Quiz 2

February 15, 2017

NAME (last,first): ____________________________________

Question 1. (10 points) Let T_A and B be the following linear transformation and matrix:

$$T_A\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2x_1 + x_2 + 4x_4 \\ x_1 + x_3 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 2 & 1 \\ 0 & 1 \end{pmatrix}$$

(a) [1 point] Write down the matrix A associated to T, i.e. such that $T(x) = A \cdot x$.

> **Solution:**
> $$A = \begin{pmatrix} 2 & 1 & 0 & 4 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

(b) [2 points] Denoted by T_B the transformation $T_B(x) = B \cdot x$, complete the following sentences:

T_A has domain $\underline{\quad\mathbb{R}^4\quad}$ and codomain $\underline{\quad\mathbb{R}^2\quad}$.
T_B has domain $\underline{\quad\mathbb{R}^2\quad}$ and codomain $\underline{\quad\mathbb{R}^4\quad}$.

(c) [2 points] For each composition that make sense, complete the following sentences:

$T_A \circ T_B$ has domain $\underline{\quad\mathbb{R}^2\quad}$ and codomain $\underline{\quad\mathbb{R}^2\quad}$.
$T_B \circ T_A$ has domain $\underline{\quad\mathbb{R}^4\quad}$ and codomain $\underline{\quad\mathbb{R}^4\quad}$.

(d) [4 points] For each composition that make sense, compute the matrices associated to the linear transformations $T_A \circ T_B$ and $T_B \circ T_A$ or say why it does not make sense.

> **Solution:** Both composition make sense since the domain of T_A coincide with the codomain of T_B and the domain of T_B coincide with the domain of T_A.
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> $$T_A \circ T_B = T_{A \cdot B} \qquad A \cdot B = \begin{pmatrix} 2 & 1 & 0 & 4 \\ 1 & 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 2 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 9 \\ 3 & 3 \end{pmatrix}$$
>
> $$T_B \circ T_A = T_{B \cdot A} \qquad B \cdot A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 2 & 1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 & 0 & 4 \\ 1 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 4 & 1 & 2 & 4 \\ 1 & 0 & 1 & 0 \\ 5 & 2 & 1 & 8 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

(e) [1 point] If the linear transformation $T_A \circ T_B$ make sense, is it invertible? Explain why or why not!

> **Solution:** Since $A \cdot B$ it is a 2×2 matrix it is invertible since its determinant is $6 - 27 = -21 \neq 0$. Therefore $T_{A \cdot B} = T_A \circ T_B$ is invertible.