

Discrete Structures— Graphs: Euler Paths and Circuits

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Plan for Today

- Final Project
- Recap graph definitions
- Euler Paths & Circuits

Final Project

- Questions?
- Remember, your proposal is due on Thursday ***before class***

Warm Up: Recap

Form 5 Groups

- Each group will get a set of topics (listed to the right)
- For your topics:
 - Define each
 - Show an example of each
 - Choose 1 and provide a practice problem
 - You will walk us all through your definitions, examples, and practice problem

Group 1:

- Graph definition, drawing
- Multigraph
- Connected graph
- Complete graph

Group 2:

- Graph equality
- Graph isomorphism

Group 3:

- Subgraph
- Induced subgraph

Group 4:

- Handshake lemma
- Proposition: In any graph, the number of vertices with odd degree must be even.

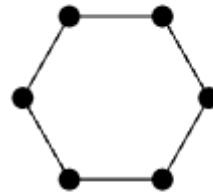
Group 5:

- Bipartite graph
- Complete bipartite graph

Named Graphs

C_n : The cycle on n vertices, just one big loop.

P_n : The path on $n + 1$ vertices (so n edges), just one long path.



C_6

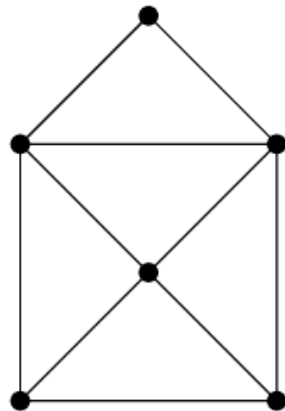


P_5

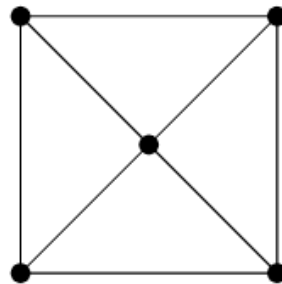
Motivation

Form 4 groups. With your group consider the graph you were assigned...

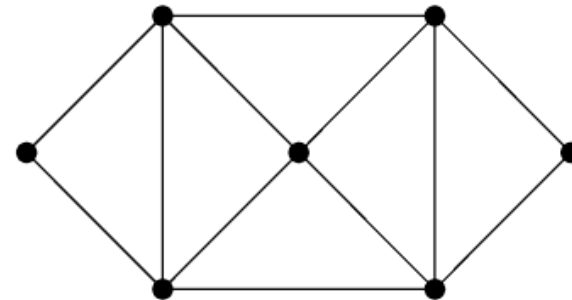
- How many nodes does your graph have? How many edges?
- Can you make a path through the graph that uses every edge exactly once?
- Can you do the above but also start and end at the same vertex?



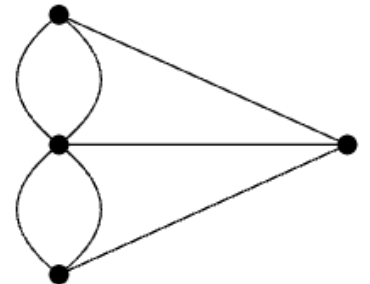
A



B



C



D

Euler Paths & Circuits

An ***Euler path***, in a graph or multigraph, is a path through the graph which uses every edge exactly once.

An ***Euler circuit*** is a Euler path which stops and starts at the same vertex.

Euler Paths & Circuits

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Based on what we saw earlier, do you think there is an easy and quick way to determine *if* a graph has an Euler path or circuit?

(Notice I said *if* an Euler path or circuit exists, not what the path or circuit is.)

Hamiltonian Paths

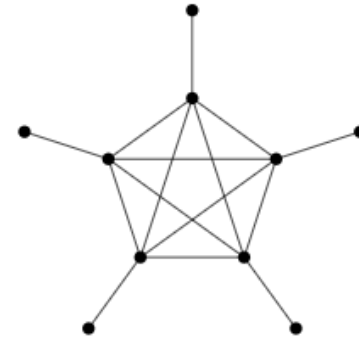
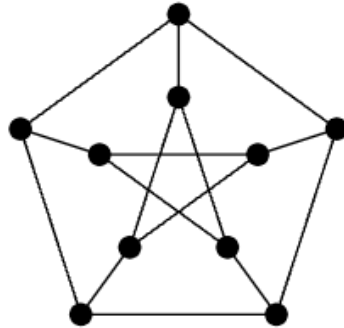
A ***Hamiltonian path*** is a path which visits every vertex exactly once.

A ***Hamiltonian cycle*** is a Hamiltonian path that starts and stops at the same vertex.

Hamiltonian Paths

A **Hamiltonian path** is a path which visits every vertex exactly once.

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Do these graphs have a Hamiltonian path?

Hamiltonian Paths

A ***Hamiltonian path*** is a path which visits every vertex exactly once.

A ***Hamiltonian cycle*** is a Hamiltonian path that starts and stops at the same vertex.

There is no known simple test for determining whether a graph has a Hamiltonian path! This is an example of an NP-complete problem, which means it is (currently) too difficult for even a computer to solve!