Richard Abele October 29, 2024

Numerical Methods in Physics and Astrophysics
Problem Set 1; Problem 2

We are given a differential equation of the form

$$y'' + \lambda^2 y = 0.$$

In order to find the Eigenvalues of the given equation, we will first solve it analytically as suggested.

A general solution to this type of differential equation is give by

$$y(x) = A\cos(\lambda x) + B\sin(\lambda x)$$

with $A, B \in \mathbb{R}$ and determined by the boundary conditions.

The given boundary conditions are

(a) $y(0) = 0$

(b) and $y(1) = y'(1)$.

From boundary condition (a) it follows that:

$$\begin{aligned} y(0) &= A\cos(0) + B\sin(0) = A = 0 \\ \Rightarrow y(x) &= B\sin(\lambda x) \\ \Rightarrow y'(x) &= B\lambda\cos(\lambda x) \end{aligned}$$

We can now apply the second boundary condition as follows:

$$\begin{aligned} y(1) &= B\sin(\lambda) \\ y'(1) &= B\lambda\cos(\lambda) \\ \Rightarrow B\sin(\lambda) &= B\lambda\cos(\lambda) \\ \Rightarrow \sin(\lambda) &= \lambda\cos(\lambda) \end{aligned}$$

This equation can then be rearranged and solved using the Newton Method from Problem 1 (see parent directory).

$$\lambda\cos(\lambda) - \sin(\lambda) = 0$$

Looking at a plot of the function in mathematica confirms that it is an oscillating odd function with infinite solutions.

The first non-zero solution yielded with seven iterations of the newton method is

$$x_n = 4.493409$$