HOMEWORK 5 - MATH402B

DUE: WEDNESDAY NOVEMBER 1ST

(1) Goodman 2.2.16

(2) Goodman 2.2.19

(3) Consider the two functions $f, g : \mathbb{Z}_6 \to \mathbb{Z}_8$ defined as

$$f([x]_6) = [2x]_8 \qquad g([x]_6) = [4x]_8$$

Discuss whether f, g are well defined.

(4) Goodman 2.3.2

(5) Goodman 2.3.4

(6) Goodman 2.3.6

(7) Goodman 2.3.7

(8) Let $(G, \cdot)$ be a group and $g, x \in G$. We define

$$c_g(x) := g \cdot x \cdot g^{-1}$$

and we call $c_g(x)$ a conjugate of x. Show that

(a) If $y = c_g(x)$ then there exists $h \in G$ such that $x = c_h(y)$.

(b) If $g, h \in G$ and $y = c_g(x)$, $z = c_h(y)$ then there exists an element $k \in G$ such that $z = c_k(x)$.

(c) If $g \in G$ the set $\{c_g(x^n) : n \in \mathbb{Z}\}$ is a subgroup of G called the *conjugate* of $\langle x \rangle$.

(d) Generalizing the previous point show that, given a subgroup H of G and $g \in G$, the *conjugate* of H, i.e. the set $\{c_g(h) : h \in H\}$ is a subgroup of G .

(9) Let $G = \{a + bi : a, b \in \mathbb{Q}\} \subset \mathbb{C}$ be the subset of $\mathbb{C}$ of complex numbers with rational real and imaginary part. Show that G is a group under multiplication and compute which of the following subgroups have finite order: $\langle 1 + i \rangle$, $\langle \frac{1}{2} i \rangle$, and $\langle -1 \rangle$.

(10) Let $(G, \cdot)$ be a group. Show that the set $C(G) = \{z \in G : zg = gz \text{ for all } g \in G\}$ is a subgroup of G called the *center* of G. Compute $C(G)$ in the case where G is a cyclic group (of any order).