $\overline{\theta}(t) = \hat{\theta}(t) + \omega(t)$. The only restriction we impose on the envelope is that we require it to be continuously differentiable, that is $(\hat{\theta}(t), \omega(t)) \in \mathcal{C}^1(\mathbb{R})$.

Consider the following SDE, in which we drop the explicit dependency of the different functions on time, that is $\theta(t)$, $\hat{\theta}(t)$ and $\omega(t)$ will be noted θ, $\hat{\theta}$ and ω:

$$d\theta = \left[(\hat{\theta} - \omega - \theta) + \left(1 + \frac{\tau}{\omega}\frac{d\omega}{dt_r}\right)(\hat{\theta} + \omega - \theta) + \tau \left(\frac{d\hat{\theta}}{dt_r} - \frac{d\omega}{dt_r} \right) \right] \cdot dt_r + \sqrt{\left(1 + \frac{\tau}{\omega}\frac{d\omega}{dt_r}\right)(\theta - \hat{\theta} + \omega)(\hat{\theta} + \omega - \theta)} \cdot dB_{t_r} \tag{1.29}$$

with τ a rescaling parameter allowing to change the rate at which the brownian process blurs information pertaining an initial condition. This parameter is of the order of the cutoff timescale Δt_c (a few seconds at most). We rescale time as we change this parameter, so that time t and the rescaled time t_r verify $t = \tau t_r$. By assumption, Δt_c is much smaller than the typical timescale of variation of strategies, therefore by hypothesis $\left(1 + \frac{\tau}{\omega}\frac{d\omega}{dt_r}\right) > 0$. The distribution of the shocks can be obtained through F^kker-Planck's equation 1.5 and we obtain:

$$f(\theta, t_r) = \frac{6}{\omega(t_r)^3}(\theta(t_r) - \hat{\theta}(t_r) + \omega(t_r))(\hat{\theta}(t_r) + \omega(t_r) - \theta(t_r))$$

1.6.1 Results

Monopoly dynamics

Proposition 1.6.1 *In the case of an envelope evolving with time, that is shocks belong to the bounded support $[\hat{\theta} - \omega, \hat{\theta} + \omega]$, there exists a unique optimal solution to the monopoly problem. It can be expressed as:*

$$p^* = \frac{4\gamma\left(1 + \frac{\tau}{\omega}\frac{d\omega}{dt}\right) + 1 + \lambda}{4\gamma\left(1 + \frac{\tau}{\omega}\frac{d\omega}{dt}\right) + 2 + \lambda} \cdot \theta - \frac{4\gamma\left(1 + \frac{\tau}{\omega}\frac{d\omega}{dt}\right)}{(2 + \lambda)\left(4\gamma\left(1 + \frac{\tau}{\omega}\frac{d\omega}{dt}\right) + 1 + \lambda\right)} \cdot \hat{\theta} \tag{1.30}$$

The corresponding optimal supply schedule writes as:

$$S^*(p) = \frac{1}{4\gamma\left(1 + \frac{\tau}{\omega}\frac{d\omega}{dt}\right) + 1 + \lambda} \left(p + \frac{4\gamma\left(1 + \frac{\tau}{\omega}\frac{d\omega}{dt}\right)}{2 + \lambda} \cdot \hat{\theta} \right) \tag{1.31}$$

Proof See appendix 1.C, to be written. ■

We start by describing the dynamics of the monopoly case because the oligopoly case is not richer dynamically, but it is more complex to describe. Note that if $\frac{d\omega}{dt} = 0$ equations 1.30 and 1.31 are equal to equations 1.18 and 1.19 respectively as expected.

The optimal supply schedule depends on the relative rate of change of the width $\frac{1}{\omega}\frac{d\omega}{dt}$ and on the average shock $\hat{\theta}$. More precisely, with a constant width, the optimal supply schedule varies according to variations in the expected average value of the shocks. This is quite standard, if demand is higher, the price and quantities both increase, and here this increase occurs with a constant slope. The behavior of the supply schedule when the width varies is less trivial.

Remember that when describing the slope of the schedule, we are considering the plane $(quantity, price)$ while the schedule as defined by $S^*(p)$ represents the same curve but in the plane $(price, quantity)$. An increase in width is equivalent to a higher ramping cost parameter while a decrease in width is equivalent to a lower ramping cost parameter. These results are illustrated in Fig. 1.3.

To understand the economic intuition behind this result, consider first an increase in the width of the envelope at date t_1. Consider now one possible value of $\theta(t_1)$. At $t_1 + dt$, had the width been constant there would have been a given level of uncertainty about the values that $\theta(t_1 + dt)$, and thus the ramping costs, could have taken. If the width of the envelope is increasing then there is more uncertainty regarding the potential values that could be taken by $\theta(t_1 + dt)$, therefore more expected ramping costs incurred, and a higher slope to hedge these costs. On the other hand, when the width decreases, the situation is reversed. In that case, we move towards a situation in which there is less uncertainty about the ramping costs, so that the slope is smaller than for a constant envelope. This difference between increasing and decreasing width is illustrated by comparing the two regions of the envelope displayed in (full) black line in Fig. 1.3. In addition, when contrasting the left and the right side of the figure one sees that the change in the informativeness of the envelope is captured by the relative change of the width: for the same rate of change, if the width is larger (right) then the change in informativeness is smaller (the change in the area captured by the (dashed) red and (full) green arrows).

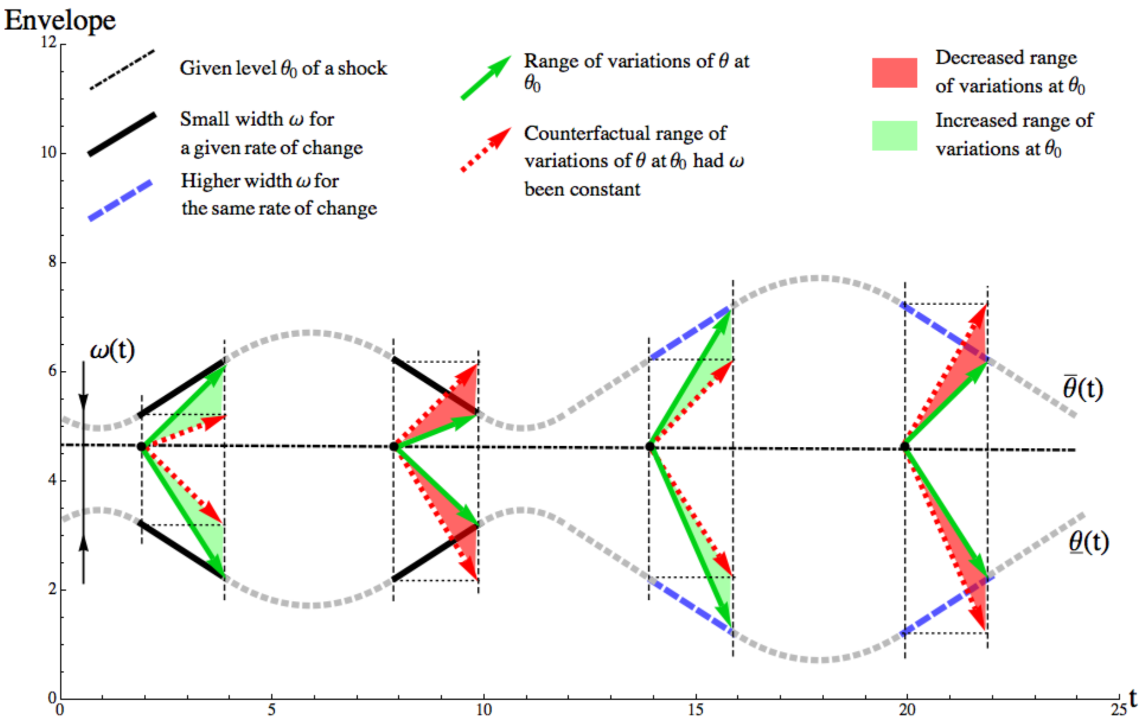


Figure 1.3: This graph plots an envelope of constant average value but varying width $\omega(t)$. By comparing regions of increasing or decreasing width, respectively the left or right side of a lobe, one sees that the informativeness of the envelope is being respectively reduced or increased with respect to a situation where the width would be kept constant. The change in informativeness is represented by the area between the (full) green arrows (observed level of informativeness) compared to the area between the (dashed) red arrows (level of informativeness had the width been constant). In addition, by comparing the left lobe to the right one, it is possible to see why the relative variation of the width, and not the absolute variation of the width, matters. For a larger width (right lobe) and the same rate of change in the width, there is less change in informativeness than for a smaller width (left lobe), i.e. the same rate of change matters less for the right lobe than for the left lobe.

Oligopoly dynamics

Proposition 1.6.2 *The solution exists, is unique, and has the following form:*

$$\forall \theta \in [-1,1],\ \ p^*(\theta) = aK_1(t)\theta + bK_2(t) \tag{1.32}$$

with

$$K_1(t) = \frac{n\sqrt{\left(4\gamma\left(1+\frac{\tau}{\omega}\frac{d\omega}{dt}\right)+\lambda+n\right)^2 - 4n + 4} - \left(4\gamma\left(1+\frac{\tau}{\omega}\frac{d\omega}{dt}\right)+\lambda+n\right)(n-2)}{2\left(4\gamma\left(1+\frac{\tau}{\omega}\frac{d\omega}{dt}\right)+\lambda+2n\right)} \tag{1.33}$$

$$K_2(t) = \frac{\lambda(n-1) + K_1(t)(\lambda+n)}{(\lambda+n)(n-1) + K_1(t)(\lambda+2n)} \tag{1.34}$$

and the supply schedule has the following expression:

$$S^*(p,t) = \frac{1}{n}\left(p\left(\frac{1}{K_1(t)} - 1\right) + \hat{\theta}\left(1 - \frac{K_2(t)}{K_1(t)}\right)\right) \tag{1.35}$$

Proof See Annex 1.D, to be written. ■

The dynamic behavior is the same as that of the monopoly situation presented above, and variations of the optimal schedule with respect to the other parameters are the same as in the case of a constant envelope, as described in section 1.5.

1.7 Discussion

DIFFERENTIATING DAY-AHEAD AND INTRADAY MARKETS.
TRANSITION BETWEEN KM AND THIS MODEL FOR DISCRETE TIME AND RAMPING COSTS: RATE OF CONVERGENCE TOWARDS THE UNIQUE SOLUTION OF THE CONTINUOUS CASE?

1.8 Concluding Remarks

By introducing technological constraints previously neglected we are able to take into account the effects of the dynamics of demand shocks on the supply function framework. We restrict ourselves to linear demand. The optimal supply schedules obtained are unique, and their slope increase with the ramping costs, congruent with the idea that too much variation in production is costly. We also capture the dynamics of the bids themselves.

Although mathematically more demanding than the traditional model by Klemperer and Meyer, we consider that this new model, while conceptually sparing (we only add ramping costs) allows for a richer, more realistic description of the electricity market, and opens new research avenues. It yields precise and testable predictions on the dynamics of the electricity market with tractable functional forms, at least in the linear demand case. In addition, by explicitly modeling the dynamics, our work opens the possibility to explore interactions between intraday and day-ahead markets, markets that were indistinguishible in the previous framework. We are also able to account for negative prices which was impossible in the previous framework. Such negative prices are actually observed, although rarely, on the market: producers prefer to subsidize consumption instead of decreasing it by a lot.

Next, we want to generalize the existence and uniqueness results to non linear demand functions and develop more comprehensive comparative statics. It would also be important to study the robustness of our solutions to modifications of the general form of the stochastic differential equation governing the dynamics of the system. Finally, and more generally, we think that this concept of dynamic costs, the fact that change is costly, is ubiquitous and could fuel interesting research into the dynamics of a large range of markets. Such avenues have been pursued in the case of stochastic optimal control, that is, instantaneous reactions to stochastic shocks. Here we are describing a market on which agents are forced to optimize in advance, so that they have to react to continuous changes in the anticipated shocks, but not the shocks themselves, which can be understood as stochastic optimisation with periodic commitment.

Appendix

Appendix 1.A Proof of Proposition 1.4.1

Define the following Hamiltonian:

$$\begin{aligned} H(p(\theta),\dot{p}(\theta),\mu(\theta),\theta) = f(\theta)\Bigg(& p(\theta)(a\theta+b-p(\theta)) - \frac{\lambda}{2}(a\theta+b-p(\theta))^2 \\ & -\frac{\gamma}{2}(1-\theta^2)\left(a-u(\theta)\right)^2\Bigg) + \mu(\theta)u(\theta) \end{aligned} \tag{1.A.1}$$

where $u(\theta)$ is the control variable defined through the following equation of motion: $u(\theta) = \dot{p}(\theta)$, $u(\theta) \in [0,a]$. We do not consider the non-negative demand constraint and will check ex-post that our solution verifies this condition.

Now note that:

$$\forall \theta \in (-1,1), \quad \frac{\partial^2 H}{\partial p^2} = -(2+\lambda)f(\theta) < 0 \tag{1.A.2}$$

$$\frac{\partial^2 H}{\partial u^2} = -\gamma(1-\theta^2)f(\theta) < 0 \tag{1.A.3}$$

The Hamiltonian is therefore strictly concave in $p(\theta)$ and $u(\theta)$. Let $(p^*(\theta), u^*(\theta))$ be an admissible pair to the problem, that is a pair such that $u^*(\theta) = \dot{p}^*(\theta)$. If there exists a continuous and piecewise continuously differentiable function $\mu(\theta)$ such that:

$$\dot{\mu}(\theta) = -\frac{\partial H^*}{\partial p} \tag{1.A.4}$$

$$\mu(-1) = \mu(1) = 0 \qquad \text{in order for prices to be free at the boundaries} \tag{1.A.5}$$

$$\forall (\theta, u) \in [-1,1]\times[0,a], \ \frac{\partial H^*}{\partial u}\left(u^*(\theta) - u\right) \geq 0 \tag{1.A.6}$$

with $\frac{\partial H}{\partial u}^* = \frac{\partial H}{\partial u}(p^*(\theta), u^*(\theta), \mu(\theta), \theta)$, then the Mangasarian sufficiency theorem ensures that $(p^*(\theta), u^*(\theta))$ is the optimal solution [Seierstad and Sydsaeter, 1987, p.105]. Let us check that eq. 1.18 defines the optimal solution.

Equation 1.A.4 defines $\mu(\theta)$ up to a constant. Through direct integration we obtain:

$$\mu(\theta) = 3a\left((2+\lambda)\frac{4\gamma+1+\lambda}{4\gamma+2+\lambda} - 1 - \lambda\right)(2\theta^2 - \theta^4) + const.$$

This expression is symmetric in θ therefore by choosing the adequate value for the constant, we ensure that eq. 1.A.5 is satisfied. The slope of the proposed p^* is in $[0, a]$ therefore eq. 1.A.6 requires $\frac{\partial H}{\partial u}$ to be null.

$$\begin{aligned} &\forall\theta \in [-1,1],\ \frac{\partial H}{\partial u} = 0 \implies \frac{d}{d\theta}\frac{\partial H}{\partial u} = 0 \\ \text{i.e. } &\dot{u}(\theta) = -\frac{4\theta}{1-\theta^2}(a - u(\theta)) - \frac{(1+\lambda)(a\theta+b)}{\gamma(1-\theta^2)} + \frac{(2+\lambda)p(\theta)}{\gamma(1-\theta^2)} \end{aligned} \tag{1.A.7}$$

It is straightforward to see that the proposed solution satisfies this differential equation, thus we know that $\frac{\partial H}{\partial u}$ is a constant and as $\mu(-1) = 0$ it is in fact null. Lastly, we see that $p^*(\theta) \leq a\theta + b$.

The proposed $p^*(\theta)$ therefore defines the unique optimal supply function, i.e. the parametrized curve $(a\theta + b - p^*(\theta), p^*(\theta))$.

Appendix 1.B Proof of Proposition 1.5.1

As for eq. 1.24, for the sake of concision, we do not write the explicit depencies of the different functions on θ, thus $f(\theta)$, $p(\theta)$, $u(\theta)$, $\mu(\theta)$ and $S(p(\theta))$ will be written as f, p, u, μ and S respectively. Define the following Hamiltonian:

$$\begin{aligned} H(p,u,\mu,\theta) = f\Bigg(&p(a\theta+b-p-(n-1)S) - \frac{\lambda}{2}(a\theta+b-p-(n-1)S)^2 \\ &-\frac{\gamma}{2}(1-\theta^2)\left(a - u(1+(n-1)S')\right)^2\Bigg) + \mu u \end{aligned} \tag{1.B.1}$$

where u is the control variable defined through the following equation of motion: $u = \dot{p}$, $u \in [0, a]$. We do not consider the non-negative demand constraint and will check ex-post that our solution verifies this condition.

If a symmetric equilibria exists, eqs. 1.20 through 1.23 imply that the regular conditions for an admissible pair to be optimal write :

$$u = \dot{p} \in [0, a] \tag{1.B.2}$$

$$\partial_u H < 0 \implies u = 0 \tag{1.B.3}$$

$$\partial_u H > 0 \implies u = a \tag{1.B.4}$$

$$\begin{aligned} &\partial_u H = 0 \implies u \in [0, a] \text{ and} \\ \ddot{p} = &-\frac{4\theta(a - \dot{p})}{1 - \theta^2} - \frac{\lambda(a\theta + b - p)}{\gamma(1 - \theta^2)} - n\frac{\dot{p}(a\theta + b - 2p) - a(n-1)p}{\gamma(1 - \theta^2)(a(n-1) + \dot{p})} \end{aligned} \tag{1.B.5}$$

$$\dot{\mu} = -\partial_p H \tag{1.B.6}$$

$$\mu(-1) = \mu(1) = 0 \tag{1.B.7}$$

It is easy to check that $(K_1, K_2) \in (0, 1)$ and that the solution 1.25 solves eq. 1.B.5 subject to the boundary conditions 1.B.7. The supply schedule is therefore also linear, with equation :

$$S(p) = \frac{1}{n}\left(p\left(\frac{1}{K_1} - 1\right) + b\left(1 - \frac{K_2}{K_1}\right)\right) \tag{1.B.8}$$

We can now use the Mangasarian theorem to obtain that our admissible pair is indeed solution, $H(p, u, \mu, \theta)$ being concave in (p, u) for linear supply schedules. However the Mangasarian cannot yield that this solution is unique because for a symmetric equilibria, if supply schedules are modified, the hamiltonian changes alongside and we are faced with a new maximisation program.

To obtain that the solution is unique we are going to show explicitly that no other candidate solution exists.

First, note that :

$$\begin{aligned} \dot{\mu} = -f\Bigg(&\frac{a\theta + b - 2p}{n} - a\frac{(n-1)p}{n\dot{p}} \\ &+\lambda\frac{a\theta + b - p}{n} \cdot \frac{a(n-1) + \dot{p}}{n\dot{p}} - \gamma(1 - \theta^2)(n-1)\frac{a - \dot{p}}{n} \cdot \frac{a\ddot{p}}{n\dot{p}^2}\Bigg) \end{aligned} \tag{1.B.9}$$

If (p^*, u^*) maximises the program then the maximum principle implies that there exists a continuous and piecewise continuously differentiable function μ, as shown in [Seierstad and Sydsaeter, 19 Theorem 2 p.85]. This combined with the above equation implies that $\dot{p} \neq 0$ a.e.

Assume now a solution of the form $\forall \theta \in [-1, 1],\ p = a\theta + \beta$, by injecting this expression in eq. 1.B.9 there is no β such that the boundary conditions 1.B.7 are verified.

In addition:

$$\forall\theta \in (-1,1),\ \frac{\partial^2 H}{\partial u^2} = -f\gamma(1-\theta^2)(1+(n-1)S')^2 < 0 \tag{1.B.10}$$

The Hamiltonian is therefore strictly concave in u and $[0,a]$ is convex. These two properties yield that u^* is continuous, as shown in [Seierstad and Sydsaeter, 1987, Note 2.b. p.86]. We have proved the following result :

Lemma 1.B.1 *For any symmetric equilibrium $\exists A \subseteq [-1,1]$ s.t. A is the union of segments of $[-1,1]$ and $\forall\theta \in A$, $\partial_u H = 0$*

Assume the following hypothesis is true, $H_1 : \exists\theta_c \in (-1,1)$ s.t. $[-1,\theta_c] \subseteq A$, then knowing that $\dot{p} \in \mathcal{C}^0([-1,1],[0,a])$ we can rewrite differential equation 1.B.5 around the value $\theta = -1$ by defining $\theta = -1+\epsilon$ with $\epsilon = o(1)$:

$$\frac{d^2p}{d\epsilon^2} = \frac{C}{\epsilon} + o(1) \text{ with } C \neq 0 \text{ if } p(\theta) \neq aK_1\theta + bK_2 \tag{1.B.11}$$

This means that locally around -1, any solution to eq. 1.B.5 but solution 1.25 diverges. Hypothesis H_1 is therefore wrong and $\exists\theta_c \in (-1,1)$ s.t. $\forall\theta \in [-1,\theta_c]$, $\exists\beta$ s.t. $p(\theta) = a\theta + \beta$.

At θ_c we have $\partial_u H = 0$ and as $\dot{p}$ is continuous, $\dot{p}(\theta_c) = a$. For the solution to be interior we need $\ddot{p}(\theta_c) \leq 0$.

$$\partial_{\dot{p}} H(p,\dot{p},\mu,\theta_c) = 0 \Leftrightarrow \mu(\theta_c) = 0 \tag{1.B.12}$$

$$\ddot{p}(\theta_c) \leq 0 \Leftrightarrow b(1+\lambda) - \beta(n+1+\lambda) \geq na\theta \tag{1.B.13}$$

Straightforward computations show that both conditions are mutually exclusive, therefore there doesn't exist another candidate symmetric equilibria, and our solution is unique.

Lastly, to compute the optimal supply function, we inverse the optimal price in order to get the shock as a function of the price at the equilibrium, and we inject this expression in Eq. 1.21.

Appendix 1.C Proof of proposition 1.6.1

Appendix 1.D Proof of proposition 1.6.2

Chapter 2

Investigating the Impact of Uncertainty on Firms with Dynamic Costs : A Case Study of the French Electricity Market

In the last chapter, we have given some attention to a methodology that allows us to use functional data for reduced form analysis. In this chapter, we focus on the economic questions that can be asked using such a methodology. Specifically, we focus on an investigation of the effect of uncertainty on the behaviour of electricity producers.

There exists a consensus that dynamic costs, also referred to as ramping or adjustment costs, are important on the electricity market[1]. These are the costs incurred by a producer when production varies. The importance of uncertainty for the expectation of dynamic costs is shown in [Bergès and Martimort, 2014]. Uncertainty itself on the electricity market has been studied by [Wolak, 2007]. We focus on two sources of uncertainty for traditional electricity suppliers, namely uncertainty about the realisation of the market demand and uncertainty from the inherently unpredictable meteorological situation (which affects renewables generation). We propose a methodology to measure this uncertainty and its impact on firm strategies on the electricity market.

Electricity as a market is very important in and of itself ($2 trillion in worldwide sales in 2010). It is also a crucial input for many industries; power outages induce very large costs to society ([LaCommare and Eto, 2004], [Reichl et al., 2013]). The electricity market is, however, quite different from the markets for other commodities in a few respects. First, electricity cannot be efficiently stored. As a consequence, electricity markets are high frequency (prices can update down to 15-min intervals) and firm strategies are purer as they are free of stock management considerations.

Second and in addition to non-storability, a generation surplus cannot be disposed of freely[2]. Thus, generation of electricity must always be matched with consumption in real time (modulo a small tolerance). This represents a hard constraint on the market[3] and forces suppliers to be reactive. However, this reactivity is costly as plant operators incur dynamic costs when adjusting production and the larger the adjustment made, the larger the cost. Hence, suppliers face a trade-off between cheap generation of electricity and costly reactivity to the demand realisation. Indeed, no single generation technology exists that satisfies both cheap generation and sufficient reactivity to allow production fluctuations at a reasonable price . Existing generation techniques are either cheap and unresponsive, e.g. nuclear plants, or expensive and flexible, e.g. gas turbines.

Interestingly, we also observe negative prices. In France for example, during the weekend of the 15^{th} June 2013, the price per MWh dropped to -200 €. This contrasts to the yearly average of approx. 45 €/MWh and is generally understood as a sign that subsidising consumption temporarily is cheaper for a supplier than shutting down a plant [EPEX, 2014][4]. The increase of the share of renewable generation in the energy mix

[1] [Anderson and Xu, 2005], [Hobbs, 2001], [Hortacsu and Puller, 2008], [Reguant, 2011], [Sewalt and De Jong, 2003].

[2] The common assumption of free disposal as made in standard microeconomics is violated.

[3] Mismatches between consumption and generation ultimately result in power outages.

[4] "Negative prices are a price signal on the power wholesale market that occurs when a high inflexible

contributes to the occurrence of negative prices on the market. The intermittency of renewables causes large residual demand shocks [EPEX, 2014]. The unreliability of renewable generation also means that more flexible plants (i.e. plants with lower dynamic costs) are required to provide rapid responses to fluctuations in production from renewables [REN21, 2013].

Furthermore, uncertainty arises from the fact that renewable production is a local and dispersed production, but feeds into a national market with a single price. When meteorological conditions change, the geographic production profile also changes. This further complicates the predictability of renewables generation and contributes to the uncertainty that electricity producers face when playing on the electricity market [Meibom et al., 2009].

This paper explores the effect that the absolute level of uncertainty about residual demand has on players' strategies on the electricity market. In the light of the existence of dynamic costs, which are inherent to the production technologies, uncertainty is costly to suppliers [Bergès and Martimort, 2014]. Thus when faced with uncertainty, we expect that electricity producers smooth production volume over time in order to minimise dynamic costs. In a single market interaction with a symmetric oligopoly and linear demand functions this translates to playing a steeper supply function when uncertainty is high. The detailed intuition behind the predictions tested is given in section 2.0.2.

We show that uncertainty does impact supplier strategies. However, this prediction and result only apply locally to the central, flat and linear part of the supply bid function. Towards the high and low volume extremities of the bid functions when capacity constraints start to matter, bid functions become vertical and the effect of uncertainty vanishes. Furthermore, we observe results that indicate that demand-side bidding is also impacted by uncertainty.

We focus on the French one-day ahead market, EPEX Spot. This market is a divisible goods auction and particularly suited for our analysis as we observe data on the full aggregate bid functions for both supply and demand. We introduce the market's auction format and rules in section 2.1. The dataset and its sources are presented in section 2.2. We develop our identification methodology in section **??**. Our empirical strategy relies on the non-parametric, comparable point selection technique presented in chapter 2.2. We reuse the selected points of the previous chapter for our analysis here. We present and interpret the results in section **??**. Finally, we discuss some overarching points in section **??** and conclude in section **??**.

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power generation meets low demand. Inflexible power sources can't be shut down and restarted in a quick and cost-efficient manner. Renewables do count in, as they are dependent from external factors (wind, sun)."

2.0.1 Literature review and contribution

There exists a literature on supply function equilibria initiated by [Klemperer and Meyer, 1989]. In traditional models, firms choose between quantities (Cournot) or prices (Bertrand) as their strategic quantities. In the intermediate case, firms choose a relationship between quantities and prices, namely a supply function. This is the focus of the supply function equilibrium models. A key ingredient of these models is uncertainty.

Supply function equilibrium models are very relevant for the analysis of electricity markets, since many electricity market designs allow firms to submit a price-volume function rather than a specific price or quantity. [Green and Newbery, 1992], [Newbery, 1998] and [Bolle, 1992] have used these models to analyse competition on the electricity markets. These papers have contributed to a broader investigation of the competition on the electricity markets, which has also been looked at from empirical perspectives [Wolfram, 1998, Borenstein et al., 2002]. While those initial papers have focussed on the supply function equilibria of the market, they have abstracted from some technological specificities for the sake for simplification.

One such aspect that we are interested in and that has been the subject of research in recent years is the importance of dynamic costs for electricity production. [Bergès and Martimort, 2014] extend [Klemperer and Meyer, 1989] to derive predictions on firms facing dynamic costs in a supply function oligopoly under uncertainty. They find that when varying production is costly, suppliers take these costs into consideration by submitting steeper functions when facing more uncertainty, in order to limit the range of variation in production. [Reguant, 2011] develops a model and an empirical strategy to measure dynamic costs on the Spanish one-day-ahead electricity market. She finds that "complex bids", which allow firms to minimise dynamic costs by linking production in one time period to production in a subsequent time period, reduce the volatility and the level of prices on the market. Her work is also unique in terms of data availability. By using individual bid functions she is able to produce estimates of start-up and ramping costs per production technology. In order to quantify dynamic costs on the Australian electricity market, [Wolak, 2007] derives a methodology to recover estimates of the parameters of parametric cost functions at the level of the production unit. His identification is based on the assumption that each profit maximising supplier knows the distribution of shocks on the demand function when playing on the market. Uncertainty is thus an explicit ingredient of his paper and he captures two sources of uncertainty in a single index: (i) the uncertainty from not knowing the aggregate supply function served by all other suppliers and (ii) the uncertainty about the realisation of the market demand. The recovered cost functions quantify the cost of varying output. Forward contracts are useful to avoid output variations. By comparing the observed level of forward contracting (assumed to be the profit maximising choice for production variation) with the theoretical minimum cost production pattern, he does not find support for ramping costs.

We contribute to this literature by providing an empirical analysis of the French electricity market. Specifically, we look at the impact of uncertainty on supplier strategies and take this as evidence that dynamic costs matter. Our approach to separate out the uncertainty from market demand expectations and predictability of renewables generation is novel. Both proxies for uncertainty used are new, uncertainty from market demand is inferred from the prediction errors that firms make in a demand estimation and uncertainty from renewable production is computed in a bottom-up approach from local weather forecasts. Instead of opting for a time series regression, we understand all hourly auctions as a cross-sectional dataset and control for the time of the day by using continuous transition variables for daytime periods. Similarly, we control for seasonality using continuous variables rather than dummies. Thereby, we are able to leverage our dataset and increase the sample size for each of our regressions and improve the precision of our estimates.

Furthermore, our work contributes to the empirical literature testing strategic behaviour of market participants. Generally, these studies focus on point-wise analyses for reasons of data availability. Not only does this cause endogeneity problems when the data used is equilibrium data, but also the analysis is restricted to an understanding of the usually observed outcomes of the market. In our setting, we benefit from an interesting dataset in which we observe full aggregate bid functions of players. The functions describe the players' behaviour both in the region where the equilibrium is likely to occur as well as in regions that rarely have an impact on the equilibrium outcome. As such, they provide a much fuller description of the firms' strategies. The additional information contained in the full aggregate bid functions has been used extensively in theoretical work (notably in the supply function equilibria literature mentioned above). However, few papers exploit these full bid functions empirically. For the government bond market, [Préget and Waelbroeck, 2005] and [Özcan, 2004] use a parametric approach to this functional data for a description of the variation of bid functions with respect to exogenous factors and an investigation of the revenue superiority of the uniform or discriminatory multi-unit auction mechanism, respectively. On the electricity market, [Wolfram, 1999] leaves the analysis of equilibrium data to investigate duopoly power of firms on the UK day-ahead spot market. Instead, she uses information from the whole aggregate supply function to investigate the impact of price caps for electricity producers. Using an analysis conditioned on 25 different demand levels, she shows that the introduction of price caps resulted in a counter-clockwise rotation of the aggregate supply function. She relates these results to produce a lower bound on the extent to which firms can increase their prices above marginal costs when regulatory pressure makes it advantageous to do so. Thereby, she contributes empirical evidence for the distorting effects of price caps.

Our work adds to this empirical literature using the information contained in the full bid functions by developing a non-parametric approach which allows to condition our analyses on multiple, representative points of the bid functions. The statistical ingredients rely on [Silverman and Ramsay, 2005] and are detailed in chapter 2.2. Thereby we are able to leverage our dataset, increase the sample size in individual regressions as well as obtain a fuller picture of the effects of exogenous variables on the behaviour of electricity producing firms. We emphasize that out approach allows to overcome structural restrictions underlying previous parametric approaches, e.g. the symmetry of the logistic function used in [Préget and Waelbroeck, 2005].

2.0.2 Theoretical prediction

We test the impact of uncertainty of supplier strategies by testing the prediction that suppliers bid steeper supply bid functions when faced with a larger uncertainty concerning the outcome of the (residual) demand realisation.

In a discontinuous setting, where the supplier produces volume Q_H of electricity in hour H, we assume that he faces a cost function $C_i(.)$ for each production plant i. This cost function depends on both marginal costs of production as well as the dynamic costs for changing production rapidly: $C_i\big((Q_H), (Q_H - Q_{H-1})^2\big)$. The larger the variation in production between hours, the larger the dynamic costs. Even when the expected residual demand is constant, there are still fluctuations in the production due to possible shocks to the residual demand. The larger the shocks, the larger the change in production and thus the larger the dynamic costs. Consequently, increased uncertainty (as represented by shocks on the demand function) translates into increased expected dynamic costs. We assume that the profit maximising supplier knows the distribution of shocks on the demand function when choosing his supply function. In order to minimise these costs, the producer can choose a steeper supply function when uncertainty is high. We want to test this prediction.

We illustrate the intuition behind this prediction using a stylised case in figure 2.0.1. The graphs depict a situation in which a single, risk-neutral supplier bids a supply function to supply electricity in the hours 9 and 10 of the next day. For both hours, the supplier faces a constant expected residual demand function represented by $E(D)$. In a static optimisation problem, the supplier would bid a supply function S_0 in both auctions.

The uncertainty in the market is represented by the width of the envelope of shocks that affect the residual demand function (represented by the arrows on $E(D)$). Thus, in each hour, the residual demand fluctuates between D_{min} and D_{max}, where the range between the extremal demands may vary from one hour to the next.

Before submitting a supply function to the market, the supplier estimates the distribution of probabilities of demand shocks that he will face. In hour 9, the supplier is able to rather precisely predict the realisation of the demand function in the auction, i.e. it

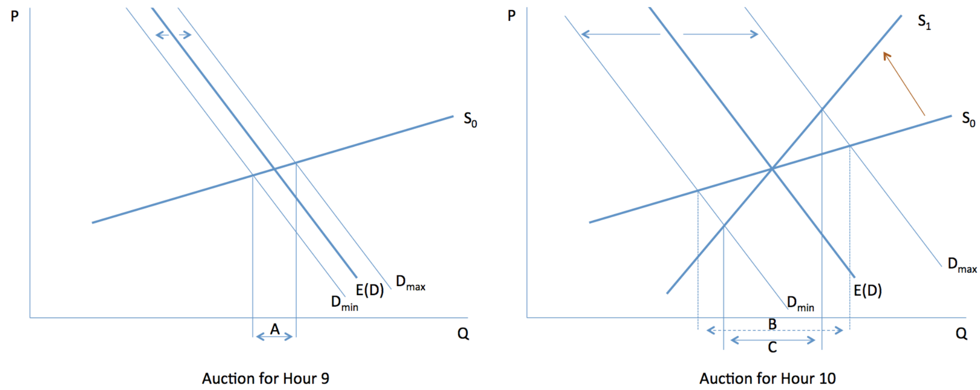

Figure 2.0.1: Illustrating the effect of increased uncertainty.

realises within a tight confidence interval. In hour 10, however, uncertainty in predicting the outcome of the demand realisation has grown strongly as represented by the much wider confidence interval on the demand realisation.

Given a fixed supply bid function S_0, the possible range of quantities to be produced by the supplier when going from hour 9 to hour 10 has increased due to the increase in the size of the uncertainty (interval on the Q-axis has grown from length A for hour 9 to the dotted length B in hour 10).

Now, we assume that the supplier faces dynamic costs, i.e. it is costly for production to vary on top of any traditional marginal cost consideration and the larger the variation, the larger the cost. Then in the case of a fixed supply bid function (S_0 in both auctions), an increase in uncertainty implies an increase in expected dynamic costs.

The supplier's reaction to increased uncertainty is therefore to bid a steeper supply function S_1 in order to trade-off static optimality and dynamic effects. As a consequence, the range of volumes produced in equilibrium is reduced (the firm produces in the range C instead of B). When seen over time, these considerations lead to a smoother production as compared to a constant supply curve: demand shocks are absorbed through a higher price volatility and a lower production volatility.

If cautious behaviour under high uncertainty is true for all firms on the market and each firm has the same expectation of the probability distribution of the uncertainty, then the reaction of bidding a more price inelastic supply function to increased uncertainty should be observable on the aggregated supply function.

We emphasize that this prediction relies on linear demand and supply functions and does not incorporate capacity constraint considerations (both upper and lower bounds on the production volume of plants), which are also important on the market. Furthermore, we have outlined our prediction using a discrete time-setting. The continuous version of this analysis on dynamic costs is explored in detail by [Bergès and Martimort, 2014].

The present paper tests this mechanism empirically and understands an increase in the slope of aggregate supply bid functions due to an increased level of uncertainty as evidence that firms minimise dynamic costs across auctions.

2.1 The EPEX spot market

2.1.1 General background

The EPEX Spot market is an auction market, which allows firms to trade electricity 12-36h ahead of delivery. It covers France, Germany with Austria and Switzerland. The volume traded on Epex Spot represents 12%, 40% and 30% of the total electricity consumption in these countries respectively in 2013 [EPEX, 2014].

The EPEX Spot market has considerably gained in importance over time and the daily trading volume has almost quadrupled since 2005, whereas the total electricity consumption has essentially remained constant. The graph in figure 2.1.1 shows these trends very clearly. Furthermore, it shows the significant volatility of the market trading volume (as indicated by the width of the grey-shaded confidence interval).

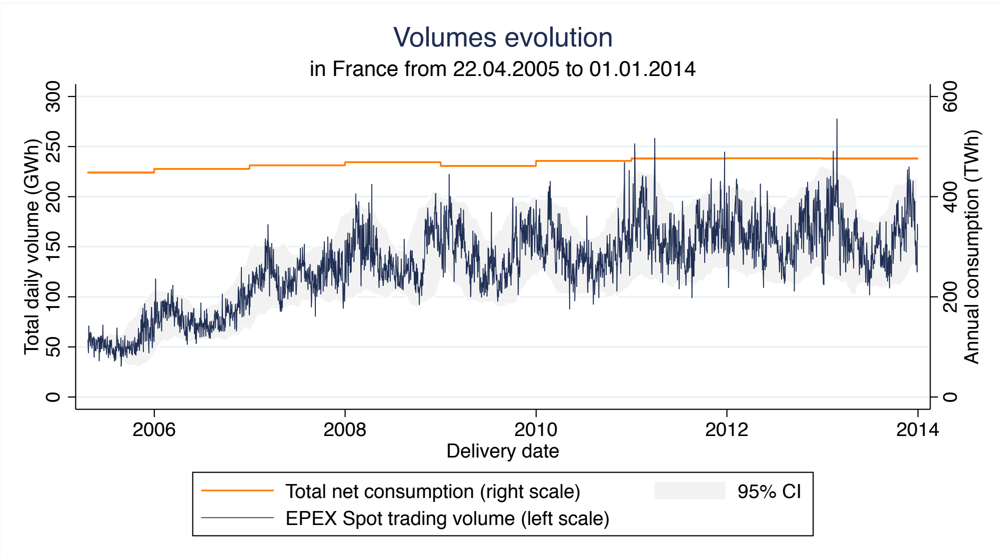

Figure 2.1.1: Traded volume plotted against total annual consumption
Note: Total consumption is netted of the electricity withdrawal at the level of the production unit. The 95% confidence interval is based on a 150-days moving window and assumes that volumes are normally distributed in the time window. GWh and TWh stand for giga and terawatt hours, respectively.

On the EPEX Spot market, the participants submit supply or demand bid functions to be able to meet their next day's supply commitment. This market is important, because it allows the firms to adjust their portfolio to the upcoming demand. The market matches business to business trades, where producers (the suppliers and transmission system operators) and industrial consumers may participate.

The EPEX Spot market settles in a three-pronged market that firms use to achieve their desired power position: The long-term bilateral contracting market, the day-ahead market and the intra-day market. Energy cannot be stored, thus an precise power position must be achieved at each point in time. Firms thus face a trade-off between cheap up-front sourcing and costly uncertainty. The closer the market gets to the delivery of its power, the less uncertainty does the firm face in determining its power requirements (pushing firms to wait until the last minute to fill their energy position). However, the imperfect flexibility of the electricity production landscape cannot satisfy the whole demand short-term at a reasonable price, hence firms must anticipate their requirements in order to obtain cheaper power. Consequently, these three markets complement each other to allow firms to gather a power position at a reasonable price.

2.1.2 Auction rules and mechanism

The EPEX Spot auction occurs daily, all year-round, and proceeds as follows: the order book closes every day at noon for contracts of the following day, results are published two hours later. Bids may be submitted 24/7 from 45 days prior until the closing of the books.

Tradable contracts exist for each hour of the day and firms submit an individual bid function for each of these hours, i.e. a separate, simultaneous auction is run for all hours of the following day and trading is specific for each of these hourly tranches.

The bid submission must be a supply function (or a demand function depending on the position of the firm) with at least 2 and at most 256 price/quantity combinations for single contract orders. The final bid function, thus, consists of the explicitly submitted points and all linearly interpolated points between them. The bid curves must be monotonically increasing for a supply function and vice versa for a demand function. Orders are transmitted via an online IT-platform and a redundant confirmation process aims to avoid erroneous bids. Bids are anonymous and the final electricity distribution is done via the French distribution network controlled by RTE EDF Transport SA.

Prices are specified in €/MWh with two decimal digits and must range from -3000€/MWh to +3000€/MWh. Quantities are specified in whole MWh. In addition to single contract orders for an individual hour, bidders may submit block orders. These are combined single contract orders with a minimum of two consecutive hours. The vital difference with multiple single contract orders is the "All-or-None" condition, namely that the executions of the individual contract orders forming the block are dependent on one another. That is for a block order covering hours 17 to 20, the quantity demanded for the hour 17 is only awarded if the corresponding quantity is also awarded for the hours 18, 19 and 20. Each registered bidder account is limited to a maximum of 40 block orders per delivery day, each of which is limited in volume to 400 MWh (approx. equal to 0.25% of total daily volume traded on EPEX Spot).

The price-quantity determining mechanism is a uniform price, multi-unit auction mechanism: the summed demand and supply curves are computed and the intersection of these gives the equilibrium price and quantity pair. The market clearing mechanism takes into account single and block orders simultaneously and hence solves the corresponding programme by an algorithm of full enumeration of possible solutions, where each partial solution is verified to provide real, compatible prices. The mechanism works under a time limit. In the case of a curtailment, i.e. a disequilibrium with disproportionate prices due to unmatched supply and demand or an abnormal price for a specific hourly contract, the system proceeds to a second price fixing.

Of particular interest is the clear distribution of information. Ex-ante bidding, firms in the market know the identities of the rival bidders they face (but neither their individual bid functions nor their results in past auctions), the history of aggregated equilibrium prices and quantities up to that day, their clients' past demand realisations and their individual long term contracting position. Upon the clearing of the market, the aggregated supply and demand bid functions, equilibrium quantity and the equilibrium price become common knowledge. Each bidding account is informed of the contracts it has been awarded, i.e. the individual quantities to be sold and bought through the system.

2.2 Our data explained

Auction market data

We have data from the French EPEX Spot market for the period 01.01.2011 to 30.06.2013. This is the latest period, where no significant changes in the auction rules have occurred and where data for all variables can be observed.

We observe the full aggregate bid functions for the day-ahead auctions of each hourly contract for both supply and demand. We understand the dataset as a cross-section rather than a time-series[5] and focus on weekday trading contracts only. This sums up to about 31 500 observations[6]. A single aggregate bid function is the sum of the individual bid functions, which are not available. We also observe the equilibrium price and quantity for each auction.

Moreover, we observe the block bidding results at the equilibrium solution only. We ignore the blocked aspects and treat subsequent auctions as independent from one another.

[5]This is supported by the graph in figure 2.1.1, which shows a flat total consumption and average trading volume on EPEX Spot since 01.01.2011.

[6]31 500 observations $\approx$ 2.5 years of hourly ($*365*24$) demand and supply ($*2$) functions for weekday trading ($*5/7$).

The two graphs in figure 2.2.1 show the aggregate supply and demand bid functions for the same hour of the same day. For a glimpse at the variation of bid functions over time, see figure 2.2.2. The table 2.1 sheds some light on the raw data. For further details as well as the plotted distribution of realised market equilibria, refer to appendix ??.

Finally, we reuse the data output from chapter 2.2. Specifically, we reuse the specific points extracted from the aggregate demand and supply bid functions, which are comparable across auctions. Why these points are useful for our analysis is explained in the methodology (section ??).

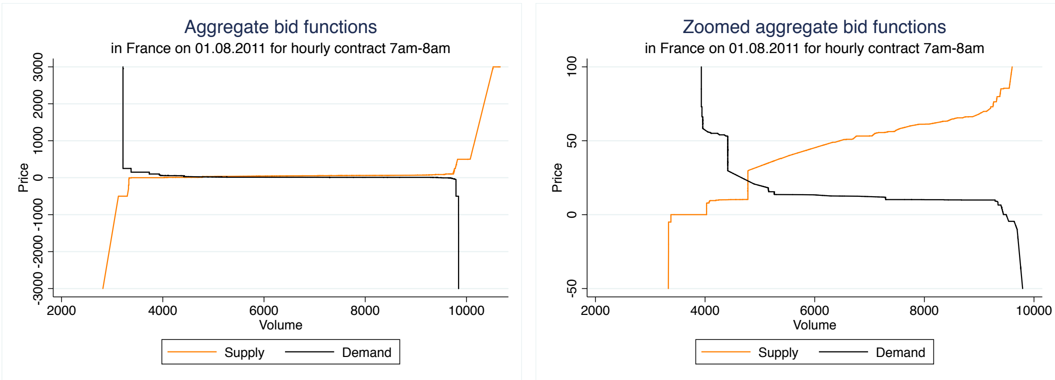

Figure 2.2.1: Example aggregate demand and supply bid functions

Note: The right-hand-side graph is a zoom of the left graph on for the price range −50 €/MWh to +100€/MWh.

	Mean	Median	Std. Dev	Min	Max
Total daily volume	161,912	159,313	25,059	99,054	277,531
Average realised daily price[7]	46.6	48.3	17.2	-39.0	381.2
Minimum demanded agg. volume[8]	5,030	4,968	1,467	914	11301
Maximum demanded agg. volume	13,327	13,222	2,212	4,990	23,254
Minimum supplied agg. volume	3,721	3,526	1,344	618	10594
Maximum supplied agg. volume	14,390	14,142	3,051	6,580	35,356
Bid points per demand function	543	531	163	115	1,253
Bid points per supply function	640	632	143	184	1,283
Bidders per auction[9]	-	-	-	1	101

Table 2.1: Some descriptive statistics

[7] Average price is volume weighted over the 24 hourly contracts of the delivery day.

[8] Minimum and maximum volumes for both demand and supply refer to the aggregate volume bid on the market for a single hour contract at the extremal prices of +3000€/MWh or −3000€/MWh.

[9] Due to the anonymity of the auction procedure, it is unknown which bidders submitted bids. Consequently, it cannot be deduced how many bid steps a typical bidder submits. Number of registered bidders for the French EPEX Spot market as of 01.10.2014.

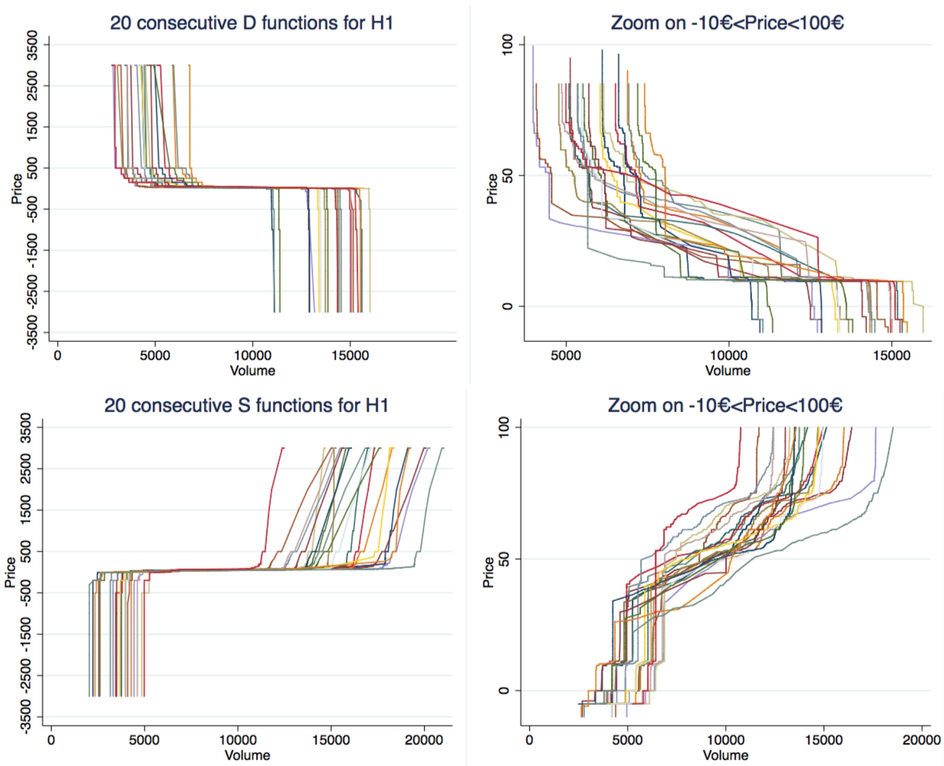

Figure 2.2.2: Aggregate bid functions for 20 consecutive days

Note: The graph shows 20 consecutive aggregate demand and supply functions for the contracts on hour 1 (between 12am and 1am) for the time period 11/12/2011 to 31/12/2011. The graph on the right is a zoom on the price elastic region of the curves on the left.

Exogenous factors

Regarding weather statistics, we have hourly previsions for temperature, wind and cloudiness from the GFS (Global Forecast System) as well as hourly observations for these quantities and luminosity from MétéoFrance . The previsions from the GFS are in the form of weather maps that are outputted from simulations that run one-day ahead at 6 am. This is the weather information that market participants have access to when bidding on EPEX Spot[10]. The weather observations are in the form of tables for specific weather stations (between 100 and 200 depending on the specific parameter of interest).

Moreover, we have the location of the total installed capacity per generation type (i.e. wind turbines, solar panels, etc.) at the level of the postcode, that is roughly a 3km precision. We obtain this data from the SOeS, a branch of the French government producing data on environmental issues at large.

[10]The next weather simulation run takes place at 12 noon, and is therefore not being used by the bidders on the EPEX day-ahead market, as the deadline for submitting bids is precisely 12 noon.

Population data and data on the level of the domestic production from the manufacturing industry is obtained in monthly steps from the French National Institute of Statistics and Economic Studies (INSEE). From the same source, we obtain the spot prices for petrol and natural gas as well as the import prices at the border for coal, which we use as a proxy for the domestic prices. Prices for the European CO2 emission certificates are taken from the Portuguese secondary market (SENDECO$_2$) for European Unit Allowances (EUA)[11].

As a very coarse proxy for generation from hydro power plants, we have the total weekly stock of water in domestic dams (in the form of the summed height of all dam water levels in France) from RTE the grid operator.

2.3 The economic model

2.4 The political model

2.5 Property rights institutions

2.6 Conclusion

Appendix

Appendix 2.A Feasible investments (proof of Prop. n°)

[11]Each unit EUA permit allows one tonne of CO2 emissions.

References

[Anderson and Xu, 2005] Anderson, E. J. and Xu, H. (2005). Supply function equilibrium in electricity spot markets with contracts and price caps. *Journal of Optimization Theory and Applications*, 124(2):257–283.

[APX, 2017] APX (2017). Type of orders on the epex day-ahead auction, uk. https://www.apxgroup.com/trading-clearing/uk-half-hour-day-ahead-1530-auction/.

[Arima, 2012] Arima, K. (2012). Dispatch modelling: Quantifying long term benefits via high resolution analysis. Technical report, Wärtsilä Power Plants.

[Bergès and Martimort, 2014] Bergès, A. and Martimort, D. (2014). Dynamics of the electricity day-ahead market : Supply function equilibria and ramping costs. Technical report, Paris School of Economics.

[Bolle, 1992] Bolle, F. (1992). Supply function equilibria and the danger of tacit collusion: the case of spot markets for electricity. *Energy economics*, 14(2):94–102.

[Borenstein et al., 2002] Borenstein, S., Bushnell, J. B., and Wolak, F. A. (2002). Measuring market inefficiencies in california's restructured wholesale electricity market. *American Economic Review*, 92(5):1376–1405.

[Boyle, 2007] Boyle, G. (2007). *Renewable Electricity and the Grid, the Challenge of Variability*. Routledge.

[EPEX, 2014] EPEX (2014). Website of the european power exchange.

[EPEX, 2015] EPEX (2015). Type of orders on the epex day-ahead auction, france. https://www.epexspot.com/en/product-info/auction.

[GE, 2015] GE (2015). Heavy-duty gas turbine operating and maintenance considerations. Technical report, General Electrics Energy.

[Green and Newbery, 1992] Green, R. and Newbery, D. (1992). Competition in the british electricity spot market. *JPE*.

[Hobbs, 2001] Hobbs, B. F. (2001). *The next generation of electric power unit commitment models*, volume 36. Springer.

[Hortacsu and Puller, 2008] Hortacsu, A. and Puller, S. L. (2008). Understanding strategic bidding in multi-unit auctions: a case study of the texas electricity spot market. *The RAND Journal of Economics*, 39(1):86–114.

[Klemperer and Meyer, 1989] Klemperer, P. and Meyer, M. (1989). Supply function equilibria in oligopoly under uncertainty. *Econometrica*.

[LaCommare and Eto, 2004] LaCommare, K. H. and Eto, J. H. (2004). Understanding the cost of power interruptions to us electricity consumers. *Lawrence Berkeley National Laboratory.*

[Meibom et al., 2009] Meibom, P., Weber, C., Barth, R., and Brand, H. (2009). Operational costs induced by fluctuating wind power production in germany and scandinavia. *Renewable Power Generation, IET*, 3(1):75–83.

[Newbery, 1998] Newbery, D. M. (1998). Competition, contracts, and entry in the electricity spot market. *The RAND Journal of Economics*, pages 726–749.

[Özcan, 2004] Özcan, R. (2004). The logistic function approach to discriminatory and uniform price treasury auctions.

[Préget and Waelbroeck, 2005] Préget, R. and Waelbroeck, P. (2005). Treasury bill auction procedures: Empirical perspectives from french market bid functions. *Journal of International Money and Finance*, 24:1054–1072.

[Reguant, 2011] Reguant, M. (2011). The welfare effects of complementary bidding mechanisms: An application to electricity markets. Technical report, Working paper.

[Reichl et al., 2013] Reichl, J., Schmidthaler, M., and Schneider, F. (2013). Power outage cost evaluation: Reasoning, methods and an application. *Journal of Scientific Research & Reports*, 2(1):249–276.

[REN21, 2013] REN21, S. C. (2013). Renewables 2013, global status report.

[Seierstad and Sydsaeter, 1987] Seierstad, A. and Sydsaeter, K. (1987). *Optimal Control Theory with Economics Applications.* North-Holland.

[Sewalt and De Jong, 2003] Sewalt, M. and De Jong, C. (2003). Negative prices in electricity markets. *Commodities Now*, 2:74–79.

[Silverman and Ramsay, 2005] Silverman, B. and Ramsay, J. (2005). *Functional Data Analysis.* Springer.

[Wolak, 2007] Wolak, F. A. (2007). Quantifying the supply-side benefits from forward contracting in wholesale electricity markets. *Journal of Applied Econometrics*, 22(7):1179–1209.

[Wolfram, 1998] Wolfram, C. D. (1998). Strategic bidding in a multiunit auction: An empirical analysis of bids to supply electricity in england and wales. *The Rand Journal of Economics*, pages 703–725.

[Wolfram, 1999] Wolfram, C. D. (1999). Measuring duopoly power in the british electricity spot market. *American Economic Review*, pages 805–826.

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