习题答案与提示

第　八　章

习题 8－1（第 13 页）

1. $5\boldsymbol{a}-11\boldsymbol{b}+7\boldsymbol{c}$.

2. 略.

3. $\overrightarrow{D_1A}=-\left(\boldsymbol{c}+\frac{1}{5}\boldsymbol{a}\right)$, $\overrightarrow{D_2A}=-\left(\boldsymbol{c}+\frac{2}{5}\boldsymbol{a}\right)$, $\overrightarrow{D_3A}=-\left(\boldsymbol{c}+\frac{3}{5}\boldsymbol{a}\right)$,

   $\overrightarrow{D_4A}=-\left(\boldsymbol{c}+\frac{4}{5}\boldsymbol{a}\right)$.

4. $(1,-2,-2)$, $(-2,4,4)$.

5. $\left(\frac{6}{11},\frac{7}{11},-\frac{6}{11}\right)$或$\left(-\frac{6}{11},-\frac{7}{11},\frac{6}{11}\right)$.

6. A:Ⅳ, B:Ⅴ, C:Ⅷ, D:Ⅲ.

7. A 在 xOy 面上, B 在 yOz 面上, C 在 x 轴上, D 在 y 轴上.

8. (1) $(a,b,-c)$, $(-a,b,c)$, $(a,-b,c)$;

   (2) $(a,-b,-c)$, $(-a,b,-c)$, $(-a,-b,c)$;

   (3) $(-a,-b,-c)$.

9. xOy 面:$(x_0,y_0,0)$, yOz 面:$(0,y_0,z_0)$, xOz 面:$(x_0,0,z_0)$;

   x 轴:$(x_0,0,0)$, y 轴:$(0,y_0,0)$, z 轴:$(0,0,z_0)$.

10. 略.

11. $\left(\frac{\sqrt{2}}{2}a,0,0\right)$, $\left(-\frac{\sqrt{2}}{2}a,0,0\right)$, $\left(0,\frac{\sqrt{2}}{2}a,0\right)$, $\left(0,-\frac{\sqrt{2}}{2}a,0\right)$, $\left(\frac{\sqrt{2}}{2}a,0,a\right)$,

    $\left(-\frac{\sqrt{2}}{2}a,0,a\right)$, $\left(0,\frac{\sqrt{2}}{2}a,a\right)$, $\left(0,-\frac{\sqrt{2}}{2}a,a\right)$.

12. x 轴: $\sqrt{34}$, y 轴: $\sqrt{41}$, z 轴:5.

13. $(0,1,-2)$.

14. 略.

15. 模:2;方向余弦: $-\frac{1}{2}$, $-\frac{\sqrt{2}}{2}$, $\frac{1}{2}$;方向角: $\frac{2\pi}{3}$, $\frac{3\pi}{4}$, $\frac{\pi}{3}$.

16. （1）垂直于 x 轴，平行于 yOz 平面；

    （2）指向与 y 轴正向一致，垂直于 xOz 平面；

    （3）平行于 z 轴，垂直于 xOy 平面.

17. 2.

18. $A(-2,3,0)$.

19. $13, 7\boldsymbol{j}$.

习题 8-2（第 23 页）

1. （1）$3, 5\boldsymbol{i}+\boldsymbol{j}+7\boldsymbol{k}$；　（2）$-18, 10\boldsymbol{i}+2\boldsymbol{j}+14\boldsymbol{k}$；　（3）$\cos(\widehat{\boldsymbol{a},\boldsymbol{b}})=\dfrac{3}{2\sqrt{21}}$.

2. $-\dfrac{3}{2}$.

3. $\pm\dfrac{1}{\sqrt{17}}(3\boldsymbol{i}-2\boldsymbol{j}-2\boldsymbol{k})$.

4. 5 880 J.

5. $|\boldsymbol{F}_1|x_1\sin\theta_1=|\boldsymbol{F}_2|x_2\sin\theta_2$.

6. 2.

7. $\lambda=2\mu$.

8. 略.

9. （1）$-8\boldsymbol{j}-24\boldsymbol{k}$；　（2）$-\boldsymbol{j}-\boldsymbol{k}$；　（3）2.

10. $\dfrac{1}{2}\sqrt{19}$.

*11—12. 略.

习题 8-3（第 29 页）

1. $3x-7y+5z-4=0$.

2. $2x+9y-6z-121=0$.

3. $x-3y-2z=0$.

4. （1）yOz 面；　（2）平行于 xOz 面的平面；

   （3）平行于 z 轴的平面；　（4）通过 z 轴的平面；

   （5）平行于 x 轴的平面；　（6）通过 y 轴的平面；

   （7）通过原点的平面.

5. $\dfrac{1}{3},\dfrac{2}{3},\dfrac{2}{3}$.

6. $x+y-3z-4=0$.

7. $(1,-1,3)$.

8. (1) $y+5=0$; (2) $x+3y=0$; (3) $9y-z-2=0$.

9. 1.

习题 8－4(第 36 页)

1. $\dfrac{x-4}{2}=\dfrac{y+1}{1}=\dfrac{z-3}{5}$.

2. $\dfrac{x-3}{-4}=\dfrac{y+2}{2}=\dfrac{z-1}{1}$.

3. $\dfrac{x-1}{-2}=\dfrac{y-1}{1}=\dfrac{z-1}{3}$, $\begin{cases}x=1-2t,\\ y=1+t,\\ z=1+3t\end{cases}$ (t 为任意常数).

4. $16x-14y-11z-65=0$.

5. $\cos\varphi=0$.

6. 略.

7. $\dfrac{x}{-2}=\dfrac{y-2}{3}=\dfrac{z-4}{1}$.

8. $8x-9y-22z-59=0$.

9. $\varphi=0$.

10. (1) 平行; (2) 垂直; (3) 直线在平面上.

11. $x-y+z=0$.

12. $\left(-\dfrac{5}{3},\dfrac{2}{3},\dfrac{2}{3}\right)$.

13. $\dfrac{3\sqrt{2}}{2}$.

14. 略.

15. $\begin{cases}17x+31y-37z-117=0,\\ 4x-y+z-1=0.\end{cases}$

16. 略.

习题 8－5(第 44 页)

1. $x^2+y^2+z^2-4x-2y+4z=0$,球心为$(2,1,-2)$,$R=3$.

2. $x^2+y^2+z^2-2x-6y+4z=0$.

3. 以点$(1,-2,-1)$为球心,半径为$\sqrt{6}$的球面.

4. $\left(x+\frac{2}{3}\right)^2+(y+1)^2+\left(z+\frac{4}{3}\right)^2=\frac{116}{9}$,它表示一球面,球心为$\left(-\frac{2}{3},-1,-\frac{4}{3}\right)$,半径为$\frac{2}{3}\sqrt{29}$.

5. $y^2+z^2=5x$.

6. $x^2+y^2+z^2=9$.

7. 绕 x 轴:$4x^2-9(y^2+z^2)=36$,绕 y 轴:$4(x^2+z^2)-9y^2=36$.

8—9. 略.

10. (1) xOy 平面上的椭圆$\frac{x^2}{4}+\frac{y^2}{9}=1$ 绕 x 轴旋转一周;

(2) xOy 平面上的双曲线 $x^2-\frac{y^2}{4}=1$ 绕 y 轴旋转一周;

(3) xOy 平面上的双曲线 $x^2-y^2=1$ 绕 x 轴旋转一周;

(4) yOz 平面上的直线 $z=y+a$ 绕 z 轴旋转一周.

注:本题各小题均有多个答案,以上给出的均是其中一个答案.

11—12. 略.

习题 8-6(第 51 页)

1—2. 略.

3. 母线平行于 x 轴的柱面方程为 $3y^2-z^2=16$,

母线平行于 y 轴的柱面方程为 $3x^2+2z^2=16$.

4. $\begin{cases}2x^2-2x+y^2=8,\\ z=0.\end{cases}$

5. (1) $\begin{cases}x=\frac{3}{\sqrt{2}}\cos t,\\ y=\frac{3}{\sqrt{2}}\cos t,\\ z=3\sin t\end{cases}$ $(0\leqslant t\leqslant 2\pi)$;　(2) $\begin{cases}x=1+\sqrt{3}\cos\theta,\\ y=\sqrt{3}\sin\theta,\\ z=0\end{cases}$ $(0\leqslant\theta\leqslant 2\pi)$.

6. $\begin{cases}x^2+y^2=a^2,\\ z=0,\end{cases}$ $\begin{cases}y=a\sin\frac{z}{b},\\ x=0,\end{cases}$ $\begin{cases}x=a\cos\frac{z}{b},\\ y=0.\end{cases}$

7. $x^2+y^2\leqslant ax$;$x^2+z^2\leqslant a^2,x\geqslant 0,z\geqslant 0$.

8. $x^2+y^2\leqslant 4,x^2\leqslant z\leqslant 4,y^2\leqslant z\leqslant 4$.

总习题八(第 51 页)

1. (1) $M(x-x_0,y-y_0,z-z_0)$,$\overrightarrow{OM}=(x,y,z)$; (2) 共面; (3) 3; (4) 36.

2. (1)(A);　　(2)(B).

3. (0,2,0).

4. $\sqrt{30}$.

5. $\overrightarrow{AD}=\boldsymbol{c}+\frac{1}{2}\boldsymbol{a},\overrightarrow{BE}=\boldsymbol{a}+\frac{1}{2}\boldsymbol{b},\overrightarrow{CF}=\boldsymbol{b}+\frac{1}{2}\boldsymbol{c}$.

6. 略.

7. 1.

8. $\arccos\frac{2}{\sqrt{7}}$.

9. $\frac{\pi}{3}$.

10. $z=-4,\theta_{\min}=\frac{\pi}{4}$.

11. 30.

12. (14,10,2).

13. $\boldsymbol{c}=5\boldsymbol{a}+\boldsymbol{b}$.

14. $4(z-1)=(x-1)^2+(y+1)^2$.

15. (1) $\begin{cases}x=0,\\ z=2y^2,\end{cases}$ z 轴;　　(2) $\begin{cases}x=0,\\ \frac{y^2}{9}+\frac{z^2}{36}=1,\end{cases}$ y 轴;

(3) $\begin{cases}x=0,\\ z=\sqrt{3}y,\end{cases}$ z 轴;　　(4) $\begin{cases}z=0,\\ x^2-\frac{y^2}{4}=1,\end{cases}$ x 轴.

16. $x+\sqrt{26}y+3z-3=0$ 或 $x-\sqrt{26}y+3z-3=0$.

17. $x+2y+1=0$.

18. $\frac{x+1}{16}=\frac{y}{19}=\frac{z-4}{28}$.

19. $\left(0,0,\frac{1}{5}\right)$.

20. $z=0,x^2+y^2=x+y;x=0,2y^2+2yz+z^2-4y-3z+2=0;$
$y=0,2x^2+2xz+z^2-4x-3z+2=0$.

21. $z=0,(x-1)^2+y^2\leqslant 1;x=0,\left(\frac{z^2}{2}-1\right)^2+y^2\leqslant 1,z\geqslant 0;y=0,x\leqslant z\leqslant\sqrt{2x}$.

22. 略.

第九章

习题 9－1(第 64 页)

1. (1) 开集,无界集,导集:$\mathbf{R}^2$,边界:$\{(x,y)\mid x=0$ 或 $y=0\}$;
   (2) 既非开集,又非闭集,有界集,导集:$\{(x,y)\mid 1\leqslant x^2+y^2\leqslant 4\}$,
   边界:$\{(x,y)\mid x^2+y^2=1\}\cup\{(x,y)\mid x^2+y^2=4\}$;
   (3) 开集,区域,无界集,导集:$\{(x,y)\mid y\geqslant x^2\}$,边界:$\{(x,y)\mid y=x^2\}$;
   (4) 闭集,有界集,导集:集合本身,
   边界:$\{(x,y)\mid x^2+(y-1)^2=1\}\cup\{(x,y)\mid x^2+(y-2)^2=4\}$.
2. $t^2f(x,y)$.
3. 略.
4. $(x+y)^{xy}+(xy)^{2x}$.
5. (1) $\{(x,y)\mid y^2-2x+1>0\}$;
   (2) $\{(x,y)\mid x+y>0,x-y>0\}$;
   (3) $\{(x,y)\mid x\geqslant 0,y\geqslant 0,x^2\geqslant y\}$;
   (4) $\{(x,y)\mid y-x>0,x\geqslant 0,x^2+y^2<1\}$;
   (5) $\{(x,y,z)\mid r^2<x^2+y^2+z^2\leqslant R^2\}$;
   (6) $\{(x,y,z)\mid x^2+y^2-z^2\geqslant 0,x^2+y^2\neq 0\}$.
6. (1) 1;　(2) $\ln 2$;　(3) $-\dfrac{1}{4}$;　(4) -2;　(5) 2;　(6) 0.

*7. 略.

8. $\{(x,y)\mid y^2-2x=0\}$.

*9. 提示:$|xy|\leqslant\dfrac{x^2+y^2}{2}$.

*10. 略.

习题 9－2(第 71 页)

1. (1) $\dfrac{\partial z}{\partial x}=3x^2y-y^3$, $\dfrac{\partial z}{\partial y}=x^3-3xy^2$;
   (2) $\dfrac{\partial s}{\partial u}=\dfrac{1}{v}-\dfrac{v}{u^2}$, $\dfrac{\partial s}{\partial v}=\dfrac{1}{u}-\dfrac{u}{v^2}$;
   (3) $\dfrac{\partial z}{\partial x}=\dfrac{1}{2x\sqrt{\ln(xy)}}$, $\dfrac{\partial z}{\partial y}=\dfrac{1}{2y\sqrt{\ln(xy)}}$;

(4) $\dfrac{\partial z}{\partial x}=y[\cos(xy)-\sin(2xy)]$, $\dfrac{\partial z}{\partial y}=x[\cos(xy)-\sin(2xy)]$;

(5) $\dfrac{\partial z}{\partial x}=\dfrac{2}{y}\csc\dfrac{2x}{y}$, $\dfrac{\partial z}{\partial y}=-\dfrac{2x}{y^2}\csc\dfrac{2x}{y}$;

(6) $\dfrac{\partial z}{\partial x}=y^2(1+xy)^{y-1}$, $\dfrac{\partial z}{\partial y}=(1+xy)^y\left[\ln(1+xy)+\dfrac{xy}{1+xy}\right]$;

(7) $\dfrac{\partial u}{\partial x}=\dfrac{y}{z}x^{\frac{y}{z}-1}$, $\dfrac{\partial u}{\partial y}=\dfrac{1}{z}x^{\frac{y}{z}}\cdot\ln x$, $\dfrac{\partial u}{\partial z}=-\dfrac{y}{z^2}x^{\frac{y}{z}}\cdot\ln x$;

(8) $\dfrac{\partial u}{\partial x}=\dfrac{z(x-y)^{z-1}}{1+(x-y)^{2z}}$, $\dfrac{\partial u}{\partial y}=-\dfrac{z(x-y)^{z-1}}{1+(x-y)^{2z}}$,

$\dfrac{\partial u}{\partial z}=\dfrac{(x-y)^z\ln(x-y)}{1+(x-y)^{2z}}$.

2—3. 略.

4. $f_x(x,1)=1$.

5. $\dfrac{\pi}{4}$.

6. (1) $\dfrac{\partial^2 z}{\partial x^2}=12x^2-8y^2,\dfrac{\partial^2 z}{\partial y^2}=12y^2-8x^2,\dfrac{\partial^2 z}{\partial x\partial y}=-16xy$;

(2) $\dfrac{\partial^2 z}{\partial x^2}=\dfrac{2xy}{(x^2+y^2)^2},\dfrac{\partial^2 z}{\partial y^2}=-\dfrac{2xy}{(x^2+y^2)^2},\dfrac{\partial^2 z}{\partial x\partial y}=\dfrac{y^2-x^2}{(x^2+y^2)^2}$;

(3) $\dfrac{\partial^2 z}{\partial x^2}=y^x\cdot\ln^2 y,\dfrac{\partial^2 z}{\partial y^2}=x(x-1)y^{x-2},\dfrac{\partial^2 z}{\partial x\partial y}=y^{x-1}(1+x\ln y)$.

7. $f_{xx}(0,0,1)=2,f_{xz}(1,0,2)=2,f_{yz}(0,-1,0)=0,f_{zzx}(2,0,1)=0$.

8. $\dfrac{\partial^3 z}{\partial x^2\partial y}=0$, $\dfrac{\partial^3 z}{\partial x\partial y^2}=-\dfrac{1}{y^2}$.

9. 略.

习题 9－3(第 77 页)

1. (1) $\left(y+\dfrac{1}{y}\right)\mathrm{d}x+x\left(1-\dfrac{1}{y^2}\right)\mathrm{d}y$; (2) $-\dfrac{1}{x}\mathrm{e}^{\frac{y}{x}}\left(\dfrac{y}{x}\mathrm{d}x-\mathrm{d}y\right)$;

(3) $-\dfrac{x}{(x^2+y^2)^{3/2}}(y\mathrm{d}x-x\mathrm{d}y)$; (4) $yzx^{yz-1}\mathrm{d}x+zx^{yz}\cdot\ln x\mathrm{d}y+yx^{yz}\cdot\ln x\mathrm{d}z$.

2. $\dfrac{1}{3}\mathrm{d}x+\dfrac{2}{3}\mathrm{d}y$.

3. $\Delta z=-0.119$, $\mathrm{d}z=-0.125$.

4. $0.25\mathrm{e}$.

5. (A).

*6. 2.95.

*7. 2.039.

*8. -5 cm.

*9. 55.3 cm^3.

*10. 0.124 cm.

*11. 2 128 m^2, 27.6 m^2, 1.30%.

*12—*13. 略.

习题 9-4(第 84 页)

1. $\dfrac{\partial z}{\partial x}=4x$, $\dfrac{\partial z}{\partial y}=4y$.

2. $\dfrac{\partial z}{\partial x}=\dfrac{2x}{y^2}\ln(3x-2y)+\dfrac{3x^2}{(3x-2y)y^2}$, $\dfrac{\partial z}{\partial y}=-\dfrac{2x^2}{y^3}\ln(3x-2y)-\dfrac{2x^2}{(3x-2y)y^2}$.

3. $\mathrm{e}^{\sin t-2t^3}(\cos t-6t^2)$.

4. $\dfrac{3(1-4t^2)}{\sqrt{1-(3t-4t^3)^2}}$.

5. $\dfrac{\mathrm{e}^x(1+x)}{1+x^2\mathrm{e}^{2x}}$.

6. $\mathrm{e}^{ax}\sin x$.

7. 略.

8. (1) $\dfrac{\partial u}{\partial x}=2xf'_1+y\mathrm{e}^{xy}f'_2$, $\dfrac{\partial u}{\partial y}=-2yf'_1+x\mathrm{e}^{xy}f'_2$;

(2) $\dfrac{\partial u}{\partial x}=\dfrac{1}{y}f'_1$, $\dfrac{\partial u}{\partial y}=-\dfrac{x}{y^2}f'_1+\dfrac{1}{z}f'_2$, $\dfrac{\partial u}{\partial z}=-\dfrac{y}{z^2}f'_2$;

(3) $\dfrac{\partial u}{\partial x}=f'_1+yf'_2+yzf'_3$, $\dfrac{\partial u}{\partial y}=xf'_2+xzf'_3$, $\dfrac{\partial u}{\partial z}=xyf'_3$.

9—10. 略.

11. $\dfrac{\partial^2 z}{\partial x^2}=2f'+4x^2f''$, $\dfrac{\partial^2 z}{\partial x\partial y}=4xyf''$, $\dfrac{\partial^2 z}{\partial y^2}=2f'+4y^2f''$.

*12. (1) $\dfrac{\partial^2 z}{\partial x^2}=y^2f''_{11}$, $\dfrac{\partial^2 z}{\partial x\partial y}=f'_1+y(xf''_{11}+f''_{12})$, $\dfrac{\partial^2 z}{\partial y^2}=x^2f''_{11}+2xf''_{12}+f''_{22}$;

(2) $\dfrac{\partial^2 z}{\partial x^2}=f''_{11}+\dfrac{2}{y}f''_{12}+\dfrac{1}{y^2}f''_{22}$, $\dfrac{\partial^2 z}{\partial x\partial y}=-\dfrac{x}{y^2}\left(f''_{12}+\dfrac{1}{y}f''_{22}\right)-\dfrac{1}{y^2}f'_2$,

$\dfrac{\partial^2 z}{\partial y^2}=\dfrac{2x}{y^3}f'_2+\dfrac{x^2}{y^4}f''_{22}$;

(3) $\frac{\partial^2 z}{\partial x^2}=2yf'_2+y^4f''_{11}+4xy^3f''_{12}+4x^2y^2f''_{22}$,

$\frac{\partial^2 z}{\partial x\partial y}=2yf'_1+2xf'_2+2xy^3f''_{11}+2x^3yf''_{22}+5x^2y^2f''_{12}$,

$\frac{\partial^2 z}{\partial y^2}=2xf'_1+4x^2y^2f''_{11}+4x^3yf''_{12}+x^4f''_{22}$;

(4) $\frac{\partial^2 z}{\partial x^2}=\mathrm{e}^{x+y}f'_3-\sin xf'_1+\cos^2 xf''_{11}+2\mathrm{e}^{x+y}\cos xf''_{13}+\mathrm{e}^{2(x+y)}f''_{33}$,

$\frac{\partial^2 z}{\partial x\partial y}=\mathrm{e}^{x+y}f'_3-\cos x\sin yf''_{12}+\mathrm{e}^{x+y}\cos xf''_{13}-\mathrm{e}^{x+y}\sin yf''_{32}+\mathrm{e}^{2(x+y)}f''_{33}$,

$\frac{\partial^2 z}{\partial y^2}=\mathrm{e}^{x+y}f'_3-\cos yf'_2+\sin^2 yf''_{22}-2\mathrm{e}^{x+y}\sin yf''_{23}+\mathrm{e}^{2(x+y)}f''_{33}$.

*13. 略.

习题 9－5(第 91 页)

1. $\frac{y^2-\mathrm{e}^x}{\cos y-2xy}$

2. $\frac{x+y}{x-y}$.

3. $\frac{\partial z}{\partial x}=\frac{yz-\sqrt{xyz}}{\sqrt{xyz}-xy}$, $\frac{\partial z}{\partial y}=\frac{xz-2\sqrt{xyz}}{\sqrt{xyz}-xy}$.

4. $\frac{\partial z}{\partial x}=\frac{z}{x+z}$, $\frac{\partial z}{\partial y}=\frac{z^2}{y(x+z)}$.

5—7. 略.

*8. $\frac{2y^2z\mathrm{e}^z-2xy^3z-y^2z^2\mathrm{e}^z}{(\mathrm{e}^z-xy)^3}$.

*9. $\frac{z(z^4-2xyz^2-x^2y^2)}{(z^2-xy)^3}$.

10. (1) $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{x(6z+1)}{2y(3z+1)}$, $\frac{\mathrm{d}z}{\mathrm{d}x}=\frac{x}{3z+1}$;

(2) $\frac{\mathrm{d}x}{\mathrm{d}z}=\frac{y-z}{x-y}$, $\frac{\mathrm{d}y}{\mathrm{d}z}=\frac{z-x}{x-y}$;

(3) $\frac{\partial u}{\partial x}=\frac{-uf'_1(2yvg'_2-1)-f'_2\cdot g'_1}{(xf'_1-1)(2yvg'_2-1)-f'_2\cdot g'_1}$,

$\frac{\partial v}{\partial x}=\frac{g'_1(xf'_1+uf'_1-1)}{(xf'_1-1)(2yvg'_2-1)-f'_2\cdot g'_1}$;

(4) $\dfrac{\partial u}{\partial x}=\dfrac{\sin v}{e^{u}(\sin v-\cos v)+1}$, $\dfrac{\partial u}{\partial y}=\dfrac{-\cos v}{e^{u}(\sin v-\cos v)+1}$,

$\dfrac{\partial v}{\partial x}=\dfrac{\cos v-e^{u}}{u[e^{u}(\sin v-\cos v)+1]}$, $\dfrac{\partial v}{\partial y}=\dfrac{\sin v+e^{u}}{u[e^{u}(\sin v-\cos v)+1]}$.

11. 略.

习题 9-6(第 102 页)

1. 略.

2. (1) $\boldsymbol{v}_0=\boldsymbol{i}+2\boldsymbol{j}+2\boldsymbol{k}$, $\boldsymbol{a}_0=2\boldsymbol{j}$, $|\boldsymbol{v}(t)|=\sqrt{5+4t^2}$;

(2) $\boldsymbol{v}_0=-2\boldsymbol{i}+4\boldsymbol{k}$, $\boldsymbol{a}_0=-3\boldsymbol{j}$, $|\boldsymbol{v}(t)|=\sqrt{20+5\cos^2 t}$;

(3) $\boldsymbol{v}_0=\boldsymbol{i}+2\boldsymbol{j}+\boldsymbol{k}$, $\boldsymbol{a}_0=-\dfrac{1}{2}\boldsymbol{i}+2\boldsymbol{j}+\boldsymbol{k}$, $|\boldsymbol{v}(t)|=\sqrt{5t^2+\dfrac{4}{(t+1)^2}}$.

3. 切线方程：$\dfrac{x-\left(\dfrac{\pi}{2}-1\right)}{1}=\dfrac{y-1}{1}=\dfrac{z-2\sqrt{2}}{\sqrt{2}}$,

法平面方程：$x+y+\sqrt{2}z=\dfrac{\pi}{2}+4$.

4. 切线方程：$\dfrac{x-\dfrac{1}{2}}{1}=\dfrac{y-2}{-4}=\dfrac{z-1}{8}$,法平面方程：$2x-8y+16z-1=0$.

5. 切线方程：$\dfrac{x-x_0}{1}=\dfrac{y-y_0}{\dfrac{m}{y_0}}=\dfrac{z-z_0}{-\dfrac{1}{2z_0}}$,

法平面方程：$(x-x_0)+\dfrac{m}{y_0}(y-y_0)-\dfrac{1}{2z_0}(z-z_0)=0$.

6. 切线方程：$\dfrac{x-1}{16}=\dfrac{y-1}{9}=\dfrac{z-1}{-1}$,法平面方程：$16x+9y-z-24=0$.

7. $P_1(-1,1,-1)$及$P_2\left(-\dfrac{1}{3},\dfrac{1}{9},-\dfrac{1}{27}\right)$.

8. 切平面方程：$x+2y-4=0$,法线方程：$\begin{cases}\dfrac{x-2}{1}=\dfrac{y-1}{2},\\ z=0.\end{cases}$

9. 切平面方程：$ax_0x+by_0y+cz_0z=1$,法线方程：$\dfrac{x-x_0}{ax_0}=\dfrac{y-y_0}{by_0}=\dfrac{z-z_0}{cz_0}$.

10. 切平面方程：$x-y+2z=\pm\sqrt{\dfrac{11}{2}}$.

11. $\cos\gamma=\dfrac{3}{\sqrt{22}}$.

12—13. 略

习题 9-7(第 111 页)

1. $1+2\sqrt{3}$.
2. $\dfrac{\sqrt{2}}{3}$.
3. $\dfrac{1}{ab}\sqrt{2(a^2+b^2)}$.
4. 5.
5. $\dfrac{98}{13}$.
6. $\dfrac{6}{7}\sqrt{14}$.
7. $x_0+y_0+z_0$.
8. $\mathbf{grad}\,f(0,0,0)=3\boldsymbol{i}-2\boldsymbol{j}-6\boldsymbol{k}$, $\mathbf{grad}\,f(1,1,1)=6\boldsymbol{i}+3\boldsymbol{j}$.
9. 略.
10. 增加最快的方向为 $\boldsymbol{n}=\dfrac{1}{\sqrt{21}}(2\boldsymbol{i}-4\boldsymbol{j}+\boldsymbol{k})$,方向导数为 $\sqrt{21}$;

    减少最快的方向为 $-\boldsymbol{n}=\dfrac{1}{\sqrt{21}}(-2\boldsymbol{i}+4\boldsymbol{j}-\boldsymbol{k})$,方向导数为 $-\sqrt{21}$.

习题 9-8(第 121 页)

1. (A).
2. 极大值:$f(2,-2)=8$.
3. 极大值:$f(3,2)=36$.
4. 极小值:$f\left(\dfrac{1}{2},-1\right)=-\dfrac{e}{2}$.
5. 极大值:$z\left(\dfrac{1}{2},\dfrac{1}{2}\right)=\dfrac{1}{4}$.
6. 当两直角边都是$\dfrac{l}{\sqrt{2}}$时,可得最大的周长.
7. 当长、宽都是 $\sqrt[3]{2k}$,而高为$\dfrac{1}{2}\sqrt[3]{2k}$时,水池的表面积最小.

8. $\left(\frac{8}{5},\frac{16}{5}\right)$.

9. 当矩形的边长分别为$\frac{2p}{3}$及$\frac{p}{3}$时,绕短边旋转所得圆柱体的体积最大.

10. 当长、宽、高都是$\frac{2a}{\sqrt{3}}$时,可得最大的体积.

11. 最大值为$\sqrt{9+5\sqrt{3}}$,最小值为$\sqrt{9-5\sqrt{3}}$.

12. 最热点在$\left(-\frac{1}{2},\pm\frac{\sqrt{3}}{2}\right)$,最冷点在$\left(\frac{1}{2},0\right)$.

13. 最热点在$\left(\pm\frac{4}{3},-\frac{4}{3},-\frac{4}{3}\right)$.

*习题 9－9(第 127 页)

1. $f(x,y)=5+2(x-1)^2-(x-1)(y+2)-(y+2)^2$.

2. $e^x\ln(1+y)=y+\frac{1}{2!}(2xy-y^2)+\frac{1}{3!}(3x^2y-3xy^2+2y^3)+R_3$,其中 $R_3=$
$$\frac{e^{\theta x}}{24}\left[x^4\ln(1+\theta y)+\frac{4x^3y}{1+\theta y}-\frac{6x^2y^2}{(1+\theta y)^2}+\frac{8xy^3}{(1+\theta y)^3}-\frac{6y^4}{(1+\theta y)^4}\right]\ (0<\theta<1).$$

3. $$\sin x\sin y=\frac{1}{2}+\frac{1}{2}\left(x-\frac{\pi}{4}\right)+\frac{1}{2}\left(y-\frac{\pi}{4}\right)-$$
$$\frac{1}{4}\left[\left(x-\frac{\pi}{4}\right)^2-2\left(x-\frac{\pi}{4}\right)\left(y-\frac{\pi}{4}\right)+\left(y-\frac{\pi}{4}\right)^2\right]+R_2,$$
其中 $$R_2=-\frac{1}{6}\left[\cos\xi\sin\eta\left(x-\frac{\pi}{4}\right)^3+3\sin\xi\cos\eta\left(x-\frac{\pi}{4}\right)^2\left(y-\frac{\pi}{4}\right)+\right.$$
$$\left.3\cos\xi\sin\eta\left(x-\frac{\pi}{4}\right)\left(y-\frac{\pi}{4}\right)^2+\sin\xi\cos\eta\left(y-\frac{\pi}{4}\right)^3\right],$$
且 $\xi=\frac{\pi}{4}+\theta\left(x-\frac{\pi}{4}\right),\eta=\frac{\pi}{4}+\theta\left(y-\frac{\pi}{4}\right)\quad(0<\theta<1)$.

4. $x^y=1+(x-1)+(x-1)(y-1)+\frac{1}{2}(x-1)^2(y-1)+R_3$,

   $1.1^{1.02}\approx1.1021$.

5. $e^{x+y}=1+(x+y)+\frac{1}{2!}(x^2+2xy+y^2)+\cdots+\frac{1}{n!}(x^n+C_n^1x^{n-1}y+\cdots+y^n)+R_n$,

其中 $R_n=\frac{e^{\theta(x+y)}}{(n+1)!}(x^{n+1}+C_{n+1}^1x^ny+\cdots+y^{n+1})$, $0<\theta<1$.

* 习题 9－10(第 132 页)

1. $\theta = 2.234p + 95.33$.

2. $$\begin{cases} a\sum_{i=1}^{n} x_i^4 + b\sum_{i=1}^{n} x_i^3 + c\sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i^2 y_i, \\ a\sum_{i=1}^{n} x_i^3 + b\sum_{i=1}^{n} x_i^2 + c\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i y_i, \\ a\sum_{i=1}^{n} x_i^2 + b\sum_{i=1}^{n} x_i + nc = \sum_{i=1}^{n} y_i. \end{cases}$$

总习题九(第 132 页)

1. (1) 充分,必要; (2) 必要,充分; (3) 充分; (4) 充分.
2. (C).
3. $\{(x,y)\mid 0<x^2+y^2<1, y^2\leqslant 4x\}$, $\dfrac{\sqrt{2}}{\ln\dfrac{3}{4}}$.

*4. 略.

5. $$f_x(x,y)=\begin{cases} \dfrac{2xy^3}{(x^2+y^2)^2}, & x^2+y^2\neq 0, \\ 0, & x^2+y^2=0; \end{cases}$$

$$f_y(x,y)=\begin{cases} \dfrac{x^2(x^2-y^2)}{(x^2+y^2)^2}, & x^2+y^2\neq 0, \\ 0, & x^2+y^2=0. \end{cases}$$

6. (1) $\dfrac{\partial z}{\partial x}=\dfrac{1}{x+y^2}$, $\dfrac{\partial z}{\partial y}=\dfrac{2y}{x+y^2}$, $\dfrac{\partial^2 z}{\partial x^2}=-\dfrac{1}{(x+y^2)^2}$,

$\dfrac{\partial^2 z}{\partial x\partial y}=-\dfrac{2y}{(x+y^2)^2}$, $\dfrac{\partial^2 z}{\partial y^2}=\dfrac{2(x-y^2)}{(x+y^2)^2}$;

(2) $\dfrac{\partial z}{\partial x}=yx^{y-1}$, $\dfrac{\partial z}{\partial y}=x^y\ln x$, $\dfrac{\partial^2 z}{\partial x^2}=y(y-1)x^{y-2}$,

$\dfrac{\partial^2 z}{\partial x\partial y}=x^{y-1}(1+y\ln x)$, $\dfrac{\partial^2 z}{\partial y^2}=x^y(\ln x)^2$.

7. $\Delta z=0.02$, $\mathrm{d}z=0.03$.

*8. 略.

9. $\dfrac{\mathrm{d}u}{\mathrm{d}t}=yx^{y-1}\varphi'(t)+x^y\ln x\psi'(t)$.

10. $\frac{\partial z}{\partial \xi} = -\frac{\partial z}{\partial v} + \frac{\partial z}{\partial w}$, $\frac{\partial z}{\partial \eta} = \frac{\partial z}{\partial u} - \frac{\partial z}{\partial w}$, $\frac{\partial z}{\partial \zeta} = -\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v}$.

11. $\frac{\partial^2 z}{\partial x \partial y} = x e^{2y} f''_{uu} + e^y f''_{uy} + x e^y f''_{xu} + f''_{xy} + e^y f'_u$.

12. $\frac{\partial z}{\partial x} = (v\cos v - u\sin v) e^{-u}$, $\frac{\partial z}{\partial y} = (u\cos v + v\sin v) e^{-u}$.

13. 切线方程$\begin{cases} x = a, \\ by - az = 0; \end{cases}$ 法平面方程 $ay + bz = 0$.

14. $(-3, -1, 3)$, $\frac{x+3}{1} = \frac{y+1}{3} = \frac{z-3}{1}$.

15. $\frac{\partial f}{\partial l} = \cos\theta + \sin\theta$, (1) $\theta = \frac{\pi}{4}$, (2) $\theta = \frac{5\pi}{4}$, (3) $\theta = \frac{3\pi}{4}$及$\frac{7\pi}{4}$.

16. $\frac{\partial u}{\partial n} = \frac{2}{\sqrt{\frac{x_0^2}{a^4} + \frac{y_0^2}{b^4} + \frac{z_0^2}{c^4}}}$.

17. $\left(\frac{4}{5}, \frac{3}{5}, \frac{35}{12}\right)$.

18. 切点$\left(\frac{a}{\sqrt{3}}, \frac{b}{\sqrt{3}}, \frac{c}{\sqrt{3}}\right)$, $V_{\min} = \frac{\sqrt{3}}{2} abc$.

19. 当 $p_1 = 80, p_2 = 120$ 时,总利润最大,最大总利润为 605.

20. (1) $g(x_0, y_0) = \sqrt{5x_0^2 + 5y_0^2 - 8x_0 y_0}$;

(2) 攀岩的起点可取为 $M_1(5, -5)$或 $M_2(-5, 5)$.

第 十 章

习题 10 - 1(第 139 页)

1. $\iint\limits_D \mu(x, y) \mathrm{d}\sigma$.

2. $I_1 = 4I_2$.

3. 略.

4. $D = \{(x, y) \mid 2x^2 + y^2 \leqslant 1\}$.

5. (1) $\iint\limits_D (x+y)^2 \mathrm{d}\sigma \geqslant \iint\limits_D (x+y)^3 \mathrm{d}\sigma$; (2) $\iint\limits_D (x+y)^3 \mathrm{d}\sigma \geqslant \iint\limits_D (x+y)^2 \mathrm{d}\sigma$;

(3) $\iint\limits_D \ln(x+y) \mathrm{d}\sigma \geqslant \iint\limits_D [\ln(x+y)]^2 \mathrm{d}\sigma$;

(4) $\iint\limits_D [\ln(x+y)]^2 d\sigma \geqslant \iint\limits_D \ln(x+y)\,d\sigma$.

6. (1) $0\leqslant I\leqslant 2$; (2) $0\leqslant I\leqslant \pi^2$; (3) $2\leqslant I\leqslant 8$; (4) $36\pi\leqslant I\leqslant 100\pi$.

习题 10-2(第 156 页)

1. (1) $\frac{8}{3}$; (2) $\frac{20}{3}$; (3) 1; (4) $-\frac{3\pi}{2}$.

2. (1) $\frac{6}{55}$; (2) $\frac{64}{15}$; (3) $e-e^{-1}$; (4) $\frac{13}{6}$.

3. 略.

4. (1) $\int_0^4 dx\int_x^{2\sqrt{x}} f(x,y)\,dy$ 或 $\int_0^4 dy\int_{\frac{y^2}{4}}^{y} f(x,y)\,dx$;

(2) $\int_{-r}^{r} dx\int_0^{\sqrt{r^2-x^2}} f(x,y)\,dy$ 或 $\int_0^r dy\int_{-\sqrt{r^2-y^2}}^{\sqrt{r^2-y^2}} f(x,y)\,dx$;

(3) $\int_1^2 dx\int_{\frac{1}{x}}^{x} f(x,y)\,dy$ 或 $\int_{\frac{1}{2}}^{1} dy\int_{\frac{1}{y}}^{2} f(x,y)\,dx+\int_1^2 dy\int_y^2 f(x,y)\,dx$;

(4) $\int_{-1}^{1} dx\int_{\sqrt{1-x^2}}^{\sqrt{4-x^2}} f(x,y)\,dy+\int_{-1}^{1} dx\int_{-\sqrt{4-x^2}}^{-\sqrt{1-x^2}} f(x,y)\,dy+$

$\int_{-2}^{-1} dx\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} f(x,y)\,dy+\int_1^2 dx\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} f(x,y)\,dy$

或 $\int_1^2 dy\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} f(x,y)\,dx+\int_{-2}^{-1} dy\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} f(x,y)\,dx+$

$\int_{-1}^{1} dy\int_{-\sqrt{4-y^2}}^{-\sqrt{1-y^2}} f(x,y)\,dx+\int_{-1}^{1} dy\int_{\sqrt{1-y^2}}^{\sqrt{4-y^2}} f(x,y)\,dx$.

5. 略.

6. (1) $\int_0^1 dx\int_x^1 f(x,y)\,dy$; (2) $\int_0^4 dx\int_{\frac{x}{2}}^{\sqrt{x}} f(x,y)\,dy$;

(3) $\int_{-1}^{1} dx\int_0^{\sqrt{1-x^2}} f(x,y)\,dy$; (4) $\int_0^1 dy\int_{2-y}^{1+\sqrt{1-y^2}} f(x,y)\,dx$;

(5) $\int_0^1 dy\int_{e^y}^{e} f(x,y)\,dx$;

(6) $\int_{-1}^{0} dy\int_{-2\arcsin y}^{\pi} f(x,y)\,dx+\int_0^1 dy\int_{\arcsin y}^{\pi-\arcsin y} f(x,y)\,dx$.

7. $\frac{4}{3}$.

8. $\frac{7}{2}$.

9. $\frac{17}{6}$

10. 6π.

11. (1) $\int_0^{2\pi} d\theta \int_0^a f(\rho\cos\theta, \rho\sin\theta)\rho d\rho$;

(2) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta \int_0^{2\cos\theta} f(\rho\cos\theta, \rho\sin\theta)\rho d\rho$;

(3) $\int_0^{2\pi} d\theta \int_a^b f(\rho\cos\theta, \rho\sin\theta)\rho d\rho$;

(4) $\int_0^{\frac{\pi}{2}} d\theta \int_0^{(\cos\theta+\sin\theta)^{-1}} f(\rho\cos\theta, \rho\sin\theta)\rho d\rho$.

12. (1) $\int_0^{\frac{\pi}{4}} d\theta \int_0^{\sec\theta} f(\rho\cos\theta, \rho\sin\theta)\rho d\rho + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\theta \int_0^{\csc\theta} f(\rho\cos\theta, \rho\sin\theta)\rho d\rho$;

(2) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} d\theta \int_0^{2\sec\theta} f(\rho)\rho d\rho$;　(3) $\int_0^{\frac{\pi}{2}} d\theta \int_{(\cos\theta+\sin\theta)^{-1}}^{1} f(\rho\cos\theta, \rho\sin\theta)\rho d\rho$;

(4) $\int_0^{\frac{\pi}{4}} d\theta \int_{\sec\theta\tan\theta}^{\sec\theta} f(\rho\cos\theta, \rho\sin\theta)\rho d\rho$.

13. (1) $\frac{3}{4}\pi a^4$;　(2) $\frac{1}{6}a^3[\sqrt{2}+\ln(1+\sqrt{2})]$;　(3) $\sqrt{2}-1$;　(4) $\frac{1}{8}\pi a^4$.

14. (1) $\pi(e^4-1)$;　(2) $\frac{\pi}{4}(2\ln 2-1)$;　(3) $\frac{3}{64}\pi^2$.

15. (1) $\frac{9}{4}$;　(2) $\frac{\pi}{8}(\pi-2)$;　(3) $14a^4$;　(4) $\frac{2}{3}\pi(b^3-a^3)$.

16. $\frac{1}{40}\pi^5$.

17. $\frac{1}{3}R^3\arctan k$.

18. $\frac{3}{32}\pi a^4$.

*19. (1) $\frac{\pi^4}{3}$;　(2) $\frac{7}{3}\ln 2$;　(3) $\frac{e-1}{2}$;

(4) $\frac{1}{2}\pi ab$. 提示:作变换 $x=a\rho\cos\theta, y=b\rho\sin\theta$.

*20. (1) $2\ln 3$;　(2) $\frac{1}{8}$.

*21. 略.

*22. (1) 略;　(2) 提示:作变换 $x=\frac{au-bv}{\sqrt{a^2+b^2}}, y=\frac{bu+av}{\sqrt{a^2+b^2}}$.

习题 10－3(第 166 页)

1. (1) $\int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \int_0^{xy} f(x,y,z)\,\mathrm{d}z$;　(2) $\int_{-1}^1 \mathrm{d}x \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \mathrm{d}y \int_{x^2+y^2}^1 f(x,y,z)\,\mathrm{d}z$;

   (3) $\int_{-1}^1 \mathrm{d}x \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \mathrm{d}y \int_{x^2+2y^2}^{2-x^2} f(x,y,z)\,\mathrm{d}z$;

   (4) $\int_0^a \mathrm{d}x \int_0^{b\sqrt{1-x^2/a^2}} \mathrm{d}y \int_0^{xy/c} f(x,y,z)\,\mathrm{d}z$.

2. $\frac{3}{2}$.

3. 略.

4. $\frac{1}{364}$.

5. $\frac{1}{2}\left(\ln 2-\frac{5}{8}\right)$.

6. $\frac{1}{48}$.

7. 0.

8. $\frac{\pi}{4}h^2R^2$.

9. (1) $\frac{7\pi}{12}$;　(2) $\frac{16}{3}\pi$.

*10. (1) $\frac{4\pi}{5}$;　(2) $\frac{7}{6}\pi a^4$.

11. (1) $\frac{1}{8}$;　*(2) $\frac{\pi}{10}$;　(3) 8π;　*(4) $\frac{4\pi}{15}(A^5-a^5)$.

12. (1) $\frac{32}{3}\pi$;　*(2) πa^3;　(3) $\frac{\pi}{6}$;　(4) $\frac{2}{3}\pi(5\sqrt{5}-4)$.

*13. $\frac{2}{3}\pi a^3$.

14. $\frac{8\sqrt{2}-7}{6}\pi$.

*15. $k\pi R^4$.

习题 10－4(第 177 页)

1. $2a^2(\pi-2)$.

2. $\sqrt{2}\pi$.

3. $16R^2$.

4. (1) $\bar{x}=\dfrac{3}{5}x_0$; $\bar{y}=\dfrac{3}{8}y_0$; (2) $\bar{x}=0$, $\bar{y}=\dfrac{4b}{3\pi}$; (3) $\bar{x}=\dfrac{b^2+ab+a^2}{2(a+b)}$, $\bar{y}=0$.

5. $\bar{x}=\dfrac{35}{48}$, $\bar{y}=\dfrac{35}{54}$.

6. $\bar{x}=\dfrac{2}{5}a$, $\bar{y}=\dfrac{2}{5}a$.

7. (1) $\left(0,0,\dfrac{3}{4}\right)$; *(2) $\left(0,0,\dfrac{3(A^4-a^4)}{8(A^3-a^3)}\right)$; (3) $\left(\dfrac{2}{5}a,\dfrac{2}{5}a,\dfrac{7}{30}a^2\right)$.

*8. $\left(0,0,\dfrac{5}{4}R\right)$.

9. (1) $I_y=\dfrac{1}{4}\pi a^3b$; (2) $I_x=\dfrac{72}{5}$, $I_y=\dfrac{96}{7}$; (3) $I_x=\dfrac{1}{3}ab^3$, $I_y=\dfrac{1}{3}ba^3$.

10. $\dfrac{1}{12}Mh^2$, $\dfrac{1}{12}Mb^2$ ($M=bh\mu$ 为矩形板的质量).

11. (1) $\dfrac{8}{3}a^4$; (2) $\bar{x}=\bar{y}=0$, $\bar{z}=\dfrac{7}{15}a^2$; (3) $\dfrac{112}{45}a^6\rho$.

12. $\dfrac{1}{2}a^2M$ ($M=\pi a^2h\rho$ 为圆柱体的质量).

13. $$\boldsymbol{F}=\left(2G\mu\left(\ln\frac{R_2+\sqrt{R_2^2+a^2}}{R_1+\sqrt{R_1^2+a^2}}-\frac{R_2}{\sqrt{R_2^2+a^2}}+\frac{R_1}{\sqrt{R_1^2+a^2}}\right),0,\right.$$
$$\left.\pi Ga\mu\left(\frac{1}{\sqrt{R_2^2+a^2}}-\frac{1}{\sqrt{R_1^2+a^2}}\right)\right).$$

14. $F_x=F_y=0$, $F_z=-2\pi G\rho\left[\sqrt{(h-a)^2+R^2}-\sqrt{R^2+a^2}+h\right]$.

*习题 10-5(第 184 页)

1. (1) $\dfrac{\pi}{4}$; (2) 1; (3) $\dfrac{8}{3}$.

2. (1) $\dfrac{1}{3}\cos x(\cos x-\sin x)(1+2\sin 2x)$; (2) $\dfrac{2}{x}\ln(1+x^2)$;

   (3) $\ln\sqrt{\dfrac{x^2+1}{x^4+1}}+3x^2\arctan x^2-2x\arctan x$; (4) $2xe^{-x^5}-e^{-x^3}-\displaystyle\int_x^{x^2}y^2e^{-xy^2}dy$.

3. $3f(x)+2xf'(x)$.

4. (1) $\pi\arcsin a$;

(2) $\pi\ln\dfrac{1+a}{2}$. 提示:设 $\varphi(\alpha)=\int_0^{\frac{\pi}{2}}\ln(\cos^2x+\alpha^2\sin^2x)\,dx$, $I=\varphi(a)$.

5. (1) $\dfrac{\pi}{2}\ln(1+\sqrt{2})$;提示:利用公式$\dfrac{\arctan x}{x}=\int_0^1\dfrac{dy}{1+x^2y^2}$.

(2) $\arctan(1+b)-\arctan(1+a)$. 提示:利用公式$\dfrac{x^b-x^a}{\ln x}=\int_a^b x^y\,dy$.

总习题十(第 185 页)

1. (1) $\dfrac{1}{2}(1-e^{-4})$;　(2) $\dfrac{\pi}{4}R^4\left(\dfrac{1}{a^2}+\dfrac{1}{b^2}\right)$.

2. (1) (C);　(2) (A);　(3) (B).

3. (1) $\dfrac{3}{2}+\cos 1+\sin 1-\cos 2-2\sin 2$;　(2) $\pi^2-\dfrac{40}{9}$;

(3) $\dfrac{1}{3}R^3\left(\pi-\dfrac{4}{3}\right)$;　(4) $\dfrac{\pi}{4}R^4+9\pi R^2$.

4. (1) $\int_{-2}^{0}dx\int_{2x+4}^{4-x^2}f(x,y)\,dy$;　(2) $\int_0^2 dx\int_{\frac{1}{2}x}^{3-x}f(x,y)\,dy$;

(3) $\int_0^1 dy\int_0^{y^2}f(x,y)\,dx+\int_1^2 dy\int_0^{\sqrt{2y-y^2}}f(x,y)\,dx$.

5. 略.

6. $\int_0^{\frac{\pi}{4}}d\theta\int_0^{\sec\theta\tan\theta}f(\rho\cos\theta,\rho\sin\theta)\rho\,d\rho+\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}d\theta\int_0^{\csc\theta}f(\rho\cos\theta,\rho\sin\theta)\rho\,d\rho+$

$\int_{\frac{3\pi}{4}}^{\pi}d\theta\int_0^{\sec\theta\tan\theta}f(\rho\cos\theta,\rho\sin\theta)\rho\,d\rho$.

7. $f(x,y)=\sqrt{1-x^2-y^2}+\dfrac{8}{9\pi}-\dfrac{2}{3}$.

8. $\int_{-1}^{1}dx\int_{x^2}^{1}dy\int_0^{x^2+y^2}f(x,y,z)\,dz$.

9. (1) $\dfrac{59}{480}\pi R^5$;　(2) 0;　(3) $\dfrac{250}{3}\pi$.

*10. (1) $F(t)$在$(0,+\infty)$内单调增加;　(2) 略.

11. $\dfrac{1}{2}\sqrt{a^2b^2+b^2c^2+c^2a^2}$.

12. $\sqrt{\dfrac{2}{3}}R$　(R 为圆的半径).

13. $I=\dfrac{368}{105}\mu$.

14. $\boldsymbol{F}=(F_x,F_y,F_z)$，其中 $F_x=0$，$F_y=\dfrac{4GmM}{\pi R^2}\left(\ln\dfrac{R+\sqrt{R^2+a^2}}{a}-\dfrac{R}{\sqrt{R^2+a^2}}\right)$，

$F_z=-\dfrac{2GmM}{R^2}\left(1-\dfrac{a}{\sqrt{R^2+a^2}}\right)$.

15. $\left(0,0,\dfrac{3}{8}b\right)$.

*16. $\mu\big|_{r=0}=\dfrac{3M}{\pi R^3}$.

第十一章

习题 11－1（第 193 页）

1. (1) $I_x=\int_L y^2\mu(x,y)\,\mathrm{d}s$，$I_y=\int_L x^2\mu(x,y)\,\mathrm{d}s$；

(2) $\bar{x}=\dfrac{\int_L x\mu(x,y)\,\mathrm{d}s}{\int_L\mu(x,y)\,\mathrm{d}s}$，$\bar{y}=\dfrac{\int_L y\mu(x,y)\,\mathrm{d}s}{\int_L\mu(x,y)\,\mathrm{d}s}$.

2. 略.

3. (1) $2\pi a^{2n+1}$； (2) $\sqrt{2}$； (3) $\dfrac{1}{12}(5\sqrt{5}+6\sqrt{2}-1)$； (4) $\mathrm{e}^a\left(2+\dfrac{\pi}{4}a\right)-2$；

(5) $\dfrac{\sqrt{3}}{2}(1-\mathrm{e}^{-2})$； (6) 9； (7) $\dfrac{256}{15}a^3$； (8) $2\pi^2a^3(1+2\pi^2)$.

4. 质心在扇形的对称轴上且与圆心距离$\dfrac{a\sin\varphi}{\varphi}$处.

5. (1) $I_z=\dfrac{2}{3}\pi a^2\sqrt{a^2+k^2}(3a^2+4\pi^2k^2)$；

(2) $\bar{x}=\dfrac{6ak^2}{3a^2+4\pi^2k^2}$，$\bar{y}=\dfrac{-6\pi ak^2}{3a^2+4\pi^2k^2}$，$\bar{z}=\dfrac{3k(\pi a^2+2\pi^3k^2)}{3a^2+4\pi^2k^2}$.

习题 11－2（第 203 页）

1—2. 略.

3. (1) $-\dfrac{56}{15}$； (2) $-\dfrac{\pi}{2}a^3$； (3) 0； (4) -2π；

(5) $\dfrac{k^3\pi^3}{3}-a^2\pi$； (6) 13； (7) $\dfrac{1}{2}$； (8) $-\dfrac{14}{15}$.

4. (1) $\frac{34}{3}$; (2) 11; (3) 14; (4) $\frac{32}{3}$.

5. $-|\boldsymbol{F}|R$.

6. $mg(z_2-z_1)$.

7. (1) $\int_L \frac{P(x,y)+Q(x,y)}{\sqrt{2}}\mathrm{d}s$; (2) $\int_L \frac{P(x,y)+2xQ(x,y)}{\sqrt{1+4x^2}}\mathrm{d}s$;

(3) $\int_L [\sqrt{2x-x^2}P(x,y)+(1-x)Q(x,y)]\mathrm{d}s$.

8. $\int_\Gamma \frac{P+2xQ+3yR}{\sqrt{1+4x^2+9y^2}}\mathrm{d}s$.

习题 11－3(第 216 页)

1. (1) $\frac{1}{30}$; (2) 8.

2. (1) $\frac{3}{8}\pi a^2$; (2) 12π; (3) πa^2.

3. $-\pi$.

4. C 为椭圆 $2x^2+y^2=1$,沿逆时针方向.

5. 提示:利用面积公式 $A=\frac{1}{2}\oint_C x\mathrm{d}y-y\mathrm{d}x$,再逐条边地计算此曲线积分.

6. (1) $\frac{5}{2}$; (2) 236; (3) 5.

7. (1) 12; (2) 0; (3) $\frac{\pi^2}{4}$; (4) $\frac{\sin 2}{4}-\frac{7}{6}$.

8. (1) $\frac{1}{2}x^2+2xy+\frac{1}{2}y^2$; (2) x^2y; (3) $-\cos 2x\cdot\sin 3y$;

(4) $x^3y+4x^2y^2-12\mathrm{e}^y+12y\mathrm{e}^y$; (5) $y^2\sin x+x^2\cos y$.

9. 略.

*10. (1) $x^3+3x^2y^2+\frac{4}{3}y^3=C$; (2) $a^2x-x^2y-xy^2-\frac{1}{3}y^3=C$;

(3) $x\mathrm{e}^y-y^2=C$; (4) $x\sin y+y\cos x=C$;

(5) $xy-\frac{1}{3}x^3=C$; (6) 不是全微分方程;

(7) $\rho(1+\mathrm{e}^{2\theta})=C$; (8) 不是全微分方程.

11. $\lambda=-1$, $u(x,y)=-\arctan\frac{y}{x^2}+C$.

习题 11－4(第 222 页)

1. $I_x = \iint\limits_{\Sigma} (y^2 + z^2)\mu(x,y,z)\,\mathrm{d}S$.

2—3. 略.

4. (1) $\frac{13}{3}\pi$;　(2) $\frac{149}{30}\pi$;　(3) $\frac{111}{10}\pi$.

5. (1) $\frac{1+\sqrt{2}}{2}\pi$;　(2) 9π.

6. (1) $4\sqrt{61}$;　(2) $-\frac{27}{4}$;　(3) $\pi a(a^2 - h^2)$;　(4) $\frac{64}{15}\sqrt{2}a^4$.

7. $\frac{2\pi}{15}(6\sqrt{3}+1)$.

8. $\frac{4}{3}\mu_0 \pi a^4$.

习题 11－5(第 231 页)

1—2. 略.

3. (1) $\frac{2}{105}\pi R^7$;　(2) $\frac{3}{2}\pi$;　(3) $\frac{1}{2}$;　(4) $\frac{1}{8}$.

4. (1) $\iint\limits_{\Sigma}\left(\frac{3}{5}P + \frac{2}{5}Q + \frac{2\sqrt{3}}{5}R\right)\mathrm{d}S$;　(2) $\iint\limits_{\Sigma}\frac{2xP + 2yQ + R}{\sqrt{1+4x^2+4y^2}}\mathrm{d}S$.

习题 11－6(第 239 页)

1. (1) $3a^4$;　*(2) $\frac{12}{5}\pi a^5$;　*(3) $\frac{2}{5}\pi a^5$;　(4) 81π;　(5) $\frac{3}{2}$.

*2. (1) 0;　(2) $a^3\left(2-\frac{a^2}{6}\right)$;　(3) 108π.

*3. (1) $\operatorname{div} \boldsymbol{A} = 2x + 2y + 2z$;　(2) $\operatorname{div} \boldsymbol{A} = y\mathrm{e}^{xy} - x\sin(xy) - 2xz\sin(xz^2)$;
　(3) $\operatorname{div} \boldsymbol{A} = 2x$.

4. 略.

*5. 提示:取液面为 xOy 面,z 轴铅直向下. 这物体表面 Σ 上点 (x,y,z) 处单位面积上所受液体的压力为 $(-\nu_0 z\cos\alpha, -\nu_0 z\cos\beta, -\nu_0 z\cos\gamma)$,其中 ν_0 为液体单位体积的重力,$\cos\alpha$、$\cos\beta$、$\cos\gamma$ 为点 (x,y,z) 处 Σ 的外法线的方向余弦.

习题 11－7（第 248 页）

1. 略.

*2. （1）$-\sqrt{3}\pi a^2$； （2）$-2\pi a(a+b)$； （3）-20π； （4）9π.

*3. （1）$\mathbf{rot}\ \boldsymbol{A}=2\boldsymbol{i}+4\boldsymbol{j}+6\boldsymbol{k}$； （2）$\mathbf{rot}\ \boldsymbol{A}=\boldsymbol{i}+\boldsymbol{j}$；

（3）$\mathbf{rot}\ \boldsymbol{A}=[x\sin(\cos z)-xy^2\cos(xz)]\boldsymbol{i}-y\sin(\cos z)\boldsymbol{j}+[y^2z\cos(xz)-x^2\cos y]\boldsymbol{k}$.

*4. （1）0； （2）-4.

*5. （1）2π； （2）12π.

*6. 略.

*7. $\mathbf{0}$.

总习题十一（第 249 页）

1. （1）$\int_\Gamma (P\cos\alpha+Q\cos\beta+R\cos\gamma)\,\mathrm{d}s$，切向量；

（2）$\iint\limits_\Sigma (P\cos\alpha+Q\cos\beta+R\cos\gamma)\,\mathrm{d}S$，法向量.

2. （C）.

3. （1）$2a^2$； （2）$\dfrac{(2+t_0^2)^{\frac{3}{2}}-2\sqrt{2}}{3}$； （3）$-2\pi a^2$； （4）$\dfrac{1}{35}$； （5）$\pi a^2$；

（6）$\dfrac{\sqrt{2}}{16}\pi$.

4. （1）$2\pi\arctan\dfrac{H}{R}$； （2）$-\dfrac{\pi}{4}h^4$； （3）$2\pi R^3$； （4）$\dfrac{2}{15}$.

5. $\dfrac{1}{2}\ln(x^2+y^2)$.

6. 略.

7. （1）略； （2）$\dfrac{c}{d}-\dfrac{a}{b}$.

8. $\left(0,0,\dfrac{a}{2}\right)$.

9. 略.

*10. 3.

11. $\dfrac{3}{2}$.

第 十 二 章

习题 12 - 1(第 258 页)

1. (1) $\frac{1+1}{1+1^2}+\frac{1+2}{1+2^2}+\frac{1+3}{1+3^2}+\frac{1+4}{1+4^2}+\frac{1+5}{1+5^2}+\cdots$;

(2) $\frac{1}{2}+\frac{1\cdot3}{2\cdot4}+\frac{1\cdot3\cdot5}{2\cdot4\cdot6}+\frac{1\cdot3\cdot5\cdot7}{2\cdot4\cdot6\cdot8}+\frac{1\cdot3\cdot5\cdot7\cdot9}{2\cdot4\cdot6\cdot8\cdot10}+\cdots$;

(3) $\frac{1}{5}-\frac{1}{5^2}+\frac{1}{5^3}-\frac{1}{5^4}+\frac{1}{5^5}-\cdots$;

(4) $\frac{1!}{1^1}+\frac{2!}{2^2}+\frac{3!}{3^3}+\frac{4!}{4^4}+\frac{5!}{5^5}+\cdots$.

2. (1) 发散; (2) 收敛; (3) 发散. 提示:先乘 $2\sin\frac{\pi}{12}$,再将一般项分解为两个余弦函数之差; (4) 发散.

3. (1) 收敛; (2) 发散; (3) 发散; (4) 发散; (5) 收敛.

*4. (1) 收敛; (2) 发散; (3) 收敛; (4) 发散.

习题 12 - 2(第 271 页)

1. (1) 发散; (2) 发散; (3) 收敛; (4) 收敛;
(5) $a>1$ 时收敛,$a\leqslant 1$ 时发散.

2. (1) 发散; (2) 收敛; (3) 收敛; (4) 收敛.

*3. (1) 收敛; (2) 收敛; (3) 收敛;
(4) 当 $b<a$ 时收敛,当 $b>a$ 时发散,当 $b=a$ 时不能肯定.

4. (1) 收敛; (2) 收敛; (3) 发散; (4) 收敛; (5) 发散; (6) 发散.

5. (1) 条件收敛; (2) 绝对收敛; (3) 绝对收敛; (4) 条件收敛;
(5) 发散.

习题 12 - 3(第 281 页)

1. (1) $(-1,1)$; (2) $(-1,1)$; (3) $(-\infty,+\infty)$; (4) $(-3,3)$;
(5) $\left(-\frac{1}{2},\frac{1}{2}\right)$; (6) $(-1,1)$; (7) $(-\sqrt{2},\sqrt{2})$; (8) $(4,6)$.

2. (1) $\frac{1}{(1-x)^2}$ $(-1<x<1)$;

(2) $\frac{1}{4}\ln\frac{1+x}{1-x}+\frac{1}{2}\arctan x-x \quad (-1<x<1)$;

(3) $\frac{1}{2}\ln\frac{1+x}{1-x} \quad (-1<x<1)$.

(4) $\frac{x^2}{(1-x)^2}-x^2-2x^3 \quad (-1<x<1)$.

习题 12－4(第 289 页)

1. $\cos x=\cos x_0+\cos\left(x_0+\frac{\pi}{2}\right)(x-x_0)+\cdots+\frac{\cos\left(x_0+\frac{n\pi}{2}\right)}{n!}(x-x_0)^n+\cdots$

$(-\infty,+\infty)$.

2. (1) $\frac{e^x-e^{-x}}{2}=\sum_{n=1}^{\infty}\frac{x^{2n-1}}{(2n-1)!}$, $(-\infty,+\infty)$;

(2) $\ln(a+x)=\ln a+\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n}\left(\frac{x}{a}\right)^n$, $(-a,a]$;

(3) $a^x=\sum_{n=0}^{\infty}\frac{(x\ln a)^n}{n!}$, $(-\infty,+\infty)$;

(4) $\sin^2 x=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(2x)^{2n}}{2(2n)!}$, $(-\infty,+\infty)$;

(5) $(1+x)\ln(1+x)=x+\sum_{n=2}^{\infty}\frac{(-1)^n x^n}{n(n-1)}$, $(-1,1]$;

(6) $\frac{x}{\sqrt{1+x^2}}=x+\sum_{n=1}^{\infty}(-1)^n\frac{2(2n)!}{(n!)^2}\left(\frac{x}{2}\right)^{2n+1}$, $[-1,1]$.

3. (1) $\sqrt{x^3}=1+\frac{3}{2}(x-1)+\sum_{n=0}^{\infty}(-1)^n\frac{(2n)!}{(n!)^2}\frac{3}{(n+1)(n+2)2^n}\left(\frac{x-1}{2}\right)^{n+2}$,

$[0,2]$;

(2) $\lg x=\frac{1}{\ln 10}\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(x-1)^n}{n}$, $(0,2]$.

4. $\cos x=\frac{1}{2}\sum_{n=0}^{\infty}(-1)^n\left[\frac{\left(x+\frac{\pi}{3}\right)^{2n}}{(2n)!}+\sqrt{3}\frac{\left(x+\frac{\pi}{3}\right)^{2n+1}}{(2n+1)!}\right]$, $(-\infty,+\infty)$.

5. $\frac{1}{x}=\frac{1}{3}\sum_{n=0}^{\infty}(-1)^n\frac{(x-3)^n}{3^n}$, $(0,6)$.

6. $\frac{1}{x^2+3x+2}=\sum_{n=0}^{\infty}\left(\frac{1}{2^{n+1}}-\frac{1}{3^{n+1}}\right)(x+4)^n$, $(-6,-2)$.

习题 12-5(第 298 页)

1. (1) 1.098 6； (2) 1.648； (3) 2.004 30； (4) 0.999 4.
2. (1) 0.494 0； (2) 0.487.
3. (1) $y=Ce^{\frac{x^2}{2}}+\left[-1+x+\frac{1}{1\cdot 3}x^3+\cdots+\frac{x^{2n-1}}{1\cdot 3\cdot 5\cdots(2n-1)}+\cdots\right]$；

   (2) $y=a_0e^{-\frac{x^2}{2}}+a_1\left[x-\frac{x^3}{1\cdot 3}+\frac{x^5}{1\cdot 3\cdot 5}-\cdots+(-1)^{n-1}\frac{x^{2n-1}}{1\cdot 3\cdot 5\cdots(2n-1)}+\cdots\right]$；

   (3) $y=C(1-x)+x^3\left[\frac{1}{3}+\frac{1}{6}x+\frac{1}{10}x^2+\cdots+\frac{2}{(n+2)(n+3)}x^n+\cdots\right]$.
4. (1) $y=\frac{1}{2}+\frac{1}{4}x+\frac{1}{8}x^2+\frac{1}{16}x^3+\frac{9}{32}x^4+\cdots$；

   (2) $y=x+\frac{1}{1\cdot 2}x^2+\frac{1}{2\cdot 3}x^3+\frac{1}{3\cdot 4}x^4+\cdots+\frac{1}{n(n-1)}x^n+\cdots$.
5. 和函数为 $y(x)=\frac{2}{3}e^{-\frac{x}{2}}\cos\frac{\sqrt{3}}{2}x+\frac{1}{3}e^x\ (-\infty<x<+\infty)$.
6. $e^x\cos x=\sum\limits_{n=0}^{\infty}2^{\frac{n}{2}}\cos\frac{n\pi}{4}\cdot\frac{x^n}{n!},(-\infty,+\infty)$.

提示：$e^x\cos x=\mathrm{Re}\ e^{(1+i)x}=\mathrm{Re}\ e^{\sqrt{2}(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})x}$.

*习题 12-6(第 307 页)

1. (1) 取正整数 $N\geqslant\frac{|x|}{\varepsilon}$； (2) 略.
2. (1) $s(x)=\begin{cases}0, & x=0,\\ 1+x^2, & x\neq 0;\end{cases}$

   (2) 当 $x\neq 0$ 时取正整数 $N\geqslant\dfrac{\ln\frac{1}{\varepsilon}}{\ln(1+x^2)}$，当 $x=0$ 时取 $N=1$；

   (3) 在 $[0,1]$ 上不一致收敛，在 $\left[\frac{1}{2},1\right]$ 上一致收敛.
3. (1) 一致收敛； (2) 不一致收敛.
4. 略.

习题 12-7(第 320 页)

1. (1) $f(x)=\pi^2+1+12\sum\limits_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos nx,(-\infty,+\infty)$；

(2) $f(x)=\frac{e^{2\pi}-e^{-2\pi}}{\pi}\left[\frac{1}{4}+\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2+4}(2\cos nx-n\sin nx)\right]$,

$(x\neq(2n+1)\pi, n=0,\pm1,\pm2,\cdots)$;

(3) $f(x)=\frac{a-b}{4}\pi+\sum_{n=1}^{\infty}\left\{\frac{[1-(-1)^n](b-a)}{n^2\pi}\cos nx+\frac{(-1)^{n-1}(a+b)}{n}\sin nx\right\}$,

$(x\neq(2n+1)\pi, n=0,\pm1,\pm2,\cdots)$.

2. (1) $2\sin\frac{x}{3}=\frac{18\sqrt{3}}{\pi}\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n\sin nx}{9n^2-1}$, $(-\pi,\pi)$;

(2) $f(x)=\frac{1+\pi-e^{-\pi}}{2\pi}+\frac{1}{\pi}\sum_{n=1}^{\infty}\left\{\frac{1-(-1)^n e^{-\pi}}{1+n^2}\cos nx+\right.$

$\left.\left[\frac{-n+(-1)^n ne^{-\pi}}{1+n^2}+\frac{1}{n}(1-(-1)^n)\right]\sin nx\right\}$, $(-\pi,\pi)$.

3. $\cos\frac{x}{2}=\frac{2}{\pi}+\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{4n^2-1}\cos nx$, $[-\pi,\pi]$.

4. $f(x)=\frac{2}{\pi}\sum_{n=1}^{\infty}\left[\frac{1}{n^2}\sin\frac{n\pi}{2}+(-1)^{n+1}\frac{\pi}{2n}\right]\sin nx$ $(x\neq(2n+1)\pi, n=0,\pm1,$ $\pm2,\cdots)$

5. $\frac{\pi-x}{2}=\sum_{n=1}^{\infty}\frac{1}{n}\sin nx$, $(0,\pi]$.

6. $2x^2=\frac{4}{\pi}\sum_{n=1}^{\infty}\left[-\frac{2}{n^3}+(-1)^n\left(\frac{2}{n^3}-\frac{\pi^2}{n}\right)\right]\sin nx$, $[0,\pi)$;

$2x^2=\frac{2}{3}\pi^2+8\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos nx$, $[0,\pi]$.

7. 略.

习题 12－8(第 327 页)

1. (1) $f(x)=\frac{11}{12}+\frac{1}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}\cos 2n\pi x$, $(-\infty,+\infty)$;

(2) $f(x)=-\frac{1}{4}+\sum_{n=1}^{\infty}\left\{\left[\frac{1-(-1)^n}{n^2\pi^2}+\frac{2\sin\frac{n\pi}{2}}{n\pi}\right]\cos n\pi x+\frac{1-2\cos\frac{n\pi}{2}}{n\pi}\sin n\pi x\right\}$,

$\left(x\neq 2k, 2k+\frac{1}{2}, k=0,\pm1,\pm2,\cdots\right)$;

(3) $f(x)=-\frac{1}{2}+\sum_{n=1}^{\infty}\left\{\frac{6}{n^2\pi^2}[1-(-1)^n]\cos\frac{n\pi x}{3}+\frac{6}{n\pi}(-1)^{n+1}\sin\frac{n\pi x}{3}\right\}$,

$(x \neq 3(2k+1), k=0, \pm 1, \pm 2, \cdots)$.

2. (1) $f(x)=\frac{4l}{\pi^2}\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{(2k-1)^2}\sin\frac{(2k-1)\pi x}{l}, [0,l]$,

$f(x)=\frac{l}{4}-\frac{2l}{\pi^2}\sum_{k=1}^{\infty}\frac{1}{(2k-1)^2}\cos\frac{2(2k-1)\pi x}{l}, [0,l]$;

(2) $f(x)=\frac{8}{\pi}\sum_{n=1}^{\infty}\left\{\frac{(-1)^{n+1}}{n}+\frac{2}{n^3\pi^2}[(-1)^n-1]\right\}\sin\frac{n\pi x}{2}, [0,2)$,

$f(x)=\frac{4}{3}+\frac{16}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos\frac{n\pi x}{2}, [0,2]$.

*3. $f(x)=\operatorname{sh}1\sum_{n=-\infty}^{\infty}\frac{(-1)^n(1-n\pi \mathrm{i})}{1+(n\pi)^2}\mathrm{e}^{n\pi x\mathrm{i}}$ $(x\neq 2k+1, k=0, \pm 1, \pm 2, \cdots)$.

*4. $u(t)=\frac{h\tau}{T}+\frac{2h}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{n\tau\pi}{T}\cos\frac{2n\pi t}{T}$ $(-\infty, +\infty)$.

总习题十二(第 327 页)

1. (1) 必要,充分; (2) 充分必要; (3) 收敛,发散.
2. (A).
3. (1) 发散; (2) 发散; (3) 收敛; (4) 发散;
   (5) $a<1$ 时收敛,$a>1$ 时发散,$a=1$ 时,$s>1$ 收敛,$s\leqslant 1$ 发散.
4. 略.
5. 不一定. 考虑级数 $\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt{n}}$ 及 $\sum_{n=1}^{\infty}\left((-1)^n\frac{1}{\sqrt{n}}+\frac{1}{n}\right)$.
6. (1) $p>1$ 时绝对收敛,$0<p\leqslant 1$ 时条件收敛,$p\leqslant 0$ 时发散;
   (2) 绝对收敛; (3) 条件收敛; (4) 绝对收敛.
7. (1) 0; (2) $\sqrt[4]{8}$. 提示:化成 $2^{\frac{1}{3}+\frac{2}{3^2}+\cdots+\frac{n}{3^n}+\cdots}$.
8. (1) $\left(-\frac{1}{5},\frac{1}{5}\right)$; (2) $\left(-\frac{1}{\mathrm{e}},\frac{1}{\mathrm{e}}\right)$; (3) $(-2,0)$; (4) $(-\sqrt{2},\sqrt{2})$.
9. (1) $s(x)=\frac{2+x^2}{(2-x^2)^2}, (-\sqrt{2},\sqrt{2})$; *(2) $s(x)=\arctan x, [-1,1]$;

   (3) $s(x)=\frac{x-1}{(2-x)^2}, (0,2)$;

   *(4) $s(x)=\begin{cases}1+\left(\frac{1}{x}-1\right)\ln(1-x), & x\in[-1,0)\cup(0,1),\\ 0, & x=0,\\ 1, & x=1.\end{cases}$

10. (1) 2e;　(2) $\frac{1}{2}(\cos 1+\sin 1)$. 提示:利用 cos 1 和 sin 1 的展开式.

11. (1) $\ln(x+\sqrt{x^2+1})=x+\sum_{n=1}^{\infty}(-1)^n\frac{(2n-1)!!}{(2n)!!}\frac{x^{2n+1}}{2n+1}, x\in[-1,1]$,

提示:利用积分 $\int_0^x \frac{\mathrm{d}t}{\sqrt{t^2+1}}$;

(2) $\frac{1}{(2-x)^2}=\sum_{n=1}^{\infty}\frac{n}{2^{n+1}}x^{n-1}, x\in(-2,2)$.

12. $$f(x)=\frac{\mathrm{e}^{\pi}-1}{2\pi}+\frac{1}{\pi}\sum_{n=1}^{\infty}\left[\frac{(-1)^n\mathrm{e}^{\pi}-1}{n^2+1}\cos nx+\frac{n((-1)^{n+1}\mathrm{e}^{\pi}+1)}{n^2+1}\sin nx\right],$$

$$-\infty<x<+\infty \text{ 且 } x\neq n\pi,\ n=0,\pm 1,\pm 2,\cdots.$$

13. $$f(x)=\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1-\cos nh}{n}\sin nx,\ x\in(0,h)\cup(h,\pi];$$

$$f(x)=\frac{h}{\pi}+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\sin nh}{n}\cos nx,\ x\in[0,h)\cup(h,\pi].$$

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