HOMEWORK 9 - MATH402B

DUE: WEDNESDAY DECEMBER 6TH

(1) Goodman 3.1.9

(2) Goodman 3.1.10

(3) Goodman 3.1.11

(4) Let $\varphi : G \to G_1$ and $\psi : H \to H_1$ two homomorphism. Let $\Phi : G \times H \to G_1 \times H_1$ be the function defined by $\Phi(g, h) = (\varphi(g), \psi(h))$. Prove that

- Φ is an homomorphism;
- Φ is injective if and only if φ and ψ are injective.
- Φ is surjective if and only if φ and ψ are surjective.
- Φ is an isomorphism if and only if φ and ψ are isomorphisms.

(5) Let G be an abelian group of order 9 which is NOT cyclic. Show that $G \cong \mathbb{Z}_3 \times \mathbb{Z}_3$.

(6) Let G be a group and $x \in G$. Consider the set

$$Cent(x) = \{g \in G : g \cdot x \cdot g^{-1} = x\}$$

Prove that $Cent(x)$ is a subgroup and for every $h \in G$ the following holds:

$$h \cdot Cent(x) \cdot h^{-1} = Cent(h \cdot x \cdot h^{-1})$$

That is, "conjugating a centralizer is the centralizer of the conjugate". It means if you know the centralizer subgroup of one element in a conjugacy class, you know it for any other element in the conjugacy class.

(7) Are the groups $G_1 = \mathbb{Z}_{30} \times \mathbb{Z}_{20} \times \mathbb{Z}_4$ and $G_2 = \mathbb{Z}_{15} \times \mathbb{Z}_{10} \times \mathbb{Z}_{16}$ isomorphic?

(8) Determine all abelian groups of order 24 up to isomorphism.

(9) Let p be a prime number. Determine all abelian groups of order p^5