

Discrete Structures–Sets

Dr. Ab Mosca (they/them)

Plan for Today

- Sets
 - Notation
 - Relationships
 - Operations

Warm Up: Induction

To perform a proof by **induction** on $P(n)$:

1. Start with your **base case**
 - Prove $P(n)$ is true for the smallest value of n possible
2. Perform the **inductive step**
 - Prove that $P(k) \rightarrow P(k + 1)$ for all k greater than or equal to the smallest possible value of n .
 - Note this is an if ... then ... proof, so we start by assuming $P(k)$ is true. This is called the **inductive hypothesis** for this type of proof.

Practice: Prove the following...

For all natural numbers, n , $6^n - 1$ is a multiple of 5.

Set Vocab

A **set** is an unordered collection of objects.

Ex. Snacks in your pantry

Ex. Natural numbers between 1 and 10 inclusive

Set Vocab

A **set** is an unordered collection of objects.

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Ex. Natural numbers between 1 and 10 inclusive

Notation:

- A set is denoted with $\{ \}$
- We usually use a capital letter to represent a set

Ex. $A = \{chips, pretzels, cookies, peanuts\}$

Ex. $B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Set Vocab

A **set** is an unordered collection of objects.

Ex. Snacks in your pantry

Ex. Natural numbers between 1 and 10 inclusive

Notation:

- We usually use a lowercase letter to represent an element of a set
- \in is read “is an element of” and \notin is read “is not an element of”

Ex. *cookies* $\in A$

Ex. 11 $\notin B$

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- A set can be an element of another set.

Ex. $A = \{a, b, \{x, y, z\}\}$

A contains elements a , b , and $\{x, y, z\}$

Set Vocab

Special Sets

There are some sets we use so often that we give them special names and symbols.

- \emptyset is the ***empty set***, i.e. the set containing no elements ($\{\}$)
- \mathbb{N} is the set of ***natural numbers***, $\{0, 1, 2, 3, \dots\}$
- \mathbb{Z} is the set of ***integers***, $\{\dots, -2, -1, 0, 1, 2, \dots\}$
- \mathbb{Q} is the set of ***rational numbers***
- \mathbb{R} is the set of ***real numbers***

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Practice: Write the following sets symbolically

- The set of classes you are taking this semester
- The set of all special sets listed above
- The set of even numbers

Set Builder Notation

Sometimes, listing all elements of a set is hard or imprecise

Ex. Let A be the set of all even natural numbers

$$A = \{0, 2, 4, 6, \dots\}$$

In these cases, we use ***set builder notation***

$$A = \{x \in \mathbb{N} : x \text{ is even}\}$$

This is read, “ A is the set of all natural numbers x such that x is even”

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Practice: Write the set builder notation for the following sets

- $\{x : x + 3 \in \mathbb{N}\}$
- $\{x \in \mathbb{N} : x + 3 \in \mathbb{N}\}$
- $\{x : x \in \mathbb{N} \vee -x \in \mathbb{N}\}$
- $\{x^2 : x \in \mathbb{N}\}$

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Practice: List the elements or describe the following sets

- The set of natural numbers between 10 and 30 (including 30)
- The set of primes
- The set of odds
- The set of positive odds

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Relationships Between Sets

Sets A and B are **equal**, if they have exactly the same elements

$$\text{Ex. } \{1, 2, 3\} = \{1, 1 + 1, 1 + 1 + 1\}$$

Set A is a **subset** of set B if every element in A is also an element of B

$$\text{Ex. } \{1, 2, 3\} \subseteq \{1, 2, 3, 4\}$$

The **power set** of A is the set of all subsets of A

$$\text{Ex. } A = \{1, 2, 3\},$$

$$\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$$

empty set is
always a subset!

Relationships Between Sets

Sets A and B are **equal**, if they have exactly the same elements

Practice:

What is the power set of $A = \{a, b\}$?

What is the power set of $A = \{a, b, c\}$?

What is the power set of $B = \{a, b, c, d\}$?

The **power set** of A is the set of all subsets of A

Ex. $A = \{1, 2, 3\}$,

$$\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$$

empty set is
always a subset!

The size of a set (i.e. the number of elements in a set) is called it ***cardinality***.

$$\text{Ex. } A = \{a, b\}, |A| = 2$$

$$A = \emptyset, |A| = 0$$

Cardinality

Practice: What are the cardinalities of the power sets you found?

power set of $A = \{a, b\}$?

power set of $A = \{a, b, c\}$?

power set of $B = \{a, b, c, d\}$?

Do you notice a pattern?

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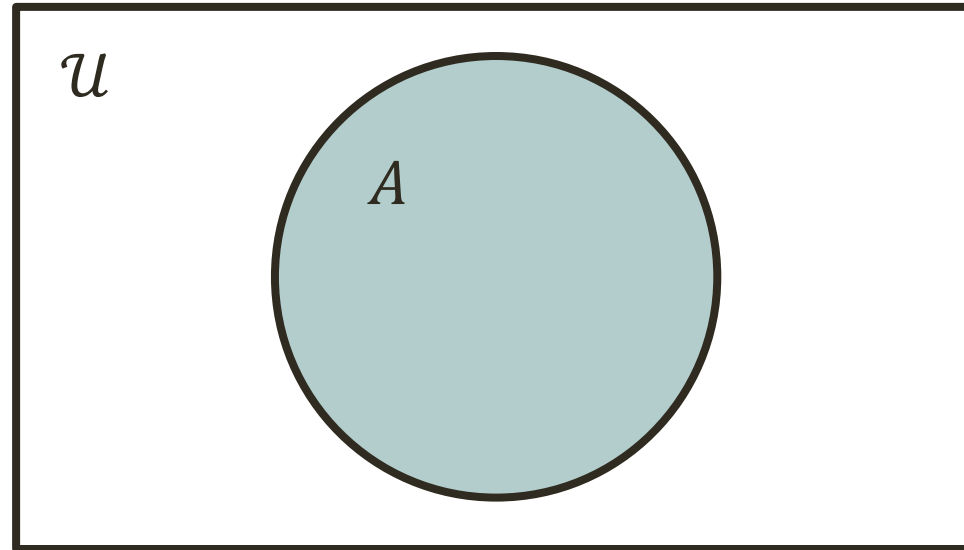
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Cardinality

Let $|A| = n$, the cardinality of $\mathcal{P}(A) = 2^n$.

Often, when we deal with sets we have some notion of what “everything” is. We call this the **universe**, \mathcal{U} .

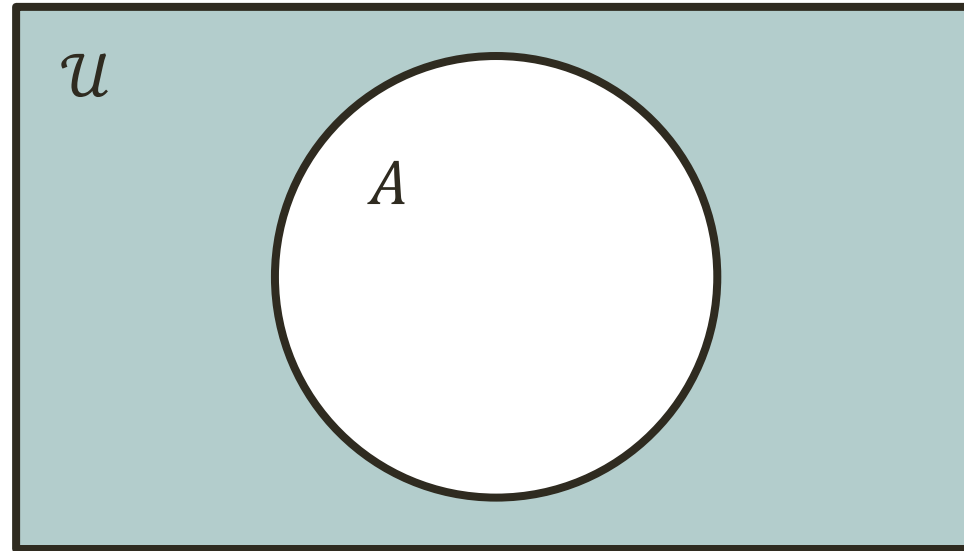
We visualize sets within the universe:



Set Operations

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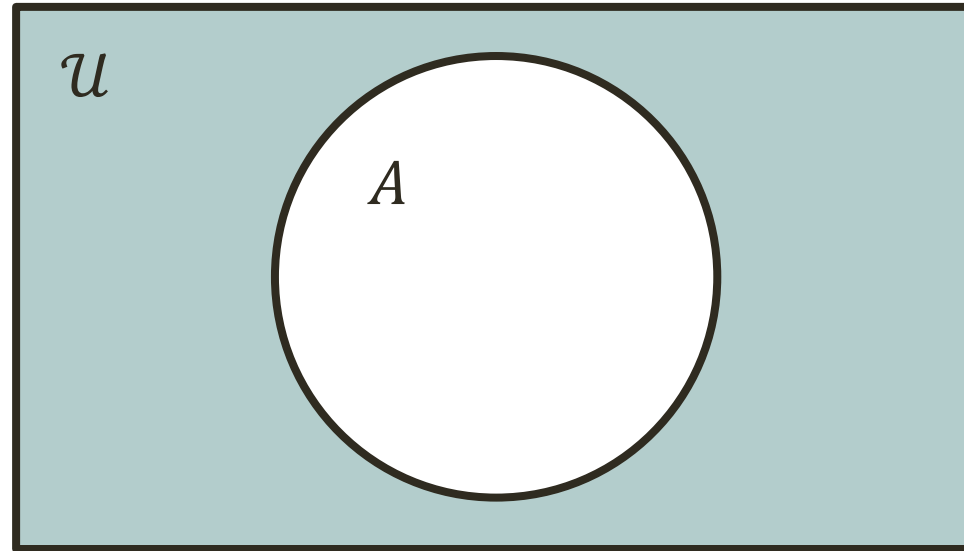
The **complement** of A (denoted \bar{A}) is the set of all elements in the universe that are not in A



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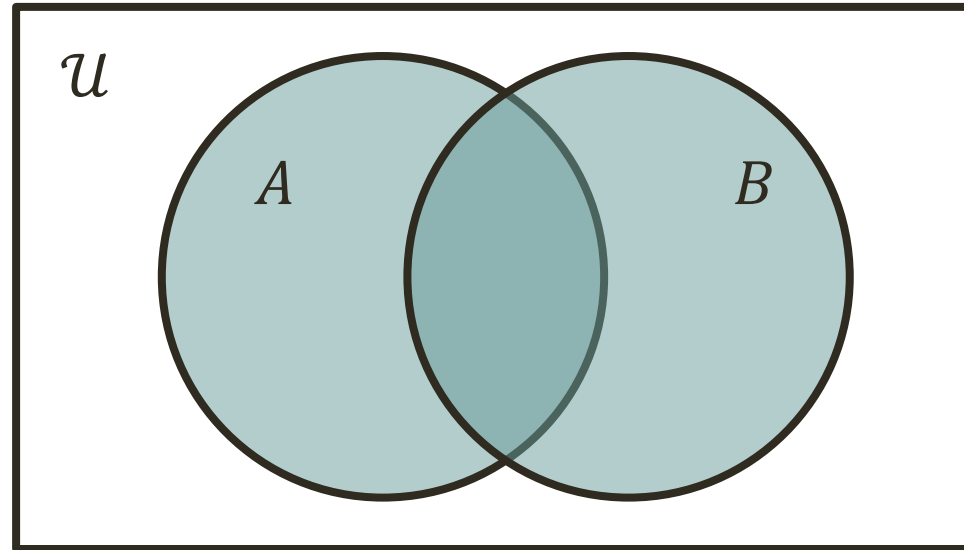
Set Operations

Practice: Let $A = \{x : x \in \mathbb{N} \text{ and } x = 2k \text{ for some } k \in \mathbb{N}\}$
What is \bar{A} ?

Set Operations

Let A, B be sets.

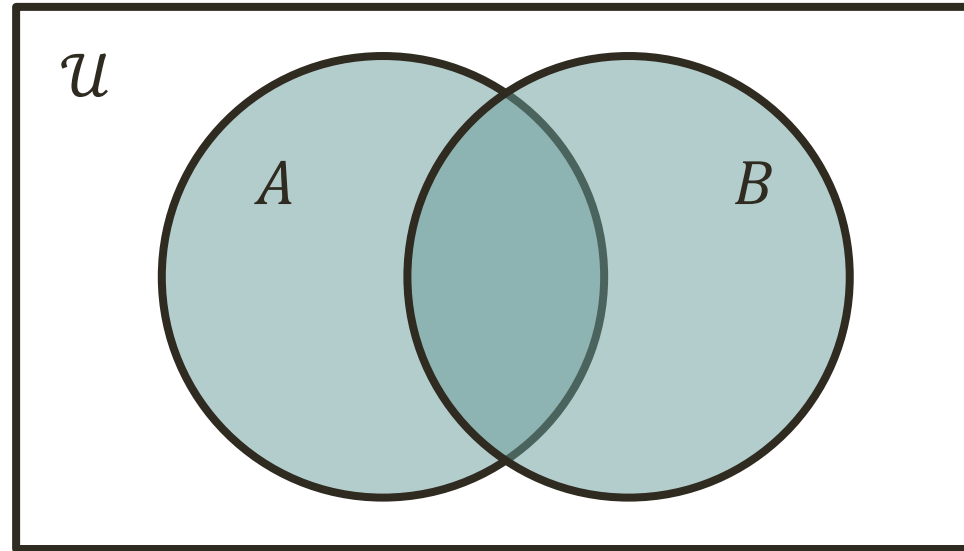
The **union** of A and B (denoted $A \cup B$) is the set of all elements in A , or B , or both



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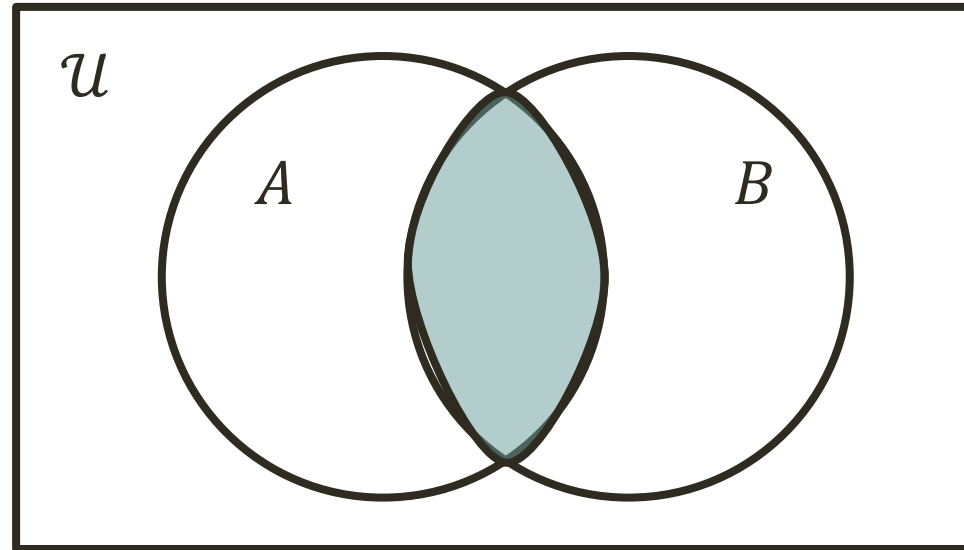


Practice: Let $A = \{x : x \in \mathbb{N} \text{ and } x = 2k \text{ for some } k \in \mathbb{N}\}$, and $B = \{x : x \in \mathbb{N} \text{ and } x = 3k \text{ for some } k \in \mathbb{N}\}$
What is $A \cup B$?

Set Operations

Let A, B be sets.

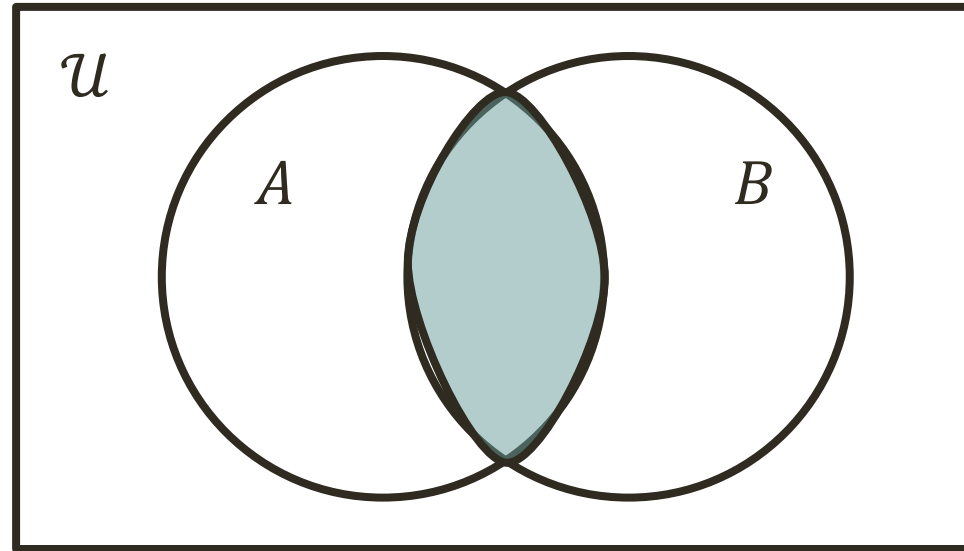
The ***intersection*** of A and B (denoted $A \cap B$) is the set of all elements in A and B



Set Operations

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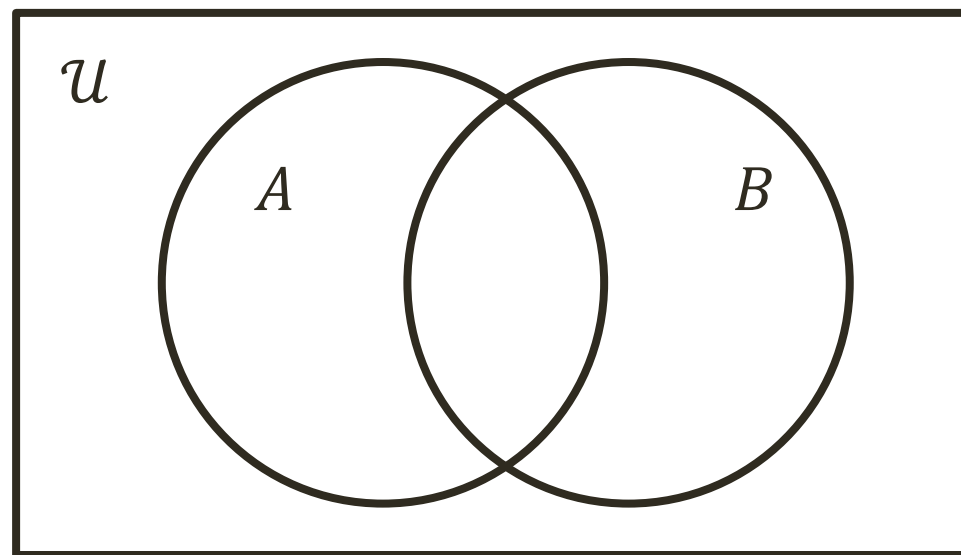


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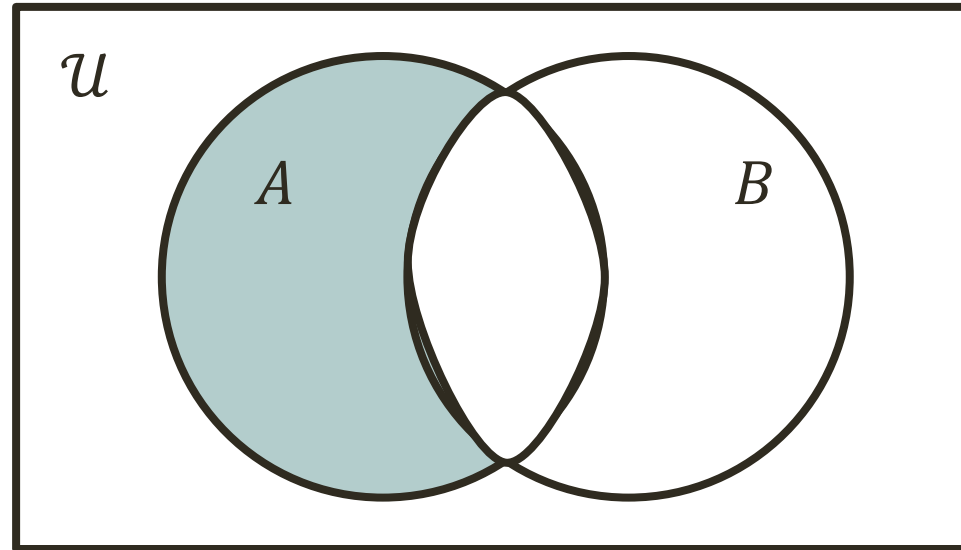


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What is $A \cap B$? Visualize it with a Venn diagram.

Set Operations

Let A, B be sets.

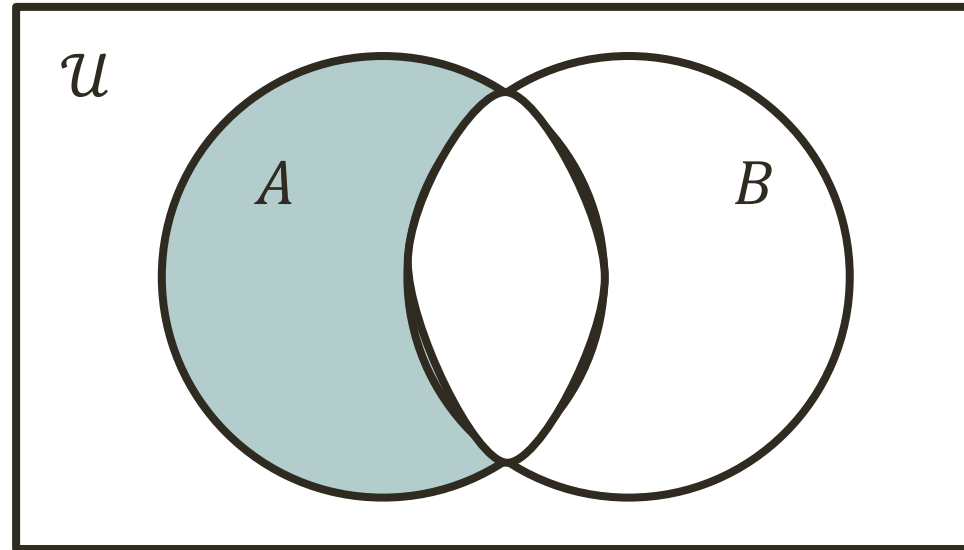
The **set difference** of A and B (denoted $A \setminus B$) is the set of all elements in A that are not in B



Set Operations

Let A, B be sets.

The **set difference** of A and B (denoted $A \setminus B$) is the set of all elements in A that are not in B



Practice: Is set difference of A and B equivalent to $A \cap \bar{B}$?

Set Operations

Let A, B be sets.

The ***cartesian product*** of A and B (denoted $A \times B$) is the set of all ordered pairs where the first element is from A and the second is from B .

$$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$$

Practice: What will the cardinality of $A \times B$ be?

Set Operations

Practice: Let $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 6\}$, $C = \{1, 2, 3\}$, $D = \{7, 8, 9\}$, $\mathcal{U} = \{1, 2, \dots, 10\}$. Find:

1. $A \cup B$
2. $A \cap B$
3. $B \cap C$
4. $A \cap D$
5. $\overline{B \cup C}$
6. $A \setminus B$
7. $(D \cap \bar{C}) \cup \overline{A \cap B}$
8. $\emptyset \cup C$
9. $\emptyset \cap C$
10. $B \times C$

Visualize each with a Venn diagram.