

Quiz 1

Math 308 - April 12, 2017

NAME (last,first): _____

Question 1. (10 points) Consider the following linear system

$$\begin{aligned}x + 2y &= 0 \\ x + 3y + z &= 0\end{aligned}$$

- (a) [1 point] Without doing any computation, what can you say about the solution set?

Solution: Since the system is homogeneous the solution set will always be non empty given the presence of the trivial solution.

- (b) [4 points] Using the Gauss-Jordan Algorithm, compute the Reduced Echelon Form of the coefficient matrix associated to the system.

Solution:

$$\begin{pmatrix} 1 & 2 & 0 \\ 1 & 3 & 1 \end{pmatrix} \xrightarrow{\text{II}-\text{I}} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \end{pmatrix} \xrightarrow{\text{I}-2\text{II}} \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \end{pmatrix}$$

- (c) [1 point] Given the reduced echelon form how many solutions will the linear system have? Why?

Solution: It will have infinitely many solutions since it has a column without a leading term and hence a free parameter will appear in the solution.

- (d) [2 point] Write down the solution set in vector form. What is the dimension of the solution set?

Solution: The solution set is given by $x - 2z = 0$ and $y + z = 0$. Setting $z = s$ we get

$$\left\{ \begin{pmatrix} 2s \\ -s \\ s \end{pmatrix} : s \in \mathbb{R} \right\} = \left\{ s \cdot \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} : s \in \mathbb{R} \right\}$$

The dimension will be 1 since there is one free parameter.

- (e) [2 point] If possible write the solution set as a span of some vectors and describe geometrically the solution set (i.e. is it a line a plane or an hyperplane in some \mathbb{R}^n).

Solution: By the previous point we can see that the solution set can be written as

$$\text{span} \left(\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \right)$$

and therefore will be a line in \mathbb{R}^3 defined by the vector $\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$.