MMSNP: Towards a Proof of the Universal-Algebraic Dichotomy Conjecture

Manuel Bodirsky, **Antoine Mottet**

September 25, 2018

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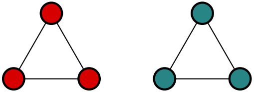
- Question: can one colour the vertices of G in a way to avoid the following patterns:



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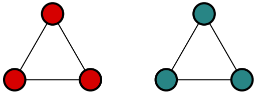

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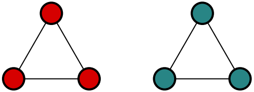

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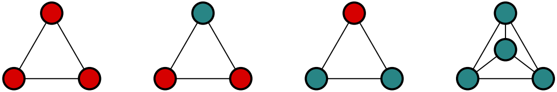

- Complexity: in P.

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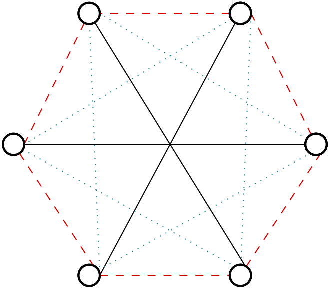
MMSNP and FPP are computationally equivalent.

Def: Structure is clique-like if its Gaifman graph is a clique.



Clique-like structures

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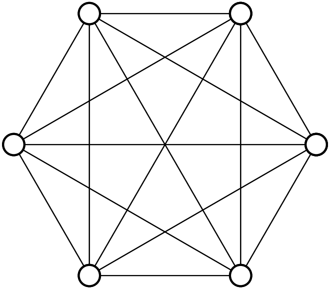

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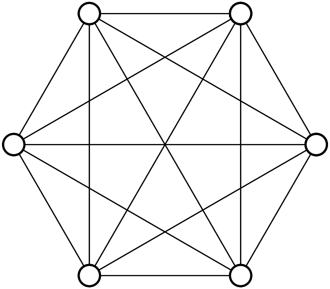

For general structures: Gaifman graph is obtained by replacing tuples by cliques.

New proofs of known results:

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- Partial confirmation of conjecture by Bodirsky-M (LICS'16).

Outline

Definition

$\mathfrak{B} = (B; E)$ a graph.
CSP($\mathfrak{B}$) is the problem:

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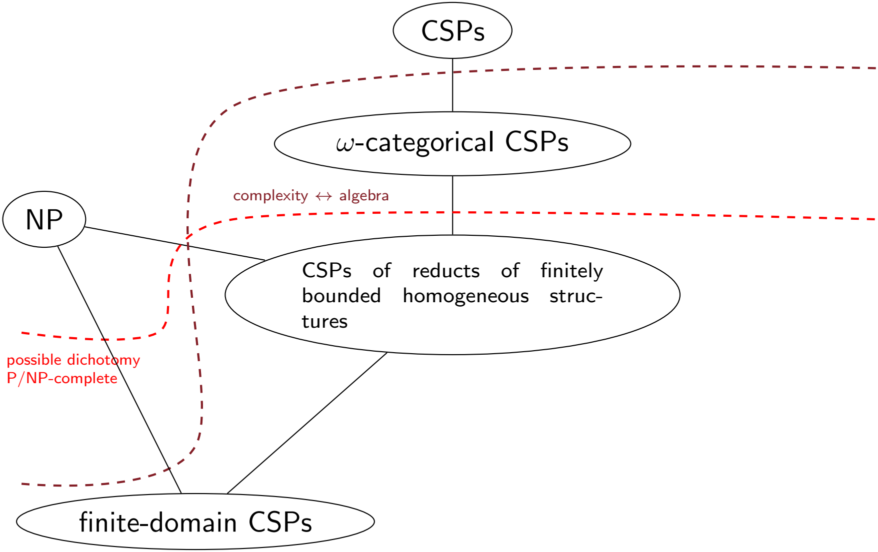
Everything in this talk generalises to relational structures.
In general, the forbidden patterns problem (FPP) for $\mathcal{F}$ is not a CSP, but a finite union of CSPs.

Proposition

Every FPP reduces in polynomial-time to a finite number of FPP of connected structures.

Context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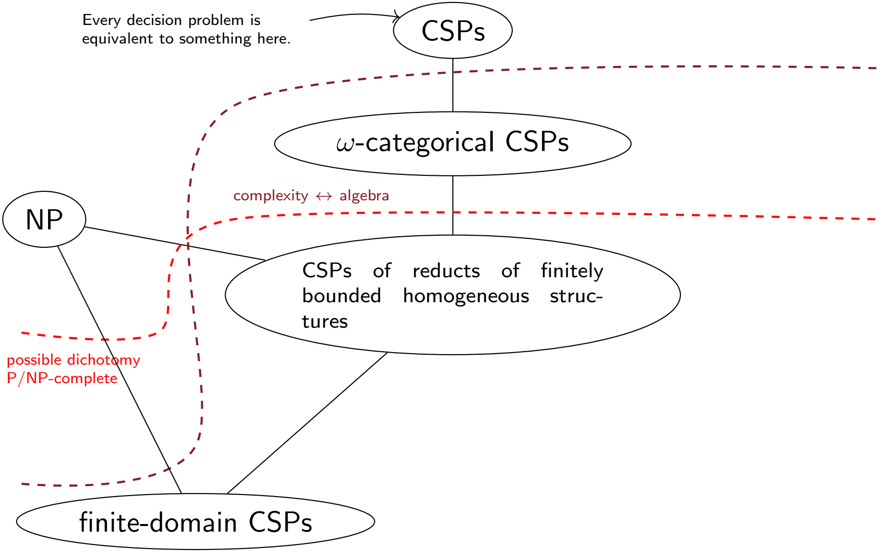
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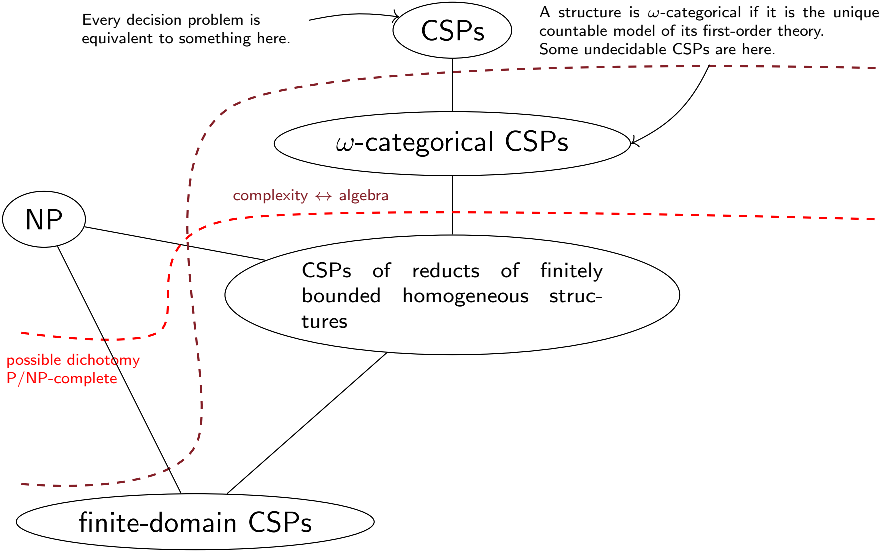
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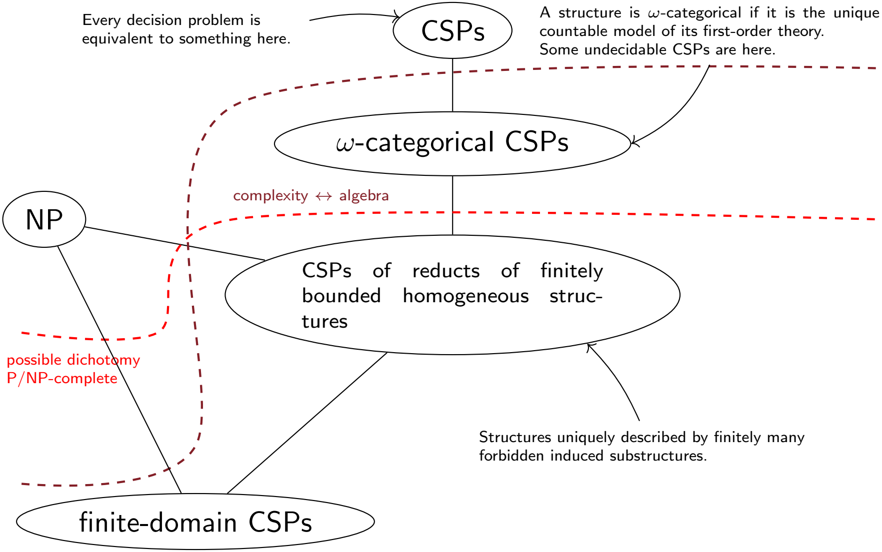
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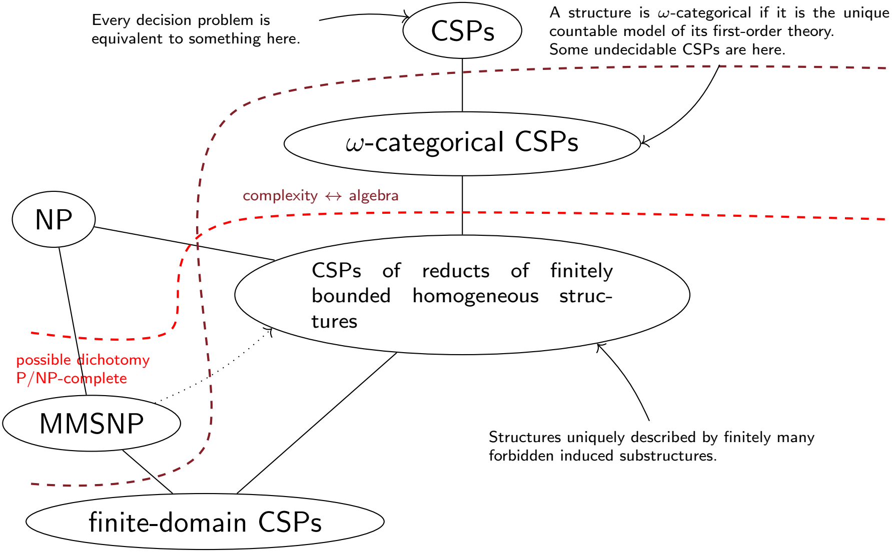
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MMSNP is essentially a subset of CSP

Theorem (Cherlin-Shelah-Shi, Adv. Appl. Math. 1999)

For every finite set $\mathcal{F}$ of finite connected coloured graphs, there exists an ω-categorical partially coloured graph $\mathfrak{B}^*$ such that $\mathfrak{A}^* \to \mathfrak{B}^*$ iff $\mathfrak{A}^*$ avoids $\mathcal{F}$.

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$$\Downarrow$$

CSPs in the BP class

Example

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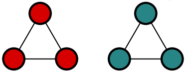

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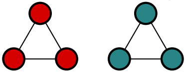

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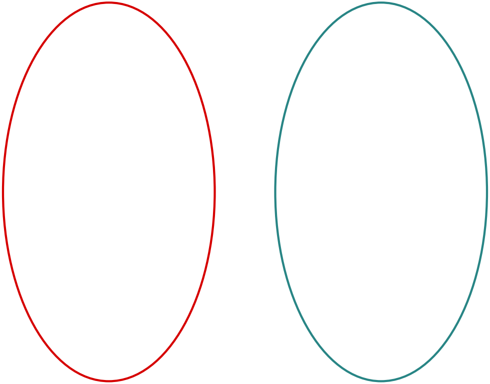

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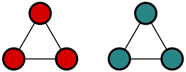

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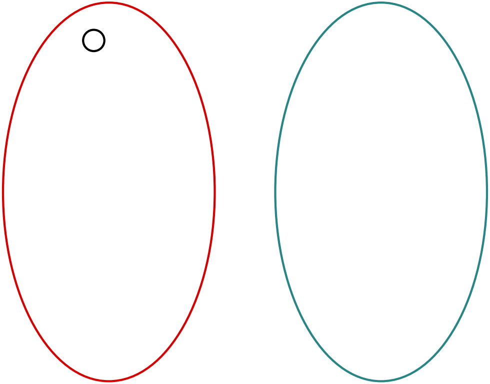

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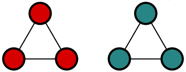

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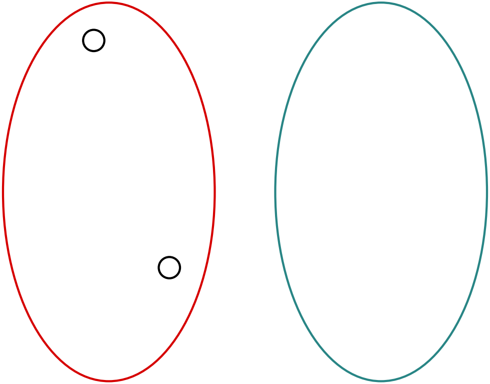

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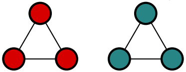

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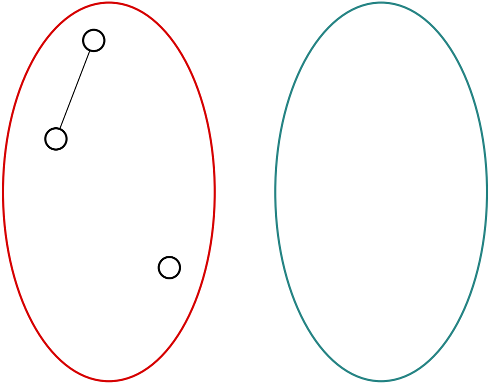

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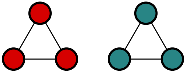

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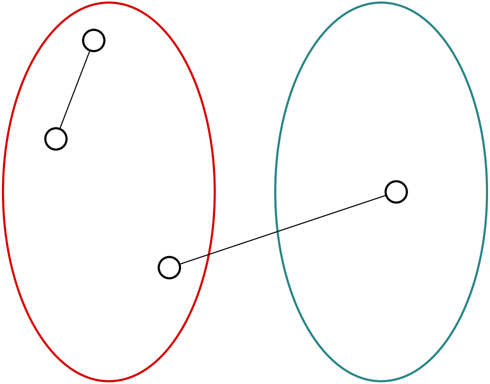

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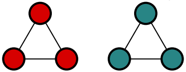

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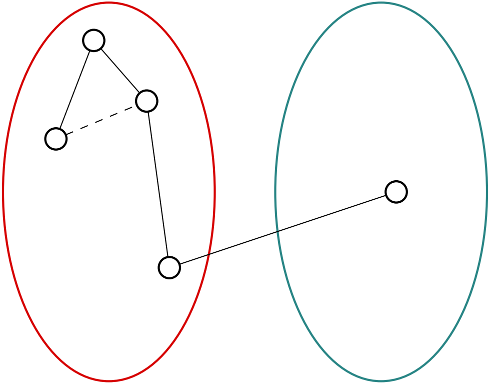

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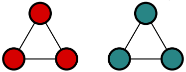

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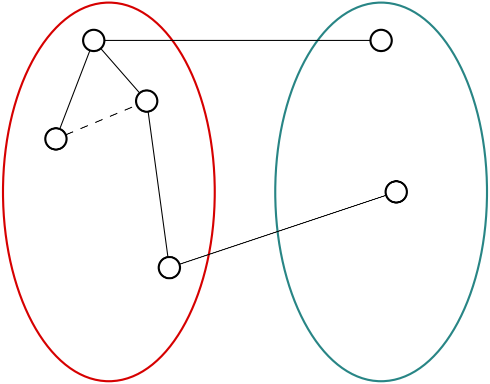

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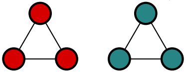

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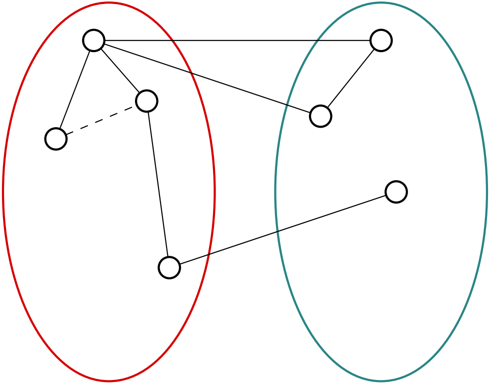

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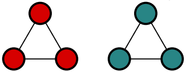

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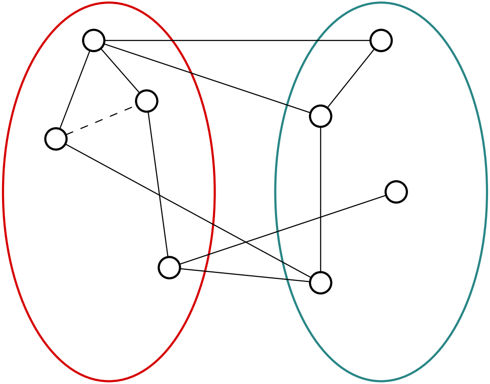

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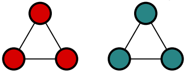

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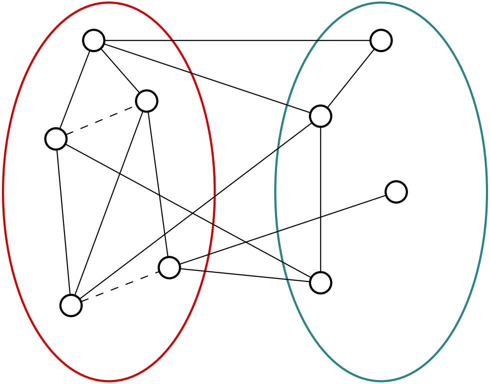

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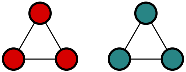

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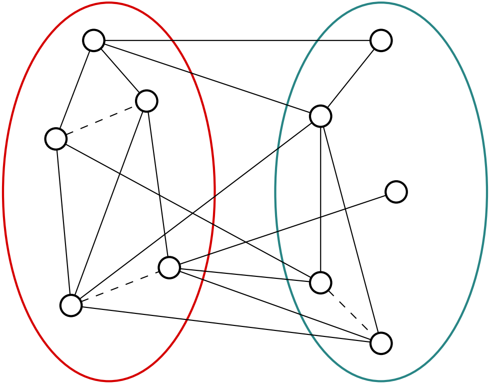

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Theorem (Barto-Opršal-Pinsker, Israel Jour. Math. 2015)

If $\mathfrak{B}$ is ω-categorical and Pol($\mathfrak{B}$) *is trivial,* CSP($\mathfrak{B}$) *is NP-hard.*

Conjecture (Bodirsky-Pinsker, 2011)

Let $\mathfrak{B}$ *be a reduct of a finitely bounded homogeneous structure.*
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Theorem (Bodirsky-M, 2017)

Let $\mathfrak{B}$ be ω-categorical such that $\mathrm{CSP}(\mathfrak{B})$ *is in MMSNP with clique-like obstructions. If* $\mathrm{Pol}(\mathfrak{B})$ *is not trivial, then* $\mathrm{CSP}(\mathfrak{B})$ *is in P.*

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- finite-domain CSPs satisfy the conjecture (Bulatov-Zhuk, FOCS'17).

Outline

Introduction

MMSNP "$\subseteq$" CSP

A Dichotomy Conjecture for Infinite-Domain CSPs

Precoloured MMSNP

Canonical Functions

Theorem and Example

Conclusion

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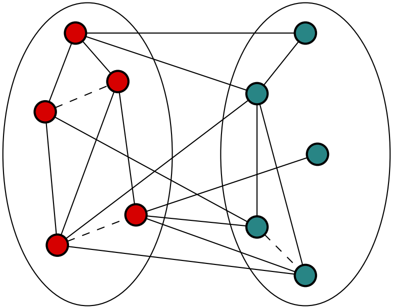
Rephrased: do CSP($\mathfrak{B}$, ●, ●) and CSP($\mathfrak{B}$) have same complexity?



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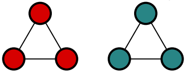
Good news: we can choose the MMSNP structure $\mathfrak{B}$ so that $(\mathfrak{B}, \neq)$ is an ω-categorical model-complete core.

Precolouring gadget

Obstructions $\mathcal{F}$:



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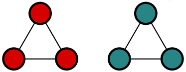

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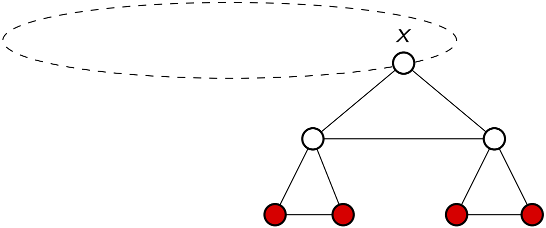

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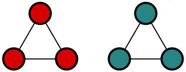

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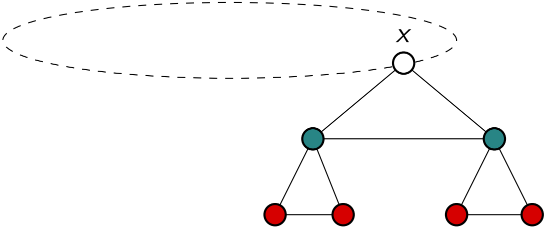

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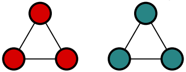

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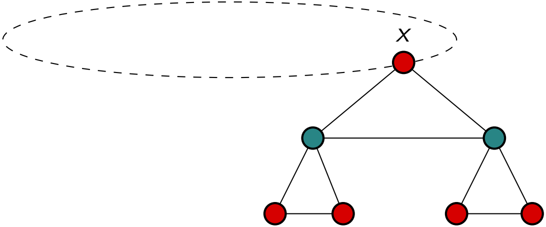

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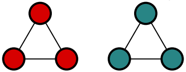

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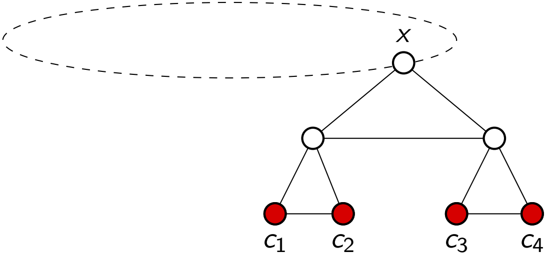

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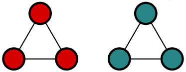

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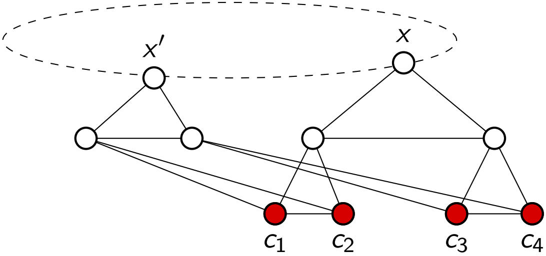

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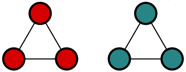

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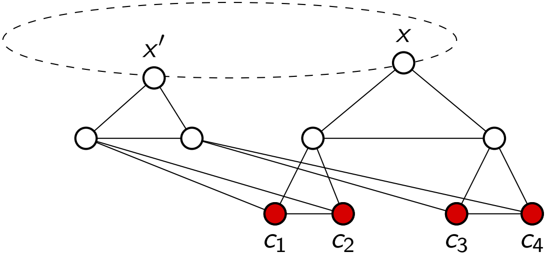

Proposition

The input precoloured graph is colourable iff the graph obtained by adding the gadgets is colourable.

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$$\begin{aligned} & \mathrm{Pol}(\mathfrak{B}) \text{ nontrivial} \\ \Rightarrow\ & \mathrm{Pol}(\mathfrak{B}, \bullet, \bullet) \text{ nontrivial} \qquad\qquad \text{(precolouring)} \end{aligned}$$

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Pseudo-Siggers

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To prove: if $\mathrm{Pol}(\mathfrak{B})$ contains a pseudo-Siggers, then $\mathrm{CSP}(\mathfrak{B})$ is in P.

Outline

Definition

$f\colon B^k \to B$, a group $\mathcal{G}$ acting on B. f is canonical (wrt $\mathcal{G}$) if for every finite subset $S \subseteq B$ of B and $\alpha_1, \ldots, \alpha_k \in \mathcal{G}$, there exists $\beta \in \mathcal{G}$ such that $\beta \circ f|_S = f \circ (\alpha_1, \ldots, \alpha_k)|_S$.

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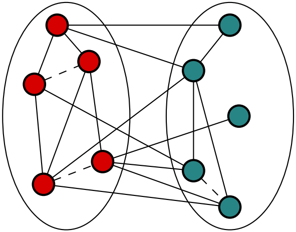

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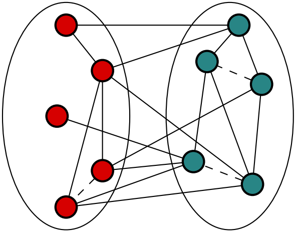
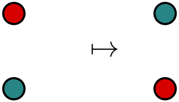


Every canonical function determines a behaviour on colours.



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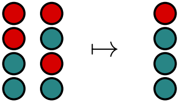

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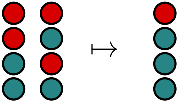

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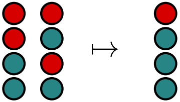

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Theorem (Bodirsky-M, LICS'16)

Let $\mathfrak{B}$ be in the BP class. If $\mathrm{Pol}(\mathfrak{B})$ *contains a pseudo-Siggers operation modulo* $\overline{\mathrm{Aut}(\mathfrak{A})}$ *that is canonical with respect to* $\mathfrak{A}$*, then* $\mathrm{CSP}(\mathfrak{B})$ *is in P.*

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$\Rightarrow$	$\mathrm{Pol}(\mathfrak{B}, {\color{red}\bullet}, {\color{teal}\bullet})$ contains a pseudo-Siggers	(Barto et al.)
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Getting canonical functions

Theorem (Hubička-Nešetřil, 2016)

Let $\mathfrak{B}$ be an MMSNP structure. Then there is an expansion $\mathfrak{C}$ of $\mathfrak{B}$ that is ω-categorical, ordered, and Ramsey.

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Problems:

- g is not unique,
- if f is pseudo-Siggers, we might get a g that is not pseudo-Siggers.

Getting canonical functions

Theorem (Hubička-Nešetřil, 2016)

Let $\mathfrak{B}$ be an MMSNP structure. Then there is an expansion $\mathfrak{C}$ of $\mathfrak{B}$ that is ω-categorical, ordered, and Ramsey.

Theorem (Bodirsky-Pinsker-Tsankov, 2010)

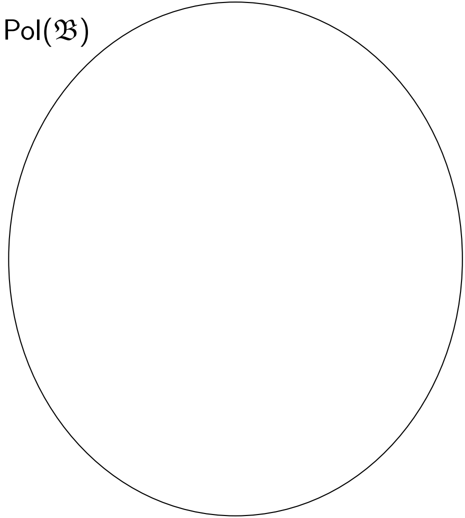
Suppose that $\mathcal{G}$ is the automorphism group of an ω-categorical ordered Ramsey structure. For every $f\colon B^k \to B$, there exists a function $g \in \overline{\mathcal{G}f\mathcal{G}}$ that is canonical with respect to $\mathcal{G}$.

Problems:

- g is not unique,
- if f is pseudo-Siggers, we might get a g that is not pseudo-Siggers.

(Partial) solution: Mashups!

The Mashup Property



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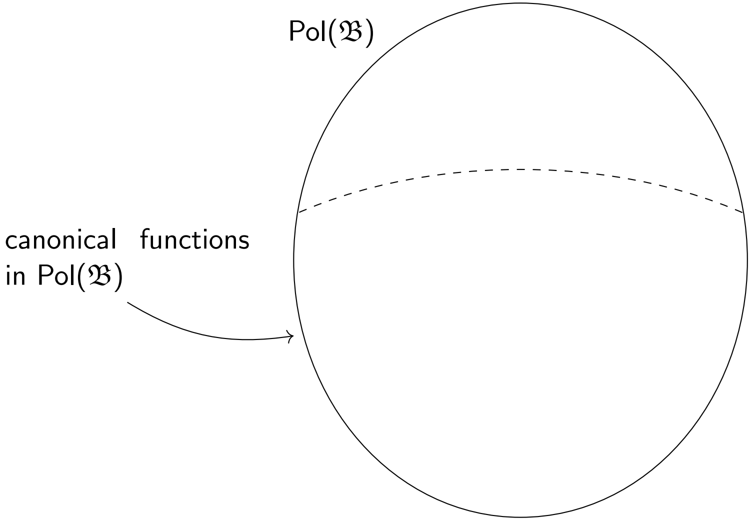
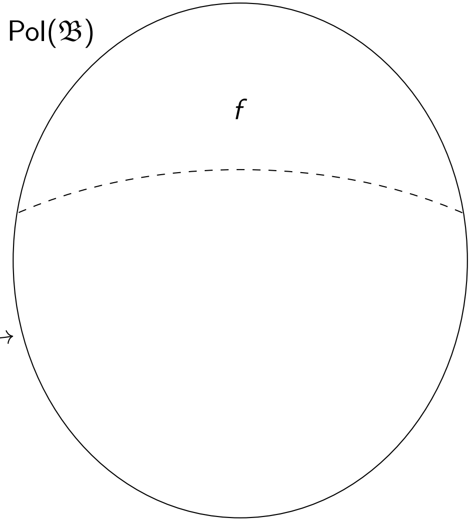


The Mashup Property

canonical functions in Pol($\mathfrak{B}$)



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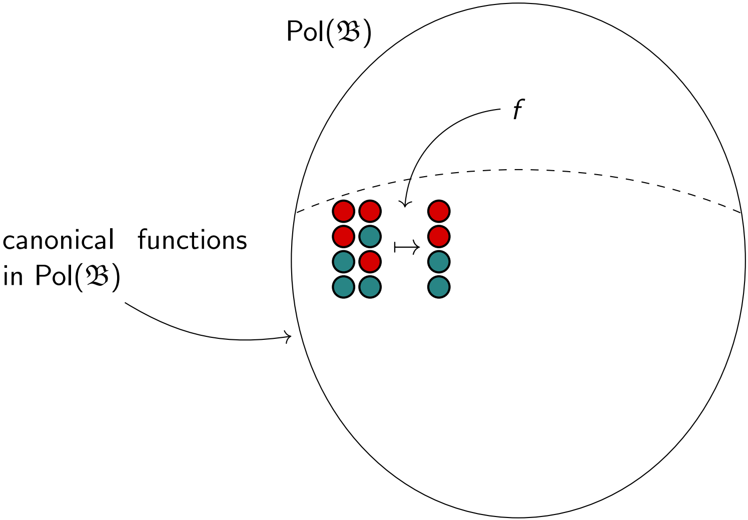
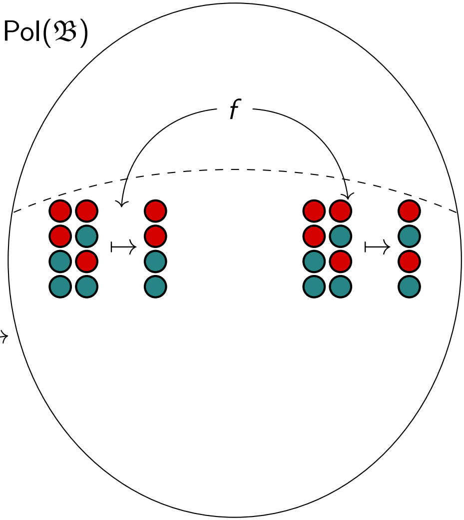


The Mashup Property

canonical functions in $\mathrm{Pol}(\mathfrak{B})$



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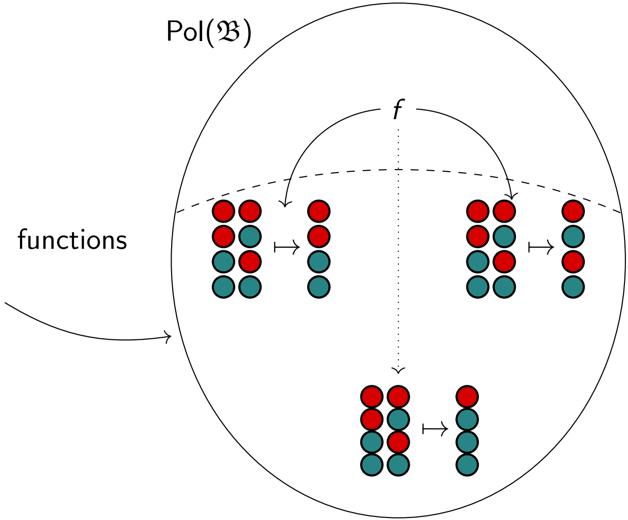

Theorem (Bodirsky-M, LICS'16)

If $\mathrm{Pol}(\mathfrak{B})$ *has the mashup property,*

$\mathrm{Pol}(\mathfrak{B})$ *has pseudo-Siggers* $\Leftrightarrow$ $\mathrm{Pol}(\mathfrak{B})$ *has a colour-canonical Siggers.*

Theorem (Bodirsky-M, LICS'16)

If $\mathrm{Pol}(\mathfrak{B})$ *has the mashup property,*

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What we have now: if Φ is an MMSNP sentence with clique-like obstructions, the corresponding $\mathrm{Pol}(\mathfrak{B})$ has the mashup property.

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Conjecture

For every MMSNP structure, $\mathrm{Pol}(\mathfrak{B})$ *has the mashup property.*

Outline

Summing up:

Theorem

Let $\mathfrak{B}$ be an MMSNP structure with clique-like obstructions. Then either the following equivalent statements hold:

1. $\mathrm{Pol}(\mathfrak{B})$ *is nontrivial,*

and $\mathrm{CSP}(\mathfrak{B})$ *is in P, or* $\mathrm{CSP}(\mathfrak{B})$ *is NP-complete.*

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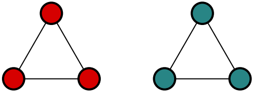
Items 3. and 4. can be checked effectively.

No monochromatic triangle

- Input: undirected graph G,
- Question: can one colour the vertices of G in a way to avoid the following patterns:



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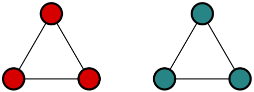

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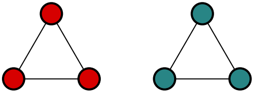

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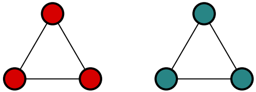

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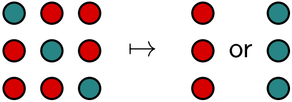

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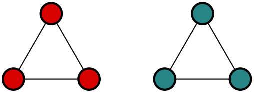

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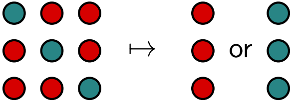
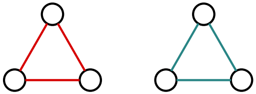


No Siggers $\Rightarrow$ the problem is NP-complete.

Outline

Open problems

- Solve the general MMSNP case (natural test-case for the Bodirsky-Pinsker conjecture).
- When is CSP($\mathfrak{B}$) in Datalog? Is it decidable? (Rewritability of MMSNP into Datalog programs, Lutz et al.)
- MMSNP$_2$: instead of colouring vertices, we colour edges. It is more expressive than MMSNP, but it is open whether it has a complexity dichotomy (Lutz et al.).
  Example: is it to possible to colour the edges of an input graph and avoid:



Since last year:

- Result from last year "Reasoning with Discrete Time" (i.e., quantitative temporal reasoning) submitted to Journal of the ACM and in second round of reviewing,
- LICS'16 result (reduction infinite-domain CSPs to finite-domain + mashup technique) strengthened and submitted to LMCS (reports received yesterday).