Promises and Infinite-Domain Constraint Satisfaction

Antoine Mottet
TU Hamburg

CSL 2024

Currently hiring a PhD student!

antoine.mottet@tuhh.de

Constraint Satisfaction Problems

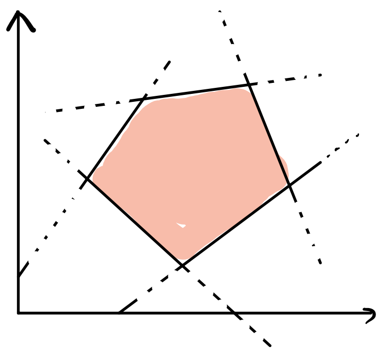
variables $\rightsquigarrow$ domain satisfying some constraints

Constraint Satisfaction Problems

Solve over $\mathbb{Z}$:

$$\begin{cases} 5x + y - z = 1 \\ x - y + z = 2 \end{cases}$$

Solve $Ax \geqslant b$ over $\mathbb{R}_{\geqslant 0}$:



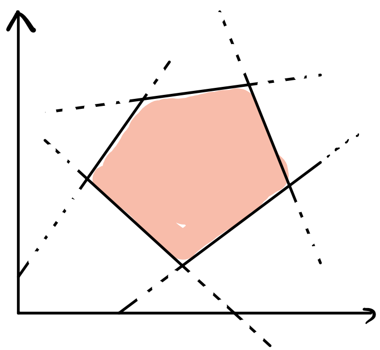
variables $\rightsquigarrow$ domain satisfying some constraints

Constraint Satisfaction Problems

Solve over $\mathbb{Z}$:

$$\begin{cases} 5x + y - z = 1 \\ x - y + z = 2 \end{cases}$$

Solve $Ax \geq b$ over $\mathbb{R}_{\geq 0}$:



Solve over $\{true, false\}$:

$$(p \vee q \vee \bar{r}) \wedge (r \vee \bar{p}) \wedge (r \vee \bar{q})$$

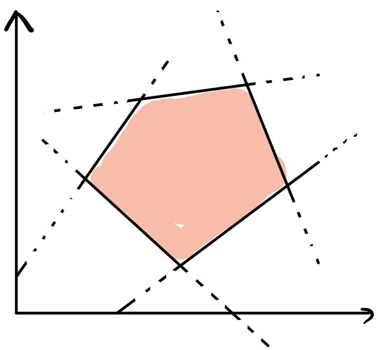
variables $\rightsquigarrow$ domain satisfying some constraints

Constraint Satisfaction Problems

Solve over $\mathbb{Z}$:

$$\begin{cases} 5x + y - z = 1 \\ x - y + z = 2 \end{cases}$$

Solve $Ax \geq b$ over $\mathbb{R}_{\geq 0}$:



Solve over $\{true, false\}$:

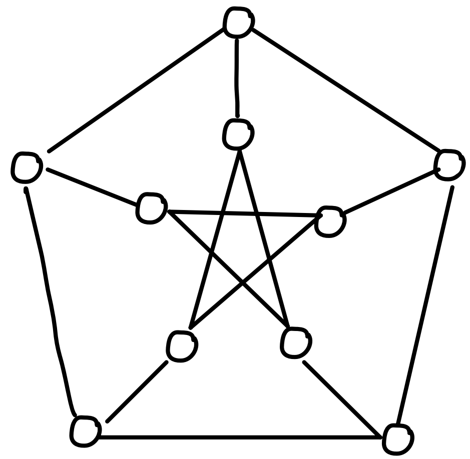
$$(p \lor q \lor \bar{r}) \land (r \lor \bar{p}) \land (r \lor \bar{q})$$

variables $\rightsquigarrow$ domain satisfying some constraints

Solve over $\{1, \ldots, 9\}$:

3			8		1			2
2		1		3		6		4
			2		4			
8		9				1		6
	6						5	
7		2				4		9
9			5		9			
		4		8		7		5
6			1		7			3

"Solve" over $\{\circ, \circ, \circ\}$:



Constraint Satisfaction Problems

Input: D ... domain $\qquad V$ ... variables

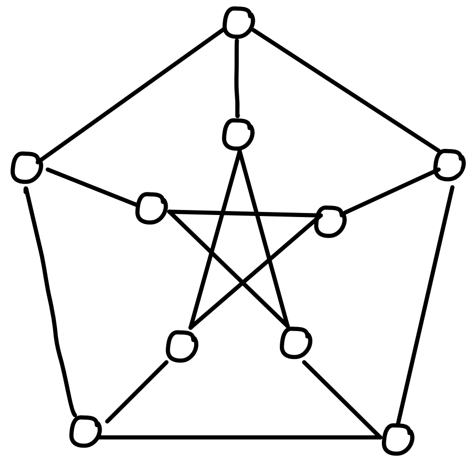
C ... constraints: $(v_1, \dots, v_r) \to \{\text{list of allowed assignments}\}$

Question: $\exists\, h: V \to D$ satisfying all constraints?

Solve over $\{1, \dots, 9\}$:

3			8		1			2
2		1		3		6		4
			2		4			
8		9				1		6
	6						5	
7		2				4		9
9			5		9			
		4		8		7		5
6			1		7			3

"Solve" over {●, ●, ●}:

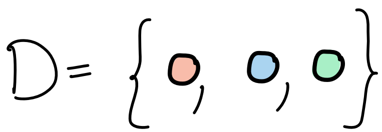


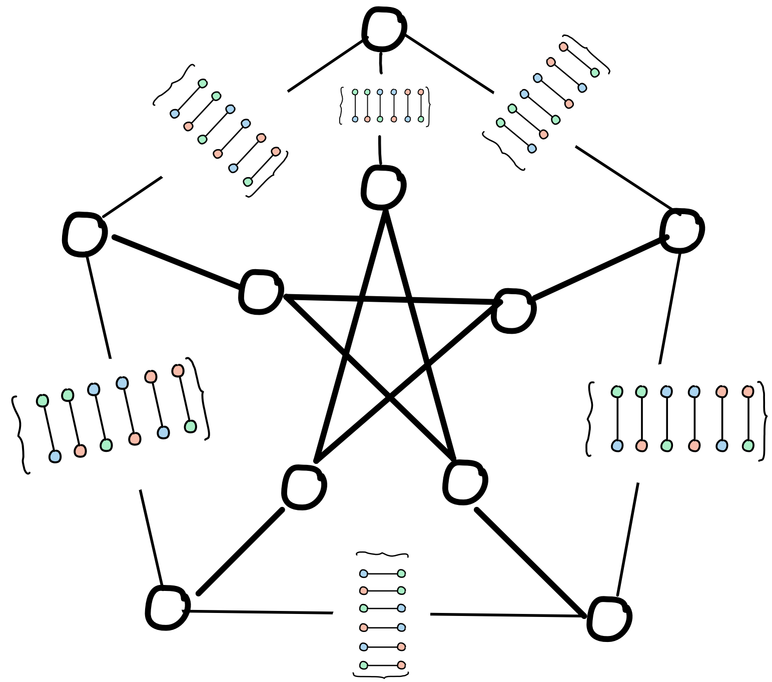
Constraint Satisfaction Problems

<u>Input:</u> D … domain V … variables

C … constraints: $(v_1, \dots, v_r) \to \{\text{list of allowed assignments}\}$

<u>Question:</u> $\exists\, h : V \to D$ satisfying all constraints?





Constraint Satisfaction Problems

Input of CSP($\mathbb{D}$): V ... variables

C ... constraints: $(v_1, \dots, v_r) \to E$

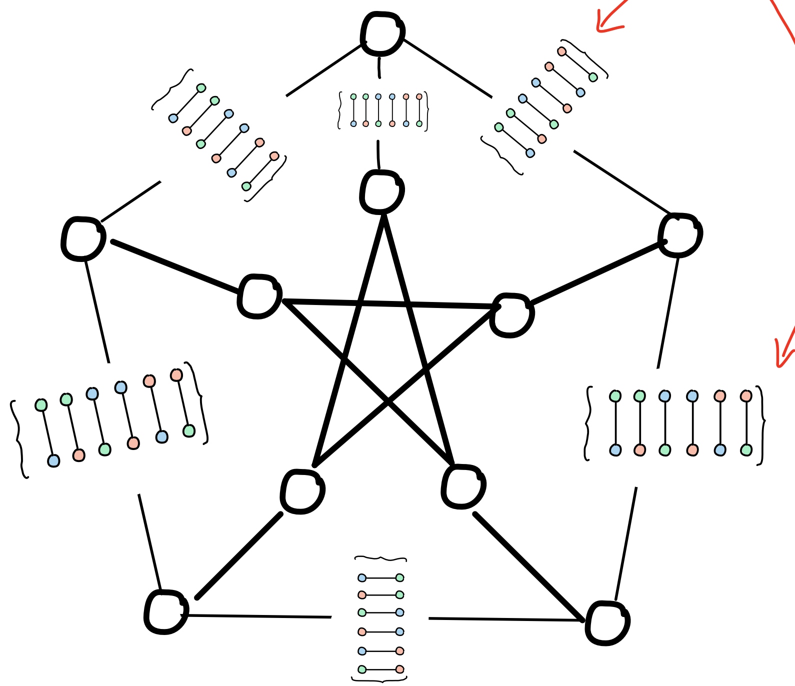
Question: $\exists\, h: V \to D$ satisfying all constraints?

$D = \{$ ●, ●, ● $\}$

all instances of 3-coloring use the same constraint relation

$E \subseteq D^2$

$\mathbb{D} = (D, E)$



Constraint Satisfaction Problems

Input of CSP($\mathbb{D}$): V … variables
C … constraints: $(v_1, \dots, v_r) \to E$

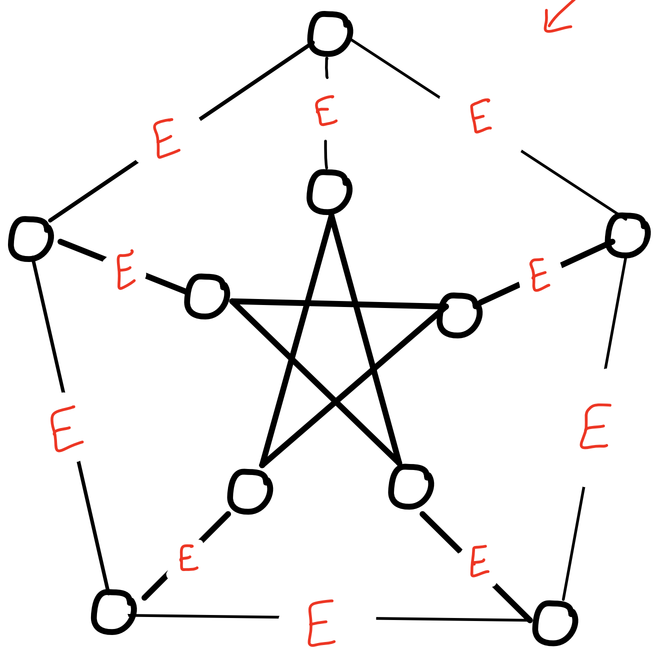
itself a relational structure $\mathbb{X}$

Question: $\exists\, h: V \to D$ satisfying all constraints? $\exists$ homomorphism $\mathbb{X} \to \mathbb{D}$?

$D = \{ \circ, \circ, \circ \}$

all instances of 3-coloring use the same constraint relation $E \subseteq D^2$

$\mathbb{D} = (D, E)$



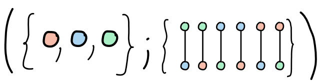
Constraint Satisfaction Problems

<u>Input of CSP($\mathbb{D}$):</u> V ... variables

C ... constraints: $(v_1, \dots, v_r) \to E$ } itself a relational structure $\mathbb{X}$

<u>Question:</u> $\exists\, h: V \to D$ satisfying all constraints? } $\exists$ homomorphism $\mathbb{X} \to \mathbb{D}$?

3-COL



1 in 3-SAT

$\left(\{\text{True}, \text{False}\};\ \left\{\begin{pmatrix}\text{True}\\ \text{False}\\ \text{False}\end{pmatrix}, \begin{pmatrix}\text{False}\\ \text{True}\\ \text{False}\end{pmatrix}, \begin{pmatrix}\text{False}\\ \text{False}\\ \text{True}\end{pmatrix}\right\}\right)$

Lin-Eq($\mathbb{F}_2$)

$\left(\{0,1\};\ \left\{\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix}\right\}, \{0\}\right)$

NOT-ALL-EQUAL-SAT

$\left(\{\text{True}, \text{False}\};\ \left\{\begin{pmatrix}\text{True}\\ \text{False}\\ \text{False}\end{pmatrix}, \begin{pmatrix}\text{False}\\ \text{True}\\ \text{False}\end{pmatrix}, \begin{pmatrix}\text{False}\\ \text{False}\\ \text{True}\end{pmatrix}, \begin{pmatrix}\text{True}\\ \text{True}\\ \text{False}\end{pmatrix}, \begin{pmatrix}\text{True}\\ \text{False}\\ \text{True}\end{pmatrix}, \begin{pmatrix}\text{False}\\ \text{True}\\ \text{True}\end{pmatrix}\right\}\right)$

Horn-SAT

$\left(\{\text{True}, \text{False}\};\ \text{all except} \begin{pmatrix}\text{True}\\ \text{True}\\ \text{False}\end{pmatrix}\right)$

<u>Theorem:</u> (Bulatov - Zhuk '17)

For every finite $\mathbb{D}$, CSP($\mathbb{D}$) is in P or NP-complete.

Extensions

High-dimensional constraints

Infinite Domains

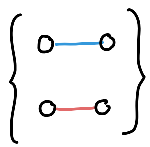
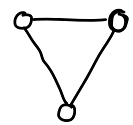
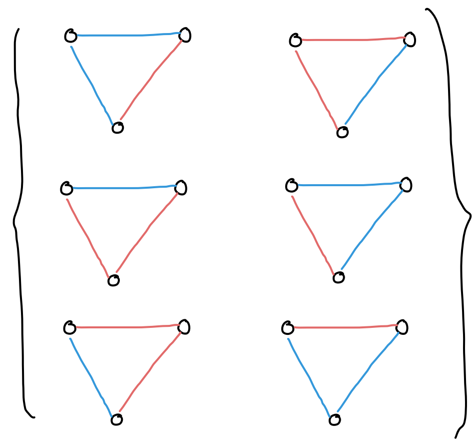
Approximations (qualitative / quantitative)

Extensions

High-dimensional constraints

Infinite Domains

Approximations (qualitative/quantitative)

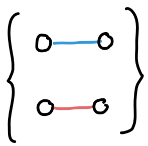
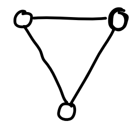
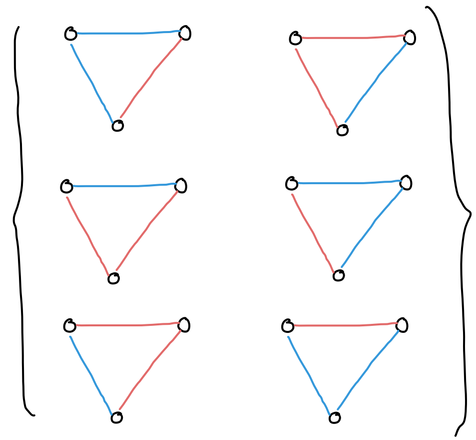
Find assignment **edges** →  s.t.  → 

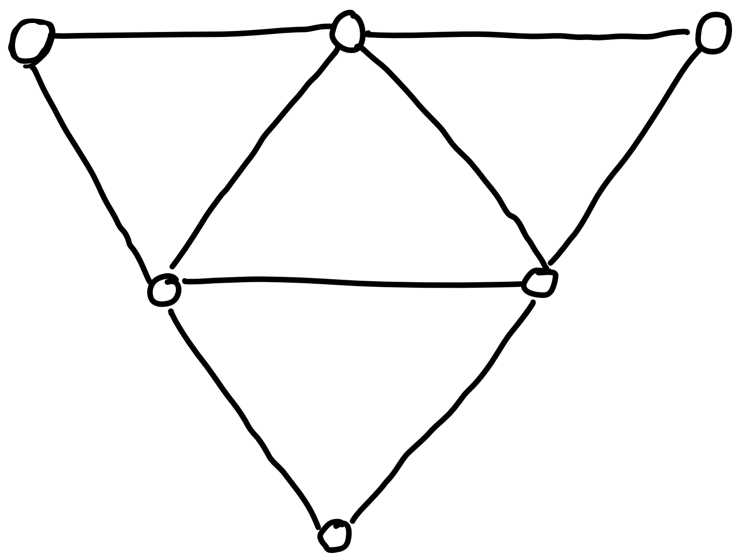
Extensions

High-dimensional constraints

Infinite Domains

Approximations (qualitative/quantitative)

Find assignment **edges** →  s.t.  → 

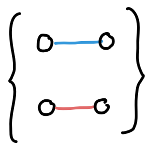
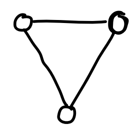
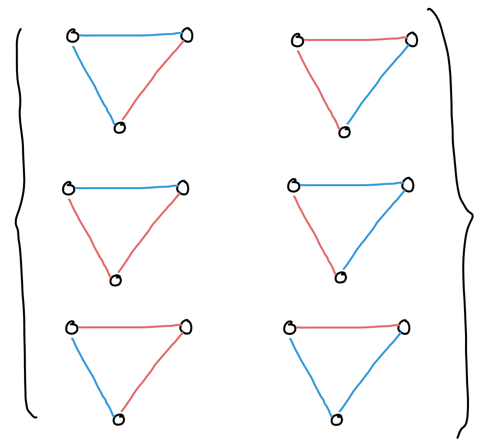


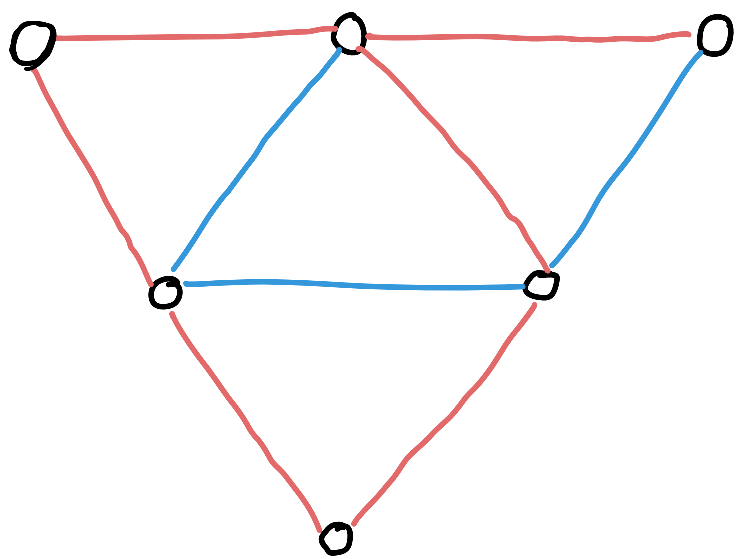
Extensions

High-dimensional constraints

Infinite Domains

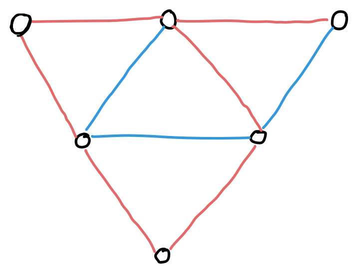
Approximations (qualitative/quantitative)

Find assignment **edges** →  s.t.  → 



Extensions

High-dimensional constraints



Infinite Domains

Approximations (qualitative/quantitative)

Domains and constraints are not part of the input

$\rightsquigarrow$ they can be infinite!

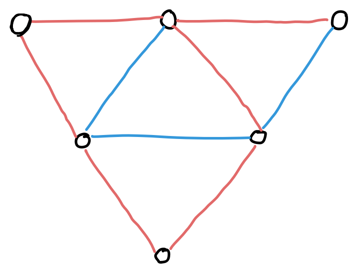
Example:

$\exists h: V \to \mathbb{Q}$ satisfying some constraints

$(u, v, w) \to \{(a, b, c) \mid a < \max(b, c)\}$?

Extensions

High-dimensional constraints



Infinite Domains

Approximations (qualitative/quantitative)

Domains and constraints are not part of the input

$\rightsquigarrow$ they can be infinite!

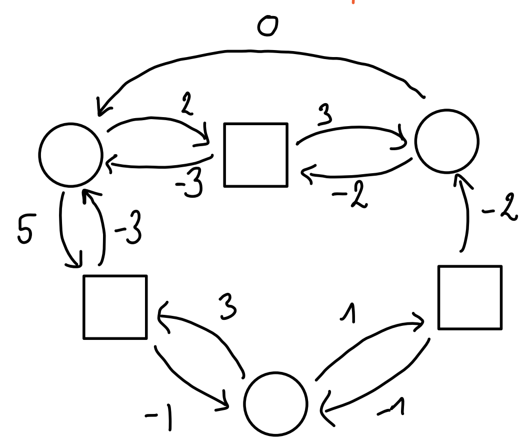
Example:

$\exists h : V \to \mathbb{Q}$ satisfying some constraints

$(u, v, w) \to \{(a, b, c) \mid a < \max(b, c)\}$?

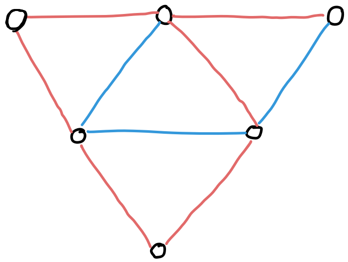
closely related to

mean-payoff games, tropical linear programming

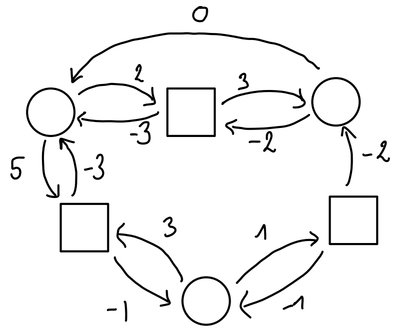


Extensions

High-dimensional constraints



Infinite Domains

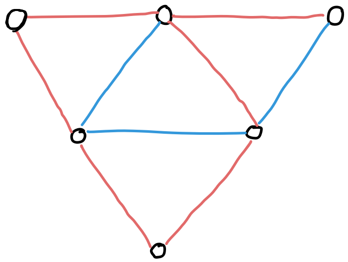


Approximations (qualitative/quantitative)

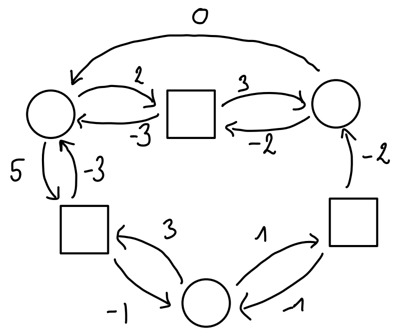
- satisfy as many constraints as possible
- satisfy all constraints in a weakened form

Extensions

High-dimensional constraints



Infinite Domains

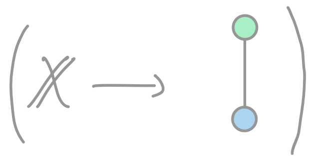


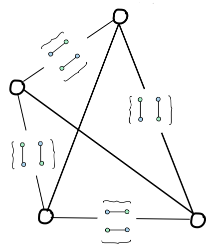
Approximations (qualitative/quantitative)

satisfy as many constraints as possible

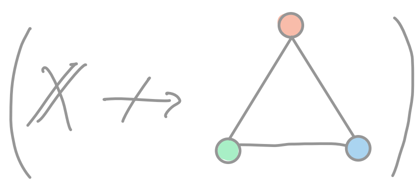
satisfy all constraints in a weakened form

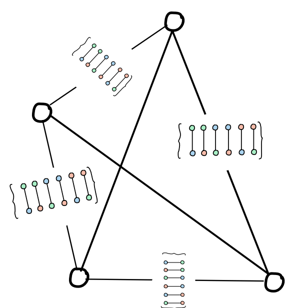
Yes-instances: strongly satisfiable





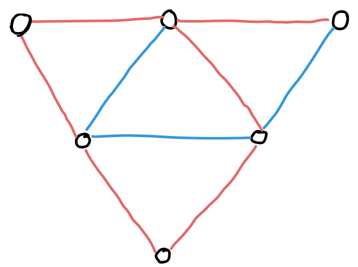
No-instances: not weakly satisfiable



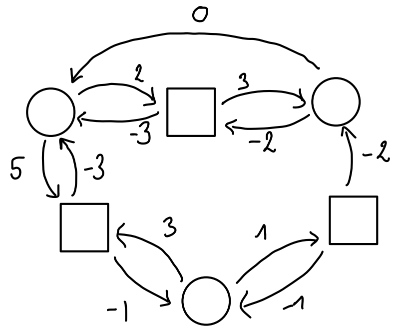


Extensions

High-dimensional constraints



Infinite Domains

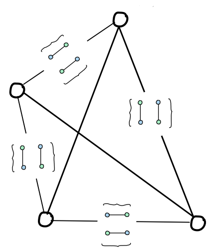


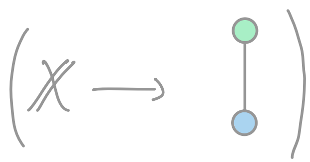
Approximations (qualitative/quantitative)

satisfy as many constraints as possible

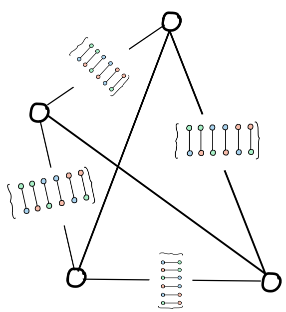
satisfy all constraints in a weakened form

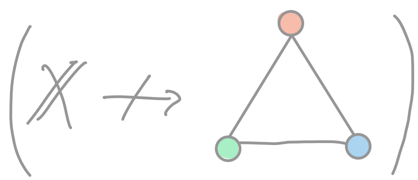
Yes-instances: strongly satisfiable



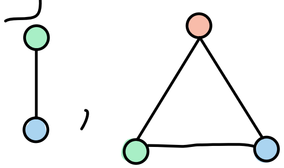


No-instances: not weakly satisfiable





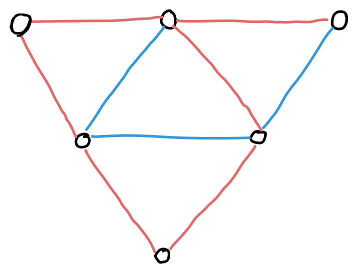
Denoted by

PCSP()

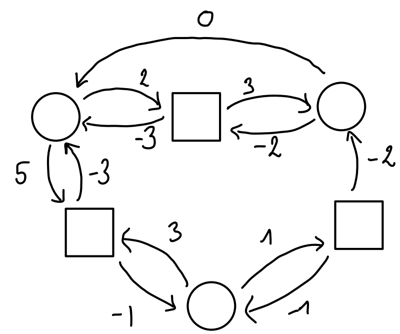
Promise

Extensions

High-dimensional constraints



Infinite Domains

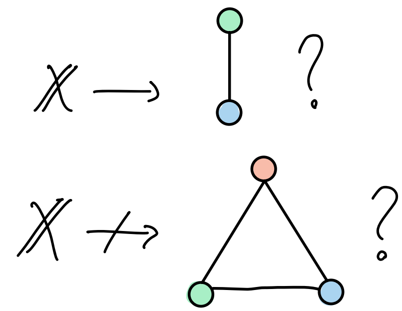


Very similar: in many cases, high-dimensional constraints are 1-dimensional constraints on an infinite set

$\updownarrow$

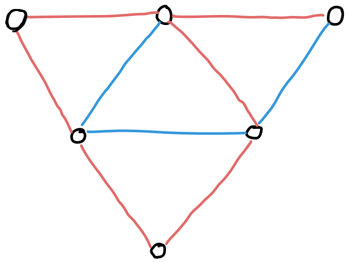
Fraïssé theory of Generic structures

Approximations (qualitative/quantitative)

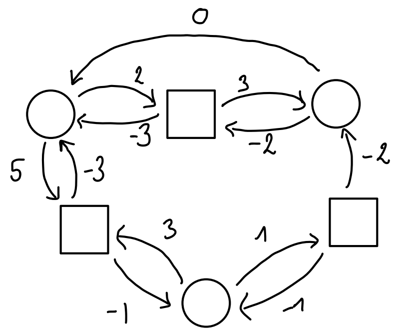


Extensions

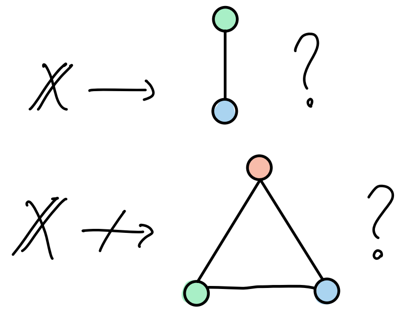
High-dimensional constraints



Infinite Domains



Approximations (qualitative/quantitative)



Very similar: in many cases, high-dimensional constraints are 1-dimensional constraints on an infinite set

↕

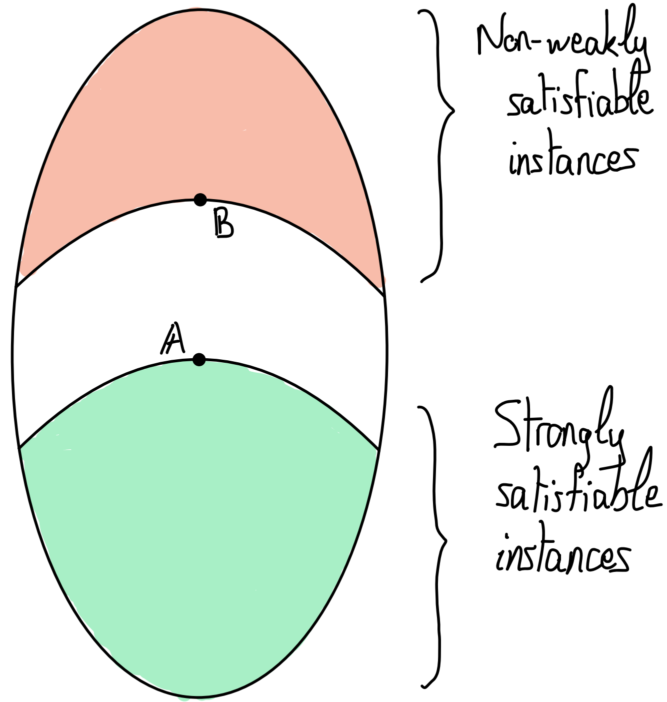
Fraïssé theory of Generic structures

⟷ Connections ?

Logical Solvability of PCSPs

PCSP($\mathbb{A},\mathbb{B}$) : Given $\mathbb{X}$, decide if

- Yes : $\mathbb{X}$ is strongly satisfiable ($\mathbb{X} \to \mathbb{A}$)
- No : $\mathbb{X}$ is not weakly satisfiable ($\mathbb{X} \not\to \mathbb{B}$)



Logical Solvability of PCSPs

PCSP($\mathbb{A}$,$\mathbb{B}$) : Given $\mathbb{X}$, decide if

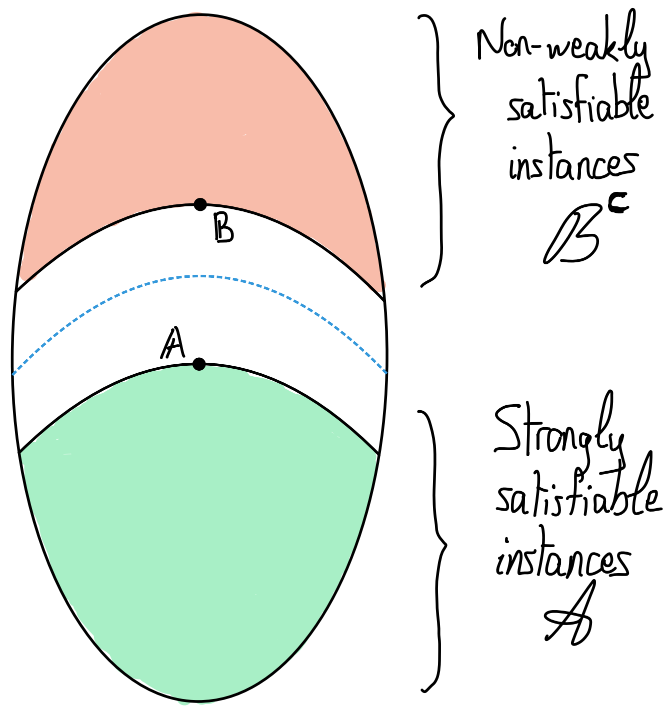
- Yes : $\mathbb{X}$ is strongly satisfiable ($\mathbb{X} \to \mathbb{A}$)
- No : $\mathbb{X}$ is not weakly satisfiable ($\mathbb{X} \not\to \mathbb{B}$)

An algorithm solving PCSP($\mathbb{A}$,$\mathbb{B}$) realizes a separation between

$$\mathcal{A} = \{ \mathbb{X} \mid \mathbb{X} \to \mathbb{A} \}$$

and

$$\mathcal{B}^c = \{ \mathbb{X} \mid \mathbb{X} \not\to \mathbb{B} \}$$



Logical Solvability of PCSPs

PCSP($\mathbb{A},\mathbb{B}$): Given $\mathbb{X}$, decide if

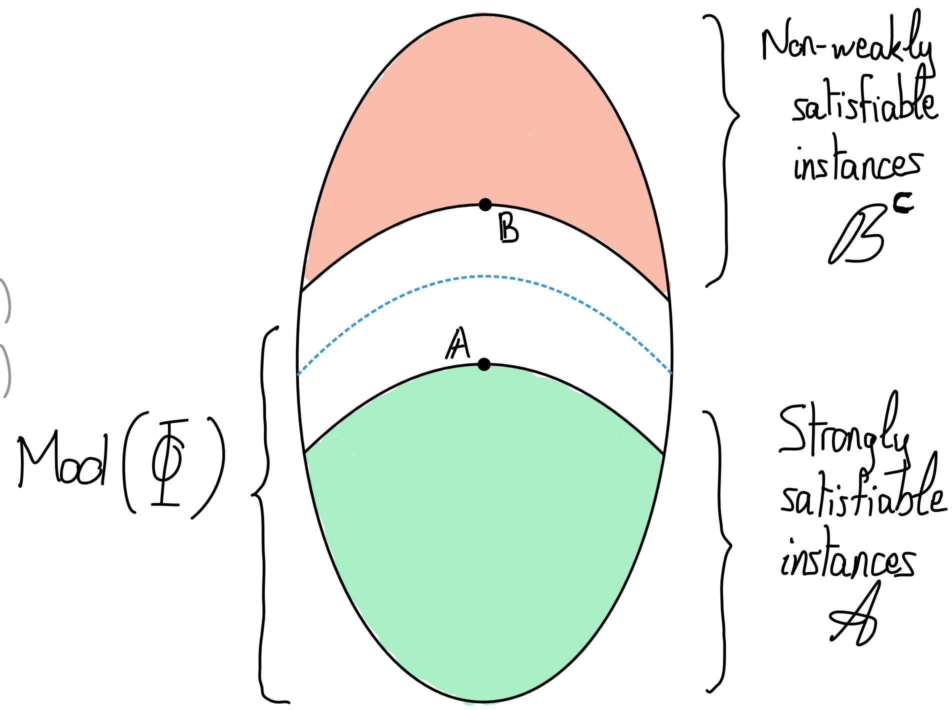
- Yes: $\mathbb{X}$ is strongly satisfiable ($\mathbb{X} \to \mathbb{A}$)
- No: $\mathbb{X}$ is not weakly satisfiable ($\mathbb{X} \not\to \mathbb{B}$)

An algorithm solving PCSP($\mathbb{A},\mathbb{B}$) realizes a separation between

$$\mathcal{A} = \{ \mathbb{X} \mid \mathbb{X} \to \mathbb{A} \}$$

and

$$\mathcal{B}^c = \{ \mathbb{X} \mid \mathbb{X} \not\to \mathbb{B} \}$$



Questions: For a logic $\mathcal{L}$ and fixed problem PCSP($\mathbb{A},\mathbb{B}$):

- is there a formula $\Phi \in \mathcal{L}$ s.t.

$$\mathcal{A} \subseteq \mathrm{Mod}(\Phi) \subseteq \mathcal{B} \;?$$

Logical Solvability of PCSPs

<u>$\mathrm{PCSP}(\mathbb{A},\mathbb{B})$</u> : Given $\mathbb{X}$, decide if

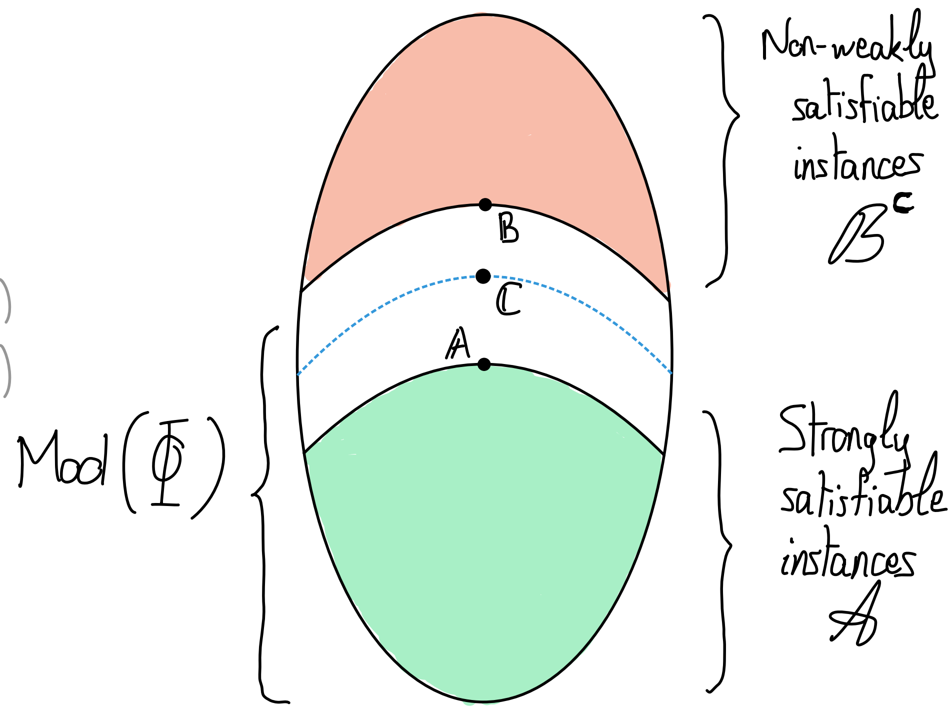
- Yes : $\mathbb{X}$ is strongly satisfiable $(\mathbb{X} \to \mathbb{A})$
- No : $\mathbb{X}$ is not weakly satisfiable $(\mathbb{X} \not\to \mathbb{B})$

An algorithm solving $\mathrm{PCSP}(\mathbb{A},\mathbb{B})$ realizes a separation between

$$\mathcal{A} = \{ \mathbb{X} \mid \mathbb{X} \to \mathbb{A} \}$$

and

$$\mathcal{B}^c = \{ \mathbb{X} \mid \mathbb{X} \not\to \mathbb{B} \}$$



<u>Questions</u> : For a logic $\mathcal{L}$ and fixed problem $\mathrm{PCSP}(\mathbb{A},\mathbb{B})$:

- is there a formula $\Phi \in \mathcal{L}$ s.t.

$$\mathcal{A} \subseteq \mathrm{Mod}(\Phi) \subseteq \mathcal{B} \ ?$$

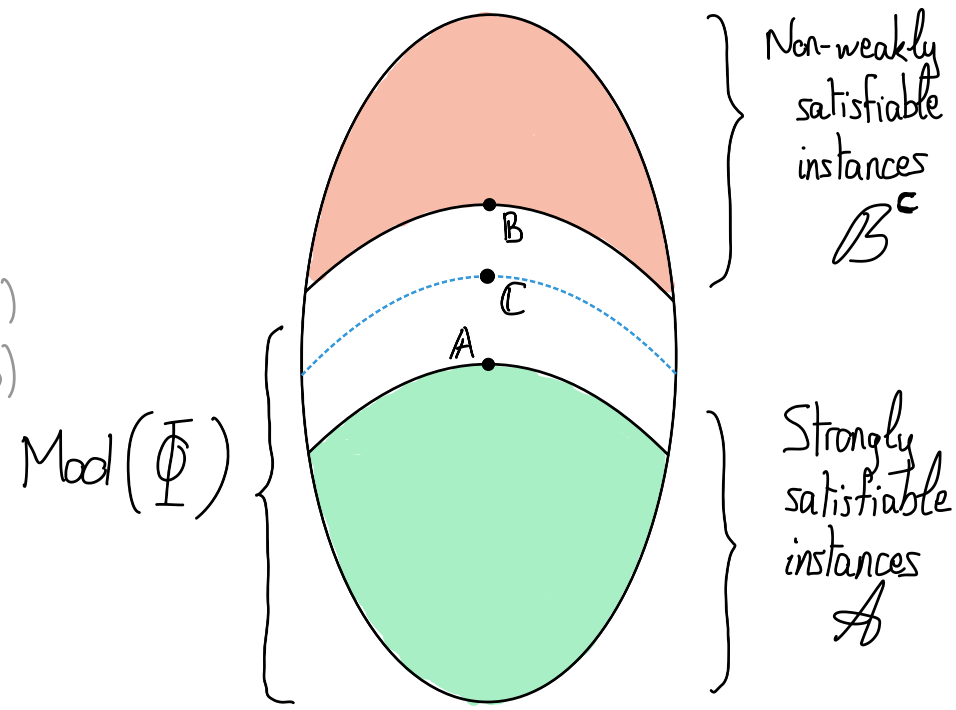
- is there a (finite) structure $\mathbb{C}$ s.t.

$$\mathcal{A} \subseteq \mathcal{C} \subseteq \mathcal{B} \text{ and } \mathrm{CSP}(\mathbb{C}) \text{ definable in } \mathcal{L} \ ?$$

Logical Solvability of PCSPs

PCSP(A, B): Given $\mathbb{X}$, decide if

- Yes: $\mathbb{X}$ is strongly satisfiable $(\mathbb{X} \to \mathbb{A})$
- No: $\mathbb{X}$ is not weakly satisfiable $(\mathbb{X} \not\to \mathbb{B})$



An algorithm solving PCSP(A, B) realizes a separation between

$$\mathcal{A} = \{ \mathbb{X} \mid \mathbb{X} \to \mathbb{A} \}$$

and

$$\mathcal{B}^c = \{ \mathbb{X} \mid \mathbb{X} \not\to \mathbb{B} \}$$

Questions: For a logic $\mathcal{L}$ and fixed problem PCSP(A, B):

- is there a formula $\Phi \in \mathcal{L}$ s.t.

$$\mathcal{A} \subseteq \mathrm{Mod}(\Phi) \subseteq \mathcal{B} \ ?$$

- is there a (finite) structure $\mathbb{C}$ s.t.

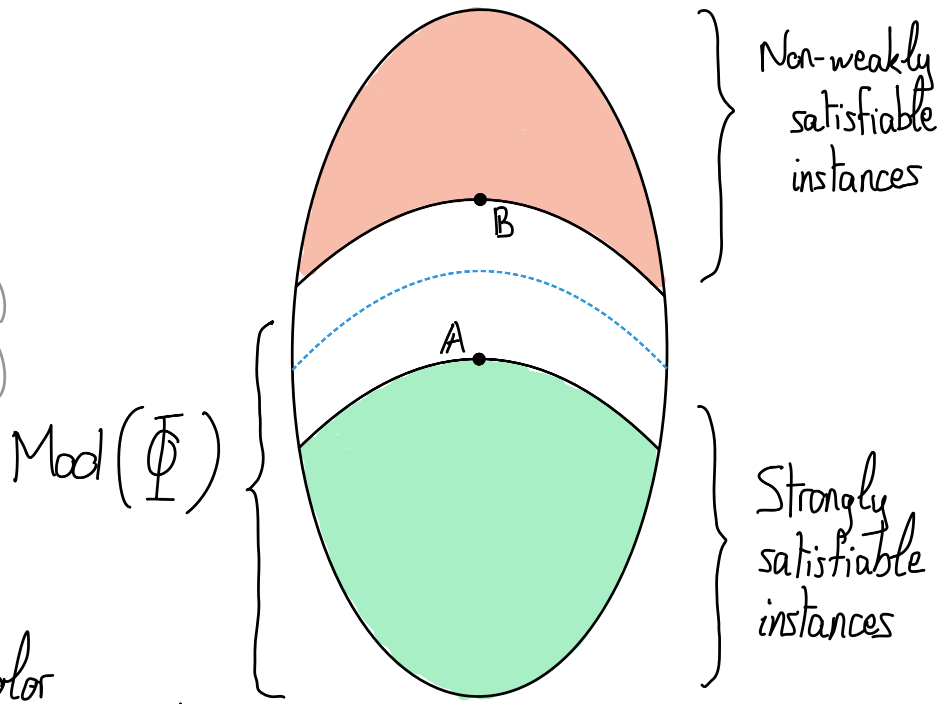
$$\mathcal{A} \subseteq \mathcal{C} \subseteq \mathcal{B} \text{ and } \mathrm{CSP}(\mathbb{C}) \text{ definable in } \mathcal{L} \ ?$$

$\mathrm{PCSP}(\mathbb{A}, \mathbb{B}) \in \mathsf{L} \Leftarrow$

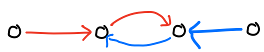
Logical Solvability of PCSPs

PCSP($\mathbb{A},\mathbb{B}$) : Given $\mathbb{X}$, decide if

- Yes : $\mathbb{X}$ is strongly satisfiable ($\mathbb{X} \to \mathbb{A}$)
- No : $\mathbb{X}$ is not weakly satisfiable ($\mathbb{X} \not\to \mathbb{B}$)



Example : $\mathbb{A}$ = 

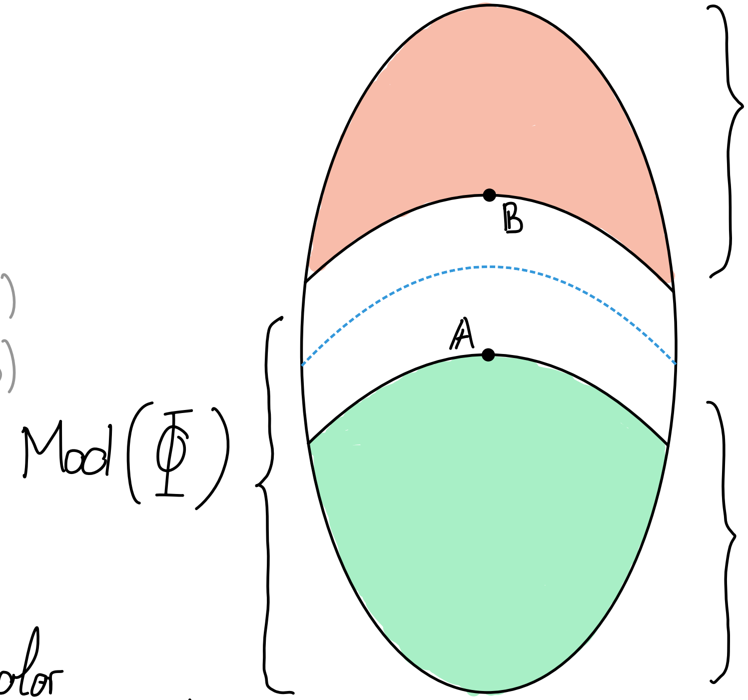
$\mathbb{B}$ = 

Φ :
- no two consecutive edges of the same color
- no edges of different colors arriving at a vertex
- no edges of different colors leaving a vertex

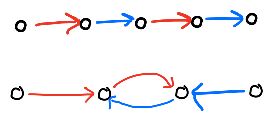
Logical Solvability of PCSPs

<ins>PCSP(A, B)</ins>: Given X, decide if

- **Yes**: X is strongly satisfiable $(X \to A)$
- **No**: X is not weakly satisfiable $(X \not\to B)$



<ins>Example</ins>: A = , B =



Φ: · no two consecutive edges of the same color
· no edges of different colors arriving at a vertex
· no edges of different colors leaving a vertex

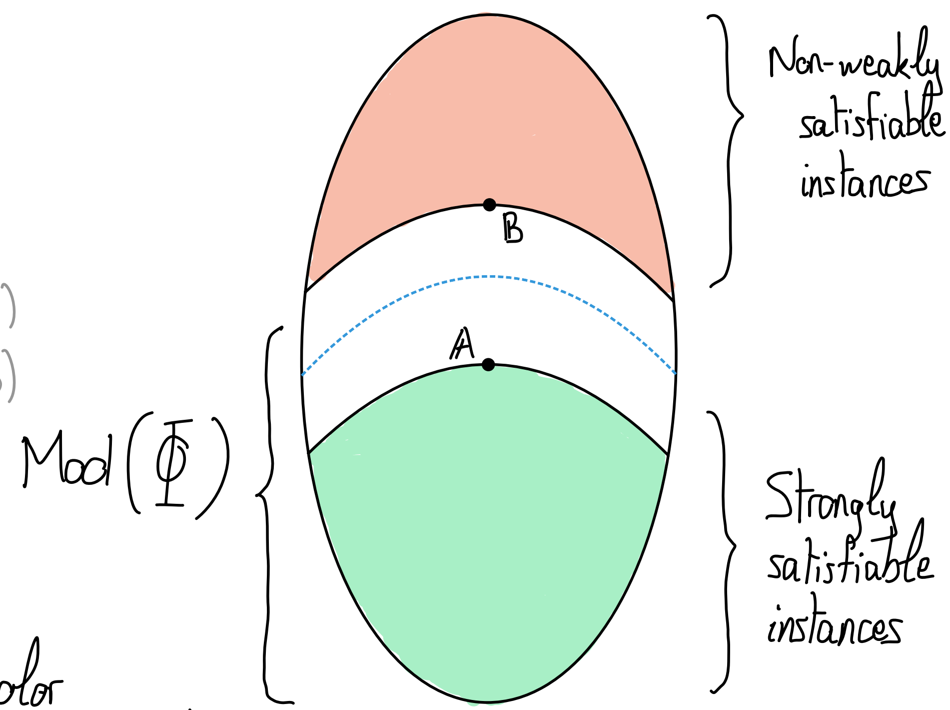
· If X strongly satisfiable $(X \to A)$ then $X \models \Phi$

· If $X \models \Phi$ then X weakly satisfiable $(X \to B)$

Logical Solvability of PCSPs

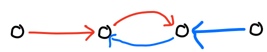
<u>PCSP(A, B)</u>: Given X, decide if

- Yes: X is strongly satisfiable $(X \to A)$
- No: X is not weakly satisfiable $(X \not\to B)$



<u>Example</u>: $\mathbb{A}$ =

$\mathbb{B}$ =



Φ: · no two consecutive edges of the same color
· no edges of different colors arriving at a vertex
· no edges of different colors leaving a vertex

· If X strongly satisfiable $(X \to \mathbb{A})$ then $X \models \Phi$

· If $X \models \Phi$ then X weakly satisfiable $(X \to \mathbb{B})$

(Exercise: no first-order formula characterizes strongly satisfiable instances
________ " ________ weakly ________ . ________)

The case of first-order logic

<u>The result</u> For every $\mathrm{PCSP}(\mathbb{A}, \mathbb{B})$, the following are equivalent:

- $\mathrm{PCSP}(\mathbb{A}, \mathbb{B})$ is first-order solvable,
- there exists a structure $\mathbb{C}$ s.t. $\mathbb{A} \to \mathbb{C} \to \mathbb{B}$ and $\mathrm{CSP}(\mathbb{C})$ is first-order definable,
- there exists a FINITE structure $\mathbb{C}$ s.t. $\mathbb{A} \to \mathbb{C} \to \mathbb{B}$ and $\mathrm{CSP}(\mathbb{C})$ is first-order definable.

The case of first-order logic

<u>The result</u> For every $\mathrm{PCSP}(\mathbb{A},\mathbb{B})$, the following are equivalent:

- $\mathrm{PCSP}(\mathbb{A},\mathbb{B})$ is first-order solvable,
- there exists a structure $\mathbb{C}$ s.t. $\mathbb{A} \to \mathbb{C} \to \mathbb{B}$ and $\mathrm{CSP}(\mathbb{C})$ is first-order definable,
- there exists a FINITE structure $\mathbb{C}$ s.t. $\mathbb{A} \to \mathbb{C} \to \mathbb{B}$ and $\mathrm{CSP}(\mathbb{C})$ is first-order definable.

<u>The proof</u>

$\mathrm{PCSP}(\mathbb{A},\mathbb{B})$ first-order solvable $\overset{\text{[Rossmann]}}{\Longrightarrow}$ $\mathrm{PCSP}(\mathbb{A},\mathbb{B})$ has a finite duality

The case of first-order logic

The result For every $PCSP(\mathbb{A},\mathbb{B})$, the following are equivalent:

- $PCSP(\mathbb{A},\mathbb{B})$ is first-order solvable,
- there exists a structure $\mathbb{C}$ s.t. $\mathbb{A} \to \mathbb{C} \to \mathbb{B}$ and $CSP(\mathbb{C})$ is first-order definable,
- there exists a FINITE structure $\mathbb{C}$ s.t. $\mathbb{A} \to \mathbb{C} \to \mathbb{B}$ and $CSP(\mathbb{C})$ is first-order definable.

The proof

$PCSP(\mathbb{A},\mathbb{B})$ first-order solvable $\overset{\text{[Rossmann]}}{\Longrightarrow}$ $PCSP(\mathbb{A},\mathbb{B})$ has a finite duality $\xRightarrow[\text{[Hubička - Nešetřil]}]{\text{[Cherlin-Shelah-Shi]}}$ there exists a structure $\mathbb{C}$ "ω-categorical" "Ramsey" s.t.

$CSP(\mathbb{C})$ in FO and $\mathbb{A} \to \mathbb{C} \to \mathbb{B}$

The case of first-order logic

The result For every $\mathrm{PCSP}(\mathbb{A},\mathbb{B})$, the following are equivalent:

- $\mathrm{PCSP}(\mathbb{A},\mathbb{B})$ is first-order solvable,
- there exists a structure $\mathbb{C}$ s.t. $\mathbb{A} \to \mathbb{C} \to \mathbb{B}$ and $\mathrm{CSP}(\mathbb{C})$ is first-order definable,
- there exists a FINITE structure $\mathbb{C}$ s.t. $\mathbb{A} \to \mathbb{C} \to \mathbb{B}$ and $\mathrm{CSP}(\mathbb{C})$ is first-order definable.

The proof

$\mathrm{PCSP}(\mathbb{A},\mathbb{B})$ first-order solvable $\overset{\text{[Rossmann]}}{\Longrightarrow}$ $\mathrm{PCSP}(\mathbb{A},\mathbb{B})$ has a finite duality $\overset{\text{[Cherlin-Shelah-Shi] [Hubička-Nešetřil]}}{\Longrightarrow}$ there exists a structure $\mathbb{C}$ "ω-categorical" "Ramsey" s.t.

$\mathrm{CSP}(\mathbb{C})$ in FO and $\mathbb{A} \to \mathbb{C} \to \mathbb{B}$

$\Downarrow$

witnessed by "tolerant polymorphisms"

$\mathbb{C}/\sim$

The case of first-order logic

The result For every $\mathrm{PCSP}(\mathbb{A},\mathbb{B})$, the following are equivalent:

- $\mathrm{PCSP}(\mathbb{A},\mathbb{B})$ is first-order solvable,
- there exists a structure $\mathbb{C}$ s.t. $\mathbb{A}\to\mathbb{C}\to\mathbb{B}$ and $\mathrm{CSP}(\mathbb{C})$ is first-order definable,
- there exists a FINITE structure $\mathbb{C}$ s.t. $\mathbb{A}\to\mathbb{C}\to\mathbb{B}$ and $\mathrm{CSP}(\mathbb{C})$ is first-order definable.

The proof

$\mathrm{PCSP}(\mathbb{A},\mathbb{B})$ first-order solvable $\overset{\text{[Rossmann]}}{\Longrightarrow}$ $\mathrm{PCSP}(\mathbb{A},\mathbb{B})$ has a finite duality $\overset{\text{[Cherlin-Shelah-Shi] [Hubička-Nešetřil]}}{\Longrightarrow}$ there exists a structure $\mathbb{C}$ "ω-categorical" "Ramsey" s.t.

$\mathrm{CSP}(\mathbb{C})$ in FO and $\mathbb{A}\to\mathbb{C}\to\mathbb{B}$

$$\begin{tikzcd} \mathbb{A} \arrow[r] & \mathbb{C} \arrow[r] \arrow[d] & \mathbb{B} \\ & \mathbb{C}/_\sim \arrow[ur] & \end{tikzcd}$$

$\Downarrow$

witnessed by "tolerant polymorphisms" $\Longrightarrow$ witnessed by "canonical" "tolerant polymorphisms"

The case of first-order logic

The result For every PCSP($\mathbb{A},\mathbb{B}$), the following are equivalent:

- PCSP($\mathbb{A},\mathbb{B}$) is first-order solvable,
- there exists a structure $\mathbb{C}$ s.t. $\mathbb{A} \to \mathbb{C} \to \mathbb{B}$ and CSP($\mathbb{C}$) is first-order definable,
- there exists a FINITE structure $\mathbb{C}$ s.t. $\mathbb{A} \to \mathbb{C} \to \mathbb{B}$ and CSP($\mathbb{C}$) is first-order definable.

The proof

PCSP($\mathbb{A},\mathbb{B}$) first-order solvable $\xRightarrow{\text{[Rossmann]}}$ PCSP($\mathbb{A},\mathbb{B}$) has a finite duality $\xRightarrow{\text{[Cherlin-Shelah-Shi] [Hubička-Nešetřil]}}$ there exists a structure $\mathbb{C}$ "ω-categorical" "Ramsey" s.t.

CSP($\mathbb{C}$) in FO and $\mathbb{A} \to \mathbb{C} \to \mathbb{B}$, $\mathbb{C} \to \mathbb{C}/_\sim \to \mathbb{B}$

$\Downarrow$

witnessed by "tolerant polymorphisms" $\Rightarrow$ witnessed by "canonical" "tolerant polymorphisms" $\xRightarrow{\text{[Larose-Loten-Tardif]}}$ CSP($\mathbb{C}/_\sim$) is first-order definable. $\square$

Other results/observations

- The Hubička-Nešetřil construction is a Fraïssé limit

Other results/observations

- The Hubička-Nešetřil construction is a Fraïssé limit
- Fraïssé limits can be obtained for algorithms other than those definable in first-order logic:

arc consistency (+ arc consistency reductions)

basic linear programming relaxation

affine integer programming relaxation

Other results/observations

- The Hubička-Nešetřil construction is a Fraïssé limit
- Fraïssé limits can be obtained for algorithms other than those definable in first-order logic:

arc consistency (+ arc consistency reductions)

basic linear programming relaxation

affine integer programming relaxation

<u>Meta result</u> The following are equivalent:

- PCSP($\mathbb{A}$, $\mathbb{B}$) is solvable by ...
- there exists a structure $\mathbb{C}$ s.t. $\mathbb{A} \to \mathbb{C} \to \mathbb{B}$ and other "good" properties

$$\text{Algorithmic results} \rightleftarrows \text{Structural properties of generic structures}$$

Other results/observations

- The Hubička-Nešetřil construction is a Fraïssé limit
- Fraïssé limits can be obtained for algorithms other than those definable in first-order logic:

arc consistency (+ arc consistency reductions) | basic linear programming relaxation | affine integer programming relaxation

<u>Meta result</u> The following are equivalent:

- PCSP($\mathbb{A}$, $\mathbb{B}$) is solvable by ...
- there exists a structure $\mathbb{C}$ s.t. $\mathbb{A} \to \mathbb{C} \to \mathbb{B}$ and other "good" properties

Algorithmic results $\rightleftarrows$ Structural properties of generic structures

<u>Open</u>:

- Consistency, Sherali-Adams, and Lasserre hierarchies
- Fixpoint logics
- Decidability of "solvable by ⟨insert algorithm here⟩?"

Take away

- Connections between finite promise CSPs and infinite classical CSPs

Take away

- Connections between finite promise CSPs and infinite classical CSPs
- Meta result: partial recipe for characterizing the power of algorithmic techniques to solve PCSPs.

Take away

- Connections between finite promise CSPs and infinite classical CSPs
- Meta result: partial recipe for characterizing the power of algorithmic techniques to solve PCSPs.
- Extending this partial recipe could solve important problems in promise constraint satisfaction

Take away

- Connections between finite promise CSPs and infinite classical CSPs
- Meta result: partial recipe for characterizing the power of algorithmic techniques to solve PCSPs.
- Extending this partial recipe could solve important problems in promise constraint satisfaction

Thank You!

(Did I mention I'm hiring?)