Discrete Structures– Counting Pt. 2

Dr. Ab Mosca (they/them)

Plan for Today

- Counting
  - Combinations
  - Permutations

Warm Up: Counting

The ***additive principle*** states that if event A can occur in m ways and event B can occur in n *disjoint* ways, then the event "A or B" can occur in m+n ways.

The ***multiplicative principle*** states that if event A can occur in m ways and each possibility for A allows for exactly n ways for event B, then the event "A and B" can occur in m*n ways.

You have a bunch of chips which come in five different colors: blue, black, purple, white, and green.

How many different two-chip stacks can you make if the bottom chip must be blue or black? Explain your answer using both the additive and multiplicative principles.

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You have a bunch of chips which come in five different colors: blue, black, purple, white, and green.

How many different three-chip stacks can you make if the bottom chip must be blue or black and the top chip must be purple, white, or green? How does this problem relate to the previous one?

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You have a bunch of chips which come in five different colors: blue, black, purple, white, and green.

How many different three-chip stacks are there in which no color is repeated? What about four-chip stacks?

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You have a bunch of chips which come in five different colors: blue, black, purple, white, and green.

Suppose you wanted to take three different colored chips and put them in your pocket. How many different choices do you have? What if you wanted four different colored chips? How do these problems relate to the previous one?

Permutations

A ***permutation*** is a (possible) rearrangement of objects.

Ex. consider the letters a, b, c

There are 6 permutations of these letters:

abc, acb, bac, bca, cab, cba

3 choices for the 1st letter, 2 for the second, 1 for the third: 3*2*1 = 6

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How many permutations are there of a, b, c, d, e, f?

Do you notice a pattern?

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How many functions $f: \{1, 2, \ldots, 8\} \rightarrow \{1, 2, \ldots, 8\}$ are bijective?

Hint: How can permutations help you answer this?

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We write this $P(n, k)$, an call it a ***k-permutation of n elements***

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How many four letter strings can you make from the characters a, b, c, d, e, f with no repeated characters?

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We write this $P(n,k)$, an call it a ***k-permutation of n elements***

$$P(n,k) = \frac{n!}{(n-k)!} = n(n-1)(n-2)\ldots(n-(k-1))$$

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How many functions $f: \{1,2,3\} \to \{1,2,3,4,5,6,7,8\}$ are injective?

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Combinations

What if order doesn't matter?

In other words, what if you have n objects and you want to know how many different ways you can choose k of them?

A ***combination*** is the number of ways to choose k objects from n.

We write this $C(n, k)$ or $\binom{n}{k}$, and read both ***n choose k***.

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How can we calculate $\binom{n}{k}$? Hint: Start with $P(n, k)$

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Combinations

A pizza parlor offers 10 toppings. how many 3-topping pizzas could the put on their menu? (The toppings must all be different, no repeats.)

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