

## Electric Field of a Uniformly Charged Rod

### Objectives:

In this lab you will:

- Diagram the plane for a complex VPython program.
- Create a program to display the E field from a charged rod at any point.
- Practice calculating the E field due to charged objects.

From class and the text, you have learned that there is a relatively simple formula for finding the Electric field on the central, perpendicular axis of a uniformly charge rod:

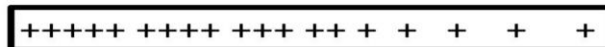
$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r\sqrt{r^2 + \left(\frac{L}{2}\right)^2}} \approx \frac{1}{4\pi\epsilon_0} \frac{2Q/L}{r},$$

where  $r$  is the distance from the midpoint of the rod to the observation location,  $Q$  is the total charge of the rod, and  $L$  is the length of the rod. If the rod is significantly longer than the distance  $r$ , then the Electric Field is approximately equal to the right-most expression above. If  $Q$  is positive, the field is radially away from the rod, and if  $Q$  is negative, the E field is directed radially inward.

To calculate the electric field at other locations, you must perform a difficult integral. You will make VPython do something approaching an integral by modeling a charged rod as a line of point charges. If you then make the number of point charges very large, VPython will do a summation that is almost as accurate as an integral. You will write a VPython program to allow you to easily calculate the electric field, due to a charged rod, at any point in space.

### 1: Warm-Up Problem

Redraw the figure below in your work space and show an approximate location where the electric field will be perpendicular to the rod (note the non-uniform charge distribution).



**Checkpoint 1: Ask an instructor to check your work for credit. You may proceed while you wait to be checked off.**

## **2: Program Design**

The rod you will be using has a length 2 m, and a total charge of  $3 \times 10^{-8}$  C. You will divide the rod into N number of pieces (6 to begin), which you will approximate by point charges. Then you will apply the superposition principle to get the net field at the observation location.

In your program, you will write a loop to step through the rod piece by piece, starting at the left end and moving to the right. You will find the E field from each small piece and then sum those fields to get the net field. The following tasks and questions will help you create algebraic expressions for important quantities to use in the program.

Consider a rod of length 2m, oriented along the x-axis, with the center of the rod at the origin. The rod is positively charged. Initially, you will divide it into 6 equal length segments. The observation location will be  $x = 0.3, 0.4, 0.5$  m.

- (a) Draw a set of axes, the rod divided into 6 segments, the observation location, and the expected net electric field at this observation location.
  - (i) What is the length of each point-like segment of the rod?
  - (ii) What is the general symbolic expression for the calculation you just did? If the length of the rod is L, and the number of pieces is N, what is a symbolic expression for  $\Delta x$ , the length of one piece, in terms of L and N?
  - (iii) What is the amount of charge on each of the 6 pieces?
  - (iv) What is the general symbolic expression for the calculation you just did? If the total charge of the rod is Q, and the number of pieces is N, what is a symbolic expression for  $\Delta Q$ , the amount of charge on one piece, in terms of Q and N?
  - (v) What is the position vector of the center of the leftmost piece of the rod (relative to the origin)?
  - (vi) What is the general symbolic expression for the calculation you just did? If the length of the rod is L, and the length of one piece is x, what is the position of the center of the leftmost piece of the rod? Your answer should be a vector relative to the origin and should be in terms of  $\Delta x$  and L (Hint: Think of what is the x - coordinate of the left end of the rod, then think of how far in from the end is the center of the left most piece)
  - (vii) What is the position vector of the center of piece #2 (the second one from the left end)?
  - (viii) What quantity did you have to add to the x-coordinate of piece #1 to get the x - coordinate of piece #2?
  - (ix) What is the symbol for the quantity you would add to the x - coordinate of the current source location to get the x - coordinate of the next piece?

**Checkpoint 2: Ask an instructor to check your work for credit. You may proceed while you wait to be checked off.**

### 3: The Program

(a) Open VPython and type the basic skeleton of your program as follows:

```
from __future__ import division
from visual import *

# Define Constants
e=1.6e-19
k=9e9
scalefactor=1

# Initial Values

# Create Objects

# Calculations
```

In the define constants section:

- (b) Define the variable “L” as the length of your rod. Make it 2m.
- (c) Define the variable “N” as the number of sections of the rod. Make it 6.
- (d) Define the variable “Q” as the total charge on the rod. Make it  $3 \times 10^{-8}$  C.
- (e) Define the variable “deltax” to be the length of each segment. Define it in terms of L and N.
- (f) Define the variable “deltaq” to be the charge on each segment. Define it in terms of N and Q.
- (g) Define the variable “obslocation” to be  $\langle 0.3, 0.4, 0 \rangle$  m (remember it needs to be a

vector). In the initial values section:

- (h) Define the variable “x” to initially be the value of the x-coordinate of the center of the left most piece of the rod (in terms of symbols NOT NUMBERS). If you need help look back to 2.a.vi.
- (i) Define a new variable “Enet”. Set it to be a vector with zero magnitude.

In the create objects section, you will create a rod out of several spheres.

(j) Construct the while loop like so:

```
while x < L/2:
    sphere(pos=(x,0,0), radius=0.1, color=color.cyan)
    x = x+deltax
```

This loop creates a sphere at the location of the left most segment of the rod then steps to the right one segment length and creates another. It continues doing this until the x-coordinate of the sphere is larger than the x-coordinate of the right most segment of the rod.

- (k) Run the program. It should display 6 spheres in a line on the x-axis.
- (l) Change the value of N to 15. Your program should now display 15 spheres.

In the calculations section, you will calculate the electric field from each segment of the rod and sum them up to get the net electric field. The x variable is now at the right end of the rod. Before the new loop you are about to create you need to reset that variable.

- (m) Reset the x variable by defining it to again be the value of the x-coordinate of the center of the left most piece of the rod.
- (n) Create a while loop that will terminate when x is larger than the value of the x-coordinate of the right most piece of the rod.
- (o) In this loop calculate the electric field (name it “E”), at the observation location, from the sphere with the current x value as its x-coordinate. Add that E contribution to the net E. Step x forward by one segment length. That is the end of the loop.
- (p) Outside the loop, i.e. not indented, define an arrow “Earrow”. Place it at the obslocation and make it represent the net electric field. Remember to include your scale factor which you will need changed in order to see the arrow. If you do not remember how to calculate an appropriate scale factor see Lab #1, Section 2.
- (q) Insert a line of code to print out the net E field and the magnitude of the net E field.
- (r) Run your program
- (s) Place the observation location at various other places in the xy-plane (at least 6). Sketch the E field at these locations in your workspace.
- (t) **Set N equal to the following numbers: 5, 10, 25, 50, 100. Record the magnitudes the electric field for each N.**

**In which jump between Ns does the magnitude of the electric field start changing by less than 0.1%.**

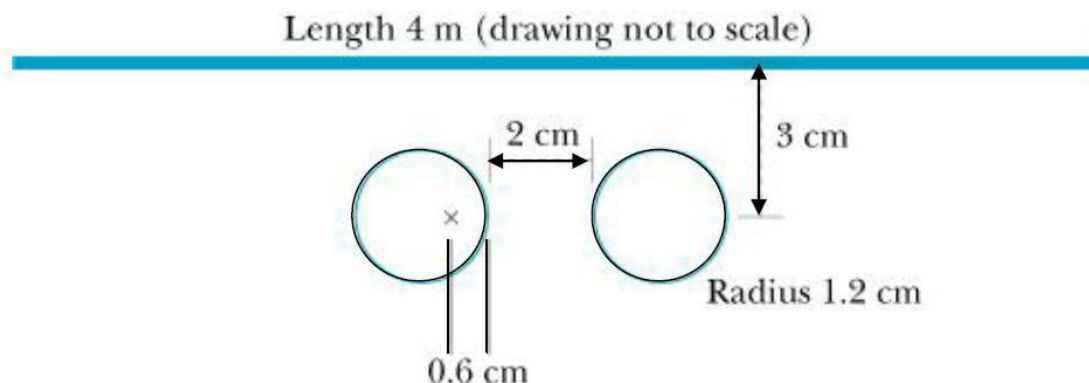
**Checkpoint 3: Ask an instructor to check your work for credit. You may proceed while you wait to be checked off.**

#### **4: Final Problem**

A very thin glass rod 4 meters long is rubbed all over with a silk cloth. It gains a uniformly distributed charge of  $+12 \times 10^{-6}$  C. Two small spherical rubber balloons are rubbed all over with wool. They each gain a uniformly distributed charge of  $-2 \times 10^{-8}$  C. The balloons are near the

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midpoint of the glass rod, with their centers 3 cm from the rod. The balloons are 2 cm apart (4.4 cm between centers).



Find the electric field at the location marked by the x, 0.6 cm to the right of the center of the left balloon. Also calculate the angle the electric field makes with the horizontal. Show all your work, including vectors. State any approximations you are making.

**Checkpoint 4: Ask an instructor to check your work for credit.**