Measurement of Resistance

- Bridge circuits are widely used for measuring the resistance, capacitance, inductance, impedance, etc.
- They provide very high accuracy because of null-indication method
- Only used for determining the unknown components not for calibrating meters/other components

Wheatstone Bridge

- It contain four resistive arm, a source of emf (battery), and a null detector
- Null detectors are usually galvanometer or other sensitive current meter
- Current through the galvanometer depends on the potential difference between points **c** and ***d***
- Bridge is balanced when the no current flows through the galvanometer
- Potential difference is across the galvanometer is 0V when voltage from point ***c*** to point **a** is equal to voltage from point ***d*** to point ***a***

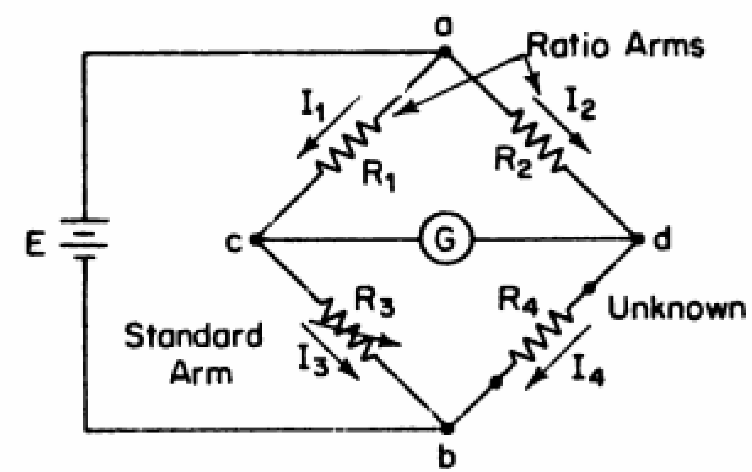


Figure: Wheatstone Bridge

Measurement of Resistance

- When the bridge is balanced

$$I_1 R_1 = I_2 R_2$$

- When the galvanometer current is zero

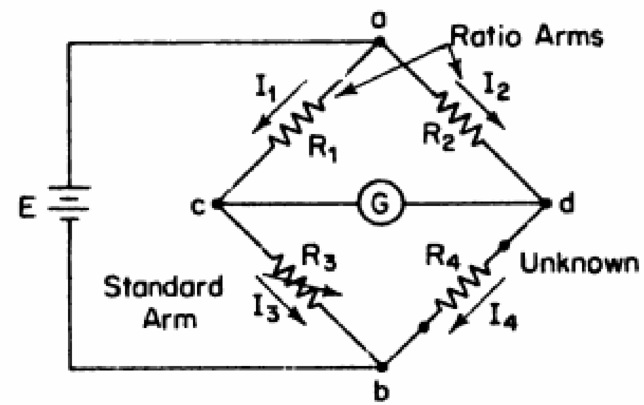


Figure: Wheatstone Bridge

$$I_1 = I_3 = \frac{E}{R_1 + R_3}$$

and $$I_2 = I_4 = \frac{E}{R_2 + R_4}$$

Combining above three equation we find

$$\frac{R_1}{R_1 + R_3} = \frac{R_2}{R_2 + R_4}$$

Therefore, $$R_1 R_4 = R_2 R_3$$

- The above equation is called expression for balance of the Wheatstone bridge
- **Unknown resistance can be found by** $R_4 = R_3 \frac{R_2}{R_1}$
- Resistance R_3 is called standard arm of the bridge and R_1 and R_2 are called ratio arm

Measurement of Resistance

- The Wheatstone bridge is widely used for precision measurement of resistance from 1 Ω to the MΩ (megohm) range

Measurement Error/ Limitation of Wheatstone Bridge

- Insufficient sensitivity of the null detector

- Change in resistance of the bridge arm due to heating effect ($I^2R\ loss$) caused by the current through the resistors

- Thermal emf in the bridge circuit or the galvanometer circuit may cause problem for measuring the low resistance. Galvanometer contain copper coils and copper suspension system to avoid having dissimilar metals in contact with one another generating thermal emfs

- Error due to the resistance of leads and contact exterior to the actual bridge circuit affect the measurement of very low resistance

Sensitivity of Wheatstone Bridge

- Calculation of current through the galvanometer due to unbalance condition is required to determine whether the galvanometer has the sensitivity required to detect the unbalance
- Different galvanometer has different internal resistance
- To find the current, Thevenin equivalent circuit is determined by looking into the galvanometer terminals, points **c** and ***d***

Two steps:

- Finding the equivalent voltage appearing at terminals **c** and ***d*** when galvanometer is removed from the circuit
- Finding the equivalent resistance looking into terminals **c** and ***d*** with the battery replaced with only the internal resistance
- The Thevenin or open circuit voltage is found as

$$E_{cd} = E_{ac}\text{-}E_{ad}=\boldsymbol{I_1R_1 - I_2R_2}$$

Where $\boldsymbol{I_1} = \frac{\boldsymbol{E}}{\boldsymbol{R_1+R_3}}$ and $\boldsymbol{I_2} = \frac{\boldsymbol{E}}{\boldsymbol{R_2+R_4}}$

Therefore, $E_{cd} = E(\frac{\boldsymbol{R_1}}{\boldsymbol{R_1+R_3}} - \frac{\boldsymbol{R_2}}{\boldsymbol{R_1+R_3}}$ **); This is Thevenin equivalent voltage**

Sensitivity of Wheatstone Bridge

Figure: Wheatstone Bridge

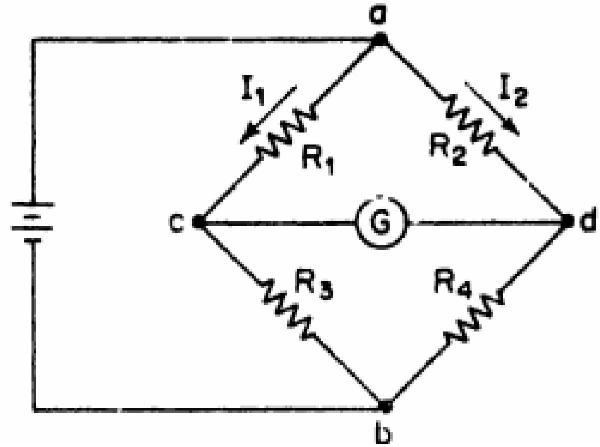
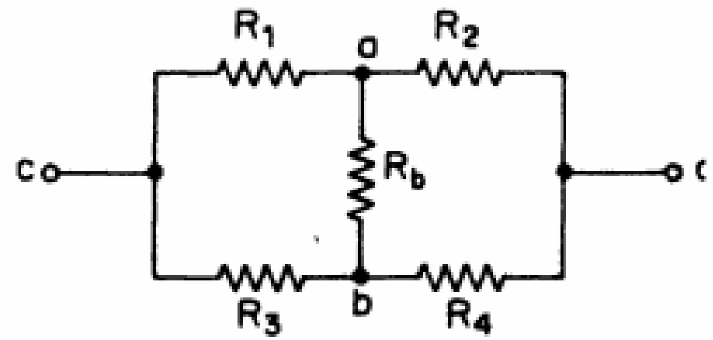


Figure: Thevenin resistance looking into terminal c and d



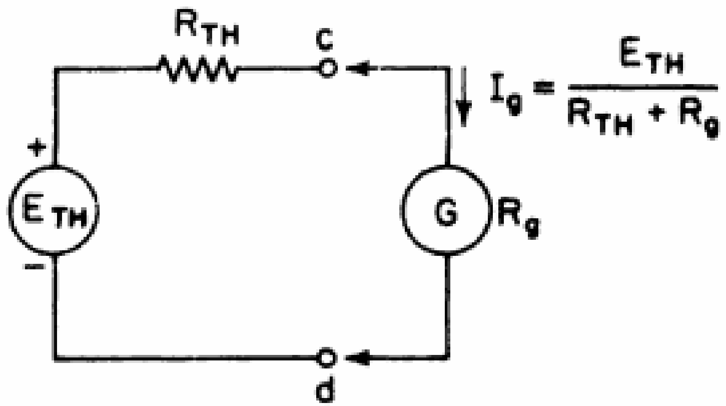


Figure: Thevenin circuit with galvanometer connected to terminal c and d

Sensitivity of Wheatstone Bridge

- Resistance of Thevenin equivalent circuit is found by looking back ***c*** and ***d*** with the battery replaced with only the internal resistance
- Using Delta-wye transformation theorem, Thevenin resistance can be found by

$$R_{Th} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4}$$

- When the null detector is connected to the output terminals of the Thevenin equivalent circuit the galvanometer current is found as

$$I_g = \frac{E_{Th}}{R_{Th} + R_g}$$

Where I_g is the galvanometer current and R_g its resistance

Example 7-1: Figure 7-3(a) shows the schematic diagram of a Wheatstone bridge with values of the bridge elements as shown. The battery has an emf of 5 V and negligible internal resistance. The galvanometer has a current sensitivity of 10 mm/μA and an internal resistance of 100 Ω. Calculate the deflection of the galvanometer caused by the 5-Ω unbalance in arm *BC*.

Solution:

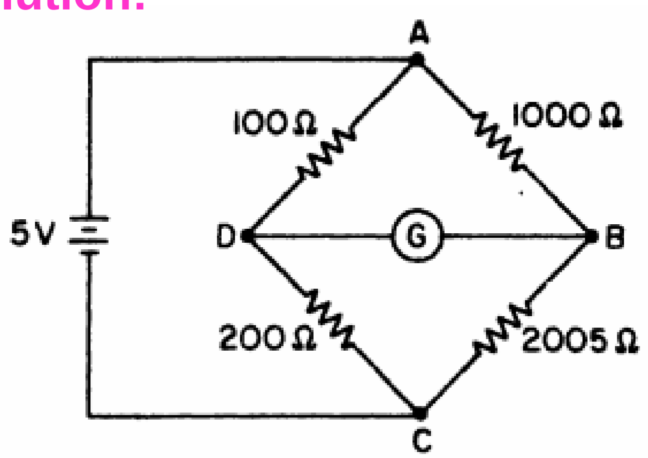
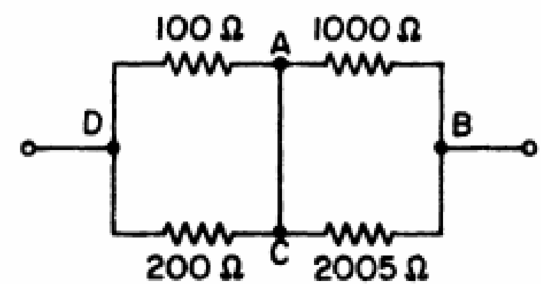


Figure: Wheatstone Bridge

Figure: Thevenin resistance looking into terminal D and B



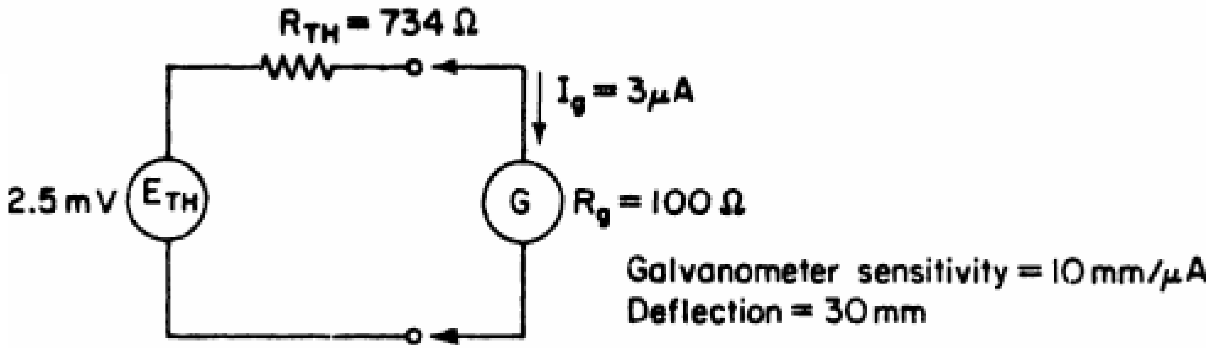


Figure: Thevenin circuit with galvanometer connected to terminal D and B

Solution:

Thevenin equivalent voltage is found as

$$E_{TH} = E_{AD} - E_{AB} = 5\text{ V} \times \left[\frac{100}{100 + 200} - \frac{1{,}000}{1{,}000 + 2{,}005}\right]$$
$$= 2.5\text{ mV}$$

Equivalent Thevenin resistance can be found as

$$R_{TH} = \frac{100 \times 200}{300} + \frac{1{,}000 \times 2{,}005}{3{,}005} = 734\ \Omega$$

Current through the galvanometer is found as

$$I_g = \frac{E_{TH}}{R_{TH} + R_g} = \frac{2.5\text{ mA}}{743\ \Omega + 100\ \Omega} = 3\ \mu\text{A}$$

The galvanometer deflection is

$$d = 3\ \mu\text{A} \times \frac{10\text{ mm}}{\mu\text{A}} = 30\text{ mm}$$

Kelvin Bridge

- Kelvin bridge is the modified version of Wheatstone bridge
- Provide accurate measurement of low value resistance below **1 Ω**
- R_y represent the resistance of connecting lead from R_3 to R_x
- Two galvanometer connections are possible to point **C** or to point **D**
- When galvanometer is connected to point **D**, the resistance of the connecting lead R_y is added to the unknown resistance R_x, resulting in too high an indication for R_x
- When galvanometer is connected to point C, resistance of the connecting lead R_y is added to bridge arm R_3 and the resulting measurement of R_x is lower than it should be

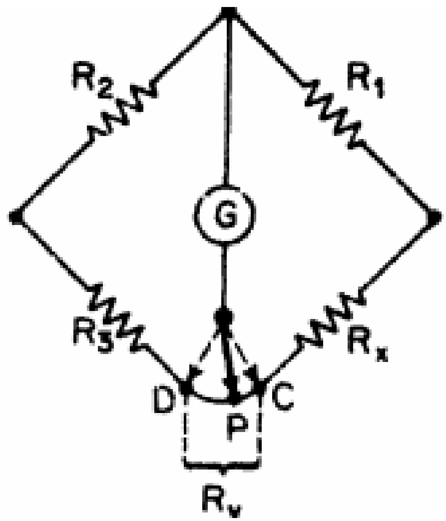


Figure: Wheatstone Bridge showing resistance of the lead from point C to D

- If galvanometer is connected to a point **P** in between two points **C** and **D**, in such a way that ratio of resistance from **C** to **P** and from **D** to **P** equals the ratio of the resistor R_1 and R_2

$$\frac{R_{CP}}{R_{DP}} = \frac{R_1}{R_2}$$

Kelvin Bridge

- The usual balance equation for the bridge it is found as

$$\boldsymbol{R_x + R_{CP} = \frac{R_1}{R_2}(R_3 + R_{DP})}$$

- Combining the above two equation

$$\boldsymbol{R_x + \left(\frac{R_1}{R_1+R_2}\right)R_y = \frac{R_1}{R_2}\left(R_3 + \left(\frac{R_2}{R_1+R_2}\right)R_y\right)}$$

- Therefore,

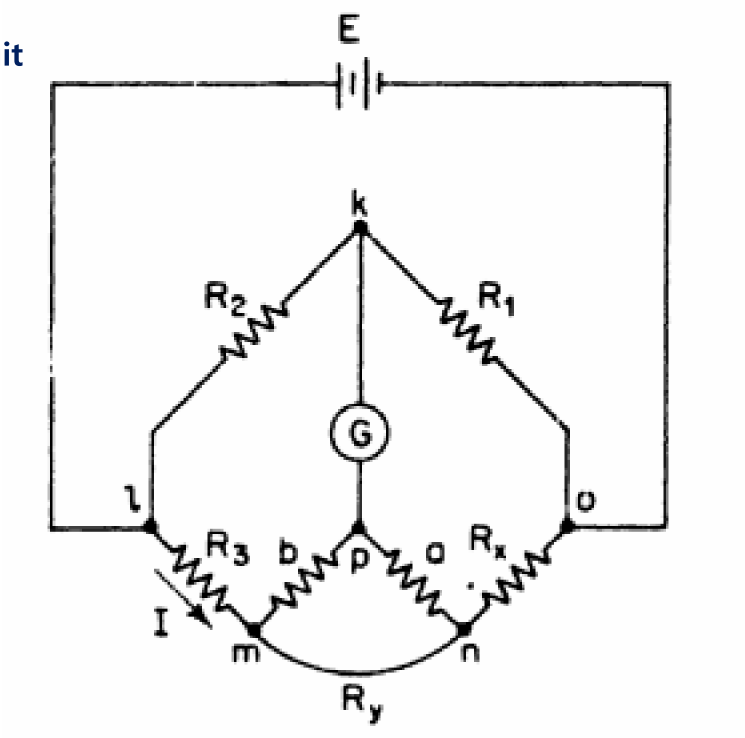
$$\boldsymbol{R_x = \frac{R_1}{R_2}R_3}$$

- The above equation is the usual balance equation for the Wheatstone bridge and it indicates the effect of the resistance of the connecting lead from point C to D by connecting the galvanometer to intermediate position P

Kelvin Bridge

- The Kelvin bridge is also called double bridge as second set of ratio arm has been used

Figure: Kelvin double Bridge circuit



- Second set of arms a and b are used to connect the galvanometer to a point P at appropriate potential between C and D, eliminating the effect of C-to-D connection
- The ratio of resistance of arms **a** and **b** is the same as ratio of $\boldsymbol{R_1\ and\ R_2}$

Measurement of Resistance

■ Kelvin Bridge

- The galvanometer indication will be 0V when $E_{kl} = E_{lmy}$

$$E_{kl} = \frac{R_2}{R_1 + R_2} E = \frac{R_2}{R_1 + R_2} I\left[R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y}\right] \quad \text{and}$$

$$E_{lmp} = I\left[R_3 + \frac{b}{a+b}\left\{\frac{(a+b)R_y}{a+b+R_y}\right\}\right]$$

Since, for zero galvanometer deflection, $E_{kl} = E_{lmp}$, we can use Eqs. and (7-15) to solve for R_x, obtaining

$$R_x = R_3\frac{R_1}{R_2} + \frac{bR_y}{a+b+R_y}\left\{\frac{R_1}{R_2} - \frac{a}{b}\right\}$$

Using the initially established condition that $a/b = R_1/R_2$, Eq. (7-16) reduces to the well-known relationship:

$$R_x = R_3\frac{R_1}{R_2}$$

The above equation is called working equation for the Kelvin bridge and it indicates that the resistance of yoke has no effect on the measurement, provided that the two set of ratio arms have equal ratios

Kelvin Bridge

- the Kelvin bridge is used to measure extremely low resistance as low as 0.00001 Ω when correct bridge components are selected

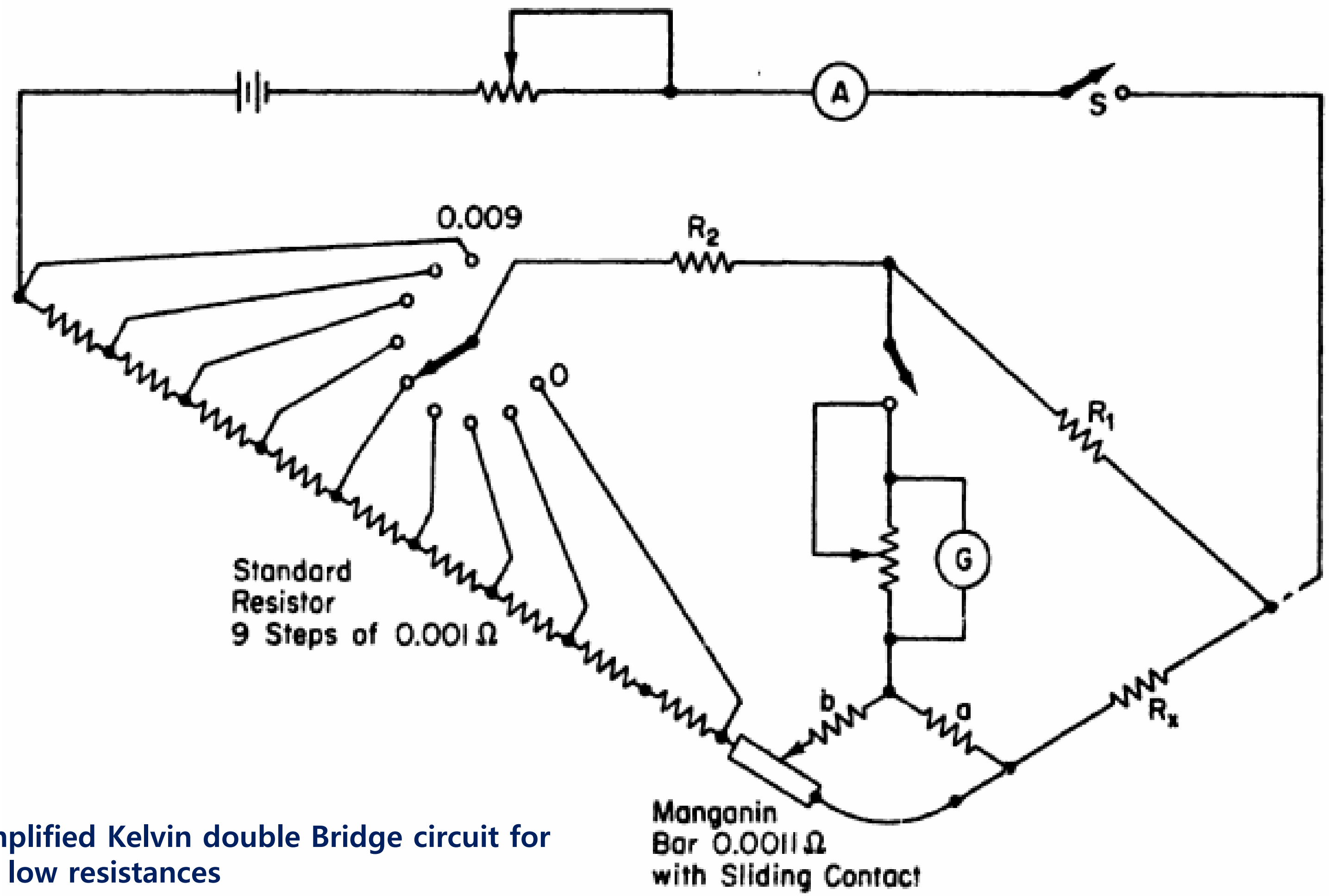


Figure: Simplified Kelvin double Bridge circuit for measuring low resistances

Steps (0.001 Ω) of R_3 reduces the error due to contact potential in the measuring circuit