Discrete Structures– Counting Pt. 1

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Plan for Today

- Counting
  - Additive and Multiplicative Principles
  - Sets
  - Inclusion/Exclusion Principles

Warm Up: Functions

A ***function*** is a rule that assigns each input exactly one output.

- When a function maps the domain to everything in the codomain (i.e the codomain is the range) we say it is ***onto***.
- When a function is such that each element of the codomain is the image of at most one element of the domain, we say it is ***one-to-one***.
- An onto *and* one-to-one function is a ***bijective*** function.

Practice: Are the following functions? If yes, are they onto, one-to-one, or bijective?

1. $f: \{1,2,3,4,5,6\} \to \{a,b,c,d\}$,
$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & a & b & b & b & c \end{pmatrix}$$
2. $\mathrm{g}: \mathbb{Z} \to \mathbb{Z}, \mathrm{g}(n) = n^2 + 1$

Motivation

A restaurant offers 8 appetizers and 14 entrées. How many choices do you have if:

- You will eat one dish, either an appetizer or entrée?
- You will each two dishes, one appetizer and one entrée?

Answer these questions then think about what rules you used to answer them. Write down the rules you have.

Motivation

Answer these questions using the rules you developed. Do they work?

A standard deck of playing cards has 26 red cards and 12 face cards.

- How many ways can you select a card that is either red or a face?
- How many ways can you select a card which is red and a face?
- How many ways can you select two cards so that the first one is red and the second is a face?

Additive Principle

The ***additive principle*** states that if event A can occur in m ways and event B can occur in n *disjoint* ways, then the event “A or B” can occur in m+n ways.

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- How many ways can you select a card that is either red or a face?
  - Red and face are *not disjoint* which is why the answer to this question is not 26 + 12 = 38

- How many ways can you select a card that is either a Jack or a Queen?
  - Jack and queen are *disjoint* which is why the answer to this question is 4 + 4 = 8

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Practice: How many two letter, alphabetical, lower-case strings start with either A or B?

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Practice: Suppose you are going out for ice cream. There are 6 ice cream flavors and 4 toppings. How many choices do you have if you must choose an ice cream and a topping?

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5 * 26 = 130

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Practice: How many license plates can you make out of three letters followed by 3 numerical digits?

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Practice: How many passwords can you make out of four letters followed by four numerical digits followed by two punctuation (. , ? ; : !)?

Sets & Counting

We will represent events as sets of possible outcomes.

Ex. Let A = {set of 9 shirts}, B = {set of 5 pants}

To make an outfit we need 1 shirt and one pant, how many outfits can we make?

9*5

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Notice, this question is asking for pairs of shirts and pants, i.e. the Cartesian product of A and B.

From before, we know $|A \times B| = |A| * |B|$

Sets & Counting

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Ex. Let A = {set of 9 shirts}, B = {set of 5 pants}

Let’s say you’re staying in and want to wear a shirt or pants (only one). How many outfits can you make?

$9 + 5$

Sets & Counting

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Ex. Let A = {set of 9 shirts}, B = {set of 5 pants}

Let's say you're staying in and want to wear a shirt or pants (only one). How many outfits can you make?

9 + 5

Notice, the question is asking $|A \cup B|$. Since there is no overlap, $|A \cup B| = |A| + |B|$

Sets & Counting

We will represent events as sets of possible outcomes.

Practice: Conceptualize the following as set problems.

- How many two letter, alphabetical, lower-case strings start with one of the 5 vowels?
- Suppose you are going out for ice cream. There are 6 ice cream flavors and 4 toppings. How many choices do you have if you must choose an ice cream and a topping?
- How many license plates can you make out of three letters followed by 3 numerical digits?
- How many passwords can you make out of four letters followed by four numerical digits followed by two punctuation (. , ? ; : !)?

Inclusion/ Exclusion

A recent buzz marketing campaign for Village Inn surveyed patrons on their pie preferences. People were asked whether they enjoyed (A) Apple, (B) Blueberry or (C) Cherry pie (respondents answered yes or no to each type of pie, and could say yes to more than one type). The following table shows the results of the survey.

Pies enjoyed:	A	B	C	AB	AC	BC	ABC
Number of people:	20	13	26	9	15	7	5

Represent these results with a Venn Diagram

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How many people enjoy at least one of the kinds of pie?

Inclusion/ Exclusion

An examination in three subjects, Algebra, Biology, and Chemistry, was taken by 41 students. The following table shows how many students failed in each single subject and in their various combinations:

Subject:	A	B	C	AB	AC	BC	ABC
Failed:	12	5	8	2	6	3	1

Represent these results with a Venn Diagram

How many students failed at least one subject?

Inclusion/ Exclusion

What do you notice about calculating $|A \cup B \cup C|$ from the last two examples?

How does $|A \cup B \cup C|$ related to the cardinality of each individual set and the cardinality of their intersections?

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For finite sets, A, B, C

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

Count all elements in each set

Remove double counts

Add back anything removed in all 3 double count subtractions

Practice: A group of college students were asked about their TV watching habits. Of those surveyed, 28 students watch *The Walking Dead*, 19 watch *The Blacklist*, and 24 watch *Game of Thrones*. Additionally, 16 watch *The Walking Dead* and *The Blacklist*, 14 watch *The Walking Dead* and *Game of Thrones*, and 10 watch *The Blacklist* and *Game of Thrones*. There are 8 students who watch all three shows. How many students surveyed watched at least one of the shows?

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Putting it all together

Practice: Consider all 5 character strings made from letters a – h.

- How many of these strings are there total?
- How many of these strings contain no repeated letters?
- How many of these strings start with the substring "aha"?
- How many of these strings either start with "aha" or end with "bah" or both?
- How many of the strings containing no repeats also do not contain the substring "bad"?