

# Discrete Structures— Graphs: BFS and DFS

Dr. Ab Mosca (they/them)

# Plan for Today

- Trees
  - Breadth first search (BFS)
  - Depth first search (DFS)
- Algorithm Analysis

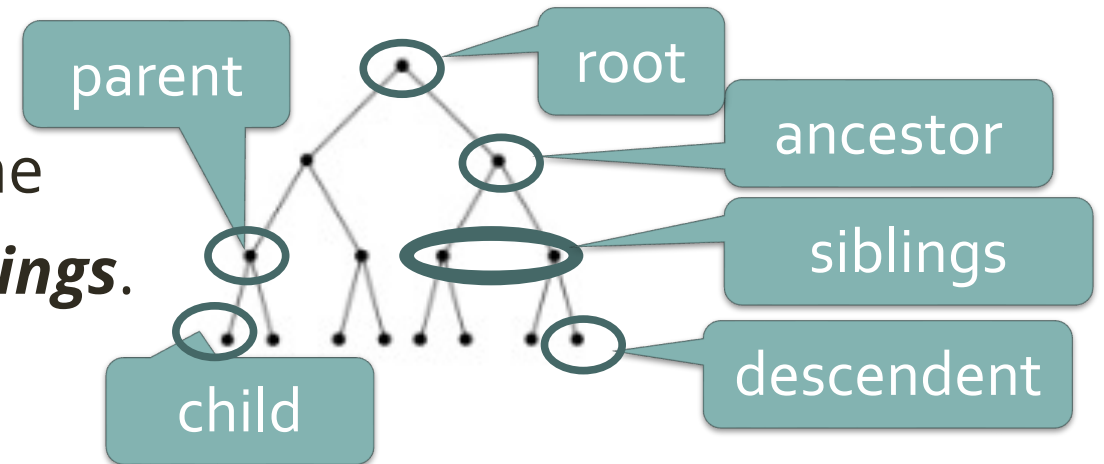
## Warm Up: Rooted Trees

We can identify one vertex in a tree as the **root**. Then, every other vertex on the tree can be characterized by its position relative to the root.

If two vertices are adjacent, we say the one closer to the root is the **parent**, and the other is the **child**.

In general, we say a vertex,  $v$ , is a **descendent** of a vertex,  $u$ , provided  $u$  is a vertex on the path from  $v$  to the root. Then, we would call  $u$  an **ancestor** of  $v$ .

Vertices with the same parent are called **siblings**.



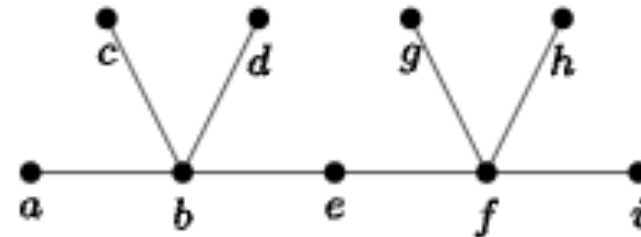
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Let  $e$  be the root. Label the other vertices.

# Navigating Trees

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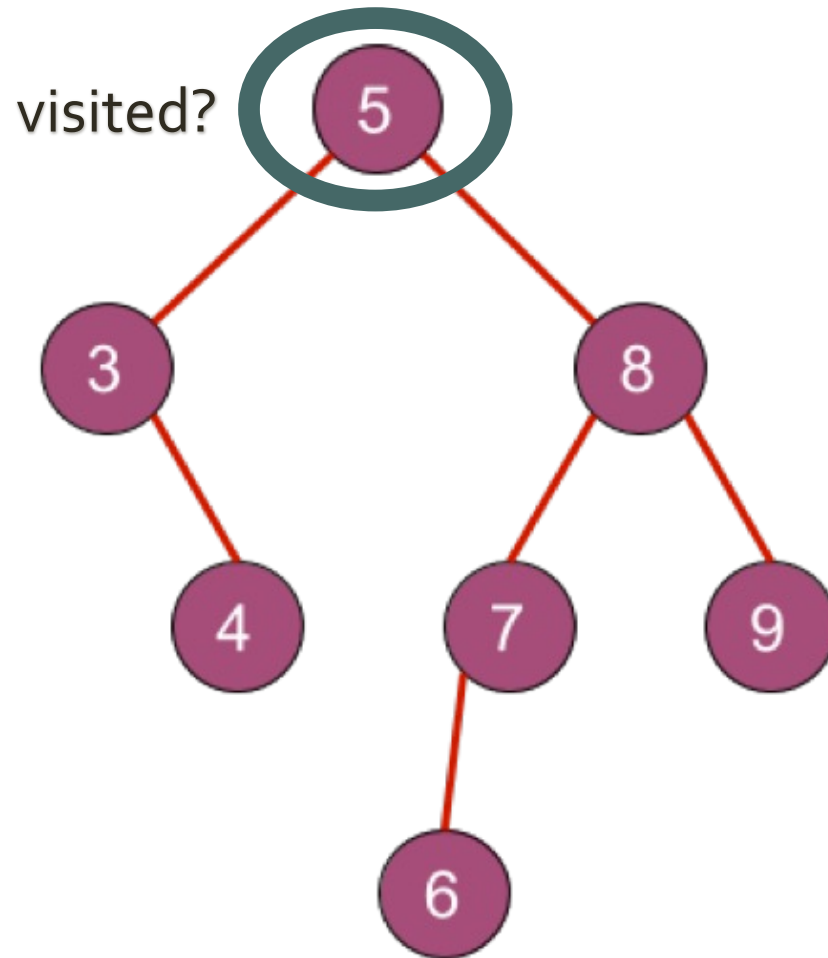
There are two common algorithms for obtaining these goals: breadth first search (BFS), and depth first search (DFS).

# BFS

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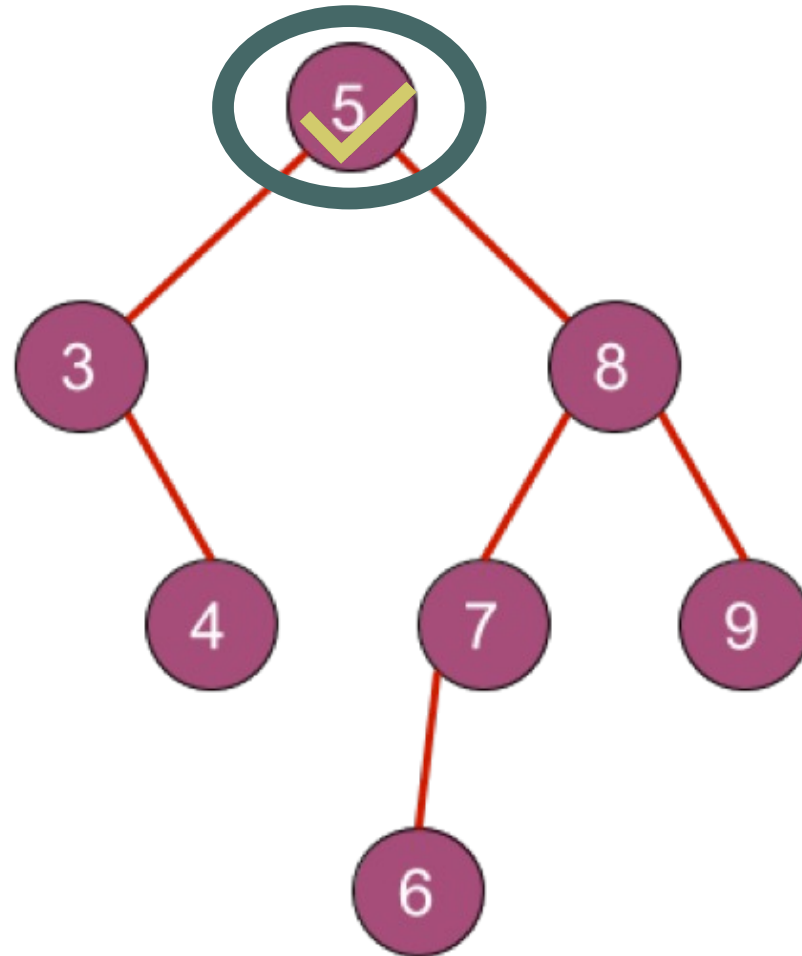
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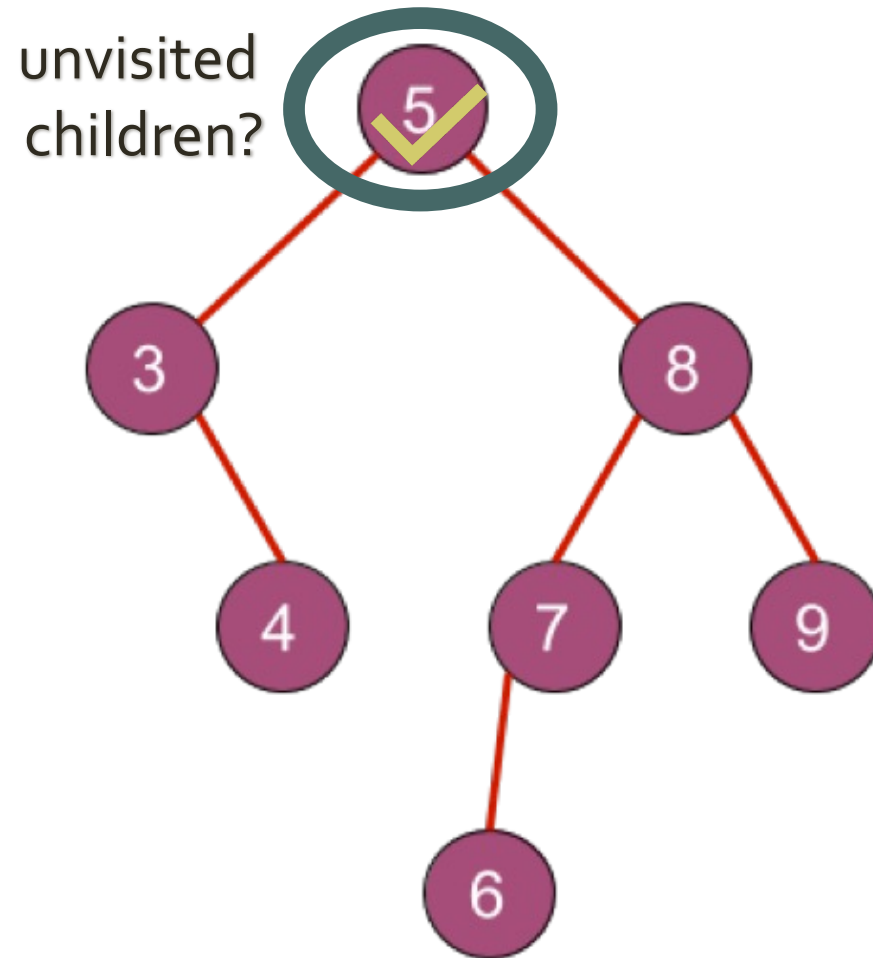
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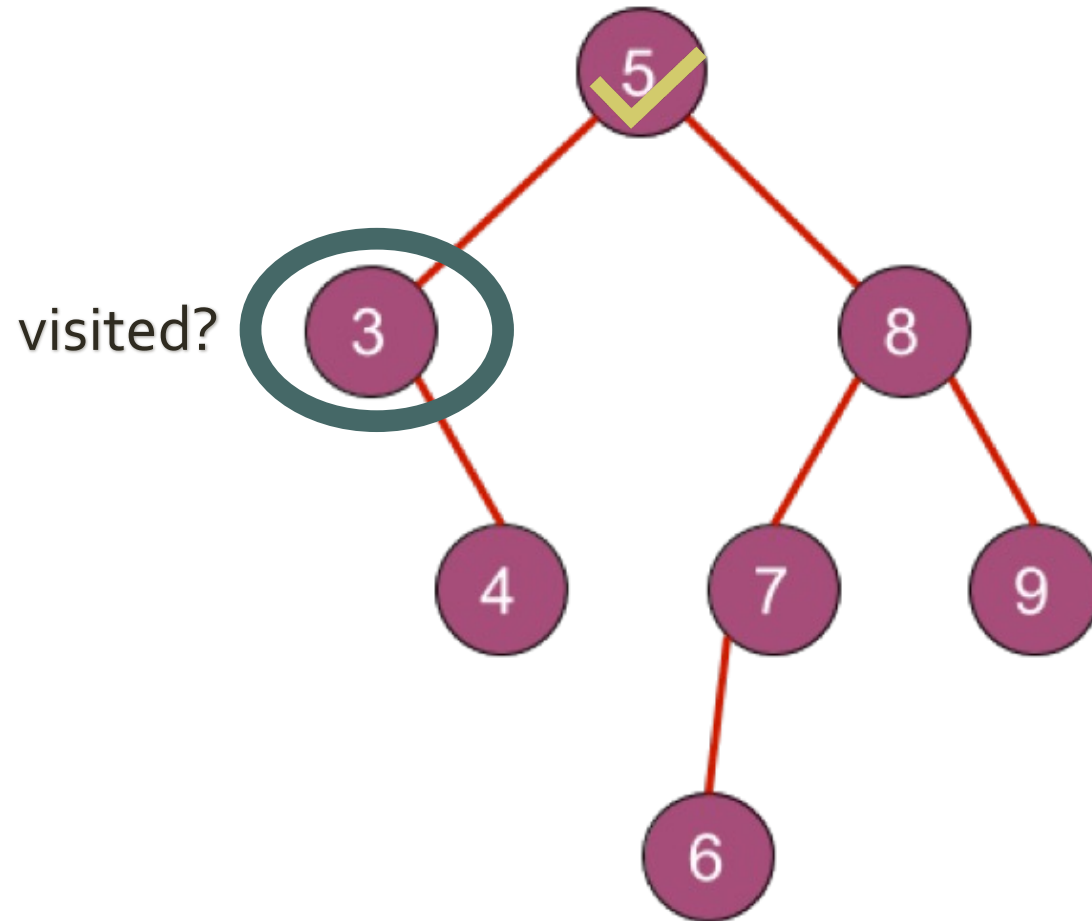
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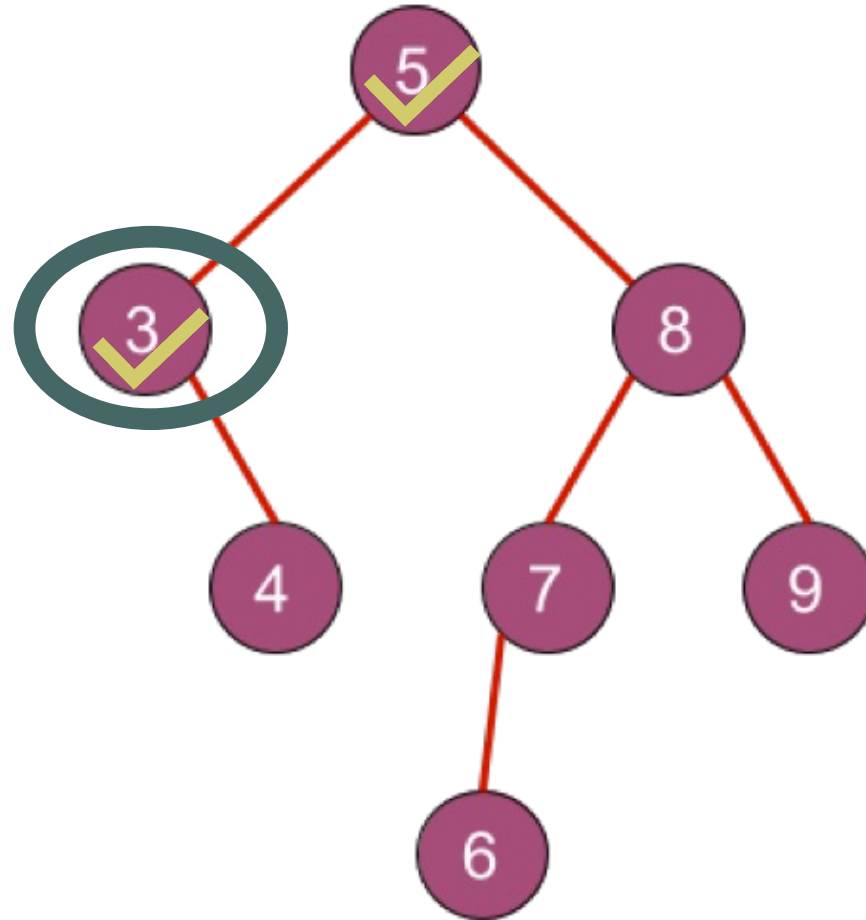
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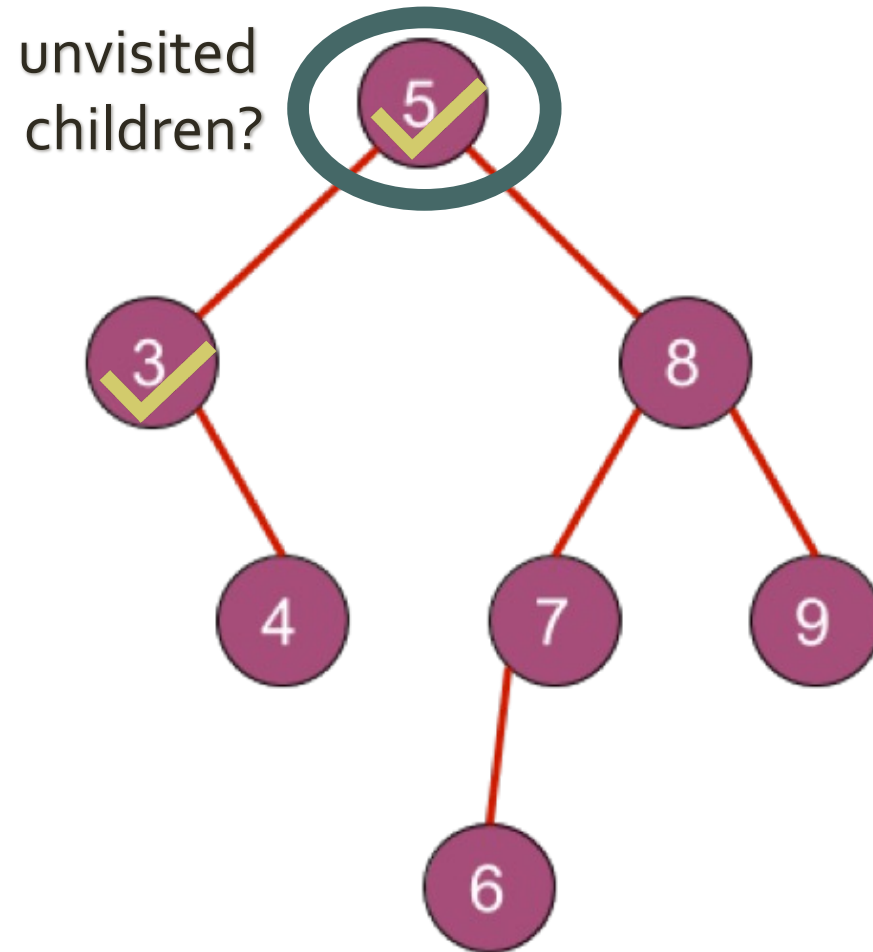
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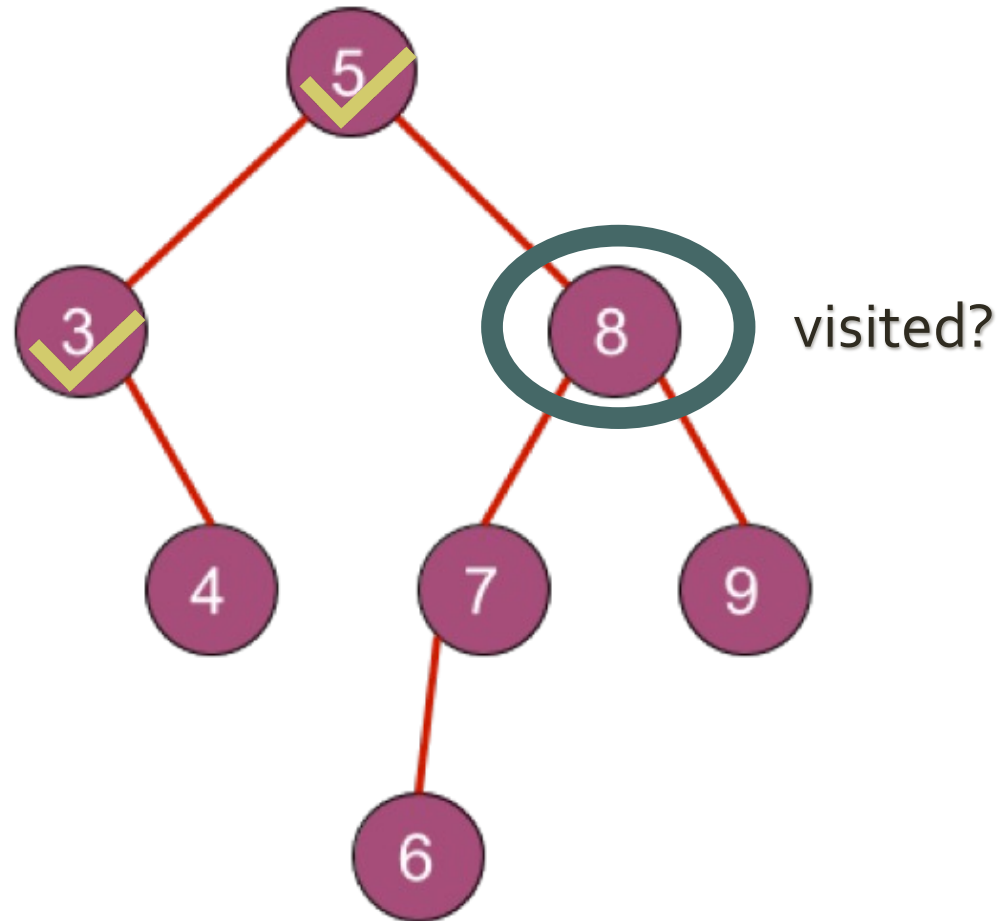
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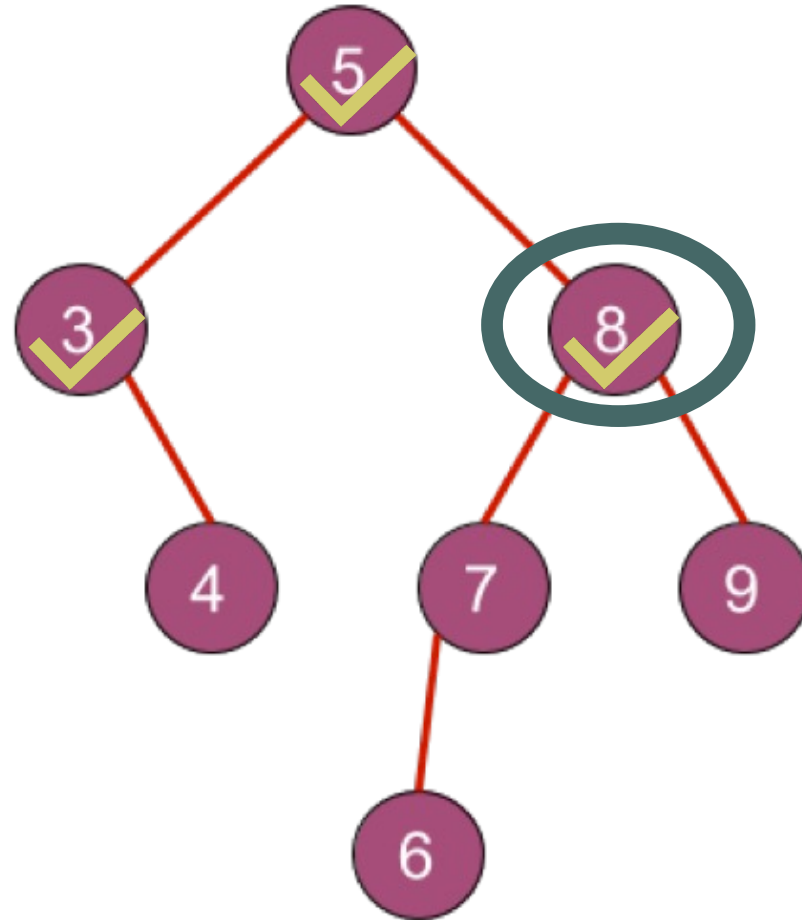
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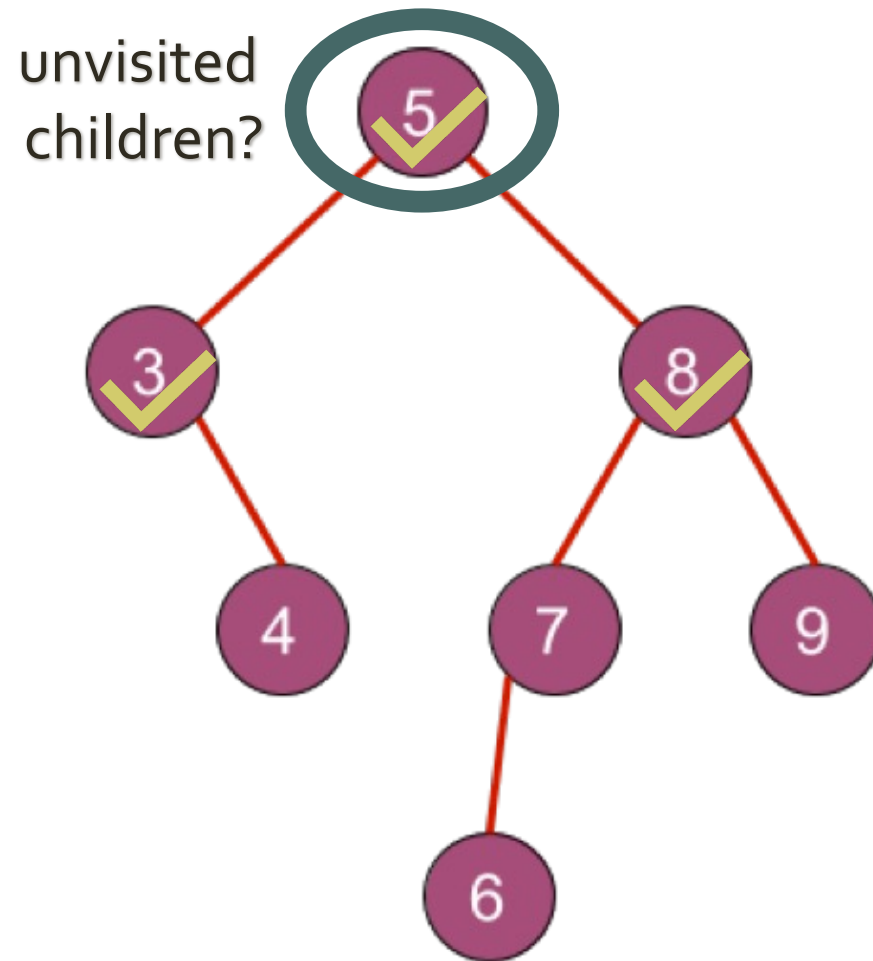
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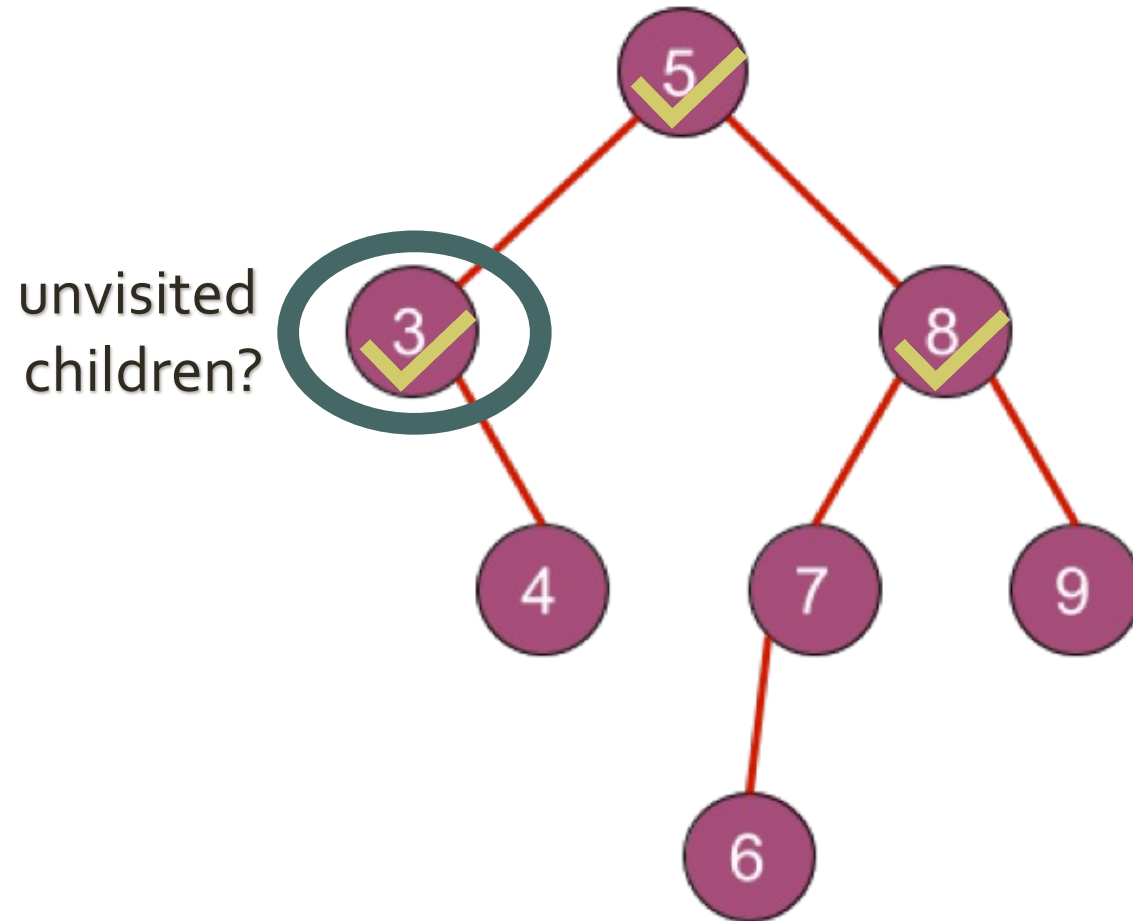
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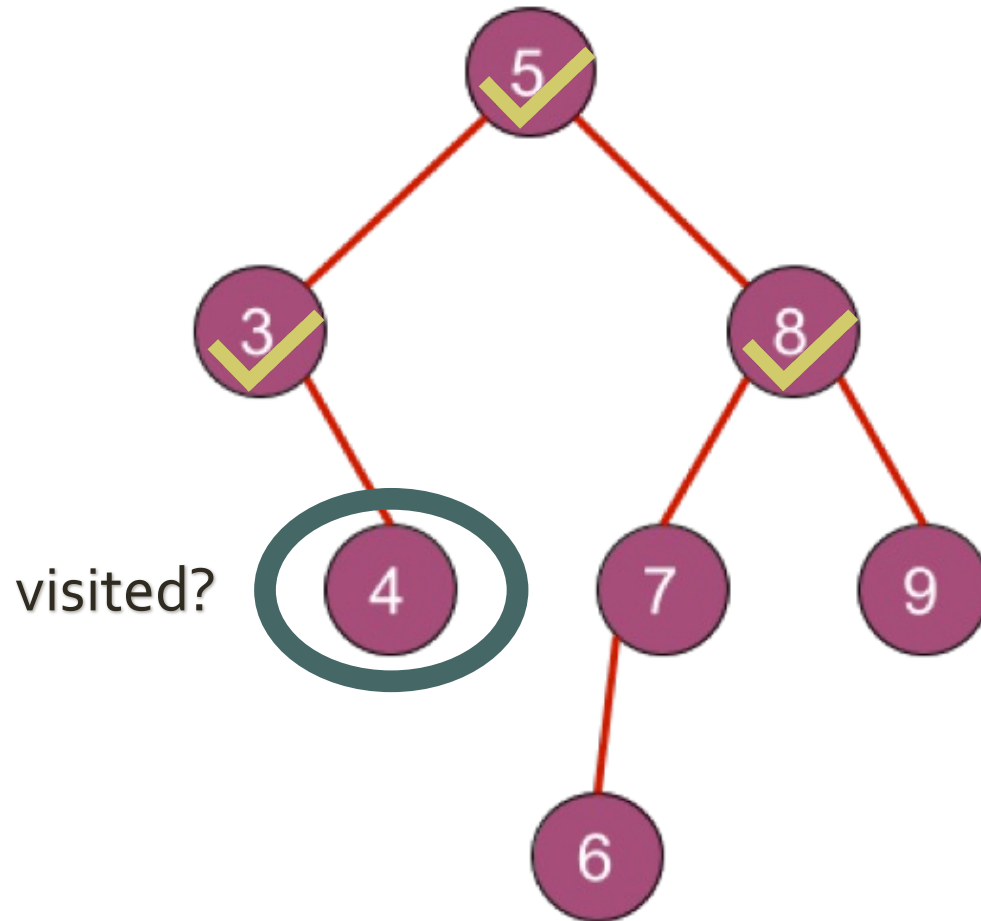
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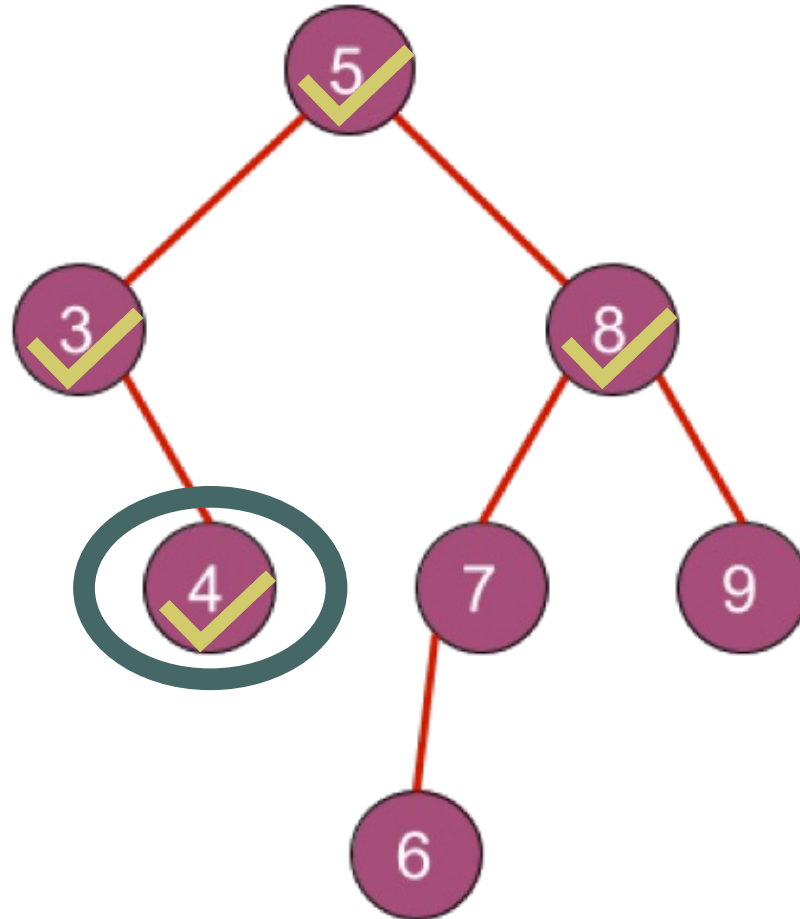
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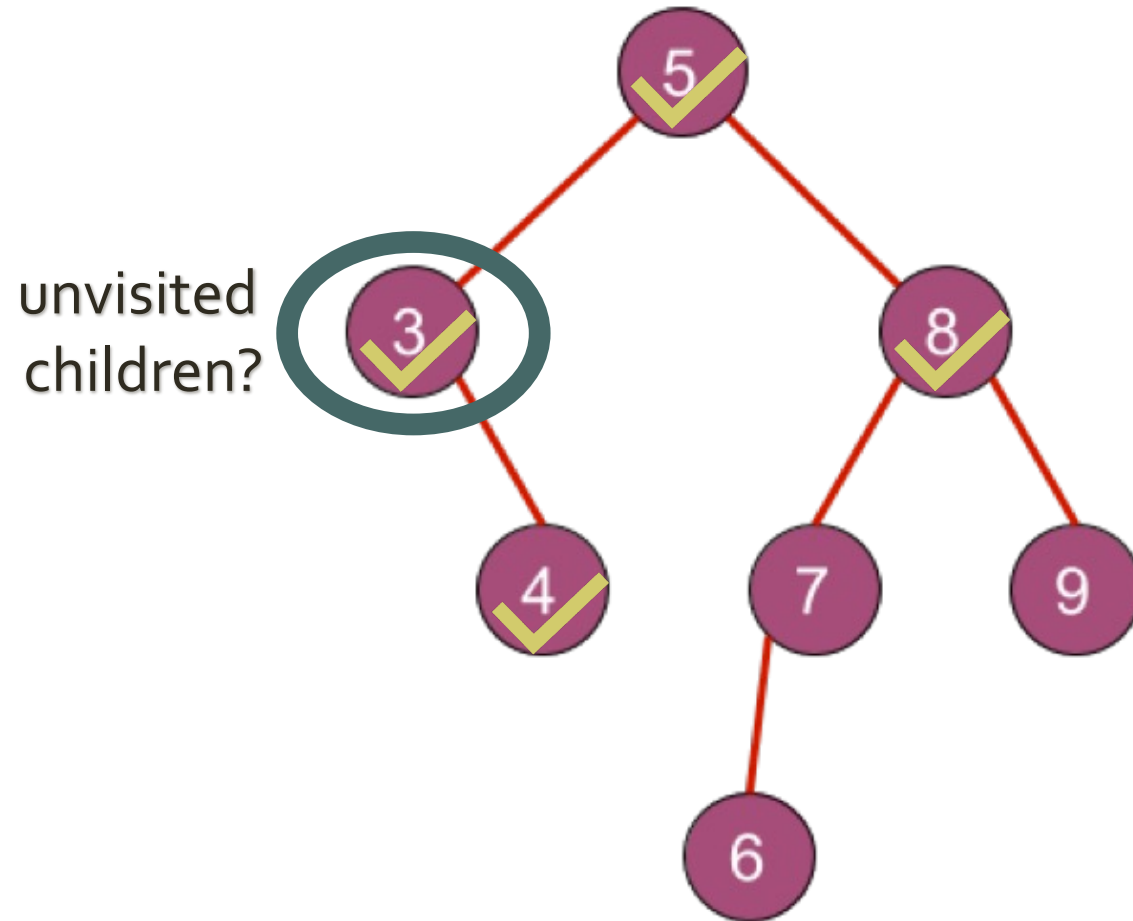
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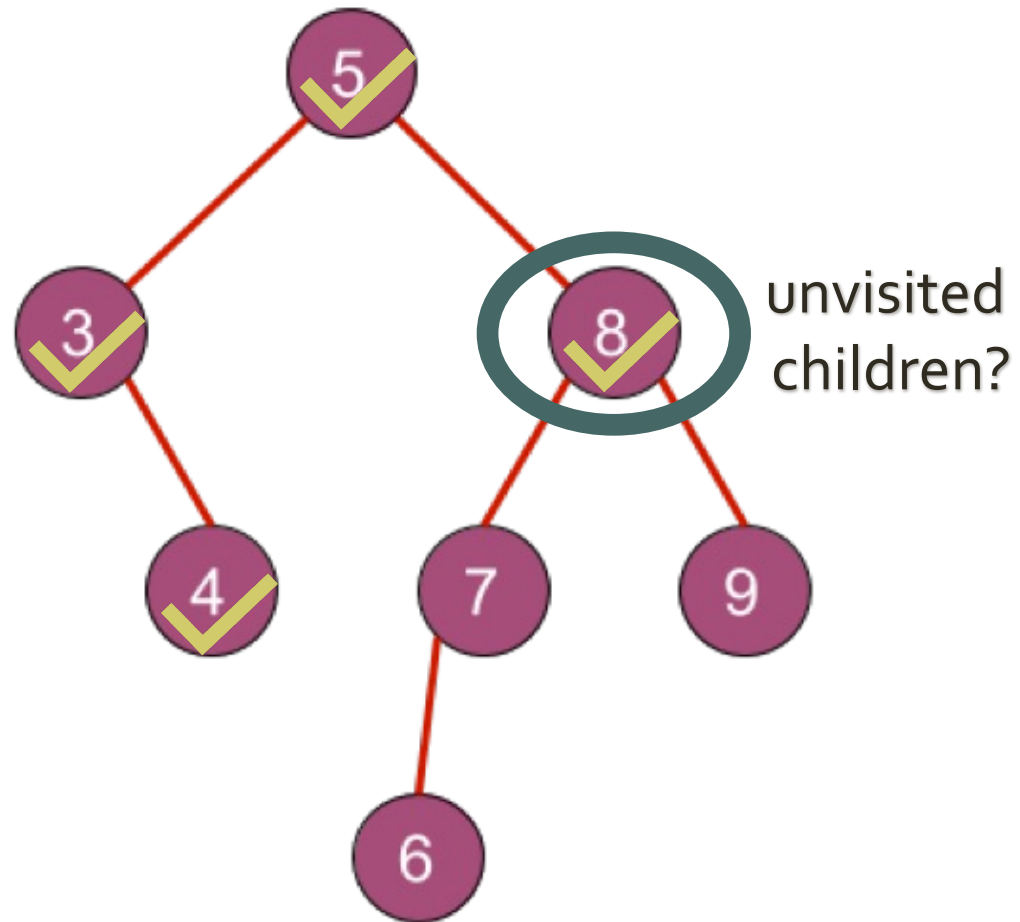
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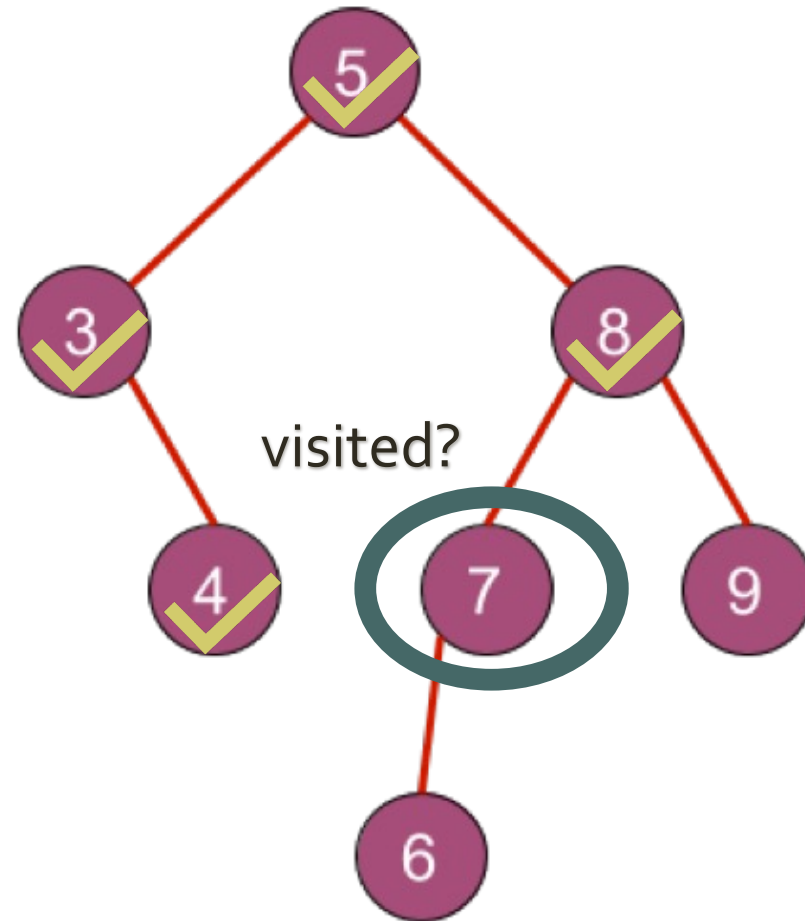
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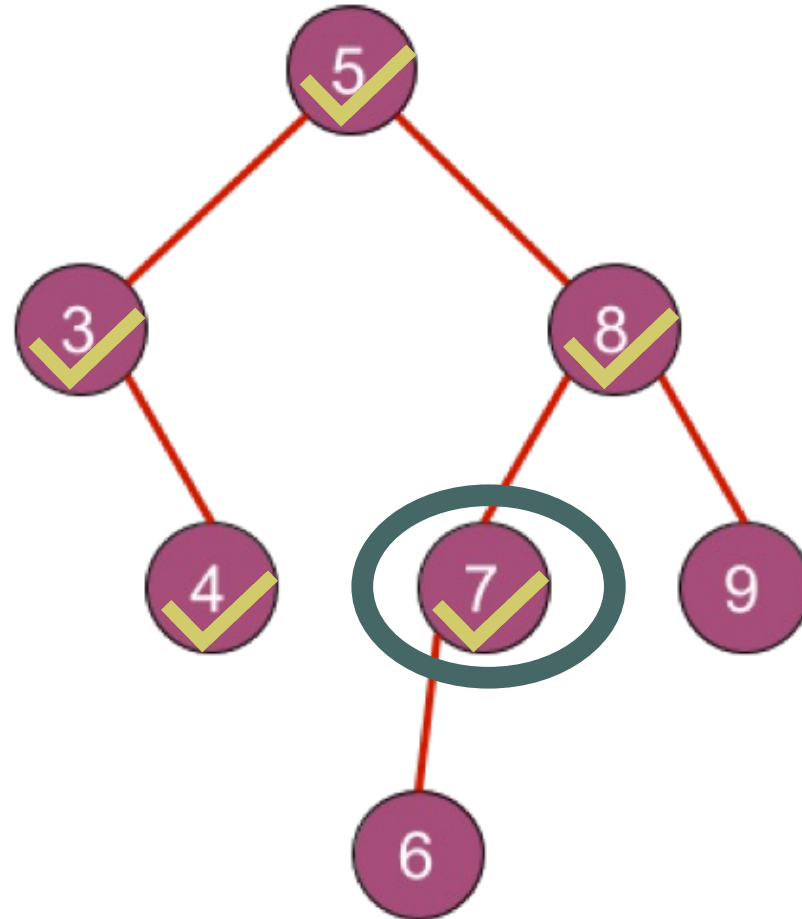
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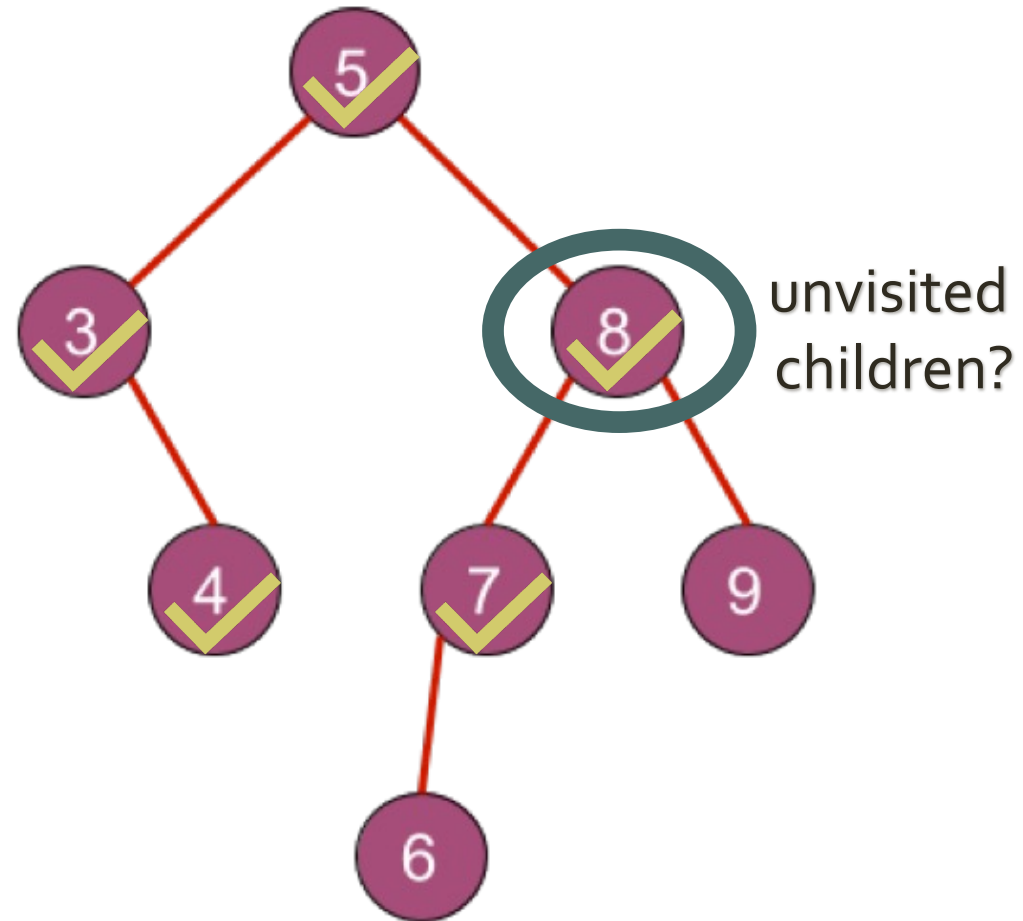
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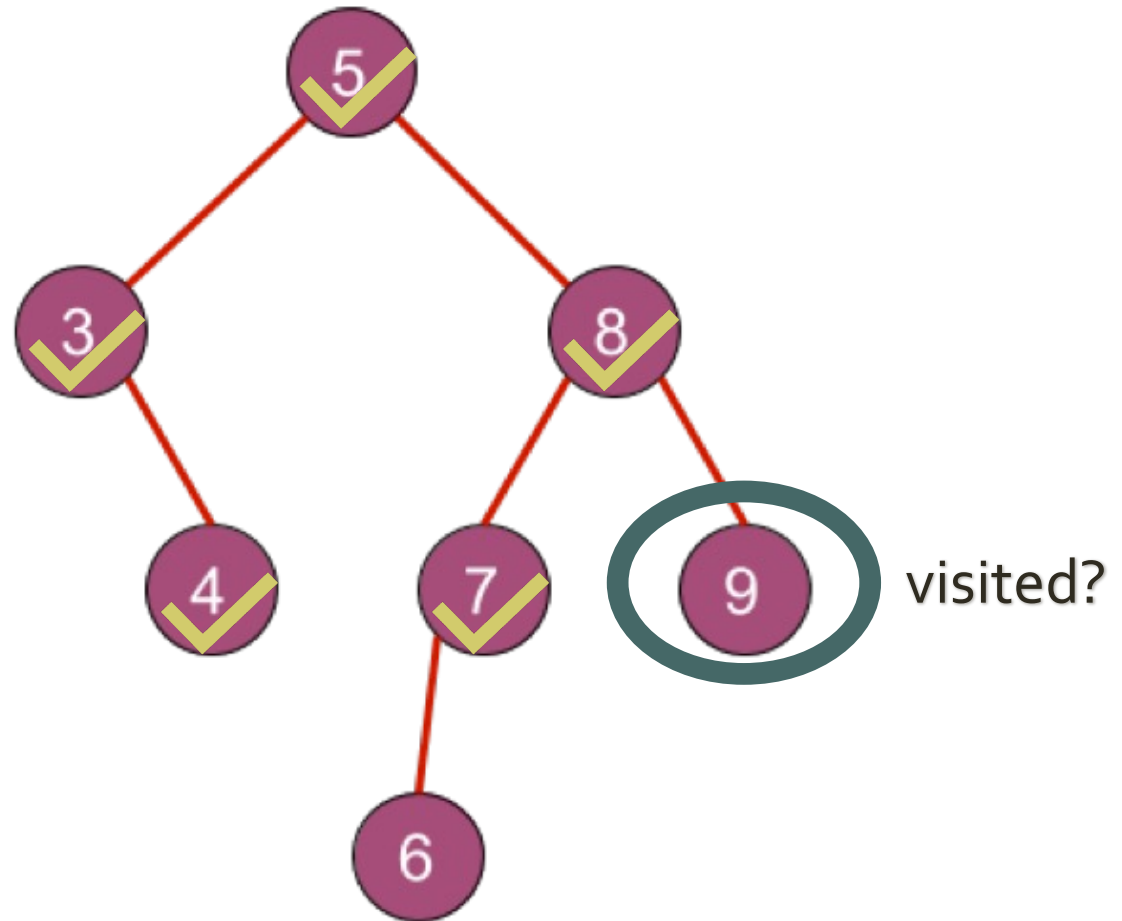
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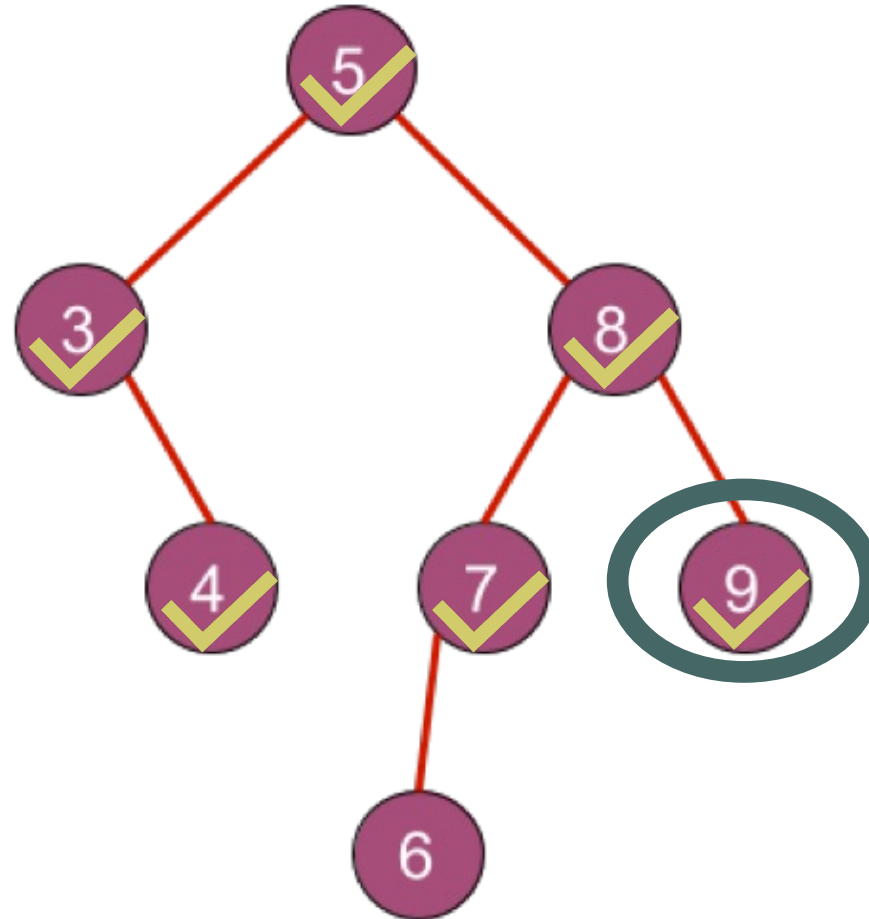
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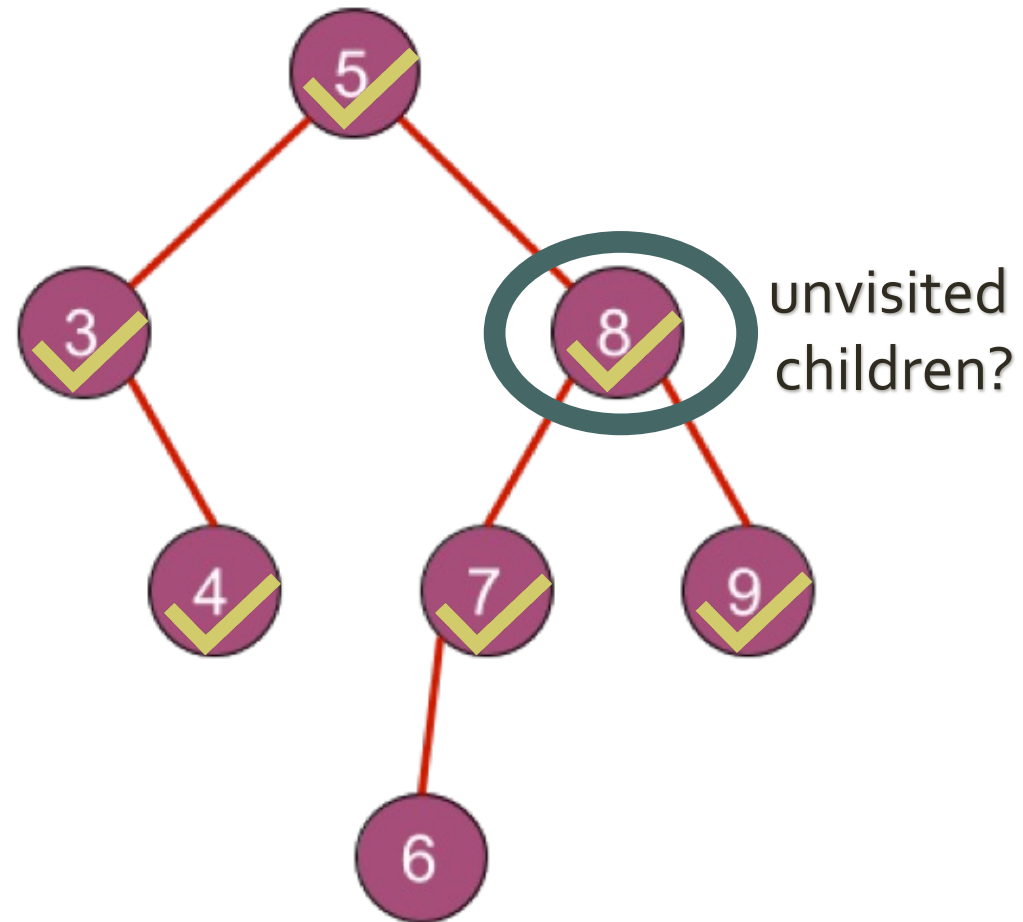
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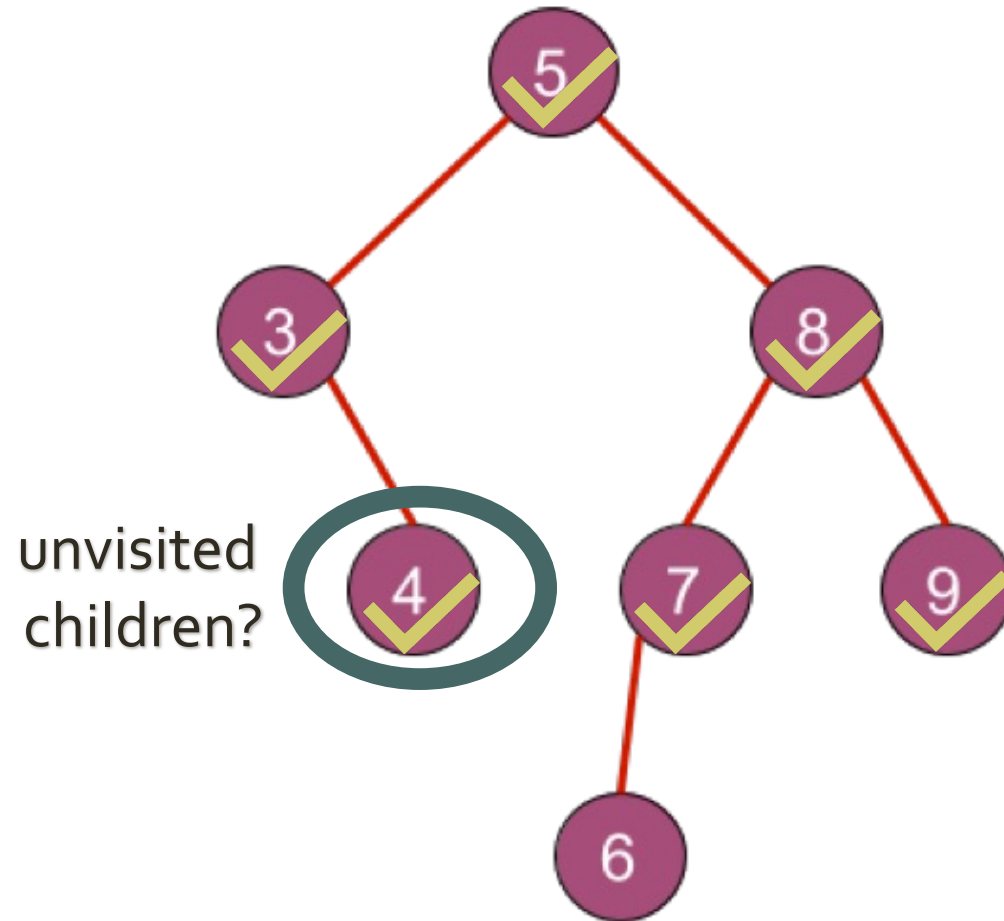
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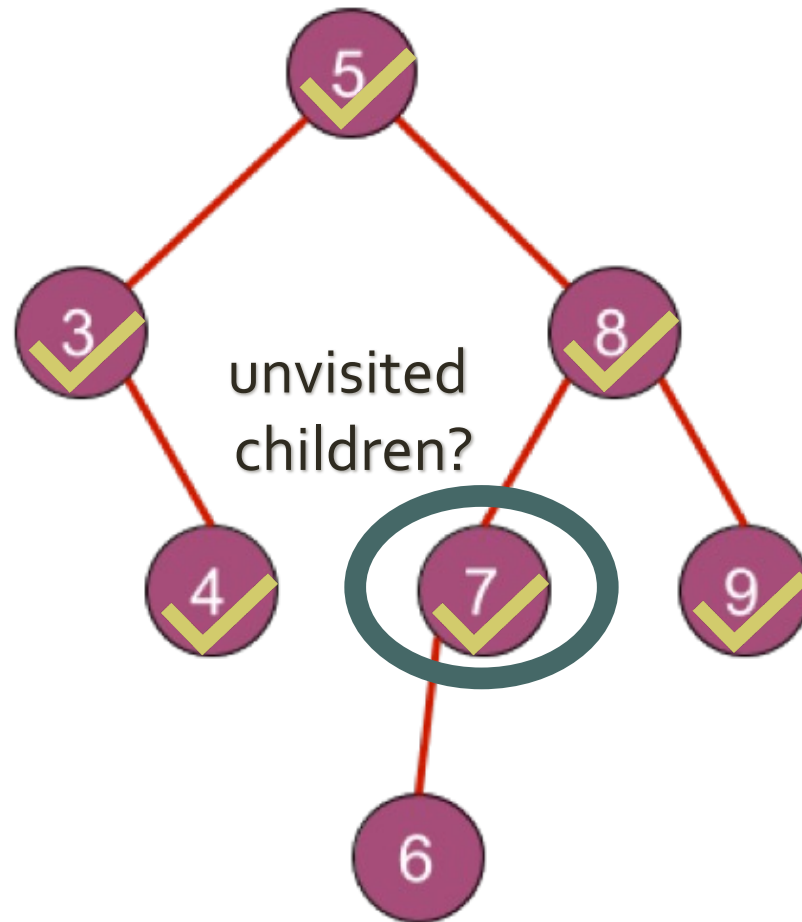
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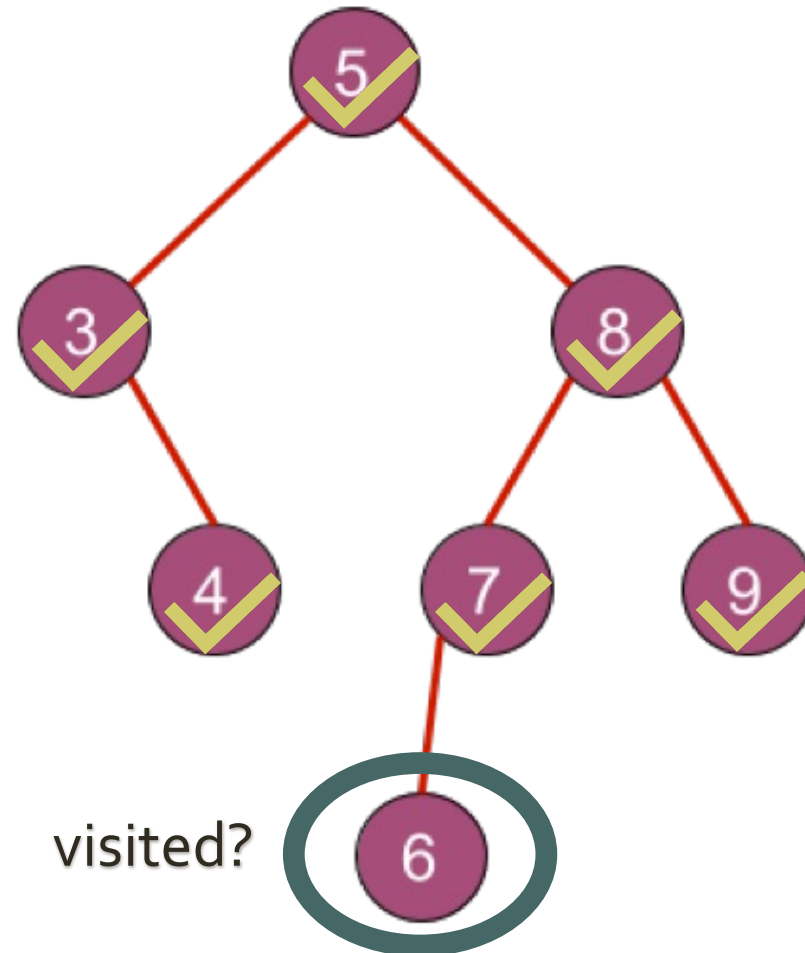
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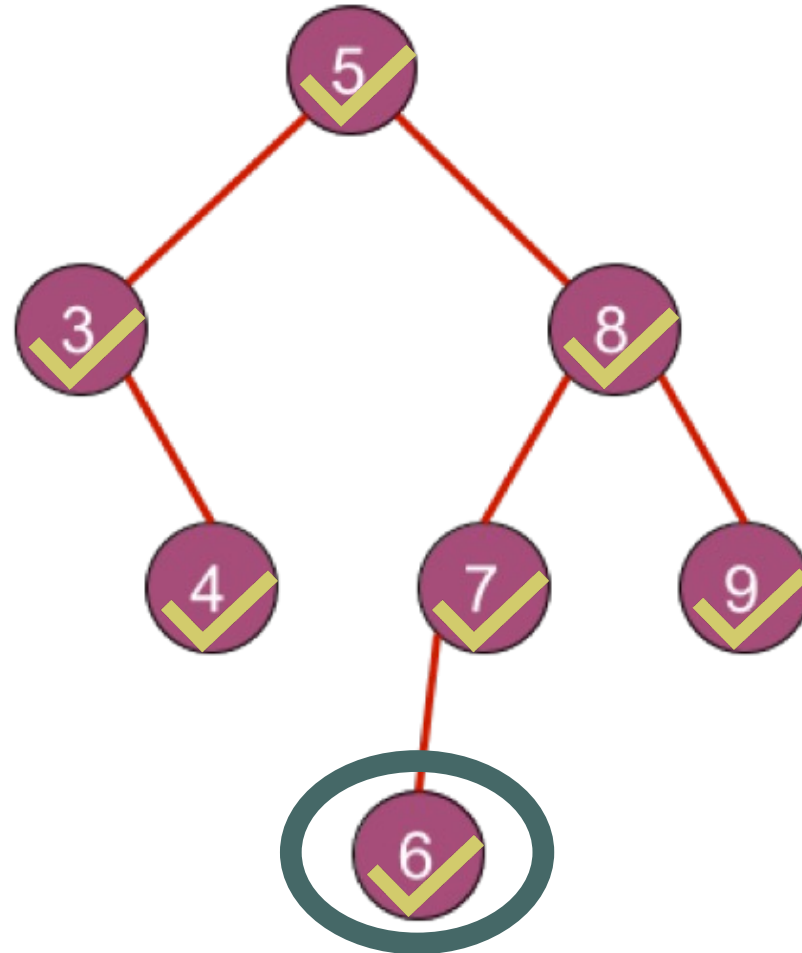
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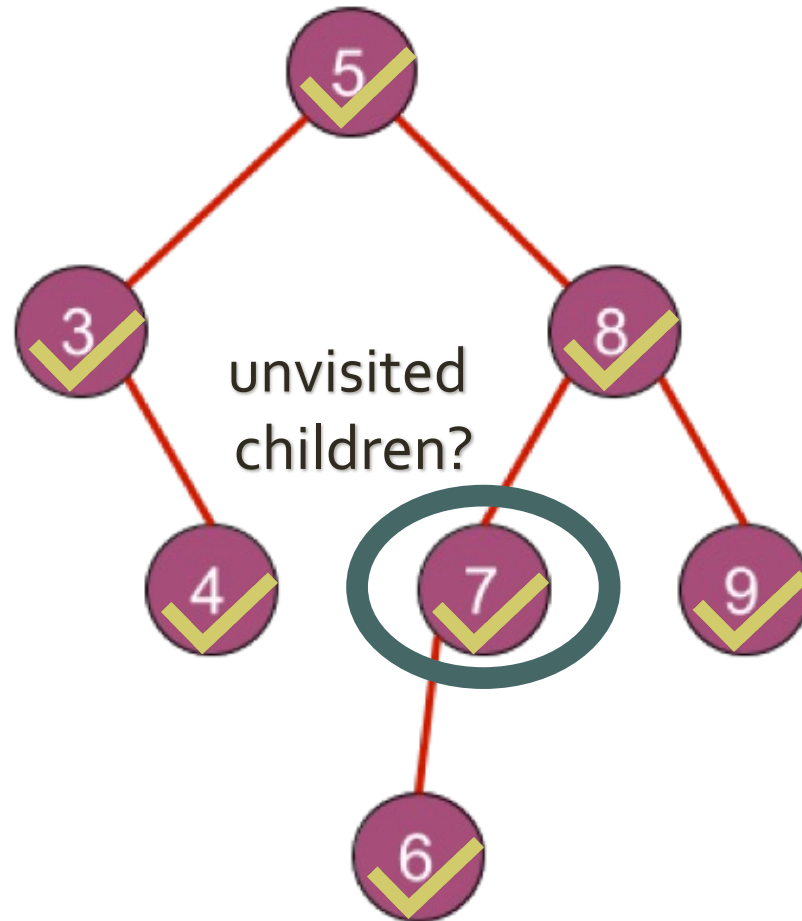
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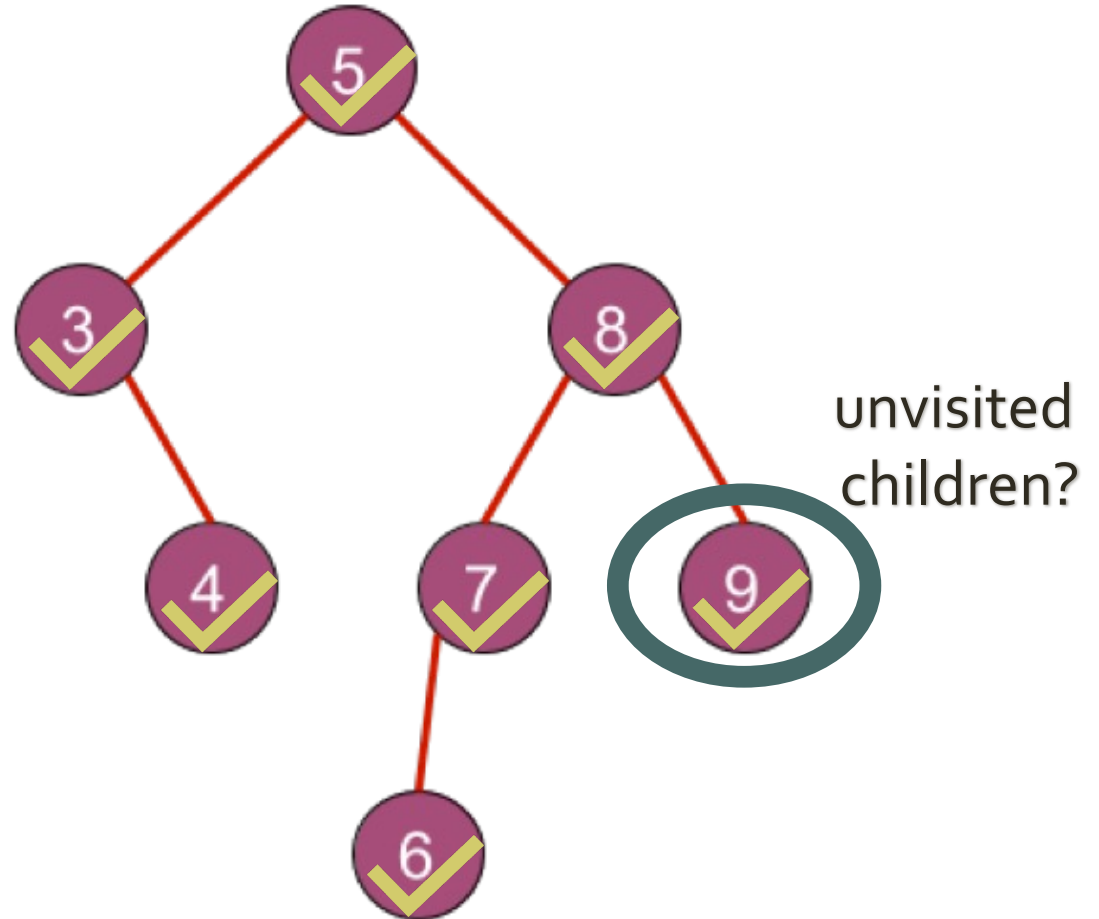
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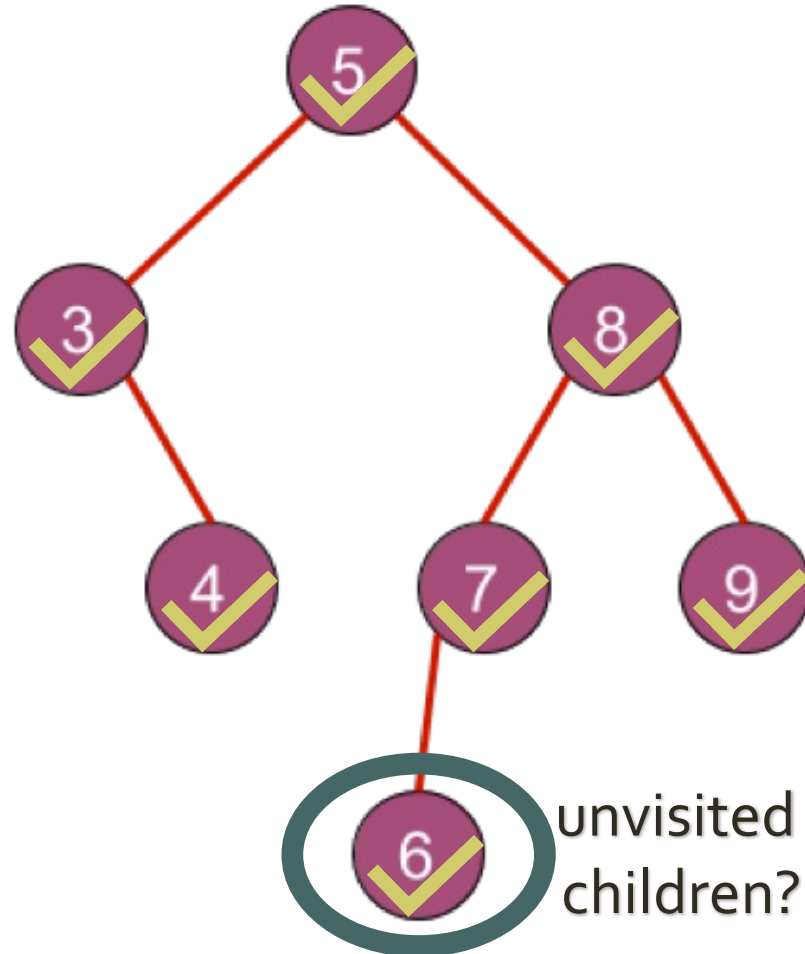
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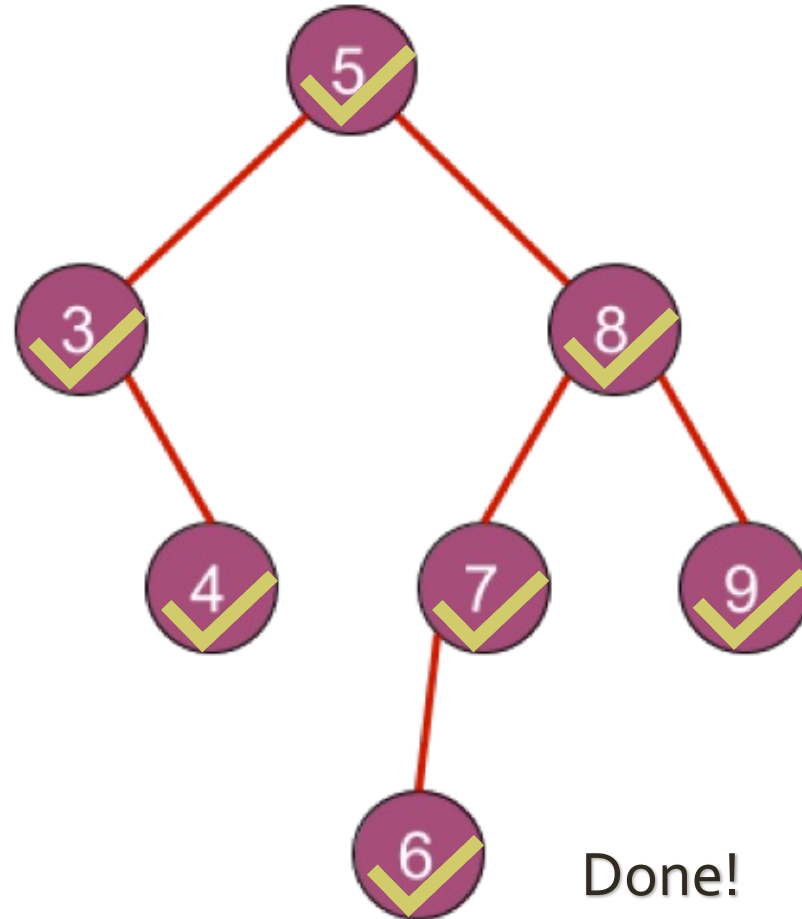
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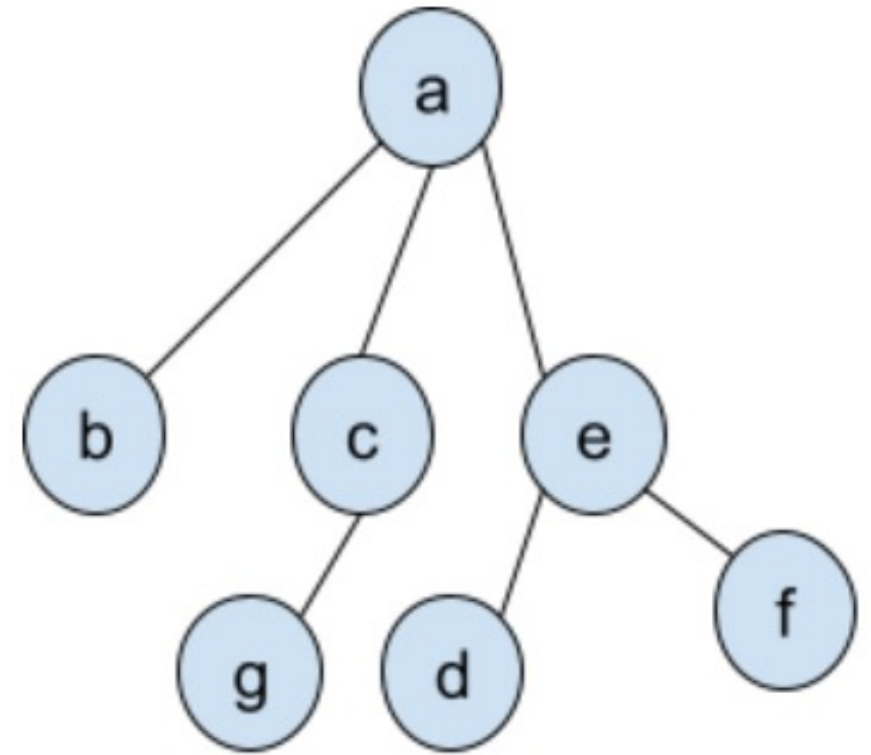
Algorithm:

```
BFS(G, v)
    create a queue Q
    mark v as visited and add to Q
    while Q is non-empty
        remove the head u of Q
        mark and enqueue all unvisited neighbors of u
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Draw this graph on the board and do a BFS. Count how many steps the BFS takes.



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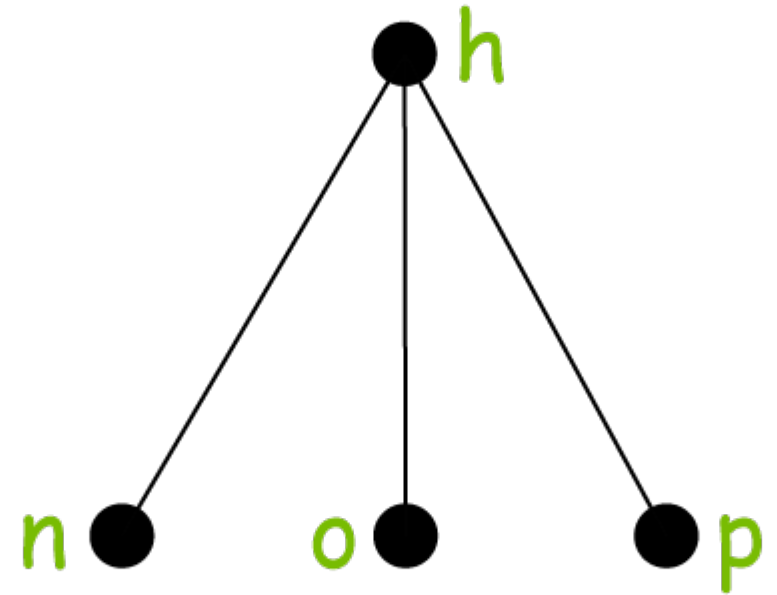
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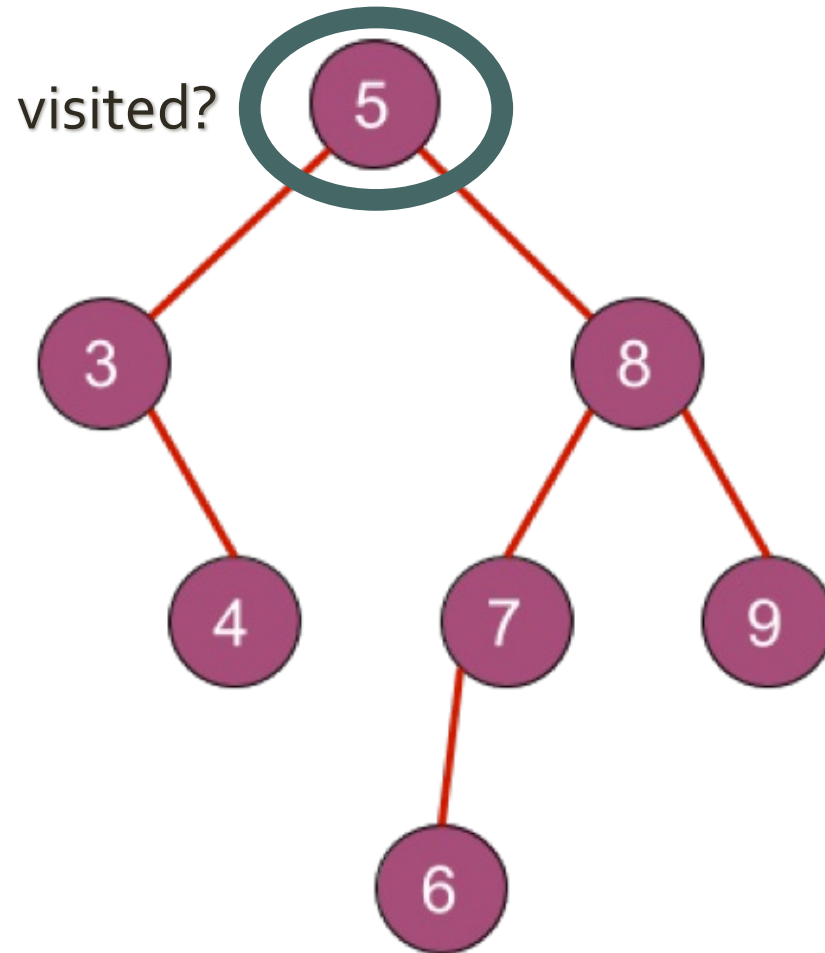
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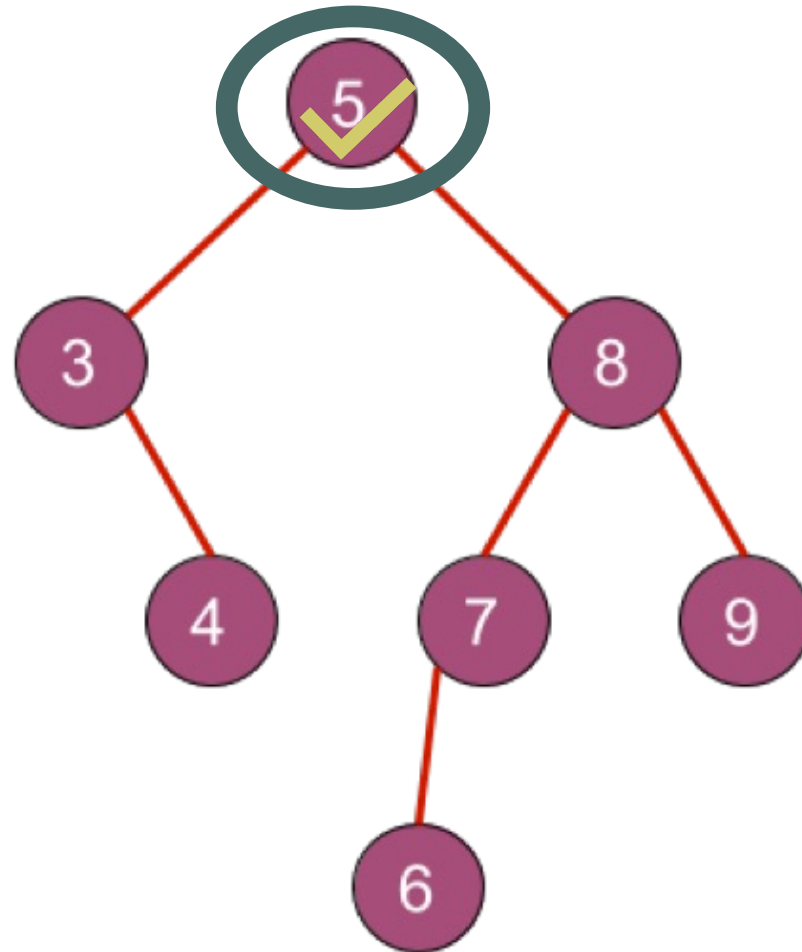
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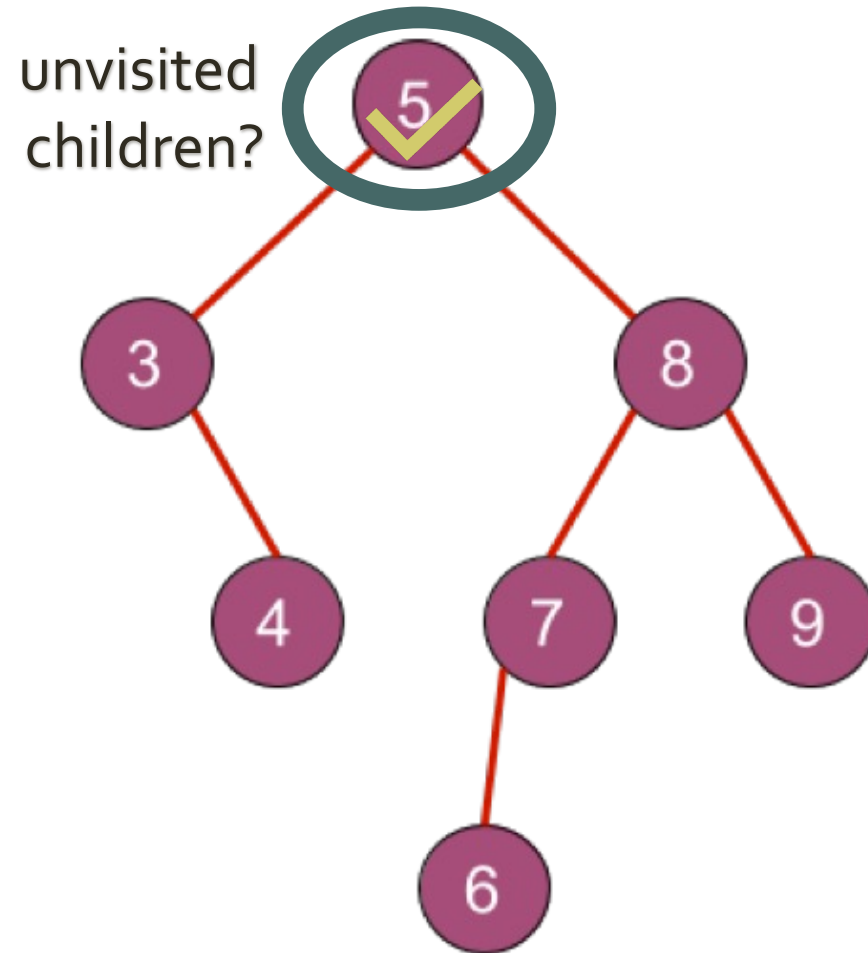
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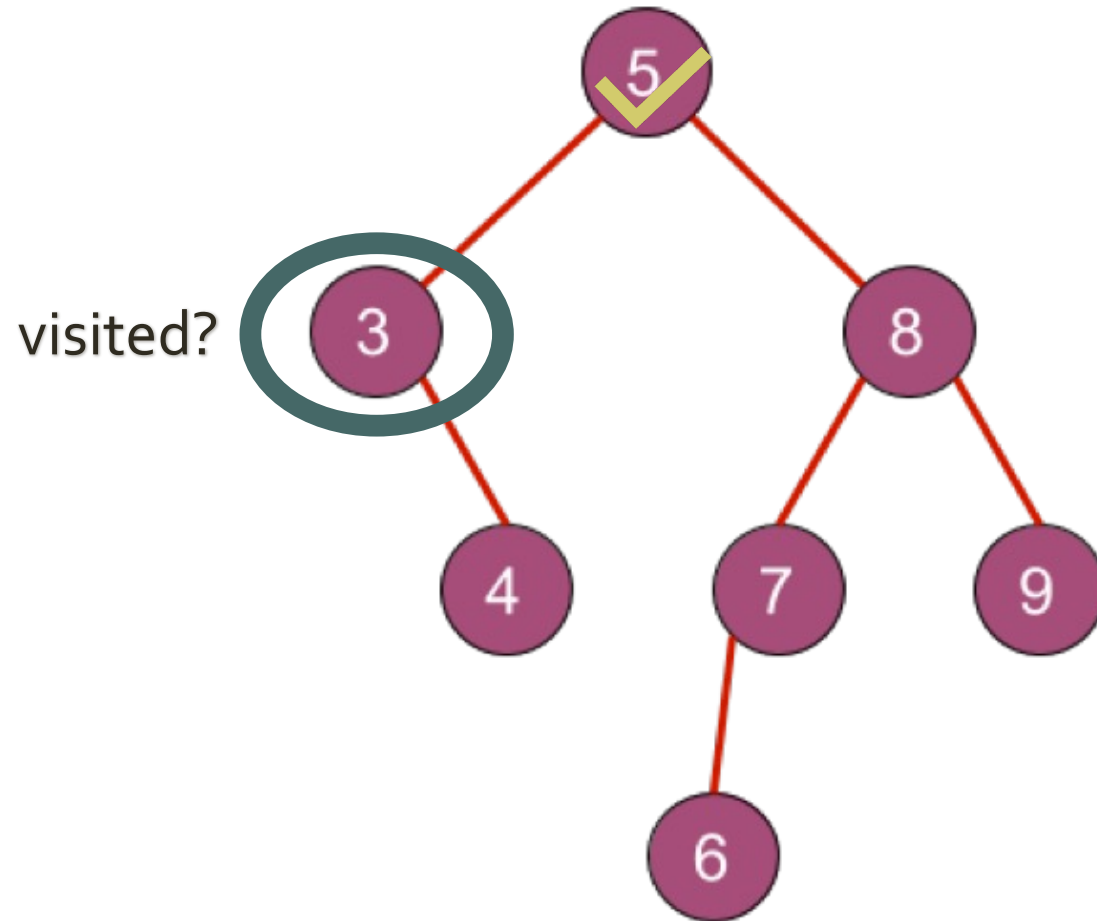
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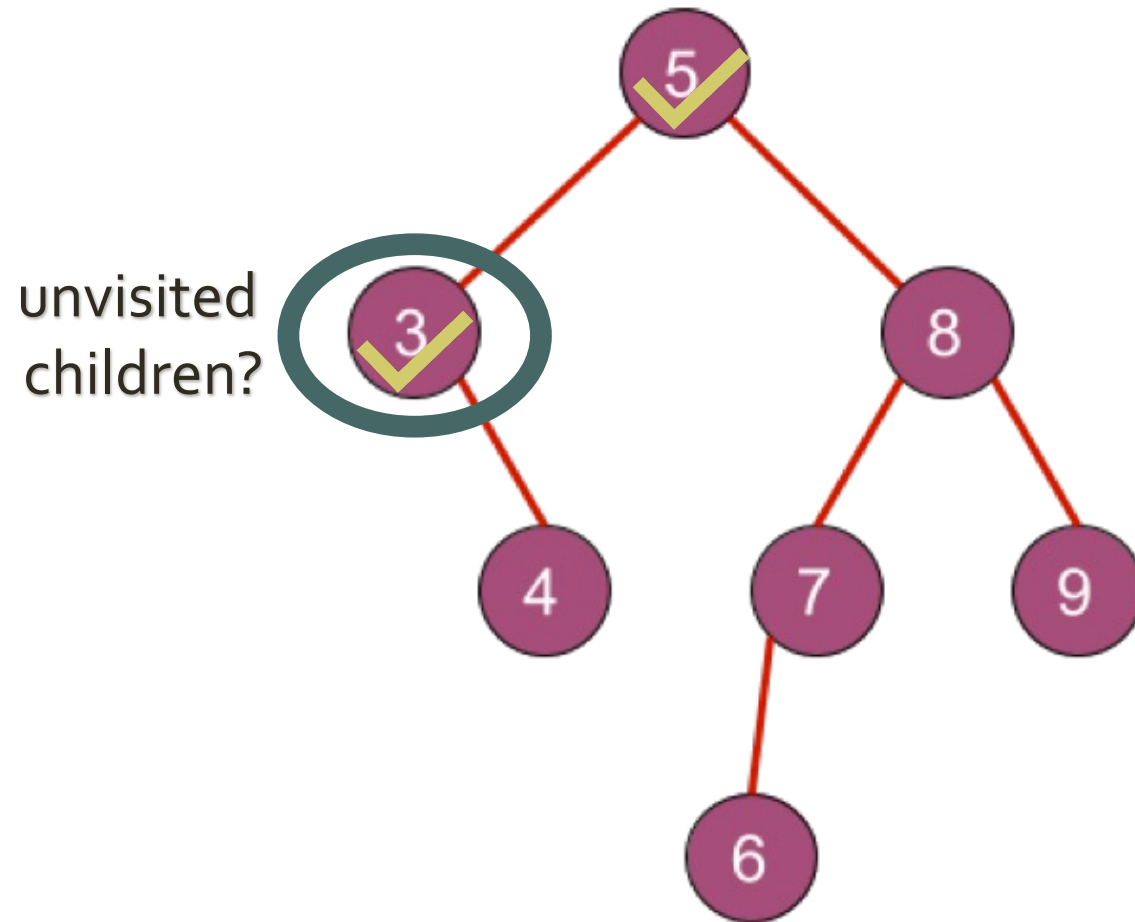
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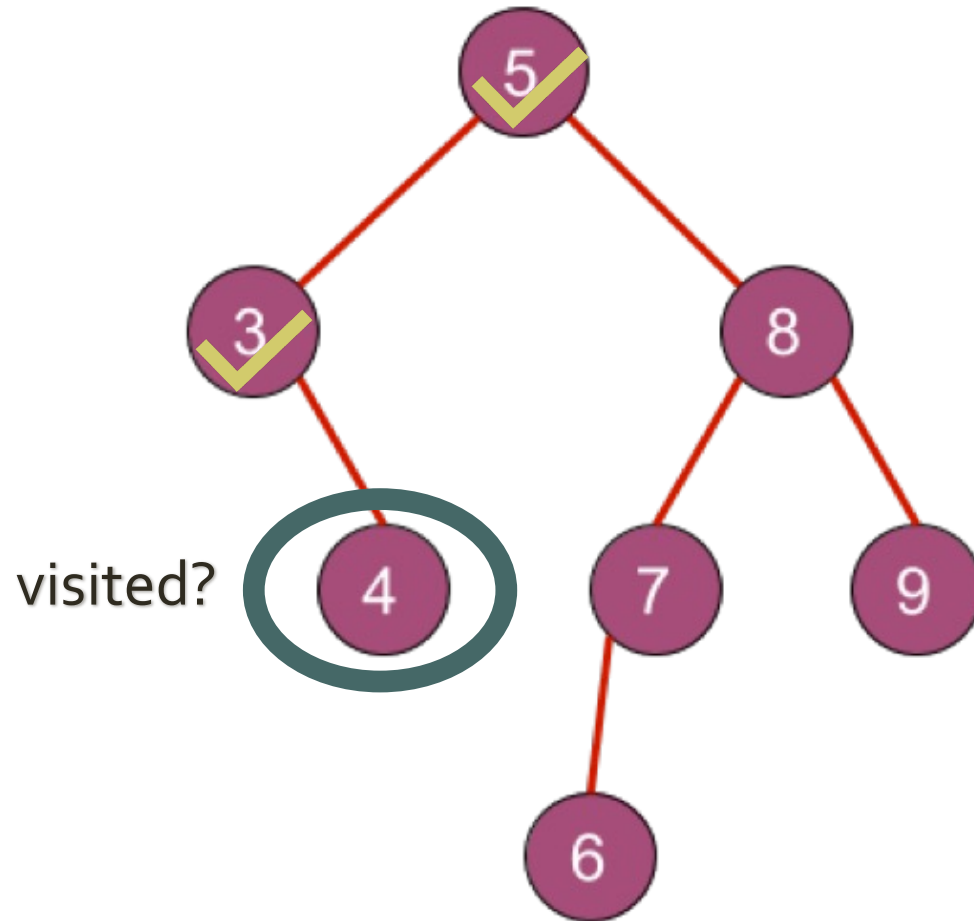
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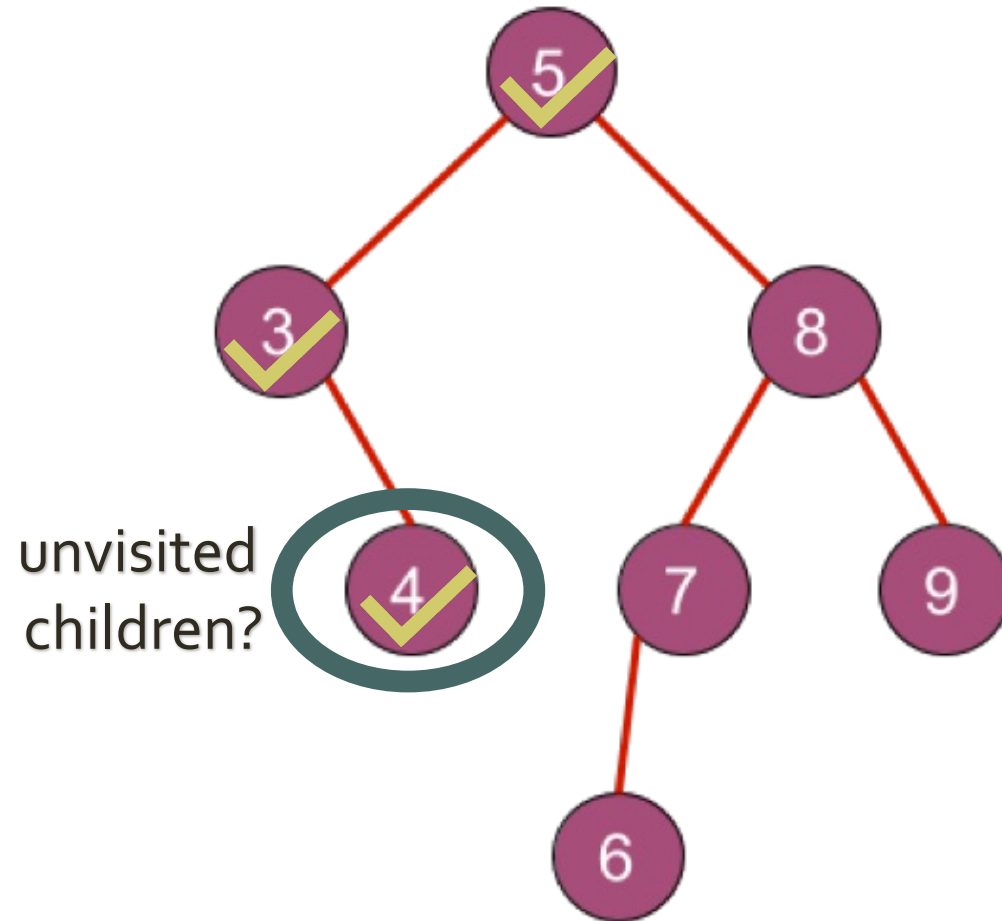
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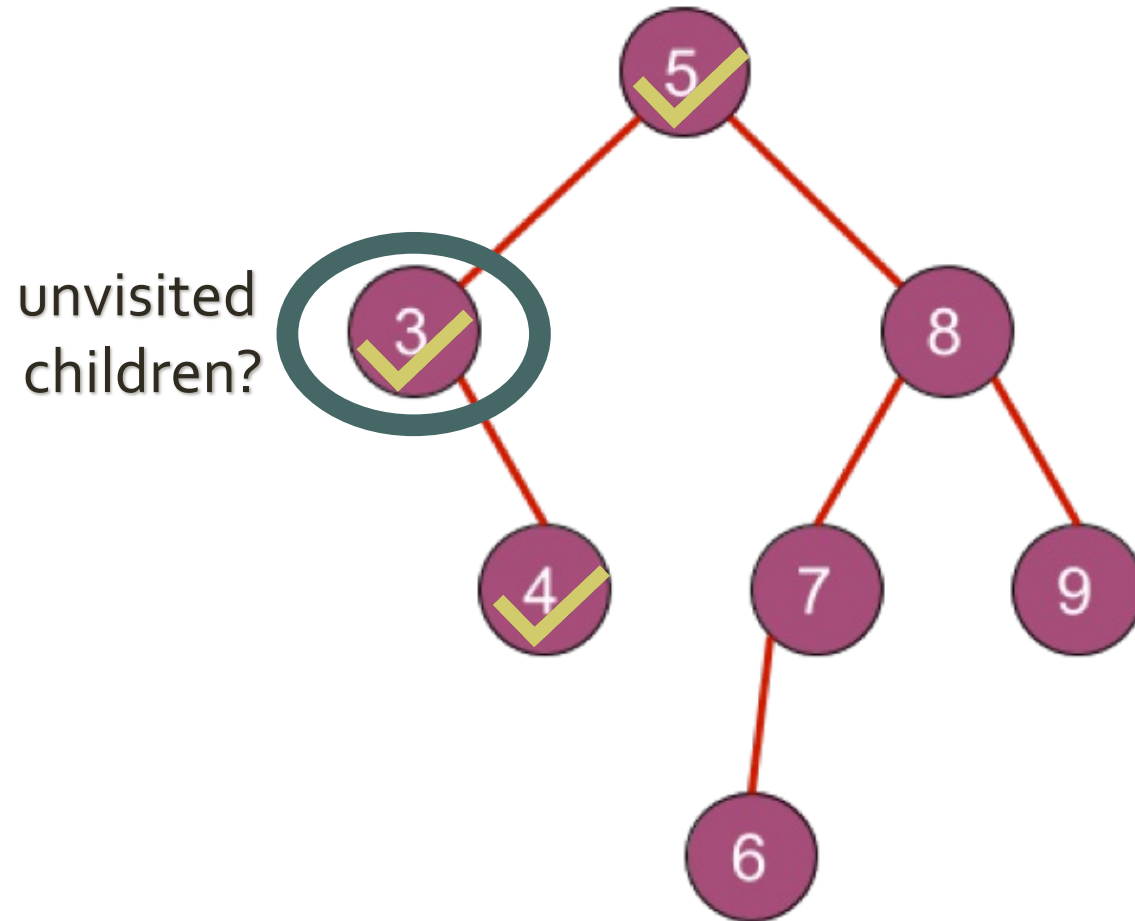
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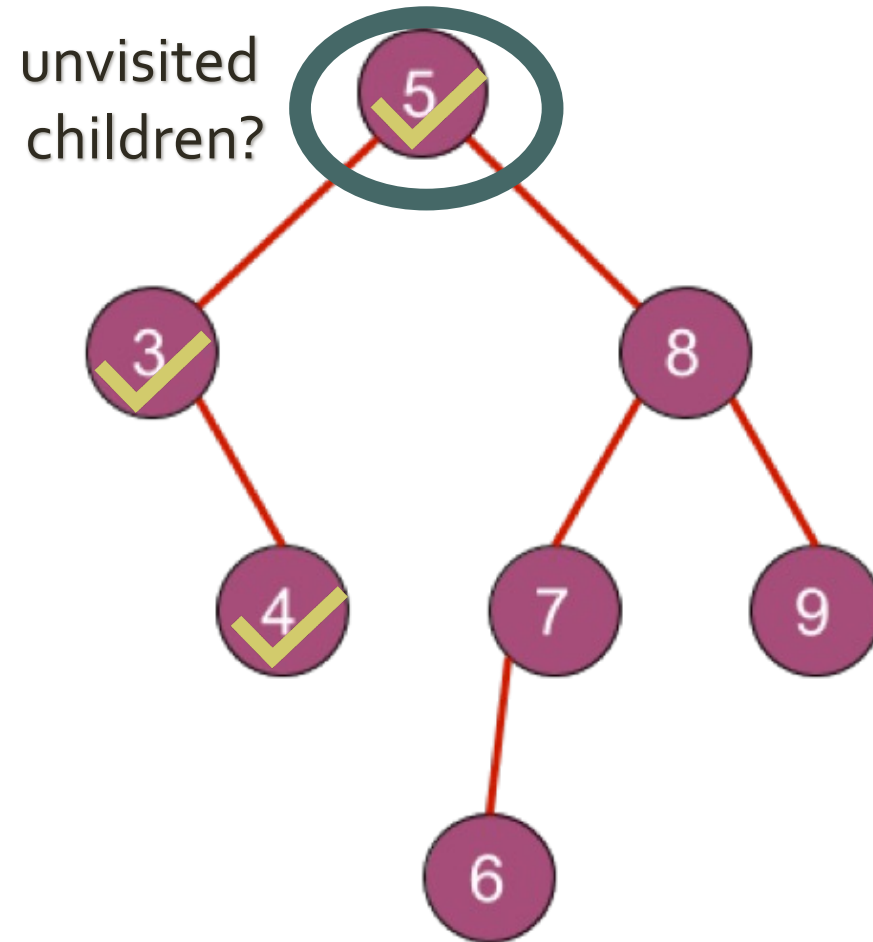
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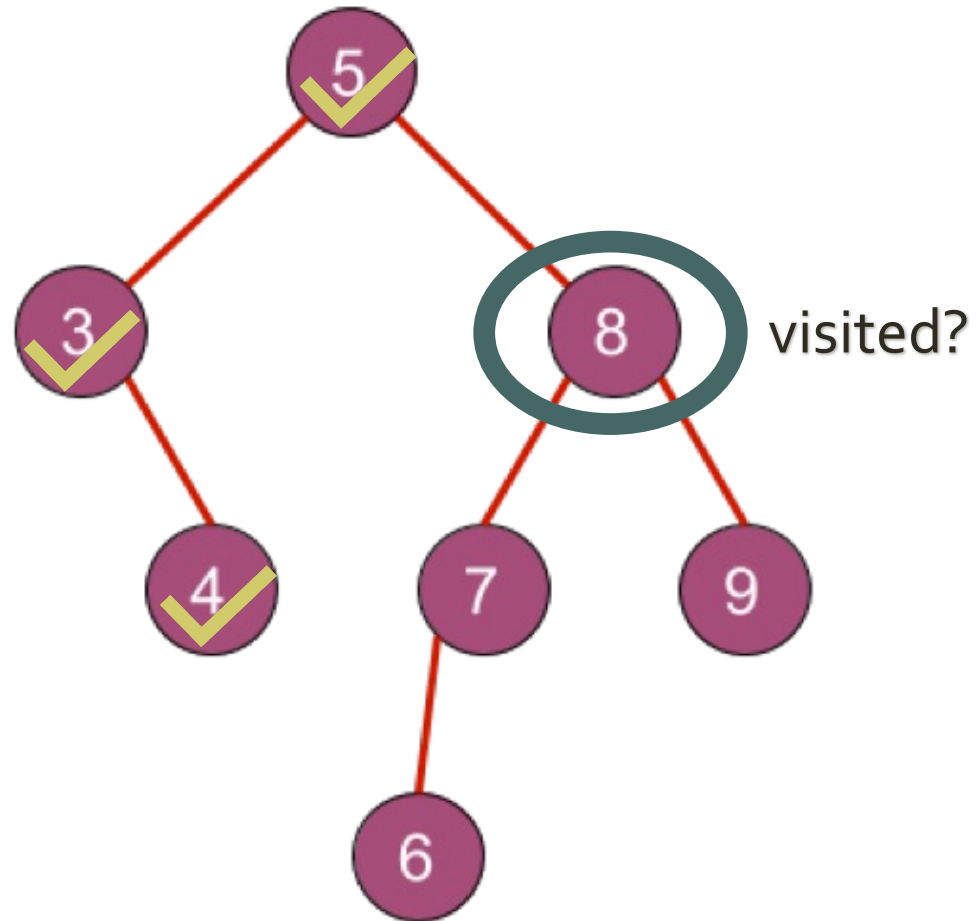
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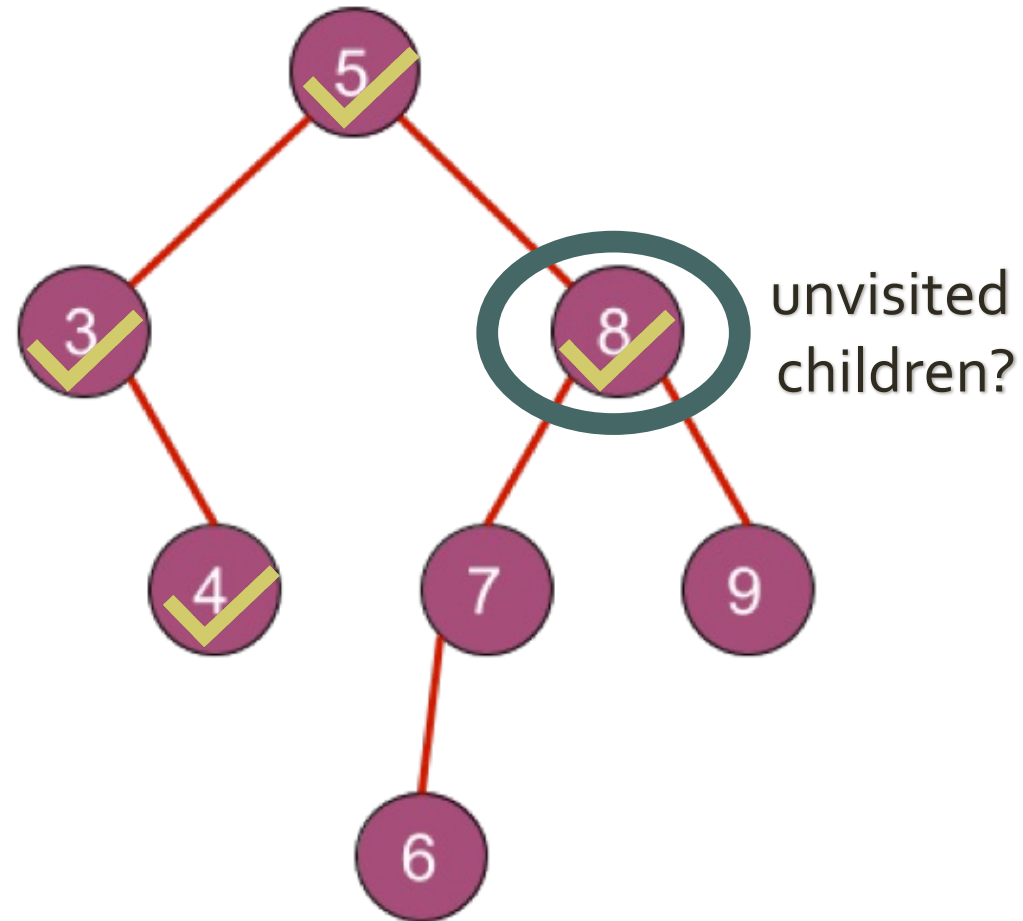
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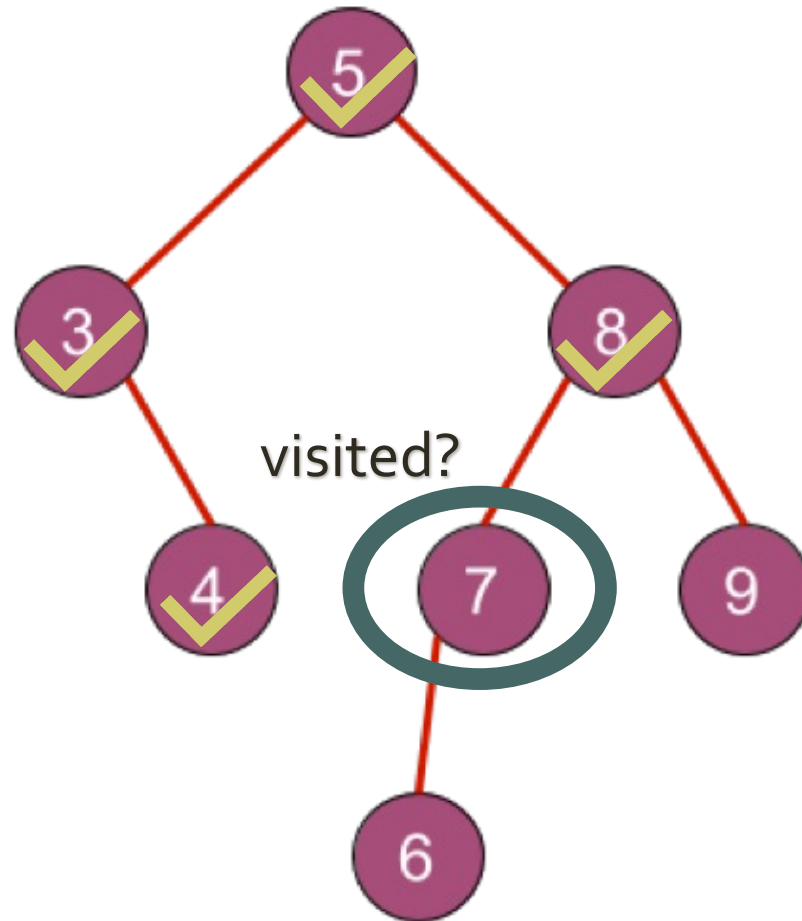
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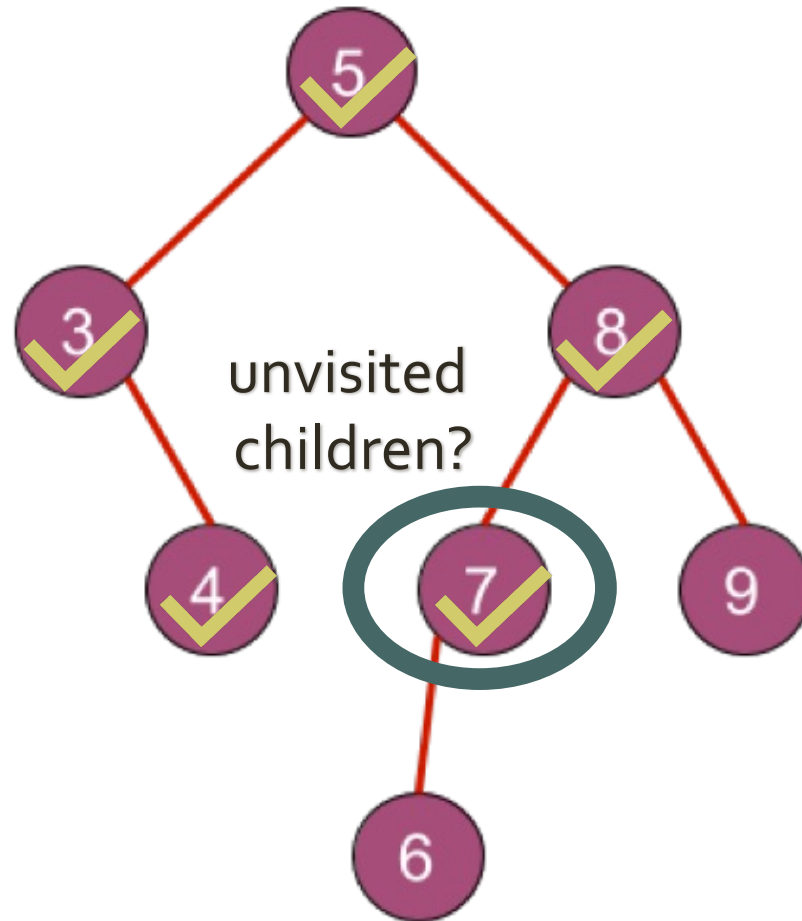
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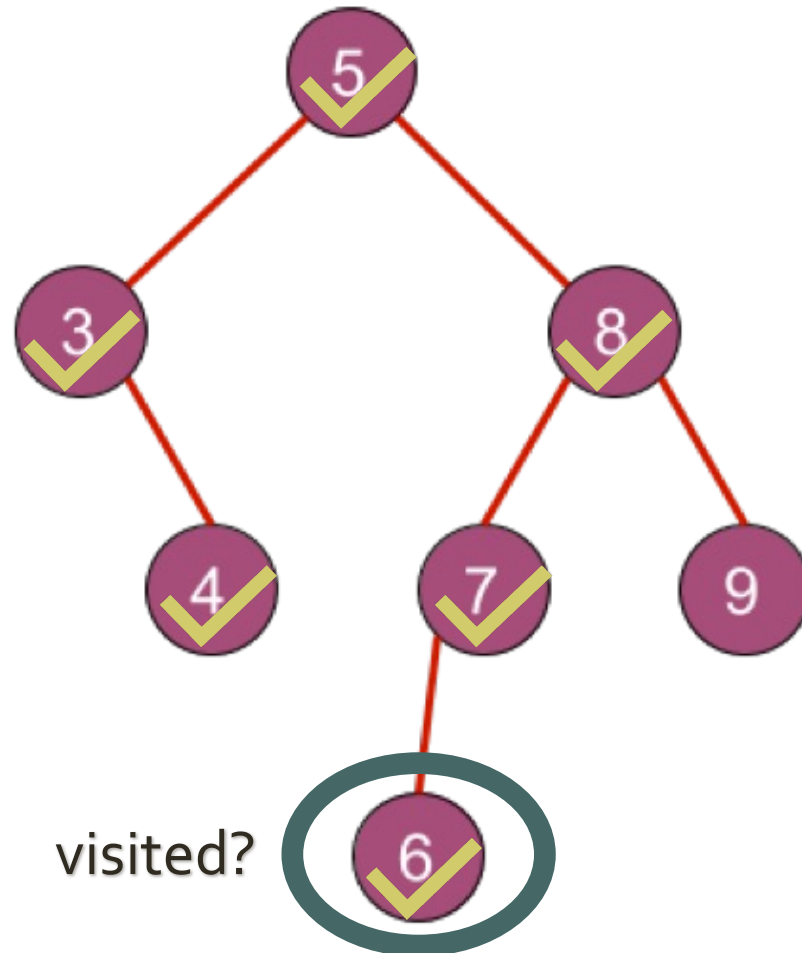
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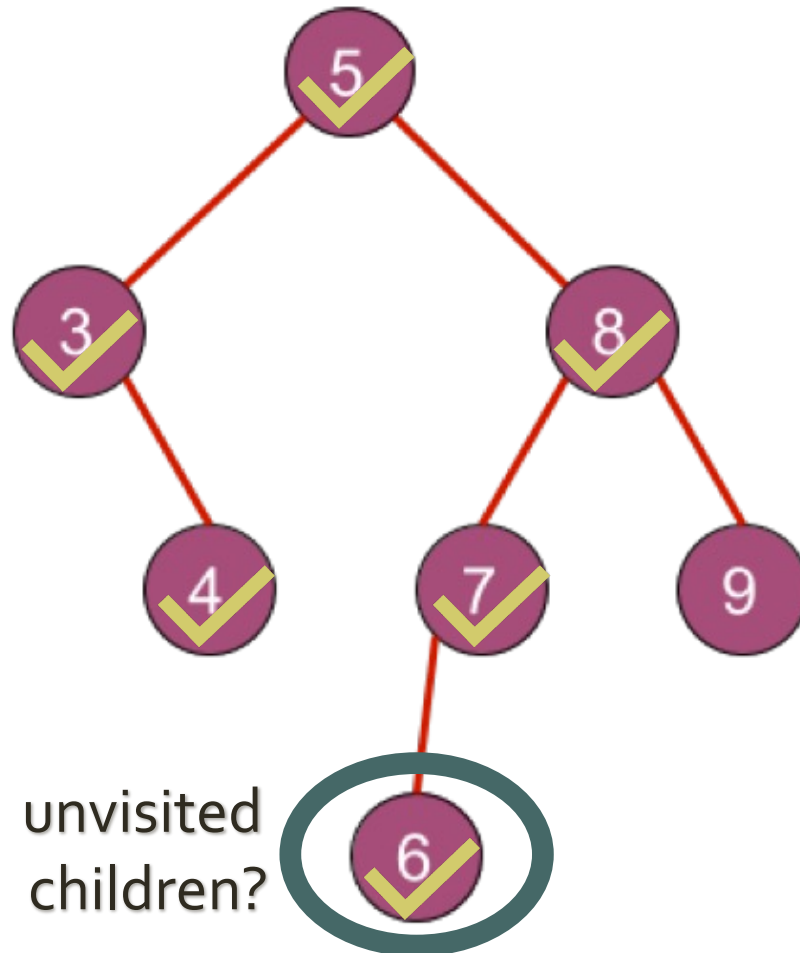
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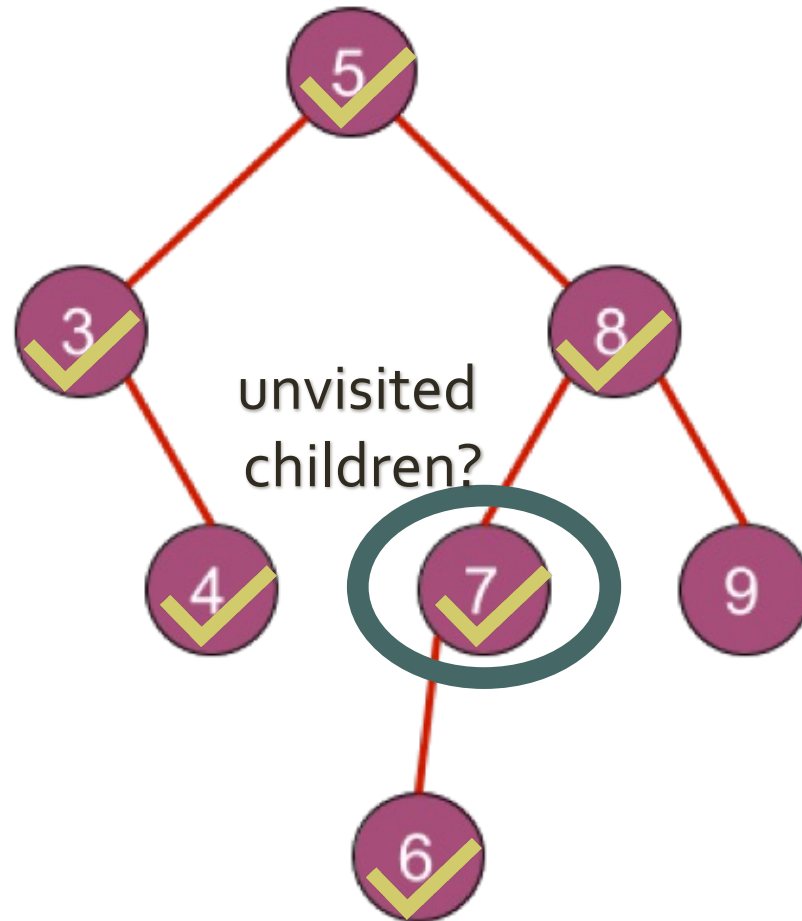
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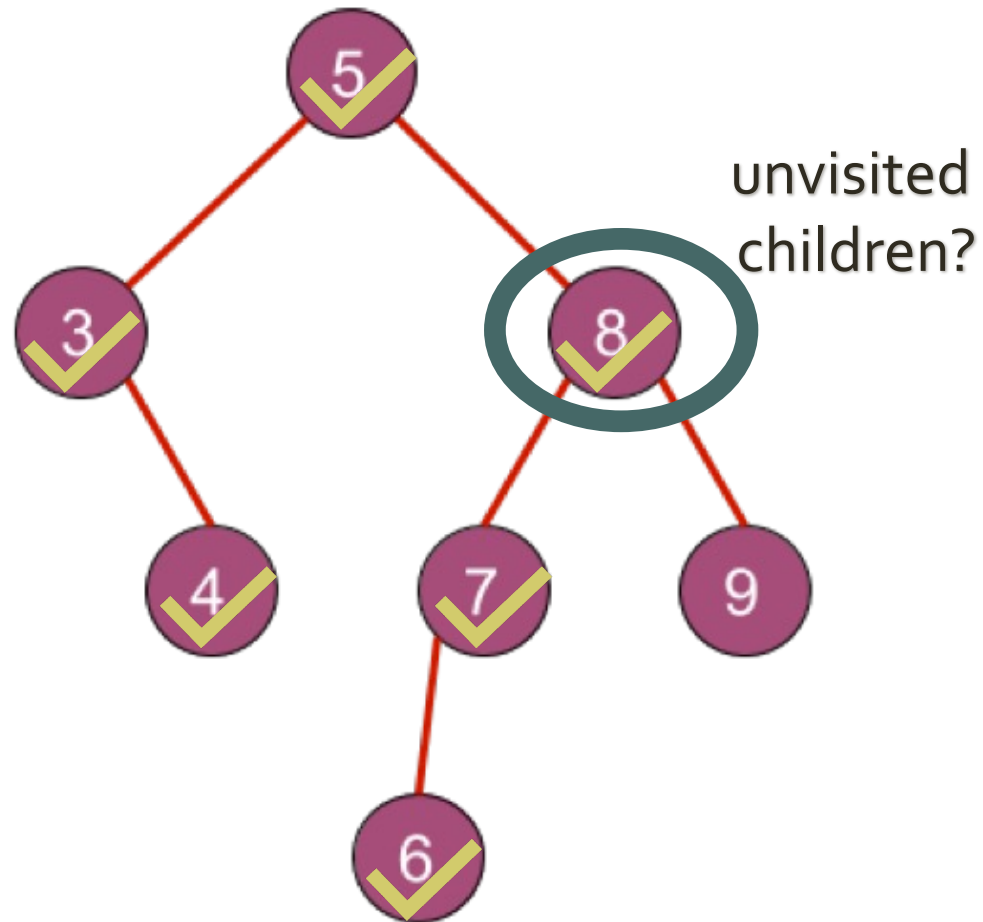
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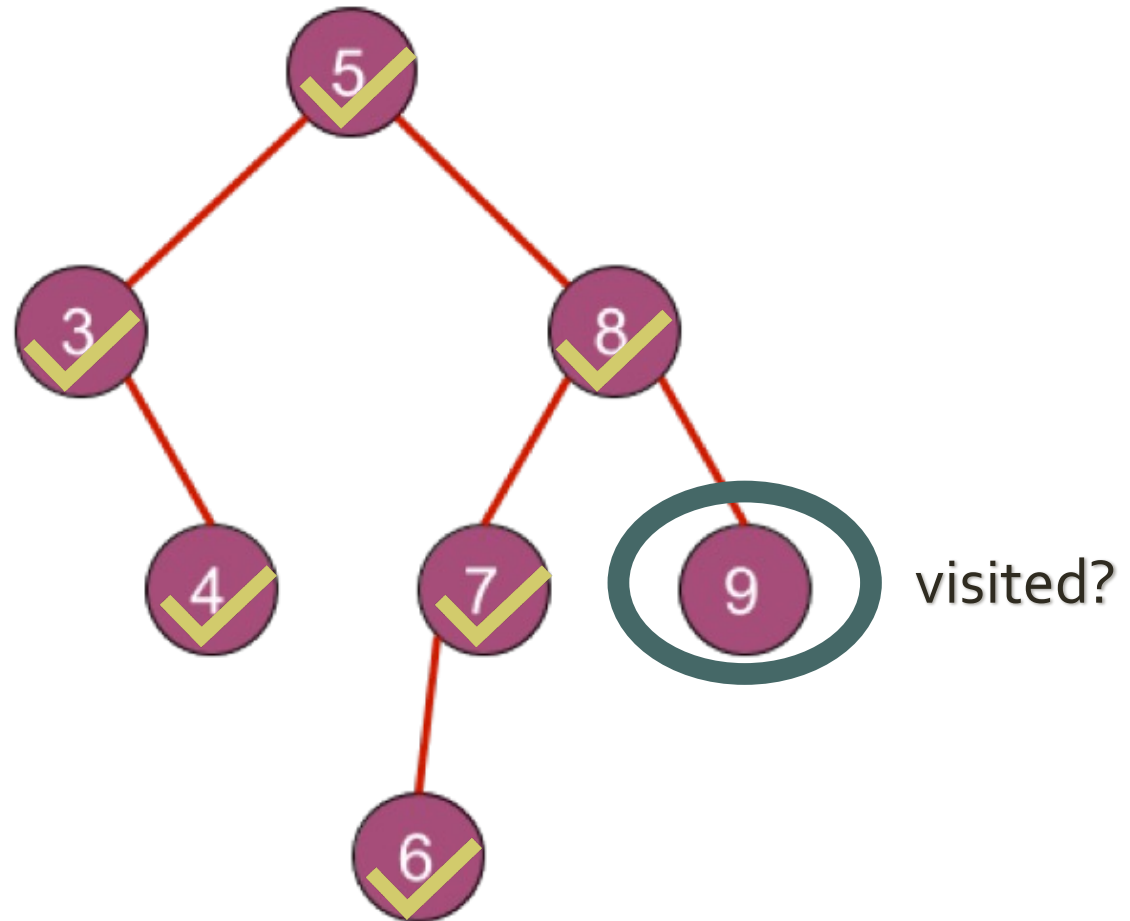
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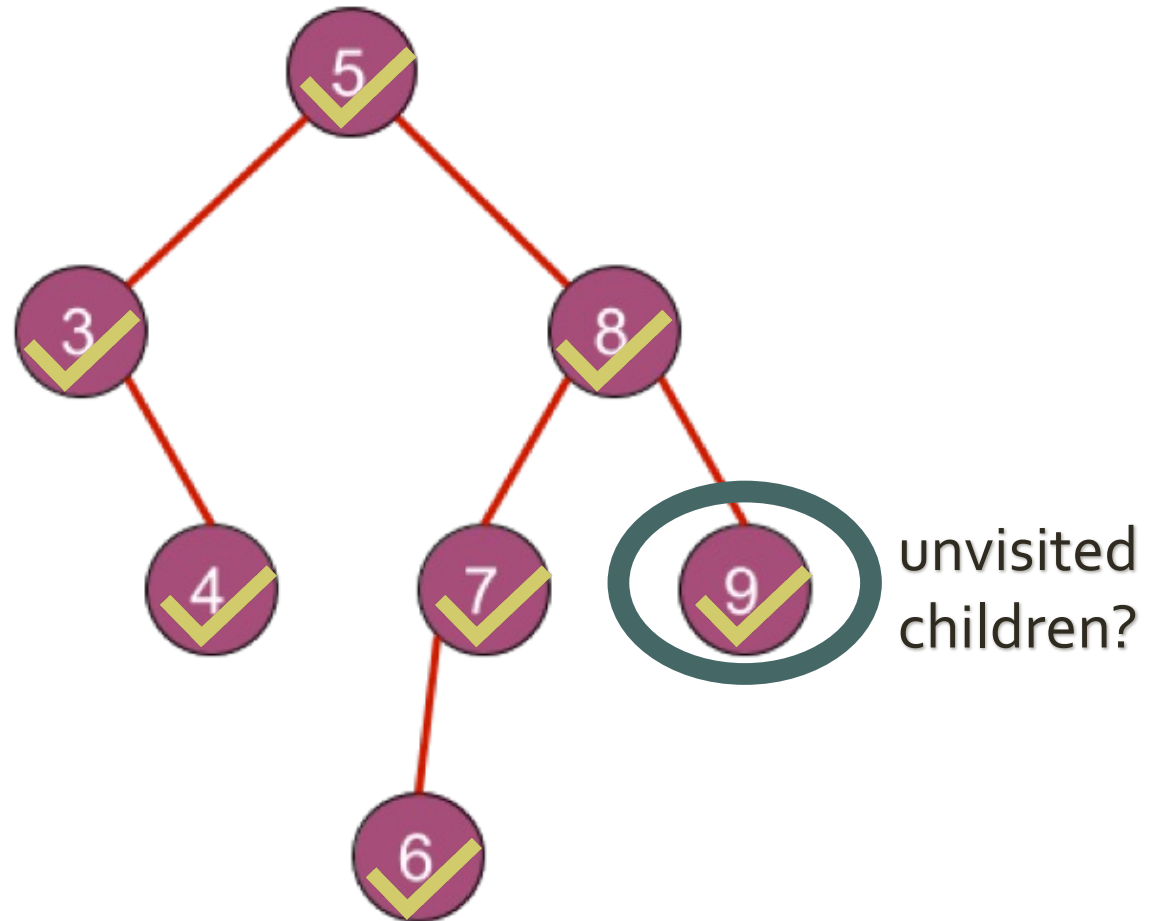
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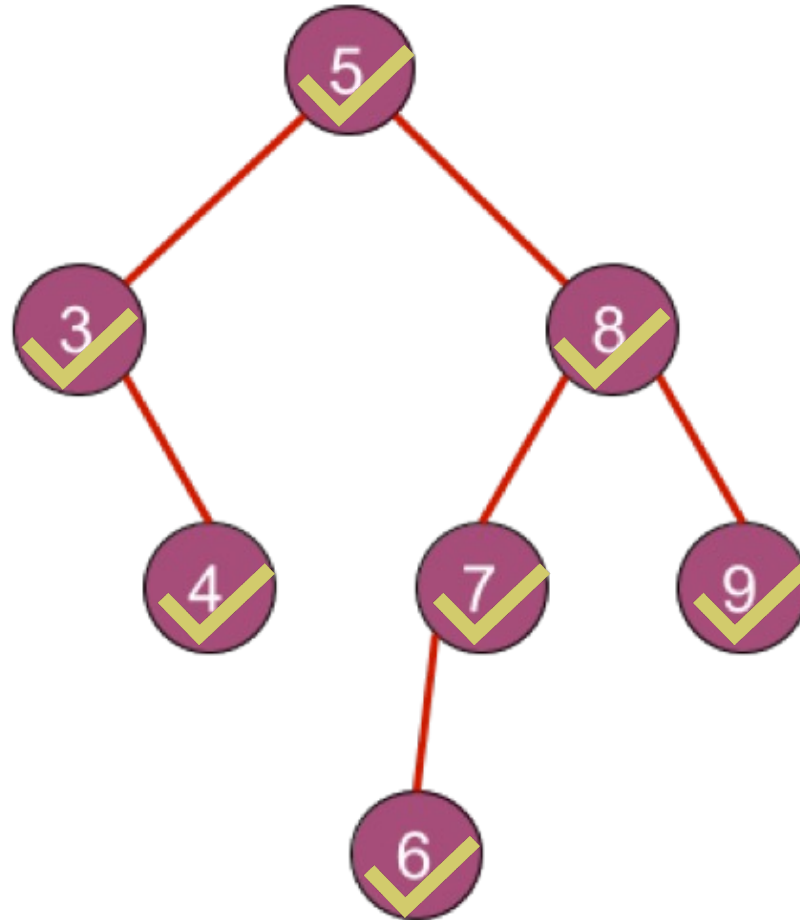
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Done!

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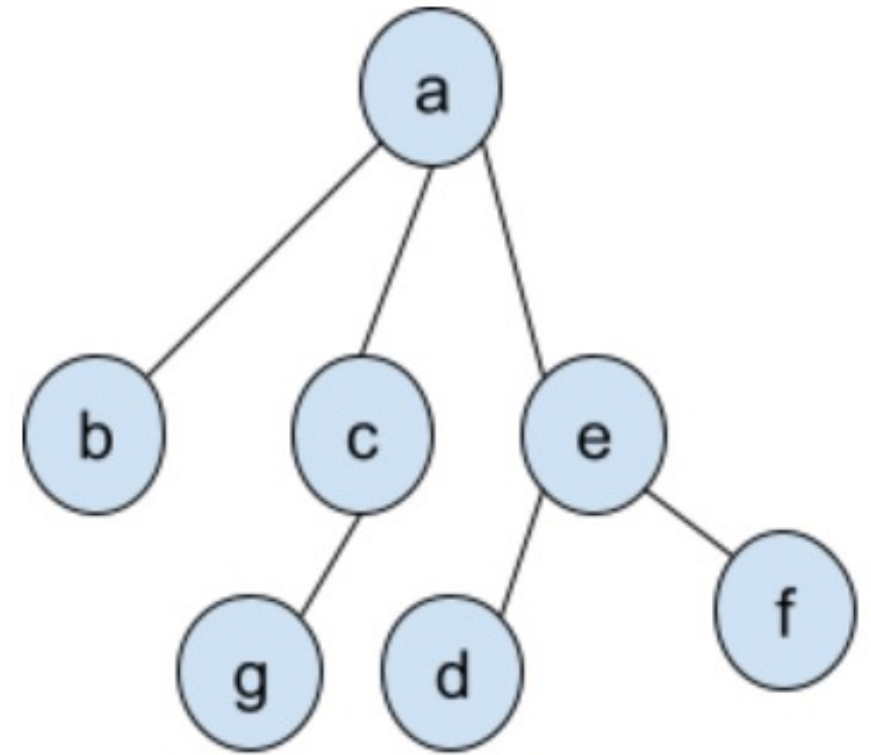
Algorithm:

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DFS(G, u)
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Draw this graph on the board and do a DFS. Count how many steps the BFS takes.



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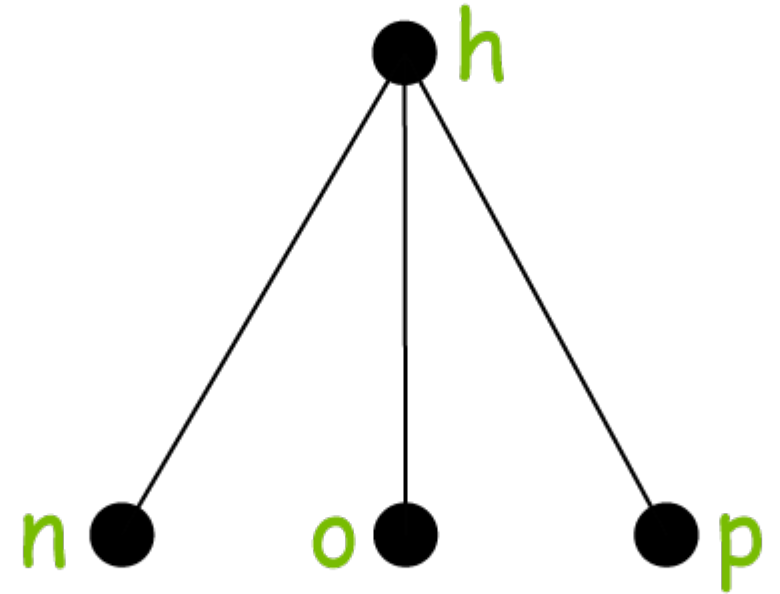
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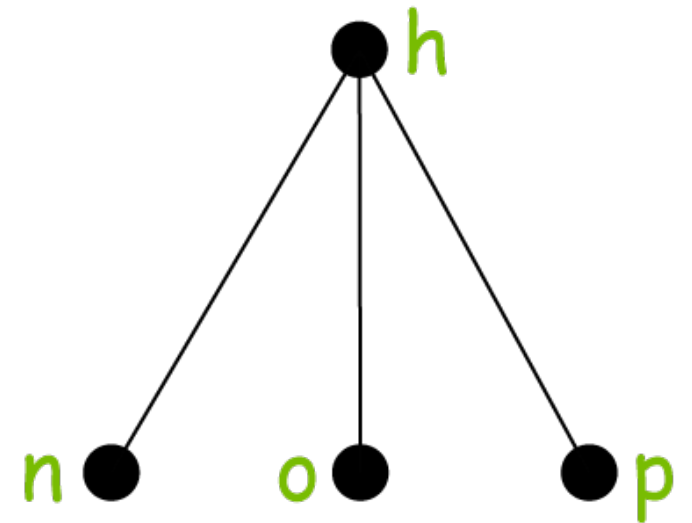
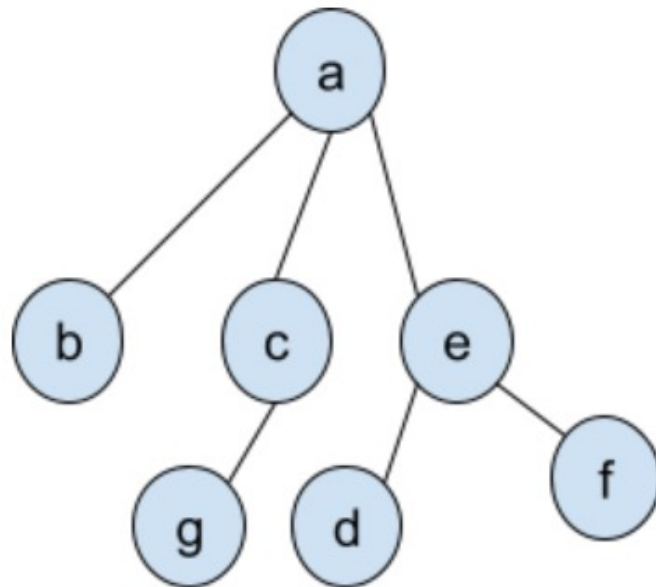
Draw this graph on the board and do a BFS. Count how many steps the DFS takes.





For each graph below, which took more steps, BFS or DFS?

BFS vs. DFS



For each graph below, which took more steps, BFS or DFS?

For each algorithm, what seems to impact number of steps?

## BFS vs. DFS

