Discrete Structures– Propositional Logic

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Plan for Today

- Truth conditions for connectives
- Truth Tables
- Logical Equivalence

Warm Up

statement: any declarative sentence which is either true or false

- ***atomic***: a statement that *cannot* be divided into smaller statements
- ***molecular***: a statement that *can* be divided into smaller statements

logical connectives:

- conjunction- $P \wedge Q$ (read: P and Q)
- disjunction- $P \vee Q$ (read: P or Q)
- implication or conditional- $P \rightarrow Q$ (read: if P then Q)
- biconditional- $P \leftrightarrow Q$ (read: P if and only if Q)
- negation- $\neg P$ (read: not P)

Take these two statements: (1) it snowed today, and (2) class is cancelled.

For each logical connective, write a molecular statement made up of these two statements. Write each molecular statement in English and symbolically.

Ex.

It snowed today and class is cancelled (English)

Let P = It snowed today, Q = class is cancelled, $P \wedge Q$ (symbols)

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  - $P \wedge Q$ is true when both P and Q are true.
  - $P \vee Q$ is true when P or Q or both are true.
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  - $P \leftrightarrow Q$ is true when P and Q are both true or both false.
  - $\neg P$ is true when P is false.
- Note: This or is called ***inclusive or***, there is also an ***exclusive or*** which is true when P or Q is true but not when both are true.

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Practice: Let P = It snowed today, Q = class is cancelled, what is the truth value for each of the statements you wrote earlier **based on the truth of P and Q today**.

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P
T
F

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Let P be a statement. The truth table for P is:

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Let Q be a statement. The truth table for Q is:

Q
T
F

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One way we model possible truth values is with ***truth tables***

When we combine statements with conjunctives, there is a finite number of truth value combinations:

Let P and Q be statements. One of these must be the case:

P is true and **Q is true**

P is true and **Q is false**

P is false and **Q is true**

P is false and **Q is false**

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Conjunction: $P \wedge Q$ is true when both P and Q are true.

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

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Disjunction: $P \vee Q$ is true when P or Q or both are true.

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

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Implication: $P \rightarrow Q$ is true when P is false or Q is true or both.

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

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Biconditional: $P \leftrightarrow Q$ is true when P and Q are both true or both false.

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

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Negation: $\neg P$ is true when P is false.

P	$\neg P$
T	F
F	T

Tru

Practice: Form five groups. Each will be assigned a conjunction. Come up with a real world scenario that your conjunction accurately represents. Be ready to share your answers!

Ex. If I had conjunction and were one of you, I might say: Let P = I study discrete math, and Q = I am taking MATH220

The truth table and real work scenarios are:

P	Q	$P \land Q$	
T	T	T	I study discrete math AND I am taking MATH220.
T	F	F	I study discrete math AND I am not taking MATH220.
F	T	F	I do not study discrete math AND I am taking MATH220.
F	F	F	I do not study discrete math AND I am not taking MATH220.

Truth Tables

Larger statements can be made up of more than two smaller parts and/or more than one conjunction.

Truth tables are helpful for deducing when these larger statements are true and false.

Ex. When is $\neg P \vee Q$ true?

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T	T
T	F
F	T
F	F

Start the table with all possible combinations of T/F for smallest statements

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Ex. When is $\neg P \vee Q$ true?

P	Q	$\neg P$
T	T	
T	F	
F	T	
F	F	

Build up the truth table with additional columns for slightly larger statements

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T	T	
T	F	
F	T	
F	F	

We know the truth values for NOT something from earlier

P	$\neg P$
T	F
F	T

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Build up the truth table with additional columns for larger statements

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T	T	F	
T	F	F	
F	T	T	
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T	F	F	
F	T	T	
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We know the truth values for something OR something from earlier

P	Q	$P \vee Q$
T	T	T
T	F	T
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T	T	$\boxed{\text{T}}$
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T	F	F	$\boxed{\text{F}}$
F	T	T	T
F	F	T	T

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P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	$\boxed{\text{F}}$

Truth Tables

Practice: Finish the truth table for the statement $(P \rightarrow Q) \vee (Q \rightarrow R)$

P	Q	R
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Truth Table

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P	Q	$\boldsymbol{R}$
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Note: $(P \to Q) \vee (Q \to R)$ is a ***tautology***, a statement that is always true.

Logical Equivalence

Two molecular statements *P* and *Q* are ***logically equivalen***t provided *P* is true precisely when *Q* is true.

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T	F	F	F
F	T	T	T
F	F	T	T

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T	T	$\boxed{T}$
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T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	$\boxed{T}$

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	$\boxed{T}$

Practice: Show that $\neg(P \vee Q)$ is logically equivalent to $\neg P \wedge \neg Q$

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T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	$\boxed{T}$

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	$\boxed{T}$