QUIZ 2 - MATH 308 G - NOVEMBER 9, 2016

SOLUTION (B)

Let T_A and B be the linear transformation and the matrix defined by

$$T_A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} x_1 - x_3 \\ x_2 + x_4 \end{pmatrix} \qquad B = \begin{pmatrix} 5 & 1 \\ 6 & 7 \\ 2 & 1 \\ 1 & 2 \end{pmatrix}$$

1 pt. Write down the matrix A such that $T_A(x) = A \cdot x$..
Solution:

$$A = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

Since

$$A \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} x_1 - x_3 \\ x_2 + x_4 \end{pmatrix} = T_A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

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2 pt. Consider the two linear transformations T_A of the previous point and T_B defined by $T_B(x) = B \cdot x$. Find the domain and the codomain of T_A and T_B and complete with the appropriate exponent the following expression (you don't need to show any computation here):

$$T_A : \mathbb{R}^4 \to \mathbb{R}^2 \qquad T_B : \mathbb{R}^2 \to \mathbb{R}^4$$

2 pt. For each composition that make sense compute the domain and the codomain of the following linear transformations and complete with the appropriate exponent the following expressions (you don't need to show any computation here):

$$T_A \circ T_B : \mathbb{R}^2 \to \mathbb{R}^2 \qquad T_B \circ T_A : \mathbb{R}^4 \to \mathbb{R}^4$$

4 pt. For all the compositions that make sense compute the matrix associated to the linar transfomations $T_A \circ T_B$ and $T_B \circ T_A$ or say why it does not make sense.
Solution:

Both compositions make sense since the domain of T_A coincide with the codomain of T_B and the domain of T_B is the same as the codomain of T_A.

$$T_A \circ T_B = T_{A \cdot B} \qquad A \cdot B = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 5 & 1 \\ 6 & 7 \\ 2 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 7 & 9 \end{pmatrix}$$

$$T_B \circ T_A = T_{B \cdot A} \qquad B \cdot A = \begin{pmatrix} 5 & 1 \\ 6 & 7 \\ 2 & 1 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 1 & -5 & 1 \\ 6 & 7 & -6 & 7 \\ 2 & 1 & -2 & 1 \\ 1 & 2 & -1 & 2 \end{pmatrix}$$

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1pt. Is the matrix associated to $T_B \circ T_A$ invertible (you don't need to compute the inverse)?
Solution:

The matrix is not invertible since two of its column are the same. This implies that the columns are not linearly independent, hence the associated linear function $T_B \circ T_A$ is not 1-to-1. This implies that it is not invertible proving that its associated matrix is not invertible either. ■