We have a graph G, with incidence matrix E, n nodes, m edges. (TO:DO Put graph in)

0.1 Derivation

$$\nabla = E^T = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 0 & -1 \end{pmatrix}$$

We have $n + m$ equations that connect heights H, fluxes Q and nodes q:

$$\nabla^T Q = q$$

$$\nabla H = -aQ|Q| = -a \begin{pmatrix} Q_1|Q_1| \\ Q_2|Q_2| \\ Q_3|Q_3| \\ Q_4|Q_4| \\ Q_5|Q_5| \end{pmatrix}$$

Unknowns: $2n + m$

$$\begin{matrix} Q & m \\ H & n \\ q & n \end{matrix}$$

Suppose that some q_0 and H_0 are known. We can separate them from the initial equations by splitting ∇. Let ∇_0 denote the $m \times n$ such that:

$$\{\nabla - \nabla_0\} H + aQ|Q| = -\nabla_0 H_0$$
$$\nabla^T - q = q_0$$

(TO:DO Put derivation in here)
The differential equations from the continuum method are

$$\frac{d}{dt}\left(Q_5 - q_5\right) = -\left(Q_5^* - q_5^*\right)$$
$$\frac{d}{dt}\left(Q_5 - Q_1 - Q_2 - q_1\right) = -\left(Q_5^* - Q_1^* - Q_2^* - q_1^*\right)$$
$$\dots$$
$$\frac{d}{dt}\left(Q_4 - q_4\right) = -\left(Q_4^* - q_4^*\right)$$
$$\frac{d}{dt}\left(H_2 - H_1 + aQ_1|Q_1|\right) = -\left(H_2^* - H_1^* + aQ_1^*|Q_1^*|\right)$$
$$\dots$$
$$\frac{d}{dt}\left(H_1 - H_5 + aQ_5|Q_5|\right) = -\left(H_1^* - H_5^* + aQ_5^*|Q_5^*|\right)$$

The discrete laplacian is
$$\Lambda = \nabla.\nabla^T$$

In operator form our equations are

$$\nabla^T Q_t - q_t = -(\nabla^T Q - q^*)$$
$$\nabla H_t + a\left[Q|Q|\right]_t = -(\nabla H^* + aQ^*|Q^*|)$$

(TO:DO) Newton method for this
Algorithm is:

1. Find initial guess $x^*,\ f(x^*)$
2. Get x_t from $f'(x)x_t = -f(x^*)$
3. $x^{**} = x^* + x_t$
4. Go to 2 until stop criterion

0.2 q doesn't change with time

As a first approximation, put $q_t = 0$.
We choose $q_{init} = (0, 2, -1, -0.5, -0.5)^T$ and compute $\nabla^T Q = q$ and $\nabla H = -aQ|Q|$
It is apparent that with least squares we get the optimal solution for these q.