Current-Mode Control Small-Signal Model[1]

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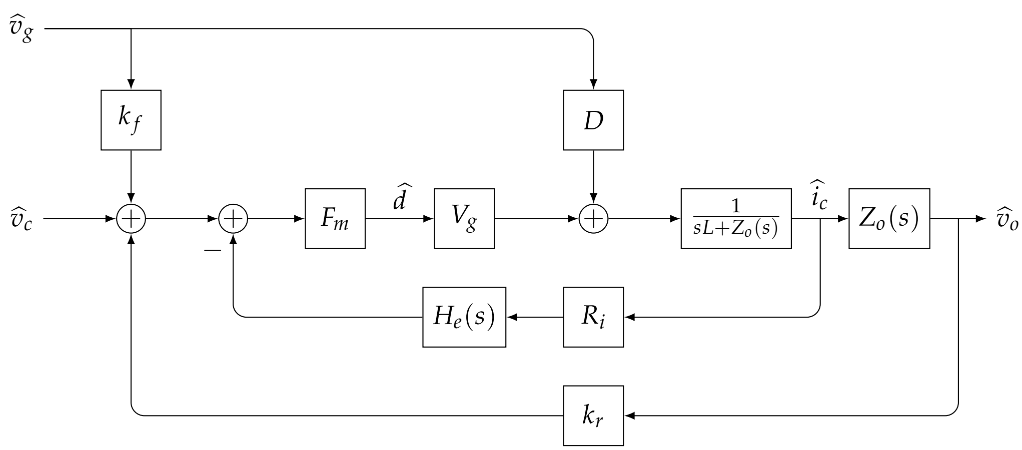
[1] Original derivations by Dr. Raymond B. Ridley of Ridley Engineering.

This document seeks to clarify the block diagrams and equations presented in Dr. Raymond B. Ridley's PhD dissertation, "A New Small-Signal Model for Current-Mode Control." All equations presented herein are as applied to a buck converter with constant-frequency, trailing-edge peak current-mode control.

Small-Signal Block Diagram



Dr. Ridley's diagrams generally mix transfer function blocks with circuit element symbols. As an alternative, a complete transfer function block diagram is provided above. Transfer function descriptions and key parameter definitions are in the margins.

$$F_m = \frac{1}{(S_n + S_e)T_s}$$

$$H_e(s) = 1 - s\frac{T_s}{2} + s^2\left(\frac{T_s}{\pi}\right)^2$$

$$k_f = \frac{-DT_sR_i}{L}\left(1 - \frac{D}{2}\right)$$

$$k_r = \frac{T_sR_i}{2L}$$

Multiple forms of the modulation gain F_m can be found in literature; their differences stem from how the average inductor current is defined relative to the peak. Dr. Ridley has experimentally verified the modulation gain used in this model is correct.

V_g steady-state input voltage
V_o steady-state output voltage
$D = V_o/V_g$ steady-state duty cycle
$\widehat{v}_g$ small-signal input voltage
$\widehat{v}_o$ small-signal output voltage
$\widehat{d}$ small-signal duty cycle
$\widehat{i}_c$ small-signal inductor current
$\widehat{v}_c$ small-signal control voltage
sL inductor impedance
$Z_o(s)$ output impedance
R_i sense resistor
$H_e(s)$ sample-and-hold effect
F_m modulation gain
k_f input feed-forward gain
k_r output feed-forward gain
S_e slope compensation ramp
$S_n = (V_g - V_o)R_i/L$ on-time ramp
$S_f = V_oR_i/L$ off-time ramp
T_s switching period

$H_e(s)$ is an approximation that is only valid up to one-half the switching frequency. Such an approximation will suffice because a stable system must have a crossover frequency beneath the Nyquist frequency of the system.

In order to simplify the design of the outer voltage loop feedback compensation network, a transfer function from the duty cycle to inductor current F_i can be defined that is independent of the output impedance $Z_o(s)$. Such a transfer function will only be valid for frequencies where $Z_o(s) \ll sL$. Assuming the output impedance is predominantly capacitive, this transfer function would apply above the resonant frequency.

$$F_i = \frac{\widehat{i_c}}{\widehat{d}} = \frac{V_g}{sL} = \frac{S_n + S_f}{sR_i}$$

$$\alpha = \frac{S_f - S_e}{S_n + S_e}$$

$$F_m F_i = \frac{1+\alpha}{sR_i T_s}$$

The output voltage feed-forward gain k_r can be ignored unless analyzing the low frequency gain of a converter operating near discontinuous conduction mode. The transfer function from the control voltage to inductor current $F(s)$ then becomes rather simple.

$$F(s) = \frac{\widehat{i_c}}{\widehat{v}_c} = \frac{F_m F_i}{1 + F_m F_i R_i H_e(s)}$$