Quiz 3

Math 308 - May 31st, 2017

NAME (last,first): ______________________________

Question 1. (10 points) Consider the following matrix

$$A = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 2 & 2 \end{pmatrix}$$

(a) [2 point] Compute the eigenvalues of A and their multiplicities.

> **Solution:** Using the properties of the determinant we obtain
>
> $$\det(A - \lambda I_4) = (2-\lambda)^2 \cdot (\lambda - 4) \cdot (\lambda + 1).$$
>
> This shows that the eigenvalues of A are $2, 4, -1$ with multiplicities $2, 1, 1$ respectively.

(b) [3 points] Compute a basis $\mathcal{B}$ of the eigenspace associated to the eigenvalue with maximal multiplicity.

> **Solution:** The eigenvalue with maximal multiplicity is $\lambda = 2$. In this case
>
> $$E_2(A) = (A - 2I_4) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$
>
> From this we can see that
>
> $$E_2(A) = \text{span}\{\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}\} \Rightarrow \mathcal{B} = \{e_1, e_2\}$$

(c) [2 point] Given the above computation, is A diagonalizable? Why?

> **Solution:** Since $\dim E_4(A) = 1$ and $\dim E_{-1}(A) = 1$ we have that the sum of the dimensions of the eigenspaces equal the number of rows (and columns) of A, hence A is diagonalizable.

(d) [3 point] Let $\mathcal{A} = \{u, v\}$ where $u = e_1 + e_2$ and $v = -e_2$, be another basis for the eigenspace you computed above. Express each vector of $\mathcal{B}$ as a linear combination of u and v and compute the matrix of the change of basis $M_{\mathcal{A},\mathcal{B}}$.

> **Solution:** It is easy to see that $e_1 = u + v$ and $e_2 = -v$. From this we have
>
> $$M_{\mathcal{A},\mathcal{B}} = ([e_1]_{\mathcal{A}}, [e_2]_{\mathcal{A}}) = \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix}$$