## Discrete Structures— Propositional Logic

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#### Plan for Today

- Truth conditions for connectives
- Truth Tables
- Logical Equivalence

#### Warm Up

**statement**: any declarative sentence which is either true or false

- atomic: a statement that cannot be divided into smaller statements
- molecular: a statement that can be divided into smaller statements

#### logical connectives:

- conjunction-  $P \wedge Q$  (read: P and Q)
- disjunction- P v Q (read: P or Q)
- implication or conditional-  $P \rightarrow Q$  (read: if P then Q)
- biconditional-  $P \leftrightarrow Q$  (read: P if and only if Q)
- negation-  $\neg P$  (read: not P)

Take these two statements: (1) it snowed today, and (2) class is cancelled.

For each logical connective, write a molecular statement made up of these two statements. Write each molecular statement in English and symbolically.

Ex.

It snowed today and class is cancelled (English)

Let P = It snowed today, Q = class is cancelled,  $P \land Q$  (symbols)

• All statements have a *truth value*: true (T or 1) or false (F or 0)

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  - (1) the truth values of its parts and
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  - $P \wedge Q$  is true when both P and Q are true.
  - P v Q is true when P or Q or both are true.
  - $P \rightarrow Q$  is true when P is false or Q is true or both.
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  - $P \leftrightarrow Q$  is true when P and Q are both true or both false.
  - $\neg P$  is true when P is false.
- Note: This or is called *inclusive or*, there is also an exclusive or which is true when P or Q is true but not when both are true.

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- Truth conditions:
  - $P \wedge Q$  is true when both P and Q are true.
  - $P \lor Q$  is true when P or Q or both are true.
  - $P \rightarrow Q$  is true when P is false or Q is true or both.
  - $P \leftrightarrow Q$  is true when P and Q are both true or both false.
  - $\neg P$  is true when P is false.

**Practice**: Let P = It snowed today, Q = class is cancelled, what is the truth value for each of the statements you wrote earlier **based on the truth of P and Q today**.

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- For any atomic statement there are only two possible truth values, that means when we string statements together with connectives there is a finite number of possible combinations of truth values
- One way we model possible truth values is with truth tables

Let P be a statement. The truth table for P is:



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Let Q be a statement. The truth table for Q is:





When we combine statements with conjunctives, there is a finite number of truth value combinations:

#### Truth Tables

Let P and Q be statements. One of these must be the case:

P is true and Q is true

P is true and Q is false

P is false and Q is true

P is false and Q is false

When we combine statements with conjunctives, there is a finite number of truth value combinations:

#### Truth Tables

Let P and Q be statements. One of these must be the case:

P is true and Q is true

P is true and Q is false

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When we use connectives, the truth value of the resulting statement depends on the true values of its parts and the connective. We use truth tables to model this.

**Conjunction**:  $P \wedge Q$  is true when both P and Q are true.

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Truth Tables

**Disjunction**:  $P \lor Q$  is true when P or Q or both are true.

P	Q	$P \lor Q$
Т	Т	Т
Т	F	Т
F	Т	Т
F	F	F

#### Truth Tables

**Implication**:  $P \rightarrow Q$  is true when P is false or Q is true or both.

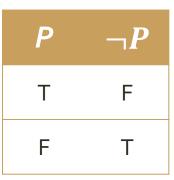
P	Q	$m{P}  ightarrow m{Q}$
Т	Т	Т
Т	F	F
F	Т	Т
F	F	Т

#### Truth Tables

**Biconditional**:  $P \leftrightarrow Q$  is true when P and Q are both true or both false.

P	Q	$P \leftrightarrow Q$
Т	Т	Т
Т	F	F
F	Т	F
F	F	Т

**Negation**:  $\neg P$  is true when P is false.



**Practice**: Form five groups. Each will be assigned a conjunction. Come up with a real world scenario that your conjunction accurately represents. Be ready to share your answers!

Ex. If I had conjunction and were one of you, I might say: Let P = I study discrete math, and Q = I am taking MATH220

The truth table and real work scenarios are:

Р	Q	$P \wedge Q$	
Т	Т	Т	I study discrete math AND I am taking MATH220.
Т	F	F	I study discrete math AND I am not taking MATH220.
F	Т	F	I do not study discrete math AND I am taking MATH220.
F	F	F	I do not study discrete math AND I am not taking MATH220.

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Truth tables are helpful for deducing when these larger statements are true and false.

Truth Tables

Ex. When is  $\neg P \lor Q$  true?

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#### Truth Tables

Ex. When is  $\neg P \lor Q$  true?

P	Q -
Т	Т
Т	F
F	Т
F	F

Start the table with all possible combinations of T/F for smallest statements

Truth tables are helpful for deducing when these larger statements are true and false.

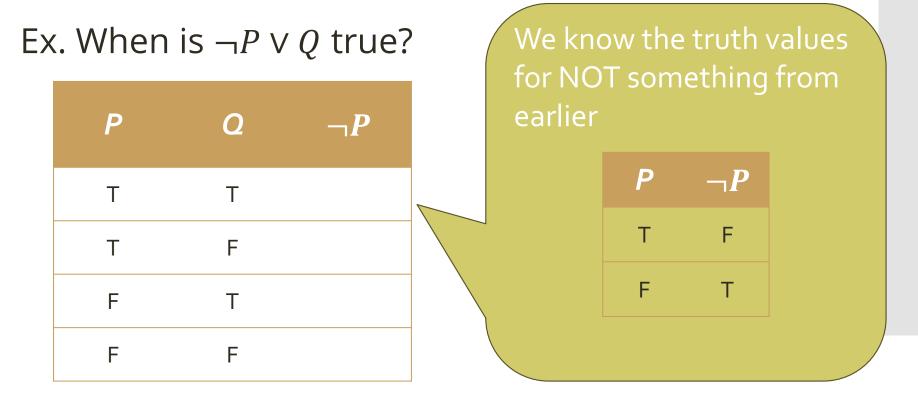
#### Truth Tables

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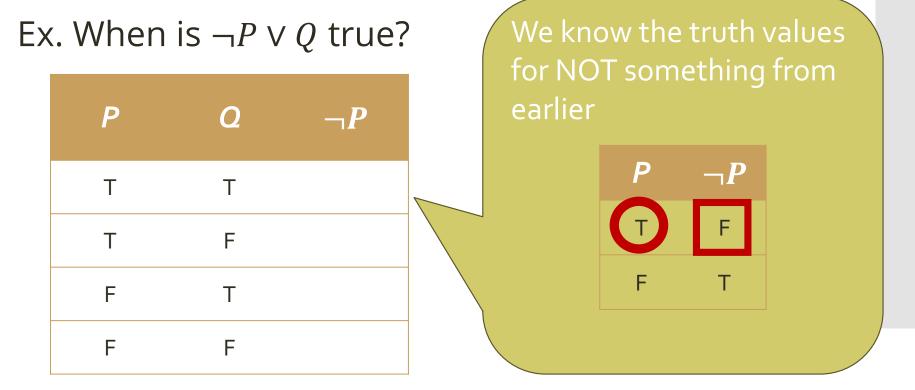
P	Q	$\neg P$
Т	Т	
Т	F	
F	Т	
F	F	

Build up the truth table with additional columns for slightly larger statements

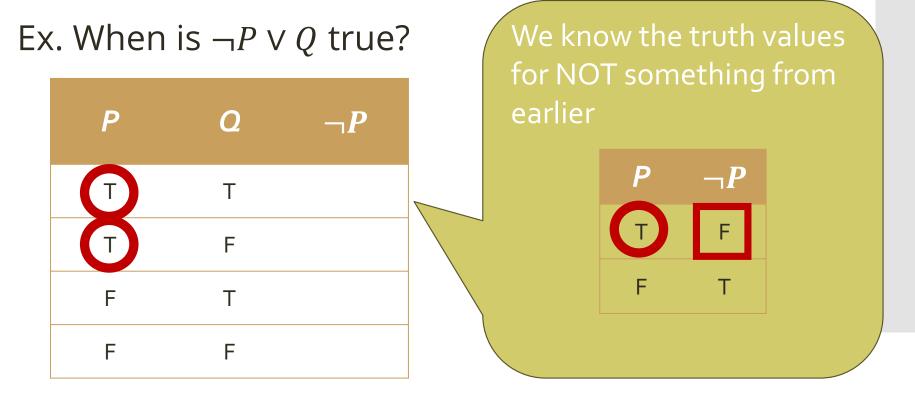
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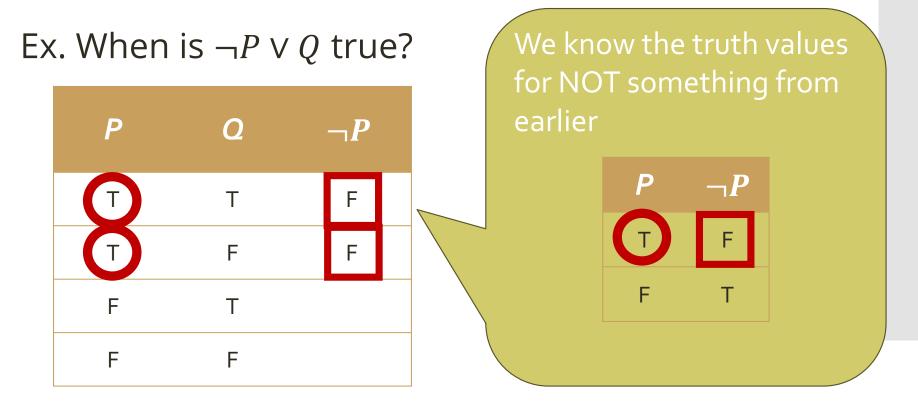
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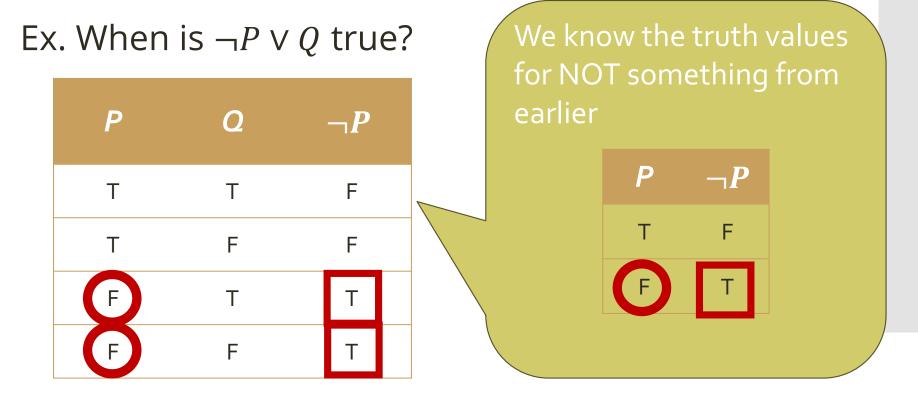
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#### Truth Tables

Ex. When is  $\neg P \lor Q$  true?

Р	Q	$\neg P$
Т	Т	F
Т	F	F
F	Т	Т
F	F	Т

Larger statements can be made up of more than two smaller parts and/or more than one conjunction.

Truth tables are helpful for deducing when these larger statements are true and false.

Ex. When is  $\neg P \lor Q$  true?

Build up the truth table with additional columns for larger statements

P	Q	$\neg P$	$\neg P \lor Q$
Т	Т	F	
Т	F	F	
F	Т	Т	
F	F	Т	

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Ex. When is  $\neg P \lor Q$  true?

P	Q	$\neg P$	$\neg P \lor Q$
Т	Т	F	
Т	F	F	
F	Т	Т	
F	F	Т	

P	Q	$P \lor Q$
Т	Т	Т
Т	F	Т
F	Т	Т
F	F	F

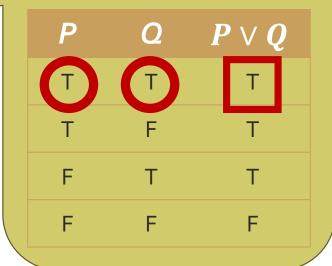
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Т	Т	F	
Т	F	F	
F	Т	Т	
F	F	Т	

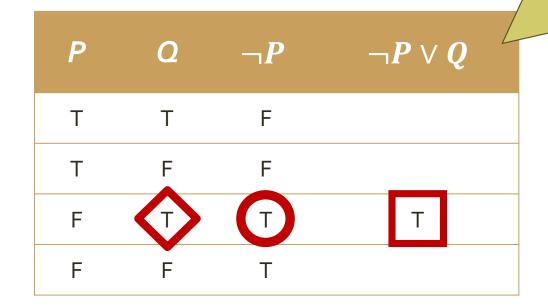


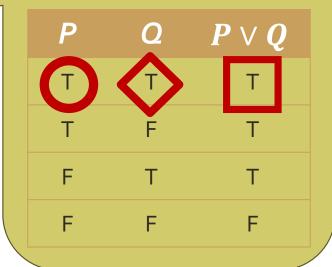
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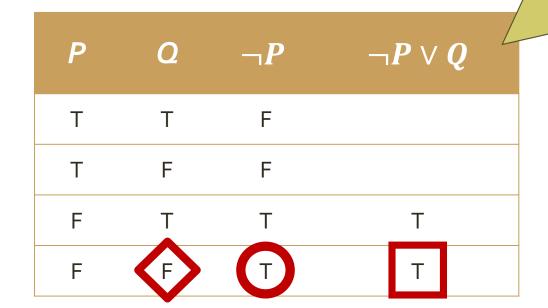


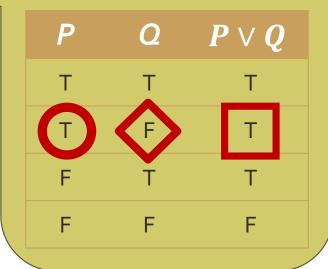
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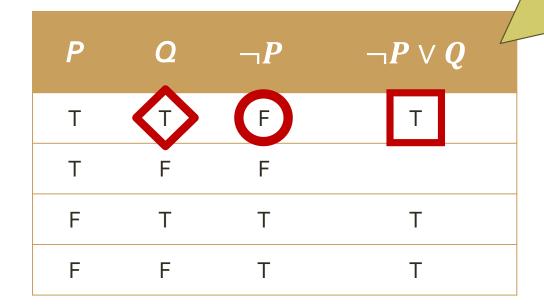


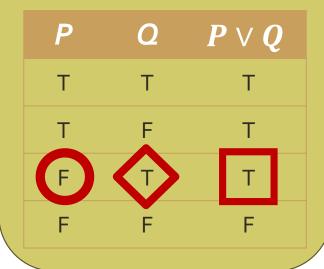
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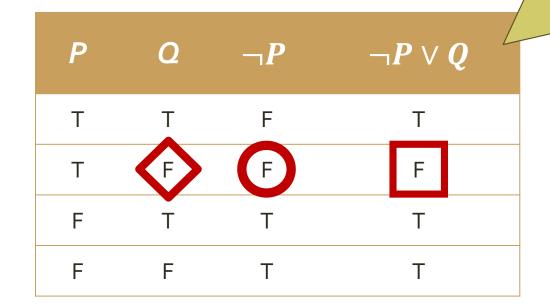
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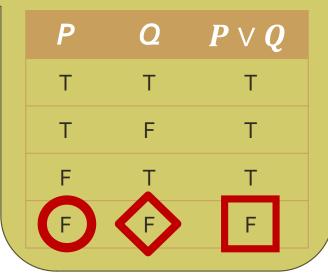
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statements are true and false.

Ex. When is  $\neg P \lor Q$  true?



We know the truth values for something OR something from earlier



**Practice**: Finish the truth table for the statement  $(P \rightarrow Q) \lor (Q \rightarrow R)$ 

#### Truth Tables

Р	Q	R
Т	Т	Т
Т	Т	F
Т	F	Т
Т	F	F
F	Т	Т
F	Т	F
F	F	Т
F	F	F

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#### Truth Table

Р	Q	R
Т	Т	Т
Т	Т	F
Т	F	Т
Т	F	F
F	Т	Т
F	Т	F
F	F	Т
F	F	F

Note:  $(P \to Q) \lor (\overline{Q \to R})$  is a **tautology**, a statement that is always true.

In other words, *P* and *Q* are *logically equivalent* if they have the same truth value for any assignment of truth values to their atomic parts.

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### Logical Equivalence

P	Q	$\neg P$	$\neg P \lor Q$
Т	Т	F	Т
Т	F	F	F
F	Т	Т	Т
F	F	Т	Т

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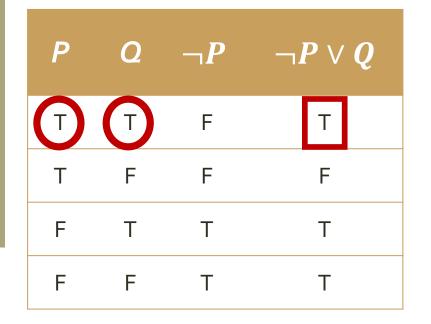
## Logical Equivalence

P	Q	$\neg P$	$\neg P \lor Q$
Т	Т	F	Т
Т	F	F	F
F	Т	Т	Т
F	F	Т	Т

P	Q	$m{P}  ightarrow m{Q}$
Т	Т	Т
Т	F	F
F	Т	Т
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### Logical Equivalence



P	Q	P  o Q
T	T	Т
Т	F	F
F	Т	Т
F	F	Т

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#### Logical Equivalence

P	Q	$\neg P$	$\neg P \lor Q$
Т	Т	F	Т
T	F	F	F
F	Т	Т	Т
F	F	Т	Т

P	Q	P  o Q
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F	Т	Т
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Т	Т	F	Т
Т	F	F	F
F	T	Т	Т
F	F	Т	Т

P	Q	$m{P}  ightarrow m{Q}$
Т	Т	Т
T	F	F
F	T	Т
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### Logical Equivalence

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Т	F	F	F
F	Т	Т	Т
F	F	Т	Т

P	Q	P  o Q
Т	Т	Т
Т	F	F
F	Т	Т
F	F	Т

#### **Practice**: Show that $\neg (P \lor Q)$ is logically equivalent to $\neg P \land \neg Q$

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## Logical Equivalence

P	Q	$\neg P$	$\neg P \lor Q$
Т	Т	F	Т
Т	F	F	F
F	Т	Т	Т
F	F	Т	Т

P	Q	$m{P}  ightarrow m{Q}$
Т	Т	Т
Т	F	F
F	Т	Т
F	F	Т