Quiz 2

February 15, 2017

NAME (last,first): ______________________________

Question 1. (10 points) Let T_A and B be the following linear transformation and matrix:

$$T_A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} x_1 + 2x_2 + 3x_4 \\ x_1 + x_3 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & 3 \\ 0 & 1 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}$$

(a) [1 point] Write down the matrix A associated to T, i.e. such that $T(x) = A \cdot x$.

Solution:

$$A = \begin{pmatrix} 1 & 2 & 0 & 3 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

(b) [2 points] Denoted by T_B the transformation $T_B(x) = B \cdot x$, complete the following sentences:

T_A has domain ____$\mathbb{R}^4$____ and codomain ____$\mathbb{R}^2$____.
T_B has domain ____$\mathbb{R}^2$____ and codomain ____$\mathbb{R}^4$____.

(c) [2 points] For each composition that make sense, complete the following sentences:

$T_A \circ T_B$ has domain ____$\mathbb{R}^2$____ and codomain ____$\mathbb{R}^2$____.
$T_B \circ T_A$ has domain ____$\mathbb{R}^4$____ and codomain ____$\mathbb{R}^4$____.

(d) [4 points] For each composition that make sense, compute the matrices associated to the linear transformations $T_A \circ T_B$ and $T_B \circ T_A$ or say why it does not make sense.

Solution: Both composition make sense since the domain of T_A coincide with the codomain of T_B and the domain of T_B coincide with the domain of T_A.

$$T_A \circ T_B = T_{A \cdot B} \qquad A \cdot B = \begin{pmatrix} 1 & 2 & 0 & 3 \\ 1 & 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 2 & 3 \\ 0 & 1 \\ 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 8 \\ 3 & 4 \end{pmatrix}$$

$$T_B \circ T_A = T_{B \cdot A} \qquad B \cdot A = \begin{pmatrix} 2 & 3 \\ 0 & 1 \\ 1 & 1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 0 & 3 \\ 1 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 5 & 4 & 3 & 6 \\ 1 & 0 & 1 & 0 \\ 2 & 2 & 1 & 3 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

(e) [1 point] If the linear transformation $T_A \circ T_B$ make sense, is it invertible? Explain why or why not!

Solution: Since $A \cdot B$ it is a 2×2 matrix it is invertible since its determinant is $8 - 24 = -16 \neq 0$. Therefore $T_{A \cdot B} = T_A \circ T_B$ is invertible.