Discrete Structures– Propositional Logic Pt. 1

Dr. Ab Mosca (they/them)

Plan for Today

- Truth conditions for connectives
- Truth Tables
- Logical Equivalence

Warm Up

statement: any declarative sentence which is either true or false

- ***atomic***: a statement that *cannot* be divided into smaller statements
- ***molecular***: a statement that *can* be divided into smaller statements

logical connectives:

- conjunction- $P \wedge Q$ (read: P and Q)
- disjunction- $P \vee Q$ (read: P or Q)
- implication or conditional- $P \rightarrow Q$ (read: if P then Q)
- biconditional- $P \leftrightarrow Q$ (read: P if and only if Q)
- negation- $\neg P$ (read: not P)

Take these two statements: (1) it snowed today, and (2) class is cancelled.

For each logical connective, write a molecular statement made up of these two statements. Write each molecular statement in English and symbolically.

Ex.

It snowed today and class is cancelled (English)

Let P = It snowed today, Q = class is cancelled, $P \wedge Q$ (symbols)

Truth Conditions

- All statements have a ***truth value***: true (T or 1) or false (F or 0)

Truth Conditions

- All statements have a ***truth value***: true (T or 1) or false (F or 0)
- The truth value of a statement depends on
  - (1) the truth values of its parts and
  - (2) its connectives

Truth Conditions

- All statements have a ***truth value***: true (T or 1) or false (F or 0)
- The truth value of a statement depends on
  - (1) the truth values of its parts and
  - (2) its connectives
- Truth conditions:
  - $P \wedge Q$ is true when both P and Q are true.
  - $P \vee Q$ is true when P or Q or both are true.
  - $P \rightarrow Q$ is true when P is false or Q is true or both.
  - $P \leftrightarrow Q$ is true when P and Q are both true or both false.
  - $\neg P$ is true when P is false.

Truth Conditions

- All statements have a ***truth value***: true (T or 1) or false (F or 0)
- The truth value of a statement depends on
  - (1) the truth values of its parts and
  - (2) its connectives
- Truth conditions:
  - $P \wedge Q$ is true when both P and Q are true.
  - $P \vee Q$ is true when P or Q or both are true.
  - $P \rightarrow Q$ is true when P is false or Q is true or both.
  - $P \leftrightarrow Q$ is true when P and Q are both true or both false.
  - $\neg P$ is true when P is false.
- Note: This or is called ***inclusive or***, there is also an ***exclusive or*** which is true when P or Q is true but not when both are true.

Truth Conditions

- All statements have a ***truth value***: true (T or 1) or false (F or 0)
- The truth value of a statement depends on
  - (1) the truth values of its parts and
  - (2) its connectives
- Truth conditions:
  - $P \wedge Q$ is true when both P and Q are true.
  - $P \vee Q$ is true when P or Q or both are true.
  - $P \rightarrow Q$ is true when P is false or Q is true or both.
  - $P \leftrightarrow Q$ is true when P and Q are both true or both false.
  - $\neg P$ is true when P is false.

Practice: Let P = It snowed today, Q = class is cancelled, what is the truth value for each of the statements you wrote earlier **based on the truth of P and Q today**.

Truth Conditions

- All statements have a ***truth value***: true (T or 1) or false (F or 0)
- The truth value of a statement depends on
  - (1) the truth values of its parts and
  - (2) its connectives

Truth Conditions

- All statements have a ***truth value***: true (T or 1) or false (F or 0)
- The truth value of a statement depends on
  - (1) the truth values of its parts and
  - (2) its connectives
- For any atomic statement there are only two possible truth values, that means when we string statements together with connectives there is a finite number of possible combinations of truth values

Truth Conditions

- All statements have a ***truth value***: true (T or 1) or false (F or 0)
- The truth value of a statement depends on
  - (1) the truth values of its parts and
  - (2) its connectives
- For any atomic statement there are only two possible truth values, that means when we string statements together with connectives there is a finite number of possible combinations of truth values
- One way we model possible truth values is with ***truth tables***

Truth Tables

One way we model possible truth values is with ***truth tables***

Let P be a statement. The truth table for P is:

P
T
F

Truth Tables

One way we model possible truth values is with ***truth tables***

Let P be a statement. The truth table for P is:

P
T
F

Let Q be a statement. The truth table for Q is:

Q
T
F

Truth Tables

One way we model possible truth values is with ***truth tables***

When we combine statements with conjunctives, there is a finite number of truth value combinations:

Let P and Q be statements. One of these must be the case:

P is true and **Q is true**

P is true and **Q is false**

P is false and **Q is true**

P is false and **Q is false**

Truth Tables

One way we model possible truth values is with ***truth tables***

When we combine statements with conjunctives, there is a finite number of truth value combinations:

Let P and Q be statements. One of these must be the case:

P is true and **Q is true**

P is true and **Q is false**

P is false and **Q is true**

P is false and **Q is false**

When we use connectives, the truth value of the resulting statement depends on the true values of its parts and the connective. We use truth tables to model this.

Truth Tables

When we use connectives, the truth value of the resulting statement depends on the true values of its parts and the connective. We use truth tables to model this.

Conjunction: $P \wedge Q$ is true when both P and Q are true.

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Truth Tables

When we use connectives, the truth value of the resulting statement depends on the true values of its parts and the connective. We use truth tables to model this.

Disjunction: $P \vee Q$ is true when P or Q or both are true.

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Truth Tables

When we use connectives, the truth value of the resulting statement depends on the true values of its parts and the connective. We use truth tables to model this.

Implication: $P \to Q$ is true when P is false or Q is true or both.

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

Truth Tables

When we use connectives, the truth value of the resulting statement depends on the true values of its parts and the connective. We use truth tables to model this.

Biconditional: $P \leftrightarrow Q$ is true when P and Q are both true or both false.

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

Truth Tables

When we use connectives, the truth value of the resulting statement depends on the true values of its parts and the connective. We use truth tables to model this.

Negation: $\neg P$ is true when P is false.

P	$\neg P$
T	F
F	T

Tr

Practice: Form five groups. Each will be assigned a conjunction. Come up with a real world scenario that your conjunction accurately represents. Be ready to share your answers!

Ex. If I had conjunction and were one of you, I might say: Let P = I study discrete math, and Q = I am taking MATH220

The truth table and real work scenarios are:

P	Q	$P \wedge Q$	
T	T	T	I study discrete math AND I am taking MATH220.
T	F	F	I study discrete math AND I am not taking MATH220.
F	T	F	I do not study discrete math AND I am taking MATH220.
F	F	F	I do not study discrete math AND I am not taking MATH220.

Truth Tables

Larger statements can be made up of more than two smaller parts and/or more than one conjunction.

Truth tables are helpful for deducing when these larger statements are true and false.

Ex. When is $\neg P \vee Q$ true?

Truth Tables

Larger statements can be made up of more than two smaller parts and/or more than one conjunction.

Truth tables are helpful for deducing when these larger statements are true and false.

Ex. When is $\neg P \vee Q$ true?

P	Q
T	T
T	F
F	T
F	F

Start the table with all possible combinations of T/F for smallest statements

Truth Tables

Larger statements can be made up of more than two smaller parts and/or more than one conjunction.

Truth tables are helpful for deducing when these larger statements are true and false.

Ex. When is $\neg P \vee Q$ true?

P	Q	$\neg P$
T	T	
T	F	
F	T	
F	F	

Build up the truth table with additional columns for slightly larger statements

Truth Tables

Larger statements can be made up of more than two smaller parts and/or more than one conjunction.

Truth tables are helpful for deducing when these larger statements are true and false.

Ex. When is $\neg P \vee Q$ true?

P	Q	$\neg P$
T	T	
T	F	
F	T	
F	F	

We know the truth values for NOT something from earlier

P	$\neg P$
T	F
F	T

Truth Tables

Larger statements can be made up of more than two smaller parts and/or more than one conjunction.

Truth tables are helpful for deducing when these larger statements are true and false.

Ex. When is $\neg P \vee Q$ true?

P	Q	$\neg P$
T	T	
T	F	
F	T	
F	F	

We know the truth values for NOT something from earlier

P	$\neg P$
T	$\boxed{F}$
F	T

Truth Tables

Larger statements can be made up of more than two smaller parts and/or more than one conjunction.

Truth tables are helpful for deducing when these larger statements are true and false.

Ex. When is $\neg P \vee Q$ true?

P	Q	$\neg P$
T	T	
T	F	
F	T	
F	F	

We know the truth values for NOT something from earlier

P	$\neg P$
T	$\boxed{\text{F}}$
F	T

Truth Tables

Larger statements can be made up of more than two smaller parts and/or more than one conjunction.

Truth tables are helpful for deducing when these larger statements are true and false.

Ex. When is $\neg P \vee Q$ true?

P	Q	$\neg P$
T	T	F
T	F	F
F	T	
F	F	

We know the truth values for NOT something from earlier

P	$\neg P$
T	F
F	T

Truth Tables

Larger statements can be made up of more than two smaller parts and/or more than one conjunction.

Truth tables are helpful for deducing when these larger statements are true and false.

Ex. When is $\neg P \vee Q$ true?

P	Q	$\neg P$
T	T	F
T	F	F
F	T	$\boxed{T}$
F	F	$\boxed{T}$

We know the truth values for NOT something from earlier

P	$\neg P$
T	F
F	$\boxed{T}$

Truth Tables

Larger statements can be made up of more than two smaller parts and/or more than one conjunction.

Truth tables are helpful for deducing when these larger statements are true and false.

Ex. When is $\neg P \vee Q$ true?

P	Q	$\neg P$
T	T	F
T	F	F
F	T	T
F	F	T

Truth Tables

Larger statements can be made up of more than two smaller parts and/or more than one conjunction.

Truth tables are helpful for deducing when these larger statements are true and false.

Ex. When is $\neg P \vee Q$ true?

Build up the truth table with additional columns for larger statements

P	Q	$\neg P$	$\neg P \vee Q$
T	T	F	
T	F	F	
F	T	T	
F	F	T	

Truth Tables

Larger statements can be made up of more than two smaller parts and/or more than one conjunction.

Truth tables are helpful for deducing when these larger statements are true and false.

Ex. When is $\neg P \vee Q$ true?

P	Q	$\neg P$	$\neg P \vee Q$
T	T	F	
T	F	F	
F	T	T	
F	F	T	

We know the truth values for something OR something from earlier

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Truth Tables

Larger statements can be made up of more than two smaller parts and/or more than one conjunction.

Truth tables are helpful for deducing when these larger statements are true and false.

Ex. When is $\neg P \vee Q$ true?

P	Q	$\neg P$	$\neg P \vee Q$
T	T	F	
T	F	F	
F	T	T	
F	F	T	

We know the truth values for something OR something from earlier

P	Q	$P \vee Q$
T	T	$\boxed{\text{T}}$
T	F	T
F	T	T
F	F	F

Truth Tables

Larger statements can be made up of more than two smaller parts and/or more than one conjunction.

Truth tables are helpful for deducing when these larger statements are true and false.

Ex. When is $\neg P \vee Q$ true?

P	Q	$\neg P$	$\neg P \vee Q$
T	T	F	
T	F	F	
F	T	T	$\boxed{T}$
F	F	T	

We know the truth values for something OR something from earlier

P	Q	$P \vee Q$
T	T	$\boxed{T}$
T	F	T
F	T	T
F	F	F

Truth Tables

Larger statements can be made up of more than two smaller parts and/or more than one conjunction.

Truth tables are helpful for deducing when these larger statements are true and false.

Ex. When is $\neg P \vee Q$ true?

P	Q	$\neg P$	$\neg P \vee Q$
T	T	F	
T	F	F	
F	T	T	T
F	F	T	T

We know the truth values for something OR something from earlier

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Truth Tables

Larger statements can be made up of more than two smaller parts and/or more than one conjunction.

Truth tables are helpful for deducing when these larger statements are true and false.

Ex. When is $\neg P \vee Q$ true?

P	Q	$\neg P$	$\neg P \vee Q$
T	T	F	$\boxed{\text{T}}$
T	F	F	
F	T	T	T
F	F	T	T

We know the truth values for something OR something from earlier

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	$\boxed{\text{T}}$
F	F	F

Truth Tables

Larger statements can be made up of more than two smaller parts and/or more than one conjunction.

Truth tables are helpful for deducing when these larger statements are true and false.

Ex. When is $\neg P \vee Q$ true?

P	Q	$\neg P$	$\neg P \vee Q$
T	T	F	T
T	F	F	$\boxed{F}$
F	T	T	T
F	F	T	T

We know the truth values for something OR something from earlier

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	$\boxed{F}$

Truth Tables

Practice: Finish the truth table for the statement $(P \to Q) \vee (Q \to R)$

P	Q	R
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Truth Table

Practice: Finish the truth table for the statement $(P \to Q) \vee (Q \to R)$

P	Q	R
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Note: $(P \to Q) \vee (Q \to R)$ is a ***tautology***, a statement that is always true.

Logical Equivalence

Two molecular statements *P* and *Q* are ***logically equivalen***t provided *P* is true precisely when *Q* is true.

In other words, *P* and *Q* are ***logically equivalent*** if they have the same truth value for any assignment of truth values to their atomic parts.

Logical Equivalence

Two molecular statements *P* and *Q* are ***logically equivalen***t provided *P* is true precisely when *Q* is true.

In other words, *P* and *Q* are ***logically equivalent*** if they have the same truth value for any assignment of truth values to their atomic parts.

Ex. We can use truth tables to show $\neg P \vee Q$ and $P \rightarrow Q$ are logically equivalent.

Logical Equivalence

Two molecular statements P and Q are ***logically equivalent*** provided P is true precisely when Q is true.

In other words, P and Q are ***logically equivalent*** if they have the same truth value for any assignment of truth values to their atomic parts.

Ex. We can use truth tables to show $\neg P \vee Q$ and $P \to Q$ are logically equivalent.

P	Q	$\neg P$	$\neg P \vee Q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

Logical Equivalence

Two molecular statements P and Q are ***logically equivalent*** provided P is true precisely when Q is true.

In other words, P and Q are ***logically equivalent*** if they have the same truth value for any assignment of truth values to their atomic parts.

Ex. We can use truth tables to show $\neg P \vee Q$ and $P \to Q$ are logically equivalent.

P	Q	$\neg P$	$\neg P \vee Q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

Logical Equivalence

Two molecular statements P and Q are ***logically equivalent*** provided P is true precisely when Q is true.

In other words, P and Q are ***logically equivalent*** if they have the same truth value for any assignment of truth values to their atomic parts.

Ex. We can use truth tables to show $\neg P \vee Q$ and $P \to Q$ are logically equivalent.

P	Q	$\neg P$	$\neg P \vee Q$
T	T	F	$\boxed{\text{T}}$
T	F	F	F
F	T	T	T
F	F	T	T

P	Q	$P \to Q$
T	T	$\boxed{\text{T}}$
T	F	F
F	T	T
F	F	T

Logical Equivalence

Two molecular statements P and Q are ***logically equivalen***t provided P is true precisely when Q is true.

In other words, P and Q are ***logically equivalent*** if they have the same truth value for any assignment of truth values to their atomic parts.

Ex. We can use truth tables to show $\neg P \vee Q$ and $P \rightarrow Q$ are logically equivalent.

P	Q	$\neg P$	$\neg P \vee Q$
T	T	F	T
T	F	F	$\boxed{F}$
F	T	T	T
F	F	T	T

P	Q	$P \rightarrow Q$
T	T	T
T	F	$\boxed{F}$
F	T	T
F	F	T

Logical Equivalence

Two molecular statements *P* and *Q* are ***logically equivalent*** provided *P* is true precisely when *Q* is true.

In other words, *P* and *Q* are ***logically equivalent*** if they have the same truth value for any assignment of truth values to their atomic parts.

Ex. We can use truth tables to show $\neg P \vee Q$ and $P \rightarrow Q$ are logically equivalent.

P	Q	$\neg P$	$\neg P \vee Q$
T	T	F	T
T	F	F	F
F	T	T	$\boxed{T}$
F	F	T	T

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	$\boxed{T}$
F	F	T

Logical Equivalence

Two molecular statements *P* and *Q* are ***logically equivalent*** provided *P* is true precisely when *Q* is true.

In other words, *P* and *Q* are ***logically equivalent*** if they have the same truth value for any assignment of truth values to their atomic parts.

Ex. We can use truth tables to show $\neg P \vee Q$ and $P \rightarrow Q$ are logically equivalent.

P	Q	$\neg P$	$\neg P \vee Q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	$\boxed{\text{T}}$

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	$\boxed{\text{T}}$

Practice: Show that $\neg(P \vee Q)$ is logically equivalent to $\neg P \wedge \neg Q$

Logical Equivalence

In other words, P and Q are ***logically equivalent*** if they have the same truth value for any assignment of truth values to their atomic parts.

Ex. We can use truth tables to show $\neg P \vee Q$ and $P \rightarrow Q$ are logically equivalent.

P	Q	$\neg P$	$\neg P \vee Q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	$\boxed{T}$

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	$\boxed{T}$