MATH220: Discrete Structures

Spring 2024

In-class Activity 03: Sequences

Assignment is DUE as indicated on the course schedule. This assignment is designed to be completed in class.

*This is a **group assignment**. Split into 4 groups, and submit as a group on Gradescope.*

Notes

Your final submission must be readable. It is your responsibility to write up your answers in a way that is easy to read and follow.

Towers of Hanoi

For today's activity you will investigate recurrences through a real work example, a game called the Towers of Hanoi.

In the Towers of Hanoi game there are three pegs and rings of decreasing diameter. The objective of the game it to move the tower of rings from peg A (the start) to peg C (the end). You can use the remaining peg B as a helper (or auxiliary). When moving rings you must follow 2 rules:

1. You may only move one disk at a time
2. You may never move a larger disk onto a smaller one

Play the game with all the disks, see if you can solve it. Pay attention for patterns.

Once you are familiar with the game, move on to your tasks.

Tasks

Your overarching goal is to be able to answer the question: How many moves are necessary and sufficient to win the Towers of Hanoi game?

Work up to answering this question by answering the following:

- Play the game with 0, 1, and 2 disks. How many moves does it take to optimally win (i.e. win with the minimum number of moves) in each case? Record these values in a document that you will turn in at the end of this project.
- Now, play the game with 3 disks. How many moves does it take to optimally win?
- Think about the Towers of Hanoi as a ***recursive problem***. What is the base case and what is the recursive step? What does this tell you about the number of moves needed to optimally win the game in general (in terms of moves for the base case + moves for the recursive step)? Write a recurrence relation describing this relationship.
- Solve the recurrence relation to get a closed formula for number of moves. (You can use any method you want to solve the recurrence relation.) Use induction to prove that your closed formula is correct.

Neatly write up your answers to all of the above and submit as a group on Gradescope.

Submission

Submit on Gradescope. Remember to tag your groupmates!

Rubric

	Missing / Not Complete (0)	Approaching (2)	Meets (4)	Exceeds (5)
Readability	Assignment is unreadable or not submitted.	Assignment includes formatting, but significant improvements could be made. For example, clear labeling of problems and subparts, proofreading.	Assignment includes formatting, but minor improvements could be made. For example, clear labeling of problems and subparts, proofreading.	Assignment is well formatted and easy to read. Text has been proofread.
Completeness	Less than half of assignment is attempted.	Roughly half of assignment has been attempted. On the problems that have been completed, effort is evident. OR All of the assignment has been attempted, but effort is not evident in many parts.	At least 80% of assignment has been attempted. On the problems that have been completed, effort is evident. OR All of the assignment has been attempted, but effort is not evident a few parts.	All of the assignment has been attempted, and effort evident throughout.
Correctness	All answers are incorrect or missing.	Of the complete problems, at least half have been approached and completed correctly.	Of the complete problems, at least 80% have been approached and completed correctly.	All complete problems are approached and completed correctly.