Promises and Infinite-Domain Constraint Satisfaction

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CSL 2024

Currently hiring a PhD student!

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Constraint Satisfaction Problems

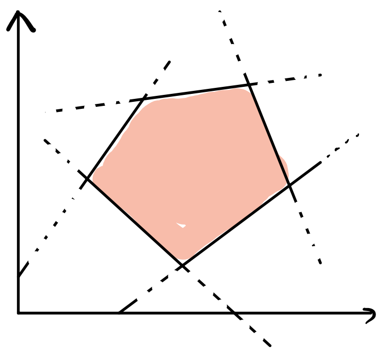
variables $\rightsquigarrow$ domain satisfying some constraints

Constraint Satisfaction Problems

Solve over $\mathbb{Z}$:

$$\begin{cases} 5x + y - z = 1 \\ x - y + z = 2 \end{cases}$$

Solve $Ax \geqslant b$ over $\mathbb{R}_{\geqslant 0}$:



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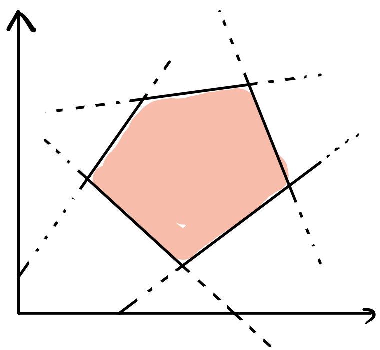

Solve over $\{true, false\}$:

$$(p \vee q \vee \bar{r}) \wedge (r \vee \bar{p}) \wedge (r \vee \bar{q})$$

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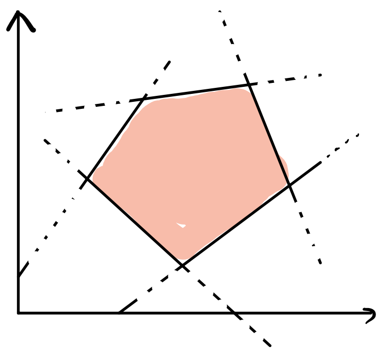

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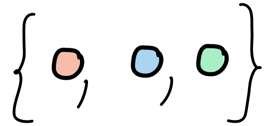
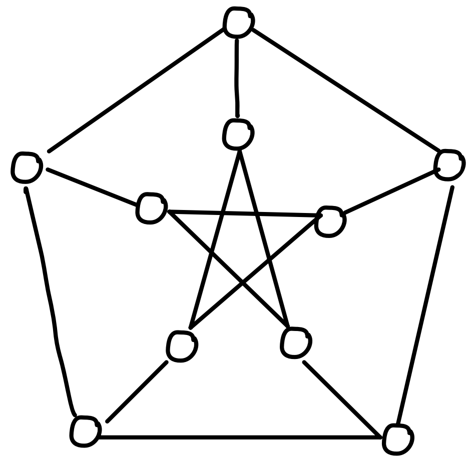
Solve over $\{1, \dots, 9\}$:

3			8		1			2
2		1		3		6		4
			2		4			
8		9				1		6
	6						5	
7		2				4		9
9			5		9			
		4		8		7		5
6			1		7			3

"Solve" over  :



Constraint Satisfaction Problems

<u>Input:</u> D ... domain V ... variables

C ... constraints: $(v_1, \ldots, v_r) \to \{$list of allowed assignments$\}$

<u>Question:</u> $\exists\, h : V \to D$ satisfying all constraints?

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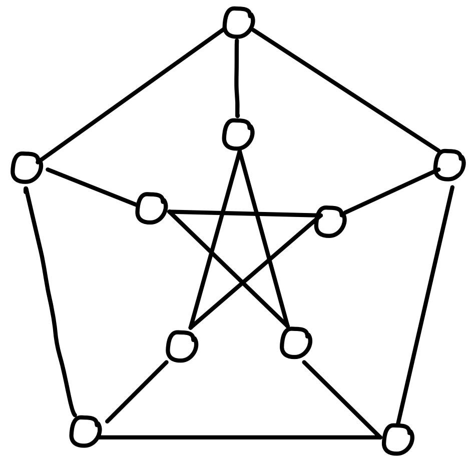
"Solve" over $\{$, , $\}$:



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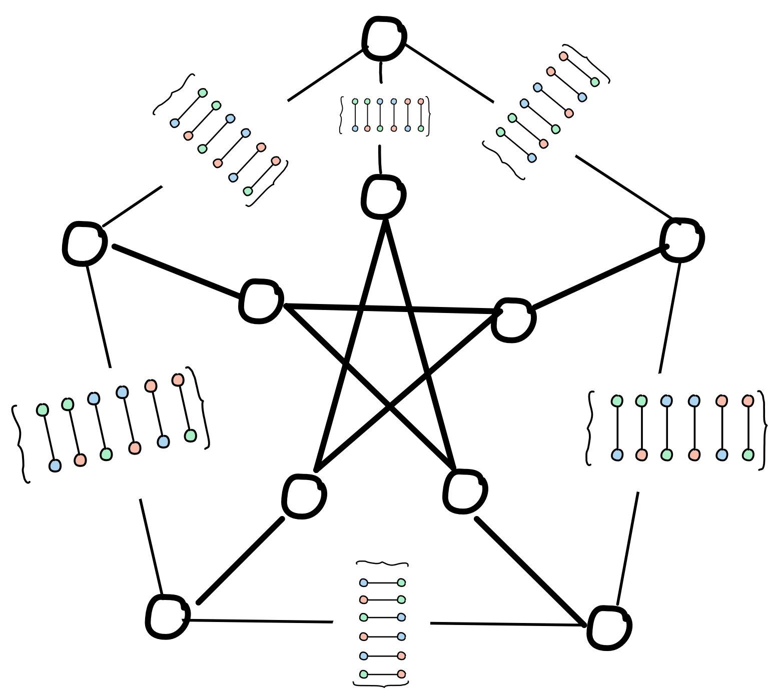
$D = \{\circ, \circ, \circ\}$



Constraint Satisfaction Problems

<u>Input of CSP($\mathbb{D}$):</u> V ... variables

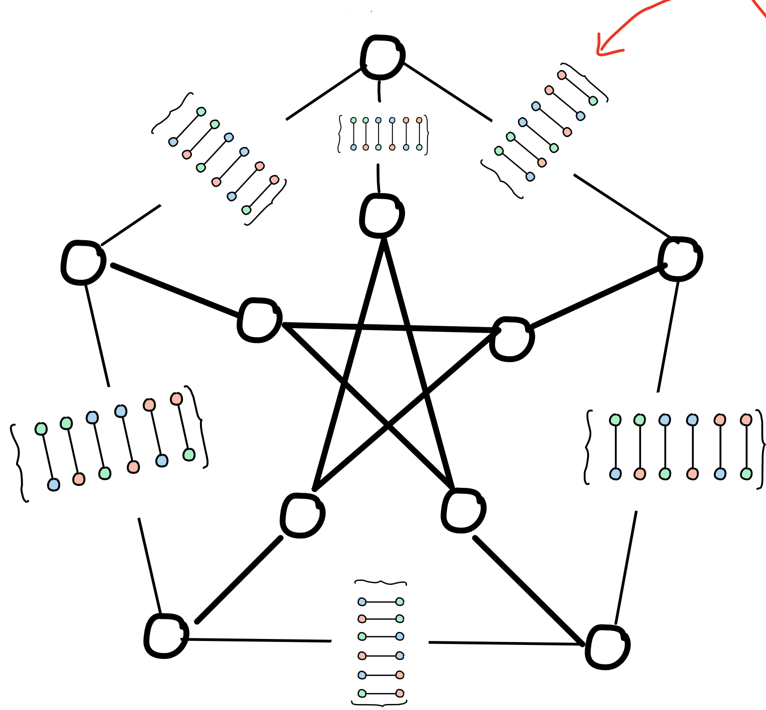
C ... constraints: $(v_1, \dots, v_r) \to E$

<u>Question:</u> $\exists\, h: V \to D$ satisfying all constraints?

$D = \{$  $\}$

all instances of 3-coloring use the same constraint relation $E \subseteq D^2$

$\mathbb{D} = (D, E)$



Constraint Satisfaction Problems

Input of CSP($\mathbb{D}$): V … variables
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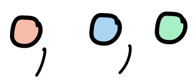
$D = \{$  $\}$

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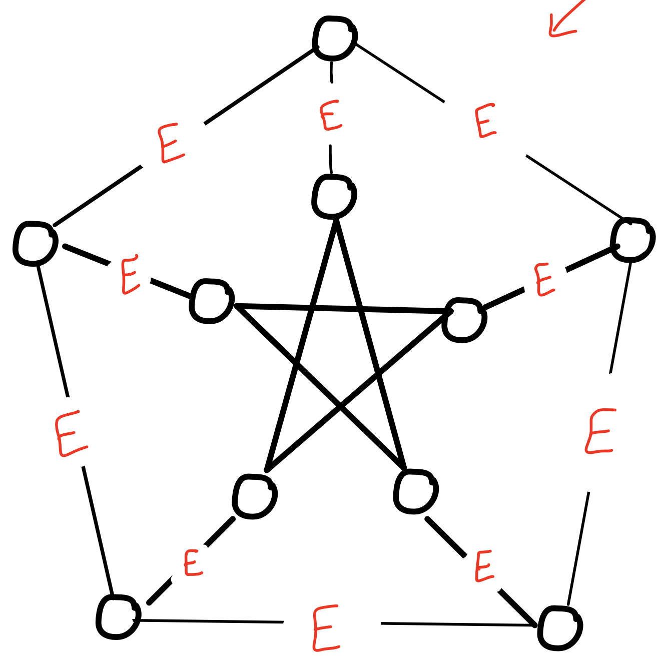

Constraint Satisfaction Problems

<u>Input of CSP($\mathbb{D}$):</u> $\left.\begin{array}{l} V \dots \text{variables} \\ C \dots \text{constraints: } (v_1, \dots, v_r) \to E \end{array}\right\}$ itself a relational structure $\mathbb{X}$

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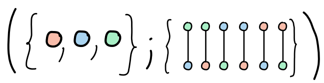
3-COL



1 in 3-SAT

$\left(\{\text{True}, \text{False}\}; \left\{ \begin{pmatrix}\text{True}\\\text{False}\\\text{False}\end{pmatrix}, \begin{pmatrix}\text{False}\\\text{True}\\\text{False}\end{pmatrix}, \begin{pmatrix}\text{False}\\\text{False}\\\text{True}\end{pmatrix} \right\}\right)$

Lin-Eq($\mathbb{F}_2$)

$\left(\{0,1\}; \left\{ \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix} \right\}, \{0\}\right)$

NOT-ALL-EQUAL-SAT

$\left(\{\text{True}, \text{False}\}; \left\{ \begin{pmatrix}\text{True}\\\text{False}\\\text{False}\end{pmatrix}, \begin{pmatrix}\text{False}\\\text{True}\\\text{False}\end{pmatrix}, \begin{pmatrix}\text{False}\\\text{False}\\\text{True}\end{pmatrix}, \begin{pmatrix}\text{True}\\\text{True}\\\text{False}\end{pmatrix}, \begin{pmatrix}\text{True}\\\text{False}\\\text{True}\end{pmatrix}, \begin{pmatrix}\text{False}\\\text{True}\\\text{True}\end{pmatrix} \right\}\right)$

Horn-SAT

$\left(\{\text{True}, \text{False}\}; \text{ all except } \begin{pmatrix}\text{True}\\\text{True}\\\text{False}\end{pmatrix}\right)$

<u>Theorem:</u> (Bulatov - Zhuk '17)

For every finite $\mathbb{D}$, CSP($\mathbb{D}$) is in P or NP-complete.

Extensions

High-dimensional constraints

Infinite Domains

Approximations (qualitative/quantitative)

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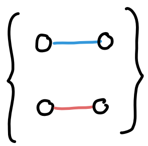
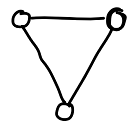
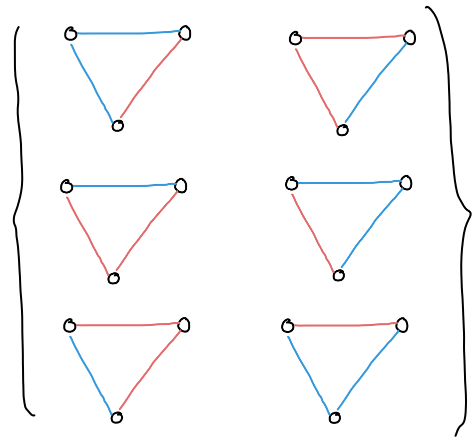
Find assignment **edges** →  s.t.  → 

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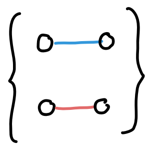
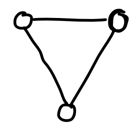
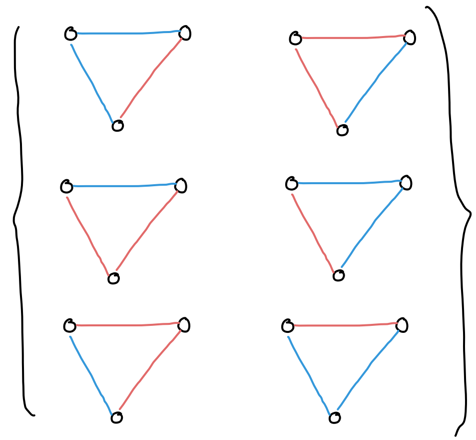
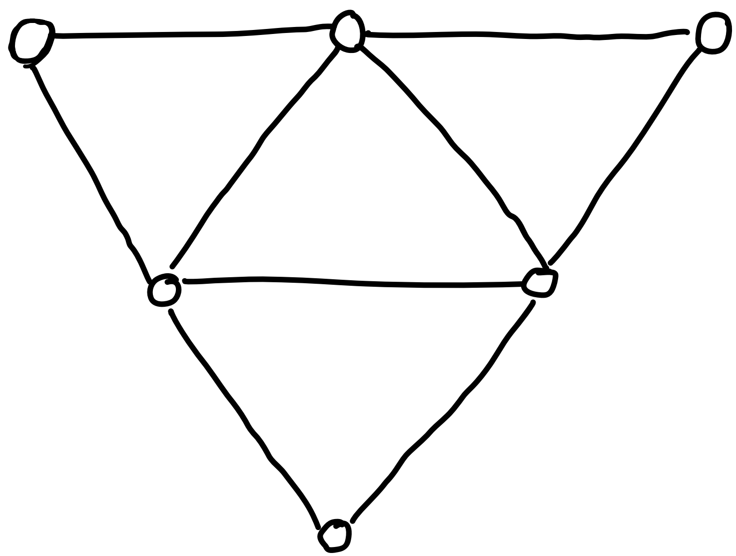
Find assignment **edges** →  s.t.  → 



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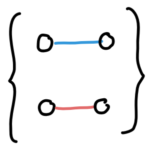
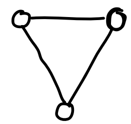
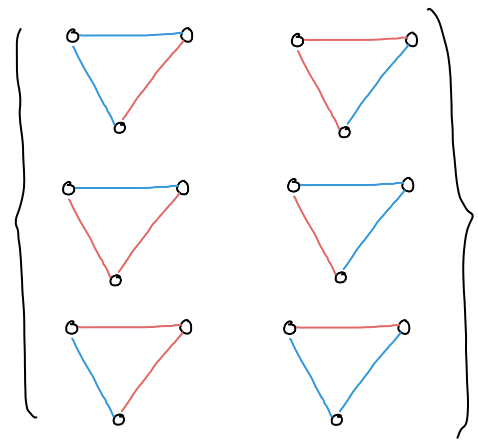
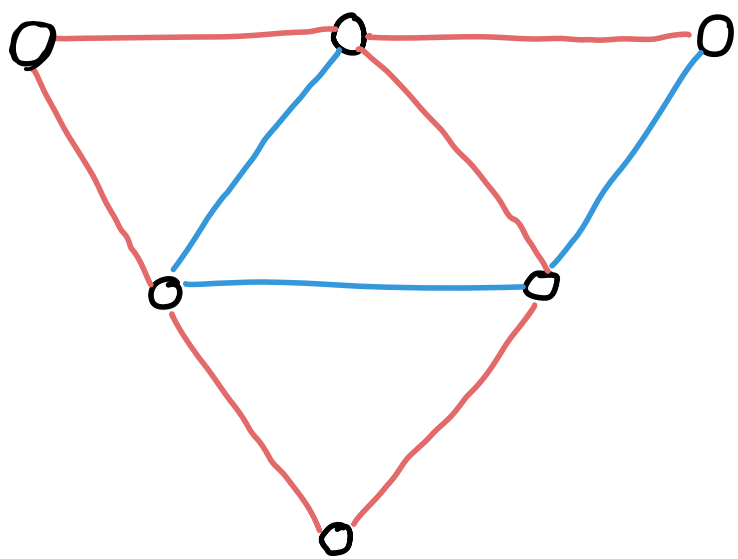
Find assignment **edges** →  s.t.  → 



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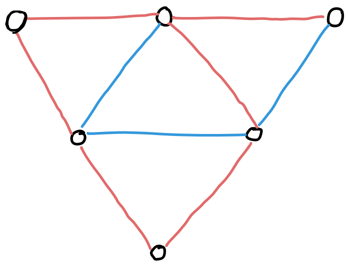


Infinite Domains

Approximations (qualitative/quantitative)

Domains and constraints are not part of the input

$\rightsquigarrow$ they can be infinite!

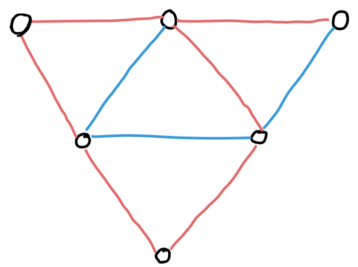
Example:

$\exists h : V \to \mathbb{Q}$ satisfying some constraints

$$(u, v, w) \to \{(a, b, c) \mid a < \max(b, c)\} \; ?$$

Extensions

High-dimensional constraints



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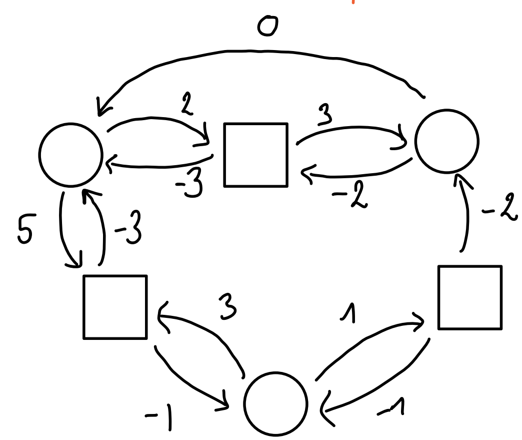
closely related to mean-payoff games, tropical linear programming



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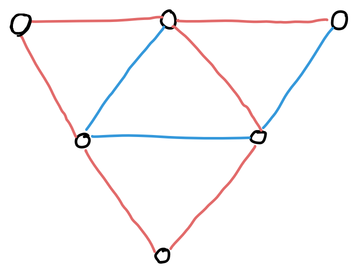

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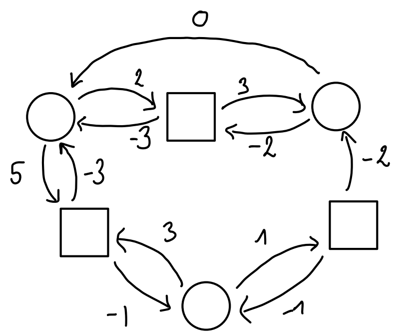

Approximations (qualitative/quantitative)

→ satisfy as many constraints as possible

→ satisfy all constraints in a weakened form

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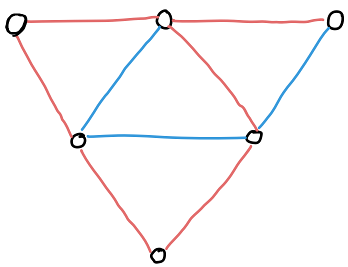

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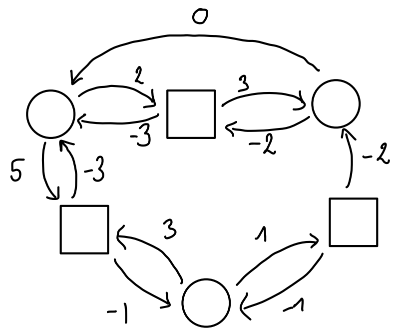
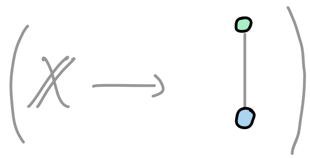


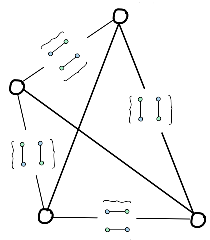
Approximations (qualitative/quantitative)

satisfy as many constraints as possible

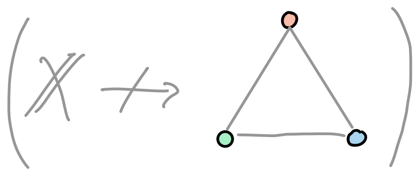
satisfy all constraints in a weakened form

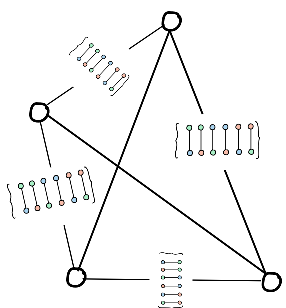
<u>Yes-instances:</u> strongly satisfiable





<u>No-instances:</u> not weakly satisfiable





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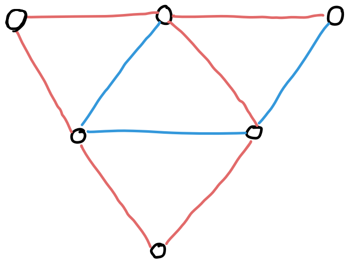

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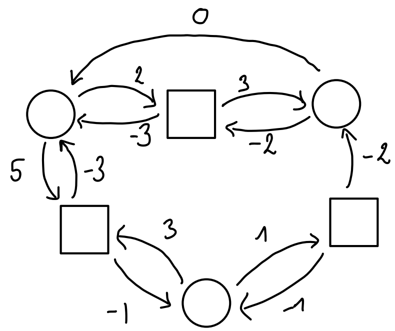

Approximations (qualitative/quantitative)

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satisfy all constraints in a *weakened* form

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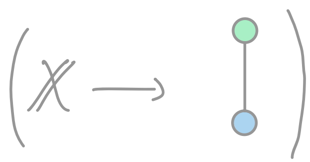
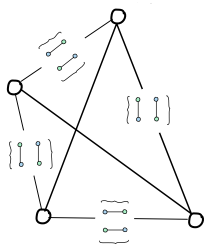
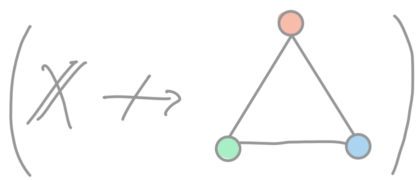
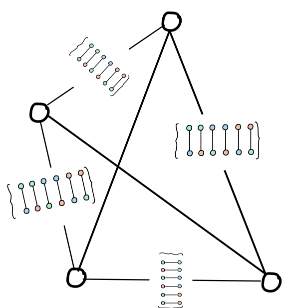


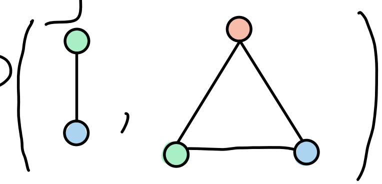


No-instances: not weakly satisfiable





Denoted by

PCSP 

Promise

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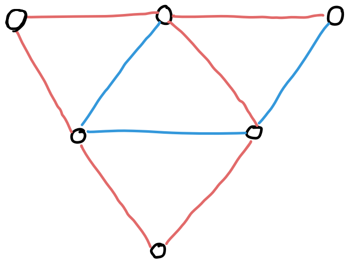

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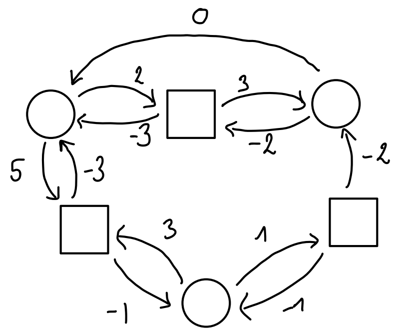
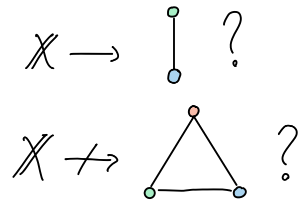


Very similar: in many cases, high-dimensional constraints are 1-dimensional constraints on an infinite set

$\updownarrow$

Fraïssé Theory of Generic structures

Approximations (qualitative/quantitative)



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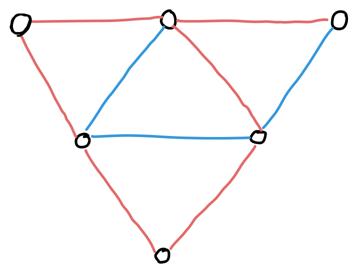

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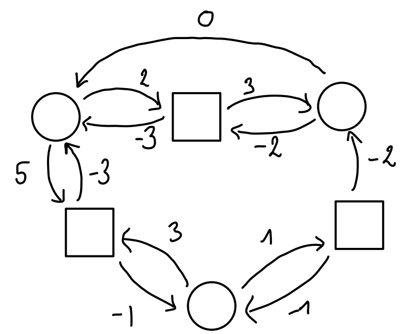

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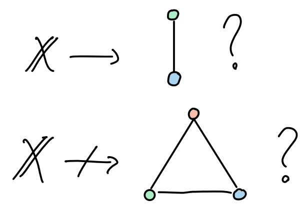
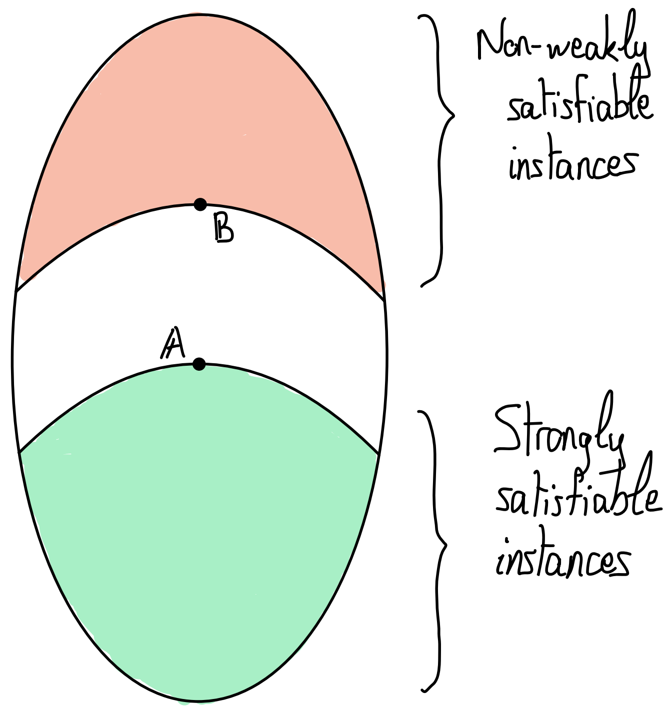


$\longleftrightarrow$ Connections?

Logical Solvability of PCSPs

PCSP($\mathbb{A}$, $\mathbb{B}$): Given $\mathbb{X}$, decide if

- Yes: $\mathbb{X}$ is strongly satisfiable ($\mathbb{X} \to \mathbb{A}$)
- No: $\mathbb{X}$ is not weakly satisfiable ($\mathbb{X} \not\to \mathbb{B}$)



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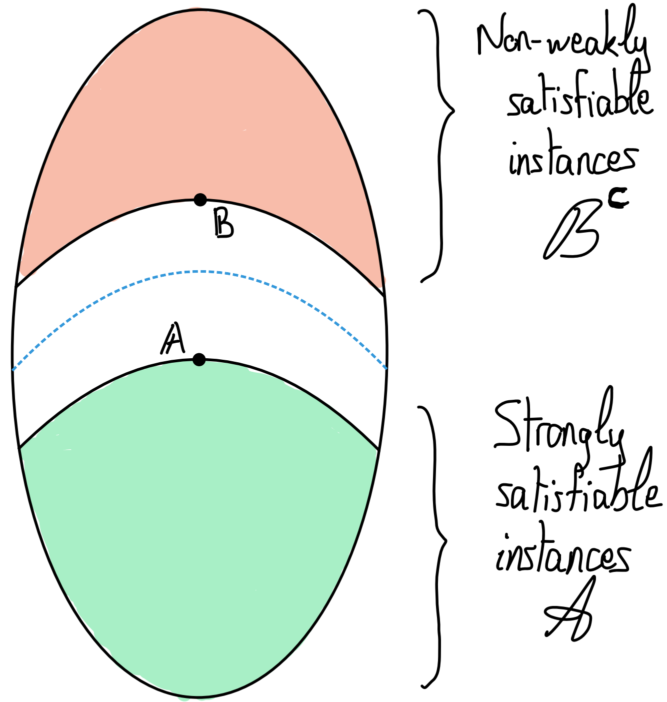
An algorithm solving $\mathrm{PCSP}(\mathbb{A},\mathbb{B})$ realizes a separation between

$$\mathcal{A} = \{ \mathbb{X} \mid \mathbb{X} \to \mathbb{A} \}$$

and

$$\mathcal{B}^c = \{ \mathbb{X} \mid \mathbb{X} \not\to \mathbb{B} \}$$



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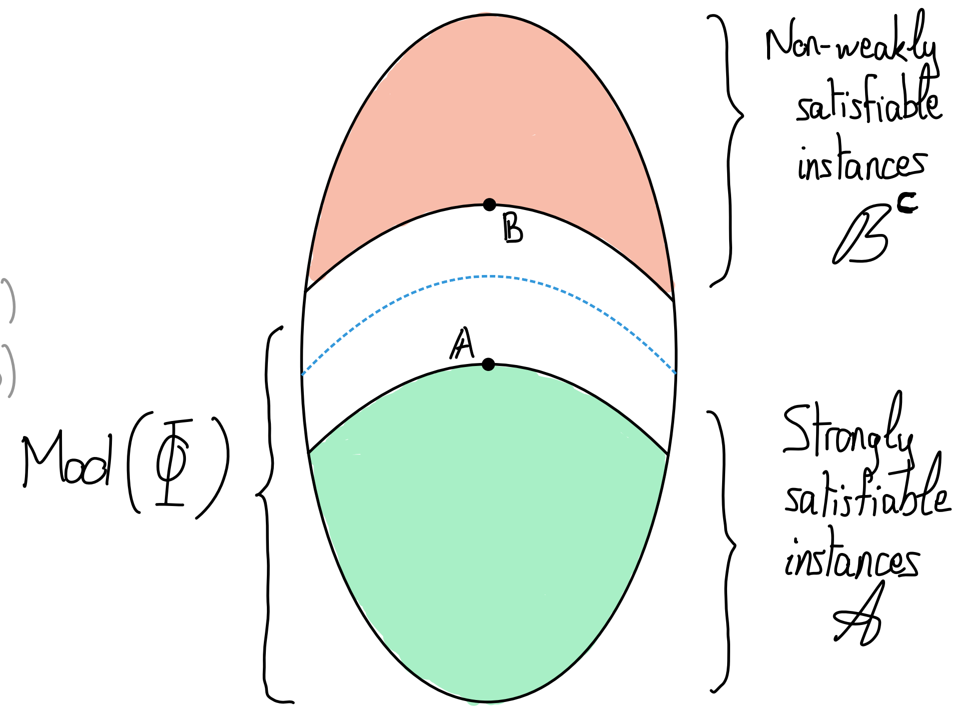
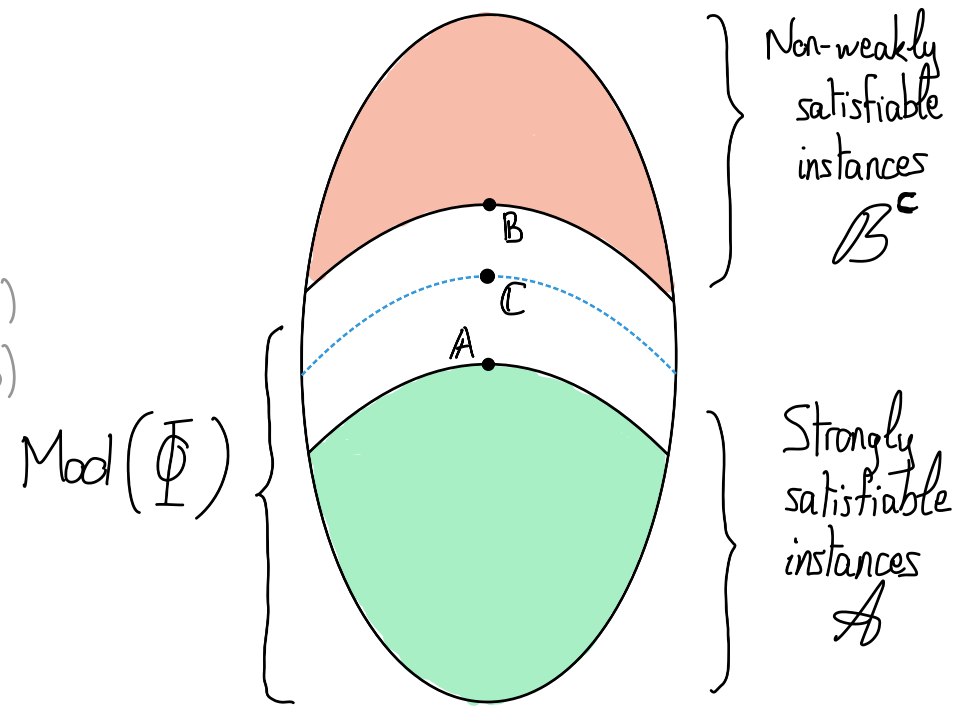


<u>Questions</u>: For a logic $\mathcal{L}$ and fixed problem PCSP(A, B):

- is there a formula $\Phi \in \mathcal{L}$ s.t.

$$\mathcal{A} \subseteq \mathrm{Mod}(\Phi) \subseteq \mathcal{B} \ ?$$

Logical Solvability of PCSPs



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and

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Questions: For a logic $\mathcal{L}$ and fixed problem PCSP($\mathbb{A}$, $\mathbb{B}$):

- is there a formula $\Phi \in \mathcal{L}$ s.t.

$$\mathcal{A} \subseteq \mathrm{Mod}(\Phi) \subseteq \mathcal{B} \ ?$$

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$$\mathcal{A} \subseteq \mathcal{C} \subseteq \mathcal{B} \text{ and } \mathrm{CSP}(\mathbb{C}) \text{ definable in } \mathcal{L}\,?$$

Logical Solvability of PCSPs

<u>$\mathrm{PCSP}(\mathbb{A},\mathbb{B})$:</u> Given $\mathbb{X}$, decide if

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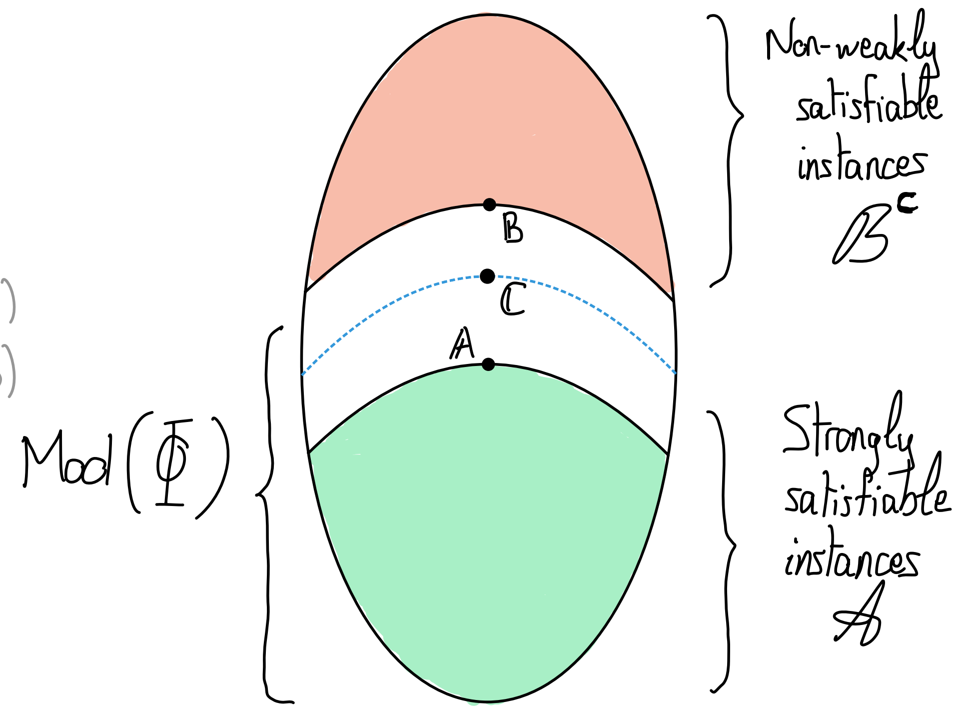

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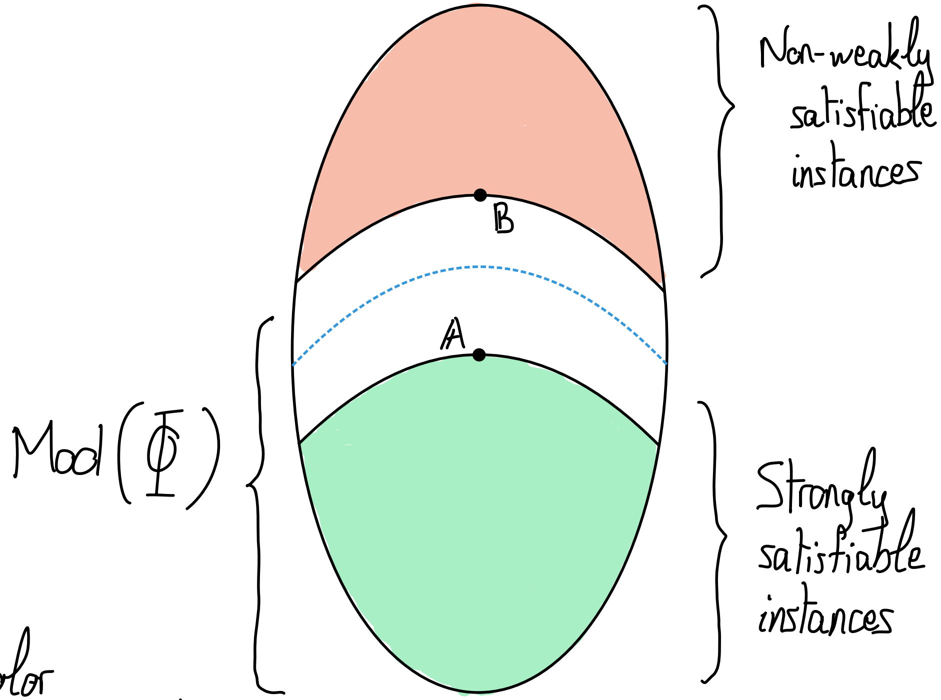
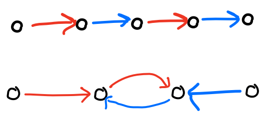


Example:

$\mathbb{A}$ =
$\mathbb{B}$ =



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- no two consecutive edges of the same color
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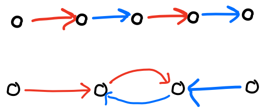

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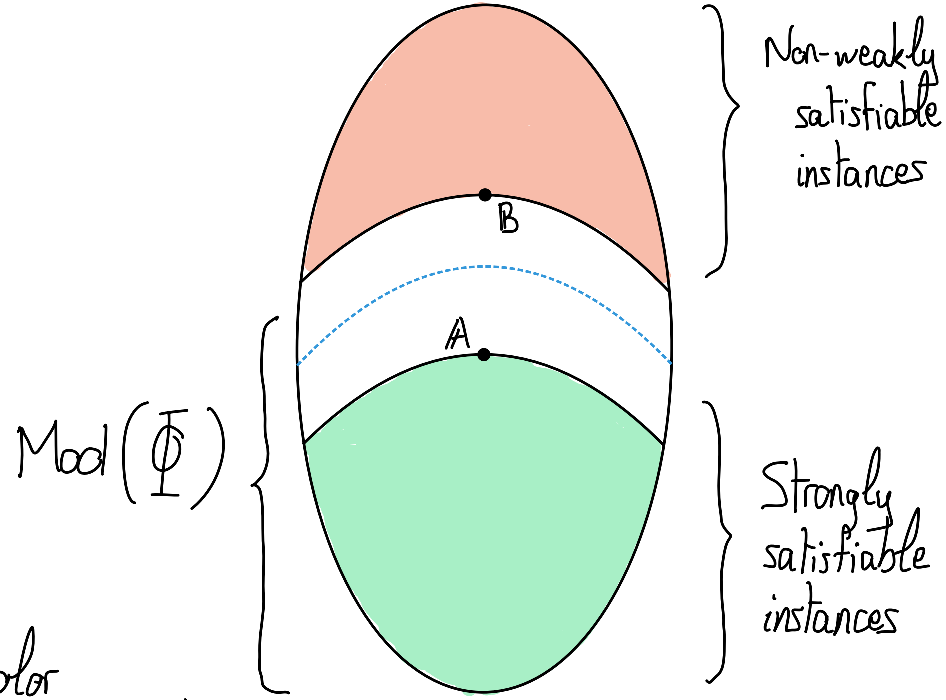

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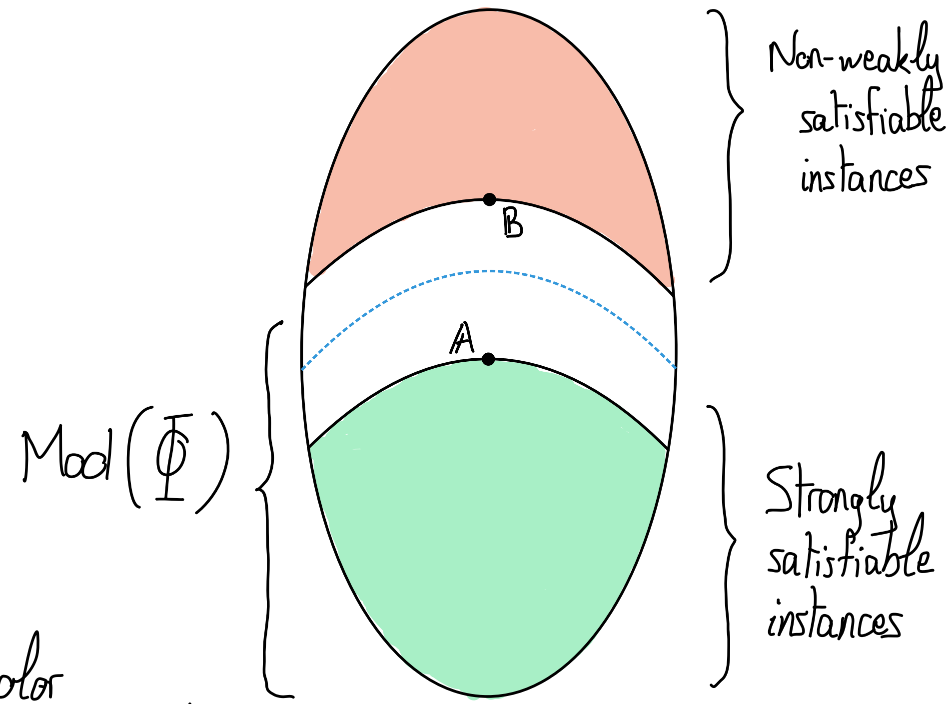

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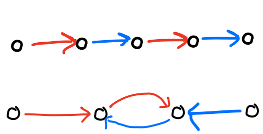

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(Exercise: no first-order formula characterizes strongly satisfiable instances
______ , ______ weakly ______ . ______)

The case of first-order logic

The result For every $\mathrm{PCSP}(\mathbb{A},\mathbb{B})$, the following are equivalent:

- $\mathrm{PCSP}(\mathbb{A},\mathbb{B})$ is first-order solvable,
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$\Downarrow$

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<u>Open</u>:

- Consistency, Sherali-Adams, and Lasserre hierarchies
- Fixpoint logics
- Decidability of "solvable by <insert algorithm here>?"

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Thank You!

(I'm hiring!)