HOMEWORK 6 - MATH402B

DUE: WEDNESDAY NOVEMBER 17TH

(1) Goodman 2.5.4

(2) Goodman 2.5.7

(3) Goodman 2.5.8

(4) Goodman 2.5.13

(5) Recall that A_n is the subgroup of S_n of even permutations (see HW 5.5). Recall that given a group G and an element $a \in G$ a conjugacy class of a is a subset of the form $\{c_g(a) = gag^{-1} : g \in G\}$ so it's the set of all conjugates of a in G (see HW 5). Compute all conjugacy classes in A_4.

(6) Let G, H, K be the following subgroups of $GL(2, \mathbb{R})$:

$$G = \{\begin{pmatrix} x & y \\ 0 & 1 \end{pmatrix} : x, y \in \mathbb{R}, x \neq 0\} \qquad H = \{\begin{pmatrix} x & 0 \\ 0 & 1 \end{pmatrix} : x > 0\} \qquad K = \{\begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} : b \in \mathbb{R}\}$$

Notice that G (that is basically the same group on our midterm) can be viewed as the plane $\mathbb{R}^2$ with the y-axis removed. Consequently, you can describe various cosets as subsets of the plane. In particular, describe the left and right cosets of H. Also describe the left and right cosets of K.

(7) Let $H, K \leq G$ be subgroups of G and consider

$$HK := \{hk : h \in H, k \in K\}.$$

Prove that

- if either H or K is normal in G then $HK = KH$ and HK is a subgroup of G.
- if $H \cap K = \{e\}$ then $|HK| = |H||K|$. (Hint: construct a bijection from $H \times K$ to HK.)

(8) Let $G = D_n$ the dihedral group or order $2n$.

- Show that the rotation subgroup $T = \{e, r, r^2, \ldots, r^{n-1}\}$ is a normal subgroup of G of order n.
- Show that any subgroup of T is a normal subgroup of G.
- Show that if $d|2n$ then G has a subgroup of order d. In other words, the converse of Lagrange Theorem holds for dihedral groups. (Hint, use the previous problem).

(9) Goodman 2.6.1