Practice Midterm 2

for Math 308-G, Autumn 2016

Total number of questions: 5

Total number of points: 50

Question 1. (10 points) Decide whether the following statements are true or false. For this you don't need to show any work (but for practicing you might want to try to do that).

(a) [1 point] If $A \cdot x = b$ is consistent, then $b \in \operatorname{range} T_A$ where T_A is the linear function $x \mapsto A \cdot x$.

◯ True ◯ False

(b) [1 point] A linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ does not change the length of the vectors, i.e. x and $T(x)$ have the same length for every $x \in \mathbb{R}^2$.

◯ True ◯ False

(c) [1 point] If $\mathcal{B}$ is a basis for a subspace S, then $\vec{0} \in \mathcal{B}$.

◯ True ◯ False

(d) [1 point] If $T : \mathbb{R}^n \to \mathbb{R}^m$ and $\operatorname{range}(T) = \mathbb{R}^m$ then T is onto.

◯ True ◯ False

(e) [1 point] If $T_A : \mathbb{R}^n \to \mathbb{R}^m$ is the linear function $x \to A \cdot x$ then A is a $n \times 1$ matrix.

◯ True ◯ False

(f) [1 point] If S is a subspace of dimension k then every set of m vectors of S with $m > k$ span S.

◯ True ◯ False

(g) [1 point] If A, B, C are $n \times n$ matrices and $A \cdot C = B \cdot C$, then $A = B$.

◯ True ◯ False

(h) [1 point] An upper triangular matrix is symmetric.

◯ True ◯ False

(i) [1 point] If $\mathcal{B}$ is a basis for a subspace S then $\dim S$ is bigger than 1.

◯ True ◯ False

(j) [1 point] The kernel of a linear transformation needs to contain the zero vector.

◯ True ◯ False

Question 2. (10 points) Let A be a $n \times n$ matrix and let T_A be the corresponding linear transformation defined by $x \mapsto A \cdot x$. Prove that the following two statements are equivalent:

(a) T_A is 1-to-1;

(b) $\ker(T_A) = \{\vec{0}\}$.

Question 3. (10 points) Let A be the following matrix:

$$A = \begin{pmatrix} 2 & 4 & 2 & 2 \\ 1 & 0 & 1 & 2 \\ 1 & 1 & 0 & 1 \\ 3 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

and let T_A be the corresponding linear transformation, $T_A : x \mapsto A \cdot x$.

(a) [4 points] Determine a basis for null(A).

(b) [4 points] Determine a basis for range(T_A).

(c) [1 point] Is T_A 1-to-1 and/or onto? Why?

(d) [1 point] What is rank(A)? And the nullity of A?

Question 4. (10 points) Let T be the linear transformation defined by

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 + 2x_3 + x_4 \\ 2x_1 + 3x_2 + 4x_3 + 3x_4 \\ x_2 + x_3 + x_4 \\ x_1 + x_2 + x_3 \end{pmatrix}$$

(a) [2 points] Identify domain and codomain of T and compute the matrix A such that $T(x) = A \cdot x$.

(b) [2 points] Suppose that the reduced echelon form of A is equal to the identity matrix I_4. Is T_A 1-to-1 and/or onto? What is the rank and the nullity of A?

(c) [6 points] In the assumption of the previous part determine if T_A is invertible, and if it possible compute the matrix associated to the inverse transformation.

Question 5. (10 points) Let A and B be the following matrices

$$A = \begin{pmatrix} 2 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \\ 3 & 3 \\ 0 & 0 \end{pmatrix}$$

(a) [2 points] Identify the domain and the codomain of the linear transformations T_A and T_B associated to A and B.

(b) [4 points] For all the compositions $T_A \circ T_B$ and $T_B \circ T_A$ explain whether they make sense and if they do, compute the matrix associated.

(c) [4 points] Compute whether the previous composition(s) are 1-to-1 and/or onto.