**10-AMALIY MASHG’ULOT. Bul funksiyalari uchun diz’yunktiv va kon’yunktiv normal shakllar (DNSh, KNSh). Mukammal diz’yunktiv va mukammal kon’yunktiv normal shakllar (MDNSh, MKNSh)**

**Reja:**

1. Bul funksiyalari uchun normal shakllar(DNSh, KNSh, MDNSh, MKNSh).
2. Mustaqil bajarish uchun masala va topshiriqlar

2.1. Formulalarni KNSh va DNSh ko‘rinishga keltiring

2.2. Formulalarni MKNSh va MDNSh ko‘rinishga keltiring

## 1. Formulalarning normal shakllari

**Mulohazalar algebrasida funksiya tushunchasi.** Oddiy algebradagi funksiya tushunchasiga o‘xshash, mulohazalar algebrasida ham **funksiya** tushunchasi kiritilishi mumkin.

Ma’lumki, oddiy algebrada funksiyaning qiymatlari turli usullar vositasida, masalan, jadval yordamida berilishi mumkin. Mulohazalar algebrasida ko‘pchilik tushunchalarni ifodalashda Chinlik jadvallari qulay vosita hisoblanadi. Chinlik jadvallarida faqat ikkita o‘zgarmas (0 va 1) ishtirok etadi. Shu tufayli $E_2 = \{0,1\}$ deb belgilaymiz.

**10.1-ta’rif.** $n$ ta Bul o‘zgaruvchisiga bog‘liq bo‘lgan $f(x_1, x_2, ..., x_n)$ funksiyaga Bul funksiyasi deyiladi. Bul funksiyalarining aniqlanish va qiymatlari sohasi $\{0, 1\}$ to‘plamdan iboratdir.

Istalgan Bul funksiyasini chinlik jadvali orqali berish mumkin, bunda o‘zgaruvchilarning mumkin bo‘lgan barcha qiymatlari to’plamiga mos mantiqiy qiymat beriladi.

O‘zgaruvchilarning mumkin bo‘lgan barcha qiymatlari to’plamida aynan bir xil qiymat qabul qiluvchi ikkita Bul funksiyasi teng kuchli funksiyalar deb ataladi.

Bitta o‘zgaruvchiga bog‘liq bo‘lgan Bul funksiyalarini chinlik jadvalini quramiz (10.1-jadval):

10.1-jadval

| $x$ | $f_1(x)$ | $f_2(x)$ | $f_3(x)$ | $f_4(x)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 |

Jadvaldan ko‘rinib turibdiki bitta o‘zgaruvchiga bog‘liq to‘rtta funksiya mavjud.

$f_1(x)$ va $f_4(x)$ mos ravishda 0 va 1 ga teng bo‘lgan o‘zgarmaslar deb ataladi.

$f_2(x)$ funksiya ayniy funksiya deyiladi:

$$f_2(x) = x.$$

$f_3(x)$ funksiya $x$ o‘zgaruvchiga teskari qiymatlarni qabul qiladi va $x$ ning inkori deb ataladi, $\bar{x}$ ko‘rinishda belgilanadi:

$$f_3(x)=\bar{x}.$$

Ikkita o‘zgaruvchiga bog‘liq bo‘lgan $f_i(x_1;x_2)=f_i$, $i=1$, …, 16 Bul funksiyalarini chinlik jadvalini quramiz (6.2-jadval):

10.2-jadval

| $x_1$ | $x_2$ | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ | $f_6$ | $f_7$ | $f_8$ | $f_9$ | $f_{10}$ | $f_{11}$ | $f_{12}$ | $f_{13}$ | $f_{14}$ | $f_{15}$ | $f_{16}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |

Jadvaldan ko‘rinib turibdiki ikkita o‘zgaruvchiga bog‘liq bo‘lgan funksiyalar soni 16 ta, $n$ o‘zgaruvchiga bog‘liq bo‘lgan funksiyalar soni $2^{2^n}$ taga teng bo‘ladi.

10.2-jadvaldan ko'rinib turibdiki, ikkita o'zgaruvchiggabog’liq funktsiyalar bitta o'zgaruvchiga bog'liq bo'lgan funktsiyalarni o'z ichiga oladi.

$f_1=0$ va $f_{16}=1$ funksiyalar mos ravishda 0 va 1 o‘zgarmaslarni beradi.

$f_4$, $f_6$, $f_{11}$, $f_{13}$ funksiyalar bitta o‘zgaruvchiga bog‘liq: $f_4=x_1$, $f_6=x_2$ – *o‘zgaruvchini o‘zini qiymatiga teng,* $f_{11}=\overline{x_2}$, $f_{13}=\overline{x_1}$ – *o‘zgaruvchilarning inkorlariga teng*.

Qolgan funksiyalarning ko‘rinishlarini yozib chiqamiz:

$f_2=x_1\wedge x_2$ – *konyunksiya,*

$f_8=x_1\vee x_2$ – *dizyunksiya,*

$f_{10}=x_1\leftrightarrow x_2$ – *ekvivalensiya,*

$f_7=x_1\oplus x_2$ – *ikki modul bo‘yicha qo‘shish yoki Jegalkin amali,*

$f_{12}=x_2\to x_1$ – *konversiya,*

$f_{14}=x_1\to x_2$ –*implikatsiya,*

$f_{15}=x_1\,|\,x_2$ –*Sheffer shtrixi,*

$f_9=x_1\downarrow x_2$ –*Pirs strelkasi,*

$f_3$ va $f_5$ funksiyalar implikatsiya va konversiyaga teskari hisoblanadi.

Bir va ikki o‘zgaruvchiga bog‘liq bo‘lgan bul funksiyalari elementar funksiyalar hisoblanadi.

*To‘plamlar ustida bajariluvchi amallar xossalari va Bul funksiyalarining xossalari orasida bog‘lanish mavjud:*

1. Birlashma va kesishmaning idempotentligi:

$$A\cup A=A, \qquad A\cap A=A,$$

xususiy xolda

$$A \cup \acute{\varnothing} = A, \quad A \cap \acute{\varnothing} = \acute{\varnothing}, \quad A \cup U = U, \quad A \cap U = A.$$

*Dizyunksiya vakonyunksiyaning idempotentligi:*

$$x \vee x = x, \qquad x \wedge x = x,$$

*xususiy holda*

$$x \vee 0 = x, \quad x \wedge 0 = 0, \quad x \vee 1 = 1, \quad x \wedge 1 = x.$$

2. Birlashma va kesishmaning kommutativligi:

$A \cup B = B \cup A$, $A \cap B = B \cap A$.

*Dizyunksiya vakonyunksiyaning* kommutativligi*:*

$$x \vee y = y \vee x, \quad x \wedge y = y \wedge x.$$

Kommutativlik ikki modul bo‘yicha qo‘shish, Pirs strelkasi va SHeffer shtrixi amallariga ham xos xususiyatdir. O‘zgaruvchilarning o‘rni almashishi funksiyaning qiymatiga ta’sir qilmaydi.

3. Birlashma va kesishmaning assotsiativligi:

$$A \cup (B \cup C) = (A \cup B) \cup C, \quad A \cap (B \cap C) = (A \cap B) \cap C.$$

*Dizyunksiya vakonyunksiya*assotsiativligi*:*

$$x \vee (y \vee z) = (x \vee y) \vee z, \quad x \wedge (y \wedge z) = (x \wedge y) \wedge z.$$

Assotsiativlik *dizyunksiya vakonyunksiyaning* bajarilish tartibi farqlanmasligini bildiradi, ikki modul bo‘yicha qo‘shish amali ham assotsiativlik qoidasiga bo‘ysunadi .

4. Birlashmaning kesishmaga nisbatan distrubutivligi va aksincha:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C), \qquad A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

*Dizyunksiyaningkonyunksiyaga* nisbatan distrubutivligi va aksincha*:*

$$x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z), \quad x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z).$$

5. Birlashma va kesishmaning yutilishi:

$$(A \cap B) \cup A = A, \qquad (A \cup B) \cap A = A.$$

*Dizyunksiya vakonyunksiyaning yutilishi:*

$$(x \wedge y) \vee y = y, \quad (x \vee y) \wedge y = y.$$

Yutilish qonunlari Bul funksiyalarini soddalashtirish imkonini beradi.

6. Involyutivlik (ikki karrali to‘ldiruvchini aniqlash):

$$\overline{\overline{A}} = A.$$

*Ikki karrali inkor qoidasi:*

$$\overline{\overline{x}} = x.$$

7. de Morgan qonuni:

$$\overline{A \cap B} = \overline{A} \cup \overline{B}, \qquad \overline{A \cup B} = \overline{A} \cap \overline{B}.$$

$$\overline{x \wedge y} = \overline{x} \vee \overline{y}, \qquad \overline{x \vee y} = \overline{x} \wedge \overline{y}.$$

de Morgan qonunlari dizyunksiyavakonyunksiya orasidagi bog‘lanishni ifodalaydi.

8. To‘ldiruvchi qonuni:

$$A \cup \overline{A} = U, \qquad A \cap \overline{A} = \varnothing.$$

*Tavtologiya yoki uchinchisi istesno qonuni:*

$$x \vee \overline{x} = 1.$$

*Muvofiqlik qonuni:*

$$x \wedge \bar{x} = 0.$$

Bul funksiyalari ustida bajariladigan amallar tartibi: eng kuchli amal – inkor, undan keyinkonyunksiya, so‘ngra – dizyunksiya, so‘ngra – implikatsiya, so‘ngra – ekvivalensiya. Qolgan amallarning bajarilish tartibi qavslar bilan ajratib ko‘rsatiladi. Konyunksiya amali algebraik ko‘paytma ko‘rinishida ham ifodalanishi mumkin. Masalan de Morgana qonunlarini quyidagicha ifodalash ham mumkin: $\overline{xy} = \bar{x} \vee \bar{y}$, $\overline{x \vee y} = \bar{x}\bar{y}$.

Matematik mantiqning 1–8 va boshqa to‘plamlar nazariyasiga bog‘liq qonunlarni quyidagi usullarda isbotlash mumkin:

*– tengliklarning ikkala tomoni uchun Eylera–Venna diagrammalarini tasvirlab ularning tengligini ko‘rsatish;*

*– chinlik jadvali yordamida;*

*– Quyidagi sxema bo‘yicha formal mulohaza yuritish bilan*.

Aytaylik to‘plamlar nazariyasi bo‘yicha $M = N$, bu erda $M$ va $N$ – qandaydir to‘plamlar.

Isbotning birinchi qismi, agar biron-bir element $M$ to'plamga tegishli bo'lsa, u to'plamga $N$ ham tegishli ekanligini ko'rsatishdir. Bu $M \subset N$ munosabatning to'g'riligini isbotlaydi.

Isbotning ikkinchi qismida, agar biron-bir element $N$ to'plamga tegishli bo'lsa, u to'plamga $M$ ham tegishli ekanligini ko'rsatish kerak. Bu esa $N \subset M$ munosabatning to'g'riligini isbotlaydi va $M = N$ ekanligi kelib chiqadi.

**10.1-misol**. $f(x_1, x_2, x_3) = x_1 \oplus x_2 \to \overline{x_3} \vee x_1 \mid (\overline{x_2} \wedge \overline{x_1})$ Funksiyani chilik jadvalini tuzing va ikkilik son ko’rinishiga keltiring.

**Yechimi**. Dastlab amallarni bajarilish tartibini aniqlab olamiz:

$$f_1 = x_1 \oplus x_2, \quad f_2 = \overline{x_2} \wedge \overline{x_1}, \quad f_3 = x_1 \mid f_2, \quad f_4 = \overline{x_3} \vee f_3, \quad f_5 = f_1 \to f_4$$

.

Chinlik jadvalini hosil qilingan qism funksiyalar tartibida hisoblab to’ldiramiz:

| $x_1$ | $x_2$ | $x_3$ | $\overline{x_1}$ | $\overline{x_2}$ | $\overline{x_3}$ | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |

Berilgan funksiyaning ikkilik son ko'rinishidagi ifodasi: $F = 11111111$.

**10.2-misol**. Berilgan $f(x_1, x_2) = \overline{x_1} \to (x_1 \to x_2)$ funksiyaning tavtologiya ekanligini isbotlang.

**Yechimi**. Funksiyaning ikkilik sonifodasi $F = 1111$ ekanligini isbotlash lozim. Chinlik jadvalini tuzamiz:

| $x_1$ | $x_2$ | $\overline{x_1}$ | $x_1 \to x_2$ | $\overline{x_1} \to (x_1 \to x_2)$ |
|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 |

**10.3-misol**. $f(x_1, x_2, x_3) = x_1 \wedge (x_1 \vee x_3) \wedge (x_2 \vee x_3)$ va $f(x_1, x_2, x_3) = (x_1 \wedge x_2) \vee (x_1 \wedge x_3)$ funksiyalarning o'zaro tengligini aniqlang.

**Yechimi**. Ikkala funksiyaning ham chinlik jadvalini tuzamiz, agarga ularning ikkilik son ifodasi aynan mos bo'lsa demak funksiyalar tengligi isbotlanadi.

| $x_1$ | $x_2$ | $x_3$ | $x_1 \vee x_3$ | $x_2 \vee x_3$ | $x_1 \wedge (x_1 \vee x_3)$ | $x_1 \wedge (x_1 \vee x_3) \wedge (x_2 \vee x_3)$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |

| $x_1$ | $x_2$ | $x_3$ | $x_1 \wedge x_2$ | $x_1 \wedge x_3$ | $(x_1 \wedge x_2) \vee (x_1 \wedge x_3)$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

Natijada $F_1 = 00000111$, $F_2 = 00000111$ qiymatlargaegabo'lamiz. Demakfunksiyalarimiztengkuchli.

**10.2-t a ' r i f .**Agar $f(x_1, x_2, ..., x_n)$ funksiyauchun $f(0,0,...,0) \equiv 1$ bo‘lsa, uholdau**0 saqlovchifunksiya**, $f(1,1,...,1) \equiv 1$bo‘lgandaesa**1 saqlovchifunksiya**debataladi.

“0 saqlovchifunksiya” iborasio‘rnida “yolg‘onqiymatsaqlovchifunksiya”, “1 saqlovchifunksiya” iborasio‘rnidaesa “chinqiymatsaqlovchifunksiya” iborasiqo‘llanilishihammumkin. $n$ ta argumentli 0 saqlovchi funksiyalar soni ga, 1 saqlovchi funksiyalarning soni ham ga teng bo‘lishini isbotlash qiyin emas.

**Funksiyalar teng kuchliligi.** Mulohazalar algebrasida teng kuchli formulalar tushunchasi kiritilgan edi. Bu yerda ham $n$ argumentli funksiyalar teng kuchliligi tushunchasini kiritish mumkin.

**10.3-t a ' r i f .** $f$ va $g$ funksiyalar mulohazalar algebrasining funksiyalari, $x_1, x_2, ..., x_n$ o‘zgaruvchilar esa ularning hech bo‘lmaganda bittasining argumentlari bo‘lsin. Agar $x_1, x_2, ..., x_n$ argumentlarning barcha qiymatlar satrlari uchun f va g funksiyalarning mos qiymatlari bir xil bo‘lsa, u holda $f$ va $g$ funksiyalar **teng kuchli funksiyalar** deb ataladi.

Agar berilgan funksiyalar teng kuchli bo‘lmasa, u holda ular **teng kuchlimas funksiyalar** deb yuritiladi.

Berilgan $f$ va $g$ funksiyalarning teng kuchliligi $f \equiv g$ shaklda yoziladi. Agar $f$ va $g$ funksiyalar teng kuchlimas funksiyalar bo‘lsa, u holda $f \not\equiv g$ yozuvdan foydalaniladi.

**10.4-t a ' r i f .** Agar $f(x_1, x_2, ..., x_n)$ funksiyaning qandaydir $x_i$ argumenti uchun

$$f(x_1, x_2, ..., x_{i-1}, 1, x_{i+1}, ..., x_n) \equiv f(x_1, x_2, ..., x_{i-1}, 0, x_{i+1}, ..., x_n)$$

sahart qolgan $x_1, x_2, ..., x_{i-1}, x_{i+1}, ..., x_n$ argumentlarning mumkin bo‘gan ixtiyoriy qiymatlarida bajarilsa, u holda $x_i$ uning **soxta argumenti,** $x_1, x_2, ..., x_{i-1}, x_{i+1}, ..., x_n$ argumentlarning mumkin bo‘gan qiymatlaridan hech bo‘lmasa bittasi uchun

$$f(x_1, x_2, ..., x_{i-1}, x_{i+1}, ..., x_n) \not\equiv f(x_1, x_2, ..., x_{i-1}, 0, x_{i+1}, ..., x_n)$$

shart bajarilganda esa $x_i$ uning **muhim argumenti** deb ataladi.

**10.1-m i s o l .** Berilgan $f(x, y) \equiv x \vee (xy)$ funksiya uchun $y$ soxta argumentdir, chunki $f(x,0) \equiv f(x,1)$ shart $x$ argumentning ixtiyoriy (0 yoki 1) qiymatida bajariladi. Lekin, $x$ o‘zgaruvchi $f(x, y)$ funksiyaning muhim argumentidir, chunki $f(0, y) \equiv 0 \not\equiv f(1, y) \equiv 1$ shart $y$ o‘zgaruvchining barcha (0 va 1) qiymatlarida o‘rinlidir.

Mulohazalar algebrasida o‘rinli bo‘lgan qonun va qoidalariga asoslanib, funksiyaning qiymatini o‘zgartirmasdan, uning argumentlari safiga istalgancha soxta argumentlarni kiritish va bu safdan istalgancha soxta argumentlarni olib tashlash mumkin.

**Funksiyalar superpozitsiyasi.** Endi formula tushunchasini funksiyalar superpozitsiyasi tushunchasi bilan bog‘liq holda o‘rganamiz.

$\phi_1(x_{11},x_{12},...,x_{1n}),\phi_2(x_{21},x_{22},...,x_{2n}),...,\phi_m(x_{m1},x_{m2},...,x_{mk_m})$

mulohazalar algebrasi funksiyalarining chekli sistemasi bo‘lsin.

**10.5-t a ’ r i f .** Quyidagi ikki usulning biri vositasida hosil qilinadigan $\psi$ funksiyaga $\Phi$ sistemadagi $\phi_1(x_1,x_2,...,x_n)$ funksiyalarning **elementar superpozitsiyasi** yoki **bir rangli superpozitsiyasi** deb ataladi:

a) biror $\phi_j \in \Phi$ funksiyaning $x_{ji}$ argumentini qayta nomlash usuli, ya’ni

$,\phi_j(x_{j1},x_{j2},...,x_{ji-1},y,x_{ji+1},...,x_{jk_j}),$

bu yerda $y$ o‘zgaruvchi, o‘zgaruvchilarning birortasi bilan mos tushishi mumkin;

b) biror $\varphi_j \in \Phi$ funksiyaning biror $x_{ji}$ argumenti o‘rniga boshqa

$\phi_m(x_{m1},x_{m2},...,x_{mk}) \in \Phi$ funksiyani qo‘yish usuli, ya’ni

$\phi_j(x_{j1},x_{j2},...,x_{ji-1},\phi_m(x_{m1},x_{m2},...,x_{mk}),x_{ji+1},...,x_{jk_j})$

10.5-ta’rifda keltirilgan usullardan birortasini berilgan $\Phi$ sistema funksiyalariga qo‘llash natijasida hosil qilingan yangi funksiyalar $\Phi^{(1)}$ sistemasini**bir rangli superpozitsiyalar sinfi** deb, $\Phi^{(1)}$ sinfi funksiyalariga qo‘llash natijasida hosil qilingan funksiyalar $\Phi^{(2)}$ sistemasini **ikki rangli superpozitsiyalari sinfi** deb, va, hokazo, $k$ **rangli superpozitsiyalar** $\Phi^{(n)}$ **sinfi** deb ataluvchi sinflarni hosil qilamiz.

Umuman olganda, $\Phi^{(k+1)} = (\Phi^n)^{(1)}$.

**10.1-i z o h .** 10.5-ta’rifning a) qismiga asosan bir xil Chinlik jadvaliga ega bo‘lib, lekin o‘zgaruvchilarning belgilanishi bilan farq qiladigan funksiyalar bir-birining superpozitsiyasi bo‘ladi.

**10.2-i z o h .** 6.5-ta’rifning a) qismiga asosan biror $x_{ji}$ o‘zgaruvchini shu funksiyaning boshqa $x_{ji}$ $(i \neq k)$ o‘zgaruvchisi bilan qayta nomlasak, natijada o‘zgaruvchilari soni kam funksiyaga ega bo‘lamiz. Bu holda $x_{ji}$ va $x_{ji}$ o‘zgaruvchilar **aynan tenglashtirildi** deb aytamiz. Masalan, $x \vee y$ va $x \vee \overline{y}$ funksiyalardagi $y$ ni $x$ bilan qayta nomlasak, u vaqtda $x \vee x = x$ va $x \wedge \overline{x} = 0$ funksiyalarni hosil qilamiz.

**10.3-i z o h .** 6.5-ta’rifning a) qismiga asosan agar $\Phi \subset \Phi^{(1)}$ bo‘lsa, u holda $\Phi^{(r)} = \Phi^{(r+1)}$ va, umuman, $r \leq s$ bo‘lganda $\Phi^{(r)} = \Phi^{(s)}$ bo‘ladi.

**10.6-t a ’ r i f .** $x$, $\overline{x}$, $xy$, $x \vee y$, $x \to y$, $x \leftrightarrow y$ *asosiy elementar funksiyalarning superpozitsiyasi vositasida hosil qilingan ifoda* ***formula*** *deb ataladi.*

**10.4-misol.** $(x \leftrightarrow y)(x \to y) \vee (x \downarrow y)$ funksiya 0 konstantani saqlashini aniqlaymiz.

**Yechimi**. $(x \leftrightarrow y)(x \to y) \vee (x \downarrow y) = 1 \wedge 1 \vee 1 = 1 \vee 1 = 1;$

Chunki,

$(x \leftrightarrow y) = (0 \leftrightarrow 0) = 1;$

$(x \to y) = (0 \to 0) = 1;$

$(x \downarrow y) = (0 \downarrow 0) = 1.$

Shunday qilib, funksiya $(x \leftrightarrow y)(x \to y) \vee (x \downarrow y) \notin T_0$, ya'ni 0 saqlamaydi.

**10.5-misol.** $(x \leftrightarrow y)(x \to y) \vee (x \downarrow y)$ funksiya 1 saqlashini aniqlaymiz.

**Yechimi**. $(x \leftrightarrow y)(x \to y) \vee (x \downarrow y) = 1 \wedge 1 \vee 0 = 1 \vee 0 = 1;$

Chunki, $(x \leftrightarrow y) = (1 \leftrightarrow 1) = 1;$

$(x \to y) = (1 \to 1) = 1;$

$(x \downarrow y) = (1 \downarrow 1) = 0.$

Shunday qilib, funksiya $(x \to y)(x \to y) \vee (x \downarrow y) \in T_1$, ya'ni 0 konstantani saqlaydi.

Teng kuchli almashtirishlar bajarib, mulohazalar algebrasining formulalarini har xil ko'rinishlarda yozish mumkin. Masalan, $\overline{A} \to$VS formulani $A \vee BC$ yoki $(A \vee B)$ $(A \vee C)$ ko'rinishlarda yoza olamiz.

Mantiq algebrasining kontakt va rele-kontaktli sxemalar, diskret texnikadagi tatbiqlarida va matematik mantiqning boshqa masalalarida formulalarning normal shakllari katta ahamiyatga ega.

Quyidagi belgilashni kiritamiz:

$$x^{\sigma} = \begin{cases} x, & агар \quad \sigma = ч, \\ \overline{x}, & агар \quad \sigma = ё. \end{cases}$$

$\sigma^{\sigma}$ = ch ekanligi aniq.

**10.6-ta'rif.**

$$x_1^{\sigma_1} \cdot x_2^{\sigma_2} \cdots x_n^{\sigma_n} \tag{2.1}$$

ko'rinishdagi formulaga elementar kon'yunksiya deb aytamiz. Bu yerda $\sigma = \{\sigma_1, \sigma_2, ..., \sigma_n\}$ ixtiyoriy qiymatlar satri va $x_i$ o'zgaruvchilar orasida bir xillari bo'lishi mumkin.

**10.7-ta'rif.**

$$x_1^{\sigma_1}\vee x_2^{\sigma_2}\vee\cdots\vee x_n^{\sigma_n} \qquad (2.2)$$

ko‘rinishdagi formulaga elementar diz’yunksiya deb aytamiz. Bu yerda ham $\sigma_1=\{\sigma_1,\sigma_2,...,\sigma_n\}$ ixtiyoriy qiymatlar satri va $x_i$ o‘zgaruvchilar orasida bir xillari bo‘lishi mumkin.

**10.8-ta'rif.** Elementar diz’yunksiyalarning kon’yunksiyasiga formulaning kon’yunktiv normal shakli (KNSh) va elementar kon’yunksiyalarning diz’yunksiyasiga formulaning diz’yunktiv normal shakli (DNSh) deb aytiladi.

KNShga $(x\vee y)\wedge(\bar{x}\vee z)\wedge(x\vee\bar{y}\vee z)$ formula va DNShga $xy\vee\bar{x}z\vee x\bar{y}z$ formula misol bo‘la oladi.

**10.1-Teorema.** Elementar mulohazalarning har bir $P$ formulasiga tengkuchli kon’yunktiv normal shakldagi $Q$ formula mavjud.

Bu teoremani isbotlashda ushbu tengkuchliliklardan foydalanamiz:

1. $\overline{A\wedge B}=\bar{A}\vee\bar{B}$; 2. $\overline{A\vee B}=\bar{A}\wedge\bar{B}$; 3. $A\to B=\bar{A}\vee B$;

4. $\overline{A\to B}=A\wedge\bar{B}$; 5. $A\leftrightarrow B=(\bar{A}\vee B)\wedge(A\vee\bar{B})$;

6. $\overline{A\leftrightarrow B}=(A\wedge\bar{B})\vee(\bar{A}\wedge B)$.

**10.2- Teorema.** $P$ formula doimo chin bo‘lishi uchun uning KNSh dagi har bir elementar diz’yunktiv hadida kamida bitta elementar mulohaza bilan birga bu mulohazaning inkori ham mavjud bo‘lishi zarur va yetarli.

**10.6- Misol.** 1. $P=x\wedge\bar{x}\to\overline{y\wedge\bar{\bar{y}}}=\overline{x\wedge\bar{\bar{x}}}\vee\overline{y\wedge\bar{\bar{y}}}=\bar{x}\vee x\vee\bar{y}\vee y$.

$P=\bar{x}\vee x\vee\bar{y}\vee y$ - aynan chindir.

2. $\overline{x\wedge\bar{\bar{x}}}\wedge(y\wedge\bar{y}\to z)=(\bar{x}\vee x)\wedge(\bar{y}\vee y)\vee z=P(\bar{x}\vee x)\wedge(\bar{y}\vee y\vee z)$ - aynan chin formuladir.

**Diz’yunktiv normal shakl.** Eslatib o‘tamizki, elementar kon’yunksiyalarning diz’yunksiyasiga formulaning diz’yunktiv normal shakli (DNSh) deb aytiladi.

**10.3-Teorema.** Elementar mulohazalarning istalgan $P$ formulasini DNShga keltirish mumkin.

**10.4-Teorema.** $P$ formula aynan yolg’on bo‘lishi uchun, uning diz’yunktiv normal shaklidagi har bir elementar kon’yunksiya ifodasida kamida bitta elementar mulohaza bilan birga bu mulohazaning inkori ham mavjud bo‘lishi zarur va yetarli.

**10.7.-Misol.** $P=\overline{\overline{\overline{(\bar{\bar{x}}\wedge x)}\to\overline{\bar{\bar{y}}\wedge y}}}=\overline{(\bar{\bar{x}}\wedge x)}\vee\overline{\bar{\bar{y}}\wedge y}=(\bar{\bar{x}}\vee\bar{x})\vee\bar{\bar{y}}\vee\bar{y}=(x\vee\bar{x})\vee(y\vee\bar{y})$

$\bar{P}=(x\vee\bar{x})\vee(y\vee\bar{y})$ - aynan chin.

$P = (\bar{x} \wedge x) \wedge (\bar{y} \wedge y)$ - aynan yolg'on.

**Mukammal kon'yunktiv va diz'yunktiv normal shakllar.** Mantiq algebrasining bitta formulasi uchun bir nechta DNSh (KNSh) mavjud bo'lishi mumkin. Masalan, $(x \vee y)$ $(x \vee z)$ formulani quyidagi $x \vee yz$, $x \vee xy \vee xz$ DNShlarga keltirish mumkin. Bular distributivlik va idempotentlik qonunlarini qo'llash natijasida hosil qilingan.

Formulalarni bir qiymatli ravishda normal shaklda tasvirlash uchun takomil diz'yunktiv normal shakl va takomil kon'yunktiv normal shakl (TDNSh va TKNSh) deb ataluvchi ko'rinishlari ishlatiladi.

$n$ ta $x_1, x_2, \ldots, x_n$ elementar mulohazalarning

$$x_1^{\sigma_1} \vee x_2^{\sigma_2} \vee \cdots \vee x_n^{\sigma_n} \tag{2.4}$$

elementar diz'yunksiyalari va

$$x_1^{\sigma_1} \wedge x_2^{\sigma_2} \wedge \cdots \wedge x_n^{\sigma_n} \tag{2.5}$$

elementar kon'yunksiyalari berilgan bo'lsin.

**10.9- Ta'rif.** (2.4) elementar diz'yunksiya ((2.5) elementar kon'yunksiya) to'g'ri elementar diz'yunksiya (elementar kon'yunksiya) deb aytiladi, shunda va faqat shundagina, qachonki (2.4)ning ((2.5)ning) ifodasida har bir elementar mulohaza $x_i$ bir marta qatnashgan bo'lsa.

**Masalan,** $x_1 \vee x_2 \vee x_3$ va $\bar{x}_1 \vee x_4 \vee x_6$ elementar diz'yunksiyalar va $x_1\ x_2\ x_3$ va $x_1\ \bar{x}_3\ x_6$ elementar kon'yunksiyalar mos ravishda to'g'ri elementar diz'yunksiyalar va elementar kon'yunksiyalar deb aytiladi.

**10.10- Ta'rif** (2.4) elementar diz'yunksiya ((2.5) elementar kon'yunksiya) $x_1, x_2, \ldots, x_n$ mulohazalarga nisbatan to'liq elementar diz'yunksiya (elementar kon'yunksiya) deb aytiladi, qachonki ularning ifodasida $x_1, x_2, \ldots, x_n$ mulohazalarning har bittasi bir martagina qatnashgan bo'lsa.

**Masalan,** $x_1 \vee \bar{x}_2 \vee x_3$ va $\bar{x}_1 \vee \bar{x}_2 \vee x_3$ elementar diz'yunksiyalar va $\bar{x}_1\ \bar{x}_2\ \bar{x}_3$, $x_1 x_2\ \bar{x}_3$ elementar kn'yunksiyalar $x_1, x_2, x_3$ mulohazalarga nisbatan to'liq elementar diz'yunksiyalar va elementar kon'yunksiyalar bo'ladi.

**10.11- Ta'rif.** Diz'yunktiv normal shakl (kon'yunktiv normal shakl) TDNSh (TKNSh) deb aytiladi, agar DNSh (KNSh) ifodasida bir xil elementar kon'yunksiyalar (elementar diz'yunksiyalar) bo'lmasa va hamma elementar kon'yunksiyalar (elementar diz'yunksiyalar) to'g'ri va to'liq bo'lsa.

Masalan, $xyz \vee xy\overline{z} \vee \overline{x}yz \vee x\overline{y}z$ DNSh $x, y, z$ mulohazalarga nisbatan TDNSh bo‘ladi. $(x \vee y)\,(x \vee \overline{y})\,(\overline{x} \vee y)$ KNSh $x, y$ mulohazalarga nisbatan TKNSh bo‘ladi.

Asosiy mantiqiy amallarning TDNSh va TKNSh ko‘rinishlari quyidagicha bo‘ladi: a) MDNSh: $\overline{x} = \overline{x}$; $xy = xy$; $x \vee y = xy \vee \overline{x}\, y \vee x\, \overline{y}$; $x \to y = xy \vee \overline{x}\, y \vee \overline{x}\, \overline{y}$; $x \to y = xy \vee \overline{x}\, \overline{y}$.

b) TKNSh: $\overline{x} = \overline{x}$; $xy = (\overline{x} \vee y)\,(x \vee \overline{y})\,(x \vee y)$;

$$x \vee y = x \vee y\,;\quad x \to y = \overline{x} \vee y\,;\quad x \to y = (\overline{x} \vee y)\,(x \vee \overline{y}).$$

**10.5- Teorema.** *$n$ ta elementar mulohazaning aynan chin formulasidan farqli har bir $A$ formulani takomil kon’yunktiv normal shaklga (TKNSh) keltirish mumkin.*

**10.8-Misol.** 1. $A = (\overline{x} \to x) \wedge (y \to \overline{y}) \vee (z \leftrightarrow u)$ formula quyidagi TKNSh ga ega bo‘ladi.

$$A = (x \vee \overline{z} \vee u) \wedge (x \vee z \vee \overline{u}) \wedge (\overline{y} \vee \overline{z} \vee u) \wedge (\overline{y} \vee z \vee \overline{u})$$

2. $A = (\overline{x \vee z}) \wedge (x \to y) = (\overline{x} \wedge \overline{y}) \wedge (\overline{x} \vee y)$

$$A = [\overline{x} \vee (y \wedge \overline{y}) \vee (z \wedge \overline{z})] \wedge [(x \wedge \overline{x}) \vee (y \wedge \overline{y}) \vee \overline{z}] \wedge$$
$$\wedge (\overline{x} \vee y \vee (z \wedge \overline{z}))] = [(\overline{x} \vee y \vee z) \wedge (\overline{x} \vee \overline{y} \vee z) \wedge$$
$$(\overline{x} \vee y \vee z) \wedge (\overline{x} \vee \overline{y} \vee \overline{z})] \wedge [(x \vee y \vee \overline{z}) \wedge (x \vee \overline{y} \vee \overline{z}) \wedge$$
$$\wedge (\overline{x} \vee y \vee \overline{z}) \wedge (\overline{x} \vee \overline{y} \vee \overline{z})] \wedge [(\overline{x} \vee y \vee z) \wedge (\overline{x} \vee y \vee \overline{z})]$$

$$A = (\overline{x} \vee y \vee z) \wedge (\overline{x} \vee \overline{y} \vee z) \wedge (\overline{x} \vee y \vee \overline{z}) \wedge (\overline{x} \vee \overline{y} \vee \overline{z}) \wedge$$
$$\wedge \qquad \overline{z}) \wedge (x \vee \overline{y} \vee \overline{z})$$

3. $A = (x \vee y) \wedge (y \vee z) \wedge (z \vee t)$

$$A = [z \vee y \vee (z \wedge \overline{z}) \vee (t \wedge \overline{t})] \wedge [(x \wedge \overline{x}) \vee y \vee z \vee (t \wedge \overline{t})] \wedge$$
$$\wedge [(x \wedge \overline{x}) \vee (y \wedge \overline{y}) \vee z \vee t) = [(x \vee y \vee z \vee t) \wedge (x \vee y \vee \overline{z} \vee t) \wedge$$
$$\wedge (x \vee y \vee z \vee \overline{t}) \wedge (x \vee y \vee \overline{z} \vee \overline{t})] \wedge [(x \vee y \vee z \vee t) \wedge$$
$$(\overline{x} \vee y \vee z \vee t) \wedge (x \vee y \vee z \vee \overline{t}) \wedge (\overline{x} \vee y \vee z \vee \overline{t})] \wedge$$
$$\wedge [(x \vee y \vee z \vee t) \wedge x \vee \overline{y} \vee z \vee t) \wedge (\overline{x} \vee y \vee z \vee t) \wedge (\overline{x} \vee \overline{y} \vee z \vee t)]$$

$n$ mulohazali mukammal kon'yunktiv normal shakl

$$\wedge\left(x_1^1 \vee x_2^1 \vee \ldots \vee x_n^1\right)$$

ifodasida $\wedge$ o'rniga $\vee$ ni va aksincha, $\vee$ o'rniga $\wedge$ ni qo'yganimizda

$$\vee\left(x_1^1 \wedge x_2^1 \wedge \ldots \wedge x_n^1\right)$$

biz $n$ mulohazali mukammal diz'yunktiv normal shaklga ega bo'lamiz.

Mukammal diz'yunktiv normal shaklning har bir $x_1^1 \wedge x_2^1 \wedge \ldots \wedge x_n^1$ hadi **kon'yunktiv konstituent** deb ataladi.

**10.6-Teorema.** *$n$ ta elementar mulohazalarning aynan yolg'on formulasidan farqli har bir $A$ formulasini mukammal diz'yunktiv normal shaklga keltirish mumkin.*

**10.9-Misol.** $A = [(\bar{x} \to x) \wedge (y \to \bar{y})] \vee (z \leftrightarrow u)$

$$A = (x \vee \bar{z} \vee u) \wedge (x \vee z \vee \bar{u}) \wedge (\bar{y} \vee \bar{z} \vee u) \wedge (\bar{y} \vee z \vee \bar{u}) \wedge$$

$$\wedge (x \vee \bar{z} \vee u) \vee (y \wedge \bar{y}) = (x \vee \bar{z} \vee u \vee y) \wedge (x \vee \bar{z} \vee u \vee \bar{y})$$

## TOPShIRIQ VARIANTLARI.

Quyidagi formulalarni DNSh, KNSh, MKNSh va MDNSh ko'rinishga keltiring.

2.1.2. $((p \to q)\&(q \to r)) \to (p \to r)$

2.1.3. $\overline{(x \to z) \to ((y \to z) \to (x \vee y \to z))}$

2.1.4. $(p_1 \to p_2) \to ((p_1 \vee p) \to (p_2 \vee p))$

2.1.5. $\overline{(p_1 \to p_2) \to ((p_1 \wedge p) \to (p_2 \wedge p))}$

2.1.6. $((p \to q)\&(q \to r)) \to (p \to r)$

2.1.7. $(x \to y)\&(y \to z) \to (z \to x)$

2.1.8. $(x \vee \bar{y} \to (z \to y \vee \bar{y} \vee x))\&(x \vee \overline{x \to (x \to x)}) \to y$

2.1.9. $\overline{\overline{x \vee y} \to \overline{x\& y}}$

2.1.10. $\overline{p_1} \to (p_1 \to p_2)$

2.1.11. $\overline{p_1 \to (p_2 \to p_1)}$

2.1.12. $\overline{\bar{x} \cdot \bar{y}} \vee (x \to y) \cdot x$

2.1.13. $\overline{(x \to z) \to ((y \to z) \to (x \vee y \to z))}$

2.1.14. $(p_1 \to p_2) \to ((p_1 \vee p) \to (p_2 \vee p))$

2.1.15. $(p_1 \to (p_2 \to p_3)) \to ((p_1 \to p_2) \to (p_1 \to p_3))$

2.1.16. $(x \to y) \& (y \to z) \to (x \to z)$

2.1.17. $((p \to q) \& (q \to r)) \to (p \to r)$

2.1.18. $(x \to y) \& (y \to z) \to (z \to x)$

2.1.19. $(x \vee \bar{y} \to (z \to y \vee \bar{y} \vee x)) \& (x \vee \overline{x \to (x \to x)}) \to y$

2.1.20. $((p \wedge q) \leftrightarrow q) \leftrightarrow (q \to p)$

2.1.21. $((p \to q) \& (q \to r)) \to (p \to r)$

2.1.22. $\overline{(p_1 \to p_2) \to ((p_1 \wedge p) \to (p_2 \wedge p))}$

2.1.23. $(x \leftrightarrow y) \& (x \vee y)$

2.1.24. $(x \to y) \& (y \to z) \to (z \to x)$

2.1.25. $(x \& \overline{x \& \bar{x} \to y \& \bar{y}} \to z) \vee x \vee (y \& z) \vee (y \& z)$

2.1.26. $(x \& (y \vee z \to y \vee z)) \vee (y \& x \& \bar{y}) \vee x \vee (y \& \overline{x \& \bar{x}})$

2.1.27. $((p \wedge q) \leftrightarrow q) \leftrightarrow (q \to p)$

2.1.28. $((p \to q) \& (q \to r)) \to (p \to r)$

2.1.29. $\overline{(x \to z) \to ((y \to z) \to (x \vee y \to z))}$

2.1.30. $(p_1 \to p_2) \to ((p_1 \vee p) \to (p_2 \vee p))$

**2.2. Quyidagi formulalarni MKNSh va MDNSh ko‘rinishga keltiring.**

2.2.1. $(x \vee \bar{y} \to x \wedge z) \to (\overline{x \to \bar{x}}) \vee y \wedge \bar{z}$;

2.2.2. $(x \wedge (y \vee z \to y \vee z)) \vee (y \wedge x \wedge \bar{y}) \vee x \vee (y \wedge \overline{x \wedge \bar{x}})$;

2.2.3. $(x \vee \bar{y} \to x \wedge z) \to (\overline{x \to \bar{x}}) \vee y \wedge \bar{z}$;

2.2.4. $(\overline{xy} \to \bar{x}) \wedge (\overline{xy \to \bar{y}})$;

2.2.5. $(\bar{a} \to c) \to ((\overline{\bar{b} \to \bar{a}})$;

2.2.6. $(x \vee \bar{y} \to x \wedge z) \to (\overline{x \to \bar{x}}) \vee y \wedge \bar{z}$

2.2.7. $(\bar{a} \to c) \to ((\overline{\bar{b} \to \bar{a}})$;

2.2.8. $\overline{(p_1 \to p_2) \to ((p_1 \wedge p) \to (p_2 \wedge p))}$.

2.2.9. $(ab \to bc) \to ((a \to b) \to (c \to b))$;

2.2.10. $(x\vee\overline{y}\to x\wedge z)\to(\overline{x\to\overline{x}})\vee y\wedge\overline{z}$

2.2.11. $(\overline{xy}\to\overline{x})\wedge(\overline{xy\to\overline{y}})$;

2.2.12. $(x\vee\overline{y}\to x\wedge z)\to(\overline{x\to\overline{x}})\vee y\wedge\overline{z}$;

2.2.13. $(\overline{a}\to c)\to((\overline{\overline{b}\to\overline{a}})$;

2.2.14. $\overline{(p_1\to p_2)\to((p_1\wedge p)\to(p_2\wedge p))}$.

2.2.15. $(x\vee\overline{y}\to x\wedge z)\to(\overline{x\to\overline{x}})\vee y\wedge\overline{z}$;

2.2.16. $(x\vee\overline{y}\to x\wedge z)\to(\overline{x\to\overline{x}})\vee y\wedge\overline{z}$;

2.2.17. $(\overline{a}\to c)\to((\overline{\overline{b}\to\overline{a}})$;

2.2.18. $(x\vee\overline{y}\to x\wedge z)\to(\overline{x\to\overline{x}})\vee y\wedge\overline{z}$;

2.2.19. $\overline{(p_1\to p_2)\to((p_1\wedge p)\to(p_2\wedge p))}$;

2.2.20. $f_1=((x\vee y)\vee z)\to((x\vee y)(x\vee z))$.

2.2.21. ) $A(x,y,z)=(00100101)$

2.2.22. $A(x,y,z)=(01111000)$

2.2.23. $((x+y)\downarrow(x\to y))\to z$

2.2.24. $(x\vee y\vee z)+(x\to(y\downarrow\overline{z}))$

2.2.25. $((x\to y)\to(x+\overline{y}))\to z$

2.2.26. $((x\mid y)\downarrow\overline{z})\to(x+y\overline{z})$

2.2.27. $(x\to(y+\overline{z}))\leftrightarrow(x\downarrow z)$

2.2.28. $(x+y+z)\leftrightarrow(x\vee y\overline{z})$

2.2.29. $(x\leftrightarrow(y\vee z))+(y\leftrightarrow xz)$

2.2.30. $(x\vee(y+\overline{z}))\leftrightarrow(y\downarrow\overline{z})$