15-AMALIY MASHG’ULOT. Graflarda yoq tushunchasi. Bog’langan va bog’lanmagan tekis graflar uchun Eyler formulasi. Yo’naltirilgan graf. Yoy tushunchasi. Yo’naltirilgan graf uchun qo’shnilik matrisasi. Graflarni bo’yash

Reja:

1.Graflar nazariyasiga oid asosiy tushunchalar.
2.Mustaqil bajarish uchun masala va topshiriqlar
3.Graflar ustida amallar

1.Graflar nazariyasiga oid asosiy tushunchalar.

Graf uchlari darajasi. Graf qirralari soni

15.1- Ta’rif . Qirraning boshi yoki oxirini ifodalovchi uchga bu qirraga **intsident uch** deyiladi.

15.2- Ta’rif. Graf uchining darajasi deb bu uchga **intsident qirralar** soniga aytiladi.

x_i uchning darajasini $P(x_i)$ bilan belgilanadi.

Boshqacha aytganda uchdan chiquvchi qirralar soni uchning darajasi hisoblanadi. Darajasi 1 ga teng uch osilgan uch bo`ladi.

15.3- Ta’rif. Hech qanday yoy yoki qirralarga ega bo`lmagan va izolyatsiyalangan uchlardan iborat graf **nol graf** deyiladi. Ko`rinib turibdiki, nol grafning uchlari darajasi nolga teng.

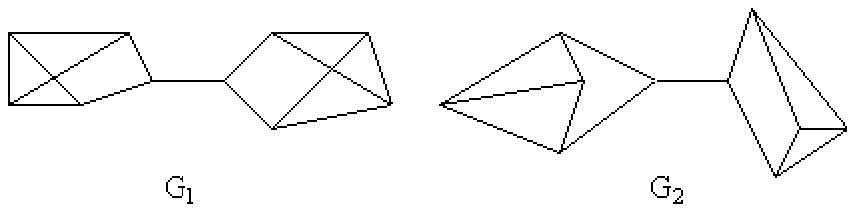
15.1- Lemma. Agar grafning barcha uchlarining darajalari 2 dan katta yoki 2 ga teng bo`lsa, graf, albatta, konturni o`z ichiga oladi.

15.4- Ta’rif 4. Agar grafning uchlari va qirralari to`plamida refleksivlik va simmetriklik xossalarini qanoatlantiruvchi binar munosabat mavjud bo`lsa, bunday graf **tolerant graf** deyiladi.

15.1- Teorema. Oriyentirlanmagan graf eyler sikli bo`lishi uchun uning uchlari juft darajalarga ega bo`lishi va uning bog`liq graf bo`lishi zarur va yetarlidir.

15.2- Teorema. Oriyentirlanmagan graf A va V uchlarni birlashtiruvchi eyler zanjiriga ega bo`ladi, faqat va faqat shu holdaki, agar graf bog`liq bo`lsa hamda faqatgina $A$ va $V$ uchlar toq darajalarga, qolgan uchlar juft darajalarga ega bo`lsa.

15.5- Ta'rif. Grafni tekislikka yotqizish mumkin bo`lsa, bunday graf **planar graf** deyiladi. Tekislikka yotqizilgan graf **tekis graf** deyiladi.



G_1 graf planar va G_2 tekis grafga izomorf.

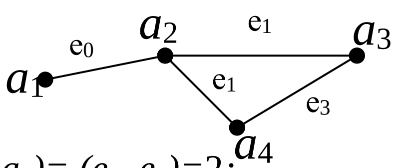
15.2- Teorema . Agar grafda karrali qirralari hamda ilmoq mavjud bo`lmasa, $n$ ta uchga ega bo`lgan va bog`liq komponentasi $K$ ga teng bo`lgan grafning qirralari soni eng ko`pi bilan aniqlanadi.

$$M=\frac{1}{2}(n-k)(n-k+1)$$

Mashrutning uzunligi deb, shu marshrutda mavjud qo`shni (e_{i-1}, e_i) qirralar soniga aytiladi.

Grafning ixtiyoriy a va ixtiyoriy v uchlari orasidagi **masofa** deb, shu uchlarni bog`lovchi eng kichik uzunlikka ega bo`lgan zanjirga aytiladi.

15.1- Misol.



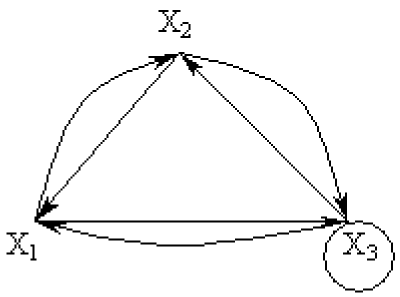
$d(a_1,a_3)=(e_0, e_1)=2;$

$d(a_1,a_4)=(e_0, e_2)=2;$

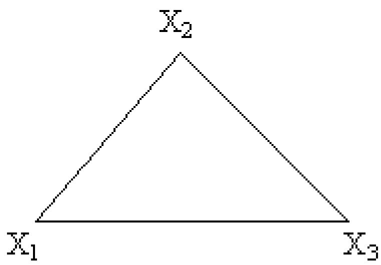
$d(a_1,a_4)=(e_0, e_1, e_3)=3$

**Yig`indi graf** ikkita qo`shiluvchi graflardan hech bo`lmaganda bittasida uchraydigan uch va qirralarni o`z ichiga oladi.

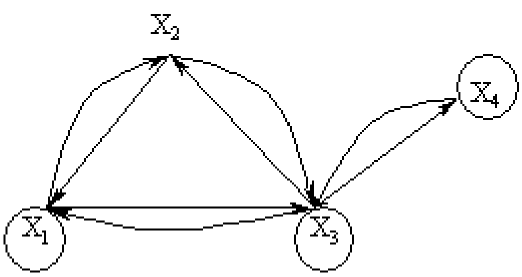
**Ko`paytma graf** ko`paytirilayotgan graflarning umumiy uchlari va qirralaridan iborat.



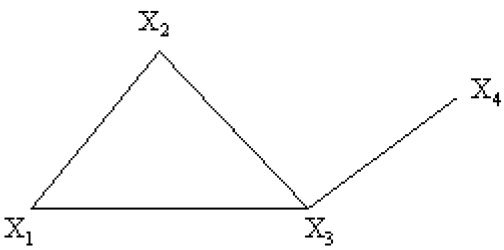
Simmetrik graf



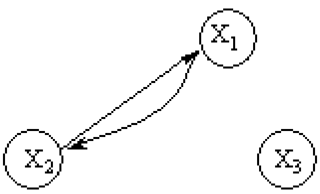
Oriyentirlanmagan graf



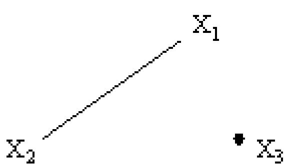
Tolerant graf



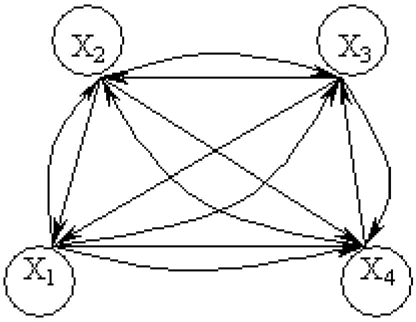
Oriyentirlanmagan graf

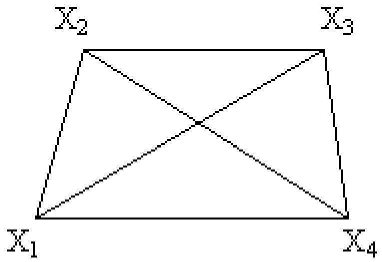


Tolerant graf



Oriyentirlanmagan graf

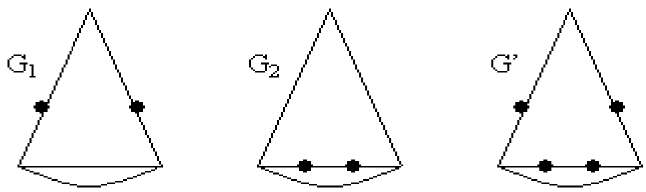




Graf-dekart ko`paytma Oriyentirlanmagan to`la graf

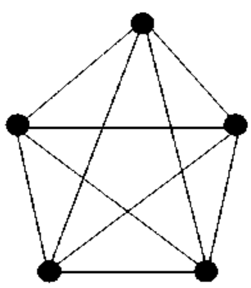
15.6- Ta’rif. Agar G_1 grafdan, shuningdek, G_2 grafdan chekli sonli martadagi qirralarni ajratish amali bilan olinishi mumkin bo`lgan shunday $G'$ graf mavjud bo`lsa, G_1, G_2 graflar **gomeomorf graf** deyiladi,

Quyidagi rasmda tasvirlangan G_1 va G_2 graflar gomeomorfdir.

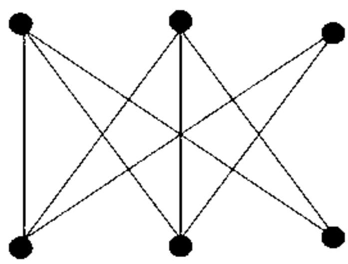


G' graf G_1 va G_2 graflardan ikki marta o`tkazilgan qirralar bo`linishi amalidan olinishi mumkin.

15.3- teorema (Pontryagin-Kuratovskiy). G graf planar bo‘ladi, faqat va faqat shu holdaki, G graf K_5 yoki $K_{3;3}$ ga gomemorof bo‘lgan, qism graflarga ega bo`lmasa.



K_5



$K_{3,3}$

Planarlik kriteriyasini ekvivalent formasini quyidagi teoremada keltirilgan.

15.4- teorema. Oriyentirlanmagan G graf K_5 yoki $K_{3\,3}$ graflarga tortiluvchi qism graflarga ega bo`lmasa.

15.5- teorema. Ko‘pi bilan 2^W uchdan iborat bo‘lgan har qanday graf R^3 fazoda uchlaridan tashqarisida yoylarining kesishmalarsiz tasvirlash mumkin.

Isboti. G’ = (МД) graf uchun | M | < 2^W bo‘lgan bo‘lsin. Unda | R < 2^W ga ega bo‘lamiz. G grafning barcha nuqtalarini biror L to‘g‘ri chiziqqa joylashtiramiz va R dagi har bir qirraga L to‘g‘ri chiziqni saqlovchi tekislikni har xil qiymatli mos qo‘yamiz.
Izlanayotgan G graf tasviri, barcha qirralami mos tekisliklarga o‘tkazgandagi keyin hosil bo‘ladi.

Planar graflaming xromatik sonining bahosi ma’lum.

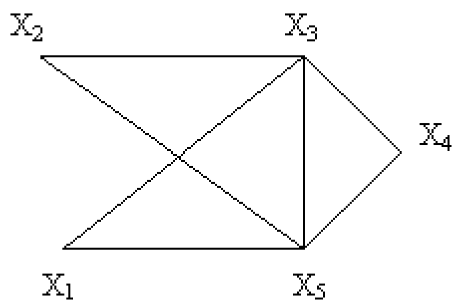
Graflarni xarakterlovchi sonlar

15.7- Ta’rif. Grafning **siklomatik soni** deb, N-n+p songa aytiladi, bu yerda N- grafning qirralari soni, n – grafning uchlari soni, P – bog`liqlik komponenti soni. Bog`liq graf uchun N-n+1.

15.6- Teorema. Grafning siklomatik soni erkli sikllarning eng katta miqdoriga teng.

15.2- Misol.

Quyidagi chizmada tasvirlangan grafning siklomatik soni 3 ga teng.



15.8- Ta’rif. Agar grafning uchlar to‘plamini o‘zaro kesishmaydigan shunday ikkita qism to‘plamlarga (bo‘laklarga) ajratish mumkin bo‘lsaki, grafning ixtiyoriy qirrasi bu to‘plamlarning biridan olingan qandaydir uchni ikkinchi to‘plamdan olingan biror uch bilan tutashtiradigan bo‘lsa, u holda bunday graf **ikki bo‘lakli graf (bixromatik** yoki **Kyonig grafi**) deb ataladi.

15.7- Teorema 1. Chekli bog`liq *G* graf daraxt bo`lishi uchun uning qirralari soni uchlari sonidan bittaga kam bo`lishi zarur va yetarli.

15.8- Teorema (*Keli*). Uchlar soni tartiblangan n ta bo`lgan daraxtlar soni $n^{n-2}$ teng. ($n$ ta elemenlardan n-2 tadan tuzilgan barcha takrorish o`rinlashtirishlar soni).

15.9- Teorema. Agar *G* graf daraxt bo`lsa, u holda uning qirralari soni $m$ va uchlari soni $n$ $m = n - 1$ munosabat bilan bog`langan.

15.10- Teorema. Quyidagi 4 ta shart teng kuchli:

- *G* graf daraxt hisoblanadi;
- Grafning qirralari soni m va uchlari soni n $m = n - 1$ munosabat bilan bog`langan;
- Grafning ixtiyoriy ikki uchi oddiy yo`l bilan bog`langan bo`lishi mumkin va bu yo`l yagonadir.
- *G* graf bog`langan va konturlarga ega emas.

Planar graflarni bo`yash masalasi graflar nazariyasining eng mashhur muammolaridan biri hisoblanadi. Ushbu masala o`tgan asrning o`rtalarida paydo bo`lgan bo'lsa ham hamon mutaxassis va qiziquvchilar e'tiboriga sazovor. Graflarni bo'yash masalasi quyidagicha paydo bo'lgan: geografik kartani bo`yash uchun ixtiyoriy 2 ta qo`shni davlatni rangi har xil bo`lishini ta'minlashda 4 xil rang yetadimi? Bunda ixtiyoriy davlat chegarasi yopiq chiziqdan iboratligi, qo`shni mamlakatlar esa umumiy chegara uzunligini tashkil etishini ko`rib chiqiladi. Keyinchalik karta tushunchasi va uning bo`yalishi boshqacharoq ko`rinishda talqin etilgan. Aytish mumkinki, ko`priklarsiz bog`langan tekis multigraf karta deb ataladi. Umumiy qirraga ega bo`lgan karta tomonlari chegaradosh hisoblanadi.

f funksiya mavjud bo`lib, unda *G*- 1 dan *k* gacha raqamlardan iborat va *f(G)*- chegara rangi, *G*- esa *k*-rang hisoblanadi( qo`shni chegaralar turli xil bo`lganda). *K*-rang mavjud bo`lsa, karta *k*- bo`yalgan deyiladi. 1879 yilda britaniyalik matematik A.Keli kartalarni bo`yash muammosini 4 ta rang gipotezasi orqali ta`riflab berdi. 4 bo`yoq farazi: har qanday karta 4 xil bo`yoq bilan bo`yaladi. Ko`pincha 4 bo`yoq farazini boshqacha ta`bir bilan foydalaniladi: har qanaqa planar graf 4 bo`yoqda bo`yaladi.

**15.9- Ta`rif.** Agar geometrik ikkilik graf *G*\* uchi k- bo`yalgan bo`lsa, karta G *k-bo`yalgan* deyiladi,.

Eslatib o`tamizki, shunday tekis graflar mavjudki, ular 4 rangdan kamroq rangda to`g`ri bo`yalgan. Masalan, $K4$ grafi.

4 ta rang gipotezasi unchalik qiyindek tuyilmadi va uning bir nechta isbotlari paydo bo`ldi.

15.11- Teorema. Ixtiyoriy 3 ta sikldan kam bo`lmagan yassi graf 3 xil rangda bo`yaladi.

Graflarning qirralarinigina emas, uchlarini ham bo`yash mumkin.

To`rt xil rang masalasi

To`rt xil rang gipotezasi o`sha davrlarda ko`pgina izlanuvchilarning diqqatiga tushgan. 1880 yilga kelib esa bu masalaning birinchi isbotini A. Kemp taqdim etdi. 1890 yilda R. Xivud bu isbotning xatosini aniqladi. Shu bilan birga u agar to`rt so`zini besh so`ziga o`zgartirilganda, uni usbotlash osonroq bo`lishini ta’kidlagan.

To`rt xil rang gipotezasi masalasini quyidagi uchta tasdiq yordamida hal qilinadi:

1. Ixtiyoriy yassi graf 4 xil rangda bo`yaladi.
2. Har bir kub karta 4 ta rangda bo`yaladi.
3. 3 xromatik indeks ixtiyoriy kub kartaga teng bo`lishi mumkin.

15.12- Teorema 1. (*to^crtta bo^cyoqlar haqida teorema*) Agar *G* planar graf bo‘lsa, unda $x(G) < 4$.

Agar *G* graf planar bo‘lmasa, uni geometrik tasvirlash uchun ayrim qirralarni olib tashlaymiz (boshqa tekislikka o‘tkaziladi).

Grafni tekislikdagi tasvirini hosil qilish uchun, olib tashlashi zarur bo‘lgan qirralarining minimal sonini *G* grafning planarlik soni deyiladi. Bu qirralami ikkinchi tekislikka o‘tkazish natijasida, grafni qismi hosil bo`ladi, lekin u tekis bo`lmasligi mumkin. U holda yana ayrim qirralami keyingi tekislikka o‘tkazish masalasi yechiladi.

Daraxt haqida tushuncha

Agar G grafda sikllar mavjud bo‘lmasa, u holda bunday graf - asiklik graf deyiladi. Asiklik grafda sikllar bo‘lmasligi, ya’ni sirtmoq, karrali qirralar bo‘lmasligi kerak.

Daraxt deb, bog‘langan asiklik G grafga aytiladi.

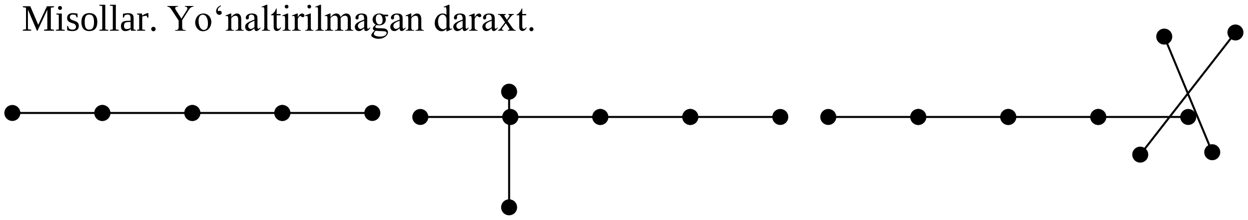
Sikllari mavjud bo‘lmagan bog‘lanmagan G graf, o‘rmon deyiladi.

O‘rmonning bog‘lovchi kompanentasi sifatida daraxt ishlatiladi. O‘rmonning yoki daraxtning ixtiyoriy šismi o‘rmon yoki daraxt bo‘lib, ular sikllarga ega emas. Bunday grafdagi ixtiyoriy zanjir oddiy zanjir bo‘ladi.

15.13-Teorema. Daraxtda ixtiyoriy ikkita a_i va a_j tugunlar bitta va faqat bita zanjir bilan bog‘langan.

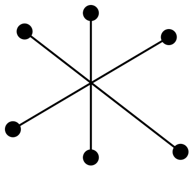
15.10-Ta’rif. Agar G grafning ixtiyoriy ikkita tuguni faqat bitta zanjir bilan bog‘langan bo‘lsa, G graf daraxt deb ataladi.

Misollar. Yo‘naltirilmagan daraxt.

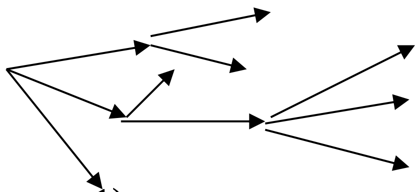


1) 2) 3)

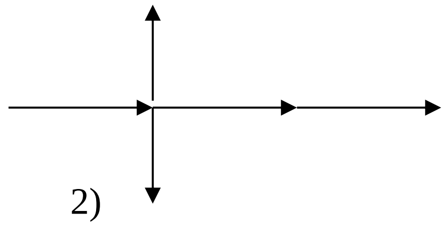
4)



15.3-Misol. Yo‘naltirilgan daraxt.



1)



2)

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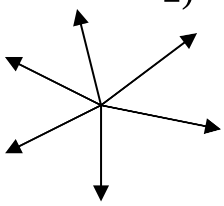
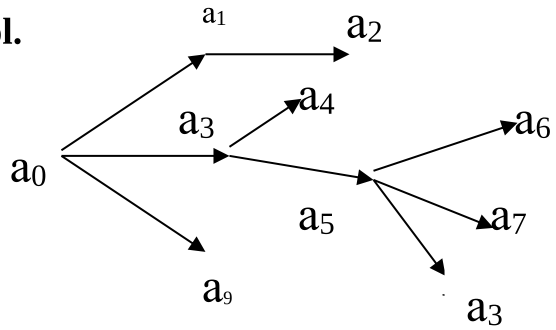


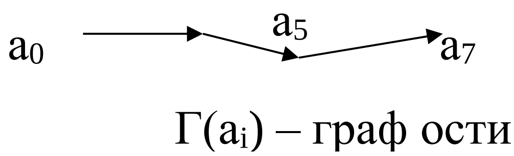
15.14-Teorema. G chekli daraxtda tugunlar soni r, qirralar soni q dan bitta ortiq, ya’ni p=q+1.

Aytaylik, a_0 ildizli G daraxtning tuguni a berilgan bo‘lsin. V (a) - a tugundan o‘tuvchi va ildiz a_0 bilan bog‘lovchi zanjirdagi tugunlar to‘plami bo‘lsin. Bu to‘plam G grafda G(a) graf ostini hosil qiladi.

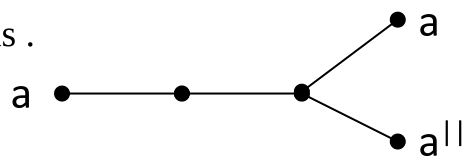
Graf osti G(a) a_0 ildizli daraxt G da a tugunning shaxobchasi deyiladi.

15.4-Misol.





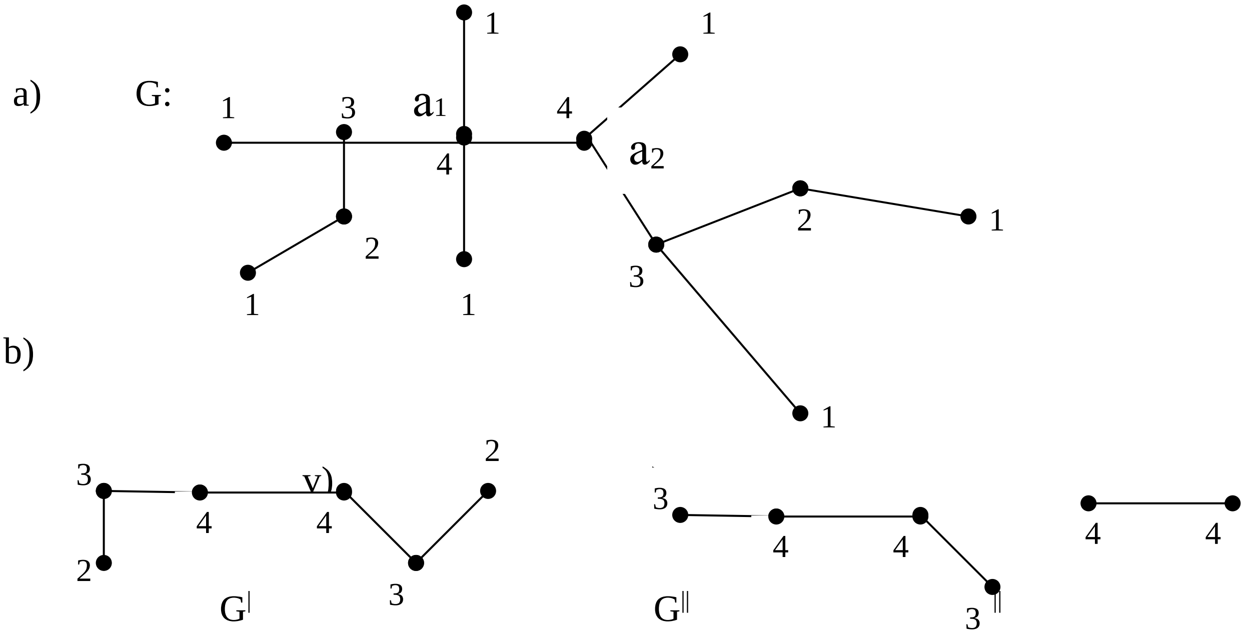
Tugunlarning tipi. Aytaylik chekli G – daraxt berilgan bo‘lsin. Bu daraxtning chekka tugunlarini 1 tipli tugunlar deb ataymiz. Ta’kidlaymizki, agar daraxtning tugunlari soni 2 tadan ortiq bo‘lsa, u holda ular orasida chekka bo‘lmagan tugunlar mavjud bo‘ladi. Haqiqatdan ham, aytaylik a_1 va a_2 – G daraxtning chekka tugunlari bo‘lsin. U holda ular zanjir bilan bog‘langan bo‘lishi kerak. Agar bu zanjir faqat bitta qirradan iborat bo‘lsa, u holda a_1 va a_2 boshqa birorta ham tugun bilan bog‘lanmagan, bu esa daraxtning bog‘langanligiga ziddir. Agar bu zanjir 2 ta yoki ko‘proq širradan iborat bo‘lsa, u holda bu zanjir lokal darajasi 2 dan katta yoki teng bo‘lgan tugunlardan o‘tadi, ya’ni bu tugunlar chekka tugunlar emas .



G daraxtdan hamma 1 tipli tugunlarni va ularga qo‘shma bo‘lgan qirralarni tashlab yuboramiz. U holda sikllarsiz bog‘langan G' daraxt qoladi.

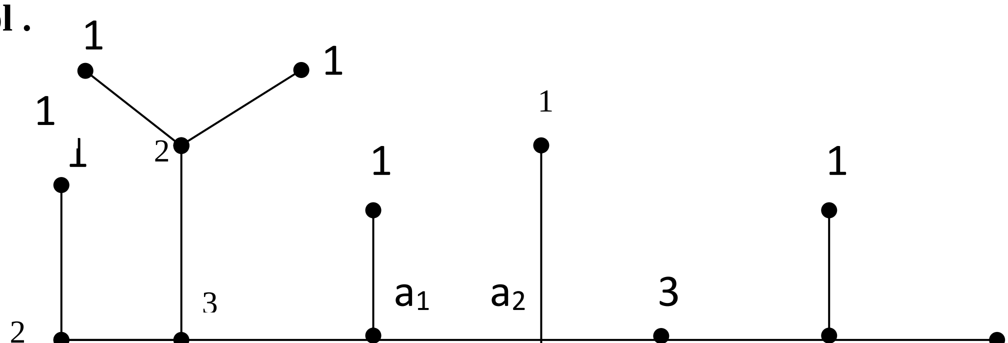
Haqiqatdan, ixtiyoriy a', $a'' \in G''$ tugunlarni birlashtiruvchi zanjir $L(a',a'')$ G daraxtning chekka tugunlaridan o‘tmaydi, balkim to‘lig‘icha G' daraxtda yotadi. G' daraxt ham o‘z navbatida chekka tugunlarga ega bo‘ladi. Bu tugunlarni G daraxtdagi 2 tipli tugunlar deb ataymiz. Xuddi shu usul bilan 3, 4 va h.k tipli tugunlarni hosil qilamiz.

15.5-Misol.



Demak G daraxtning markazi 4 –tipli a_1, a_2 tugunlardan iborat.

15.6-Misol .



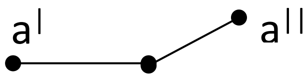
Ushbu daraxtning markazi 4-tipli a_1, a_2 tugunlardan iborat.

15.14-Teorema. G daraxtning markazi deb, eng katta K - tipga ega bo‘lgan tugunlarga aytiladi.

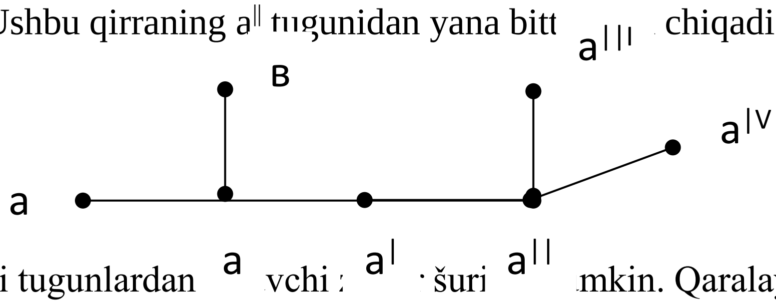
15.15-Teorema. Har bir G daraxtda bitta tugundan yoki ikkita qo‘shni tugunlardan iborat markaz mavjuddir.

G grafning a tuguni chetki tugun deyiladi, agarda uning lokal darajasi ρ (a)=1. Chetki tugunga qo‘shma bo‘lgan qirra ham chetki širra deyiladi.

15.1-Tasdiq. Agar chekli daraxtda tugunlar soni 1 tadan ortiq bo‘lsa, u holda u hech bo‘lmaganda ikkita chetki tugunga va hech bo‘lmaganda bitta chetki qirraga ega bo‘ladi.



Isboti. Haqiqatdan, aytaylik a G daraxtning tuguni bo‘lsin. Bu tugun boshqa tugunlar bilan bog‘langanligi uchun, undan hech bo‘lmaganda bitta qirra chiqadi. Agar bu qirraning ikkinchi tomoning tuguni $a^{|}$ chetki bo‘lmasa, u holda undan yana bitta qirra chiqadi. Ushbu qirraning $a^{||}$ tugunidan yana bitt chiqadi va hokazo.

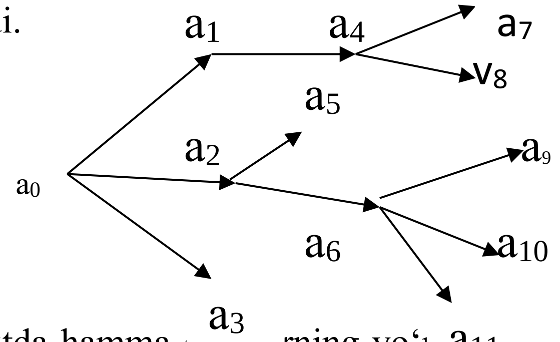


Shunday qilib, yangi tugunlardan vchi šuri mkin. Qaralayotgan G daraxt chekli bo‘lganligi uchun, zanjir qurish jaryoni ham cheklidir. Zanjirning oxirgi qirrasi va unga qo‘shma bo‘lgan tugunlardan biri chetkidir (a^{IV}).

Aytaylik, G daraxtda a_0 tugun berilgan bo‘lsin. Bu tugunni G daraxtning ildizi, G daraxt esa ildizli daraxt deyiladi. Bunday daraxtda qirralar yo‘naltirilgan bo‘ladi.

Tugun a| ni ildiz a_0 bilan L zanjir orqali birlashtirish mumkin.

Agar G daraxtda tugunlarni bog‘lovchi qirralarda yo‘nalish ko‘rsatilgan bo‘lsa va hamma qirralarning yo‘nalishi ildizdan boshlangan bo‘lsa, G daraxt yo‘naltirilgan daraxt deyiladi.



Agar yo‘naltirilgan G daraxtda hamma qirralarning yo‘ ... eskarisiga (ya’ni ildizga qarab) o‘zgartirsak, u holda yana yo‘naltirilgan daraxt hosil bo‘ladi. Boshqacha aytganda yig‘uvchi tarmoq hosil bo‘ladi.

Yo‘naltirilgan daraxtning har bir tuguniga (ildizdan tashqari) faqat bitta širra kiradi, ya’ni bu tugun fašat bitta qirraning tugashi bo‘ladi. Haqiqatdan ham, qaralayotgan tugunni ildizi bilan bog‘lovchi zanjirda bu qirra oxirgi hisoblanadi.

Ildizga birorta ham qirra kirmaydi, ildiz bilan qo‘shma bo‘lgan xamma qirralar ildizni o‘zlarining ikkita tomoni bilan bog‘laydi, demak ildiz ularning boshlanishi deyiladi.

Berilgan har bir G daraxtda ildiz sifatida ixtiyoriy tugunni olib, G daraxtni yo‘naltirilgan daraxtga aylantirish mumkin.

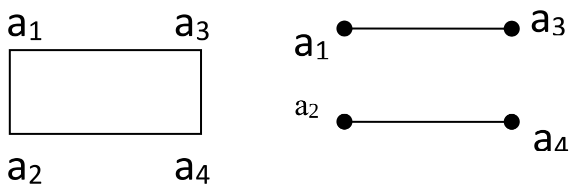
Daraxtda siklomatik son. Aytaylik G chekli yo‘naltirilmagan daraxt bo‘lsin. Uning siklomatik soni deb $\gamma(G) = \gamma_б + \gamma_к - \gamma_у$, aytiladi. Bu yerda $\gamma_б$ - grafning bog‘liqlik komponentalari soni; $\gamma_к$ - grafning qirralar soni; $\gamma_у$ - grafning tugunlari soni.

15.2-Tasdiq. G daraxtning siklomatik soni 0 (nol) ga teng.

15.3-Tasdiq. O‘rmonning siklomatik soni – bog‘langan daraxt komponentalarining yig‘indisi bo‘lib, u ham 0 (nol) ga teng.

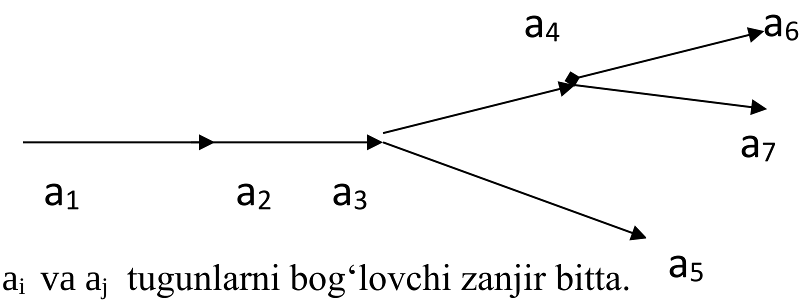
15.4-Tasdiq. Qolgan chekli graflarning siklomatik soni musbatdir, ya'ni γ (G)>0.

15.7-Misol.



γ (G)=2+4-4=2>0;

15.8-Misol.



Ixtiyoriy a_i va a_j tugunlarni bog'lovchi zanjir bitta.

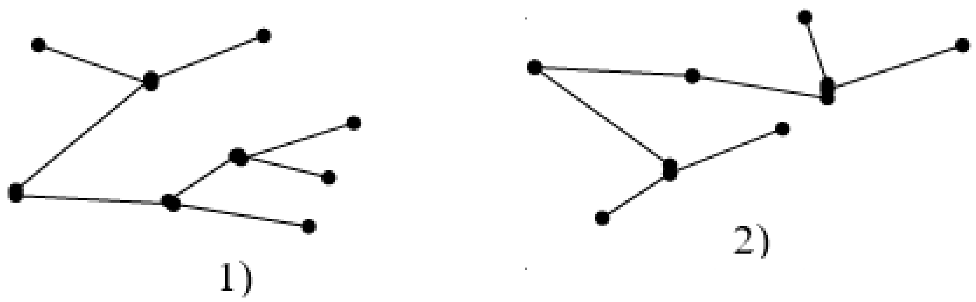
Demak γ (G) = 1+6-7=7-7=0.

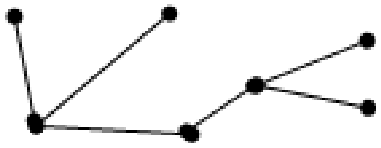
2.Mustaqil bajarish uchun masala va topshiriqlar

2.1. daraxtlar ustida amallar

Quyida keltirilgan yo'naltirilgan va yo'naltirilmagan daraxtlar uchun:

1) Markazini toping.
2) Tugunlar tipini aniklang.
3) Ildizini aniqlang.
4) Siklomatik sonini toping.





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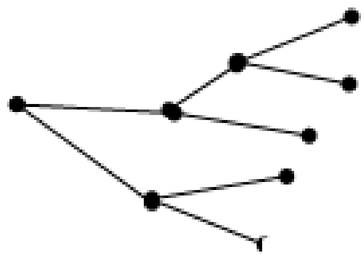

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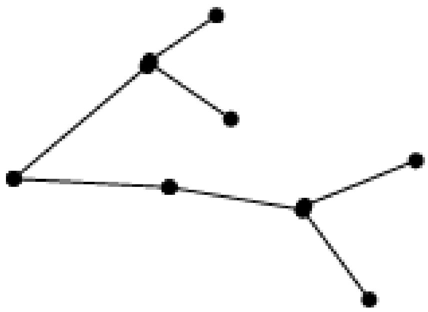

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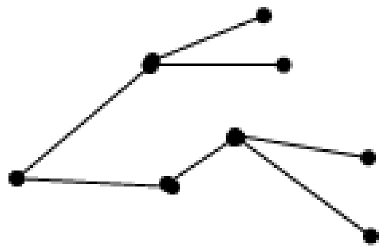

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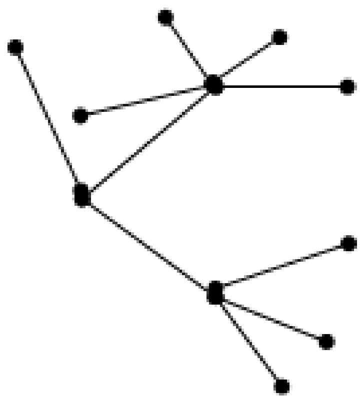

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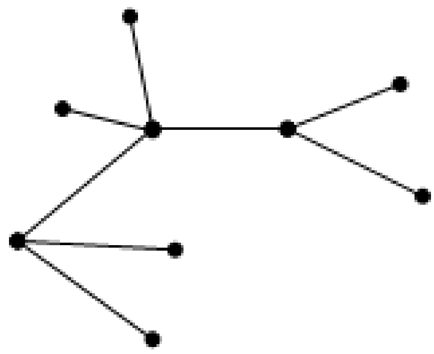

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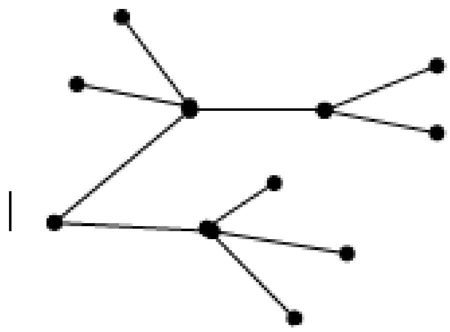

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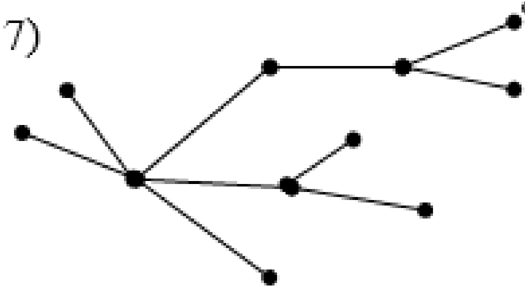

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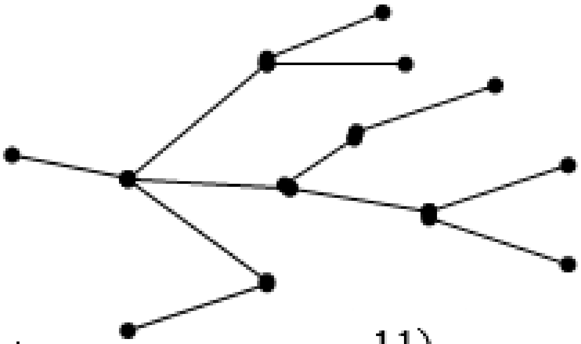

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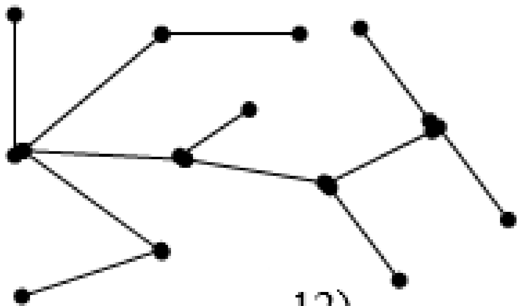

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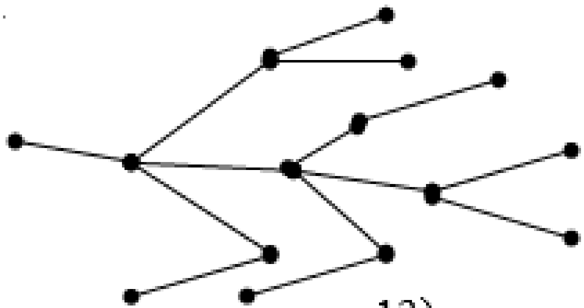

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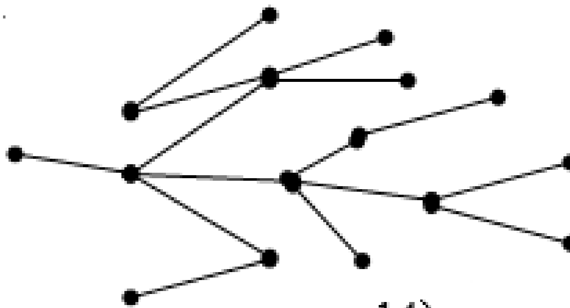

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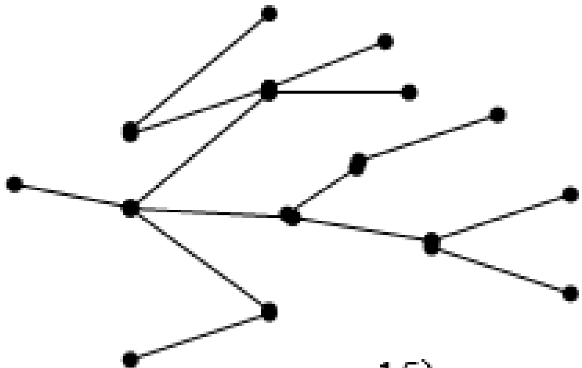

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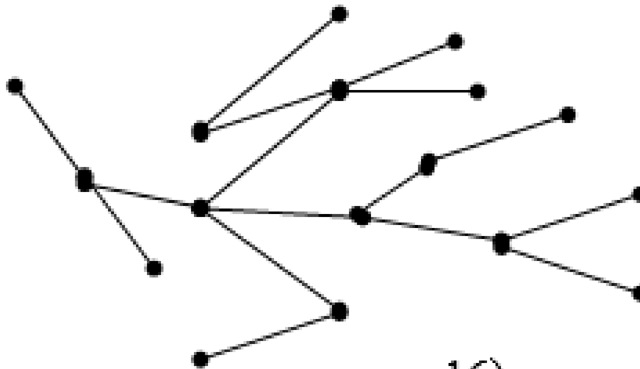

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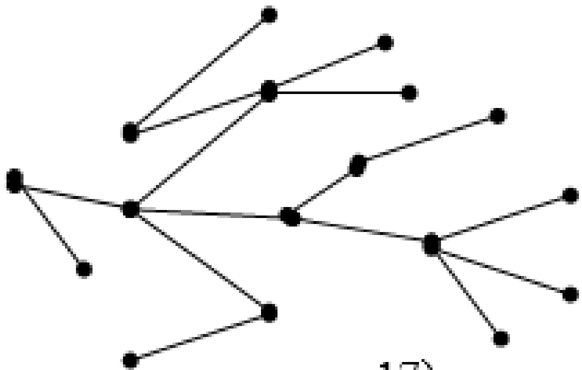

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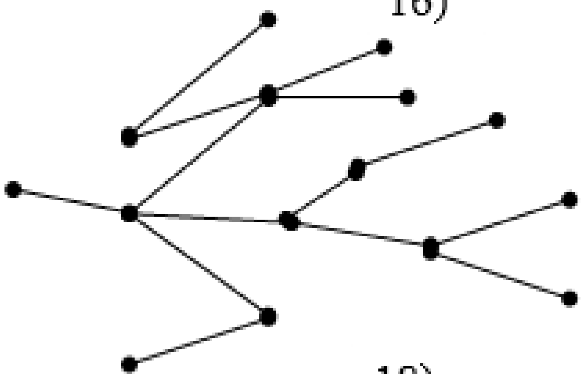

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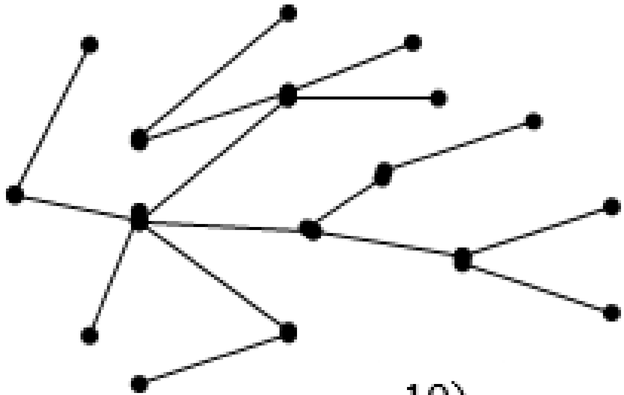

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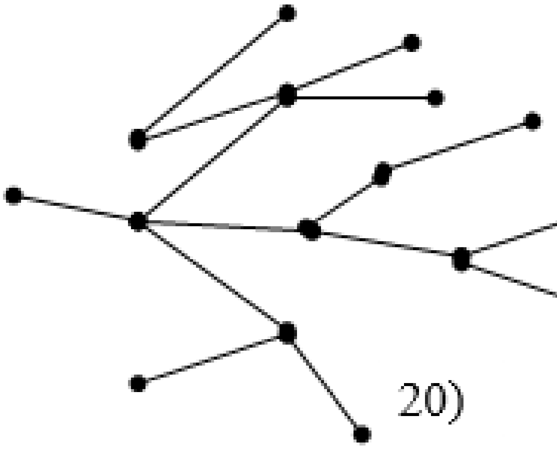

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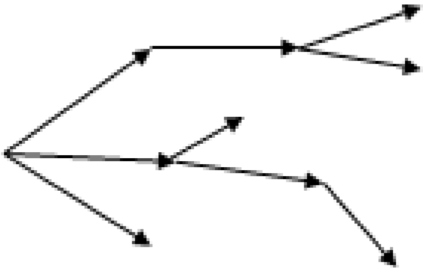

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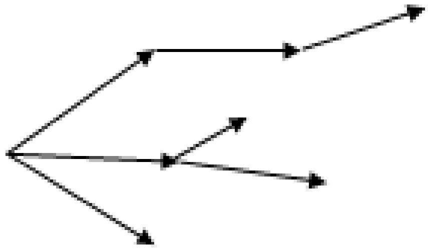

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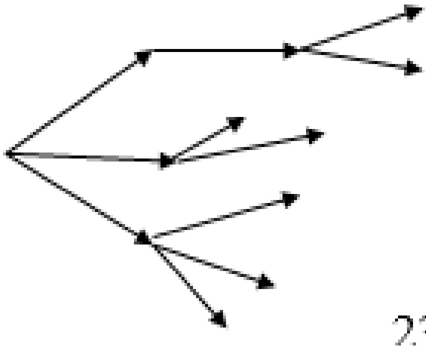

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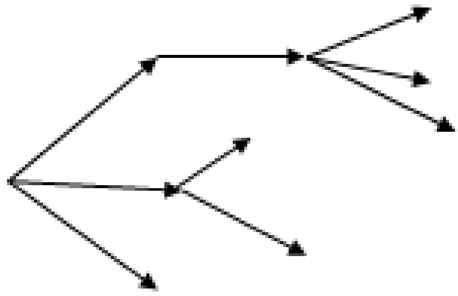

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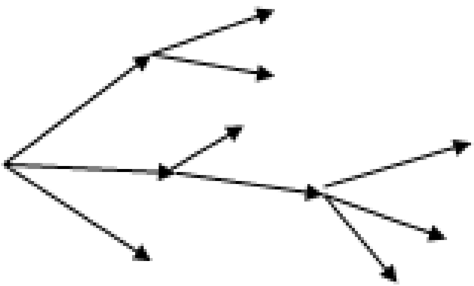

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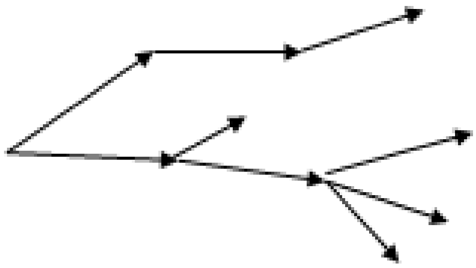

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