Discrete Structures— Propositional Logic Pt. 1

Dr. Ab Mosca (they/them)

Plan for Today

- Truth conditions for connectives
- Truth Tables
- Logical Equivalence

Warm Up

statement: any declarative sentence which is either true or false

- atomic: a statement that cannot be divided into smaller statements
- molecular: a statement that can be divided into smaller statements

logical connectives:

- conjunction- $P \wedge Q$ (read: P and Q)
- disjunction- P v Q (read: P or Q)
- implication or conditional- $P \rightarrow Q$ (read: if P then Q)
- biconditional- $P \leftrightarrow Q$ (read: P if and only if Q)
- negation- $\neg P$ (read: not P)

Take these two statements: (1) it snowed today, and (2) class is cancelled.

For each logical connective, write a molecular statement made up of these two statements. Write each molecular statement in English and symbolically.

Ex.

It snowed today and class is cancelled (English)

Let P = It snowed today, Q = class is cancelled, $P \land Q$ (symbols)

• All statements have a *truth value*: true (T or 1) or false (F or 0)

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 - (1) the truth values of its parts and
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- Truth conditions:
 - $P \wedge Q$ is true when both P and Q are true.
 - $P \lor Q$ is true when P or Q or both are true.
 - $P \rightarrow Q$ is true when P is false or Q is true or both.
 - $P \leftrightarrow Q$ is true when P and Q are both true or both false.
 - $\neg P$ is true when P is false.

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 - $P \leftrightarrow Q$ is true when P and Q are both true or both false.
 - $\neg P$ is true when P is false.
- Note: This or is called *inclusive or*, there is also an exclusive or which is true when P or Q is true but not when both are true.

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Practice: Let P = It snowed today, Q = class is cancelled, what is the truth value for each of the statements you wrote earlier **based on the truth of P and Q today**.

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 - (1) the truth values of its parts and
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- For any atomic statement there are only two possible truth values, that means when we string statements together with connectives there is a finite number of possible combinations of truth values
- One way we model possible truth values is with truth tables

Let P be a statement. The truth table for P is:



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Let Q be a statement. The truth table for Q is:





When we combine statements with conjunctives, there is a finite number of truth value combinations:

Truth Tables

Let P and Q be statements. One of these must be the case:

P is true and Q is true

P is true and Q is false

P is false and Q is true

P is false and Q is false

When we combine statements with conjunctives, there is a finite number of truth value combinations:

Truth Tables

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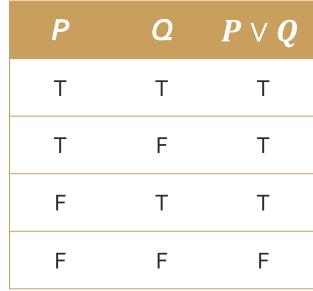
P is false and Q is false

When we use connectives, the truth value of the resulting statement depends on the true values of its parts and the connective. We use truth tables to model this.

Conjunction: $P \wedge Q$ is true when both P and Q are true.

Truth Tables

Disjunction: $P \lor Q$ is true when P or Q or both are true.



Truth Tables

Implication: $P \rightarrow Q$ is true when P is false or Q is true or both.

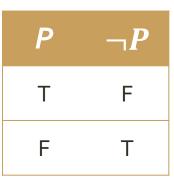
P	Q	$m{P} ightarrow m{Q}$
Т	Т	Т
Т	F	F
F	Т	Т
F	F	Т

Truth Tables

Biconditional: $P \leftrightarrow Q$ is true when P and Q are both true or both false.

P	Q	$P \leftrightarrow Q$
Т	Т	Т
Т	F	F
F	Т	F
F	F	Т

Negation: $\neg P$ is true when P is false.



Practice: Form five groups. Each will be assigned a conjunction. Come up with a real world scenario that your conjunction accurately represents. Be ready to share your answers!

Ex. If I had conjunction and were one of you, I might say: Let P = I study discrete math, and Q = I am taking MATH220

The truth table and real work scenarios are:

Р	Q	$P \wedge Q$	
Т	Т	Т	I study discrete math AND I am taking MATH220.
Т	F	F	I study discrete math AND I am not taking MATH220.
F	Т	F	I do not study discrete math AND I am taking MATH220.
F	F	F	I do not study discrete math AND I am not taking MATH220.

Tru

Truth tables are helpful for deducing when these larger statements are true and false.

Ex. When is $\neg P \lor Q$ true?

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Truth Tables

Ex. When is $\neg P \lor Q$ true?

P	Q -
Т	Т
Т	F
F	Т
F	F

Start the table with all possible combinations of T/F for smallest statements

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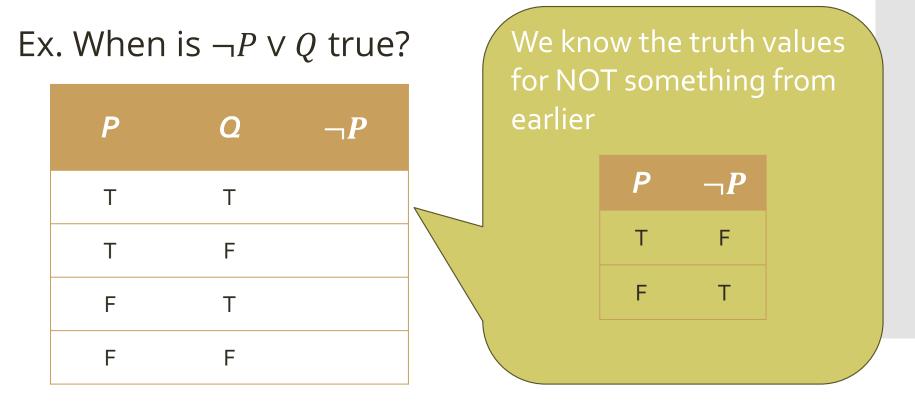
Truth Tables

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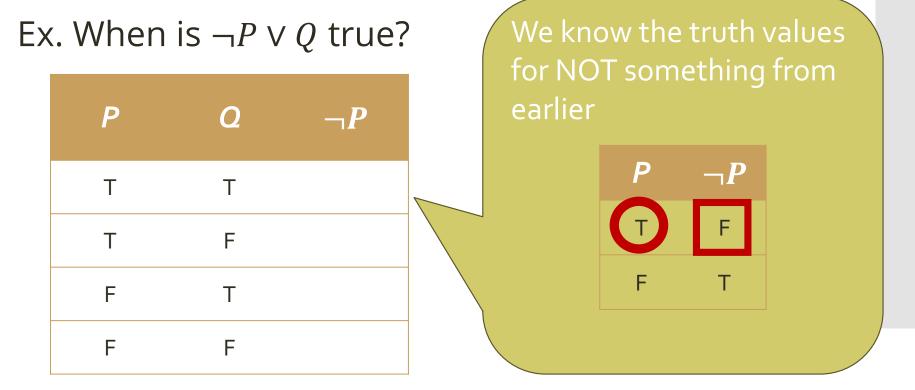
P	Q	$\neg P$
Т	Т	
Т	F	
F	Т	
F	F	

Build up the truth table with additional columns for slightly larger statements

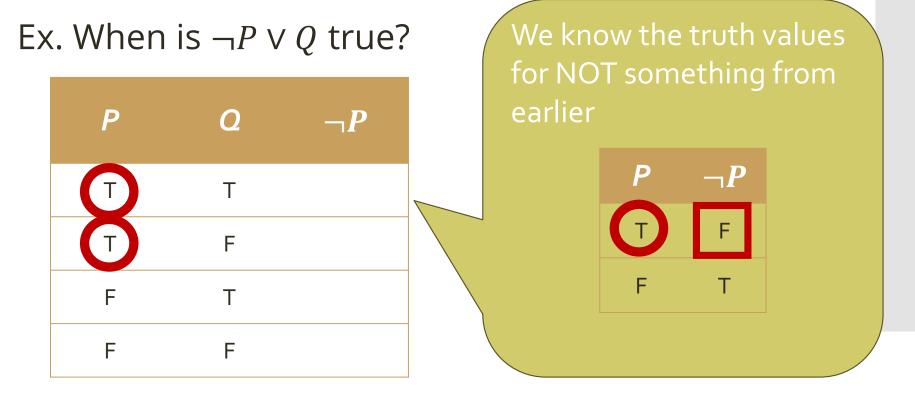
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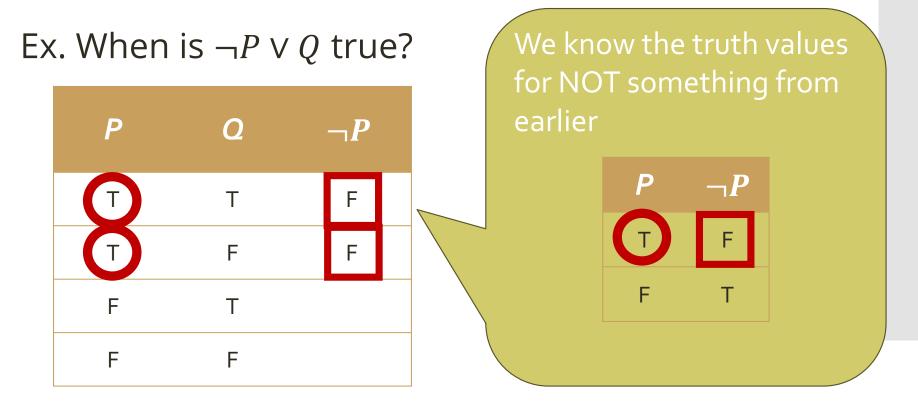
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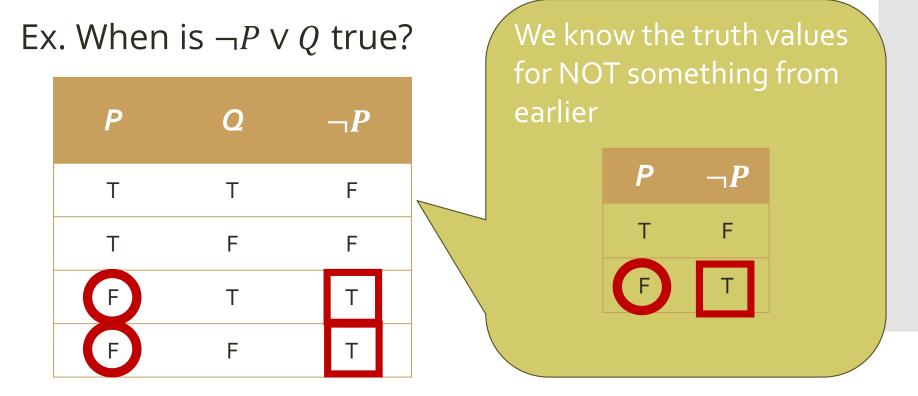
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Ex. When is $\neg P \lor Q$ true?

P	Q	$\neg P$
Т	Т	F
Т	F	F
F	Т	Т
F	F	Т

Larger statements can be made up of more than two smaller parts and/or more than one conjunction.

Truth tables are helpful for deducing when these larger statements are true and false.

Ex. When is $\neg P \lor Q$ true?

Build up the truth table with additional columns for larger statements

P	Q	$\neg P$	$\neg P \lor Q$
Т	Т	F	
Т	F	F	
F	Т	Т	
F	F	Т	

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Ex. When is $\neg P \lor Q$ true?

P	Q	$\neg P$	$\neg P \lor Q$
Т	Т	F	
Т	F	F	
F	Т	Т	
F	F	Т	

P	Q	$P \lor Q$
Т	Т	Т
Т	F	Т
F	Т	Т
F	F	F

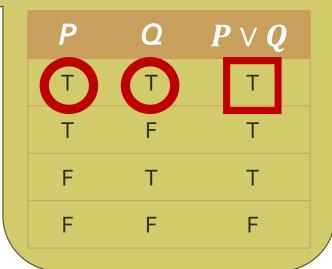
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Т	Т	F	
Т	F	F	
F	Т	Т	
F	F	Т	

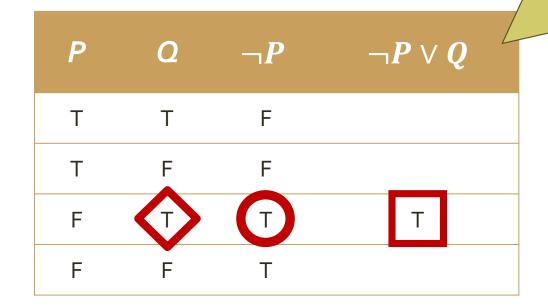


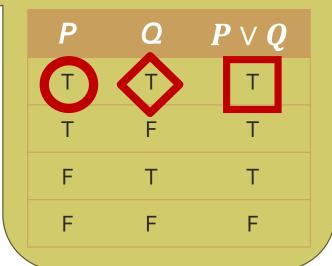
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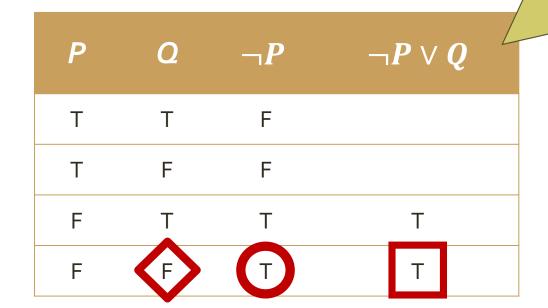


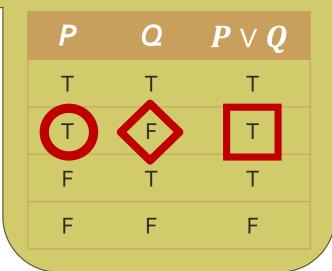
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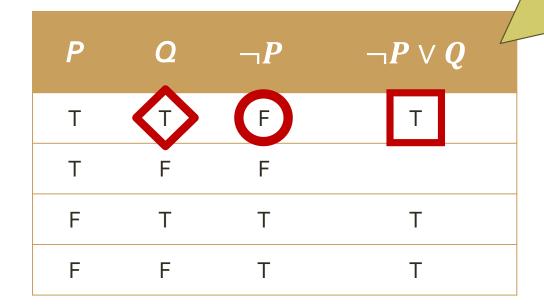


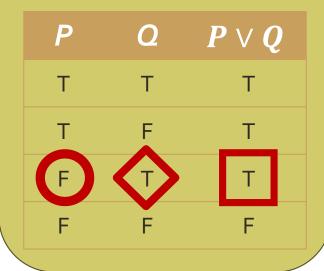
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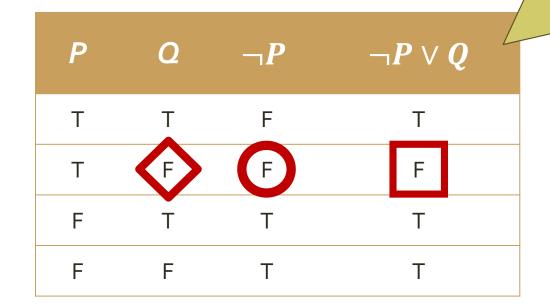
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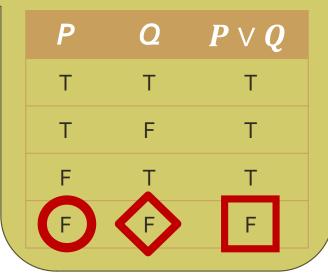
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statements are true and false.

Ex. When is $\neg P \lor Q$ true?



We know the truth values for something OR something from earlier



Practice: Finish the truth table for the statement $(P \rightarrow Q) \lor (Q \rightarrow R)$

Truth Tables

Р	Q	R
Т	Т	Т
Т	Т	F
Т	F	Т
Т	F	F
F	Т	Т
F	Т	F
F	F	Т
F	F	F

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Truth Table

Р	Q	R
Т	Т	Т
Т	Т	F
Т	F	Т
Т	F	F
F	Т	Т
F	Т	F
F	F	Т
F	F	F

Note: $(P \to Q) \lor (\overline{Q \to R})$ is a **tautology**, a statement that is always true.

In other words, *P* and *Q* are *logically equivalent* if they have the same truth value for any assignment of truth values to their atomic parts.

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Logical Equivalence

P	Q	$\neg P$	$\neg P \lor Q$
Т	Т	F	Т
Т	F	F	F
F	Т	Т	Т
F	F	Т	Т

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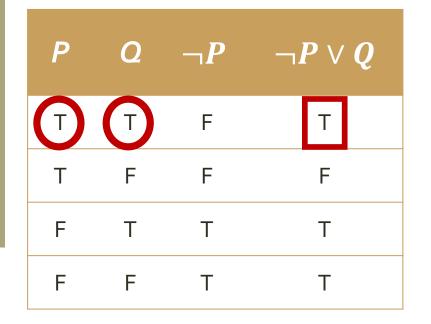
Logical Equivalence

P	Q	$\neg P$	$\neg P \lor Q$
Т	Т	F	Т
Т	F	F	F
F	Т	Т	Т
F	F	Т	Т

P	Q	$m{P} ightarrow m{Q}$
Т	Т	Т
Т	F	F
F	Т	Т
F	F	Т

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Logical Equivalence



P	Q	P o Q
T	T	Т
Т	F	F
F	Т	Т
F	F	Т

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Logical Equivalence

P	Q	$\neg P$	$\neg P \lor Q$
Т	Т	F	Т
T	F	F	F
F	Т	Т	Т
F	F	Т	Т

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Т	Т	Т
T	F	F
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Т	Т	F	Т
Т	F	F	F
F	T	Т	Т
F	F	Т	Т

P	Q	$m{P} ightarrow m{Q}$
Т	Т	Т
T	F	F
F	T	Т
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P	Q	$\neg P$	$\neg P \lor Q$
Т	Т	F	Т
Т	F	F	F
F	Т	Т	Т
F	F	Т	Т

P	Q	P o Q
Т	Т	Т
Т	F	F
F	Т	Т
F	F	Т

Practice: Show that $\neg (P \lor Q)$ is logically equivalent to $\neg P \land \neg Q$

In other words, *P* and *Q* are *logically equivalent* if they have the same truth value for any assignment of truth values to their atomic parts.

Logical Equivalence

P	Q	$\neg P$	$\neg P \lor Q$
Т	Т	F	Т
Т	F	F	F
F	Т	Т	Т
F	F	Т	Т

P	Q	$m{P} ightarrow m{Q}$
Т	Т	Т
Т	F	F
F	Т	Т
F	F	Т