Economic Planning

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Problem Introduction

Given an economy of three industries, coal, steel, and transport, we want to create 5-year production schedules with 3 different goals. We can choose to use our resources to output more industry, invest in our industries to increase productive capacities, or store our goods for a later year. Every industry requires inputs from other industries as well as manpower to create output or increase the industry's capacity. To model this we are making several assumptions. First, we are assuming that the inputs needed to create output in an industry is constant. That is, we cannot increase the input of one industry to make up for a deficiency in another. Also, this assumes that the marginal cost of an industry remains the same as we increase our output, which does not reflect real-world economies. In real economies there is a time lag between inputs and outputs. To encapsulate this we will assume inputs in each year will no become outputs until the next year. Also, any investments to production capacity will not be realized for 2 years.

Our goal is to analyze three different economic strategies. In the first the total productive capacity is maximized while meeting exogenous demands. This is comparable to improving the economy without putting the populace through any hardship. For the second we want to maximize the output on years 4 and 5 while ignoring the consumers' demand. In this situation we are willing to endure several bad years to ensure a robust economy in the long run. Finally, we want to maximize the total manpower utilized while ignoring any limits on our workforce. This strategy might be favorable in a time of high unemployment where we are willing to sacrifice economic health to provide jobs.

Problem Formulation and Variables

Due to the high volume of charts, variables, and constraints used, this project requires a clear notation. An index of the notation that we use in our report is as follows:

i : represents an industry, denoted by either C (Coal), S (Steel), or T (Transport)
t : the year number, $(t = 0, 1, 2, 3, 4, 5)$
x_{it} : the total output of industry i in year t, where
x_i : the static output from industry i in year 5 and beyond
y_{it} : the extra productive capacity for industry i effective in year t, $(t = 2, 3, \dots, 6)$. The initial productive capacity is given by Table 3
c_{ij} : the input/output coefficients for x_{jt+1}, where c_{ij} is the amount of industry i, in dollars, needed to output \$1 of industry j in year $t + 1$. Given by Table 1
d_{ij} : the input/output coefficients for y_{jt+2}, which is the amount of industry i needed to increase the productive cpacity of industry j by \$1. Given by Table 2

s_{it} : the stock level of industry i at the beginning of year t, or the amounts unused input in year $t-1$. Our initial stock levels are given by Table 3

We also have a manpower limit of $470 million per year and an exogenous demand of $60 m of coal, $60 m of steel, and $30 m of transport for every $t \geq 1$. This demand can be thought of as resources that are leaving our system to satisfy consumers' needs.

Table 1:

Inputs (year t)	Outputs (year $t+1$)		
	Coal	*Steel*	*Transport*
Coal	0.1	0.5	0.4
Steel	0.1	0.1	0.2
Transport	0.2	0.1	0.2
Manpower	0.4	0.2	0.1

Table 2:

Inputs (year t)	Outputs (year $t+1$)		
	Coal	*Steel*	*Transport*
Coal	0.1	0.5	0.4
Steel	0.1	0.1	0.2
Transport	0.2	0.1	0.2
Manpower	0.4	0.2	0.1

Table 3:

Year 0		
	Stocks	*Productive Capacity*
Coal	150	300
Steel	80	350
Transport	100	280

Input/Output Constraints

Our first constraint is our total output constraint. This constraint signals that our total supply of our coal, steel, and transport that we produce and store in our stock over the 5 years must be equivalent to our expenditures for each industrys production, increase in productive capacity, and the exogenous demands. That is, we must keep in mind the different ways our output must be allocated to different expenditures.

The constraints are as stated below:

$$x_{it} + s_{it} = \sum_{j=1,3} c_{ij}x_{it+1} + \sum_{j=1,3} d_{ij}x_{jt+2} + s_{it+1} + \text{ exogenous demand for industry } i \text{ at time } t$$

Our left hand side of the equation represents the amount of total output that is available to us. Our right hand side of the equation represents the amount industry i ($i = 1$ (Coal), 2 (Steel), 3 (Transport) is allocated to various expenditures.

$\sum_{j=1,3} c_{ij}x_{it+1}$ represents the amount the input industry i has allocated at time t for the purpose of the production for the output industries of coal, steel, and transport finished for the following year. In other words, this is the amount allocated to produce more output of each industry of coal, steel, and transport.

$\sum_{j=1,3} d_{ij}x_{jt+2}$ represents the amount industry i has allocated at time t for the purpose of adding to production capacity in 2 years.

s_{it+1} is the amount of unused industry i during year t that will be allocated to stock or storage in the beginning of the next year.

Our exogenous demand for industry i at year $t \geq 1$ are given as \$60 m of coal, \$30 m of steel, and \$30 m of transport.

We we assume $x_{it} = x_i$ for $t \geq 5$ where x_i is the static output of industry i for $t \geq 5$.

Constraints of Productive Capacity

Our economy has a limit on how much of each commodity can be produced in a given year. This can be thought of as a limit on factories, natural resources, or trade for example. However, we can invest in our infrastructure to increase our productive capacity. To make our model more realistic, there is a time lag of 2 years between investing and realizing our gained productivity. The amount needed to increase our productive capacity of good i, $i =$ coal, steel, transport, by one dollar is given by (table). Therefore, for each year t, $t = 1, \ldots, 5$, the amount of good i that can be produced is limited by the sum of initial productive capacity and the total amount of increased capacity.

$$x_{il} \leq \text{initial capacity of i} + \sum_{l \leq t} y_{il} \text{ for all } i,\ t.$$

Or in standard form:

$$x_{il} - \sum_{l \leq t} y_{il} \leq \text{initial capacity of } i \text{ for all } i,\ t.$$

This introduces 15 constraints, 5 for each year for coal, steel, and transport.

Manpower Constraints

Just as the workforce of an economy is limited, it is realistic to assume that the maximum amount of manpower that we have available is restricted. For our model we have a max of $470 m manpower. The amount of manpower necessary in year t to make $1 of output in year $t+1$ is given by Table 1 and the amount of manpower needed to increase productive capacity by $1 in year $t+2$ is given by Table 2. So our constraints are:

$$0.6x_{Ct+1} + 0.3x_{Tt_1} + 0.2x_{Tt+1} + 0.4y_{Ct+2} + 0.2y_{St+2} + 0.1y_{Tt+2} \leq 470 \text{ for all } t$$

Constraints of Demand Beyond Year 5

While our model creates a 5-year production schedule, the goal is to create an economy that meets our goals at the end of the 5 years and beyond. We can imagine scenarios where our requirements are met during the 5 years but not year 6 or later. For example if we only invest in increasing our productive capacity while letting exogenous demand drain our stocks, it is possible to find ourselves in a situation where we dont have enough commodities to satisfy our consumers demands on year 6.

To account for this we must produce enough in year 5 to meet exogenous demands on year 6. We can do this by assuming that the output of a good on year 5 or later is constant. Then

$$x_i = \sum_{j=1,3} c_{ij}x_j + \text{ endogenous demand of } i \text{ for all } i$$

Where x_i is the amount of i produced for any $t \geq 5$
This gives us a system of 3 equations and 3 unknowns:

$$\begin{gathered}(1-c_{11})x_C - c_{12}x_S - c_{13}x_T = \text{endogenous demand of coal} \\ c_{21}x_C + (1-c_{22})x_S - c_{23}x_T = \text{endogenous demand of steel} \\ c_{31}x_C - c_{32}x_S + (1-c_{33})x_T = \text{endogenous demand of transport}\end{gathered}$$

The solution to this system is $x_C = 116.3968, x_S = 105.6680, x_T = 92.3077$. So as long as we produce at least these amounts in year 5 we can meet the demands in year 6 and beyond. Our constraints give us a lower bound of

$$x_C >= 116.3968, x_S >= 105.6680, x_T >= 92.3077$$

Maximizing Productive Capacity

Another strategy that may be employed is to grow our economy without hindering quality of life. We can do this by maximizing productive capacity while still meeting exogenous demand. The resulting objective function is:

$$\text{Maximize } z = \sum_{\substack{i=1,3 \\ l\leq 5}} y_{il}$$

This gives a productive capacity of $2,142 m at the optimal point described by Table 5. This solution greatly the increases the capacity to produce coal but does not change the capacity for steel or transport. This is expected as coal is the cheapest industry to increase capacity. If our goal was a balanced economy, it may be in our interest to scale our objective function to acount for the value of each industry.

Maximizing Output in Years 4 and 5

If we want to improve our economy we may be willing to forego the consumers' demands in order to invest in our infrastructure to strengthen the economy in the future. We can model this by ignoring exogenous demands while trying to maximize total output in years 4 and 5. Note that we will still have to meet the endogenous demands for years $t \geq 6$. This gives us the objective function:

$$\text{Maximize } z = \sum_{\substack{i=1,3 \\ t=4,5}} x_{it}$$

The optimal point is given by Table 5. This gives us a maximum output of \$2619 million in years 4 and 5.One point of interest is that we are producing at max capacity for years 4 and 5. Overall this plan leaves us with a balanced and strong economy but we must be willing to endure hardships early on.

Maximizing Manpower

One economic strategy is to maximize the total manpower used. This may be beneficial during a recession where many people may be unemployed. The objective function used to achieve this is:

$$\text{Maximize } z = \sum_{t=1,\ldots,5} 0.5x_{Ct+1} + 0.3x_{St+1} + 0.2xTt + 1 + 0.4y_{St+2} + 0.2y_{St+2} + 0.1y_{Tt+2}$$

We are assuming we have unlimited manpower so we ignore any associated constraints. A total manpower of \$2,450 million is achieved at the optimum point shown in Table 6. This economic plan invests heavily on coal production and increasing the capacity of coal output as both are the most labor intensive as shown by Table 1 and 2. However, we produce relatively little steel and transport and we do not invest into increasing their capacities at all. So although this may solve problems like unemployment it may not necessarily lead to a healthy economy.

Methods and Solutions

Due to the size of our system (45 variables and 42 constraints) we found it infeasible to manually create the A matrix, b vector, and c vector by hand. Note that our A matrix contains 1890 entries. To work around this we developed a MATLAB script to automatically populate our A, b, and c. We then used MATLAB's linprog function, which uses the dual simplex method to find the minimum of a linear system, to find the optimum point. We were given 3 problems with different objective functions and modified constraints. Let x_i be the optimal point and f_i the objective function for the ith problem. Then the max value of system i is obtained at $x_i^T f_i$.

For the problem where we are maximizing productive capacity we get $x_1^T f_1 =$ \$1219 million additional capacity. Our initial capacity is \$930 million so our total capacity at the end of the 5 years is \$2,142 million. For the second problem our total output for years 4 and 5 is $x_2^T f_2 = \$2,619$ million for the third, the total manpower used is $x_3^T f_3 = \$2,450$ million.

Table 4:

Maximizing Productivity					
	Year 1	Year 2	Year 3	Year 4	Year 5
Coal Output	260.4	293.4	300	17.9	166.4
Steel Output	135.3	181.7	193.1	105.7	105.7
Transport Output	140.7	200.6	267.2	92.3	92.3
Increase in Coal Capacity	0	0	0	189.2	1,022.7
Increase in Steel Capacity	0	0	0	0	0
Increase in Transport Capacity	0	0	0	0	0
Coal Stock	0	0	0	148.4	0
Steel Stock	0	0	0	0	0
Transport Stock	0	0	0	0	0

Table 5:

Maximizing Output in Year 4 and 5					
	Year 1	Year 2	Year 3	Year 4	Year 5
Coal Output	184.6	430.5	430.5	430.5	430.5
Steel Output	86.7	153.3	182.9	359.4	359.4
Transport Output	141.3	198.4	225.9	519.4	519.4
Increase in Coal Capacity	0	130.5	0	0	0
Increase in Steel Capacity	0	0	0	9.4	0
Increase in Transport Capacity	0	0	0	239.4	0
Coal Stock	31.6	16.4	0	0	0
Steel Stock	11.5	0	0	0	176.5
Transport Stock	0	0	0	0	0

Table 6:

Maximizing Manpower					
	Year 1	Year 2	Year 3	Year 4	Year 5
Coal Output	251.8	316	319.8	366.3	859.4
Steel Output	134.8	179	224.1	223.1	220
Transport Output	143.6	181.7	280	279.1	276
Increase in Coal Capacity	0	16	4	46	493.4
Increase in Steel Capacity	0	0	0	0	0
Increase in Transport Capacity	0	0	0	0	0
Coal Stock	0	0	0	0	0
Steel Stock	11	0	0	0	0
Transport Stock	4.2	0	0	0	0
Total Manpower Used	281.1	614.3	1,154	1790.8	2,450.5

Comparison of Objective Functions and Optimal Solutions

For the first objective function we found the productive capacity in the fifth year to be $2142 million The second solution was the sum of the optimal production output margins for all industries of years 4 and 5 ($2619 million). This profit was different from the first one because we had to consider the last 2 years and excluded the exogenous demand values In the third objective function we maximized manpower requirements and got a solution of $2450 million. All these solutions are optimal in their own way, and are feasible based on the realistic and dynamic constraints that we placed on the system. They provide different scenarios that can be useful to discern optimal possible fiscal results.

While our solution is a dynamic one that considers time as a variable, we are only focusing on maximizing one objective function at a time. This means is we are focusing on either output , capacity, or maximizing expenditure of workers to increase employment. We have three different results, or solutions that can all be strategic or not based on the economies current needs. The report, or rather its methodology, should therefore be used situationally by an economy whose goal is shadowed by the scope of this paper.