List of Topics of Math 308

Linear Systems - CH 1 in the book (sec 1.1, 1.2)

Notions:

- Coefficient Matrix and Augmented Matrix of a System;
- Vector Form and Matrix Form of a system;
- Solution Set (also in vector form);
- Dimension of the Solution Set;
- Triangular Systems and Systems in Ecehelon Form;
- Matrices in Echelon Form and in Reduced Echelon Form;
- Row Operations and Gauss-Jordan Algorithm.

Abilities:

- I can determine whether a point or a vector is a solution to a system of linear equations.
- I can convert linear systems to augmented matrices and vice versa. I can solve systems of linear equations using row operations. I can use Gaussian elimination with back-substitution to solve systems of linear equations. I can use Gauss-Jordan elimination to solve systems of linear equations.
- I can express solutions to linear systems in general form and vector form and, in 2 or 3 variables, characterize the solution sets geometrically.
- I can characterize the solutions to systems of linear equations using appropriate notation and vocabulary (e.g., consistent or inconsistent systems, free parameters or variables, leading variables). I understand the trichotomy of possible solutions algebraically. I can recognize when a linear system is in echelon form or reduced echelon form algebraically.
- I understand the trichotomy of possible solutions to systems of linear equations geometrically. In 2 or 3 variables, I can recognize when a consistent linear system is in echelon form geometrically.

Vectors - CH 2 in the book (sec. 2.1, 2.2, 2.3)

Notions:

- Vectors and coordinates, $\mathbb{R}^m$;
- Span of a set of vectors and properties;
- Linear Independence and properties;

- Relation with linear systems;
- The Big Theorem version 1.

Abilities:

- I understand addition and scalar multiplication of vectors algebraically and geometrically. I know the algebraic properties that vector addition and scalar multiplication satisfy. I know how to multiply an $n \times m$ matrix A by a vector $x \in \mathbb{R}^m$.
- I can state the definition of the span of a set of vectors. I can determine if a vector in $\mathbb{R}^m$ is in the span of a set of vectors. I can determine if a set of vectors spans $\mathbb{R}^m$.
- I can state the definition of linear dependence and linear independence. I can determine whether a set of vectors is linearly independent or linearly dependent.
- I have a conceptual understanding of the span of a set of vectors. I understand the connection between solving a linear system and determining if a vector is in the span of other vectors.
- I have a conceptual understanding of linear dependence and linear independence. I can rephrase the definition of linear independence or dependence in terms of span and in terms of solutions to linear equations.

Linear Transformations - CH 3 in the book (sec. 3.1, 3.2, 3.3)

Notions:

- Domain, Codomain, Range of a function;
- 1-to-1 and onto;
- Linear Transformation: definition and properties;
- Matrix of a Linear Transformation;
- Standard/Canonical basis of $\mathbb{R}^n$;
- Matrix operations;
- Composition of Linear Transformation;
- Inverse of a Linear Transformation and Inverse of a matrix;
- Big Theorem version 2.

Abilities:

- I can determine whether a function $T : \mathbb{R}^m \to \mathbb{R}^n$ is a linear transformation. I understand that a linear transformation is completely determined by its values on $e_1, e_2, \dots, e_m$ and that every linear transformation can be represented by a unique matrix. I can graphically represent linear transformations in $\mathbb{R}^n$.
- Given a linear transformation T, I can determine whether it is one-to-one and whether it is onto. I understand how a linear transformation T being one-to-one or onto translates into properties of the matrix

associated to T.

- I can perform algebraic operations with matrices, including addition, subtraction, scalar multiplication, and matrix multiplication. I can compute the transpose of matrices. I understand the connection between matrix multiplication and composition of linear transformations.
- I can find the inverse of a matrix or determine that no inverse exists using Gaussian elimination.
- I understand what it means for a matrix to be invertible, both algebraically and conceptually. I understand the relationships among a matrix being invertible, properties of the associated linear transformation, and spanning or linear independence properties of the columns of A.

Subspaces - CH 4 in the book (sec. 4.1, 4.2, 4.3)

Notions:

- Subspaces: definition and properties;
- Null space of a matrix, Kernel of a linear transformation;
- Basis of a subspace and dimension;
- Row and Column space of a matrix;
- Rank and Nullity of a matrix.

Abilities:

- I can prove whether or not a subset W of $\mathbb{R}^n$ forms a subspace. I have a geometric understanding of what subspaces look like in $\mathbb{R}^2$ and $\mathbb{R}^3$.
- I can determine whether a set of vectors forms a basis for a subspace. I can compute a basis for a given subspace and I can find its dimension. I can identify a basis for $\mathbb{R}^2$ and $\mathbb{R}^3$ pictorally.
- I can find a basis for the row space, the column space, or the null space of a matrix. I can determine the rank and nullity of a matrix.
- I can find a basis for the kernel and range of a linear transformation. I can rephrase statements about a linear transformation being one-to-one or onto in terms of statements about the kernel and range.

Determinand - CH 5 in the book (sec. 5.1, 5.2)

Notions:

- Determinant: how to compute it and properties;
- Minors and Cofactors.

Abilities:

- I can compute the determinant of 2×2 matrices using the definition or the shortcut method.

- I understand the geometric description of determinant as area and its relation to linear transformations.
- I understand how elementary row operations change the determinant. I can compute the determinant of an $n \times n$ matrix using row reduction.
- I know how to use the determinant to test whether a matrix is invertible.
- I know how the determinant of a product of matrices relates to the determinant of each factor, how the determinant of A and A^{-1} are related, and shortcuts for computing the determinant of upper (or lower) triangular and diagonal matrices.
- I can compute the determinant of an $n \times n$ matrix using cofactor expansion (row or column expansion).

Eigenvalues/vectors - CH 6 in the book (6.1,6.2)

Notions:

- Eigenvalues and Eigenvectors of a matrix: definition and properties;
- Eigenspaces of a Matrix: definition and properties;
- Characteristic Polynomial of a matrix;
- Multiplicity of an eigenvalue;
- Diagonalizable Matrix.

Abilities:

- Given a matrix A and an eigenvalue λ of A, I can compute the eigenspace $E_\lambda(A)$ of λ. I can show that the eigenspace is a subspace.
- I can compute the characteristic polynomial $p_A(\lambda)$ of a matrix A and I know its relation to the eigenvalues of the matrix. Given a factored characteristic polynomial of a matrix, I can compute the eigenvalues and eigenvectors of a matrix. I know the relationship between the dimension of an eigenspace and the multiplicity of an eigenvector.
- I understand how to diagonalize an $n \times n$ matrix A that has eigenvectors that form a basis for $\mathbb{R}^n$.
- I can compute a diagonalizable matrix given a set of eigenvectors and the corresponding eigenvalues;
- I can deduce properties of the matrix given its eigenvalues and eigenvectors;
- I can compute powers of a diagonalizable matrix;