

Functions of Linear Regression

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Given a data set $X \in \mathbb{R}^{n \times (m-1)}$ and a label set $Y \in \mathbb{R}^{n \times 1}$. Let two indices i and j be subject to $1 \leq i \leq n$ and $2 \leq j \leq m$ respectively. Thus, a data value is denoted as X_j^i and a label value is denoted as Y^i . For all X^i , insert $X_1^i = 1$ so that $X \in \mathbb{R}^{n \times m}$ and $1 \leq j \leq m$. Initialize weight matrix $W \in \mathbb{R}^{1 \times m}$ and thus a weight value is denoted as W_j .

Hypothesis function is defined in Equation (1).

$$H(X^i) = \sum_{j=1}^m W_j X_j^i \quad (1)$$

Loss function is defined in Equation (2).

$$C(X^i) = H(X^i) - Y^i \quad (2)$$

Cost function is defined in Equation (3).

$$\begin{aligned} J(W) &= \frac{1}{2n} \sum_{i=1}^n C^2(X^i) \\ &= \frac{1}{2n} \sum_{i=1}^n \left(\sum_{j=1}^m W_j X_j^i - Y^i \right)^2 \end{aligned} \quad (3)$$

The partial derivative of $J(W)$ with respect to W_j is given as follows:

$$\begin{aligned} \frac{\partial J(W)}{\partial W_j} &= \frac{1}{2n} \sum_{i=1}^n \frac{\partial C^2(X^i)}{\partial W_j} \\ &= \frac{1}{2n} \sum_{i=1}^n \frac{\partial C^2(X^i)}{\partial C(X^i)} \cdot \frac{\partial C(X^i)}{\partial H(X^i)} \cdot \frac{\partial H(X^i)}{\partial W_j} \\ &= \frac{1}{2n} \sum_{i=1}^n 2C(X^i) \cdot 1 \cdot X_j^i \\ &= \frac{1}{n} \sum_{i=1}^n (H(X^i) - Y^i) X_j^i \end{aligned}$$

Thus, for all W_j , the update function is given as follows:

$$W_j = W_j - \frac{\gamma}{n} \sum_{i=1}^n (H(X^i) - Y^i) X_j^i$$

where γ is the learning rate given by users.

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