Discrete Structures– Graphs: Euler Paths and Circuits

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Plan for Today

- Recap graph definitions
- Euler Paths & Circuits

Warm Up: Recap

Form 5 Groups

- Each group will get a set of topics (listed to the right)
- For your topics:
  - Define each
  - Show an example of each
  - Choose 1 and provide a practice problem
  - You will walk us all through your definitions, examples, and practice problem

Group 1:
- Graph definition, drawing
- Multigraph
- Connected graph
- Complete graph

Group 2:
- Graph equality
- Graph isomorphism

Group 3:
- Subgraph
- Induced subgraph

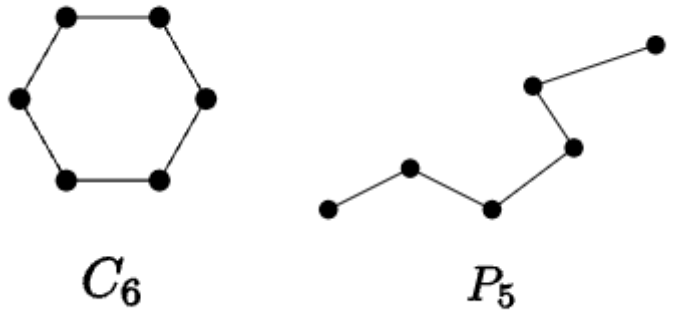
Group 4:
- Handshake lemma
- Proposition: In any graph, the number of vertices with odd degree must be even.

Group 5:
- Bipartite graph
- Complete bipartite graph

Named Graphs

C_n: The cycle on n vertices, just one big loop.

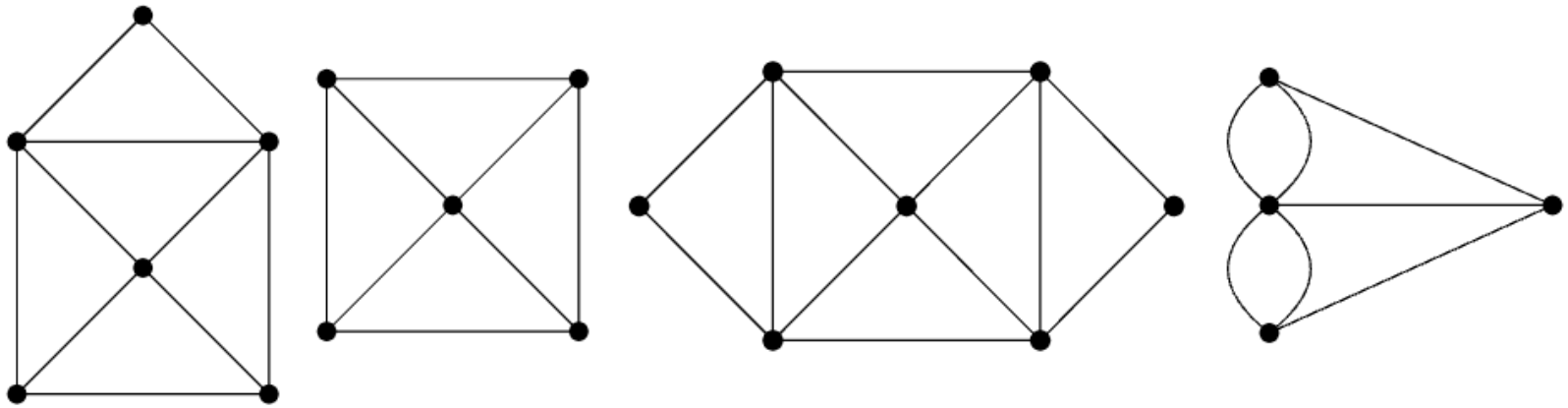
P_n: The path on $n + 1$ vertices (so n edges), just one long path.



Motivation

Form 4 groups. With your group consider the graph you were assigned…

- How many nodes does your graph have? How many edges?
- Can you make a path through the graph that uses every edge exactly once?
- Can you do the above but also start and end at the same vertex?



A B C D

Euler Paths & Circuits

An ***Euler path***, in a graph or multigraph, is a path through the graph which uses every edge exactly once.

An ***Euler circuit*** is a Euler path which stops and starts at the same vertex.

Euler Paths & Circuits

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An ***Euler circuit*** is a Euler path which stops and starts at the same vertex.

Based on what we saw earlier, do you think there is an easy and quick way to determine ***if*** a graph has an Euler path or circuit?

(Notice I said ***if*** an Euler path or circuit exists, not what the path or circuit is.)

Hamiltonian Paths

A ***Hamiltonian path*** is a path which visits every vertex exactly once.

A ***Hamiltonian cycle*** is a Hamiltonian path that starts and stops at the same vertex.

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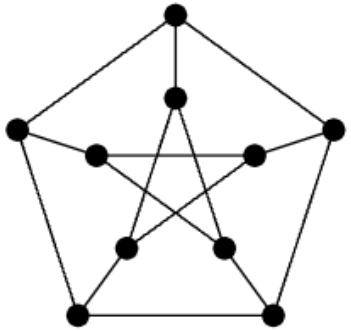

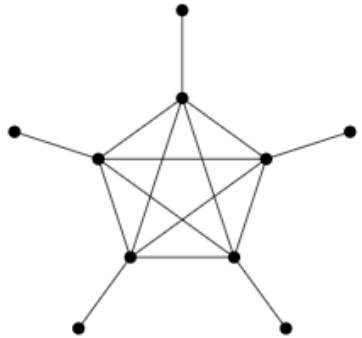

Do these graphs have a Hamiltonian path?

Hamiltonian Paths

A ***Hamiltonian path*** is a path which visits every vertex exactly once.

A ***Hamiltonian cycle*** is a Hamiltonian path that starts and stops at the same vertex.

There is no known simple test for determining whether a graph has a Hamiltonian path! This is an example of an NP-complete problem, which means it is (currently) too difficult for even a computer to solve!