



organized by Julia Brandes, Tim Browning, Oscar Marmon and Amos Turchet

## PROGRAM

TIME	Monday 17	Tuesday 18	Wednesday 19
08:45 - 09:15	Registration		
09:15 - 09:30	Welcome		
09:30 - 10:00	Laudatio: Brzezinski	Damaris Schindler	Régis de la Bretèche
10:00 - 10:30	Trevor Wooley		
10:30 - 11:00		Coffee Break	Coffee Break
11:00 - 11:30	Coffee Break	Jörg Brüdern	J.-L. Colliot-Thélène
11:30 - 12:00	Marta Pieropan		
12:00 - 12:30		Lunch Break	Roger Heath-Brown
12:30 - 13:00			
13:00 - 14:00			
14:00 - 14:30	Yuri Tschinkel	Emmanuel Peyre	
14:30 - 15:00			
15:00 - 15:30	Coffee Break	Coffee Break	
15:30 - 16:00	Short Talks	Alexei Skorobogatov	
16:00 - 16:30			
16:45 - 17:45			
Evening		SOCIAL DINNER	

# Abstracts

Time: Monday 17th 10:00;

Speaker: **Trevor Wooley** (University of Bristol)

Title: Mean values of exponential sums - beyond Vinogradov and translation invariance.

*Abstract:* Even moments of exponential sums defined by polynomials count integral solutions of systems of Diophantine equations. Upper bounds consistent with square-root cancellation in these mean values correspond, broadly speaking, to the phenomenon that diagonal solutions of the system provide the dominant contribution to the solution set. Very recently, the extent to which we can prove such conjectural behaviour has been greatly improved in translation-dilation invariant systems by the efficient congruencing method of the author, and  $l_2$ -decoupling method of Bourgain, Demeter and Guth. We will report on what can be said when translation-dilation invariance is absent. There are three approaches: the determinant method (to which Per Salberger has made substantial contributions), nested efficient congruencing, and a third method which we will reveal in the talk. The latter method is the subject of joint work with Julia Brandes.

Time: Monday 17th 11:30;

Speaker: **Marta Pieropan** (Freie Universität Berlin)

Title: On rationally connected varieties over large  $C_1$  fields of characteristic 0.

*Abstract:* In the 1950s Lang studied the properties of  $C_1$  fields, that is, fields over which every hypersurface of degree at most  $n$  in an  $n$ -dimensional projective space has a rational point. Later he conjectured that every smooth proper rationally connected variety over a  $C_1$  field has a rational point. The conjecture is proven for finite fields (Esnault) and function fields of curves over algebraically closed fields (Graber-Harris-de Jong-Starr). I use birational geometry to address the open case of Henselian fields of mixed characteristic with algebraically closed residue field.

Time: Monday 17th 14:00;

Speaker: **Yuri Tschinkel** (Courant Institute of Mathematical Sciences - New York University.)

Title: Rationality problems.

*Abstract:* I will report on recent advances in the study of rationality properties of algebraic varieties that allowed to essentially settle the problem of stable rationality of very general threefolds and led to the discovery of new effects in dimension 4 (joint work with B. Hassett, A. Kresch, and A. Pirutka).

Time: Tuesday 18th 09:30;

Speaker: **Damaris Schindler** (Utrecht University)

Title: On integral points on degree four del Pezzo surfaces.

*Abstract:* We report on our investigations concerning algebraic and transcendental Brauer-Manin obstructions to integral points on complements of a hyperplane section in degree four del Pezzo surfaces. This is joint work with Joerg Jahnel.

Time: Tuesday 18th 11:00;

Speaker: **Jörg Brüdern** (Georg-August-Universität Göttingen)

Title: Another look at Artin's conjecture.

*Abstract:* Artin has conjectured that over the  $p$ -adic numbers a form of degree  $d$  in more than  $d^2$  variables has non-trivial zeros. While this is false in general, the conjecture is known to hold for diagonal forms. In this talk we sketch an argument that shows that the conjecture also holds for forms that can be realised as linear slices of diagonal hypersurfaces. This is joint with Olivier Robert.

Time: Tuesday 18th 14:00;

Speaker: **Emmanuel Peyre** (Institut Fourier)

Title: Thin subsets and slopes.

*Abstract:* Slopes à la Bost give new invariants refining heights for rational points on varieties. The aim of this talk is to describe the links between thin accumulating subsets which are obstructions to the equidistribution of rational points of bounded height and these invariants. These links are rooted in the analogies between rational points and rational curves.

Time: Tuesday 18th 15:30;

Speaker: **Alexei Skorobogatov** (Imperial College London)

Title: Finiteness theorems for abelian varieties and K3 surfaces of CM type.

*Abstract:* This is a joint work with Martin Orr. We study abelian varieties and K3 surfaces with complex multiplication defined over number fields of bounded degree. Building on the recent proof of the average Colmez conjecture, we show that these varieties fall into finitely many isomorphism classes over an algebraic closure of  $\mathbb{Q}$ . As an application we confirm finiteness conjectures of Shafarevich and Coleman in the CM case. In addition, we prove the uniform boundedness of the Galois invariant subgroup of the geometric Brauer group for forms of a smooth projective variety satisfying the integral Mumford–Tate conjecture. When applied to K3 surfaces, this affirms a conjecture of Várilly-Alvarado in the CM case.

Time: Wednesday 19th 09:30;

Speaker: **Régis de la Bretèche** (Institut de Mathématiques de Jussieu - Université Paris Diderot)

Title: My experiences of counting rational points following Salberger’s Mathematics.

*Abstract:* I began to count rational points on algebraic varieties 20 years ago. During this 20 years I was inspired by Salberger’s Mathematics and articles. I will take some examples to express my gratitude to Per and illustrate my progress in this subject thanks to Salberger’s Mathematics.

Time: Wednesday 19th 11:00;

Speaker: **J.-L. Colliot-Thélène** (C.N.R.S. - Université de Paris-Sud, Orsay)

Title: On the local-global principle for zero-cycles.

*Abstract:* I shall describe progress since Per Salberger’s 1988 paper ”Zero-cycles on rational surfaces over number fields”.

Time: Wednesday 19th 12:00;

Speaker: **Roger Heath-Brown** (University of Oxford)

Title: Counting Rational Points on Quadric Surfaces.

*Abstract:* Let  $Q$  be a quadratic form in four variables, with rational integer coefficients. We want to count rational points of bounded height on the surface  $Q = 0$ , with a good explicit dependence on  $Q$ . This is motivated by work in progress investigating the variety  $X_0Y_0^2 + X_1Y_1^2 + X_2Y_2^2 + X_3Y_3^2 = 0$ .

## Short Talks

TIME	MVH 12	Pascal
15:30 - 15:45	Kirsti Biggs	Daniel Loughran
15:50 - 16:05	Junxian Li	Vlad Mitankin
16:10 - 16:25	Soohyun Park	Efthymios Sofos
16:45 - 17:00	Emiliano Ambrosi	Nick Rome
17:05 - 17:20	Adelina Manzateanu	Florian Wilsch
17:25 - 17:40	Lasse Grimmelt	Kévin Destagnol

**Emiliano Ambrosi:** Specialization of representations of the étale fundamental group and applications.

*Abstract:* Let  $X \rightarrow S$  be a one dimensional family of smooth projective varieties over a finitely generated field  $k$  over a smooth geometrically connected base. For every rational point  $s$  of  $S$  we have an  $l$ -adic representation of the absolute Galois group of  $k$  on the  $l$ -adic étale cohomology of the fiber  $X_s$  of the morphism. We will discuss how the image of these representations vary when  $s$  is varying in  $S(k)$ . Finally we will show how these results can be used to study problems related to the specialization of cohomological cycles.

**Kirsti Biggs:** Waring’s Problem with Shifts.

*Abstract:* In its original form, Waring’s problem asks whether every positive integer can be written as the sum of  $s$   $k$ th powers of natural numbers, where  $s$  depends only on  $k$ . In this talk, I will discuss an analogue of this problem in which we attempt to approximate a large, positive real number  $\tau$  by a sum of ‘shifted’  $k$ th powers. I will outline the Davenport–Heilbronn method, which allows us to obtain an asymptotic formula for the number of solutions to the relevant Diophantine inequality whenever  $s \geq k^2 + (3k - 1)/4$ , improving on the best previously known result. I will also show that there are arbitrarily large  $\tau$  which cannot be approximated in this way if we insist on the  $k$ th powers being too close together.

**Kévin Destagnol:** Manin’s conjecture for a family of projective hypersurfaces in higher dimension.

*Abstract:* The Manin-Peyre conjectures describe the distribution of rational points of bounded height on Fano varieties in terms of geometric invariants of the variety. Inspired by recent works of La Bretèche and of Blomer, Brüdern and Salberger, we will focus during this talk on the Manin-Peyre conjectures for the singular projective hypersurfaces in  $\mathbb{P}_{\mathbb{Q}}^{2n-1}$  defined by the following equations

$$x_1 y_2 \cdots y_n + x_2 y_1 y_3 \cdots y_n + \cdots + x_n y_1 y_2 \cdots y_{n-1} = 0$$

for all  $n \geq 3$ . The verification of Peyre’s constant relies in particular on work of Salberger providing Crepant resolutions for the previous hypersurfaces.

**Lasse Grimmelt:** Representation of Squares by Cubic Forms.

*Abstract:* Let  $n \in \mathbb{Z}$ ,  $C(x_1, \dots, x_n)$  a nonsingular cubic form with integral coefficients. We analyze the existence of integral solutions of

$$C(x_1, \dots, x_{n_1}) = y^2.$$

We apply Heath-Brown’s version of the circle method and try to prove a Hasse- principle in as few variables as possible. The case  $n \geq 7$  requires analoga of certain exponential sum bounds used by

Heath-Brown in his work on non singular cubic forms in 10 variables. They can be proved either by elementary transformation or application of variants of Deligne's bounds with an additional multiplicative character, studied by Katz. Trying the same approach for the case  $n = 6$  requires results similar to those of Hooley's paper on cubic forms in 9 indeterminates, especially one deep algebraic geometric result.

**Junxian Li:** A lower bound for the least prime in an arithmetic progression.

*Abstract:* Fix  $k$  a positive integer, and let  $\ell$  be coprime to  $k$ . Let  $p(k, \ell)$  denote the smallest prime equivalent to  $\ell \pmod{k}$ , and set  $P(k)$  to be the maximum of all the  $p(k, \ell)$ . In this joint work with Kyle Pratt and George Shakan, we show that for almost every  $k$  one has  $P(k) \gg \phi(k) \log k \log_2 k \log_4 k / \log_3 k$ . This improves an earlier bound of Pomerance, and answers a question of Ford, Green, Konyagin, Maynard, and Tao. We also give a heuristic which suggests that

$$\liminf_k \frac{P(k)}{\phi(k) \log^2 k} = 1.$$

**Daniel Loughran:** Rational points on conic bundle surfaces.

*Abstract:* We consider the problem of counting rational points of bounded height on conic bundle surfaces with respect to some anticanonical and non-anticanonical height functions. This is joint work with Christopher Frei and Efthymios Sofos.

**Adelina Manzateanu:** Rational Curves on Cubic Hypersurfaces over  $\mathbb{F}_q$ .

*Abstract:* Using a version of the Hardy – Littlewood circle method over  $\mathbb{F}_q(t)$ , one can count  $\mathbb{F}_q(t)$ -points of bounded degree on a smooth cubic hypersurface  $X \subset \mathbb{P}_{\mathbb{F}_q}^{n-1}$ . Moreover, there is a correspondence between the number of  $\mathbb{F}_q(t)$ -points of bounded height and the number of  $\mathbb{F}_q$ -points on the moduli space  $\text{Mor}_d(\mathbb{P}_{\mathbb{F}_q}^1, X)$ , which parametrises the rational maps of degree  $d$  on  $X$ . In this talk I will show that for  $n \geq 10$ , and  $q$  and  $d$  large enough, there exists a rational curve defined over  $\mathbb{F}_q$  on  $X$  passing through two fixed points, one of which must not belong to the Hessian. Moreover, I will give an asymptotic formula for the number of such curves.

**Vlad Mitankin:** Integral points on generalised affine Châtelet surfaces.

*Abstract:* For an integer  $a$  and a separable polynomial  $P(t)$  with integral coefficients we look at affine surfaces  $X$  cut out by  $y^2 - az^2 = P(t)$ . Under the assumption of Schinzel's hypothesis we show that the integral version of the Brauer-Manin obstruction is the only one for the existence of integral points on  $X$ . Furthermore, we are able to deduce an approximation argument for the Brauer-Manin set of infinity with respect to the  $t$ -variable. This work is based on a classical result of Colliot-Thélène and Sansuc regarding rational points. The new idea is to inject tools from algebraic number theory so that their argument can be modified to work for integral points.

**Soohyun Park:** Bounded gaps between primes in special sequences.

*Abstract:* We use Maynard's methods to show that there are bounded gaps between primes in the sequence  $\{\lceil n\alpha \rceil\}$ , where  $\alpha$  is an irrational number of finite type. In addition, given a superlinear function  $f$  satisfying some properties described by Leitmann, we show that for all  $m$  there are infinitely many bounded intervals containing  $m$  primes and at least one integer of the form  $\lceil f(q) \rceil$  with  $q$  a positive integer. This may also include an overview of some aspects of the history of gaps between primes/sieve theory and some recent developments.

**Nick Rome:** Counting Hasse Principle Failures in Châtelet Surfaces.

*Abstract:* Establishing local-global principles, such as the Hasse principle, is an essential tool for the study of Diophantine equations and counting rational points on surfaces. The family of Châtelet surfaces has long been known to include counter-examples to the Hasse principle. In this talk, we will investigate the frequency of such counter-examples in a family of surfaces parametrised by points on a quadric.

**Efthymios Sofos:** Fibers with a rational point.

*Abstract:* Assume we are given a fibration defined over the rational numbers that contains at least one fiber with a rational point and has no section over the ground field. A question, that lately came into prominence, regards the quantitative description of the fibers with a rational point. I will talk about recent progress in certain cases where the fibers satisfy the Hasse principle, as well as an unexpected connection with the normal distribution that might help with our understanding of problems of this type.

**Florian Wilsch:** Counting integral points on a log Fano threefold.

*Abstract:* Manin conjectured an asymptotic formula for the number of rational points of bounded height on Fano varieties. One generalization concerns the number of integral points on log Fano varieties  $(X, D)$ . In this setting, the known cases of Manin's conjecture include partial equivariant compactifications of  $\mathbb{G}_a^n$  and  $\mathbb{G}_m^n$ , proved by Chambert-Loir and Tschinkel. We prove an asymptotic formula for the number of integral points of bounded height on the complement of certain planes in  $\mathbb{P}^3$  blown up in a conic that is compatible with the existing predictions. To do so, we lift the problem to the universal torsor – a method developed by Salberger with which he counted rational points on toric varieties.