Math 308 Discussion Problems #3
Chapter 3 (after 3.3)

(1) (Geometry Question) (Note: This problem is repeated in Chapter 5 with more parts.) Suppose we are given the unit square A in the plane with corners $(0,0)$, $(1,0)$, $(1,1)$ and $(0,1)$.

(a) Find a linear transformation T that sends A to the parallelogram B with corners $(0,0)$, $(1,2)$, $(2,2)$ and $(1,0)$.
(b) Where does T send the point $(1/2,1/2)$, which was in A?
(c) Is the linear transformation T unique? Why or why not?
(d) What linear transformation T' would send A to itself?
(e) Suppose we want to not only send A to B but also push B in the horizontal direction by one unit. What map can do this?
(f) Let L be the linear span of the side of B with corners $(0,0)$ and $(1,2)$. Write L in parametric form: $\mathbf{p}+\mathbf{q}\mathbf{t}$ where t varies in some range and $\mathbf{p}$, $\mathbf{q}$ are vectors. What is the range of t and what are $\mathbf{p}$ and $\mathbf{q}$?
(g) Find the point in A that maps under T to the point $(1/2,1)$ on L. In your parametric representation of L, what is the representation of $(1/2,1)$?
(h) How can you map A to a parallelogram C of area 4 while still keeping $(0,0)$ and $(1,0)$ as two of its corners?
(i) What is the general formula for the linear transformation that sends A to a parallelogram of area k while still keeping $(0,0)$ and $(1,0)$ as two of its corners?

(2) (Geometry Question) How can you map the triangle $(1,0,0)$, $(0,1,0)$, $(0,0,1)$ to the plane so that its area is preserved and one of its corners is $(0,0)$?

(3) (after 3.2) Let

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 4 \end{bmatrix}.$$

Find a 3×2 matrix B with $AB = I_2$. Is there more than one matrix B with this property? Justify your answer.

(4) (after 3.2) Find a 2×3 matrix A and a 3×2 matrix B such that $AB = I$ but $BA \neq I$.

(5) (after 3.2) Find a 2×2 matrix A, which is not the zero or identity matrix, satisfying each of the following equations.
a) $A^2 = 0$
b) $A^2 = A$
c) $A^2 = I_2$

(6) (after 3.2) Let

$$B = \begin{bmatrix} 1 & z \\ 4 & 3 \end{bmatrix}.$$

Find all values of z such that the linear transformation T induced by B fixes no line in $\mathbb{R}^2$. (By "fixing a line" we mean that $T(\mathbf{v}) = \mathbf{v}$ for every point $\mathbf{v}$ on the line.)

(7) (after 3.3) Find a 3×2 matrix A and a 2×3 matrix B such that AB is invertible or explain why such matrices cannot exist. Answer the same question with the requirement that BA be invertible.