Current-Mode Control Small-Signal Model[1]

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November 24, 2021

[1] Original derivations by Dr. Raymond B. Ridley of Ridley Engineering.

This document seeks to clarify the block diagrams and equations presented in Dr. Raymond B. Ridley's PhD dissertation, "A New Small-Signal Model for Current-Mode Control." All equations presented herein are as applied to a buck converter with constant-frequency, trailing-edge peak current-mode control.

Small-Signal Block Diagram

Dr. Ridley's diagrams generally mix transfer function blocks with circuit element symbols. As an alternative, a complete transfer function block diagram is provided below. Transfer function descriptions are included and key parameter definitions are in the margins.

$$F_m = \frac{1}{(S_n + S_e)T_s}$$

$$H_e(s) = 1 - s\frac{T_s}{2} + s^2(\frac{T_s}{\pi})^2$$

Multiple forms of the modulation gain F_m can be found in literature; their differences stem from how the average inductor current is defined relative to the peak. Dr. Ridley has experimentally verified the modulation gain used in this model is correct.

$H_e(s)$ is an approximation that is only valid up to one-half the switching frequency. Such an approximation will suffice because a stable system must have a crossover frequency beneath the Nyquist frequency of the system.

In order to simplify the design of the outer voltage loop feedback compensation network, a transfer function from the duty cycle to inductor current F_i can be defined that is independent of the output impedance $Z_o(s)$. Such a transfer function will only be valid for frequencies where $Z_o(s) \ll sL$. Assuming the output impedance is predominantly capacitive, this transfer function would apply above the resonant frequency.

$$F_i = \frac{\widehat{i_c}}{\widehat{d}} = \frac{V_g}{sL} = \frac{S_n + S_f}{sR_i}$$

$$F_m F_i = \frac{1 + \alpha}{sR_i T_s} \ , \ \alpha = \frac{S_f - S_e}{S_n + S_e}$$

The output voltage feed-forward gain k_r can be ignored unless analyzing the low frequency gain of a converter operating near discontinuous conduction mode. The transfer function from the control voltage to inductor current $F(s)$ then becomes rather simple.

V_g steady-state input voltage
V_o steady-state output voltage
$D = V_o/V_g$ steady-state duty cycle
$\widehat{v_g}$ small-signal input voltage
$\widehat{v_o}$ small-signal output voltage
$\widehat{d}$ small-signal duty cycle
$\widehat{i_c}$ small-signal inductor current
$\widehat{v_c}$ small-signal control voltage
sL inductor impedance
$Z_o(s)$ output impedance
R_i sense resistor
$H_e(s)$ sample-and-hold effect
F_m modulation gain
k_f input feed-forward gain
k_r output feed-forward gain
S_e slope compensation ramp
$S_n = (V_g - V_o)R_i/L$ on-time ramp
$S_f = V_o R_i/L$ off-time ramp
T_s switching period

$$F(s) = \frac{\widehat{i_c}}{\widehat{v_c}} = \frac{F_m F_i}{1 + F_m F_i R_i H_e(s)}$$

Page Layout

Headings

This style provides A- and B-heads (that is, `\section` and `\subsection`), demonstrated above.

The Tufte-LaTeX classes will emit an error if you try to use `\subsubsection` and smaller headings.

In his later books,[2] Tufte starts each section with a bit of vertical space, a non-indented paragraph, and sets the first few words of the sentence in small caps. To accomplish this using this style, use the `\newthought` command:

```
\newthought{In his later books}, Tufte starts...
```

[2]

Sidenotes

One of the most prominent and distinctive features of this style is the extensive use of sidenotes. There is a wide margin to provide ample room for sidenotes and small figures. Any `\footnotes` will automatically be converted to sidenotes.[3] If you'd like to place ancillary information in the margin without the sidenote mark (the superscript number), you can use the `\marginnote` command.

[3] This is a sidenote that was entered using the `\footnote` command.

This is a margin note. Notice that there isn't a number preceding the note, and there is no number in the main text where this note was written.

The specification of the `\sidenote` command is:

`\sidenote[`⟨*number*⟩`][`⟨*offset*⟩`]{`*Sidenote text.*`}`

Both the ⟨*number*⟩ and ⟨*offset*⟩ arguments are optional. If you provide a ⟨*number*⟩ argument, then that number will be used as the sidenote number. It will change of the number of the current sidenote only and will not affect the numbering sequence of subsequent sidenotes.

Sometimes a sidenote may run over the top of other text or graphics in the margin space. If this happens, you can adjust the vertical position of the sidenote by providing a dimension in the ⟨*offset*⟩ argument. Some examples of valid dimensions are:

```
1.0in    2.54cm    254mm    6\baselineskip
```

If the dimension is positive it will push the sidenote down the page; if the dimension is negative, it will move the sidenote up the page.

While both the ⟨*number*⟩ and ⟨*offset*⟩ arguments are optional, they must be provided in order. To adjust the vertical position of the sidenote while leaving the sidenote number alone, use the following syntax:

`\sidenote[][`⟨*offset*⟩`]{`*Sidenote text.*`}`

The empty brackets tell the `\sidenote` command to use the default sidenote number.

If you *only* want to change the sidenote number, however, you may completely omit the ⟨*offset*⟩ argument:

`\sidenote[`⟨*number*⟩`]{`*Sidenote text.*`}`

The `\marginnote` command has a similar *offset* argument:

`\marginnote[`⟨*offset*⟩`]{`*Margin note text.*`}`

References

References are placed alongside their citations as sidenotes, as well. This can be accomplished using the normal `\cite` command.[4]

[4] The first paragraph of this document includes a citation.

The complete list of references may also be printed automatically by using the `\bibliography` command. (See the end of this document for an example.) If you do not want to print a bibliography at the end of your document, use the `\nobibliography` command in its place.

To enter multiple citations at one location,[5] you can provide a list of keys separated by commas and the same optional vertical offset argument: `\cite{Tufte2006,Tufte1990}`.

[5] ; and

`\cite[`⟨*offset*⟩`]{`*bibkey1*`,`*bibkey2*`,...}`

Figures and Tables

Images and graphics play an integral role in Tufte's work. In addition to the standard figure and tabular environments, this style provides special figure and table environments for full-width floats.

Full page–width figures and tables may be placed in figure* or table* environments. To place figures or tables in the margin, use the marginfigure or margintable environments as follows (see figure 1):

```
\begin{marginfigure}
  \includegraphics{helix}
  \caption{This is a margin figure.}
\end{marginfigure}
```

Figure 1: This is a margin figure. The helix is defined by $x = \cos(2\pi z)$, $y = \sin(2\pi z)$, and $z = [0, 2.7]$. The figure was drawn using Asymptote (`http://asymptote.sf.net/`).

The marginfigure and margintable environments accept an optional parameter ⟨*offset*⟩ that adjusts the vertical position of the figure or table. See the "Sidenotes" section above for examples. The specifications are:

```
\begin{marginfigure}[⟨offset⟩]
   ...
\end{marginfigure}
```

```
\begin{margintable}[⟨offset⟩]
    ...
\end{margintable}
```

Figure 2 is an example of the `figure*` environment and figure 3 is an example of the normal `figure` environment.

Figure 3: Hilbert curves of various degrees n. *Notice that this figure only takes up the main textblock width.*

Figure 2: This graph shows $y = \sin x$ from about $x = [-10, 10]$. *Notice that this figure takes up the full page width.*

Table 1 shows table created with the `booktabs` package. Notice the lack of vertical rules—they serve only to clutter the table's data.

Margin	Length
Paper width	$8\frac{1}{2}$ inches
Paper height	11 inches
Textblock width	$6\frac{1}{2}$ inches
Textblock/sidenote gutter	$\frac{3}{8}$ inches
Sidenote width	2 inches

Table 1: Here are the dimensions of the various margins used in the Tufte-handout class.

Full-width text blocks

In addition to the new float types, there is a fullwidth environment that stretches across the main text block and the sidenotes area.

```
\begin{fullwidth}
Lorem ipsum dolor sit amet...
\end{fullwidth}
```

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Typography

Typefaces

If the Palatino, Helvetica, and `Bera Mono` typefaces are installed, this style will use them automatically. Otherwise, we'll fall back on the Computer Modern typefaces.

Letterspacing

This document class includes two new commands and some improvements on existing commands for letterspacing.

When setting strings of ALL CAPS or small caps, the letterspacing—that is, the spacing between the letters—should be increased slightly.[6] The `\allcaps` command has proper letterspacing for strings of FULL CAPITAL LETTERS, and the `\smallcaps` command has letterspacing for small capital letters. These commands will also automatically convert the case of the text to upper- or lowercase, respectively.

[6]

The `\textsc` command has also been redefined to include letterspacing. The case of the `\textsc` argument is left as is, however. This allows one to use both uppercase and lowercase letters: The Initial Letters Of The Words In This Sentence Are Capitalized.

Installation

To install the Tufte-LaTeX classes, simply drop the following files into the same directory as your `.tex` file:

```
tufte-book.cls
tufte-common.def
tufte-handout.cls
tufte.bst
```

More Documentation

For more documentation on the Tufte-LaTeX document classes (including commands not mentioned in this handout), please see the sample book.

Support

The website for the Tufte-LaTeX packages is located at `http://code.google.com/p/tufte-latex/`. On our website, you'll find links to our svn repository, mailing lists, bug tracker, and documentation.