Algebraic Properties

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January 28, 2026

1 Transformations

1.1 Transformation Formatting

You can only apply transformations to visualize a graph if the expression is in transformation format.

Transformation Format: $a \cdot f(b(x-h)) + k$

f: The Parent Function
a: Vertical Stretch
b: Compress Horizontally
h: Horizontal Shift
k: Vertical Shift

These elements can be missing but if it's not in this format you can't plot a graph with transformations.

1.2 Horizontal Translation

Horizontal Translation shifts the functions output along the x axis positively or negatively by some amount, subtracting from the x value pushes the value the amount subtracted in the positive direction, and adding to the x value pushes the function in the negative direction.

$$f(x) = \frac{x-5}{2}$$

This may seem like the opposite of what should happen, but you can think about this as the x value being prolonged or pre-emptive to naturally derive the reasoning behind this. When subtracting five from x it moves the graph towards positivity five, this means that the graph starts later than normal.

1.3 Horizontal Stretch or Compression

Horizontal Stretching or Compression is caused by scaling the 'b' value of an expression in transformation format, the b value is the coefficient of the x value,

and potential horizontal translation all within one quantity which is within the parent function.

Transformation Format: $a \cdot f(b(x-h)) + k$

Compression is caused by: $|b| > 1$
It's called compression because the graph is squeezed into the y-axis, and is made narrower.

Stretching is caused by: $0 < |b| < 1$
It's called stretching because the graph stretches away from the y-axis, and is made wider.

1.4 Y-Axis Reflection (Vertical Reflection)

A reflection over the y-axis, every (x, y) value becomes (-x, y). The Y-Axis reflection is caused by a negative 'b' value in transformation format:

Transformation Format: $a \cdot f(b(x-h)) + k$

$$f(x) = 2^{-(x-2)}$$

Here, the '-' in '-'(x-2) is causing the Y-Axis transformation.

1.5 Vertical Stretch or Compression

Vertical Stretching or Compression is caused by scaling the multiplier of a functions parent function; in transformation format, the value being scaled is the 'a' variable.

Transformation Format: $a \cdot f(b(x-h)) + k$

$$f(x) = 8\sqrt{x} + 4$$

This square root function is vertically stretched by a factor of eight.

Stretching is caused by: $|a| > 1$
Compression is caused by: $0 < |a| < 1$

1.6 X-Axis Reflection

A reflection over the x-axis, every (x, y) value becomes (x, -y). The x-axis reflection is caused by a negative 'a' value in transformation format, which is

characterised by being a coefficient of the parent function:

Transformation Format: $a \cdot f(b(x-h)) + k$

$$f(x) = -\sqrt{x} + 4$$

The '-' in the above outside the parent function is what's causing the x-axis reflection.

1.7 Vertical Translation

Vertical translation is caused by the 'k' factor in function transformation format. The 'k' factor is outside of the parent function and unaffected by any scaling.

Transformation Format: $a \cdot f(b(x-h)) + k$

$$f(x) = -\sqrt{x} + 4$$

When the k factor is raised or lowered, the function is vertically translated along the y axis based on how much it is added to or subtracted from.

Raises the function: $k > 0$
Lowers the function: $k < 0$

1.8 Precedence of Transformation Graphing

1. Horizontal Translation
2. Horizontal Stretch or Compression and Y-Axis Reflection
3. Vertical Strectch or Compression and X-Axis Reflection
4. Vertical Translation