Algebraic Properties

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1 Matrix Definition

2 Augmented Matrices

A matrix can be used to represent a system of linear equations written in standard form. To do so, we extract the coefficients of each term in the equation to form an augmented matrix:

$$\begin{aligned} 3x + 2y &= 5 \\ x - y + 3z &= 1 \\ 2x + y + z &= 4 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 3 & 2 & 0 & 5 \\ 1 & -1 & 3 & 1 \\ 2 & 1 & 1 & 4 \end{array}\right]$$

3 Row Operations

3.1 Interchanging Rows

$$\left[\begin{array}{ccc|c} 3 & 2 & 0 & 5 \\ 1 & -1 & 3 & 1 \\ 2 & 1 & 1 & 4 \end{array}\right]$$

$$R_1 \Leftrightarrow R_2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 3 & 1 \\ 3 & 2 & 0 & 5 \\ 2 & 1 & 1 & 4 \end{array}\right]$$

$$2 \cdot \left[\begin{array}{ccc|c} 1 & -1 & 3 & 1 \\ 3 & 2 & 0 & 5 \\ 2 & 1 & 1 & 4 \end{array}\right] = \left[\begin{array}{ccc|c} 2 & -2 & 6 & 2 \\ 6 & 4 & 0 & 10 \\ 4 & 2 & 2 & 8 \end{array}\right]$$

4 Row-Echelon Form

A matrix is in row-echolon form if it satisfies the following conditions.

1. Any rows consisting entirely of zeros are at the bottom of the matrix
2. For all other rows, the first nonzero entry is 1. This is called the leading 1.
3. The leading 1 in each nonzero row is to the right of the leading 1 in the row immediately above.

4.1 Reduced Row-Echelon Form

A matrix is in reduced row-echelon form if it is in row-echelon form with the added condition that each row with a leading entry of 1 has zeros above the leading 1.

5 Gaussian Elimination

1. write the augmented matrix for the system.
2. use elementary row operations to write the augmented matrix in row-echelon form.
3. use back substitution to solve the resulting system of equations

The agoal of writing an augmented matrix in row-echelon form is to make the elements along the main diagonal one and the entries below the main diagonal zero. The main diagonal refers to the elements on the diagonal from the upper left to the lower irght all to the left of the vertical bar.

6 Inconsistent Systems

A system with no solutions is inconsistent. If the system reduces to a contradiction there is no solution and the system is inconsistent.

7 Systems with Dependent Equations

A solution to a system of two linear equations in two variables is a point of intersection of the lines. If the two lines are parallel, the system is inconsistent and has no solution. If the equations represent the same line there are infinite solutions which represent all the points on the line and the system is said to be dependent.