I want to prove the following Lemma or Corollary

if a^2 is even then a is even

A <u>digression on logic</u>

Let p, q be propositions. We can form a truth table by treating propositions as functions with boolean values along w/ operations of negation, and, or

p	q	$\neg p$	$p \wedge q$	$p \vee q$
T	T	F	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	F

<u>Def</u>

We define the implication $p \to q$ for the proposition ("p implies q") or "if p, then q" which is a function of p and q

We define the biconditional $p \leftrightarrow q$ or "p if and only if q" as follows

p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \to q$	$p \leftrightarrow q$
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

p is called the hypothesis

q is called the conclusion

To form the <u>contrapositive</u>, swap and negate the propositions. $(\neg q \to \neg p)$

Can check from truth table that contrapositive is equivalent to $p \to q$

p	$\neg p$	$\neg q$	q	$\neg q \to \neg p$	$p \to q$
T	F	F	T	T	T
T	F	T	F	F	F
F	T	F	T	T	T
F	T	T	F	T	T

<u>Def</u> proof by contrapositive

<u>Ex</u> if a^2 even then a even

To prove this, can use contrapositive "if a odd then a^2 odd"

<u>Remark</u>: A statement and its contrapositive are logically equivalent

$n = 2m+1$

$n^2 = (2m+1)(2m+1) = 4m^2 + 4m + 1 = 2\underbrace{(2m^2+2m)}+1$

$= 2r + 1$ which is odd by def.

Therefore if n is odd, then n^2 odd, and equivalently, if n^2 even then n even.

Back to our proof $\Rightarrow$ We know that $a^2 = 2b^2$ is even and thus a is even

Since a is even it can be written as $2k$ for $k \in \mathbb{Z}$