Elementary Statistics – Simple Linear Regression Pt 1

Dr. Ab Mosca (they/them)

Slides based off slides courtesy of Kaitlyn Cook (https://www.smith.edu/people/kaitlyn-cook)

Plan for Today

- Simple Linear Regression
    - Why?
    - Interpretation

Warm Up: EDA for two Numerical Variables

Download the Housing_Data.csv from the course website (under the Examples tab).

What is the observational unit in this dataset? What are the variables?

Create a data visualization to summarize the relationship between the LivingSpace and Price variables. (Use Excel, GoogleSheets, or some other tool of your choosing.)

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Create a data visualization to summarize the relationship between the LivingSpace and Price variables. (Use Excel, GoogleSheets, or some other tool of your choosing.)

Calculate the Pearson Correlation between Price and LivingSpace.

Do you think there is a relationship? What type? Is it strong?

Big Picture

Over the last few days, we've seen visuals and single number summaries for describing the distribution of individual variables, as well as the *form*, *direction*, and *strength* of their relationship with one another.

We can use statistical models to capture all of these features in a more precise and quantitative way!

→ In this class, we'll focus on linear regression models, where we seek to model this relationship using a straight line

Big Picture

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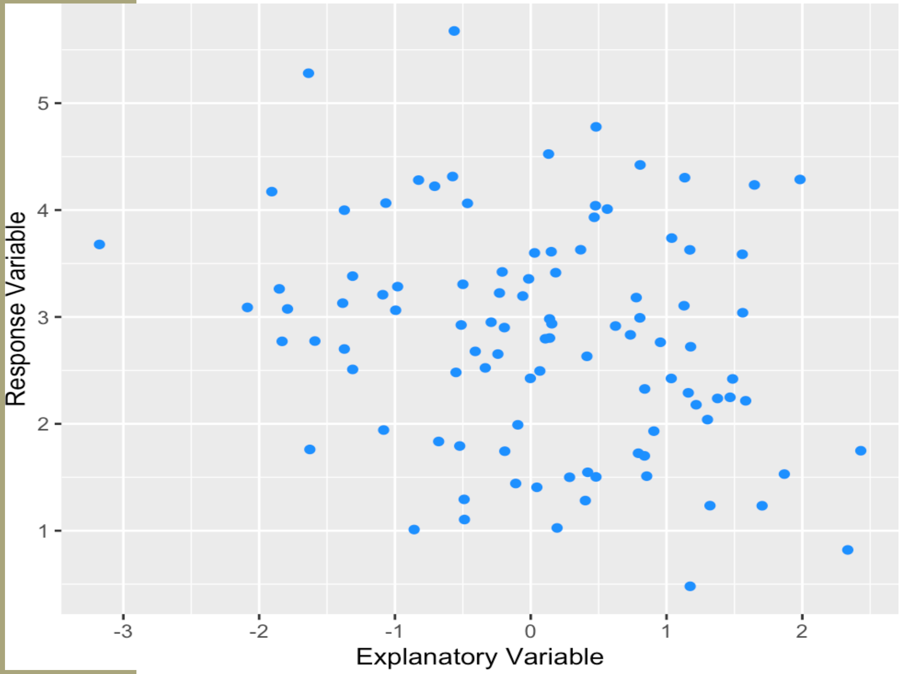

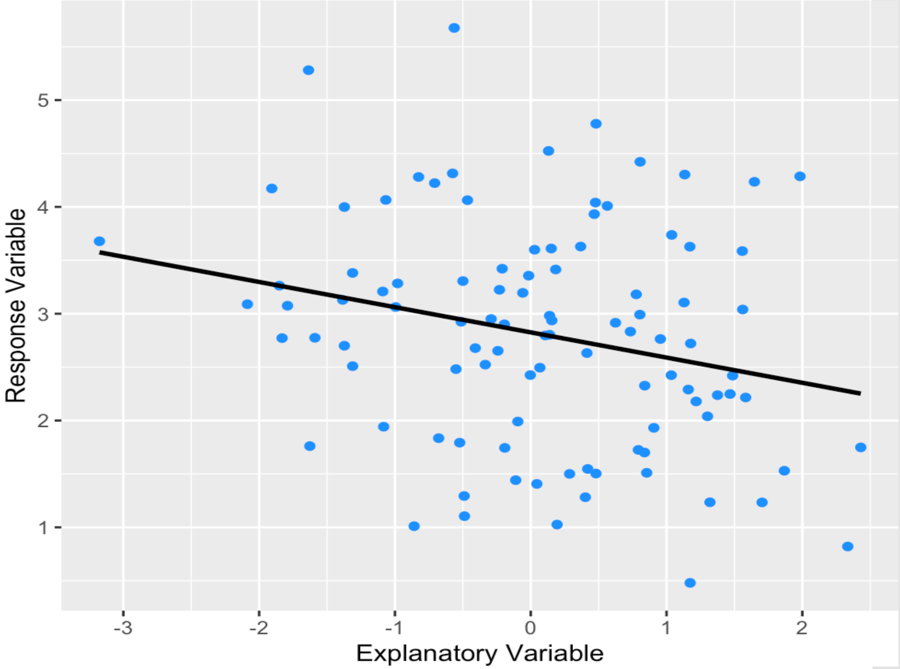

Motivating Example

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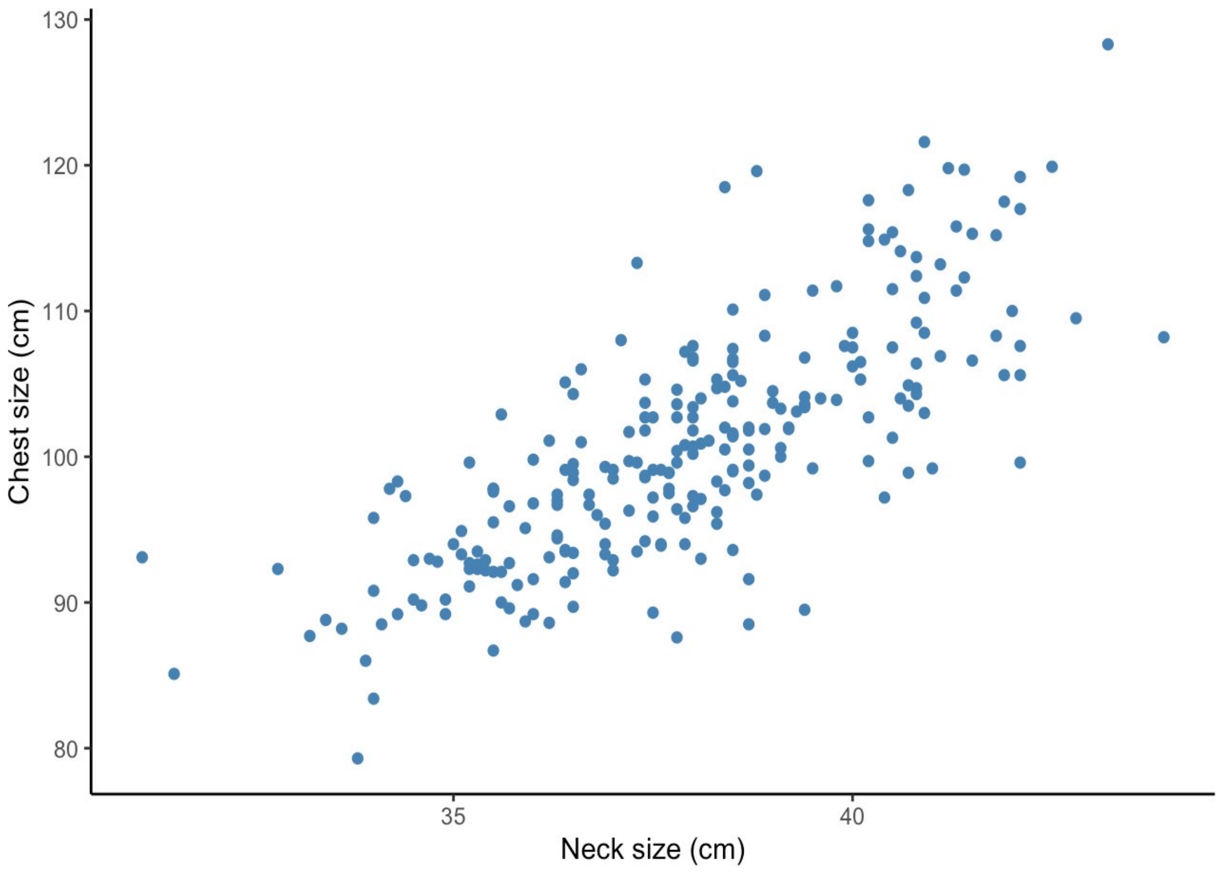
Then we may wish to build a regression model for the relationship between neck size (cm) and chest size (cm) in order to:

1. Explain and quantify the association between our variables of interest
   - How do an individual's neck and chest size relate with one another?
2. Model or predict the possible values of our response variable
   - Given that an individual has a neck size of 38 cm, what might we expect their chest size to be?

Motivating Example

The following scatterplot displays neck and chest size measurements for 251 individuals:



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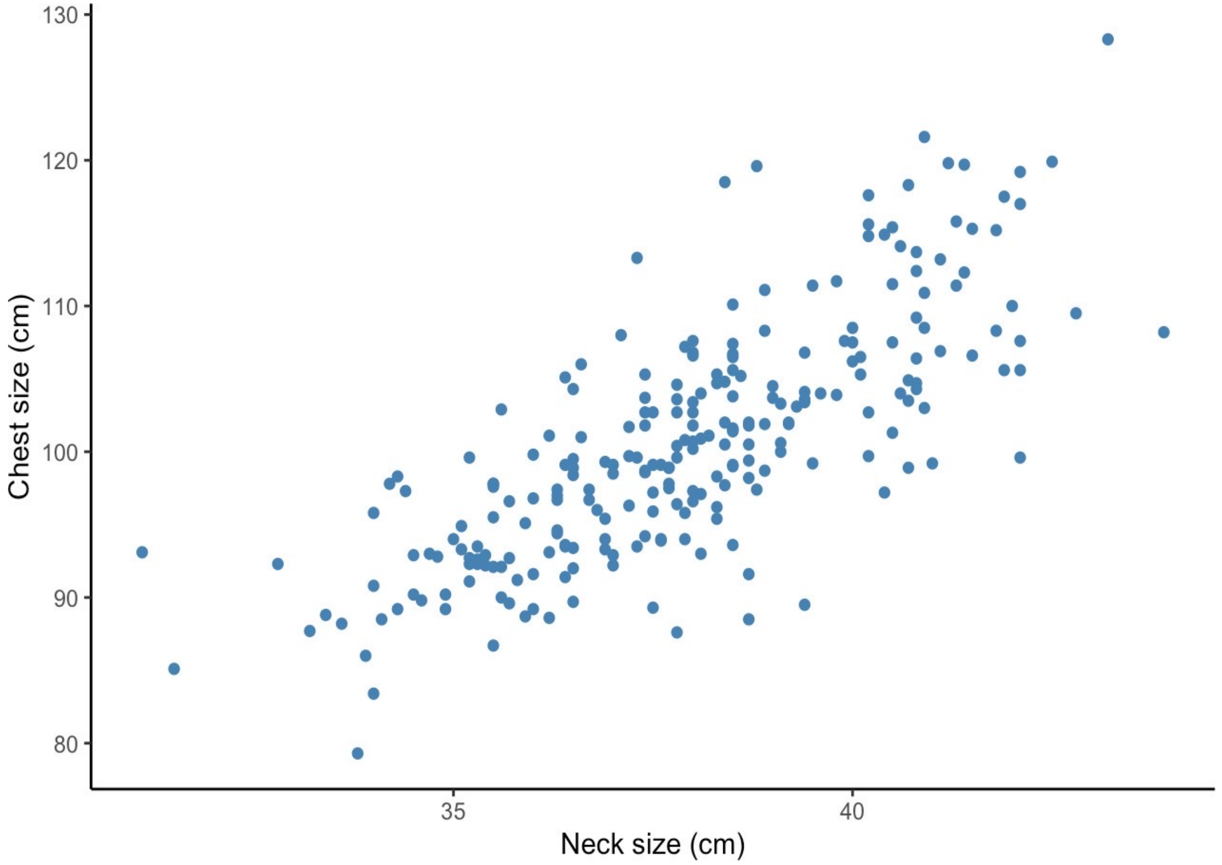

If Jackson & Connor were to offer only one shirt size, they might tailor it to fit an individual of *average* neck size ($\bar{x}$) and *average* chest size ($\bar{y}$). . .

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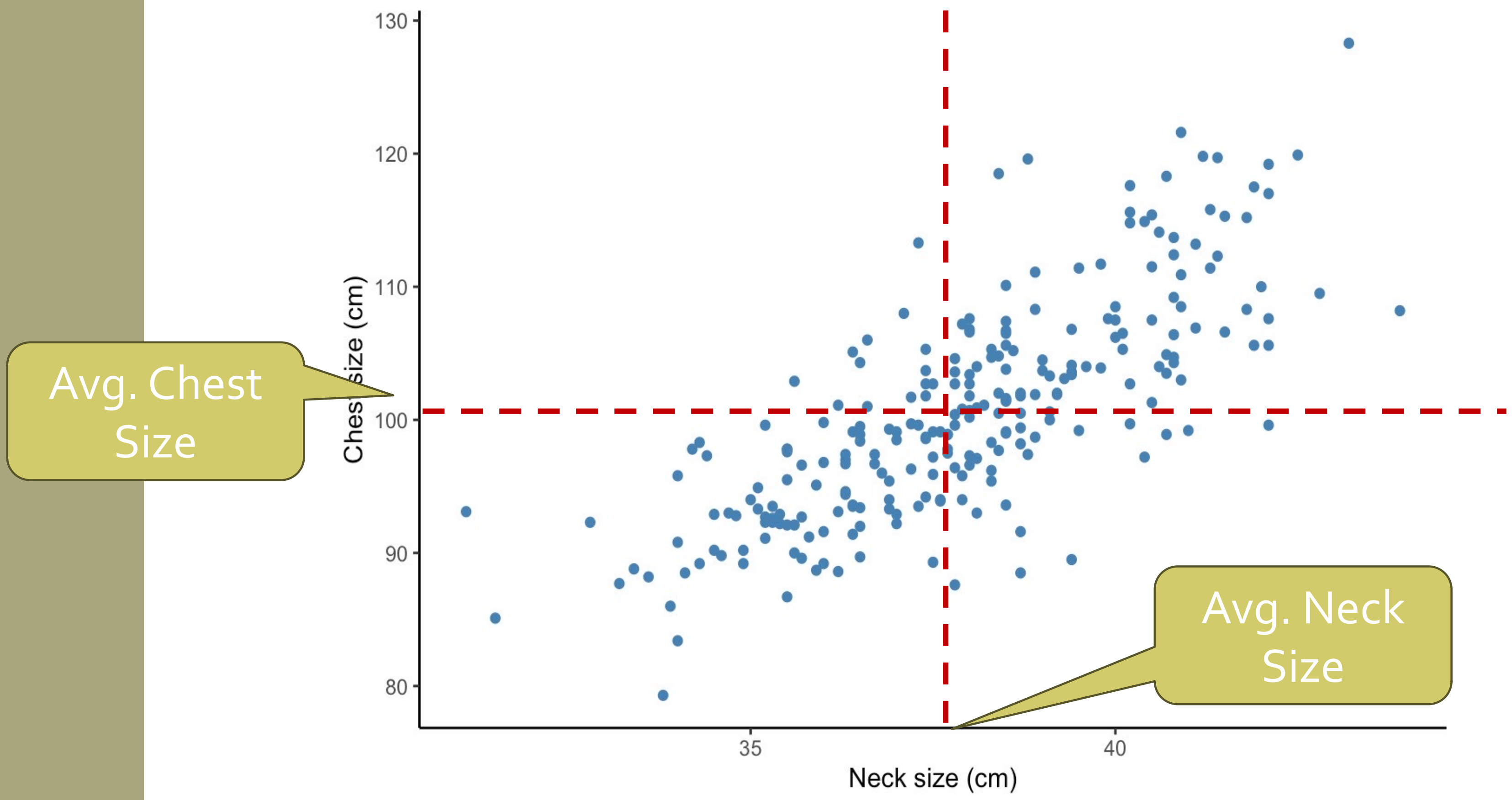

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Does this seem like the best marketing approach? Why or why not?

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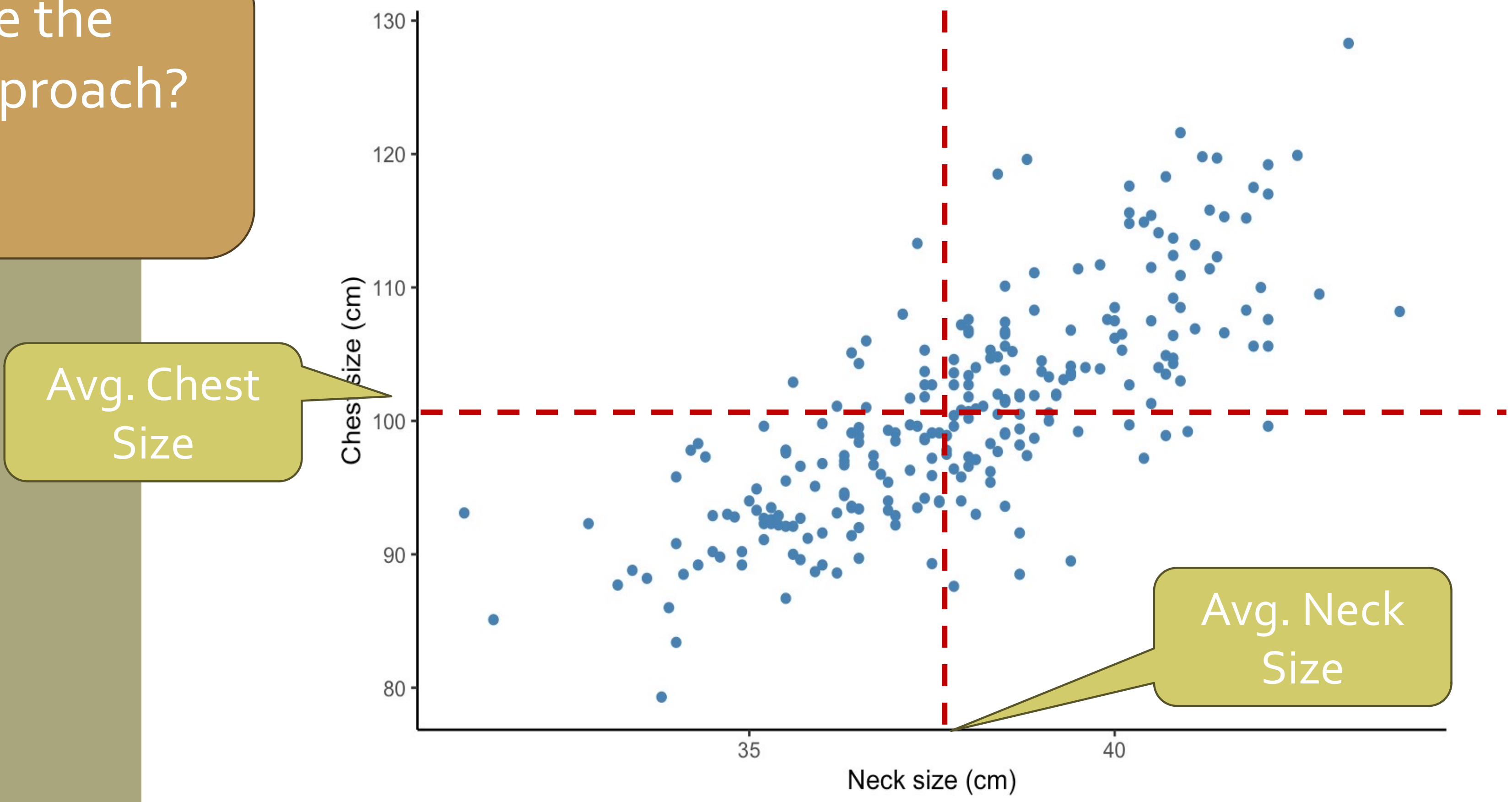

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What might work better?

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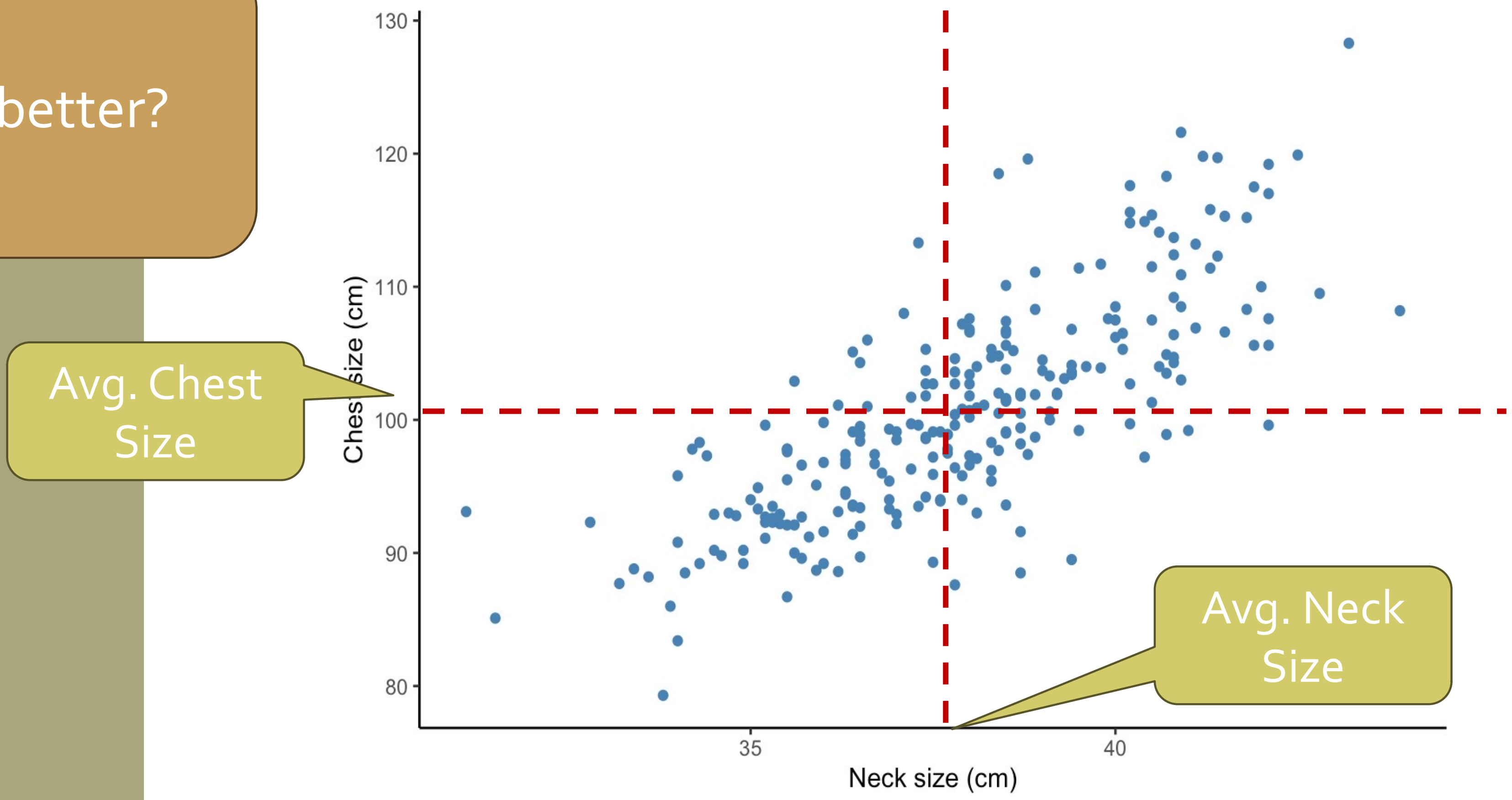

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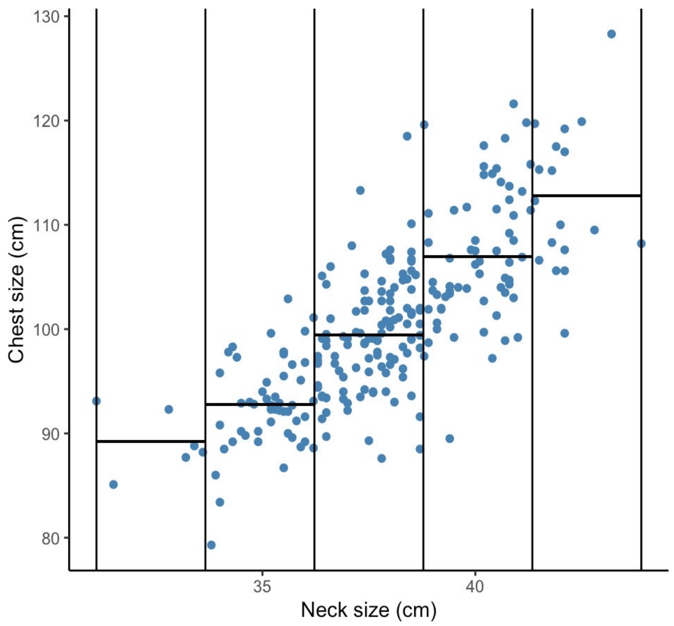
Motivating Example

In order to improve the fit of their shirts (and to allow them to be worn by more individuals!), Jackson & Connor might want to incorporate more information about how average chest sizes varies as a function of neck size:



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XS, S, M, L, XL

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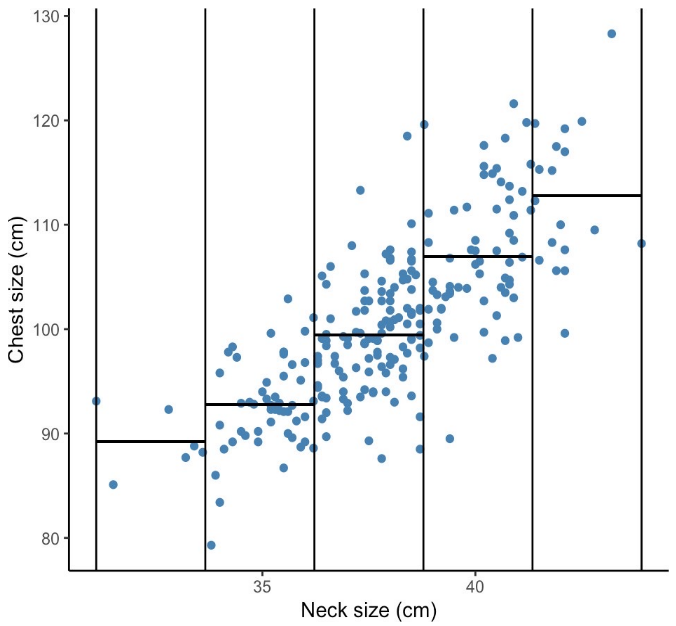

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If you were to improve it, how could you?

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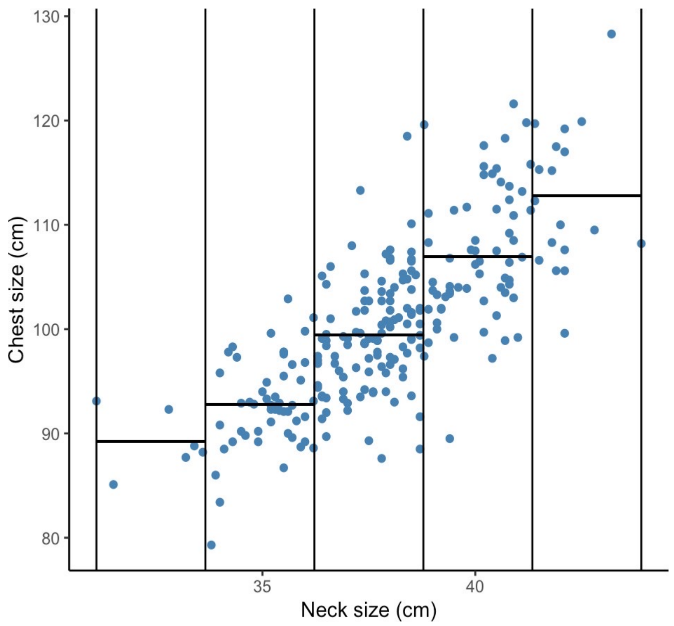

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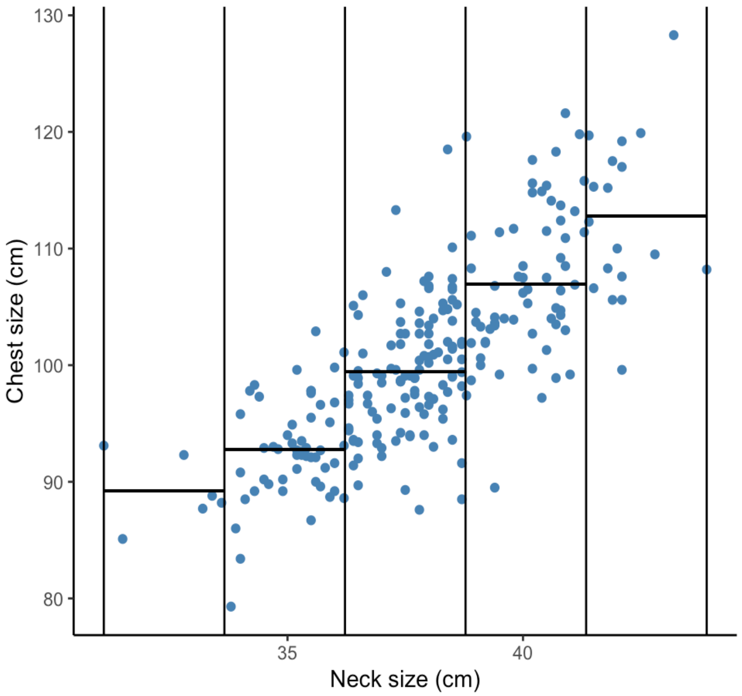

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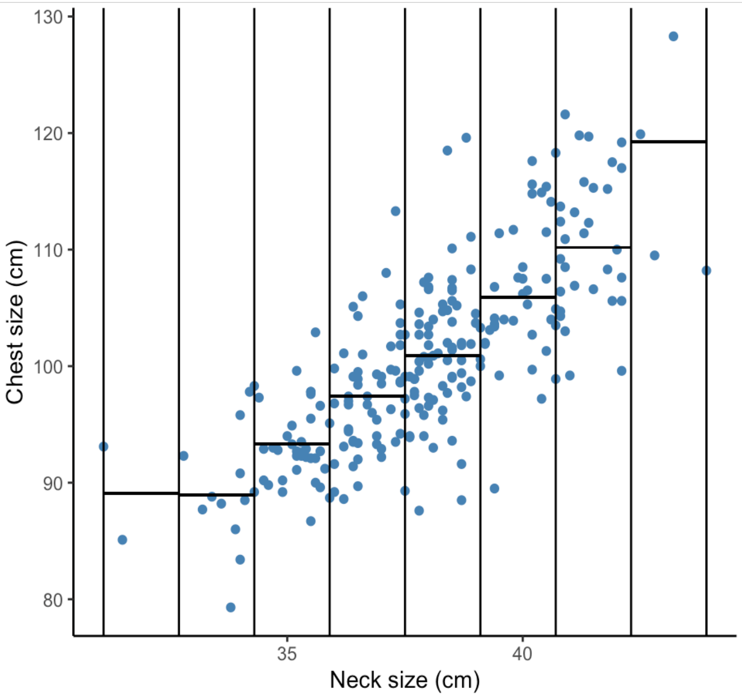

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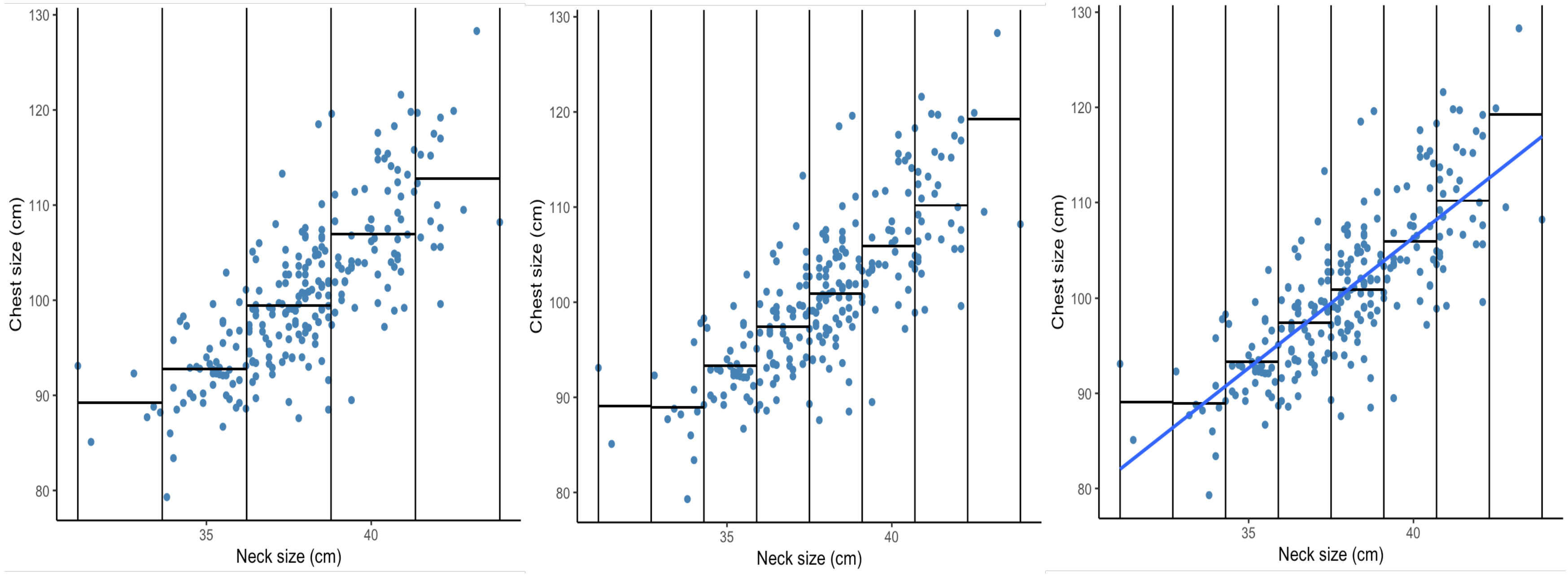

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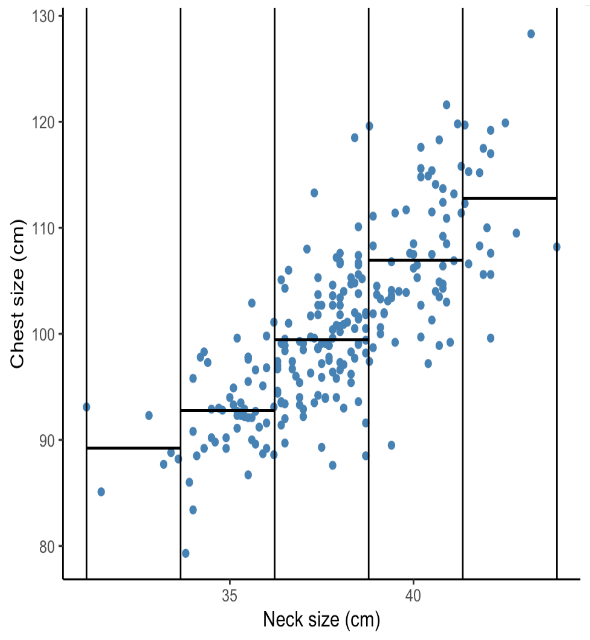

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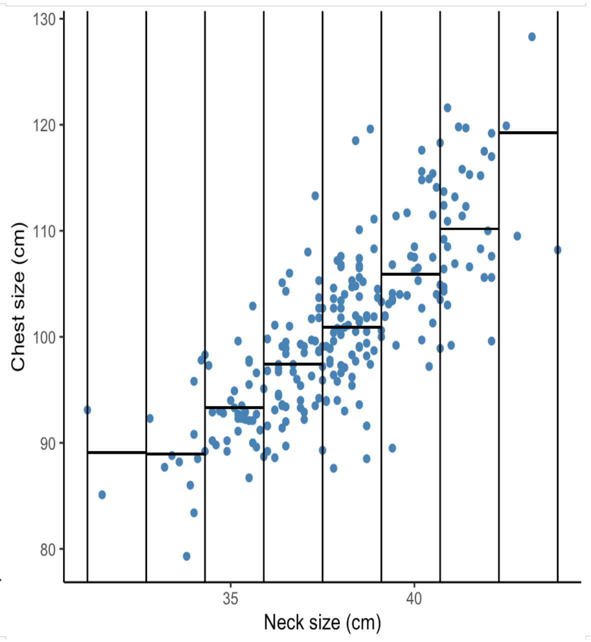

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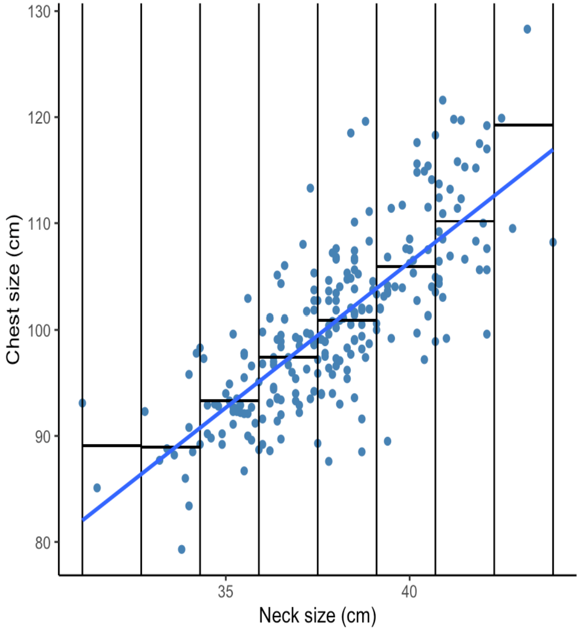

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Linear Regression!

Population Regression Line

Primary Motivation for Regression Modeling

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This line might not perfectly capture everyone in our population, but we assume that—on average—it provides an accurate picture.

Interpreting the Regression Line

Given a linear regression line where Y_i represents an individual's chest size (in cm) and X_i their neck size (in cm):

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

β_0: the intercept term captures the average chest size for the population of individuals who have a neck size of 0 cm

β_1: the slope term captures the expected (average) change in chest size associated with a one centimeter increase in neck size

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Practice: Suppose the regression is:

$$Y_i = 5 + 3X_i$$

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What is the average chest size for someone with a neck size of 0 cm?

How much would you expect chest size to change with a one cm increase in neck size?

What would the chest size for someone with a neck size of 15 cm be?

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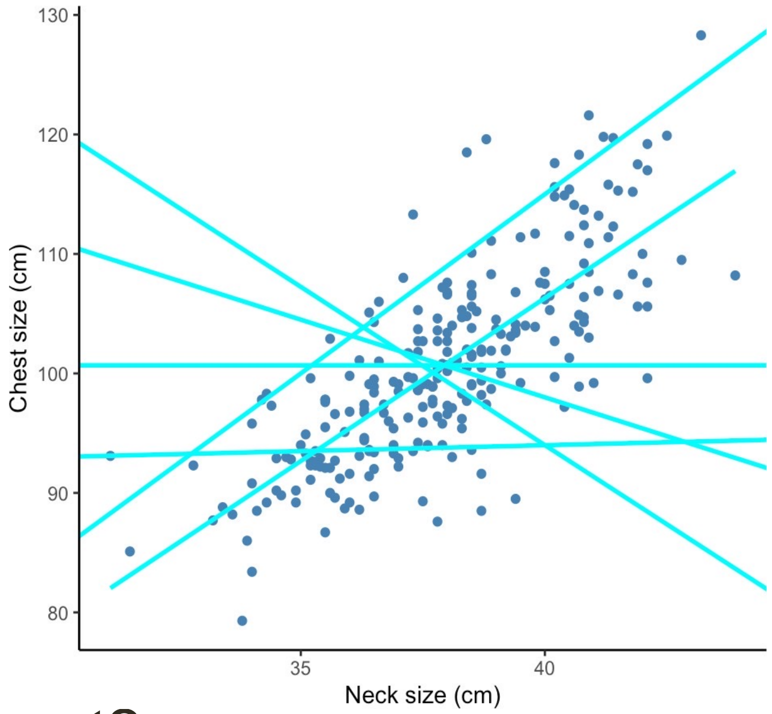
However there are many options…



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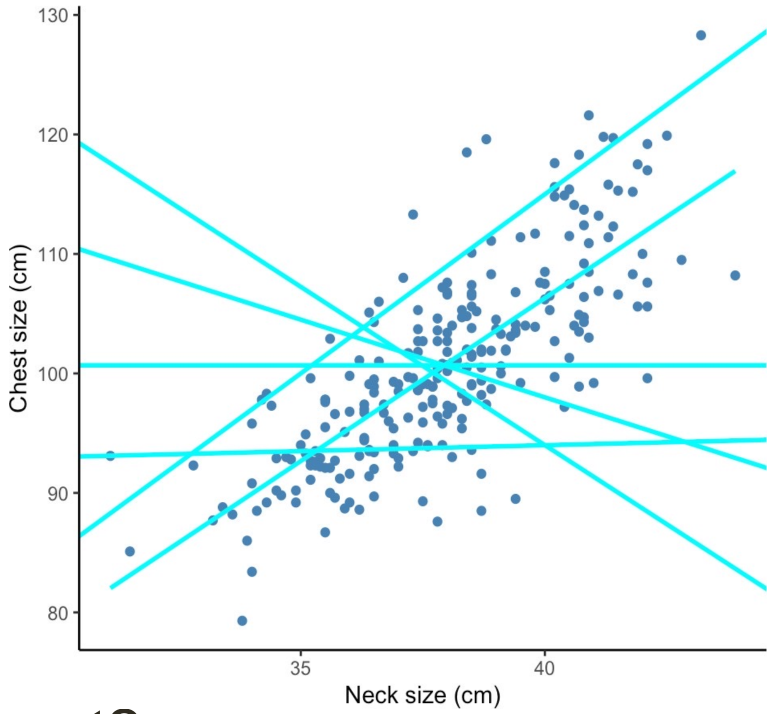

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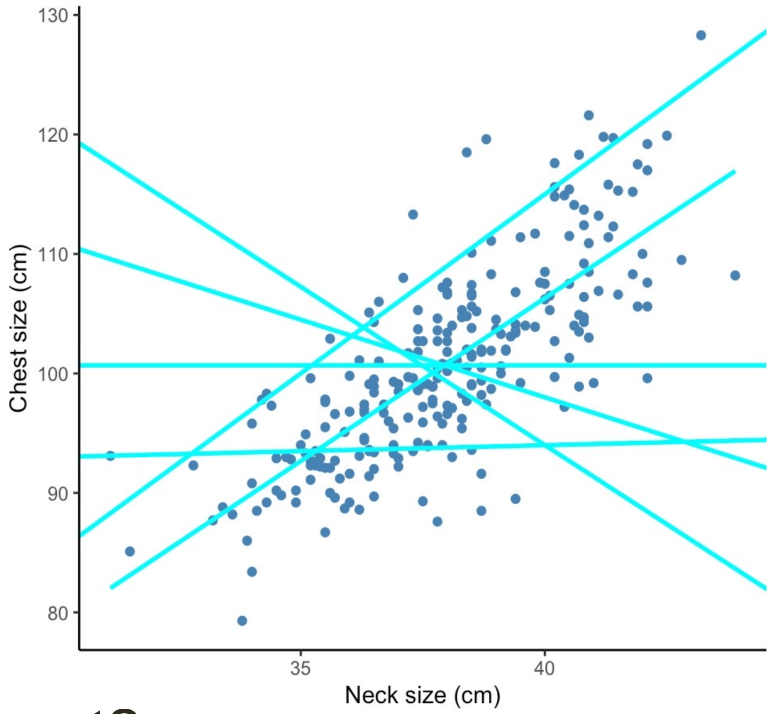

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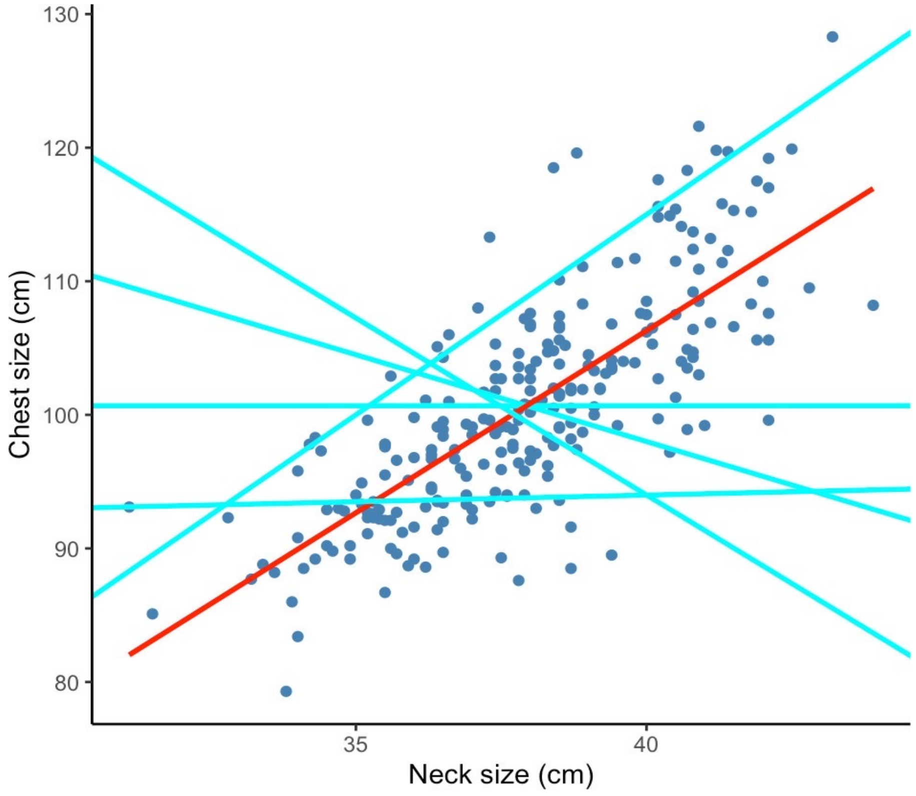
We choose the line that minimizes left over or unexplained variance in Y.



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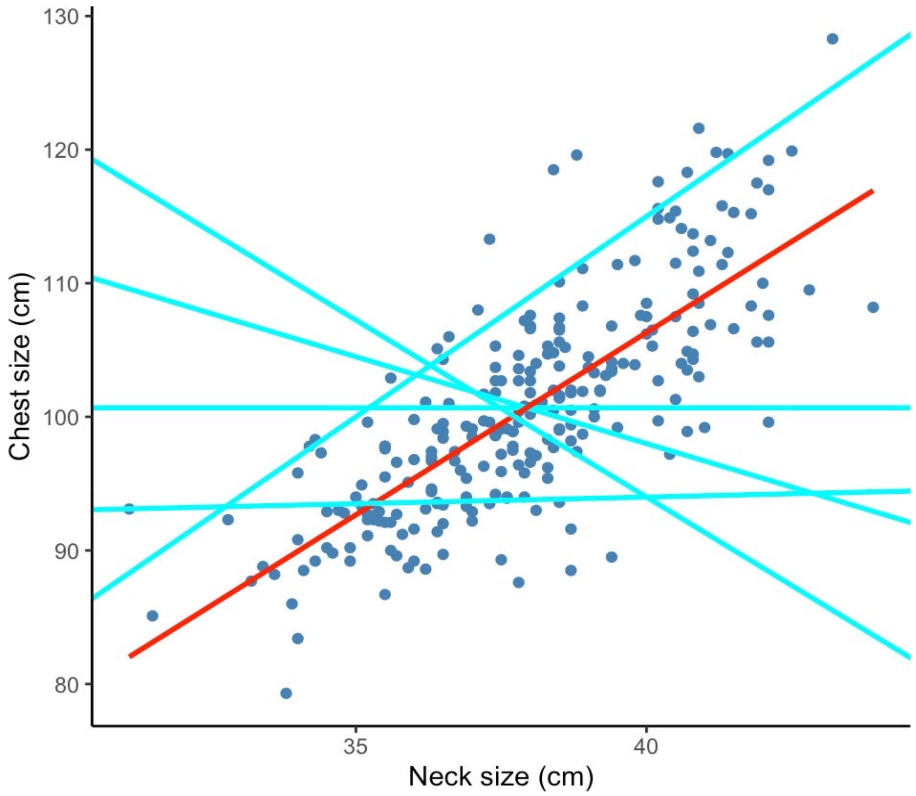
The key insight of linear regression modeling is that we can obtain summary statistics (i.e., “best guesses” or “estimates””) for β_0 and β_1 by fitting a line to our observed data!

We choose the line that minimizes left over or unexplained variance in Y.

This is called the least squares line.



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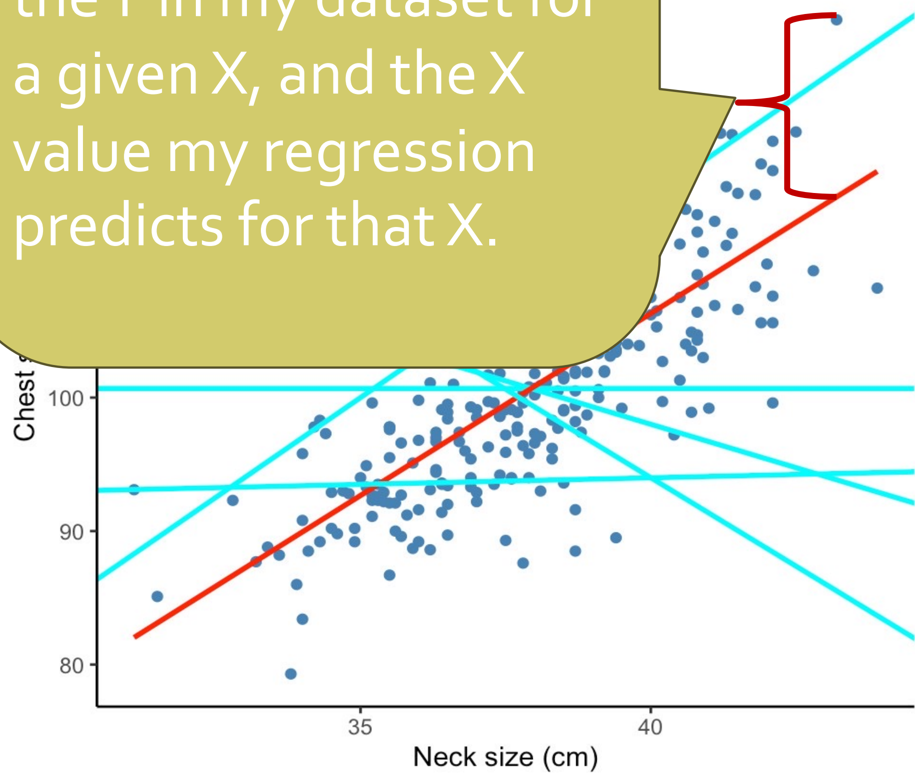

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The big idea behind least squares is that we *minimize* the value we get when we square and sum residuals for every observation in the data.

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