Tuesday
10.01.23

* Field:- If each and every point of a region there corresponds some physical quantity then the region is called field.

→ If physical quantity are vector then the field will be vector field otherwise scalar field.

→ Scalar field:- Temperature of different points in the atmosphere, height of earth surface at different points

→ Vector field:- Force acting on different point of a body, force acting on a charged body in an electric field.

* Circuit Theory/Approach	Field Theory/Approach
→ V, I, ω	→ E, H, P
→ conducting	→ non-conducting
→ $V = -\int \vec{E}\cdot\vec{dl}$	→ $\vec{E} = \frac{-dV}{dL}$
→ $I = \int \vec{H}\cdot\vec{dl}$	→ $H = \nabla \times \vec{A}$
→ Average result	→ Accurate result
→ Less variable	→ Complex / More variables

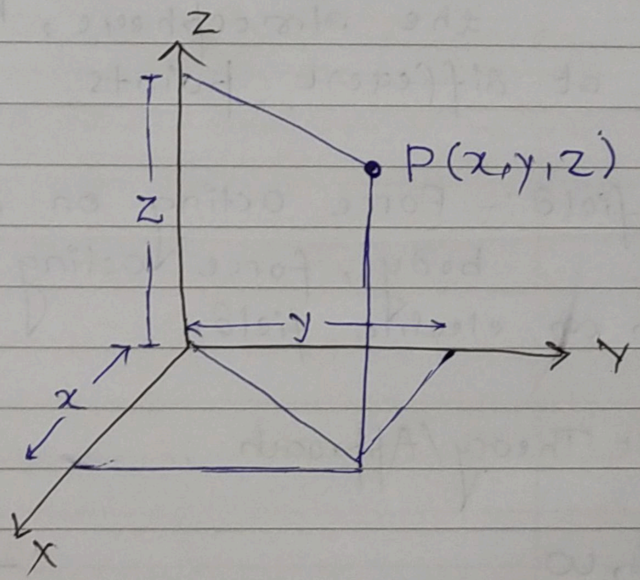
* Coordinate System:-

i) Cartesian (Rectangular) (x, y, z)

ii) Cylindrical (ρ, ϕ, z)

iii) Spherical (r, θ, ϕ)

i)



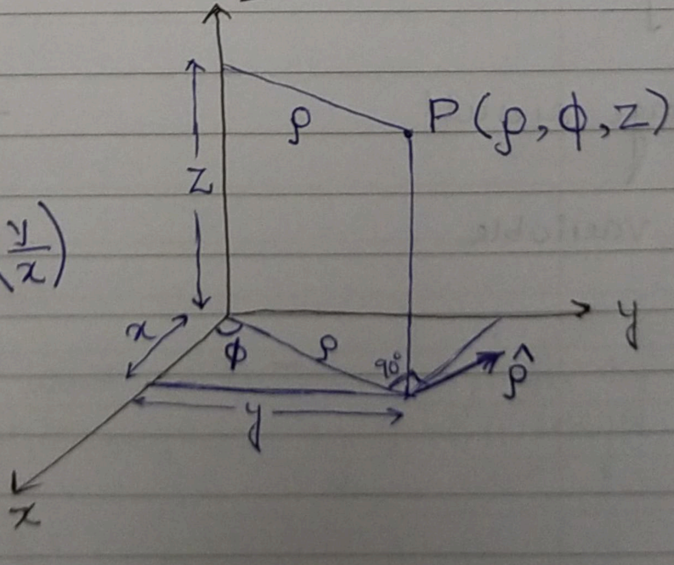
ii)

$x = \rho\cos\phi$

$y = \rho\sin\phi$

$\phi = \tan^{-1}\left(\frac{y}{x}\right)$

$\rho = \sqrt{x^2+y^2}$

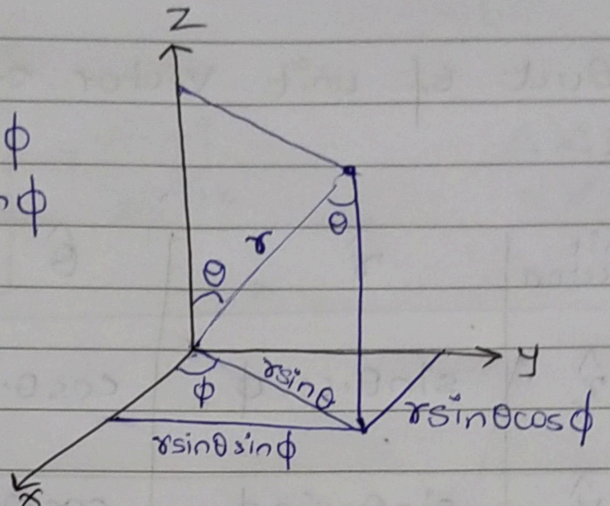


iii)

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$



$r = \sqrt{x^2+y^2+z^2}$ $\phi = \tan^{-1}\left(\frac{y}{x}\right)$

Unit vectors:-

$\hat{x}, \hat{y}, \hat{z}$

$\vec{a_x}, \vec{a_y}, \vec{a_z}$

$\hat{\rho}, \hat{\phi}, z$ (cylindrical)

$\hat{r}, \hat{\theta}, \hat{\phi}$ (spherical)

* Dot product of unit vector of rectangular & cylindrical:-

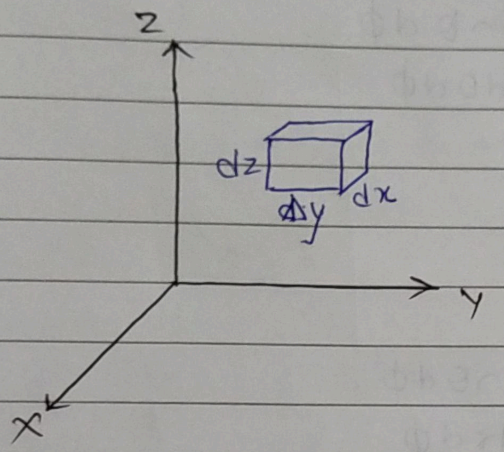
Unit Vectors	$\hat{\rho}$	$\hat{\phi}$	$\hat{z}$
$\hat{x}$	$\cos\phi$	$-\sin\phi$	0
$\hat{y}$	$\sin\phi$	$\cos\phi$	0
$\hat{z}$	0	0	1

Wed.
11.01.23

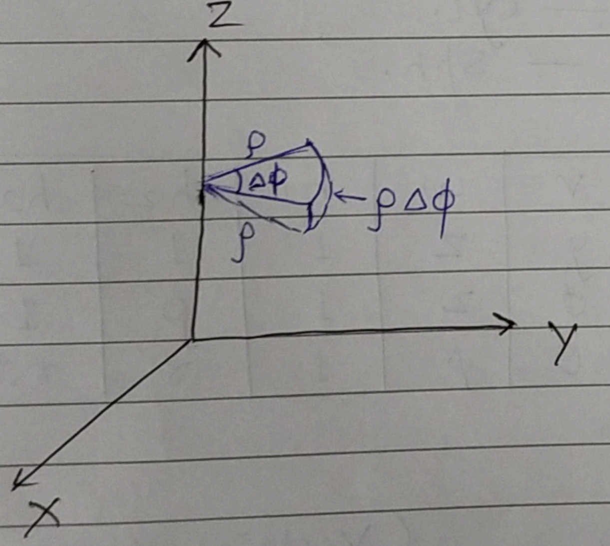
* Dot product of unit vector of rectangular & spherical:-

Unit Vectors	$\hat{r}$	$\hat{\theta}$	$\hat{\phi}$
$\hat{x}$	$\sin\theta \cdot \cos\phi$	$\cos\theta \cdot \cos\phi$	$\sin\theta \cdot \sin\phi$
$\hat{y}$	$\sin\theta \cdot \sin\phi$	$\cos\theta \cdot \sin\phi$	$\sin\theta \cos\phi$
$\hat{z}$	$\cos\theta$	$-\sin\theta$	0

Wed.
11.01.23

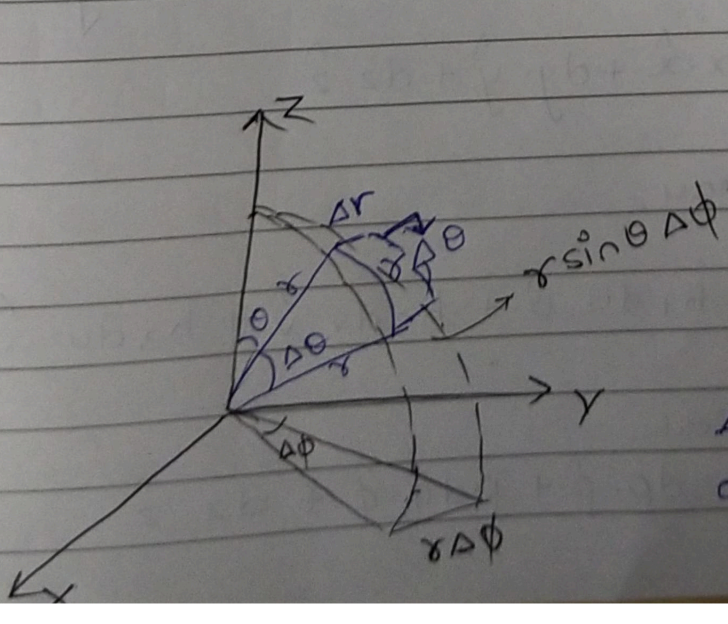


$$\Delta V = \Delta x \, \Delta y \, \Delta z$$
$$dV = dx\, dy\, dz$$



$$\Delta V = \Delta\rho \cdot \rho\,\Delta\phi \cdot \Delta z$$
$$= \rho\,\Delta\rho\,\Delta\phi\,\Delta z$$

$$dV = \rho\, d\rho\, d\phi\, dz .$$



$$\Delta V = \Delta r \cdot r\Delta\theta \cdot r\sin\theta\, \Delta\phi$$
$$\Delta V = r^2 \sin\theta \, \Delta r \, \Delta\theta \, \Delta\phi$$
$$dV = r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$dS_1 = r\,d\theta \cdot r\sin\theta\,d\phi$$
$$= r^2 \sin\theta\,d\theta\,d\phi$$

$$dS_2 = dr \cdot r\,d\theta$$
$$= r\,dr\,d\theta$$

$$dS_3 = dr \cdot r\sin\theta\,d\phi$$
$$= r\sin\theta\,dr\,d\phi$$

* Curvilinear Coordinate system. — Curv.
  Cartesian / Rectangle — Rect.
  Cylindrical — Cyl.
  Spherical — Sph.

Curv.	u	v	w	h_1	h_2	h_3
Rect.	x	y	z	1	1	1
Cyl.	ρ	ϕ	z	1	ρ	1
Sph.	r	θ	ϕ	1	r	$r\sin\theta$

* In Cartesian, (Vectorial length = $\vec{dL}$)

$$\vec{dL} = dx \cdot \hat{x} + dy \cdot \hat{y} + dz \cdot \hat{z}$$

In Curvilinear,

$$\vec{dL} = h_1\,du \cdot \hat{u} + h_2\,dv\,\hat{v} + h_3\,dw \cdot \hat{w}$$

In Cylindrical,
$$\vec{dL} = d\rho \cdot \hat{\rho} + \rho\,d\phi \cdot \hat{\phi} + dz \cdot \hat{z}$$

In Spherical,

$$\vec{dL} = dr\cdot\hat{r} + r\,d\theta\cdot\hat{\theta} + r\sin\theta\, d\phi\cdot\hat{\phi}$$

* Gradient : (∇V)

$$\nabla = \frac{\partial}{\partial x}\cdot\hat{x} + \frac{\partial}{\partial y}\cdot\hat{y} + \frac{\partial}{\partial z}\cdot\hat{z} = \text{del oper.}$$

→ Vector quantity

→ $$\nabla V = \frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\cdot\hat{y} + \frac{\partial V}{\partial z}\cdot\hat{z} \quad \text{(Cartesian)}$$

$$= \frac{1}{h_1}\frac{\partial V}{\partial u}\cdot\hat{u} + \frac{1}{h_2}\frac{\partial V}{\partial v}\hat{v} + \frac{1}{h_3}\frac{\partial V}{\partial w}\cdot\hat{\omega} \quad \text{(Curvilinear}$$

$$= \frac{\partial V}{\partial \rho}\cdot\hat{\rho} + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\cdot\hat{\phi} + \frac{\partial V}{\partial z}\cdot\hat{z} \quad \text{(Cylindrical)}$$

$$= \frac{\partial V}{\partial r}\cdot\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\cdot\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\cdot\hat{\phi} \quad \text{(Spherical)}$$

* Divergence:- $(\nabla\cdot\vec{D})$

$\vec{D}$ = Electric flux density.

$$\nabla \cdot \vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$= \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u}(h_2 h_3 D_u) + \frac{\partial}{\partial v}(h_1 h_3 D_v) + \frac{\partial}{\partial w}(h_1 h_2 D_w)\right]$$

$$= h_1 \frac{\partial D_u}{\partial u} + h_2 \frac{\partial D_v}{\partial v} + h_3 \frac{\partial D_w}{\partial w}$$

$$\nabla \cdot \vec{D} = \frac{1}{\rho}\left[\frac{\partial (\rho D_\rho)}{\partial \rho} + \frac{\partial}{\partial \phi}(D_\phi) + \frac{\partial}{\partial z}(\rho D_z)\right]$$

$$= \frac{1}{\rho}\frac{\partial D_\rho \cdot \rho}{\partial \rho} + \frac{\cancel{\rho}}{\rho}\frac{\partial D_\phi}{\partial \phi} + \frac{1}{\cancel{\rho}}\frac{\partial (D_z \cancel{\rho})}{\partial z}$$

$$\nabla \cdot \vec{D} = \frac{1}{r^2 \sin\theta}\left[\frac{\partial (r^2 \sin\theta D_r)}{\partial r} + \frac{\partial}{\partial \theta}(r \sin\theta D_\theta) + \frac{\partial}{\partial \phi}(r D_\phi)\right]$$

$$= \frac{1}{r^2 \sin\theta}\frac{\partial (D_r r^2)}{\partial r} + \frac{1}{r \sin\theta}\frac{\partial (D_\theta \sin\theta)}{\partial \theta} + \frac{1}{r \sin\theta}\frac{\partial D_\phi}{\partial \phi}$$

* Curl : $(\nabla \times \vec{H}$

$\nabla \times \vec{H} =$

$=$

* Laplacian Oper

$\nabla^2 V = \frac{\partial^2 V}{\partial x^2}$

$\nabla \cdot \nabla V =$

$\nabla \cdot \nabla V$

* Curl : $(\nabla \times \vec{H})$ H = Magnetic field.

$$\nabla \times \vec{H} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial y}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

$$= \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 \hat{u} & h_2 \hat{v} & h_3 \hat{\omega} \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial \omega} \\ h_1 H_u & h_2 H_v & h_3 H_\omega \end{vmatrix}$$

* Laplacian Operator : $(\nabla^2 V) = 0$

[If $\nabla^2 V = \rho$ (Poisson)]

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla \cdot \nabla V = \left(\frac{\partial \cancel{V}}{\partial x} + \frac{\partial \cancel{V}}{\partial y} + \frac{\partial \cancel{V}}{\partial z} \right) \left(\frac{\partial V}{\partial x} \hat{x} + \frac{\partial V}{\partial y} \hat{y} + \frac{\partial V}{\partial z} \hat{z} \right)$$

$\nabla \cdot \nabla V$ = Divergence of Gradient.

$$\nabla \cdot \nabla V = \frac{1}{h_1 h_2 h_3} \left[\frac{\partial}{\partial u} \frac{h_2 h_3}{h_1} \left(\frac{\partial V}{\partial u} \right) + \frac{\partial}{\partial v} \frac{h_1 h_3}{h_2} \left(\frac{\partial V}{\partial v} \right) + \frac{h_1 h_2}{h_3} \left(\frac{\partial V}{\partial w} \right) \right]$$