Quiz 2

Math 308 - May 3, 2017

NAME (last,first): ______________________________

Question 1. (10 points) Consider the following matrix and linear transformation:

$$A = \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \qquad T : \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mapsto \begin{pmatrix} 2x_1 + x_2 \\ 2x_1 + x_2 \\ -x_1 + 2x_2 \\ x_1 \end{pmatrix}$$

(a) [2 point] Write down a matrix B such that $T = T_B$, i.e. $T(x) = B \cdot x$.

> **Solution:**
> $$B = \begin{pmatrix} 2 & 1 \\ 2 & 1 \\ -1 & 2 \\ 1 & 0 \end{pmatrix}$$

(b) [2 points] Identify domain and codomain of the linear transformations T_A and T_B.

> **Solution:** $T_A : \mathbb{R}^4 \to \mathbb{R}^3$ and $T_B : \mathbb{R}^2 \to \mathbb{R}^4$.

(c) [1 point] Discuss which compositions $T_A \circ T_B$ and $T_B \circ T_A$ make sense and compute their domain and codomain.

> **Solution:** Since the codomain of T_B is equal to the domain of T_A the composition $T_A \circ T_B : \mathbb{R}^2 \to \mathbb{R}^3$ make sense while $T_B \circ T_A$ does not since the codomain of T_A is different from the domain of T_B.

(d) [3 points] For each composition that make sense compute the matrix associated to the linear transformations $T_A \circ T_B$ and $T_B \circ T_A$.

> **Solution:** We know that $T_A \circ T_B = T_{A \cdot B}$ and
> $$A \cdot B = \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 \\ 2 & 1 \\ -1 & 2 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 3 \\ 0 & 3 \\ 3 & 1 \end{pmatrix}$$

(e) [2 points] For each composition that make sense discuss whether the linear transformation is 1-to-1, onto and/or invertible.

> **Solution:** The columns of $A \cdot B$ are linearly independent hence $T_A \circ T_B$ is 1-to-1. On the other hand their span is not the whole $\mathbb{R}^3$ hence it is not onto nor invertible.