Potential Difference

OBJECTIVES

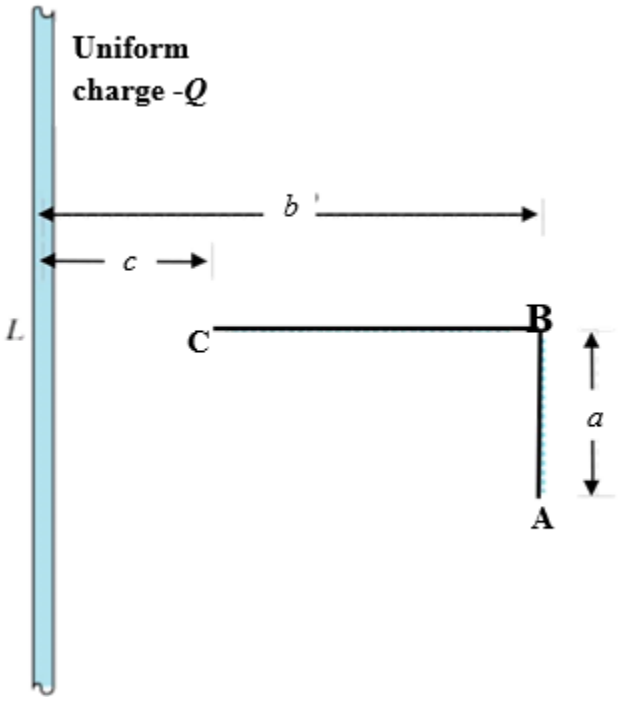
In this lab you will:

- Use VPython to graph potential differences
- Watch VPython generate a graph of potential difference as a marker moves along various paths
- Explain qualitatively the graphs you generate
- Explore the potential created by a capacitor.

Calculating the potential difference along paths near realistic charge distributions is difficult. It is particularly hard if the electric field varies significantly in the region. You will write a VPython program to calculate and display the potential difference as a marker moves along several paths near a charged rod. You will then write brief explanations of the graphs you generated.

1. Warm Up Problem

A long rod of length L carries a uniform charge –Q. Calculate the potential difference $V_A - V_C$. All the distances are small compared to L. Explain your work carefully.



Checkpoint 1: Ask an instructor to check your work for credit. You may proceed while you wait to be checked off.

2. Potential Difference Near a Charged Rod

You will use your code from Lab 2 which already contains the code to create a rod.

a) Download the given VPython code from Brightspace under Lab 3.

Most of this code is still useful. Run it once to check. The modifications to this code are tricky, so be careful. In order to calculate the change in potential over a large path, you will sum up the small changes in potential over short segments of the path. You will treat the electric field as constant over each small interval of the path. You will need to calculate the electric field in that small region. This electric field calculation will involve summing over all the contributions to the electric field that the small charge segments of the rod make. You will need to put the loop which finds the electric field inside another loop which finds the potential.

b) Add the following two lines of code to your setup statements. The statements should now be:

```
from visual import *
from visual.graph import *
from __future__ import division
scene = display(x=0, y=0, width=600, height = 600)
graph = gdisplay(x=600, y=0, width=400, height=300)
```

The second line preps VPython to make a graph. The fourth and fifth lines simply arranges the display windows so that they do not overlap.

Do the following in the define constants section.

c) Set N equal to 100. This will make it look and act like a line of charge.

d) Remove the definition of obslocation.

e) Define a time interval "deltat".

```
deltat=0.001
```

The choice of time interval is a tradeoff between how accurate you want your program to be and how fast you want it to run. If you feel your program is going too quickly, you can always shrink the time interval, however, if you feel your program is running too slowly

you can only extend the time interval up to a certain point before you start affecting the accuracy of the program.

Do the following in your initial values section.

f) Set the initial value of a time variable, "t", to be zero.

g) Set the initial value of our total potential difference, "deltaVtotal", to be zero.

You should already have the code for the x-coordinate and Enet.

Do the following in the create objects section.

h) Enter the following line of code to create a graph.

```
Vgraph=gcurve(color=color.cyan)
```

This creates a graph called "Vgraph" which has a cyan data curve. This line does not tell VPython what to put on the graph.

i) Create a marker called "marker" so as you calculate the potential change along the path, you can see where you are. This marker should be a red sphere with radius 0.05 m initially located at <0, 0.15, 0> m.

j) Give the marker a velocity by defining "marker.v" to be <0, 0.5, 0> m/s

Do the following in the calculations sections.

k) Remove the arrow and all the print statements.

l) Put your entire calculations section in a while loop that stops once the y-coordinate of the marker's position is greater than 1.15 m.

Your calculations section should now look like this:

```
while marker.pos.y < 1.15:
    x = -0.5 * L + 0.5 * deltax
    while x < L/2:
        r = obslocation - vector(x,0,0)
        rmag = mag(r)
        rhat = r / mag(r)
        E = (oofpez * deltaq / rmag**2) * rhat
        Enet = Enet + E
        x = x + deltax
```

Notice how the part which calculates the electric field is indented twice. We will call the loop involving marker.pos.y the external loop and the loop involving x the internal loop.

You took out obslocation, so you need to change that line. You are interested in the electric field at the location of the marker.

m) In place of obslocation put the marker.pos variable.

VPython will calculate the net electric field at a certain location, but the next time through the loop you will want it to calculate an entirely new electric field. So, you need to zero the Enet variable before the internal loop.

n) Set Enet back to a vector with zero magnitude right before the internal loop starts.

Now you are ready to start calculating the potential. The followings lines of code should be after the internal loop, and should only be indented once, so they are part of the external loop but not the internal loop.

You are going to calculate little changes in potential over small segments of the marker's path. On these small segments you are taking the electric field to be constant. Potential change is given by:

$$dV = -E \cdot dl$$

You could do the dot product calculation manually, however, VPython has a dot product function.

$$A \cdot B \rightarrow \text{dot}(A,B)$$

You will also need to figure out the length of the line element i.e. how far the marker traveled during the time interval. This length is simply the speed of the marker times the time interval.

o) Enter a line of code to calculate the potential change over a single time interval. Name that potential change "deltaV".

p) Like you did in finding the net electric field, add this deltaV to the value of deltaVtotal with the following line.

```
deltaVtotal=deltaVtotal+deltaV
```

q) Add one time interval to the time.

```
t=t+deltat
```

r) Use the position update formula to move the marker.

```
marker.pos=marker.pos+marker.v*deltat
```

s) Add a data point to the graph with the following line.

```
Vgraph.plot(pos=(t,deltaVtotal))
```

The graph will then be of potential difference versus time.

t) After the calculations and outside both loops put a print statement to print out deltaVtotal.

u) Run your code.

You should see the marker travel away from the rod as VPython generates a graph.

v) Record the value of deltaVtotal and sketch a copy of the graph into your workspace, be sure to give its axes labels with units and the graph a title.

w) Write an explanation of how you know the general shape of this graph is correct. Is your graph consistent with the warm up problem?

x) Now start the marker at <-1, 0.15, 0> m and move it to <1, 0.15, 0> m with a velocity of <0.5, 0, 0> m/s. BEFORE YOU RUN YOUR CODE, sketch a prediction of the graph's shape.

y) Record the value of deltaVtotal and sketch a copy of the graph into your workspace, be sure to give its axes labels with units and the graph a title.

z) Write and explanation of how you know the general shape of this graph and the value you received for deltaVtotal are correct.

Checkpoint 2: Ask an instructor to check your work for credit. You may proceed while you wait to be checked off.

3. Potential Difference Near a Charged Rod: Path Independence

a. Copy your entire calculations section and place a copy directly after the original.

b. In between the original calculations section and the copy you just made, insert a line which redefines marker.v i.e. changes the velocity of the marker.

c. Now make the marker travel diagonally from <0, 0.15, 0> m to <1, 1.15, 0> m, and then continue to <0, 1.15, 0 > m on a straight path.

d. Sketch a copy of the graph in your work space.

e. There are two distinct parts to the graph. Write a brief statement explaining the shape of each one.

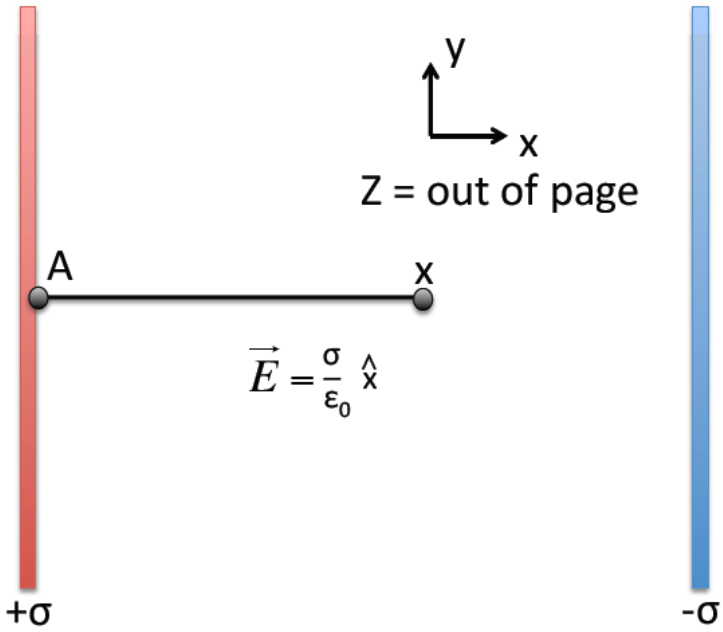
f. Compare this deltaVtotal from this run to that from your first run.

Checkpoint 3: Ask an instructor to check your work for credit. You may proceed while you wait to be checked off.

4. Final Problem:

Analytically Solve and Analyze a Capacitor's Potential

Suppose you have a capacitor with positive plate charge density +σ and negative plate charge density -σ (σ has units of $\frac{C}{m^2}$). The capacitor has large enough plates which are close enough together so that you can use $\overrightarrow{E} = \frac{\sigma}{\epsilon_0}\,\hat{x}$ between them. Let **s** be the separation distance of the plates. *Take the origin to be at the center of the capacitor, ie. at the midpoint along the axis passing from the center of each plate.* Derive the potential difference of $\vec{E}$ along a line between the centers of both plates. (Hint: Use $\Delta V = -\int_{\vec{r}_i}^{\vec{r}_f} \vec{E} \cdot \overrightarrow{dl}$, starting at a point A on the positively charged plate and stop at an arbitrary point x inside the capacitor as shown in the figure below.)



a) What is $\overrightarrow{dl}$ as a vector?

b) Write down the form of $-\int_{\vec{r}_i}^{\vec{r}_f} \vec{E} \,.\, \overrightarrow{dl}$ with the dot product evaluated and the final and initial positions substituted in.

c) Evaluate $-\int_{\vec{r}_i}^{\vec{r}_f} \vec{E} \,.\, \overrightarrow{dl}$ to obtain V(x).

d) Since we are free to re-define the 0 of energy here, add an offset to the potential to make $V(-\frac{s}{2}) = \frac{\sigma}{\epsilon_0} . \frac{s}{2}$.

e) Evaluate $-(\frac{\partial V}{\partial x})\ \hat{x}$ to recover the x component of $\vec{E}$. Analogous relationships hold for the y and z dimensions as well.

f) Suppose the two plates of a capacitor are held at a constant potential hence maintaining a constant potential difference between them at all times. They are also kept at a fixed separation. If the constant electric field between the plates is E, what is the relation between ΔV, s and E?

g) Now, if the plates are taken farther away from each other but still kept at constant potential, how does E change?

Checkpoint 4: Ask an instructor to check your work for credit.