Quiz 3

November 30, 2016

Question 1. (10 points) Let A be the following matrix:

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 3 & 0 \\ 2 & 1 & 2 \end{pmatrix}$$

(a) [3 point] Find all the eignevalues of A and their multiplicities.

Solution: To find the eigenvalues of A we need to compute the characteristic polynomial. To do that we compute

$$p_\lambda(A) = \det(A - \lambda I_3) = \det \begin{pmatrix} 1-\lambda & 0 & 1 \\ 0 & 3-\lambda & 0 \\ 2 & 1 & 2-\lambda \end{pmatrix} = -\lambda(\lambda - 3)^2$$

From this expression we can see that A has two eigenvalues, namely 0 with multiplicity 1 and 3 with multiplicity 2.

(b) [5 points] Determine bases for all the eigenspaces of A relatively to the egienvalues you found in the previous part.

Solution: For the eigenvalue 0 the eigenspace associated is $E_0(A) = \text{null}(A - 0 \cdot I_3) = \text{null}(A)$. To compute the null space of A we reduce it to echelon form

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 3 & 0 \\ 2 & 1 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The corresponding linear system has two non zero equations given by $x_1 + x_3 = 0$ and $x_2 = 0$ so the solution set is given by

$$\{ \begin{pmatrix} -s \\ 0 \\ s \end{pmatrix} : s \in \mathbb{R} \} = \text{span} \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

For the eigenvalue 3 the eigenspace associated is $E_3(A) = \text{null}(A - 3 \cdot I_3)$. Call $B = A - 3I_3$. We need to compute its null space and to do that we reduce B to echelon form

$$\begin{pmatrix} -2 & 0 & 1 \\ 0 & 0 & 0 \\ 2 & 1 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The corresponding linear system has two non zero equations given by $x_1 - 1/2x_3 = 0$ and $x_2 = 0$ so the solution set is given by

$$\{ \begin{pmatrix} \frac{1}{2}s \\ 0 \\ s \end{pmatrix} : s \in \mathbb{R} \} = \text{span} \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

(c) [1 point] Compute

$$A^{2016} \cdot \begin{pmatrix} -100 \\ 0 \\ 100 \end{pmatrix}$$

Solution: Since the vector we are multiplying is in $E_0(A)$ we know that

$$A^k \cdot \begin{pmatrix} -100 \\ 0 \\ 100 \end{pmatrix} = 0^k \cdot \begin{pmatrix} -100 \\ 0 \\ 100 \end{pmatrix} = 0$$

for any value of k. So the answer is 0.

(d) [1 point] Without doing *any* computation, can you tell if A is invertible or not? Why?

Solution: Since 0 is an eigenvalue of A we know that $\det(A) = 0$ which implies by the big theorem that A is not inveritble.