HOMEWORK 3 - MATH402B

DUE: WEDNESDAY OCTOBER 18TH

(1) Goodman 1.6.3

(2) Goodman 1.6.4

(3) Goodman 1.6.14

(4) Goodman 1.7.4

(5) Goodman 1.7.5

(6) Goodman 1.7.13

(7) Goodman 1.7.14

(8) Define for two integers

$$x \sim y \;\Leftrightarrow\; x^2 - y^2 = 3m \qquad \text{for some } m \in \mathbb{Z}$$

Show that

(a) For any $x \in \mathbb{Z}$, $x \sim x$;

(b) For any $x, y \in \mathbb{Z}$, if $x \sim y$ then $y \sim x$;

(c) For any $x, y, z \in \mathbb{Z}$, if $x \sim y$ and $y \sim z$ then $x \sim z$.

(d) Compute $\{x \in \mathbb{Z} : 5 \sim x\}$.

(e) Show that $5 \sim y$ if and only if $5 \equiv_3 \pm y$.

(9) Show that $gcd(a, b)$ divides $gcd(a - b, a + b)$.