PROPERTIES AND APPLICATIONS OF THE 2D FOURIER TRANSFORM

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Objectives

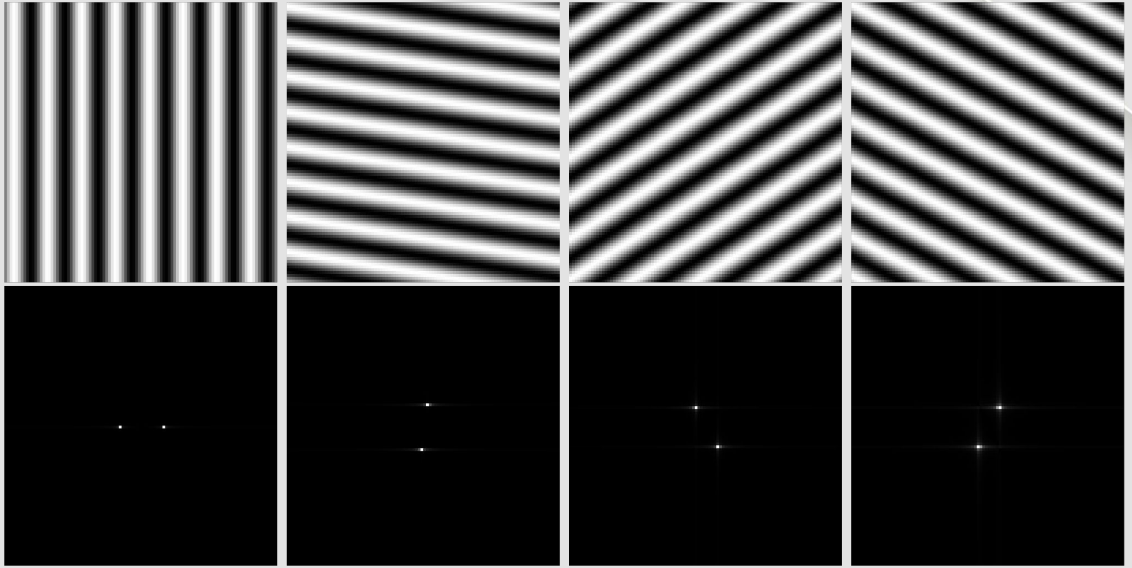
In this activity, we implement Fourier Transform techniques through various image processing applications. We have the following objectives:

- Demonstrate the rotation and spacing properties of the Fourier Transform by varying the frequencies and angles of the wave equation
- Filter unwanted patterns in the Fourier domain by using various masking techniques
- Solidify knowledge regarding convolution theorem with the 2D Fourier Transform
- Enhance fingerprint elements like ridges through frequency filtering
- Filter out specific lines in an image by masking out specific projections in the Fourier domain

Results and Analysis

In this part, we present the results, discuss their significance in line with the objectives, and analyze accordingly.

Rotation property of the FT

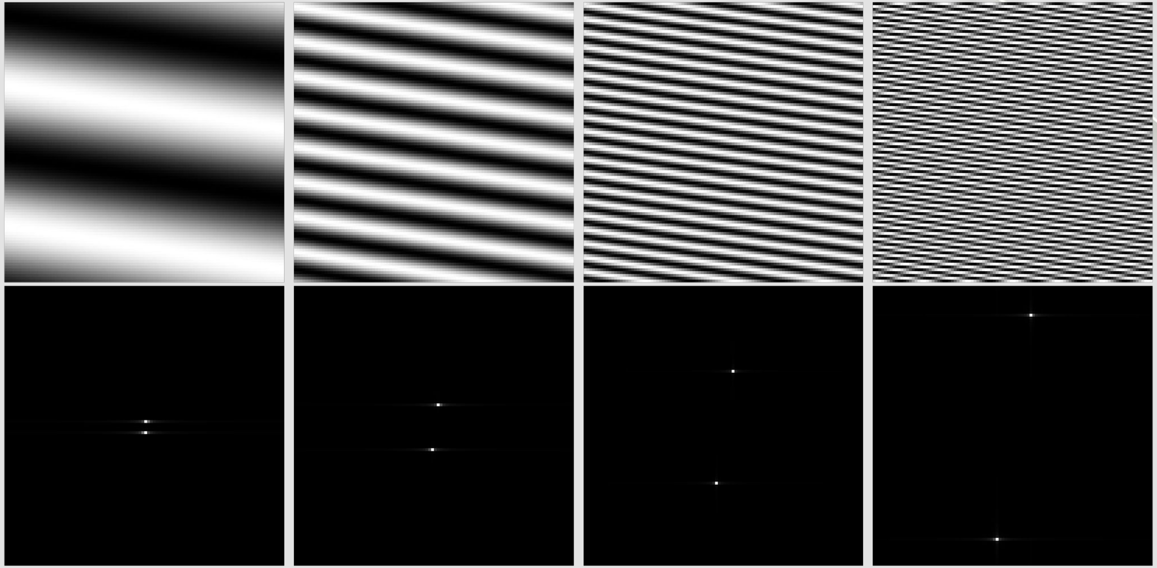


(a) $\Theta = 0°$ (b) $\Theta = 30°$ (c) $\Theta = 45°$ (d) $\Theta = 90°$

Fig. 1 Sinusoidal waves with varying angles and their corresponding FTs given f = 4.

We can see from Fig. 1 that **the rotation of the sinusoids also results to the rotation of their FTs.** This confirms the fact that **rotation in the spatial domain causes the rotation of its frequency domain.** It can also be noted that the orientation of the FT is normal with respect to its corresponding sinusoidal grating above.

Spacing property of the FT

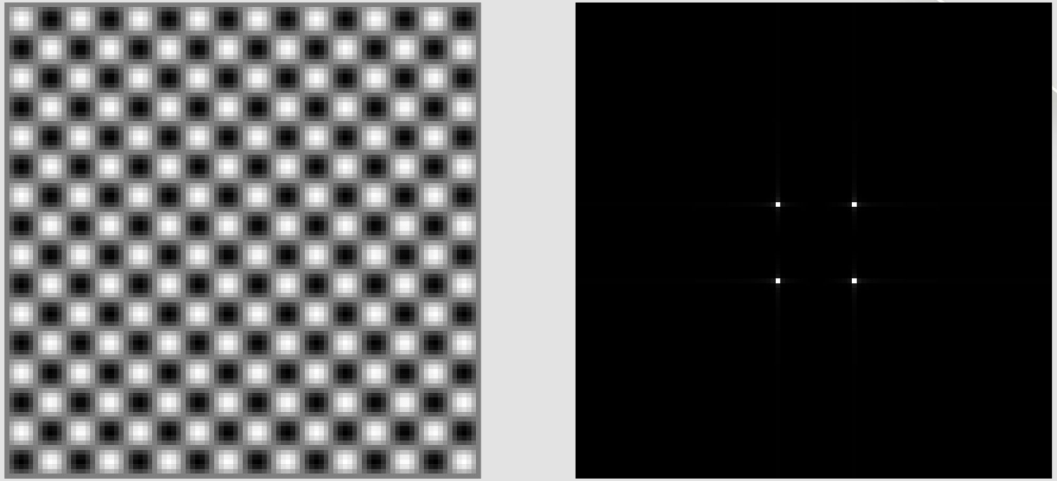


(a) f = 1　(b) f = 4　(c) f = 10　(d) f = 20

Fig. 2 Sinusoidal waves with varying frequencies and their corresponding FTs given ϴ = 30°.

We can see from Fig. 2 that **increasing the frequency of the sinusoidal wave results to an increased spacing distance between the dirac deltas in its FT.**

Combination of sinusoids

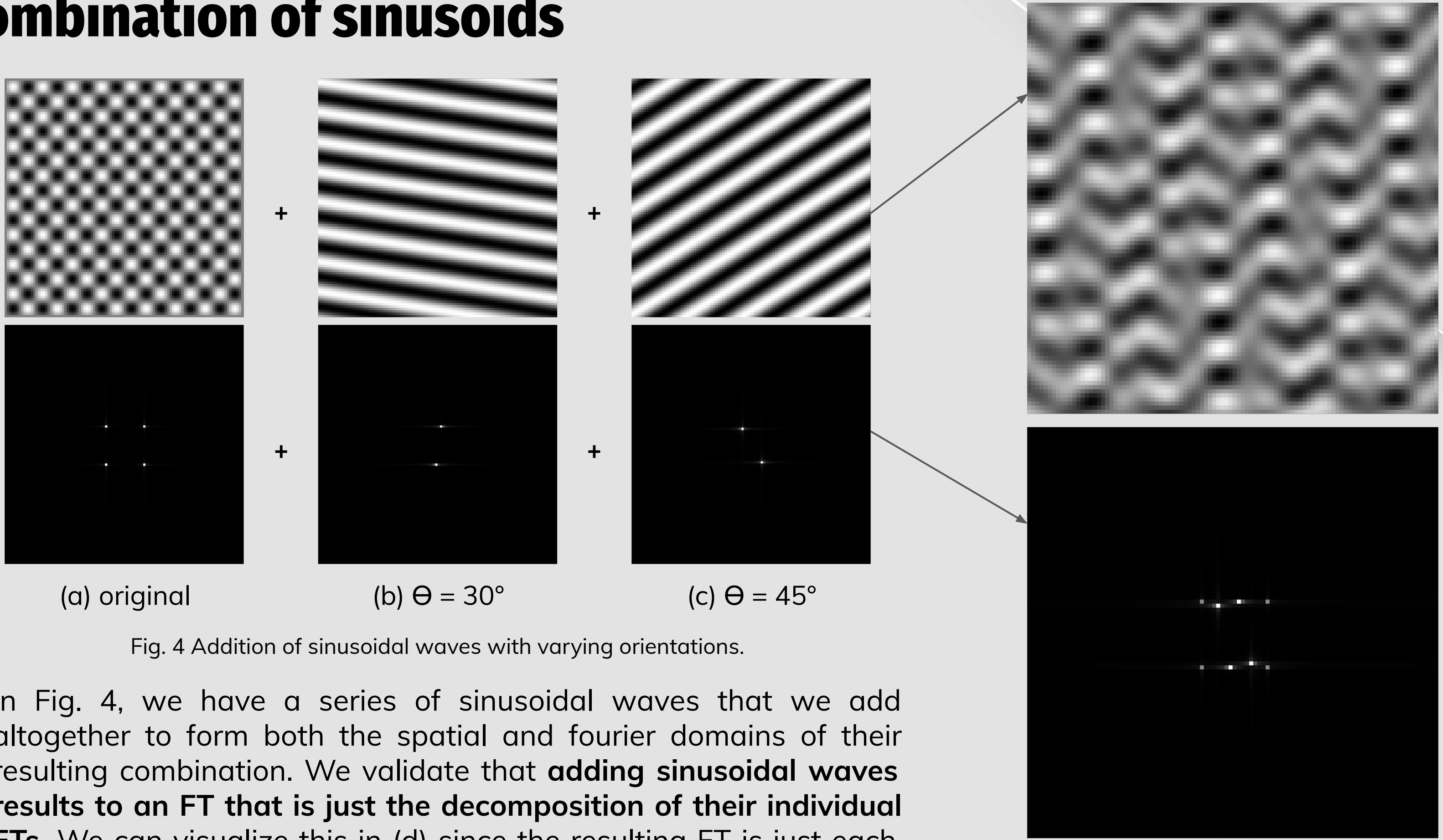


(a) (b)

Fig. 3 Sinusoidal waves with varying frequencies and their corresponding FTs given ϴ = 30°.

In Fig. 3, we have two sinusoids: one running along the x-direction and the other running along the y-direction. Multiplying these two to get the wave equation, **we obtain its corresponding FT of four dirac deltas that are evenly spaced with respect to each other.** This is due to **the nature of the sinusoidal wave wherein its product can be decomposed to a sum of sines.** Each pair of peaks in (b) corresponds to a singular sinusoidal grating, only differentiated by the orientation of their sine waves.

Combination of sinusoids

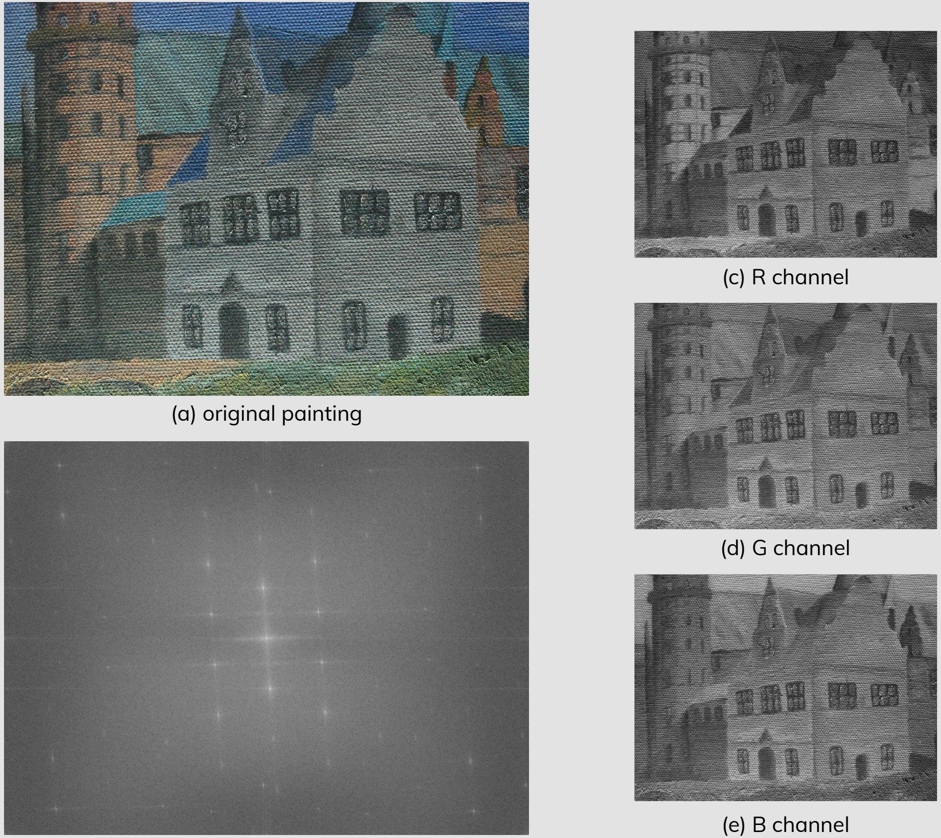


(a) original (b) Θ = 30° (c) Θ = 45° (d) combination

Fig. 4 Addition of sinusoidal waves with varying orientations.

In Fig. 4, we have a series of sinusoidal waves that we add altogether to form both the spatial and fourier domains of their resulting combination. We validate that **adding sinusoidal waves results to an FT that is just the decomposition of their individual FTs.** We can visualize this in (d) since the resulting FT is just each individual FT superimposed.

Canvas Weave Modeling and Removal



(a) original painting

(b) FT of painting

(c) R channel

(d) G channel

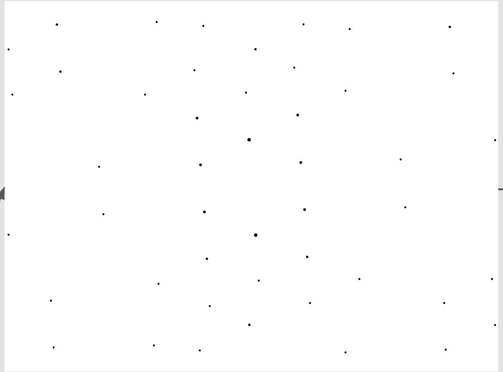
(e) B channel

Fig. 5 Fourier transform of a painting with weaving patterns. Decomposing the original image into individual RGB channels in graylevel results to (c), (d), and (e).

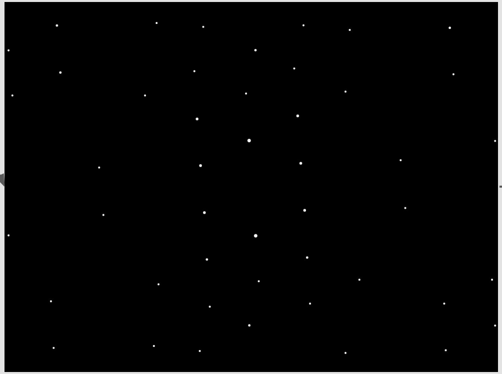
In Fig. 5, we observe the resulting FT of a painting with its original brush strokes obscured by the texture of the canvas. **The resulting dirac deltas or peaks in (b) represent these prominent ‘weaving patterns’ which are essentially just a series of sines after addition or multiplication.** Moving on, we separate the layers of (a) into its individual R, G, and B channels as we can see in (c), (d), and (e), respectively. **By manually tracing these peaks except the center (we do not erase the center since it holds most of the spectra in the image), we mask the ‘weaving patterns’ to reveal the obscured brush strokes.**

Canvas Weave Modeling and Removal

(a) original painting

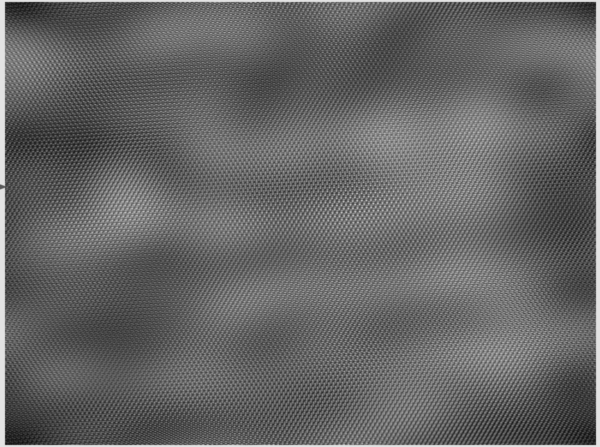


(b) original mask



(c) inverted mask

(d) deweaved painting



(e) weaving pattern from G channel

Fig. 6 Applying (b) to the painting results to the deweaved painting in (d). Applying (c) to the painting results to the weaving pattern in (e).

Canvas Weave Modeling and Removal

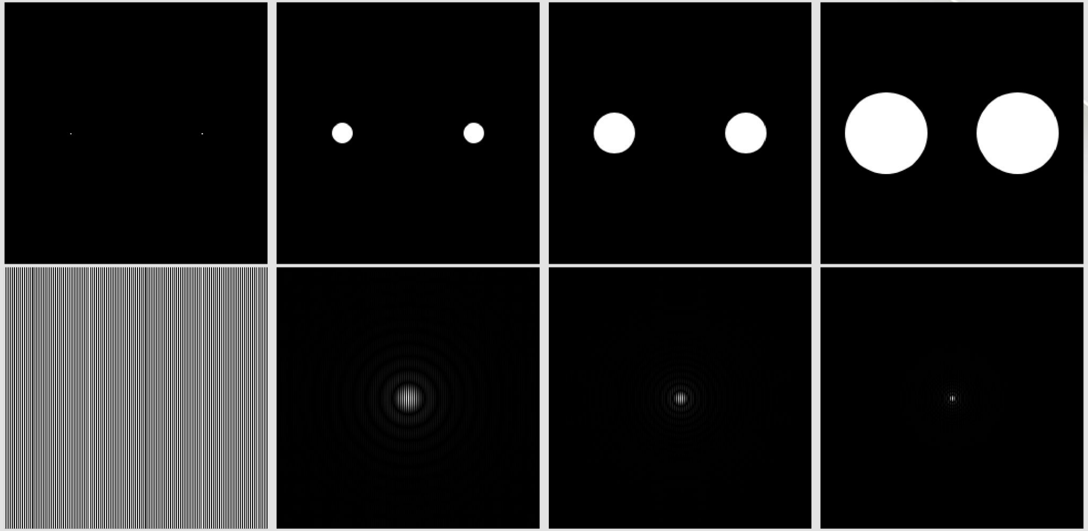
(a) original painting

(b) deweaved painting

Fig. 7 Side-by-side of the original painting and the deweaved painting after canvas weaving removal.

In Fig. 7, we can clearly see that **the weaving patterns of the original painting have been significantly filtered out.** This is because **convolving the mask with each individual RGB channel resulted to the removal of the overall weaving pattern as seen in (e) from Fig. 6.** These weaving patterns are characterized by **tiny structures of dots with varying intensities** which is accurate to the weaving pattern of the original painting. **By masking these peaks, we remove the high-frequency portions of the image corresponding to the weaving patterns which essentially smooths out the painting to reveal details like brush strokes.**

Convolution Theorem Redux

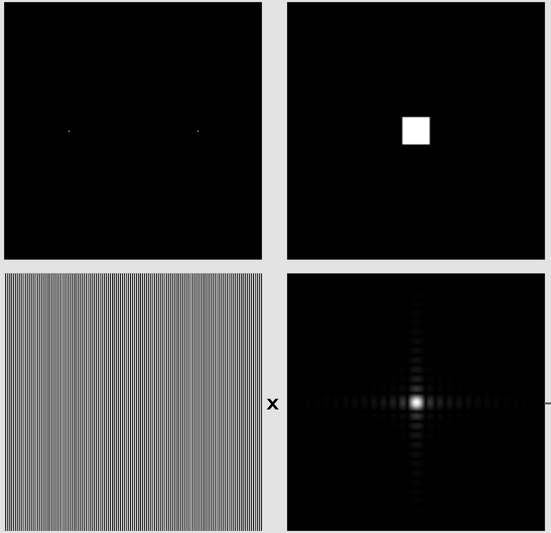


(a) 1x1 pixels (b) r = 10 (c) r = 20 (d) r = 40

Fig. 8 FT patterns of two symmetrically spaced circles with increasing radii.

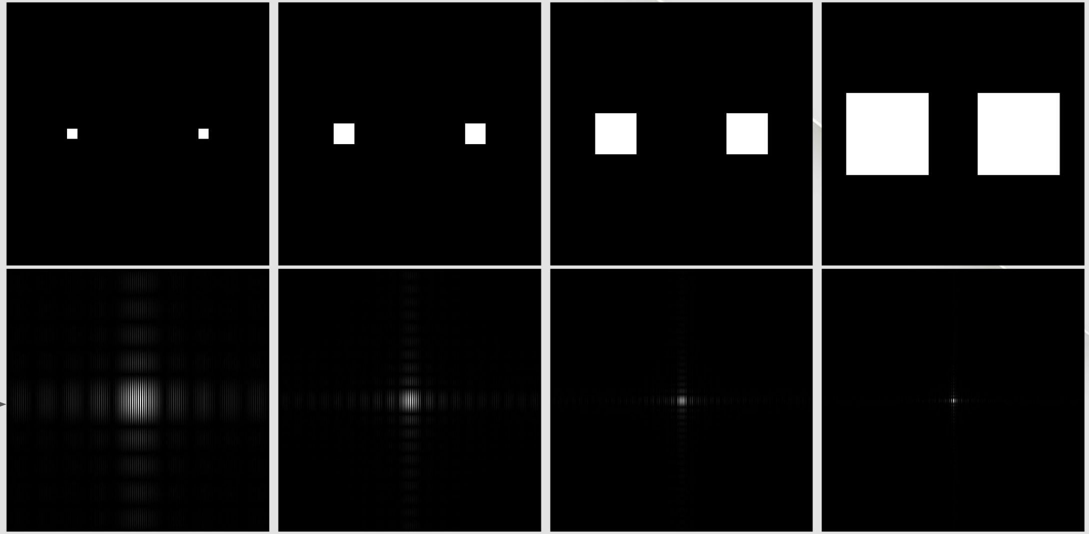
The two dirac delta approximations in (a) has an FT pattern of a sinusoidal wave along the horizontal axis. We have already established this from the previous activity along with the FT pattern of a circular aperture which results to the Airy pattern. **As such, (b), (c), and (d) exhibit both Airy and sinusoidal patterns on their FT domain since they are just symmetrically-spaced circular apertures acting as dirac deltas. Thus, their FT patterns can be visualized as a product of the individual FTs coming from the dirac deltas and the circular aperture.** From the resulting demonstration, we can confirm once again that **the FT pattern decreases in size as the radii of the circular apertures are increased.**

Convolution Theorem Redux



(a) dirac deltas (b) square

Fig. 9 Individual FT patterns.

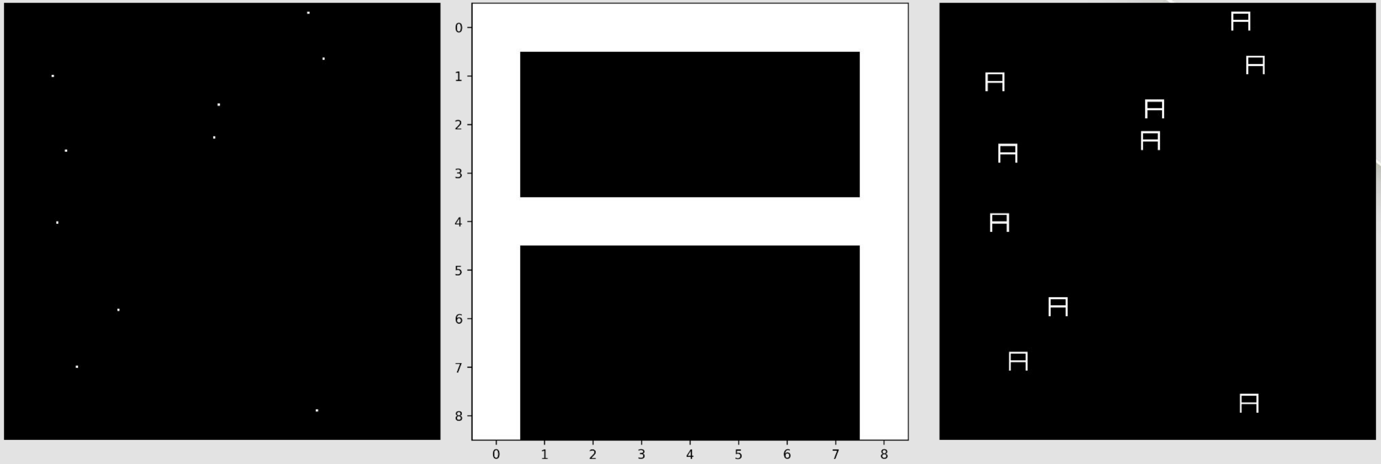


(c) r = 5 (d) r = 10 (e) r = 20 (f) r = 40

Fig. 10 FT patterns of two symmetrically spaced squares with increasing radii.

As we have previously established, **the resulting FT pattern of the given images is just a combination of each individual FT pattern comprising that image.** Since the initial images in Fig. 10 are two squares symmetrically spaced about the central horizontal axis, **their resulting FT patterns arise as a combination of the individual FT patterns from (a) and (b) in Fig. 9.** This essentially can be thought of as **a convolution of the dirac deltas and the square aperture.** Again, Fig. 10 demonstrates the inverse relationship between the sizes of the apertures and their corresponding fourier domain.

Convolution Theorem Redux



(a) randomized dirac deltas (b) 9x9 pattern (c) convolved image

Fig. 11 FT patterns of two symmetrically spaced circles with increasing radii.

As we can see from above, **convolving (a) and (b) resulted to the constructed pattern being ‘pasted’ onto the randomized dirac deltas.** This process can be inverted as we have seen in the correlation activity from before, wherein dirac deltas are placed on top of the matching pattern after convolution.

Convolution Theorem Redux

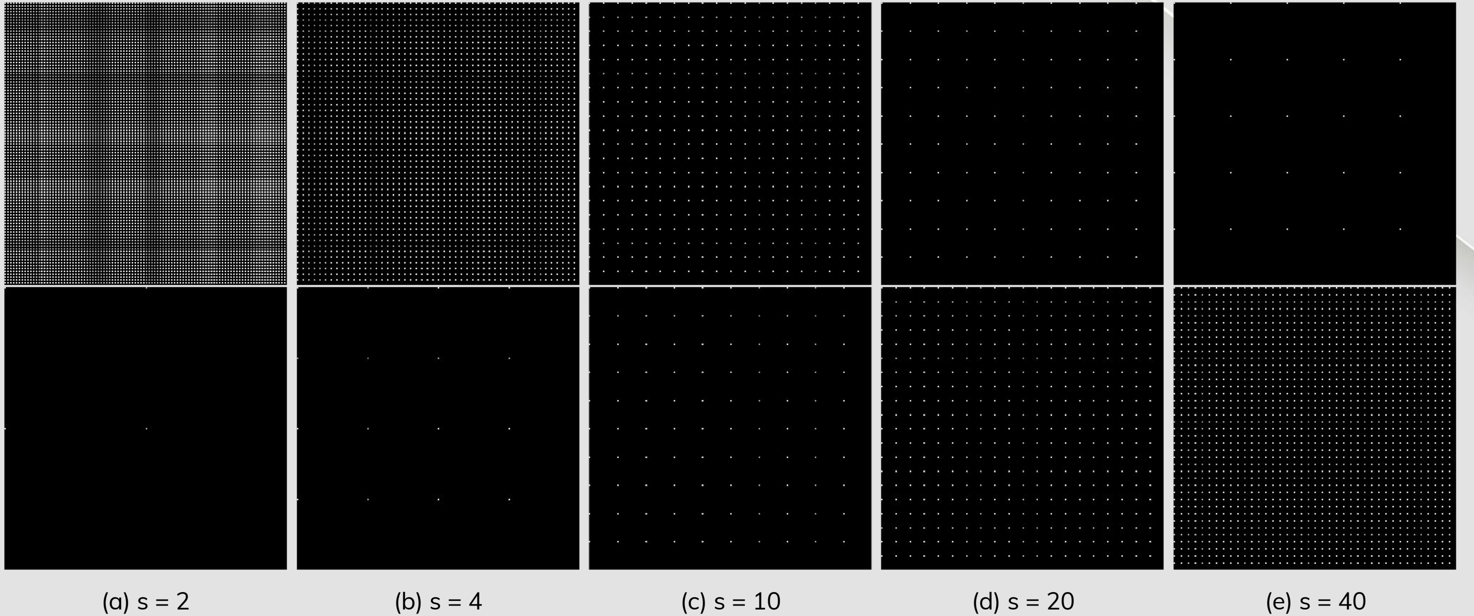


Fig. 12 FT patterns of symmetrically spaced dots with increasing pixel distances.

From the figure above, we can see that **the FT pattern of the repeating 1's approaches the appearance of a sinusoid.** This is most apparent in (a) with the tightest spacing. However, **as the spacing increases, the spacing between the FT pattern decreases.** These FT patterns can be thought of as **a superposition of sinusoids that interfere more destructively in the intensity domain, thus, the inverse relationship: less initial signal = more needed interference = more dots in the Fourier domain representing a series of sinusoids.**

Fingerprints : Ridge Enhancement

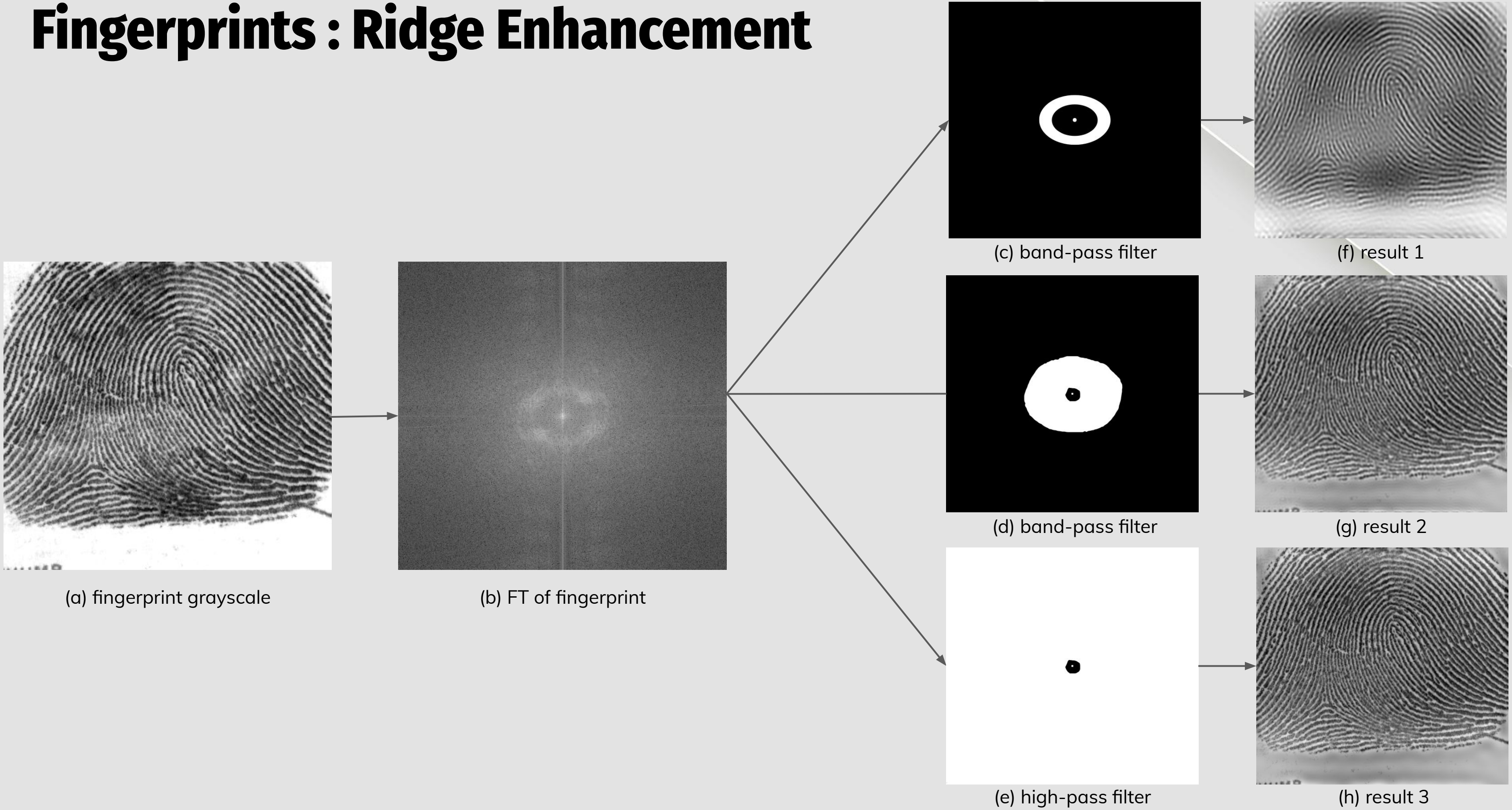


Fig. 13 Convolving (c), (d), and (e) with the fingerprint results to the ridge enhanced fingerprints in (f), (g), and (h). (h) shows the sharpest enhancement out of the three.

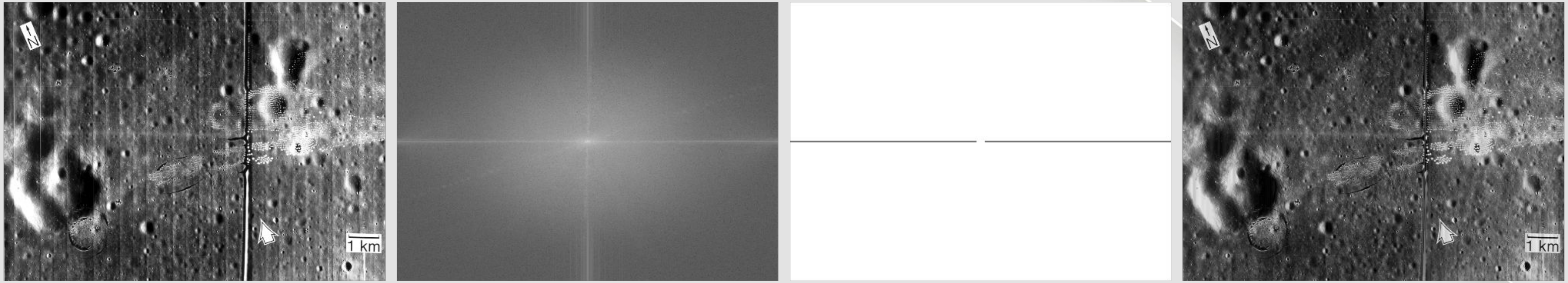
Fingerprints : Ridge Enhancement

(a) original fingerprint | (b) result from band-pass filter | (c) result from band-pass filter | (d) result from high-pass filter

Fig. 14 Ridge enhanced fingerprints after frequency filtering with various masks.

Results (b) and (c) arise from the modified band-pass filter in Fig. 13. The band-pass filter essentially filters out the high and low frequencies while keeping those in the mid-range. However, for this image, we retain the DC value in the center of the FT pattern since this holds most of the fingerprint's information like brightness and tone. By increasing the radius of the mask to retain more information in the mid-range, the fingerprint becomes sharper as seen in (c). **Result (d) arises from a modified high-pass filter in Fig. 13. The high-pass filter removes the low frequencies of the image while retaining its high frequency components.** Again, we retain the DC value for this mask. **This is commonly used in image sharpening since information like the edges are located in the high frequencies.** Analyzing Fig. 14 overall, we can see that (d) has the sharpest and most defined ridge information out of the three trials.

Lunar Landing Scanned Pictures : Line removal



(a) raw moon image (b) FT of image (c) mask (d) filtered image

Fig. 15 Vertical line filtering of the Apollo 11 Site image.

We have already established the rotation property of the Fourier Transform wherein **lines in the spatial domain are rotated by 90 degrees when projected onto the frequency domain.** Thus, to filter out the prominent vertical lines in (a), **we simply mask out the horizontal peaks in (b).** To do this, we repeat the same process in the canvas weave modelling and treat the vertical lines as weaving patterns. **Convolving the mask with the raw image, we see that we have significantly reduced the vertical lines in (d), resulting to a smoother layout of the area.** We can also see that much of the detail in the raw image has been retained. **By inverting the mask in (c) then convolving with (a), we can observe the vertical lines that have been filtered out by the algorithm in Fig. 16.**

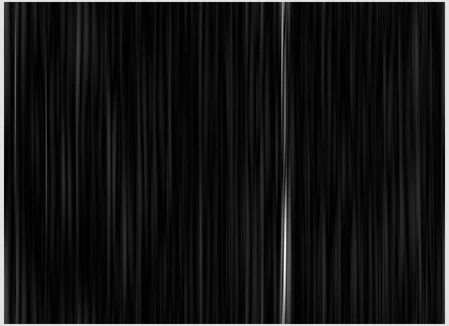


Fig. 16 Filtered-out pattern.

Reflection

This was another great activity! I believe that I am close to solidifying my knowledge about the Fourier Transform through various image enhancement techniques. I also liked how streamlined the activities were—from establishing certain properties of the Fourier Transform, to seeing them in action through practical applications. I especially loved the vertical line filtering in the moon images since you can really see the drastic side-by-side comparison of the raw image with the enhanced one.

I believe that my results are accurate and stayed true to the topic, with additional cross references and analyses. I also cross-validated my results with my peers, with my generous lab instructor, and through the internet with my references. However, if I had more time to allot, I would have tried out more frequency filters especially in the fingerprint activity.

Overall, this was a great activity!

Self-Grade

Technical Correctness: 35/35

I believe that my results are correct through math, research, and through validation with my peers and with my instructors.

Quality of Presentation: 35/35

I believe that the quality of my powerpoint is up to par while my code is much more sufficient in GitHub. I constructed the figures as instructed, and exported my data accordingly.

Self-Reflection: 30/30

I believe that I have acknowledged and reflected upon the activity well enough. I also have complete citation on the next slide.

Initiative: 10/10

I went above and beyond with my data presentation, and included extra analyses like additional frequency filters for the fingerprint and more.

References

[1] *Frequency Filters*. (n.d.). https://www.l3harrisgeospatial.com/docs/FrequencyFilters.html

[2] GeeksforGeeks. (2019). Frequency Domain Filters and its Types. *GeeksforGeeks*. https://www.geeksforgeeks.org/frequency-domain-filters-and-its-types/

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[6] Maciel, J. M., Rodríguez, F., González, M., Lecona, F. G. P., & Ramírez, V. H. V. (2017). Digital Processing Techniques for Fringe Analysis. In *InTech eBooks*. InTech. https://doi.org/10.5772/66474

[7] *Spatial Frequency Domain*. (n.d.). https://www.cs.auckland.ac.nz/courses/compsci773s1c/lectures/ImageProcessing-html/topic1.html