

Math 308 Discussion Problems #4

Chapter 4 (after 4.3)

- (1) (after 4.1) Let S be a plane in \mathbf{R}^3 passing through the origin, so that S is a two-dimensional subspace of \mathbf{R}^3 . Say that a linear transformation $T : \mathbf{R}^3 \rightarrow \mathbf{R}^3$ is a *reflection about S* if $T(\mathbf{v}) = \mathbf{v}$ for any vector \mathbf{v} in S and $T(\mathbf{n}) = -\mathbf{n}$ whenever \mathbf{n} is perpendicular to S . Let T be the linear transformation given by $T(\mathbf{x}) = A\mathbf{x}$, where A is the matrix

$$\frac{1}{3} \begin{bmatrix} -1 & -2 & 2 \\ -2 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}.$$

This linear transformation is the reflection about a plane S . Find a basis for S .

- (2) (after 4.2) (Geometry Question) In this problem we continue the Geometry Problem from Chapter 2, but now we work in \mathbf{R}^3 . Consider the infinite linear system given by the equations $ax + by = 0$ where you should think of these having a z variable with zero coefficient.
- (a) Describe the solution space of the above system.
 - (b) How many linearly independent solutions are there in this solution space? (i.e., what is the dimension of this solution space?)
 - (c) Write down a basis of the solution space.
 - (d) Express this solution space as the kernel of a finite matrix. What is the smallest size matrix that will do the job?
 - (e) If we keep on doing this example in higher and higher dimensional space, what happens to the dimension of the solution space?

- (3) (after 4.2) Expand the set $\left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$ to be a basis for the subspace

$$w + x + y + z = 0.$$

- (4) (after 4.3) Find an *invertible* $n \times n$ matrix A and an $n \times n$ matrix B such that $\text{rank}(AB) \neq \text{rank}(BA)$, or explain why such matrices cannot exist.
- (5) (after 4.3) Find a 3×4 matrix A with nullity 2 and with

$$\text{col}(A) = \text{span} \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ -3 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix} \right\},$$

or explain why such a matrix can't exist.

- (6) (after 4.3) Find a 3×3 matrix A and a 3×3 matrix B , each with nullity 1, such that AB is the 0 matrix, or explain why such matrices cannot exist.