FOURIER TRANSFORM MODEL OF IMAGE FORMATION PART 1

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Objectives

In this activity, we implement the Fourier Transform model, specifically the Fast Fourier Transform algorithm and its versatility in image processing. We look at the following:

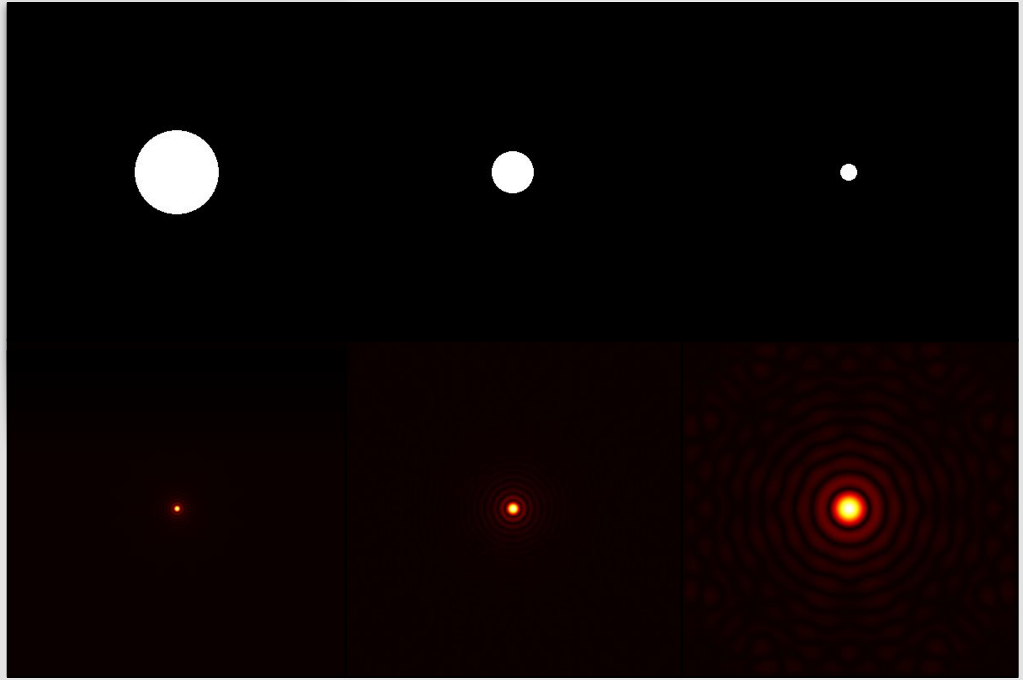
- Apply various Fast Fourier Transform algorithms (fft, fft2, ifft, ifft2, fftshift) on a set of images and analyze their respective outputs
- Simulate Fraunhofer diffraction patterns like the Airy pattern from apertures of varying sizes and shapes
- Successfully perform image convolution
- Apply correlation for template matching in images

** Note that for this activity, the author uses the terms FT pattern and Fraunhofer diffraction pattern interchangeably.*

Results and Analysis

In this part, we present the results, discuss their significance in line with the objectives, and analyze accordingly.

Familiarization with Discrete FT



(a) r = 0.5 (b) r = 0.25 (c) r = 0.1

Fig. 1 FT of circular apertures with decreasing radius sizes

As we can see from the figure, **as the radius of the original circle decreases, the resulting fourier transform becomes larger.** The fourier transform takes on the form of concentric circles whereby the brightest projection of the circle is at the center, surrounded by less intense rings which become dimmer as the distance is increased. This is also known as the **Airy pattern** which, in the real world, can be exhibited by the Poisson (Arago) spot.

Familiarization with Discrete FT

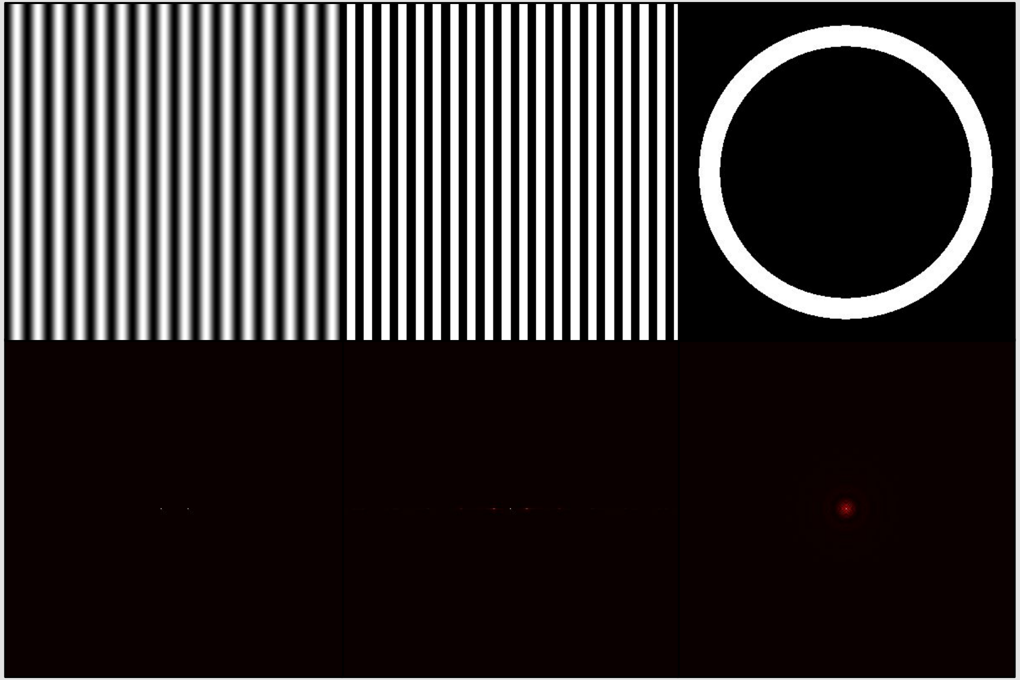

(a) sine (b) grating (c) annulus

Fig. 2 FT of various apertures

As we can see from the figure, **varying the shape of the apertures also alters the resulting fourier transform.** **(a)** For a sinusoid, the resulting FT are two dots denoting the maxima and minima of the wave, equidistant from the origin. **(b)** For grating, since these are essentially square waves in 2D, the resulting FT resembles that of a sinc function where rectangular fringes form at the x-axis. **(c)** For an annulus, the FT is similar to that of a regular circular aperture but with dampening due to the central obstruction of the original image.

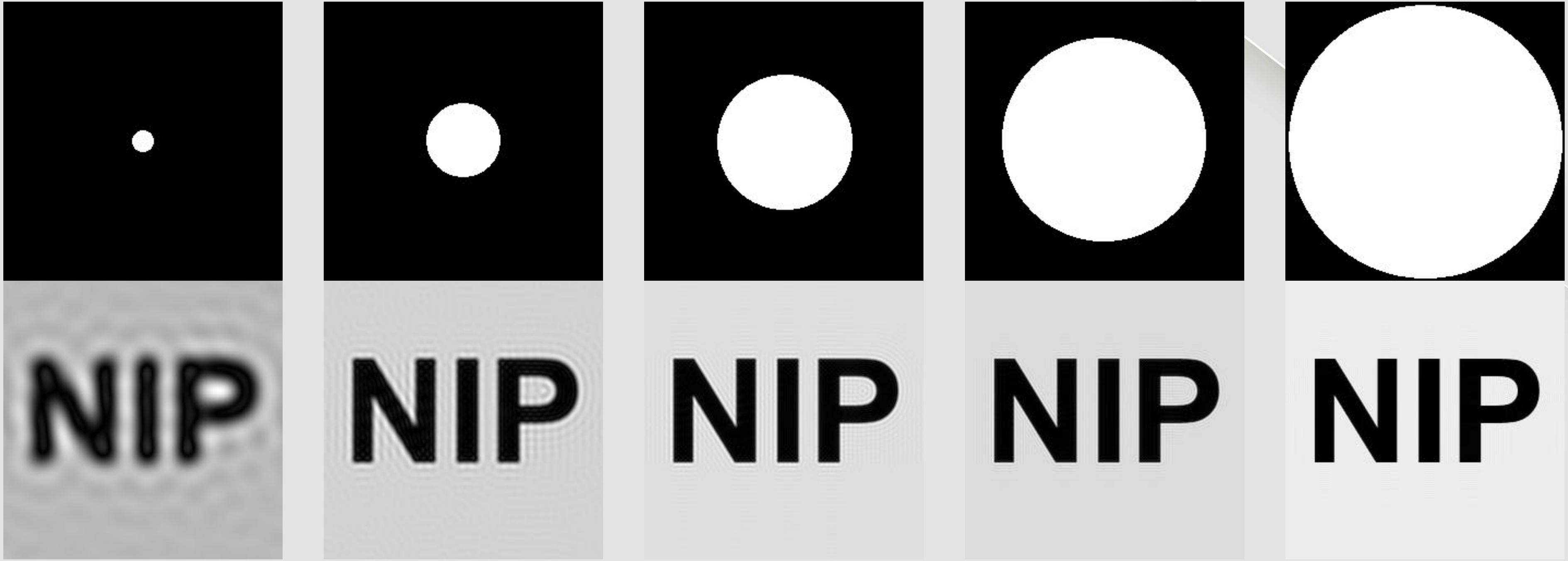
Familiarization with Discrete FT

(a) grayscale (b) fft2(fft2()) (c) ifft2(fft2())

Fig. 3 Effects of fft2 and ifft2 functions

As we can see from the figure, **taking the fft2 of the fft2 of the grayscale image rotated it by 180 degrees since its quadrants have been interchanged twice.** However, **taking the ifft2 of the fft2 of the grayscale image reverted back the process by inverting the quadrants once again.** This is why the original image is exactly like the resulting image of the inverted one.

Simulation of an Imaging System



(a) r = 0.1 (b) r = 0.25 (c) r = 0.5 (c) r = 0.75 (c) r = 1.0

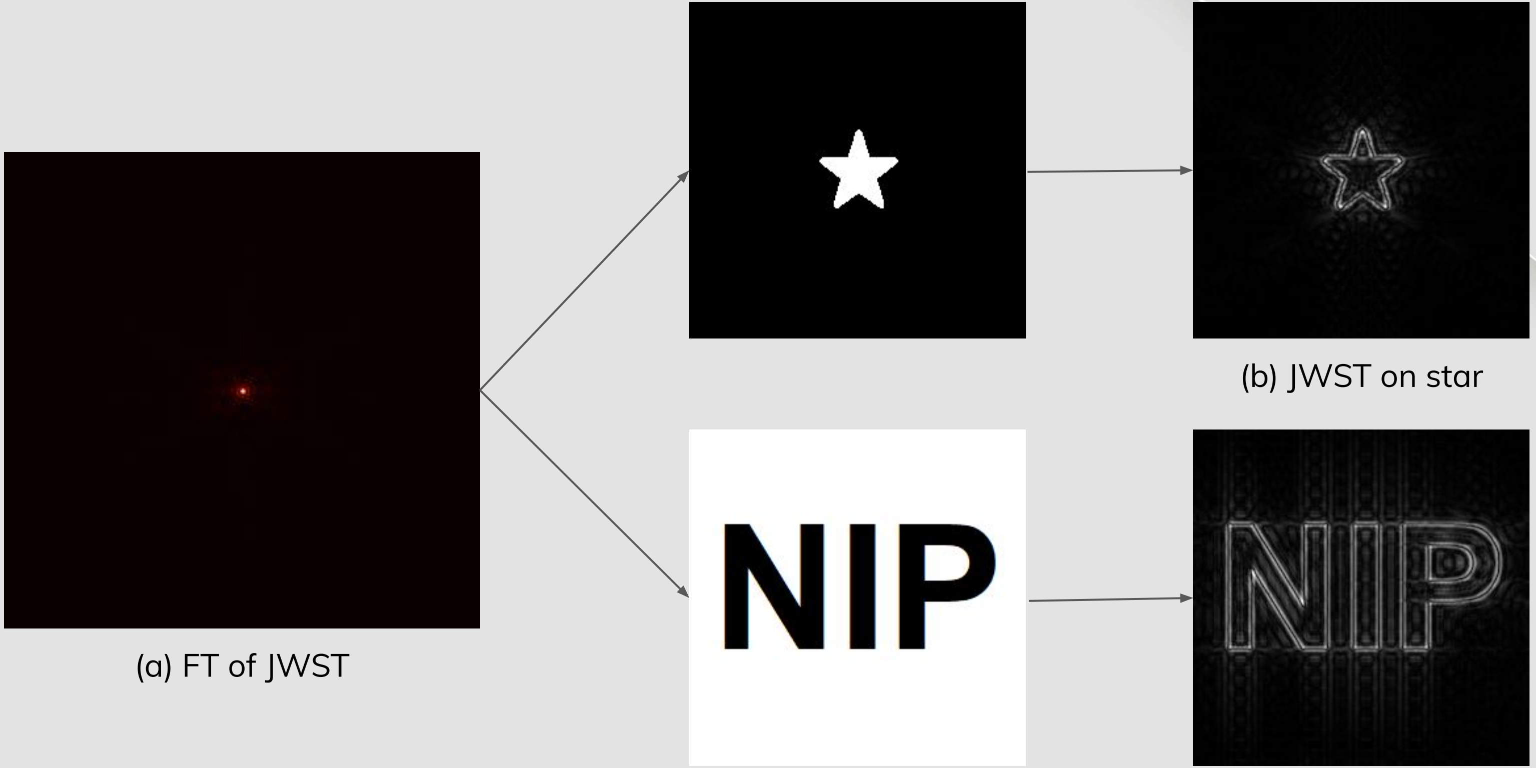
Fig. 4 Image simulation of “NIP” through convolution with circular apertures of varying sizes

Analysis

As we saw from the previous slide, the size of the aperture directly affects the quality of the resulting image simulation. **As the aperture size increases, the brighter and sharper the resulting convolved image becomes.** In the real world, this simulates the limitation of the imaging lens whereby its radius limits the amount of information it can process through convolution.

From the previous results, we have established that as the circular aperture increases, its FT pattern decreases. This means that **having a smaller circular aperture with greater FT pattern induces a smeared and blurrier convolved image.** This results to a combination of the "NIP" image and the Airy pattern wherein the letters have a slight ripple effect from the center going outwards.

JWST Convolution Results



(a) FT of JWST

(b) JWST on star

(c) JWST on “NIP”

Fig. 5 Simulation of image produced by JWST mirror convolved with various images

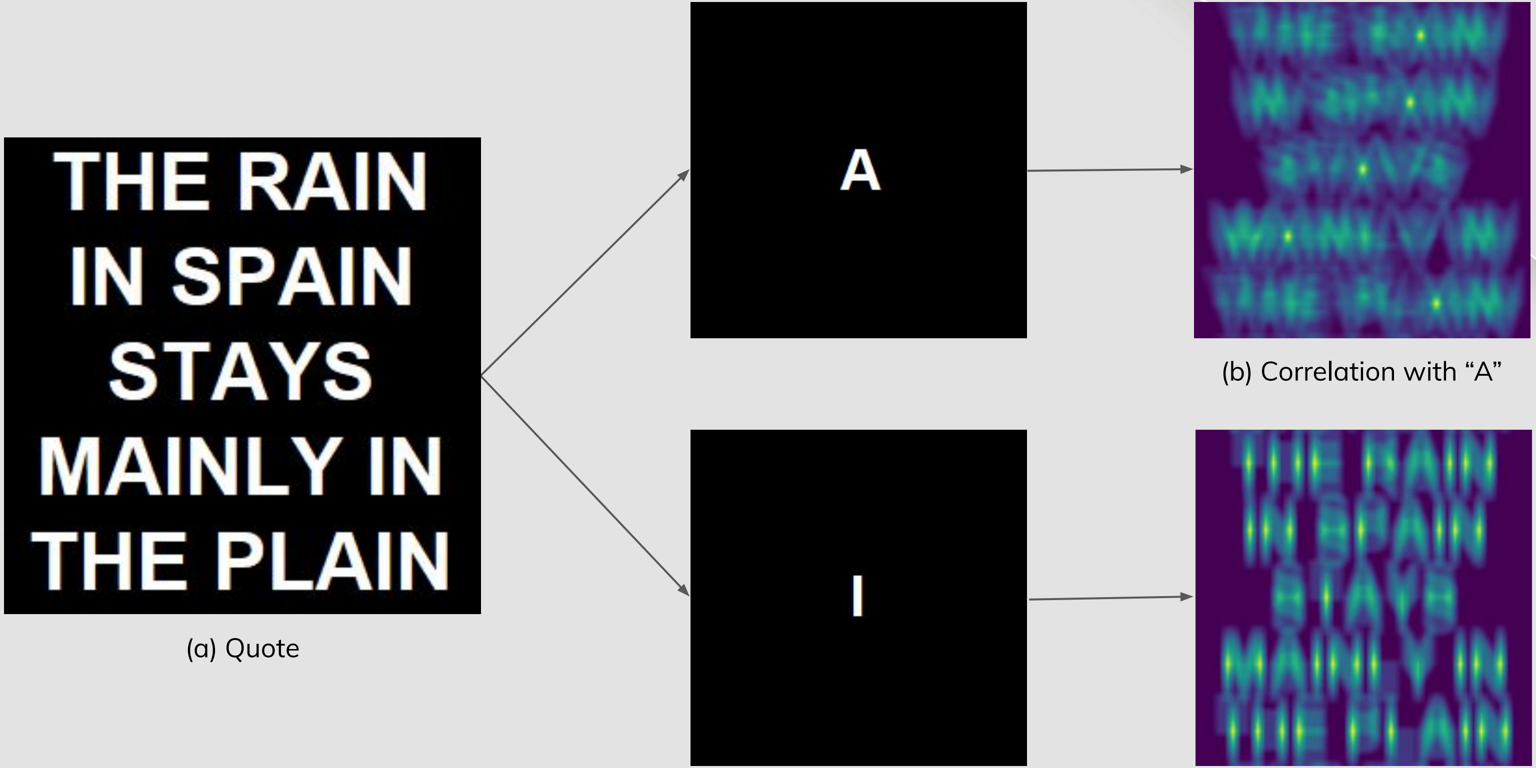
Analysis

From the FT pattern of the JWST mirror simulation, we can see that **it slightly resembles that of an Airy disk but now with a bright diamond, star-like center instead.** As we go outwards, less intense **hexagonal rings start to form but are almost instantaneously annihilated since the original aperture is too big for a uniform FT pattern to be generated.** This Fraunhofer diffraction pattern suggests that this is how a singular star would look like as imaged by the JWST.

After convolving the JWST FT pattern with an image of a 5-pointed star, we can see that it highlighted its edges and formed another ripple effect starting from its center. Rays can also be observed propagating outwards from the edges of the star.

Finally, convolving the JWST FT pattern with the “NIP” image also mainly highlighted its edges with the same ripple effect as before. However, it can also be seen that the letters are clearly being mirrored with the same pattern as reflected on the y-axis.

Template Matching Using Correlation



(a) Quote

(b) Correlation with “A”

(c) Correlation with “I”

Fig. 6 Correlation of “A” and “I” with the given quote

Analysis

From the previous slide, we have shown the correlation and degree of similarity between the templates and the given quote.

Using template “A”, five high-intensity points can be observed in the resulting image, denoting the presence of the letter “A” at these locations. However, since we multiplied their fourier transforms together, we can see that the template “A” smeared significantly across the letters of the quote, forming a blurry and unreadable image.

Using template “I”, approximately 34 high-intensity points were observed, seemingly denoting the presence of the letter “I” at these locations. However, the quote obviously does not have 34 letter I’s but instead **contain a significant amount of straight and upright letters that resemble the letter “I”.** This tells us that our correlation algorithm is relevant in pattern recognition and matching similar-looking templates on images.

Reflection

Overall, Activity 2 was much simpler, more fun, and more intuitive than Activity 1. I enjoyed having to apply various functions of the FFT algorithm on real world simulations like the JWST mirror and small lens apertures. This made me appreciate working with images because the results are basically at my fingertips, and the techniques are very easy to implement.

I believe that my results are correct since the math makes sense and I did cross-validate them with my peers and with my generous instructors during lab hours. However, if I had more time this week, I would have implemented more examples and variations for the apertures I used.

On the other hand, I tried to do more research regarding the JWST mirror and large space telescopes in general, because my current thesis involves data coming from the 4th Fermi LAT (Large Area Telescope) Catalog. Deep sky surveys really pique my interest because of all the possibilities that we can encounter and even discover once we process all these raw data.

Self-Grade

Technical Correctness: 35/35

I believe that my results are correct through math and research, and through validation with my peers and with my instructors.

Quality of Presentation: 35/35

I believe that the quality of my powerpoint is up to par while my code is much more sufficient. I constructed the figures as instructed, and exported my data accordingly.

Self-Reflection: 30/30

I believe that I have acknowledged and reflected upon the activity well enough. I have also cited my sources.

Initiative: 10/10

I applied image convolution to other apertures, and I did improve the efficiency of my code by defining functions which I am sure will be used for future activities.

References

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Mazet, V. (n.d.). *Basics of Image Processing*. Fourier transform - Basics of Image Processing. Retrieved March 24, 2023, from https://vincmazet.github.io/bip/filtering/fourier.html

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