12-AMALIY MASHG’ULOT. Mukammal diz’yunktiv normal shakldagi bul funksiyalarini karno kartalari orqali soddalashtirish. Ikkilik mantiqiy amallariga mos sxemalar tuzish

Reja:

1. Ikkilik mantiqiy amallariga mos sxemalar tuzishga oid tushunchalar
2. Mustaqil bajarish uchun masala va topshiriqlar

2.1. Mantiqiy formulalarga mos Rele kontakt sxemalarini tuzing

1. Rele - kontaktli sxemalar (RKS).

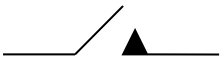
Muloxazalar algebrasining formulalarini RKS yordamida realizatsiya (ifodalash) qilish.

Aftomatik boshkarish kurilmalari va EXM larda yuzlab va minglab rele, elektron lampa, yarim o‘tkazgich va magnit elementlarini uz ichiga olgan rele – kontakti va elektron – lampa sxemalar uchraydi. Bu sxemalar avtomatik boshkarish kurilmalari va EXM tarki

bida benixoya katta tezlikda juda murakkab operatsiyalar bajarishda bevosita ishtirok etadilar va avtomatlarning barcha ish faoliyatini boshkarib turadilar.

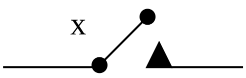
Biz kuyida Bul algebrasining yana bir modeli – rele kontakt sxemasi (RKS) bilan tanishamiz.

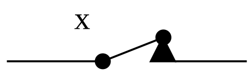
Xar bir kontakt fizik kurilma bulgan rele bilan birga boglangan bulib, rele kontaktni yopadi (ulaydi, yani kontakt orkali tok utadi) yoki ochadi (uzadi, yani kontakt orkali tok utmaydi). Biz rele – kontakt kurilmasini kontakt deb ataymiz.

Bundan buyon  sxema yordamida kontaktning xam yopik, xam ochik xolatlarini belgilaymiz.

11.1 va 11.2 – shakldagi eng oddiy sxemalarni mos ravishda ***ochik*** va ***yopik*** kontakt deyiladi. Tabiiyki, xar bir kontakt

fakat ikkita xolatda bulishi mumkin : “ochik” va “yopik” yoki “tok o‘tkazmaydi” va “tok o‘tkazadi”.





12.1 – shakl. 11.2 – shakl.

Kontaktlarning bunday xolatlari muloxazalarning xam ikki xolatda yani 0 va 1 xolatdan bulishini eslatadi. Shunday kilib, kontaktning ochik xolatiga muloxazalarning “yolgon” – “0” qiymatini, yopik xolati esa muloxazaning “rost” – “1” qiymatini mos kuyish mumkin.

Demak, barcha kontaktlar bilan barcha elementar muloxazalar orasida uzaro bir qiymatli moslik mavjud ekan. Bundan buyon x muloxazaga mos keluvchi kontaktni xam shu xarf bilan belgilaymiz.

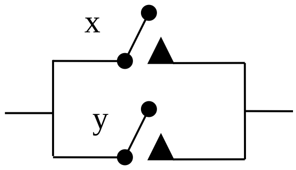
Kontaktlar ustida quyidagi operatsiyalarni kiritamiz:

Kontaktlar kompozitsiyasi.

12.1-Ta'rif. x va u kontaktlarning kompozitsiyasi deb, ularni ketma – ket ulash natijasida xosil buladigan ushbu sxemaga aytiladi:

Kontaktlar yigindisi.

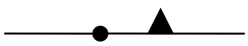
12.2-Ta'rif. x va u kontaktlarning yigindisi deb, ularni parallel ulash natijasida xosil buladigan ushbu sxemaga aytiladi:



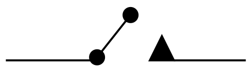
Qarama – qarshi kontakt.

12.3-Ta'rif. x kontaktga qarama – qarshi kontakt deb, x kontakt yopik bulganda ochik, x kontakt ochik bulganda yopik bo'luvchi kontaktga aytiladi.

Quyidagi kontaktlar mos ravishda doimo yopik va doimo ochik kontaktlar deyiladi:



yopik – 1



ochiq - 0

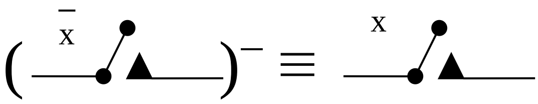
Yuqorida keltirilgan operatsiyalar yordamida uzgaruvchilari kontaktlardan iborat bulgan funksiyalar tuzish mumkin.

Xakikatan, kontaktning yopik xolatini 1, ochik xolatini esa 0 bilan belgilaylik. Bir necha kontaktlardan parallel va ketma – ket ulash natijasida tuzilgan murakkab sxema kontaktlar funksiyasi bulib, bu funksiya xam 1 yoki 0 qiymat kabul kiladi. Bundan tashkari, x va u uzgaruvchilarga mos ravishda x va u kontaktlar mos kelsa, u xolda ularning konyuksiyasi va dizyunksiyasiga kontaktlarni ketma – ket va parallel ulashdan iborat sxema mos kelishi ravshandir. Shunday kilib, muloxazalar algebrasining xar bir keltirilgan formulasiga (inkor amali fakat uzgaruvchilar ustida kelsa) mos kantaklardan tuzilgan yagona sxema mos kelar yekan. Malumki muloxazalar algebrasining xar bir formulasini keltirilgan formula kurinishiga keltirish mumkun. (teorema)

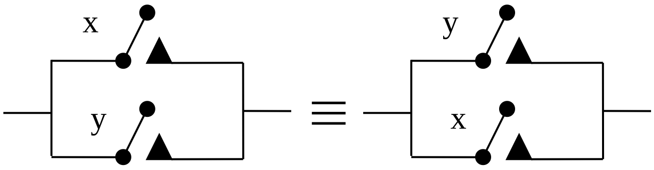
Bu esa kontaktlar tuplamida muloxazalar algebrasida malum bir manoda yekvivalent bulgan sxema kurish mumlun yekanligini kursatadi.

Kontaklar algebrasi deb ataluvchi bu algebraning bazi tengkuchliliklarini kurib chikaylik:

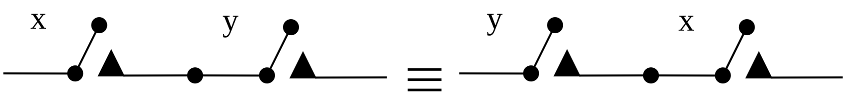
1. $\overline{(\bar{x})} \equiv x$, $\bar{\bar{x}} = x$. Ya’ni,



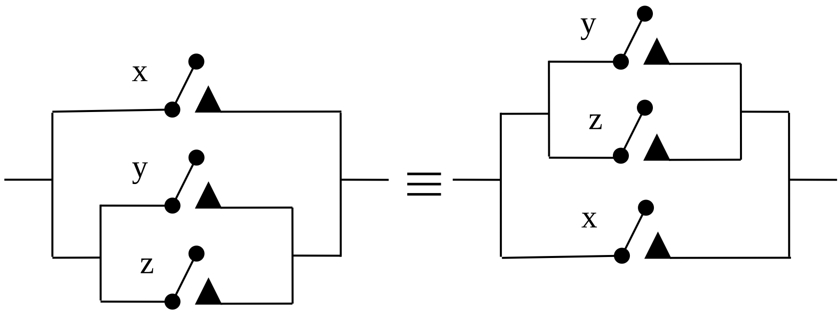
2. Kontaktlar yig‘indisi kommutativ operatsiyadir. xvy=yvx.



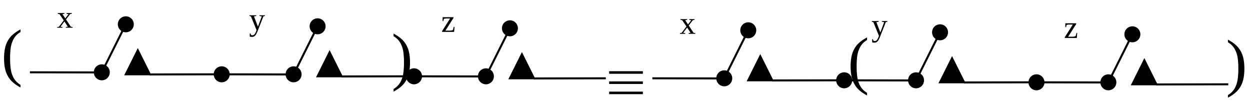
3. Kontaktlar ko‘paytmasi kommutativdir. xy = yx.



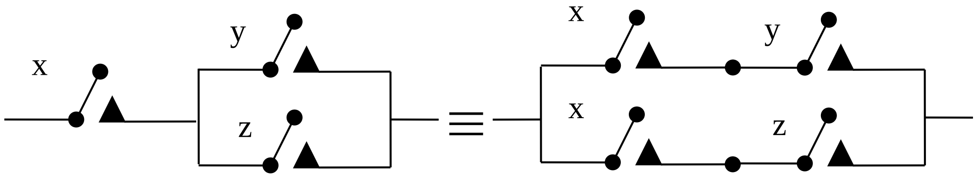
4. Kontaktlar yig‘indisi assotsiativlik xossasi. xv(yvz) = (xvy)vz.



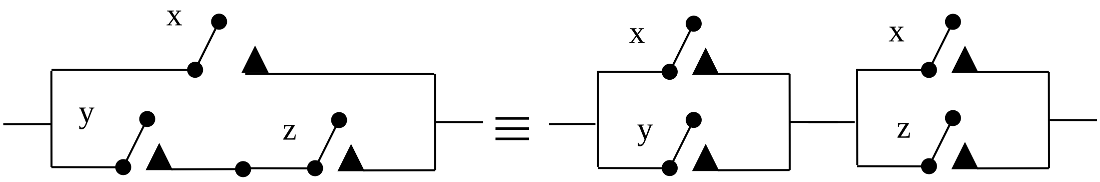
5. Kontaktlar kompozitsiyasi assotsiativligi. (xy)z=x(yz).



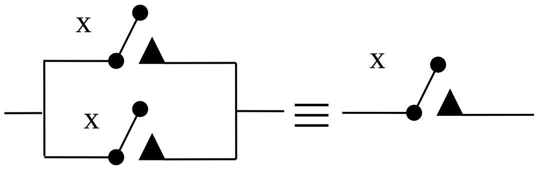
6. Kontaktlar kompozitsiyasi yig‘indiga nisbatan distributivdir. x(yvz)=xy v xz.



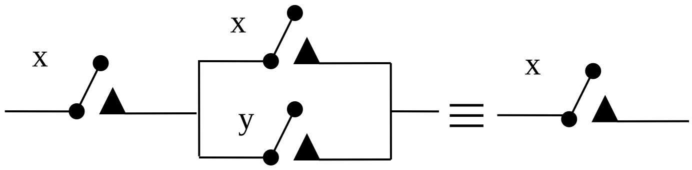
7. Kontaktlar yig‘indisi kompozitsiyasiga nisbatan disributivdir. xv(yz) = (xvy)(xvz).



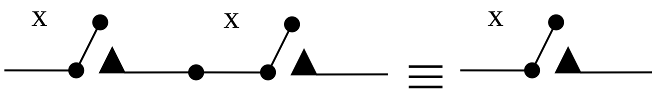
8. Kontaktlar yig‘indisi idempomentdir. x v x=x.



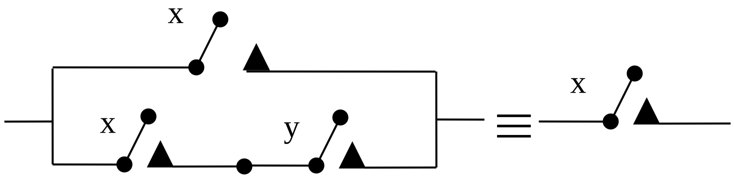
9. Kontaktlar yig‘indisi uchun yutilish qonuni. x(xvy)=x.



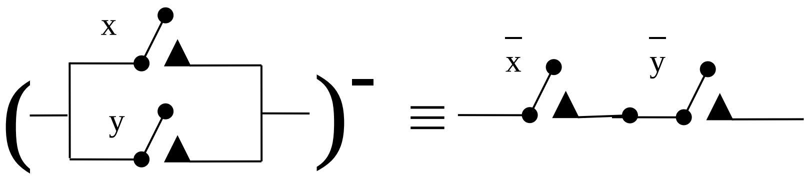
10. Kontaktlar kompozitsiyasi idempotent operatsiyadir. xx=x.



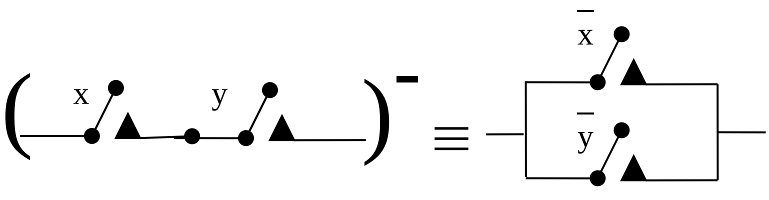
11. Kontaktlar kompozitsiyasi uchun yutilish qonuni. xv(xy)=x.



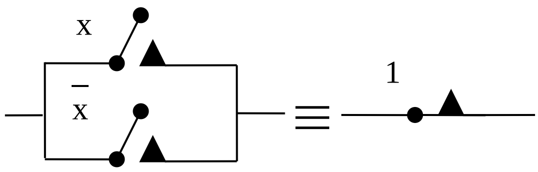
12. Kontaktlar yig‘indisi uchun De Morgan qonuni. $\overline{x v y} = \bar{x}\ \bar{y}$.



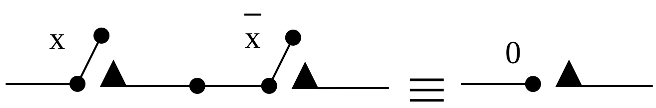
13. Kontakt kompozitsiyasi uchun De Morgan qonuni. $\overline{x y} = \bar{x} v \bar{y}.$



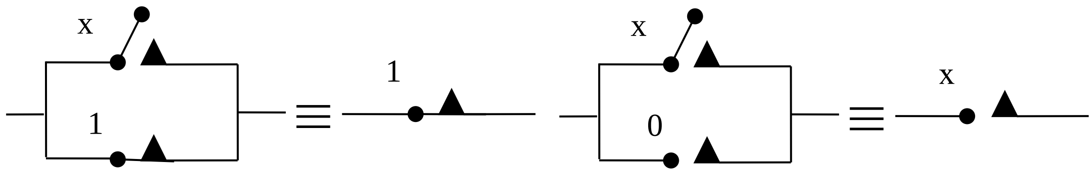
14. Uchinchisini inkor etish qonuni. $x v \bar{x} = 1$.



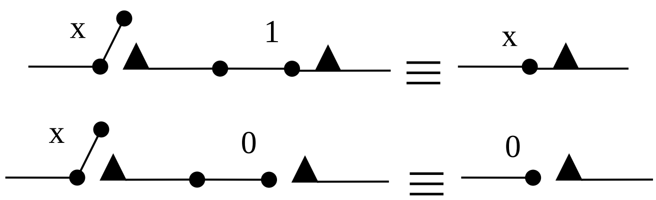
15. Qarama - qarshilik qonuni. $x\,\bar{x} = 0$.



16.



a) x v 1 = 1; b) x v 0 = x;



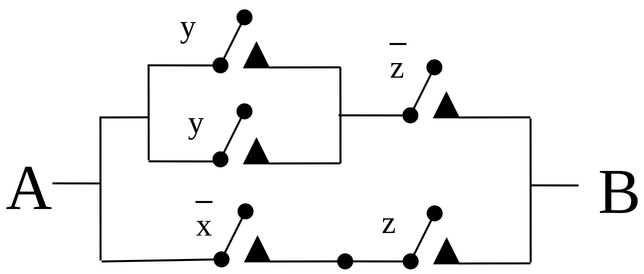
v) x * 1 = x; g) x * 0 = 0.

Yuqorida keltirilgan qonunlar barcha kontaktlar to‘plami kontaktlarni ketma-ket ulash, parallel ulash va kontaktga qarama-qarshi kontaktni topish operatsiyalariga nisbatan Bul algebrasi tashkil etishini ko‘rsatadi.

Yuqorida mulohazalar algebrasining har bir keltirilgan U(x_1, x_2, …, x_n) formulasiga x_1, x_2, …, x_n o‘zgaruvchi kontaktlardan tuzilgan yagona sxema mos kelishi aytilgan edi.

Aksincha, kontaktlarning ixtiyoriy sxemasiga mulohazalar algebrasining ma’lum bir keltirilgan formulasi mos keladi.

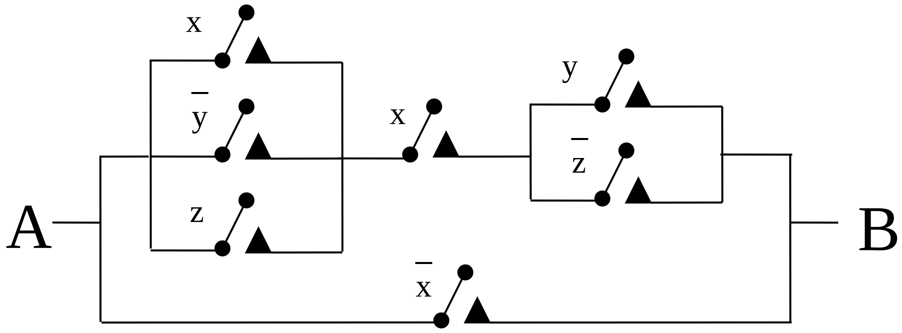
12.1-Misol 1. $U(x, y, z) = (x\,v\,y)\,\bar{z}\,v\,\bar{x}\,z$ keltirilgan formulaga mos keluvchi RKS tuzilsin.



Ushbu sxemadan ko‘rinib turibdiki, x=1, y=0, z=1 bo‘lganda AV kontaktdan tok o‘tmaydi.

Agar x=0, y=1, z=0 bo‘lsa, u holda AV kontaktdan tok o‘tadi.

12.2-Misol Quyidagi RKS ga mos keluvchi mulohazalar algebrasining keltirilgan formulasini yozing.



Bu sxemada $x, \bar{y}, z$ kontaktlar parallel ulanganligi uchun ularga mos keluvchi $x, \bar{y}, z$ o‘zgaruvchilarning dizyunsiyasi olinadi, ya’ni $x \vee \bar{y} \vee z$. Xuddi shunga o‘xshash ketma-ket:

$(x \vee \bar{y} \vee z)\,x$, $(x \vee \bar{y} \vee z)\,x\,(y \vee \bar{z})$, $(x \vee \bar{y} \vee z)\,x\,(y \vee \bar{z}) \vee \bar{x}$ formulasini hosil qilamiz.

12.4-Ta’rif. Mulohazalar algebrasining keltirilgan formulasiga mos keluvchi RKS ni tuzish mazkur formulani RKS yordamida realizatsiya qilish deyiladi.

12.5-Ta’rif. RKS ning vazni deb unga kirgan barcha kontaktlar soniga aytiladi. Misol 2 dagi sxemaning vazni 7 ga teng.

12.6-Ta’rif. Mulohazalar algebrasining U(x_1, x_2, …, x_n) formulasining uzunligi deb, bu formulaga kirgan barcha o‘zgaruvchilar soniga aytiladi (bunda o‘zgaruvchi va uning inkori turli o‘zgaruvchilar deb qaraladi).

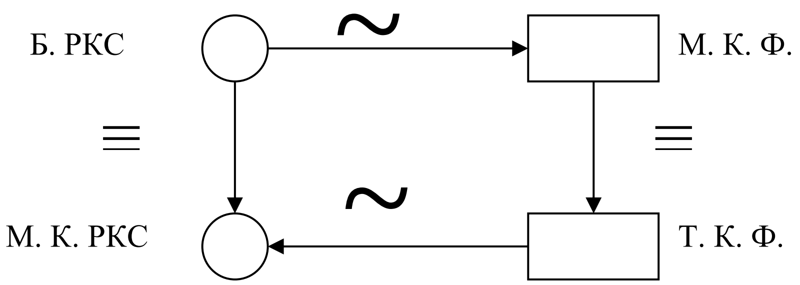
12.3-Misol. $U(x, y, z) = ((x \vee \bar{y} \to \bar{z}) \vee \bar{x}\, y \vee \bar{z}$ formulaning uzunligi 6 ga tengdir.

Har qanday RKS uchun quyidagi masalani qo‘yish mumkin.Berilgan RKS ni shunday RKS bilan almashtirish kerakki, ular teng kuchli bo‘lsin, hamda keyingi RKS ning vazni berilgan RKS ning vaznidan kichik bo‘lsin.

Bu masala RKS ni **minimizatsiyalash masalasi** deyiladi. Albatta, murakkab va katta vaznga ega bo‘lgan sxemalar uchun minimizatsiyalash masalasini bevosita sxemaning o‘zini shakl almashtirib yechish qiyin.

Bu masala odatda, berilgan RKS ga mos keluvchi mulohazalar algebrasining formulasini eng qisqa uzunlikka ega bo‘lgan teng kuchli formula bilan almashtirish yordamida yechiladi.

Ushbu protsessni sxematik ravishda quyidagicha ifodalash mumkin:

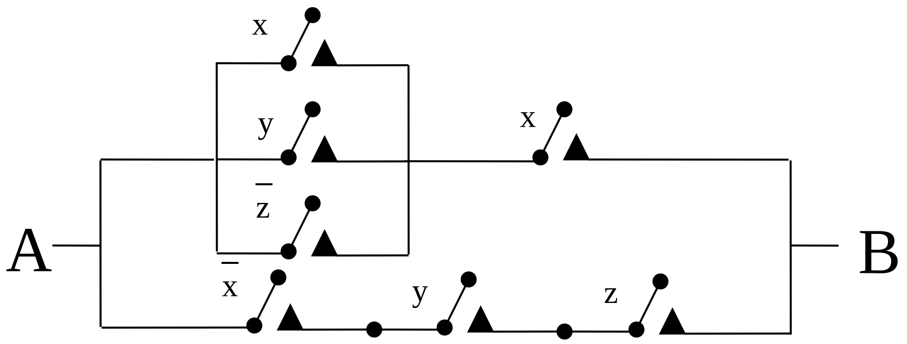


Bunda:

B.RKS – berilgan RKS
M.K.F – mos keluvchi formula
TKF – teng kuchli formula
MK.RKS – mos keluvchi RKS
~ - mos qo‘yish
≡ - teng kuchlilik munosabati

12.4-Misol.

Ushbu RKS ni minimizatsiyalang.



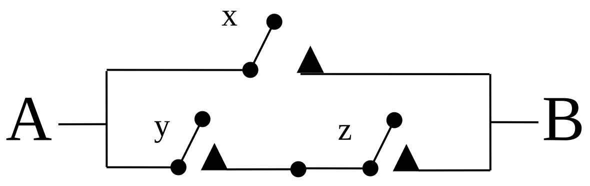
Maskur RKS ga mos keluvchi formula $U(x,y,z)=(x\,v\,y\,v\,\bar{z})\,x\,v\,\bar{x}\,y\,z.$ Berilgan kuchli almashtirishlar yordamida bu formulani quyidagi formula bilan almashtirish mumkin:

$$U(x,y,z)=(xvyv\bar{z})xv\bar{x}yz=xxvxyvx\bar{z}v\bar{x}yz=xvxyvx\bar{z}v\bar{x}yz=$$

$$=xvx\bar{z}v\bar{x}yz=xv\bar{x}yz=(xv\bar{x})(xvyz)=1*(xvyz)=xvyz;$$

Demak, $U(x,y,z)=x\,v\,y\,z;$

Hosil bo‘lgan formulaga mos keluvchi RKS:



Shunday qilib, bazni 7 ga teng bo‘lgan berilgan RKS ni vazni 3 ga teng bo‘lgan RKS bilan almashtirish mumkin emas.

2. Mustaqil bajarish uchun masala va topshiriqlar

2.1.Quyidagi mantiqiy formulalarga mos Rele kontakt sxemalarini tuzing.

2.1.1. $x \wedge (x \to y)$;

2.1.2. $(\overline{xy} \to \bar{x}) \wedge (\overline{xy \to \bar{y}})$;

2.1.3. $(x \to y) \to (y \to x)$;

2.1.4. $(x \vee \bar{z}) \to y \wedge z$;

2.1.5. $(x \vee \bar{y} \to x \wedge z) \to (\overline{x \to \bar{x}}) \vee y \wedge \bar{z}$;

2.1.6. $(ab \to bc) \to ((a \to b) \to (c \to b))$;

2.1.7. $(\bar{a} \to c) \to ((\overline{\bar{b} \to \bar{a}})$;

2.1.8. $(\bar{a} \to \bar{b}) \to ((bc \to ac)$.

2.1.9. $\overline{x\bar{y}} \leftrightarrow \bar{x} \vee xy$;

2.1.10. $(x \leftrightarrow y) \wedge (x\bar{y} \vee \bar{x}y)$;

2.1.11. $xy \to (x \to \bar{y})$;

2.1.12. $x \vee y \to (x \leftrightarrow y)$;

2.1.13. $x \vee y \to z$;

2.1.14. $(x \to z)(y \to z) \to (x \to y)$.

2.1.15. $(x \to x) \to z$;

2.1.16. $x \to (x \to y)$;

2.1.17. $\overline{\bar{x} \cdot \bar{y}} \vee (x \to y) \cdot x$;

2.1.18. $(x \leftrightarrow y) \wedge (x \vee y)$;

2.1.19. $(x \to y)(y \to z) \to (z \to x)$

2.1.20. $\left(x \vee \bar{y} \to \left(z \to y \vee \bar{y} \vee x\right)\right) \wedge \left(x \vee \overline{x \to \left(x \to x\right)}\right)$;

2.1.21. $\left(x \wedge \overline{x \wedge \bar{x} \to y \wedge \bar{y}} \to z\right) \vee x \vee (y \wedge z) \vee (y \wedge z)$;

2.1.22. $\left(x \wedge \left(y \vee z \to y \vee z\right)\right) \vee \left(y \wedge x \wedge \bar{y}\right) \vee x \vee \left(y \wedge \overline{x \wedge \bar{x}}\right)$;

2.1.23. $(x \to y)(y \to z) \to (x \to z)$;

2.1.24. $(x \wedge z) \vee (x \wedge \bar{z}) \vee (y \wedge z) \vee (\bar{x} \wedge y \wedge z)$.

2.1.25. $(x \to y) \& (y \to z) \to (x \to z)$

2.1.26. $((p \to q) \& (q \to r)) \to (p \to r)$

2.1.27. $(x \to y) \& (y \to z) \to (z \to x)$

2.1.28. $(x \vee \bar{y} \to (z \to y \vee \bar{y} \vee x)) \& (x \vee \overline{x \to (x \to x)}) \to y$

2.1.29. $((p \wedge q) \leftrightarrow q) \leftrightarrow (q \to p)$

2.1.30. $((p \to q) \& (q \to r)) \to (p \to r)$