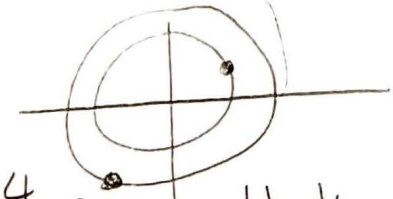


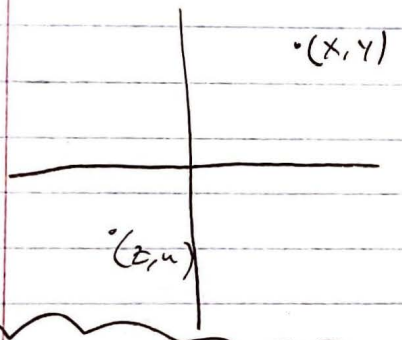
Project 4: Helium Atom

(x, y) are coordinates electron 1 at time t
(z, u) are coordinates electron 2 at time t

$$x'(t) = \begin{Bmatrix} z \\ u \\ -x(x^2+y^2)^{-3/2} \\ -y(x^2+y^2)^{-3/2} \end{Bmatrix} \text{ with I.C. } x(0) = \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{Bmatrix}$$

n.w: $x(t) = \begin{Bmatrix} x(t) \\ y(t) \\ z(t) \\ u(t) \end{Bmatrix}$

Step 1: Integrate



$x'(t) = z \Rightarrow x(t) = zt + c_1 = 1$
$y'(t) = u \Rightarrow y(t) = ut + c_2 = 0$
$z'(t) = -x(x^2+y^2)^{-3/2}$
$u'(t) = -y(x^2+y^2)^{-3/2}$

$z(t) = -tx(x^2+y^2)^{-3/2} + c_3 = 0$
$u(t) = -ty(x^2+y^2)^{-3/2} + c_4 = 1$

note: we can write $x(t)$ in x, y, t and $y(t)$ in x, y, t and we can write $z(t)$ in z, u, t and $u(t)$ in z, u, t. (manipulation)

Step 2: solve for exact eqn's

$x(t) = zt + c_1 \Rightarrow x(0) \Rightarrow c_1 = 1$
$y(t) = ut + c_2 \Rightarrow y(0) \Rightarrow 0 = c_2$
$z(t) \Rightarrow c_3 = 0$
$u(t) \Rightarrow c_4 = 1$

$x(t) = zt + 1$
$y(t) = ut$
$z(t) = -xt(x^2+y^2)^{-3/2}$
$u(t) = -ty(x^2+y^2)^{-3/2} + 1$

$x(t) = z(t)\,t + 1$

$\frac{x(t) - 1}{t} = z(t)$

Step 3:

Step 3 can be modified

$x(t) = \left(-xt(x^2+y^2)^{-3/2}\right) \cdot t + 1 = -xt^2(x^2+y^2) + 1$
$y(t) = -yt^2(x^2+y^2)^{-3/2} + t$