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Operation Research.

Cat 2.

1.

Factory/Shop	W	X	Y	Z	Supply
A	3	5	7	6	50
B	2	5	8	2	75
C	3	6	9	2	25
Demand	20	20	50	60	

(a Formulate the Linear Programming Model for the transportation problem, stating its objective functions & constraints.

(Minimize Cost):

$$\text{Min } Z = 3x_{AW} + 5x_{AX} + \dots + 2x_{CZ}$$

Constraints:

Supply: $x_{AW} + x_{AX} + x_{AY} + x_{AZ} \le 80$ (Factory A) (same to B) *

Demand: $x_{AW} + x_{BW} + x_{CW} = 20$ (Shop W) → Similar for X, Y, Z.

Non-negativity: $x_{ij} \ge 0$.

(b Is the transportation problem balanced? Explain.

- ✓ Total Supply: $80 + 75 + 25 = 180$
- ✓ Total Demand: $20 + 20 + 50 + 60 = 150$
- ✓ Conclusion: Unbalanced (supply > Demand).
  - → to fix, we add a dummy shop with demand 30 & cost = 0.

(c Initial basic feasible solution using Vogel approximation method

- Calculating penalties (Row/column Differences):
- ✓ Row A: 2 (5−3), column W: 0 (3−2=1, but 2−2=0 after adjustment).
- ✓ Allocating to maximum penalty cell (B−Z):
  - $\text{Min}(75, 60) = 60 \rightarrow x_{BZ} = 60$
- ✓ Update supply/Demand: B's supply = 15, Z's demand = 0

Max penalty (column W, Row B)
Cell B-W (Cost 2): Min (75, 20) = 20
$x_{BW} = 20.$
B's supply = 55, W's demand = 0.

Penalties after Allocation

Factory/Shop	X	Y	Z	Supply	Row penalty
A	5	7	6	80	1(6-5)
B	5	8	2	55	3(8-5)
C	6	9	2	25	4(9-6)
Demand	20	50	60		
Col. Penalty	0	1	0		

Final allocation

Factory/Shop	W	X	Y	Z	Dummy
A	0	20	50	0	10
B	20	0	0	35	0
C	0	0	0	25	0
Total					

(d) Optimal solution of the above transportation problem using MODI method.
initial feasibility.
— Number of allocated cells = 5. ($m+n-1 = 3+~~5~~-1 = 7$).
— add dummy allocation.
Dual variables u_i, v_j
$u_A = 0.$
Allocated cells:
- $u_A + v_X = 5 \rightarrow v_X = 5$
- $u_A + v_Y = 7 \rightarrow v_Y = 7$
- $u_B + v_W = 2 \rightarrow u_B = 2 - v_W$

↓

Opportunity costs

- for empty cells:
- Cell A-X: $3-(u_A+v_X) = 3-(0+0) = 3 \geq 0$
- Cell B-X: $5-(u_B+v_X) = 5-(2+5) = -2$ - negative

Loop: $B-X \Rightarrow B-Z \Rightarrow A-Z \Rightarrow A-X \Rightarrow B-X$

$\theta = \min(35, 20) = 20$

New allocation: $x_{BX} = 20, x_{BZ} = 15, x_{AX} = 0, x_{AZ} = 20$.

Total cost =

$(20 \times 2) + (20 \times 5) + (50 \times 7) + (15 \times 2) + (25 \times 2) + (10 \times 0) =$

$40 + 100 + 350 + 30 + 50 + 0 = 570/=$

Question 2

Academic staff	Responsibility Library	Time table	Exams	Research	Computing
A	40	15	25	30	35
B	30	10	30	40	35
C	35	25	40	10	45
D	10	10	55	10	50
E	35	55	50	15	45

(a linear programming model of a assignment problem.

min $Z = 40x_{A_1} + 15x_{A_2} + 25x_{A_3} + 30x_{A_4} + 35x_{A_5} + 30x_{B_1} + 10x_{B_2} +$
$30x_{B_3} + 40x_{B_4} + 35x_{B_5}$

Constraints

$x_{A_1} + x_{A_2} + x_{A_3} + x_{A_4} + x_{A_5} = 1$

$x_{B_1} + x_{B_2} + x_{B_3} + x_{B_4} + x_{B_5} = 1$

(b Is this problem balanced ?

Staff = (Rows): 2

No. Jobs (columns): 5

— Conclusion: Unbalanced (Jobs > Staff) → add 3 dummy staff with cost = 0.

(C Assign each academic staff to only one responsibility, each responsibility to only one staff. → Row reduction — subtract min in each row

Row A: Min = 15 → [25, 0, 10, 15, 20]

Row B: Min = 10 → [20, 0, 20, 30, 25]

Column Reduction — Subtracting min in each column:

✓ Column 1: Min = 20 → [5, 0, ...]

✓ Column 2: Min = 0 → [0, 0, ...]

Assign to Zeros:

- A to Time-table (cost = 15)
- B to library (cost = 30)
- Dummy staff takes remaining jobs.

minimum cost: 45 (15+30).

(3 Relationships of project activities & their predecessor activities

Activity	A	B	C	D	E	F	G	H	I	J
Time (day)	8	7	6	8	4	8	5	6	6	10
Predecessor	—	—	A	B	B	B	C	D	E	G, H, I.

(a Construct a network for the project completion activities

A(8) → C(6) → G(5) → I(6)

B (7) → E (4) → D(8) → H(6) → I (6)

B (7) → F(8)

(b Identify the critical path & duration of project completion.

- A→C→G→I: 8+6+5+6 = 25 weeks
- B→E→D→H→I: 7+4+8+6+6 = 31 weeks.
- B→F: 7+8 = 15 weeks.

The critical Path: B→E→D→H→I (31 weeks).

(c Define the following terms:

i) Critical activity. — Any activity on the critical path (B, E, D, H, I

ii) Free float — time an activity can delay without delaying its successor.

iii) Slack. — total time an activity can delay without delaying the project.

(d Differentiate CPM from PERT

CPM	PERT
Fixed durations	Probabilistic time estimates
Focus: Cost control	Focus: Time uncertainty.