7- AMALIY MASHGULOT. Takroriy guruhlash, o'rinlashtirish, o'rin almashtirish formulalarini qullab misollar yechish

Reja:

1. Takroriy guruhlash, o'rinlashtirish, o'rin almashtirish formulalari.
2. Mustaqil bajarish uchun masala va topshiriqlar

2.1.Takroriy o'rin almashtirish formulalarini qo'llashga doir topshiriqlar.

2.2.Takroriy guruhlash formulalarini qo'llashga doir topshiriqlar.

2.3.Kombinator tenglamalar doir topshiriqlar.

1.Takroriy guruhlash, o'rinlashtirish, o'rin almashtirish formulalari.

Takrorlanuvchi guruhlashlar.

7.1-Ta'rif. n ta elementli to'plamning barcha tartiblanmagan takrorlanuvchi k ta elementli qism to'plamlarini ajratish t**akrorlanuvchi guruhlash** deyiladi.

S to`plamning elementlari 1;2;…;$n$ sonlari bilan raqamlangan bo`lsin. S to`plam chekli yoki sanoqli bo`lgani uchun, har doim S to`plam elementlari va $N$ natural sonlar to`plami elementlari o`rtasida bir qiymatli moslik o`rnatish mumkin. U holda S to`plam o`rniga o'zaro bir qiymatli moslik kuchiga asosan, unga ekvivalent bo`lgan $S^/=\{1;2;...;n\}$ to`plamning C_n^k guruhlashlarini topish mumkin.

$S^/$ to`plamning har qanday tanlanmasini $\{n_1;n_2;...;n_k\}$ ko`rinishda yozish mumkin, bunda $n_1 \le n_2 \le ... \le n_k$ ketma-ketlik o'rinli bo'lib, "tenglik" amali tanlanma takrorlanuvchi bo`lishi mumkinligini bildiradi.

k ta elementli tanlanma $\{n_1;n_2;...;n_k\}$ ga k ta elementli to`plam $\{n_1;n_2+1;...;n_k+k-1\}$ ni mos qo`yamiz, bunda elementlar turlicha bo`ladi.

$\{n_1;n_2;...;n_k\}$ va $\{n_1;n_2+1;...;n_k+k-1\}$ to`plamlar orasidagi moslik yana o`zaro bir qiymatli bo`lib, $\{n_1;n_2+1;...;n_k+k-1\}$ to`plam $S^/\bigcup\{1;2;...;k-1\}$ to`plamdan $n+k-1$ tadan takrorlanmaydigan $k$ elementli guruhlash bo`ladi.

U holda takrorlanmaydigan C_{n+k-1}^k guruhlashlar soni $\tilde{C}_n^k$ takrorlanuvchi guruhlash soniga teng bo'ladi, ya`ni

$$\tilde{C}_n^k = C_{n+k-1}^k = \frac{(n+k-1)!}{k!\cdot(n-1)!} = \frac{n\cdot(n+1)\cdot...\cdot(n+k-1)}{k!}$$

7.1-Teorema. n ta elementdan k ta elementli takrorlanuvchi guruhlashlar soni $\tilde{C}_n^k = C_{n+k-1}^k$ ga teng.

Misol. 4 ta o'yin kubigini tashlab, nechta turlicha variant hosil qilish mumkin?

Yechilishi: Har bir o'yin kubigida 1 dan 6 gacha raqamlardan bittasi tushishi mumkin, ya'ni har bir kubikda 6 ta variant bo'lishi mumkin. Agar 4 ta o'yin kubigi tashlansa, har bir variantni 4 ta ob'yektning tartiblanmagan takrorlanuvchi ketma-ketligi deyish mumkin, ularning har biri uchun esa 6 ta imkoniyat bor:

$$\tilde{C}_n^k = \frac{(n+k-1)!}{k!\cdot(n-1)!} = \frac{(6+4-1)!}{4!\cdot 5!} = \frac{9!}{4!\cdot 5!} = \frac{6\cdot 7\cdot 8\cdot 9}{1\cdot 2\cdot 3\cdot 4} = 126.$$

N'yuton binomi.

Maktab kursidan qisqa ko`paytirish formulalari bilan tanishsiz, masalan ikki son yig`indisining kvadrati

$$(a+b)^2 = (a+b)\cdot(a+b) = aa+ab+ba+bb = a^2+2ab+b^2$$

yoki ikki son yig`indisining kubini topish

$$(a+b)^3 = (a+b)\cdot(a+b)\cdot(a+b) = a^3+3a^2b+3ab^2+b^3$$

kabi masalalarda a va b lar oldidagi koeffitsiyentlarni topish masalasi kelib chiqadi. Koeffitsiyentlarni topish usulini frantsuz matematigi Blez Paskal (1623 – 1662 yy) fanga kiritgan, hozirda **Paskal uchburchagi** deb ataladi:

$$\begin{array}{ccccccccccccccccl}
 & & & & & & & 1 & & & & & & & & & n=0\\
 & & & & & & 1 & & 1 & & & & & & & & n=1\\
 & & & & & 1 & & 2 & & 1 & & & & & & & n=2\\
 & & & & 1 & & 3 & & 3 & & 1 & & & & & & n=3\\
 & & & 1 & & 4 & & 6 & & 4 & & 1 & & & & & n=4\\
 & & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & & & & n=5\\
 & 1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1 & & & n=6\\
1 & & 7 & & 21 & & 35 & & 35 & & 21 & & 7 & & 1 & & n=7
\end{array}$$

. . . .

n soni yetarlicha katta bo`lganda, $(a+b)^n$ uchun Paskal uchburchagini tashkil qiluvchi sonlar C_n^k ga teng bo'ladi:

$$\begin{array}{ccccccccccc}
 & & & & & C_0^0 & & & & & \\
 & & & & C_1^0 & & C_1^1 & & & & \\
 & & & C_2^0 & & C_2^1 & & C_2^2 & & & \\
 & & C_3^0 & & C_3^1 & & C_3^2 & & C_3^3 & & \\
 & C_4^0 & & C_4^1 & & C_4^2 & & C_4^3 & & C_4^4 & \\
C_5^0 & & C_5^1 & & C_5^2 & & C_5^3 & & C_5^4 & & C_5^5\\
\dots & & \dots. & & \dots & & \dots. & & \dots & \dots. & \dots
\end{array}$$

$$C_n^0 \quad C_n^1 \quad \dots \quad \dots. \quad \dots \quad \dots. \quad \dots \quad C_n^{n-1} \quad C_n^n$$

Paskal uchburchagining tashqi tomonlaridagi sonlar har doim 1 ga teng bo'ladi, chunki $C_n^0 = C_n^n = 1.$ Paskal uchburchagining yana bir qonuniyati, uchburchakdagi 2 ta ketma-ket sonni qo'shish natijasida keyingi qatordagi shu 2 son o'rtasida turgan sonni topish mumkin. Bu xossa **Paskal formulasi** deb nomlanadi:

$$C_{n-1}^{k-1} + C_{n-1}^{k} = C_n^k$$

Bunda $0 < k < n.$

Isboti:

$$C_{n-1}^{k-1} + C_{n-1}^{k} = \frac{(n-1)!}{(n-k)!(k-1)!} + \frac{(n-1)!}{(n-k-1)!k!} = \frac{(n-1)!}{(n-k-1)!(k-1)!}\left(\frac{1}{n-k} + \frac{1}{k}\right) =$$

$$= \frac{(n-1)!}{(n-k-1)!(k-1)!} \cdot \frac{n}{(n-k)k} = \frac{n!}{(n-k)!k!} = C_n^k.$$

7.2-Teorema (Binomial teorema). Quyidagi tenglik o'rinli

$$(a+b)^n = \sum_{k=0}^{n} C_n^k \cdot a^k \cdot b^{n-k} =$$

$$= C_n^0 \cdot a^0 \cdot b^n + C_n^1 \cdot a^1 \cdot b^{n-1} + \dots + C_n^k \cdot a^k \cdot b^{n-k} + \dots + C_n^n \cdot a^n \cdot b^0$$

bu yerda C_n^k sonlarga **binomial koeffitsiyentlar**, tenglamaga esa **N'yuton binomi** deyiladi.

Isboti: Formulani matematik induktsiya metodidan foydalanib isbotlash mumkin. Haqiqatan ham,

$n=1$ bo`lganda $(a+b)^1 = C_1^0 \cdot a^0 \cdot b^1 + C_1^1 \cdot a^1 \cdot b^0 = b + a;$

$n=2$ da $(a+b)^2 = C_2^0 \cdot a^0 \cdot b^2 + C_2^1 \cdot a^1 \cdot b^{2-1} + C_2^2 \cdot a^2 \cdot b^0 = b^2 + 2ab + a^2.$

Endi formulani $n-1$ uchun o`rinli deb faraz qilib, quyidagiga ega bo`lamiz:

$$(a+b)^n = (a+b)^{n-1}(a+b) = a \cdot (a+b)^{n-1} + b \cdot (a+b)^{n-1} =$$

$$= \sum_{k=0}^{n-1} C_{n-1}^k a^{k+1} b^{(n-1)-k} + \sum_{k=0}^{n-1} C_{n-1}^k a^k b^{(n-1)-k+1}.$$

Yig'indida indekslarni almashtiramiz: $k = j-1,\ j = k+1,$ u holda

$$\sum_{k=0}^{n-1} C_{n-1}^{k} a^{k+1} b^{(n-1)-k} = \sum_{j=1}^{n} C_{n-1}^{j-1} a^{j} b^{n-j}$$

bo'ladi. Bundan

$$(a+b)^n = \sum_{k=1}^{n} C_{n-1}^{k-1} a^{k} b^{n-k} + \sum_{k=0}^{n-1} C_{n-1}^{k} a^{k} b^{n-k}.$$

Oxirgi tenglikda yig'indilar chegaralarini tenglashtiramiz. Buning uchun yordamchi $C_{n-1}^{-1} = 0, \quad C_{n-1}^{n} = 0$ tengliklarni kiritamiz, u holda

$$\sum_{k=1}^{n} C_{n-1}^{k-1} a^{k} b^{n-k} = \sum_{k=0}^{n} C_{n-1}^{k-1} a^{k} b^{n-k}$$

va

$$\sum_{k=0}^{n-1} C_{n-1}^{k} a^{k} b^{n-k} = \sum_{k=0}^{n} C_{n-1}^{k} a^{k} b^{n-k}$$

tengliklar hosil bo'ladi.

Bu tengliklarni o'rniga qo'yib, quyidagini hosil qilamiz:

$$(a+b)^n = \sum_{k=0}^{n} \left(C_{n-1}^{k-1} + C_{n-1}^{k}\right) a^{k} b^{n-k} = \sum_{k=0}^{n} C_{n}^{k} a^{k} b^{n-k}.$$

Teorema isbotlandi.

Hozirda N'yuton binomi deb yuritiladigan yuqoridagi formulani Isaak N'yuton(1643-1727 yy)gacha O'rta osiyolik olimlar, yurtdoshlarimiz: matematik, astronom, shoir Umar Xayyom (1048-1122 yy) va Mirzo Ulug'bekning shogirdi G'iyosiddin Jamshid al-Koshiy "Arifmetika kaliti" asarida yorqin misollarda ko'rsatib bergan. Yevropada esa B. Paskal o`z ishlarida qo`llagan. N'yutonning xizmati shundaki, u formulani daraja ko`psatkichi n ning butun bo'lmagan holi uchun umumlashtirdi.

$|x|<1$ uchun n ning butun bo'lmagan qiymatida N'yuton binomi formulasining ko'rinishi quyidagicha bo'ladi:

$$(1+x)^{\alpha} = 1+\alpha x+\frac{\alpha(\alpha-1)}{2!}x^2+\frac{\alpha(\alpha-1)(\alpha-2)}{3!}x^3+\ldots+\frac{\alpha(\alpha-1)(\alpha-2)\ldots(\alpha-k+1)}{k!}x^k+\ldots$$

Binom yoyilmasi ko'pgina konbinatorika formulalarida asos bo'lib xizmat qiladi, masalan:

1. $a=b=1$ bo'lganda $\sum_{k=0}^{n} C_n^k = 2^n$ hosil bo'ladi. Bu son n ta elementli S to'plamning barcha mumkin bo'lgan tartiblanmagan qism to'plamlari soniga teng.

2. $a=1,\ b=-1$ bo'lganda $\sum_{k=0}^{n} C_n^k (-1)^k = 0$ ga teng, ya'ni toq va juft o'rinda turgan binomial koeffitsiyentlar yig'indisi 2^{n-1} ga va ular o'zaro ham teng bo'ladi.

Polinomial teorema.

7.3-Teorema (N'yuton binomining umumlashgan teoremasi).

k ta qo'shiluvchiga ega bo'lgan $(a_1+a_2+...+a_k)^n$ ifoda uchun N'yuton formulasi quyidagiga teng:

$$(a_1+a_2+...+a_k)^n = \sum_{\substack{r_1\geq 0,\dots,r_k\geq 0 \\ r_1+r_2+\dots+r_k=n}} \frac{n!}{r_1!\cdot r_2!\cdot\ldots\cdot r_k!}\cdot a_1^{r_1}\cdot a_2^{r_2}\cdot\ldots\cdot a_k^{r_k}$$

ya'ni yig'indi $r_1+r_2+...+r_k=n$ tenglamaning barcha nomanfiy butun yechimlari uchun hisoblanadi.

7.1-Misol. N'yuton polinomi formulasidan foydalanib $(a+b+c)^3$ ni hisoblaymiz.

Agar qavslarni ochib, soddalashtiradigan bo'lsak, bir qancha amallarni bajargandan keyin quyidagi tenglikka kelamiz:

$$(a+b+c)^3 = a^3+b^3+c^3+3a^2b+3a^2c+3ab^2+3b^2c+3ac^2+3bc^2+6abc.$$

Barcha hisoblashlardan keyin 10 ta haddan iborat bo'lgan tenglik hosil bo'ladi.

Bu tenglikni polynomial formuladan oson topish mumkin: bizning misolda $n=3,\quad k=3$, ya'ni

$$\begin{cases} r_1\geq 0, r_2\geq 0, r_3\geq 0, \\ r_1+r_2+r_3=3. \end{cases}$$

Turli koeffitsiyentlar ham 3 ta, bular:

$$\frac{3!}{3!\cdot 0!\cdot 0!}=1,\quad \frac{3!}{2!\cdot 1!\cdot 0!}=3,\quad \frac{3!}{1!\cdot 1!\cdot 1!}=6.$$

Natijani yozish uchun chekli sondagi r_1,r_2,r_3 indekslarni barcha mumkin bo'lgan kombinatsiyalari jadvalini tuzgan ma`qul:

r_1	r_2	r_3
3	0	0
0	3	0
0	0	3
2	1	0
2	0	1

1	2	0
0	2	1
1	0	2
0	1	2
1	1	1

U holda

$$(a+b+c)^3 = 1\cdot(a^3+b^3+c^3)+3\cdot(a^2b+a^2c+ab^2+b^2c+ac^2+bc^2)+6\cdot abc.$$

hosil bo'ladi.

7.2-Misol. $(x+y+z)^9$ darajani yoyishdan hosil bo'lgan $x^3y^2z^4$ had oldidagi koeffitsiyentni toping.

Yechilishi: $\frac{9!}{3!2!4!} = 1260.$

7.3-Misol. 15 talabani nechta usulda 3 ta o'quv guruhiga 5 nafardan guruhlarga ajratish mumkin?

Yechilishi: Bizda 15 ta ob'yekt bor, ularni 5 tadan 3 ta guruhga ajratish kerak. Bu ishni

$$\frac{15!}{5!5!5!} = 68796$$

usulda bajarish mumkin.

7.4-Misol 4. "MASALA" so'zidagi harflarni necha xil usulda o'rin almashtirish mumkin?

Yechilishi: Ushbu so'z 6 ta harfdan iborat bo'lgani uchun uni 6! Usulda o'rin almashtirish mumkin. Biroq unda 3 ta "A" harfi qatnashgan, "A" harflarini o'rin almashtirgan bilan yangi so'z hosil bo'lmaydi. 3 ta harfni o'rin almashtirishlar soni 3! ga tengligidan $\frac{7!}{3!} = 840$ qiymat topiladi.

Demak, "MASALA" so'zidagi harflarni o'rin almashtirish bilan 840 ta turli "so'z" hosil qilish mumkin ekan.

Takrorlanuvchi o'rin almashtirishlar

7.4-Teorema. Aytaylik $k_1, k_2, \ldots, k_m$ - butun manfiymas sonlar bo'lib, $k_1+k_2+\ldots+k_m=n$ va A to'plam n ta elementdan iborat bo'lsin. A ni elementlari mos ravishda $k_1, k_2, \ldots, k_m$ ta bo'lgan $B_1, B_2, \ldots, B_m$ m ta to'plam ostilar yigindisi ko'rinishida ifodalash usullari soni

$$C_n(k_1,...,k_m)=\frac{n!}{k_1!*k_2!*...*k_m!}$$

ta bo‘ladi.

$C_n(k_1,...,k_m)$ sonlar ***polinomial koeffitsiyentlar*** deyiladi.

2. Mustaqil bajarish uchun masala va topshiriqlar

2.1. Takrorlanuvchi o‘rin almashtirishlarga doir topshiriqlar

2.1.0. “Matematika” so‘zidagi harflardan nechta so‘z yasash mumkin?
2.1.1. “Kombinatorika” so‘zidagi harflardan nechta so‘z yasash mumkin?
2.1.2. Familiyangizdagi harflardan nechta so‘z yasash mumkin?
2.1.3. *a,b,c* harflaridan *a* harfi ko‘pi bilan 2 marta, *b* harfi ko‘pi bilan bir marta, *c* harfi ko‘pi bilan 3 marta qatnashadigan nechta 5 ta harfli so‘z yasash mumkin?
2.1.4. $(1+x)^n$ yoyilmasida x^5 va x^{12} hadlar oldidagi koeffitsiyentlar teng bo‘lsa, *n* nimaga teng?
2.1.5. $\left(\sqrt{2}+\sqrt[4]{3}\right)^{100}$ yoyilmasida nechta ratsional had mavjud?
2.1.6. Polinomial teorema yordamida $(x+y+z)^3$ yoyilmani toping?
2.1.7. $(x+y+z)^7$ ning yoyilmasida $x^2y^3z^2$ had oldidagi koeffitsiyent nimaga teng?
2.1.8. 8 ta fanning har biridan 3, 4, 5 baholar olish mumkin. Baholar yig‘indisi 30 ga teng bo‘ladigan qilib imtixonlarni necha xil usulda topshirish mumkin?
2.1.9. Abituriyent 3 ta fandan imtixon topshirishi lozim. Har bir imtixondan ijobiy baho (3,4,5-baholar) olgandagina, keyingi imtixonga qo‘yiladi. O‘qishga kirish uchun o‘tish bali 17 ball bo‘lgan bo‘lsa, abituriyent imtixonlarni necha xil usulda topshirishi mumkin?
2.1.10. $(1+2t-3t^2)^8$ yoyilmasida t^9 oldidagi koeffitsiyent nimaga teng?
Masala 2.1.11.- 2.1.20 So‘z – o‘zbek alifbosidagi ixtiyoriy chekli harflar ketma-ketligidir. Quyida berilgan so‘zlardagi harflardan nechta so‘z yasash mumkin?
2.1.11. BISSEKTRISSA; **2.1.12.** PARABOLA; **2.1.13.** GIPERBOLA;
2.1.14. ELLIPS; **2.1.15.** SIMMETRIK; **2.1.16.** PARALEL;
2.1.17. PARALELOGRAM; **2.1.18.** PARALELOPIPED; **2.1.19.** REFLEKSIV;
2.1.20. TRANZITIV.
2.1.21. Mevalar korzinkasida 2 ta olma, 3ta nok, 4 ta apelsin bor. Har kuni bitta meva yeyish mumkin bo‘lsa, buni necha xil usulda amalga oshirish mukin?
2.1.22. Talabalar turar joyida 1 kishilik, 2, kishilik va 4 kishilik xonalar mavjud. 7 ta talabani necha xil usulda joylashtirish mumkin?
2.1.23. Shaxmat taxtasining birinchi gorizontalida oq shaxmat donalari komplekti: 1ta shox, 1ta farzin, 2 ta ot, 2 ta fil, 2 ta to‘rani necha xil usulda joylashtirish mumkin?
2.1.24. Beshta A harfi va ko‘pi bilan 3 ta B harfidan nechta so‘z yasash mumkin?

2.1.25. 7xil gul turidan 3 tadan yoki 5 tadan qilib nechta gul buketi yasash mumkin?

2.1. Takrorlanuvchi o‘rin almashtirishlarga doir topshiriq(na’muna)

2.1.0. “Matematika” so‘zidagi harflardan nechta so‘z yasash mumkin?

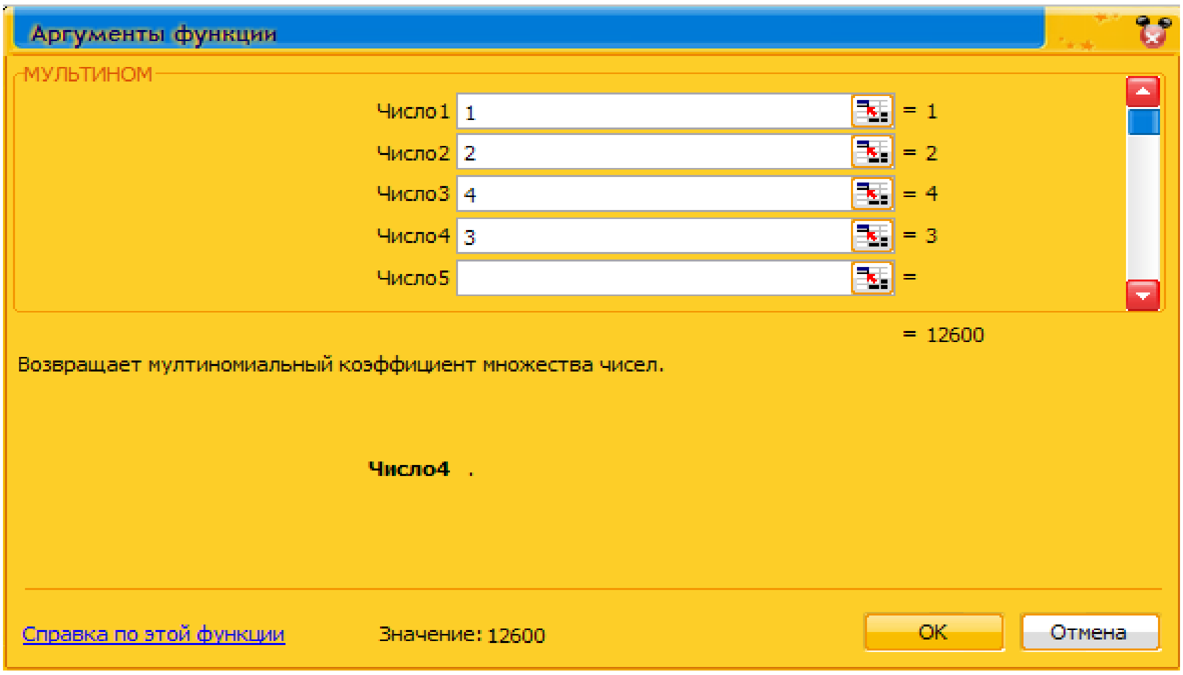
2.1. Topshiriqning bajarilishi bo’yicha na’muna

2.1.0. Misolning yechilishi. “Matematika” so‘zidagi harflardan nechta so‘z yasash mumkin?

k_1=2 (“m”- harfi), k_2 =2 (“a” – harfi), k_3 =2 (“t” - harfi), k_4=1 (“e” - harfi), k_5=1 (“i”-harfi), k_6=1 (“k”- harfi), n=10 (so‘zdagi harflar soni)

$$C_{10}(2,3,2,1,1,1) = \frac{10!}{2!*3!*2!*1!*1!*1!} = 151200$$

Shu o‘rinda eslatib o‘tamiz BMI, magistrlik dissertatsiyasi yoki ilmiy ishingizda ko‘p miqdordagi takrorlanuvchi o‘rin almashtirishlarni hisoblashga to‘g‘ri kelsa, unda Excel dasturlar paketidagi МУЛЬТИНОМ komandasidan foydalanish mumkin: Masalan $C_{10}(1,2,4,3) = \frac{10!}{1!*2!*4!*3!} = 12600$ ekanligini tezlik bilan hisoblash hech qanday qiyinchilik tug‘dirmaydi.



2.2. Takrorlanuvchi guruhlashlarga doir topshiriqlar

Teorema. n ta elementdan k ta elementli takrorlanuvchi guruhlashlar soni

$$f_n^k = C_{n+k-1}^{n-1} = C_{n+k-1}^{k}$$

ta bo‘ladi.

2.2.1. *0,1,2,3,4,5,6* raqamlaridan iborat DOMINO o‘yini toshlari nechta?

2.2.2. *0,1,2,…,k* raqamlaridan iborat DOMINO o‘yini toshlari nechta?

2.2.3. Qandalotchilik sexida 11 turdagi shirinlik mavjud. 6 ta bir xil yoki 6 ta har xil shirinlikni necha xil usulda tanlash mumkin?

2.2.4. Muzqaymoq do‘konida 8 xil turdagi muzqaymoq sotilayapti. 5 kishiga necha xil usulda muzqaymoq olish mumkin?

2.2.5. Asaka avtomobil zavodi tayyor mahsulotlar maydonchasida 15 xil rangdagi NEXIA avtomobillari turibdi. Mashina tashiydigan trallerga 8 ta mashina sig‘sa, necha xil usulda NEXIA avtomobillarini trallerga yuklash mumkin?

2.2.6. TATU da barcha viloyatlardan talabalar o‘qishadi. 5 ta talabadan iborat guruhni necha xil usulda tuzish mumkin?

Masala: Quyida berilgan tengsizliklar nechta musbat butun yechimga ega?

2.2.7. $3<x+y+z+v+w\le 7$

2.2.8. $6<x+y+z+v<10$

2.2.9. $5<x+y+z+v\le 8$

2.2.10. $11<x+y+z+v+w+t\le 14$

2.2.11. $6<x+y+z+v+w\le 10$

2.2.12. $9<x+y+z\le 12$

2.2.13. $8<x+y+z+v+w+t\le 12$

2.2.14. $3<x+y+z+v+w<6$

2.2.15. $4<x+y+z\le 9$

2.2.16. $10<x+y+z\le 14$

2.2.17. $2<x+y+z+v<5$

2.2.18. $5<x+y+z+v\le 8$

2.2.19. $2<x+y+z+v+w+t\le 5$

2.2.20. $6<x+y+z\le 9$

2.2.21. $5<x+y\le 9$

2.2.22. $2<x+y+z<5$

2.2.23. $3<x+y+z+v\le 7$

2.2.24. $8<x+y+z+v<12$

2.2.25. $2<x+y+z+v+w<6$

2.2.26. $3<x+y+z<7$

2.2.27. $11<x+y+z\le 15$

2.2.28. $5<x+y+z+v+w+t<10$

2.2.29. $7<x+y+z+v+w+t+m\le 11$

2.2.30. $9<x+y+z+v+w+t\le 12$

2.2.Takrorlanuvchi guruhlashlarga doir topshiriq(na’muna)

2.5.0. Bog‘dagi besh xil turdagi guldan 3 tadan qilib necha xil usulda buket yasash mumkin?

2.2.Topshiriqning bajarilishi bo’yicha na’muna

2.5.0. Bog‘dagi besh xil turdagi guldan 3 tadan qilib necha xil usulda buket yasash mumkin?

$f_5^3 = C_{5+3-1}^{5-1} = C_{5+3-1}^{3} = C_7^3 = \frac{7!}{3!*4!} = 35$ usulda buket yasash mumkin.

2.3. Kombinator tenglamalarga doir topshiriqlar

2.3.1. $A_{2x-1}^{x-1} \cdot P_x = x \cdot P_{2x-1}$

2.3.2. $\left(C_x^0\right)^2 + \left(C_x^1\right)^2 + \left(C_x^2\right)^2 = 5A_7^2$

2.3.3 $C_{x-2}^{x-3} : C_x^{x-1} = A_{x-1}^{x-4} : 30$

2.3.4. $A_x^{x-3} = \left(C_{x-1}^{x-3} + C_{x-1}^{x-4}\right)P_3$

2.3.5 $A_{x+1}^2 \cdot A_x^2 \cdot A_{x-1}^2 = P_3 \cdot P_{x+1}$

2.3.6 $A_x^3 = P_{x-2} + C_x^4 - P_{x-1} = 39$

2.3.7 $A_x^4 \cdot P_{x-4} = 42 \cdot P_{x-2}$

2.3.8 $1{,}5 \cdot C_x^{x-2} = 0{,}5 \cdot A_{x+1}^{x-1}$

2.3.9 $P_x = C_x^{x-2} \cdot P_4 \cdot 2!$

2.3.10 $A_x^{x-6} = x \cdot C_{x-1}^{x-6}$

2.3.11 $120 \cdot A_{2x}^{x} = (\underline{P}_x)^2 \cdot C_{2x}^{x}$

2.3.12 $3 \cdot \underline{P}_x \cdot \underline{P}_5 = x^2 \cdot A_x^{x-4}$

2.3.13 $\underline{P}_x \cdot C_x^{x-4} = C_{x-2}^{x-4} \cdot C_x^{x-2}$

2.3.14 $A_{x+2}^7 \cdot \underline{P}_{x-5} = (\underline{P}_5 - 10) \cdot \underline{P}_x$

2.3.15 $X \cdot \underline{P}_{x+2} \cdot C_{x-1}^{x-3} = 3 \cdot \underline{P}_x \cdot C_{x+2}^2 \cdot A_{x-1}^2$

2.3.16 $\underline{P}_{x+5} = \underline{P}_2 \cdot \underline{P}_3 \cdot \underline{P}_5 \cdot A_{x+3}^{x-3}$

2.3.17 $\underline{P}_{2x+1} = A_{2x-1}^3 \cdot \underline{P}_{2x-4} \cdot 110$

2.3.18 $C_x^{x-3} : C_x^{x-1} = A_{x-1}^{x-4} : A_{x-2}^{x-4}$

2.3.19 $(C_5^x - C_4^x) \cdot A_5^x = x \cdot C_4^x \cdot C_5^x$

2.3.20 $C_{x+3}^{x+1} = C_{x+1}^{x-1} + C_{x+1}^{x} + C_x^{x-2}$

2.3.21 $\underline{P}_x \cdot C_{x+3}^x = A_{x+3}^x$

2.3.22 $A_x^3 + A_{x+1}^4 = \underline{P}_x \cdot C_x^{x-1} \cdot 0{,}7$

2.3.2 $\underline{P}_x + 4P_{x+5} = A_x^2 \cdot C_{x+1}^{x-1}$

2.3.24 $A_x^{x-1} \cdot C_x^{x-2} \cdot C_x^{x-3} \cdot C_x^{x-4} =$
$= (C_x^{x-1} \cdot C_x^{x-3})^2 \cdot \underline{P}_x$

2.3.25 $C_x^{x-1} \cdot A_x^{x-3} = \underline{P}_x \cdot C_x^{x-3}$

2.3. Кombinator tenglamalarga doir topshiriqlar

2.3.0. $12C_{x+3}^{x-1} = 55A_{x+1}^2$

2.3.Topshiriqning bajarilishi bo'yicha na'muna

2.3.0. $12C_{x+3}^{x-1} = 55A_{x+1}^2$

Tenglamani yechish uchun $C_n^k = \dfrac{n!}{k!*(n-k)!}$, $A_n^k = k!*C_n^k = \dfrac{n!}{(n-k)!}$ va x birdan katta natural son bo'lishi mumkinligini e'tiborga olib, tenglamada qatnashgan mos koeffitsiyentlarni yuqoridagi formulalarga asoslanib yoyib chiqamiz:

$$12 * \frac{(x+3)!}{(x-1)!*(x+3-(x-1))!} = 55 * \frac{(x+1)!}{(x+1-2)!}$$

Soddalashtiramiz, surat va maxrajlarda qisqarishi mumkin bo'lgan faktoriallarni qisqartiramiz.

$$12 * \frac{(x+3)*(x+2)*(x+1)*x}{4!} = 55 * (x+1) * x$$

Tenglamaning ikkala tomonini *x*(x+1)* ga qisqartiramiz, 12 bilan 4!=1*2*3*4=24 ni qisqartirib, tenglamada ayrim shakl almashtirishlarni amalgam oshirib, quyidagi ko'rinishga olib kelamiz:

$$\frac{(x+3)*(x+2)}{2} = 55;$$

$$(x+2)(x+3) = 55*2 = 110 = 10*11.$$

Kvadrat tenglama yechimlari x_1=-13 bizning shartni (x>1) bajarmaydi Ø, x_2=8 yechim esa kombinator tenglamamiz yechimi bo'ladi.