Quiz 1

January 18, 2017

NAME (last,first): ______________________________

Question 1. (10 points) A *plane quadric* in $\mathbb{R}^2$ is an equation of the form

$$ax^2 + bxy + cy^2 + dx + ey + f = 0$$

where a, b, c, d, e and f are real numbers. You are tasked to find all possible plane quadrics passing through $(1, 0)$, $(1, 1)$, $(0, 1)$ and $(0, 0)$.

(a) [2 point] Write down the linear system that represents the problem.

Solution: To find the linear system we substitute the values of x and y of the given points into the equation of the quadric and we get the following system

$$\begin{cases} a + d + f = 0 \\ a + b + c + d + e + f = 0 \\ c + e + f = 0 \\ f = 0 \end{cases}$$

(b) [1 points] Without doing any computation is the system consistent or inconsistent? Why?

Solution: The system is homogeneous so it is always consistent.

(c) [5 point] Using the Gauss-Jordan Algorithm, find the solution set of the system.

Solution:

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \underset{\sim}{II-I} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \underset{\sim}{II-III} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\underset{\sim}{I-IV, II+IV, III-IV} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \rightarrow \begin{cases} a = -d \\ b = 0 \\ c = -e \\ f = 0 \end{cases}$$

This implies that, choosing $d = s_1$ and $e = s_2$ as free variables, the solution set is given by

$$\{(a, b, c, d, e, f) = (-s_1, 0, -s_2, s_1, s_2, 0) : s_1, s_2 \in \mathbb{R}\}$$

(d) [2 point] Is the solution set finite or infinite? What is the dimension of the solution set?

Solution: The solution set is infinite since every value of s_1 and s_2 is admissible. The dimension of the solution set is equal to the number of free parameters, hence it is 2.