

2- AMALIY MASHG'ULOT. Munosabatlar ustida amallar.
Munosabatlar kompozitsiyasi. Binar munosabatlar va ularning matrisalarini
toppish (4 soat)

Reja:

1. Munosabatlar haqida asosiy tushunchalar
2. Mustaqil bajarish uchun masala va topshiriqlar
 - 2.1. Munosabat va Ekvivalent munosabatlarga doir topshiriqlar
 - 2.2. Munosabatlarning aniqlanish sohasi, qiymatlar sohasi, ularni martitsalarda ifodalashga doir topshiriqlar
 - 2.3. Munosabatlar kompozitsiyasiga doir topshiriqlar

1. Munosabatlar haqida asosiy tushunchalar

2.1- Ta'rif 1. Ixtiyoriy A va B to'plamlarning **dekart** yoki **to'g'ri ko'paytmasi** deb, birinchi elementi A to'plamga, ikkinchi elementi B to'plamga tegishli bo'lgan (x, y) tartiblashgan juftliklardan iborat to'plamga aytiladi va quyidagicha belgilanadi: $A \times B = \{ (x, y), x \in A, y \in B \}$.

Bunda x va y lar (x, y) juftlikning **koordinatalari** yoki **komponentlari** deyiladi, demak mos ravishda x juftlikning birinchi koordinatasi, y esa juftlikning ikkinchi koordinatasi deyiladi.

2.1- Misol. Dekart ko'paytmaga misol qilib to'g'ri burchakli dekart koordinata sistemasida nuqtalar to'plamini olish mumkin, ya'ni tekislikda har bir nuqta ikkita koordinataga ega: abssissa va ordinata.

Misol 2. $A = \{a_1, a_2\}$ va $B = \{b_1, b_2, b_3\}$ to'plamlar berilgan bo'lsin. U holda

$$A \times B = \{a_1, a_2\} \times \{b_1, b_2, b_3\} = \{(a_1, b_1), (a_1, b_2), (a_1, b_3), (a_2, b_1), (a_2, b_2), (a_2, b_3)\}$$

2.2.-Ta'rif. $R = A \times B$ dekart ko'paytmaga **to'g'ri dekart ko'paytma**, $R^{-1} = B \times A$ ifodaga **teskari dekart ko'paytma** deyiladi.

Dekart ko'paytmaning xossalari:

1⁰. Dekart ko'paytma kommutativ emas:

$$A \times B \neq B \times A$$

2⁰. Dekart ko'paytma assotsiativ emas:

$$((A \times B) \times C) \neq (A \times (B \times C)).$$

2.3-Ta'rif . $P \subseteq A_1 \times A_2 \times \dots \times A_n$ dekart ko'paytmaning ixtiyoriy bo'sh bo'lmagan P qism to'plamiga A_1, A_2, \dots, A_n to'plamlar orasida aniqlangan n **o'rinli munosabat** yoki n o'rinli P - **predikat** deyiladi.

Agar $(a_1, a_2, \dots, a_n) \in P$ bo'lsa, P munosabat (a_1, a_2, \dots, a_n) elementlar uchun **rost munosabat** deyiladi va $P(a_1, a_2, \dots, a_n) = 1$ bo'ladi, agar $(a_1, a_2, \dots, a_n) \notin P$ bo'lsa, P munosabat **yolg'on munosabat** deyiladi va $P(a_1, a_2, \dots, a_n) = 0$ yoki $\bar{P}(a_1, a_2, \dots, a_n)$ kabi yoziladi.

2.4-Ta'rif. Agar $P \subseteq A_1 \times A_2 \times \dots \times A_n$ n o'rinli munosabatda $n=1$ bo'lsa, P munosabat A_1 to'plamning qism to'plami bo'ladi va **unar munosabat** (bir o'rinli munosabat) yoki **xossa** deyiladi.

$n=2$ bo'lganda esa **binar munosabat** (ikki o'rinli munosabat) yoki **moslik** deyiladi.

Agar $P \subseteq A^2$ bo'lsa, P ga A to'plamning elementlari orasidagi munosabat deyiladi.

2.2-Misol. Unar munosabatlarga misollar keltiramiz:

1) $A_1 = Z$ butun sonlar to'plamidan iborat bo'lsin. $P(x) \subseteq Z$ unar munosabat $P(x)=1$ shart bilan aniqlansin, bunda x – juft son, u holda P munosabat quyidagi ko'rinishda bo'ladi: $P = \{ \dots; -4; -2; 0; 2; 4; \dots \}$.

2) $A_1 = R$ haqiqiy sonlar to'plamidan iborat, $P \subseteq R$ munosabat $P(x)=1$ shart bilan aniqlansin, bunda x – irratsional son bo'lsin, u holda P munosabat quyidagi ko'rinishlarda bo'ladi:

$$P(\sqrt{2}) = P(e) = P(\pi) = 1, \quad P(0) = P(1) = P\left(-\frac{1}{3}\right) = 0.$$

3) A_1 – barcha odamlar to'plami, $P(x) \subseteq A_1$ munosabatda x – erkak kishi bo'lsin. Javob: $P(x)=1$ bo'ladi.

4) A_1 – tekislikdagi barcha uchburchaklar to'plami bo'lsa, x – teng yomli uchburchaklar bo'lsin. Javob: $P(x)=1$ bo'ladi.

2.3-Misol. Binar munosabatlarga misollar keltiramiz:

1) $P_1 \subseteq Z \times Z$ binar munosabat $P(x,y)=1$ shart bilan aniqlansin, bunda $x-y$ 3 ga bo'linadigan sonlar, u holda P munosabat quyidagi ko'rinishda bo'ladi:

$$P=\{(4;1);(5;2); (6;3);...\}.$$

2) $P_2 \subseteq Z \times Z$ munosabat $P(x,y)=1$ shart bilan aniqlansin, bunda $x+y$ 2 ga bo'linadigan sonlar bo'lsin, u holda P munosabat quyidagi ko'rinishlarda bo'ladi:

$$P=\{(1;1);(0;2); (5;3);...\}.$$

3) $P_3 \subseteq R \times R$ munosabat , $P_3(x, y)=1$ shart bilan aniqlansin, bunda $x-y$ ratsional son. U holda quyidagilar o'rinli:

$$P_3(1;4) = P_3(\sqrt{2} + 2; \sqrt{2}) = P_3(e; e-1) = 1,$$

$$P_3(1; \sqrt{2}) = P_3(1; e) = P_3(1; \pi) = 0.$$

$$P_3(\sqrt{2}; \pi) = P_3(e; \pi) = 0$$

4) A – to'plam elementlari kitob nashriyotlari nomlari bo'lsin.

B - to'plam elementlari ushbu kitoblarni sotadigan firmalar bo'lsin, u holda P -munosabatga nashriyot va firmalar o'rtasida tuzilgan shartnomalar to'plami deb, ma'no berish mumkin.

2.5-Ta'rif. Dekart ko'paytmaning ixtiyoriy bo'sh bo'lmagan qism to'plamiga **munosabat** deyiladi.

P -munosabat bo'lsin, u holda $P \subset A \times B$ bo'ladi. $\langle x, y \rangle \in R$ yozuv o'rniga ko'pincha xPy yozishadi va “ x element y ga nisbatan P munosabatda” deb o'qiladi.

2.4-Misol. $A = \{1, 2, 3\}$ va $B = \{1, 2\}$ bo'lsin, u holda

$$A \times B = \{\langle 1, 1 \rangle, \langle 1, 2 \rangle, \langle 2, 1 \rangle, \langle 2, 2 \rangle, \langle 3, 1 \rangle, \langle 3, 2 \rangle\}$$

Munosabat 1) $R_1 = \{\langle 1, 1 \rangle, \langle 3, 2 \rangle\}$

2) $R_2 = \{\langle 1, 1 \rangle, \langle 1, 2 \rangle, \langle 2, 2 \rangle\}$ ko'rinishda bo'lishi mumkin.

2.6-Ta'rif. $P \subseteq A \times B$ binar munosabat uchun $P^{-1} \subseteq B \times A$ **teskari munosabat** deyiladi, agar ixtiyoriy $x \in A$ va $y \in B$ elementlar uchun $P(x, y)=1$ dan $P^{-1}(y, x)=1$ kelib chiqsa.

2.7-Ta'rif . $x = y$ bo'lganda $I_A(x, y) = 1$ shart bajarilsa, $I_A \subseteq A \times A$ binar munosabatga **diagonal munosabat** yoki **ayniy munosabat** deyiladi. Ayniy munosabat uchun $I_A^{-1} = I_A$ tenglik o'rinli.

Binar munosabat, ya'ni moslik haqida alohida to'xtalib o'tamiz, chunki munosabatlar orasida eng ko'p uchraydigani bu moslikdir.

X va Y to'plamlar berilgan bo'lsin.

X va Y to'plamlar elementlarini qandaydir usul bilan mos qo'yib, tartiblangan juftliklarni hosil qilaylik. Agar har bir $x \in X$ element uchun $y \in Y$ element mos qo'yilgan bo'lsa, u holda X va Y to'plamlar o'rtasida **moslik o'rnatildi** deyiladi. Moslikni berish uchun quyidagilarni ko'rsatish zarur:

- 1) elementlari boshqa biror to'plam elementlari bilan mos qo'yiladigan X to'plam;
- 2) elementlari X to'plam elementlari bilan mos qo'yiladigan Y to'plam;
- 3) moslikni aniqlovchi qoida, ya'ni $R \subseteq X \times Y$ to'plam, uning elementlari moslikda qatnashuvchi barcha (x, y) juftliklardan iborat.

Shunday qilib, f moslik $f = \langle X, Y, R \rangle$ to'plamlar uchligidan iborat bo'ladi, bunda $R \subseteq X \times Y$. Agar $(x, y) \in R$ bo'lsa, y element x elementga mos qo'yilgan deyiladi.

2.5-Misol. Laboratoriya xonasida 8 ta laboratoriya qurilmasi bor: $X = \{x_1, x_2, \dots, x_8\}$. Laboratoriya ishini bajarish uchun 10 nafar talaba 5 ta guruhga ajralishdi: $Y = \{y_1, y_2, y_3, y_4, y_5\}$. U holda quyidagicha moslik bo'lishi mumkin:

$f = \{X, Y, (x_1, y_2), (x_2, y_1), (x_3, y_3), (x_5, y_4), (x_8, y_5)\}$, bu yerda (x_1, x_2, \dots, x_8) - moslikning aniqlanish sohasi, $(y_1, y_2, y_3, y_4, y_5)$ - moslikning qiymatlari sohasi bo'ladi.

Moslik 4 xilda bo'ladi:

1. **Birga-bir qiymatli moslik**, bu X va Y to'plamlar elementlari orasidagi shunday moslikki, bunda X ning har bir elementiga Y ning bitta yagona elementi mos qo'yiladi. Masalan, musbat butun sonning kvadrati butun musbat sonning o'zi bilan birga-bir mos qo'yilgan.

2. **Birga-ko'p qiymatli moslik**, bunda X ning bitta elementiga Y danikkita va undan ortiq element mos qo'yilgan bo'ladi.

Masalan, X - butun musbat sonlar to'plami bo'lsin: $X = \{4, 9, 16\}$

Y - X dan olingan kvadrat ildiz bo'lsin: $Y = \{-2, 2, -3, 3, -4, 4\}$.

3. **Ko'pga-bir qiymatli moslik**, bunda Y to'plamning har bir elementiga X to'plamdan bir nechta qiymat mos qo'yiladi. Masalan, imtihon topshiruvchi talabalar to'plami X ga baholar to'plami Y mos qo'yiladi. Bunda har bir talaba bittadab baho oladi, lekin 1 ta baho bir nechta talabaga qo'yiladi.

4. **Ko'pga-ko'p qiymatli moslik**, bunda X to'plamning bitta elementiga Y to'plamdan bir nechta qiymat mos qo'yiladi, shuningdek, Y ning bitta elementiga X dan bir nechta qiymat mos qo'yiladi. Masalan, X - biror qurilmaning bajaruvchi sxemalari, Y - esa elementlar tipi deyish mumkin.

2.6-Misol. Odamlar o'rtasidagi "qarindoshlik" munosabati binar munosabat bo'lib, bu to'plam umumiy ajdodga ega bo'lgan odamlar juftligini o'z ichiga oladi.

Binar munosabatlar 3 xil usulda beriladi:

1. Juftliklarning (sanab o'tilgan) ro'yhati.
2. Matritsa (jadval) orqali.
3. Grafik – struktura ko'rinishida.

$T \subset A \times A$ berilgan bo'lsin, bu yerda $A = \{a_1, a_2, \dots, a_n\}$. U holda, agar a va b orasida T munosabat bo'lsa, C kvadrat matritsaning i -satri va j -ustuni kesishgan joyda joylashgan q element 1 ga teng bo'ladi; aks holda $C_{ij} = 0$.

$$C_{ij} = \begin{cases} 1, & \text{если } (a_i, a_j) \in T \\ 0, & \text{если } (a_i, a_j) \notin T \end{cases}$$

2.7-Misol. $M = \{1, 2, 3, 4, 5\}$ to'plamda aniqlangan

$$T = \{(a, b) : (a - b) - \text{juft son}\}$$

munosabat berilgan bo'lsin. Munosabatni ro'yhat va matritsa bilan bering.

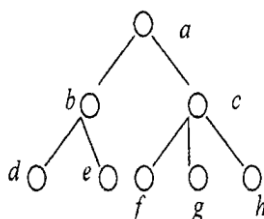
1) $T = \{(1, 1), (1, 3), (1, 5), (2, 2), (2, 4), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (5, 1), (5, 3), (5, 5)\}$.

2) Matritsa ko'rinishi:

T	1	2	3	4	5
1	1	0	1	0	1
2	0	1	0	1	0
3	1	0	1	0	1
4	0	1	0	1	0
5	1	0	1	0	1

yoki $\|T\| = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{pmatrix}$

2.8-Misol. $M = \{a, b, c, d, e, f, g, h\}$ odamlar to'plami bo'lsin va struktura ko'rinishida berilgan bo'lsin.



Quyidagi munosabatlar haqida gapirish mumkin:

a) R_1 – “yaqin o'rtoq bo'lish” munosabati:

$$R_1 = \{(a, b), (a, c), (b, d), (b, e), (c, f), (c, g), (c, h), (b, a),$$

$$(c, a), (d, b), (e, b), (f, c), (g, c), (h, c)\}$$

$$\|R_1\| = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

b) R_2 – “boshliq bo’lish” munosabati:

$$R_2 = \{(a,b), (a,c), (a,d), (a,e), (a,f), (a,g), (a,h), (b,d), (b,e), (c,f), (c,g), (c,h)\}$$

c) R_3 – “ota bo’lish” munosabati:

$$R_3 = \{(a,b), (a,c), (b,d), (b,e), (c,f), (c,g), (c,h)\}.$$

2.9-Misol 10. $A = \{4, 5, 6\}$ va $B = \{1, 2, 3, 4\}$ to’plamlar uchun $U \subseteq A \times B$ va $R \subseteq A \times B$ bo’lgan $U = \{(x, y) : x + y = 8\}$, $R = \{(x, y) : x < y\}$ binar munosabatlarni tuzing.

Yechilishi: $U = \{(4, 4), (5, 3), (6, 2)\}$ va $R = \{(x, y) : x < y\} = \emptyset$.

2. Mustaqil bajarish uchun masala va topshiriqlar

2.1. Munosabat va Ekvivalent munosabatlarga doir topshiriqlar

2.1.1. Birdan farqli natural sonlar to’plami dekart kvadratida aniqlangan $R = \{(x, y) : x \text{ va } y \text{ lar birdan farqli umumiy bo’luvchiga ega}\}$ munosabat ekvivalent munosabat bo’ladimi?

2.1.2. Odamlar o’rtasidagi “yaxshi ko’rish” munosabati ekvivalent munosabat bo’ladimi?

2.1.3. Odamlar o’rtasidagi “qarindoshlik” munosabati ekvivalent munosabat bo’ladimi?

2.1.4. $A = \{a, b, c\}$ to’plam dekart kvadratida Refleksiv bo’lgan, simmetrik, tranzitiv bo’lmagan munosabatga misol keltiring va isbotlang.

2.1.5. $A = \{a, b, c\}$ to’plam dekart kvadratida simmetrik bo’lgan, refleksiv, tranzitiv bo’lmagan munosabatga misol keltiring va isbotlang.

2.1.6. $A = \{a, b, c\}$ to’plam dekart kvadratida tranzitiv bo’lgan, refleksiv, simmetrik bo’lmagan munosabatga misol keltiring va isbotlang.

2.1.7. $A = \{a, b, c\}$ to’plam dekart kvadratida refleksiv, simmetrik bo’lgan, tranzitiv bo’lmagan munosabatga misol keltiring va isbotlang.

2.1.8. $A = \{a, b, c\}$ to’plam dekart kvadratida refleksiv, tranzitiv bo’lgan, simmetrik bo’lmagan munosabatga misol keltiring va isbotlang.

2.1.9. $A = \{a, b, c\}$ to’plam dekart kvadratida simmetrik, tranzitiv bo’lgan, refleksiv bo’lmagan munosabatga misol keltiring va isbotlang.

- 2.1.10.** $A=\{a, b, c\}$ to'plam dekart kvadratida refleksiv, simmetrik, tranzitiv bo'lmagan munosabatga misol keltiring va isbotlang.
- 2.1.11.** $A=\{a, b, c\}$ to'plam dekart kvadratida ekvivalent munosabatga misol keltiring va isbotlang.
- 2.1.12.** $A=\{a, b, c\}$ to'plam dekart kvadratida refleksiv bo'lgan, simmetrik, tranzitiv bo'lmagan munosabatga misol keltiring va isbotlang.
- 2.1.13.** Kutubxonadagi kitoblar to'plamida R munosabat quyidagicha aniqlangan: a va b kitoblar R munosabatga tegishli, agar ushbu kitoblarda bir xil adabiyotlar manbasiga murojaat qilingan bo'lsa. R munosabat 1) Refleksiv munosabat; 2) Simmetrik munosabat; 3) Ekvivalent munosabat bo'ladimi?
- 2.1.14.** Internetda qidirish uchun kalit so'zlar to'plamida R munosabat quyidagicha aniqlansin: a va b kalit so'zlar juftligi R munosabatga tegishli agar ular bir xil simvoldan boshlansa. R munosabat ekvivalent munosabat bo'ladimi?
- 2.1.15.** K -kalit so'zlar, P - web sahifalar to'plami bo'lsin, R munosabat ushbu to'plamlar dekart ko'paytmasida aniqlangan bo'lsin. (x,y) juftlik R munosabatga tegishli bo'lsin, agar x kalit so'z y web-sahifada bo'lsa. R munosabat ekvivalent munosabat bo'ladimi?
- 2.1.16.** $A=\{1,2,3,4\}$ to'plam dekart kvadratida Refleksiv bo'lgan, simmetrik, tranzitiv bo'lmagan munosabatga misol keltiring va isbotlang.
- 2.1.17.** $A=\{1,2,3,4\}$ to'plam dekart kvadratida simmetrik bo'lgan, refleksiv, tranzitiv bo'lmagan munosabatga misol keltiring va isbotlang.
- 2.1.18.** $A=\{1,2,3,4\}$ to'plam dekart kvadratida tranzitiv bo'lgan, refleksiv, simmetrik bo'lmagan munosabatga misol keltiring va isbotlang.
- 2.1.19.** $A=\{1,2,3,4\}$ to'plam dekart kvadratida refleksiv, simmetrik bo'lgan, tranzitiv bo'lmagan munosabatga misol keltiring va isbotlang.
- 2.1.20.** $A=\{1,2,3,4\}$ to'plam dekart kvadratida refleksiv, tranzitiv bo'lgan, simmetrik bo'lmagan munosabatga misol keltiring va isbotlang.
- 2.1.21.** $A=\{1,2,3,4\}$ to'plam dekart kvadratida simmetrik, tranzitiv bo'lgan, refleksiv bo'lmagan munosabatga misol keltiring va isbotlang.

2.1.22. $A=\{1,2,3,4\}$ to'plam dekart kvadratida refleksiv, simmetrik, tranzitiv bo'lmagan munosabatga misol keltiring va isbotlang.

2.1.23. $A=\{1,2,3,4\}$ to'plam dekart kvadratida ekvivalent munosabatga misol keltiring va isbotlang.

2.1.24. $A=\{1,2,3,4\}$ to'plam dekart kvadratida refleksiv bo'lgan, simmetrik, tranzitiv bo'lmagan munosabatga misol keltiring va isbotlang.

2.1. Munosabat va Ekvivalent munosabatlarga doir topshiriq(na'muna)

2.1.0. $A=\{1, 2, 3\}$ to'plamning dekart kvadratida aniqlangan $R=\{(1,1), (2,2), (3,3), (1,2), (2,1)\}$ munosabat ekvivalent munosabat ekanligi isbotlansin.

2.1. Topshiriqning bajarilishi bo'yicha na'muna

2.1.0. Munosabat ekvivalent bo'lishi uchun quyidagi uchta shart bajarilishi lozim:

1. Refleksivlik sharti: $\forall x \in A$ uchun $(x, x) \in R$ (xRx) bo'lsa;

$$1 \in A \Rightarrow (1,1) \in R$$

$$2 \in A \Rightarrow (2,2) \in R$$

$$3 \in A \Rightarrow (3,3) \in R$$

2. Simmetriklik sharti: $\forall (x, y) \in R \Rightarrow (y, x) \in R$;

$$(1,2) \in R \Rightarrow (2,1) \in R;$$

$$(2,1) \in R \Rightarrow (1,2) \in R.$$

3. Tranzitivlik sharti: $(x, y) \in R, (y,z) \in R \Rightarrow (x,z) \in R$.

$$(2,1) \in R, (1,2) \in R \Rightarrow (2,2) \in R$$

$$(1,2) \in R, (2,1) \in R \Rightarrow (1,1) \in R$$

Demak $A=\{1, 2, 3\}$ to'plamning dekart kvadratida aniqlangan $R=\{(1,1), (2,2), (3,3), (1,2), (2,1)\}$ munosabat ekvivalent munosabat bo'ladi.

2.2. Munosabatlarning aniqlanish sohasi, qiymatlar sohasi, ularni martitsalarda ifodalashga doir topshiriqlar

$A=\{a,b,c,d,e\}$, $B=\{1,2,3,4\}$ to'plamlarda quyidagicha munosabatlar berilgan:

$$R_1 \subseteq A \times B \quad \text{и} \quad R_2 \subseteq B \times B = B^2$$

1) R_1, R_2 grafik ko'rinishda ifodalansin, ularning aniqlanish va qiymatlar sohasi topilsin.

2) $R_1, R_2, R_1^{-1}, R_2^{-1}, R_2^2, R_2 \cap R_2^{-1}$ - munosabatlar matritsasi topilsin.

3) R_2 munosabatni refleksivlik, simmetriklik, antisimmetriklik, tranzitivlik xossalari tekshirilsin.

2.2.1.

$$R_1 = \{ \langle a;3 \rangle, \langle b;1 \rangle, \langle b;3 \rangle, \langle c;2 \rangle, \langle c;4 \rangle, \langle d;3 \rangle, \langle e;1 \rangle, \langle e;2 \rangle, \langle e;3 \rangle, \langle e;4 \rangle \},$$

$$R_2 = \{ \langle 1;4 \rangle, \langle 2;1 \rangle, \langle 2;2 \rangle, \langle 2;3 \rangle, \langle 3;2 \rangle, \langle 3;3 \rangle, \langle 4;1 \rangle, \langle 4;3 \rangle \}.$$

2.2.2.

$$R_1 = \{ \langle a;1 \rangle, \langle a;3 \rangle, \langle a;4 \rangle, \langle d;3 \rangle, \langle c;1 \rangle, \langle c;3 \rangle, \langle c;4 \rangle, \langle d;1 \rangle, \langle d;3 \rangle, \langle e;4 \rangle \},$$

$$R_2 = \{ \langle 1;1 \rangle, \langle 1;4 \rangle, \langle 2;1 \rangle, \langle 2;3 \rangle, \langle 3;2 \rangle, \langle 4;1 \rangle, \langle 4;3 \rangle, \langle 4;4 \rangle \}.$$

2.2.3.

$$R_1 = \{ \langle a;1 \rangle, \langle a;3 \rangle, \langle b;1 \rangle, \langle b;3 \rangle, \langle c;1 \rangle, \langle c;3 \rangle, \langle d;3 \rangle, \langle d;4 \rangle, \langle e;2 \rangle, \langle e;4 \rangle \},$$

$$R_2 = \{ \langle 1;1 \rangle, \langle 1;2 \rangle, \langle 1;4 \rangle, \langle 2;3 \rangle, \langle 3;2 \rangle, \langle 3;4 \rangle, \langle 4;1 \rangle, \langle 4;4 \rangle \}.$$

2.2.4

$$R_1 = \{ \langle a;3 \rangle, \langle b;3 \rangle, \langle c;2 \rangle, \langle c;3 \rangle, \langle c;4 \rangle, \langle d;2 \rangle, \langle d;3 \rangle, \langle d;4 \rangle, \langle e;2 \rangle, \langle e;4 \rangle \},$$

$$R_2 = \{ \langle 1;2 \rangle, \langle 1;4 \rangle, \langle 2;1 \rangle, \langle 2;3 \rangle, \langle 3;2 \rangle, \langle 3;4 \rangle, \langle 4;1 \rangle, \langle 4;3 \rangle \}.$$

2.2.5.

$$R_1 = \{ \langle a;3 \rangle, \langle a;4 \rangle, \langle b;2 \rangle, \langle b;3 \rangle, \langle c;2 \rangle, \langle c;3 \rangle, \langle c;4 \rangle, \langle d;3 \rangle, \langle d;2 \rangle, \langle d;4 \rangle \},$$

$$R_2 = \{ \langle 1;3 \rangle, \langle 1;4 \rangle, \langle 2;3 \rangle, \langle 2;4 \rangle, \langle 3;2 \rangle, \langle 3;3 \rangle, \langle 4;1 \rangle, \langle 4;3 \rangle \}.$$

2.2.6.

$$R_1 = \{ \langle a;4 \rangle, \langle b;2 \rangle, \langle b;3 \rangle, \langle b;4 \rangle, \langle c;2 \rangle, \langle c;4 \rangle, \langle d;2 \rangle, \langle d;3 \rangle, \langle d;4 \rangle, \langle e;2 \rangle \},$$

$$R_2 = \{ \langle 1;2 \rangle, \langle 1;4 \rangle, \langle 2;2 \rangle, \langle 2;3 \rangle, \langle 3;1 \rangle, \langle 3;2 \rangle, \langle 4;1 \rangle, \langle 4;2 \rangle \}.$$

2.2.7.

$$R_1 = \{ \langle b;1 \rangle, \langle b;2 \rangle, \langle b;3 \rangle, \langle c;2 \rangle, \langle c;4 \rangle, \langle d;1 \rangle, \langle d;2 \rangle, \langle d;3 \rangle, \langle e;2 \rangle, \langle e;4 \rangle \},$$

$$R_2 = \{ \langle 1;1 \rangle, \langle 1;4 \rangle, \langle 2;1 \rangle, \langle 2;3 \rangle, \langle 3;1 \rangle, \langle 3;2 \rangle, \langle 4;1 \rangle, \langle 4;2 \rangle \}.$$

2.2.8.

$$R_1 = \{ \langle b;1 \rangle, \langle b;2 \rangle, \langle b;4 \rangle, \langle c;1 \rangle, \langle c;2 \rangle, \langle c;4 \rangle, \langle d;2 \rangle, \langle d;3 \rangle, \langle e;2 \rangle, \langle e;3 \rangle \},$$

$$R_2 = \{ \langle 1;4 \rangle, \langle 2;3 \rangle, \langle 2;4 \rangle, \langle 3;2 \rangle, \langle 3;4 \rangle, \langle 4;1 \rangle, \langle 4;3 \rangle, \langle 4;4 \rangle \}.$$

2.2.9.

$$R_1 = \{ \langle a;3 \rangle, \langle b;2 \rangle, \langle b;3 \rangle, \langle c;2 \rangle, \langle c;4 \rangle, \langle d;2 \rangle, \langle d;4 \rangle, \langle e;2 \rangle, \langle e;3 \rangle, \langle e;4 \rangle \},$$

$$R_2 = \{ \langle 1;3 \rangle, \langle 1;4 \rangle, \langle 2;2 \rangle, \langle 2;3 \rangle, \langle 2;4 \rangle, \langle 3;1 \rangle, \langle 3;2 \rangle, \langle 4;1 \rangle \}.$$

2.2.10.

$$R_1 = \{ \langle a;1 \rangle, \langle a;3 \rangle, \langle b;2 \rangle, \langle b;4 \rangle, \langle c;2 \rangle, \langle c;3 \rangle, \langle d;2 \rangle, \langle d;4 \rangle, \langle e;2 \rangle, \langle e;3 \rangle \},$$

$$R_2 = \{ \langle 1;3 \rangle, \langle 1;4 \rangle, \langle 2;3 \rangle, \langle 2;4 \rangle, \langle 3;1 \rangle, \langle 3;2 \rangle, \langle 4;1 \rangle, \langle 4;2 \rangle \}.$$

2.2.11.

$$R_1 = \{ \langle a;1 \rangle, \langle a;3 \rangle, \langle b;2 \rangle, \langle c;1 \rangle, \langle c;2 \rangle, \langle c;3 \rangle, \langle d;2 \rangle, \langle e;1 \rangle, \langle e;2 \rangle, \langle e;3 \rangle \},$$

$$R_2 = \{ \langle 1;1 \rangle, \langle 1;2 \rangle, \langle 2;1 \rangle, \langle 2;2 \rangle, \langle 3;3 \rangle, \langle 3;4 \rangle, \langle 4;3 \rangle, \langle 4;4 \rangle \}.$$

2.2.25.

$$R_1 = \{ \langle a;1 \rangle, \langle a;3 \rangle, \langle b;2 \rangle, \langle b;4 \rangle, \langle c;1 \rangle, \langle c;3 \rangle, \langle d;2 \rangle, \langle d;4 \rangle, \langle e;1 \rangle, \langle e;3 \rangle \},$$

$$R_2 = \{ \langle 1;3 \rangle, \langle 1;4 \rangle, \langle 2;2 \rangle, \langle 2;4 \rangle, \langle 3;1 \rangle, \langle 3;2 \rangle, \langle 4;1 \rangle, \langle 4;2 \rangle \}.$$

2.2. Munosabatlarning aniqlanish sohasi, qiymatlar sohasi, ularni martitsalarda ifodalashga doir topshiriq(na'muna)

$A=\{a,b,c,d,e\}$, $B=\{1,2,3,4\}$ to'plamlarda quyidagicha munosabatlar berilgan:

$$R_1 \subseteq A \times B \quad \text{u} \quad R_2 \subseteq B \times B = B^2$$

1) R_1, R_2 grafik ko'rinishda ifodalansin, ularning aniqlanish va qiymatlar sohasi topilsin.

2) $R_1, R_2, R_1^{-1}, R_2^{-1}, R_2^2, R_2 \cap R_2^{-1}$ - munosabatlar matritsasi topilsin.

3) R_2 munosabatni refleksivlik, simmetriklik, antisimmetriklik, tranzitivlik xossalriga tekshirilsin.

$$R_1 = \{ \langle a;1 \rangle, \langle a;3 \rangle, \langle b;2 \rangle, \langle b;3 \rangle, \langle c;1 \rangle, \langle c;3 \rangle, \langle d;2 \rangle, \langle d;3 \rangle, \langle d;4 \rangle, \langle e;1 \rangle \},$$

$$R_2 = \{ \langle 1;3 \rangle, \langle 1;4 \rangle, \langle 2;2 \rangle, \langle 2;3 \rangle, \langle 2;4 \rangle, \langle 3;2 \rangle, \langle 3;3 \rangle, \langle 4;4 \rangle \}.$$

2.2. Topshiriqning bajarilisi bo'yicha na'muna

$$1) D_l(R_1) = \{a, b, c, d, e\} \quad D_l(R_2) = \{1, 2, 3, 4\}$$

$$D_r(R_1) = \{1, 2, 3, 4\} \quad D_r(R_2) = \{2, 3, 4\}$$

$$2) \text{ Munosabat matritsalar: } [R_1] = \begin{bmatrix} 1010 \\ 0110 \\ 1010 \\ 0111 \\ 1000 \end{bmatrix}, \quad [R_2] = \begin{bmatrix} 0011 \\ 0111 \\ 0110 \\ 0001 \end{bmatrix},$$

$$[R_2^2] = [R_2] \times [R_2],$$

$$[R_2^2] = \begin{bmatrix} 0011 \\ 0111 \\ 0110 \\ 0001 \end{bmatrix} \times \begin{bmatrix} 0011 \\ 0111 \\ 0110 \\ 0001 \end{bmatrix} = \begin{bmatrix} 0111 \\ 0111 \\ 0111 \\ 0001 \end{bmatrix}, \quad [R_1^{-1}] = \begin{bmatrix} 10101 \\ 01010 \\ 11110 \\ 00010 \end{bmatrix}, \quad [R_2^{-1}] = \begin{bmatrix} 0000 \\ 0110 \\ 1110 \\ 1101 \end{bmatrix},$$

$$[R_2 \cap R_2^{-1}] = \begin{bmatrix} 0011 \\ 0111 \\ 0110 \\ 0001 \end{bmatrix} \cap \begin{bmatrix} 0000 \\ 0110 \\ 1110 \\ 1101 \end{bmatrix} = \begin{bmatrix} 0000 \\ 0110 \\ 0110 \\ 0001 \end{bmatrix}$$

$$3) R_2 \text{ refleksiv emas, chunki } [R_2] \neq [E], \text{ bunda } [E] = \begin{bmatrix} 1000 \\ 0100 \\ 0010 \\ 0001 \end{bmatrix}.$$

R_2 simmetrik emas, chunki $[R_2] \neq [R_2^{-1}]$.

R_2 antisimmetrik emas, chunki $[R_2 \cap R_2^{-1}] \not\subseteq [E]$.

R_2 tranzitiv emas, chunki $[R_2^2] \not\subseteq [R_2]$.

2.3. Munosabatlar kompozitsiyasiga doir topshiriqlar

$A=\{a,b,c\}$, $B=\{1,2,3\}$, $C=\{\alpha,\beta,\gamma\}$ to'plamlarda aniqlangan $R_1 \subset A \times B$ va

$R_2 \subset B \times C$ binar munosabatlarning **kopaytmasi** yoki **kompozitsiyasi** topilsin:

- | | |
|--|--|
| 2.3.1. $R_1=\{(a,3),(b,2),(c,1),(c,2)\},$
$R_2=\{(1,\beta),(2,\alpha),(3,\beta),(3,\gamma)\}$ | 2.3.15. $R_1=\{(a,3),(a,2),(a,1)\},$
$R_2=\{(2,\gamma),(1,\alpha),(1,\beta)\}$ |
| 2.3.2. $R_1=\{(a,1),(a,3),(c,1),(c,3)\},$
$R_2=\{(2,\alpha),(2,\gamma),(1,\beta),(3,\alpha)\}$ | 2.3.16. $R_1=\{(a,3),(a,2),(a,1)\},$
$R_2=\{(1,\gamma),(3,\alpha),(1,\beta)\}$ |
| 2.3.3. $R_1=\{(a,2),(b,1),(c,3)\},$
$R_2=\{(1,\beta),(2,\beta),(3,\alpha)\}$ | 2.3.17. $R_1=\{(a,3),(a,2),(a,1)\},$
$R_2=\{(1,\gamma),(1,\alpha),(3,\beta)\}$ |
| 2.3.4. $R_1=\{(a,3),(b,2),(c,1)\},$
$R_2=\{(1,\gamma),(2,\alpha),(3,\alpha)\}$ | 2.3.18. $R_1=\{(a,3),(a,2),(a,1)\},$
$R_2=\{(3,\gamma),(2,\alpha),(2,\beta)\}$ |
| 2.3.5. $R_1=\{(a,2),(b,3),(c,1)\},$
$R_2=\{(1,\gamma),(2,\beta),(3,\alpha)\}$ | 2.3.19. $R_1=\{(a,3),(a,2),(a,1)\},$
$R_2=\{(2,\gamma),(3,\alpha),(2,\beta)\}$ |
| 2.3.6. $R_1=\{(b,3),(b,2),(b,1)\},$
$R_2=\{(2,\gamma),(2,\alpha),(2,\beta)\}$ | 2.3.20. $R_1=\{(a,3),(a,2),(a,1)\},$
$R_2=\{(2,\gamma),(2,\alpha),(3,\beta)\}$ |
| 2.3.7. $R_1=\{(a,1),(a,2),(a,3)\},$
$R_2=\{(3,\gamma),(3,\alpha),(3,\beta)\}$ | 2.3.21. $R_1=\{(b,3),(b,2),(b,1)\},$
$R_2=\{(3,\beta),(1,\alpha),(1,\beta)\}$ |
| 2.3.8. $R_1=\{(c,3),(c,2),(c,1)\},$
$R_2=\{(1,\gamma),(1,\alpha),(2,\beta)\}$ | 2.3.22. $R_1=\{(b,3),(b,2),(b,1)\},$
$R_2=\{(3,\beta),(1,\alpha),(1,\gamma)\}$ |
| 2.3.9. $R_1=\{(c,3),(c,2),(c,1)\},$
$R_2=\{(2,\gamma),(2,\alpha),(2,\beta)\}$ | 2.3.23. $R_1=\{(b,3),(b,2),(b,1)\},$
$R_2=\{(3,\beta),(1,\alpha),(1,\beta)\}$ |
| 2.3.10. $R_1=\{(c,3),(c,2),(c,1)\},$
$R_2=\{(3,\gamma),(3,\alpha),(3,\beta)\}$ | 2.3.24. $R_1=\{(b,3),(b,2),(b,1)\},$
$R_2=\{(3,\beta),(2,\alpha),(2,\beta)\}$ |
| 2.3.11. $R_1=\{(a,3),(a,2),(a,1)\},$
$R_2=\{(1,\gamma),(1,\alpha),(1,\beta)\}$ | 2.3.25. $R_1=\{(b,3),(b,2),(b,1)\},$
$R_2=\{(3,\beta),(2,\alpha),(2,\gamma)\}$ |

$$2.3.12. \quad R_1 = \{(a,3), (a,2), (a,1)\},$$

$$R_2 = \{(2,\gamma), (2,\alpha), (2,\beta)\}$$

$$2.3.13. \quad R_1 = \{(b,3), (b,2), (b,1)\},$$

$$R_2 = \{(1,\gamma), (1,\alpha), (1,\beta)\}$$

$$2.3.14. \quad R_1 = \{(b,3), (b,2), (b,1)\},$$

$$R_2 = \{(3,\gamma), (3,\alpha), (3,\beta)\}$$

$$2.3.26. \quad R_1 = \{(b,3), (b,2), (b,1)\},$$

$$R_2 = \{(2,\beta), (2,\gamma), (3,\alpha)\}$$

$$2.3.27. \quad R_1 = \{(b,3), (b,2), (b,1)\},$$

$$R_2 = \{(3,\beta), (3,\alpha), (2,\gamma)\}$$

$$2.3.28. \quad R_1 = \{(b,3), (b,2), (b,1)\},$$

$$R_2 = \{(1,\beta), (3,\alpha), (3,\gamma)\}$$

$$2.3.29. \quad R_1 = \{(b,3), (b,2), (b,1)\},$$

$$R_2 = \{(3,\beta), (3,\gamma), (2,\beta)\}$$

2.3. Munosabatlar kompozitsiyasiga doir topshiriq(na'muna)

$A = \{a, b, c\}$, $B = \{1, 2, 3\}$, $C = \{\alpha, \beta, \gamma\}$ to'plamlarda aniqlangan $R_1 \subset A \times B$ va

$R_2 \subset B \times C$ binar munosabatlarning **kopaytmasi** yoki **kompozitsiyasi** topilsin:

$$1.6.0. \quad R_1 = \{(a,2), (a,3), (b,1), (c,2)\}, R_2 = \{(1,\alpha), (2,\alpha), (2,\beta), (3,\gamma)\}$$

2.3. Topshiriqning bajarilishi bo'yicha na'muna

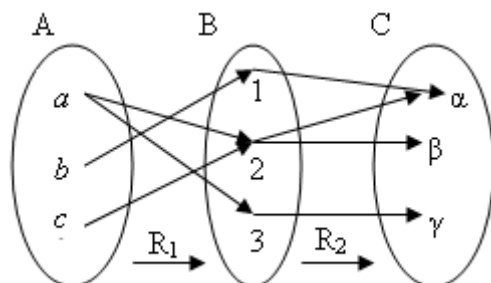
1.6.0. $R_1 \subset A \times B$ va $R_2 \subset B \times C$ binar munosabatlarning **kopaytmasi** yoki **kompozitsiyasi**,

$$R_1 \circ R_2 = \{(x, y) : x \in A, y \in C \text{ ba } \exists z \in B \text{ topiladiki } (x, z) \in R_1 \text{ va } (z, y) \in R_2\}$$

kabi aniqlanadi, shunga ko'ra:

$$R_1 \circ R_2 = \{(a,2); (a,3); (b,1); (c,2)\} \circ \{(1,\alpha); (2,\alpha); (2,\beta); (3,\gamma)\} = \\ = \{(a,\beta); (a,\alpha); (a,\gamma); (b,\alpha); (c, \alpha); (c, \beta)\}$$

2-usul. R_1 va R_2 munosabatlarni quyidagicha chizmalarda ifodalab olamiz:



A to'plam elementlarini B to'plam elementlari orqali C to'plam elementlari bilan bog'lash mumkin bo'lgan yo'llarning uchlaridan iborat bo'lgan to'plamga R_1 va R_2 munosabatlarning kompozitsiyasini tashkil qiladi.