Name: ____

Math 308
Autumn 2016
MIDTERM - 2
11/18/2016

Instructions: The exam is **9** pages long, including this title page. The number of points each problem is worth is listed after the problem number. The exam totals to **50** points. For each item, please **show your work** or **explain** how you reached your solution. Please do all the work you wish graded on the exam. Good luck !

PLEASE DO NOT WRITE ON THIS TABLE !!

Problem	Score	Points for the Problem
1		10
2		10
3		10
4		10
5		10
TOTAL		50

Statement of Ethics regarding this exam

I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature: ________________________________ **Date:**______________

Question 1. (10 points) Decide whether the following statements are true or false. For this you don't need to show any work (but you will need to justify the answers of the other questions).

(a) [1 point] If $A \cdot x = b$ is consistent, then $b \in \text{range}\,(T_A)$ where T_A is the linear transformation $x \mapsto A \cdot x$.

○ True ○ False

(b) [1 point] The function $T : \mathbb{R}^2 \to \mathbb{R}^3$ given by $T\left(\left[\begin{smallmatrix} x_1 \\ x_2 \end{smallmatrix}\right]\right) = \begin{bmatrix} x_1 \\ x_2 \\ x_1 x_2 \end{bmatrix}$ is a linear transformation.

○ True ○ False

(c) [1 point] If A is a $m \times n$ matrix, then $\text{nullity}(A) \leq n$

○ True ○ False

(d) [1 point] If $\mathcal{B}$ is a basis for a subspace S, and $u, v \in \mathcal{B}$ then $u + v \in \mathcal{B}$.

○ True ○ False

(e) [1 point] If S is a subspace of $\mathbb{R}^n$, then any basis of S consists of at least n elements.

○ True ○ False

(f) [1 point] The set $\left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^4 : 2x_1 = x_3 + 1 \right\}$ is a subspace of $\mathbb{R}^3$.

○ True ○ False

(g) [1 point] For any matrix A, the transpose of A exists.

○ True ○ False

(h) [1 point] If B is the reduced echelon form of A then T_A and T_B, their associated linear transformations, are equal.

○ True ○ False

(i) [1 point] If a linear transformation is 1-to-1 then it is also onto.

○ True ○ False

(j) [1 point] If a square matrix A is invertible then A^2 is also invertible.

○ True ○ False

Question 2. (10 points) Let A be an $n \times n$ matrix. Answer the following questions without using the Big Theorem (you may, however, use other theorems from the text).

(a) [5 points] Prove that $rank(A) = n$ if and only if $nullity(A) = 0$.

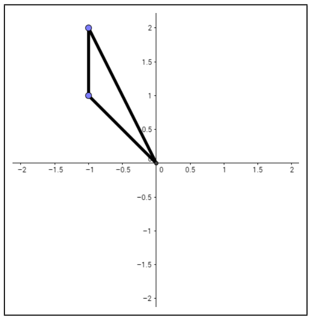
(b) [5 points] Assuming either $rank(A) = n$ *or* $nullity(A) = 0$, prove that T_A, the linear transformation associated to A, is onto.

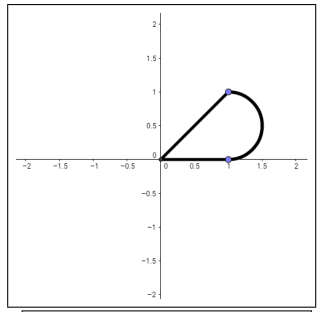
Question 3. (10 points) Let $A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 0 & -2 \end{bmatrix}$ and let T_B be the linear transformation given by $T_B\left(\left[\begin{smallmatrix} x_1 \\ x_2 \end{smallmatrix}\right]\right) = \begin{bmatrix} x_2 \\ x_1 \\ x_1 \end{bmatrix}$. Denote the linear transformation associated to A by T_A.

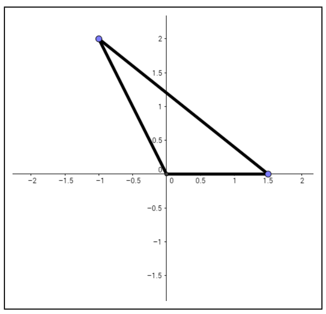
(a) [2 points] Find the matrix associated to T_B and the domains and codomains of T_A and T_B.

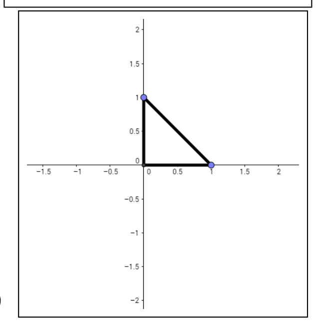
(b) [4 points] Write down matrices associated to the linear transformations $(T_A \circ T_B)$ and $(T_B \circ T_A)$. If no such matrix exists, say why.

(c) [4 points] Apply the linear transformation $(T_A \circ T_B)^{-1}$ to the triangle in the plane with vertices at the points $(0,0)$, $(0,1)$ and $(1,0)$. Check the circle next to the picture that best represents the resulting shape. Explain your reasoning in the space below.

○ 

○ 

○ 

○ 

Question 4. (10 points) Let A be the following matrix

$$A = \begin{pmatrix} 1 & 2 & 1 & 0 & 1 \\ 1 & 3 & 2 & 1 & 2 \\ 1 & 3 & 3 & 2 & 1 \\ 2 & 0 & 4 & 2 & -8 \end{pmatrix}$$

(a) [4 points] Compute a basis for $\mathrm{row}(A)$ and a basis for $\mathrm{col}(A)$.

(b) [2 points] What are the rank and the nullity of A?

(c) [2 points] Compute a basis for the null space of A.

(d) [2 points] Let T_A be the associated linear transformation, i.e. $T_A(x) = A \cdot x$. Identify the domain and codomain of T_A and decide whether T_A is 1-to-1, onto and/or invertible.

Question 5. (10 points) Let S be $\text{span}\{u_1, u_2, u_3, u_4, u_5\}$ where

$$u_1 = \begin{pmatrix} 1 \\ 2 \\ 2 \\ 1 \end{pmatrix} \quad u_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} \quad u_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad u_4 = \begin{pmatrix} 2 \\ 0 \\ 3 \\ 1 \end{pmatrix} \quad u_5 = \begin{pmatrix} 1 \\ 3 \\ 1 \\ 0 \end{pmatrix}$$

(a) [3 points] Find a basis for S.

(b) [2 points] What is the dimension of S? Is the basis you found a basis for $\mathbb{R}^4$?

(c) [1 points] Let A be the matrix whose columns are the vectors of the basis you found in the previous part. What can you say about the nullity of A without computing the null space?

(d) [4 points] Let $L = \operatorname{span} e_1 \subset \mathbb{R}^3$ be the line corresponding to the x-axis. What is $T_A(L)$, the image of L under the linar transformation T_A, where as usual $T_A : x \to A \cdot x$?