# Preliminary Housing Thermal Modeling

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## 1 Overview

A mathematical model of the system was constructed using Initial Value Problem ODEs and mathematically solved for a range of starting conditions. A Lumped Parameter model was used, simplifying the model into a form that can be easily mathematically solved. The parameters were adjusted to achieve temporal consistency with the historical data.

## 2 Model

### 2.1 Lumped Parameter Model

The lumped parameter model is appropriate for thermodynamic interactions where the principle resistance to heat transfer lies within the fluid. This is intuitive due to the relatively high conductance of glass and aluminium and low thickness, however it can be checked with the Biot number.

$$Bi \approx \frac{h_c L_c}{k_{solid}} \tag{1}$$

Lumped parameter models are appropriate for Biot numbers << 1. There are four relevant interfaces to check: water-aluminium, water-glass, air-aluminium, and air-glass.

| | | | |
|---|---|---|---|
| Glass-Air | 0.07 | Glass-Water | 0.27 |
| Aluminium-Air | 0.0004 | Aluminium-Water | 0.0025 |

The Glass-Water interface is at the limits of a lumped parameter model, but should still be approximated with reasonable accuracy. All other interfaces lend themselves well to a lumped parameter model.

In the lumped parameter model, we ignore the conduction through the solid and treat it as a single body of uniform temperature. As a result we can write energy conservation equations in the form

$$dU = mcdT = \rho c V dT \tag{2}$$

which will be used in the First Law equation

$$\dot{Q} - \dot{W} = \frac{dU}{dt} \tag{3}$$

Each interface is subect to Newton's law of cooling which states

$$\dot{Q} = -h_c A_s (T - T_\infty) \tag{4}$$

The model can then be constructed in the differential format $\frac{dX}{dt} = AX + b$:

$$\frac{dX}{dt} = \begin{bmatrix} -\frac{h_a}{\alpha_{Air}}(A_g + A_a) & \frac{h_a A_g}{\alpha_{air}} & \frac{h_a A_a}{\alpha_{air}} & 0 \\ \frac{h_a A_g}{\alpha_g} & -\frac{A_g}{\alpha_g}(h_a + h_w) & 0 & \frac{h_w A_g}{\alpha_g} \\ \frac{h_a A_g a}{\alpha_a} & 0 & -\frac{A_a}{\alpha_a}(h_a + h_w) & \frac{h_w A_a}{\alpha_a} \\ 0 & 0 & 0 & 0 \end{bmatrix} X + \begin{bmatrix} \dot{W}_{elec} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{5}$$

where

$$X = \begin{bmatrix} T_{air} \\ T_{glass} \\ T_{alum} \\ T_{sea} \end{bmatrix}$$

and

$$\alpha = \rho c V$$

## 2.2 Evaluation

The initial values used for the analytical solution were

| | | | |
|---|---|---|---|
| $\dot{W}_{elec}$ | $4.5W$ | $h_{water}$ | $75\frac{W}{m^s \cdot K}$ |
| $A_{glass}$ | $0.0162m^2$ | $h_{air}$ | $20\frac{W}{m^2 \cdot K}$ |
| $A_{cap}$ | $0.00319m^2$ | $L_{glass}$ | $0.005m$ |
| $T_{sea}$ | $30^{\circ}C$ | $L_{alum}$ | $0.015m$ |

and

| | | | | | |
|---|---|---|---|---|---|
| $\rho_{air}$ | $1.225\frac{kg}{m^3}$ | $\rho_{glass}$ | $4000\frac{kg}{m^3}$ | $\rho_{alum}$ | $2700\frac{kg}{m^3}$ |
| $V_{air}$ | $0.000617m^3$ | $V_{glass}$ | $0.000308m^3$ | $V_{alum}$ | $0.0000478m^3$ |
| $c_{air}$ | $1005\frac{J}{Kg \cdot K}$ | $c_{glass}$ | $840\frac{J}{Kg \cdot K}$ | $c_{alum}$ | $900\frac{J}{Kg \cdot K}$ |

However, after initial solutions were plotted, it was determined that an adjustment to the $\alpha_{air}$ was necessary to produce appropriate an temporal scale. The equivalent of 0.25kg of silicon ($c = 710\frac{J}{kg \cdot K}$) was added to $\alpha_{air}$ to account for the electronics components and hardware within the air cavity.

The model was solved using the python `scipy.integrate solve_ivp` function running a BFD algorithm. The solution was plotted with a 0.01 second time step for 10000 seconds. A range of initial internal temperatures were evaluated from 30 °C to 60 °C, with the aluminium and glass starting temperatures at 80% of the difference between the air temperature and the sea temperature.

# 3 Results

The solutions to the IVP were overlaid on the plot of previous thermal data. The data is in solid lines and the numeric solutions in dashed lines. Figure (1) shows all of the data overlaid with the solutions showing good general consistency and steady state conditions. Focusing on the short term behavior representing a significant majority of the data recorded, Figure (2) looks at the first 1000 seconds. The long term behavior compared to only runs of greater than 1000 seconds is in Figure (3).

# 4 Discussion

The numerical model shows good correlation to the data but is not a perfect match. Notable disparities are several instances of significant temperature rise in the first few hundred seconds that is not seen within the model. These represent only a few data points and could be considered outlier or anomalies and should be further examined for possible causes. Low starting temperatures and high starting temperatures have the poorest consistency between model and data with faster rise in temperature occurring at low starting temps in the data than model. At high starting temperatures, the inconsistencies are more varied without common theme.

While this data fits the model well, it should be noted that several parameters were adjusted to fit the model to the data, such as the additional heat capacity added within the air, and the ratio of starting temperatures. The fit validates the model as a possible explanation/description of the energy flow, but does not ensure that it is the correct one and not just a coincidence that it fits well. This could be better validated with experimental testing. Additionally, assessing this model with the data from other housings with different dimensions but similar thermal properties would support it's general accuracy. Another concern is the relatively high Biot number of the water/glass interface. The lumped

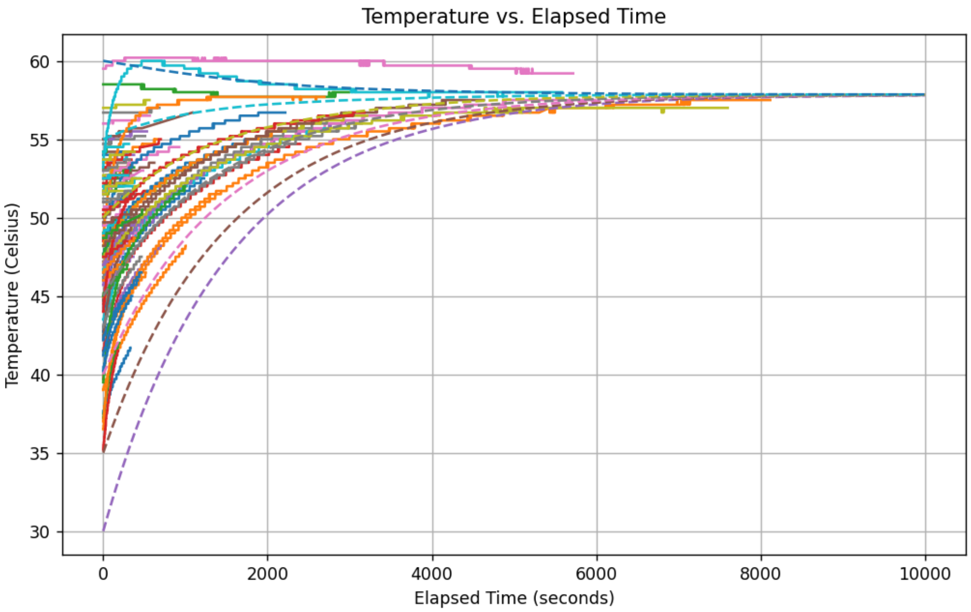


Figure 1: Plotted temperature data overlaid with numerical solutions

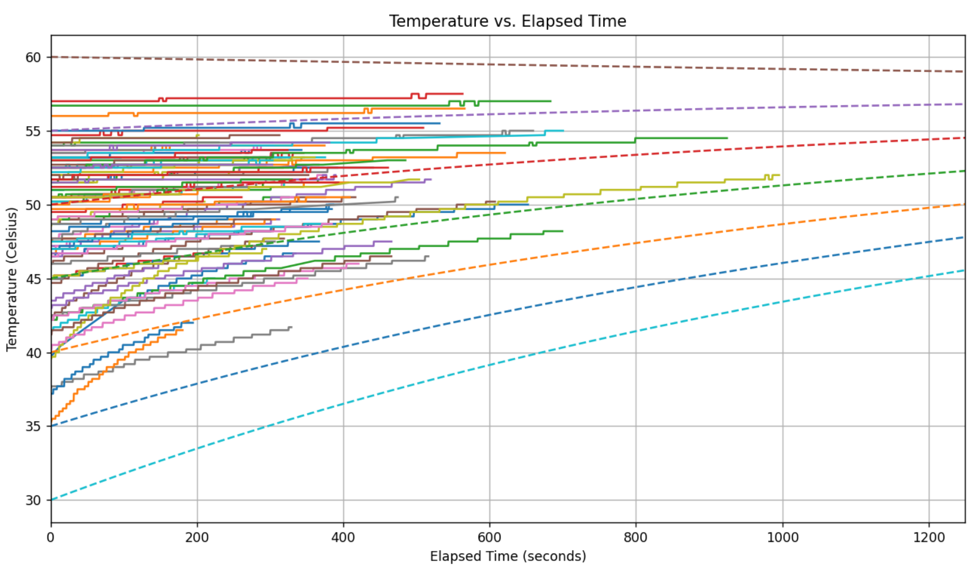


Figure 2: Plotted temperature data versus solutions for short data collections

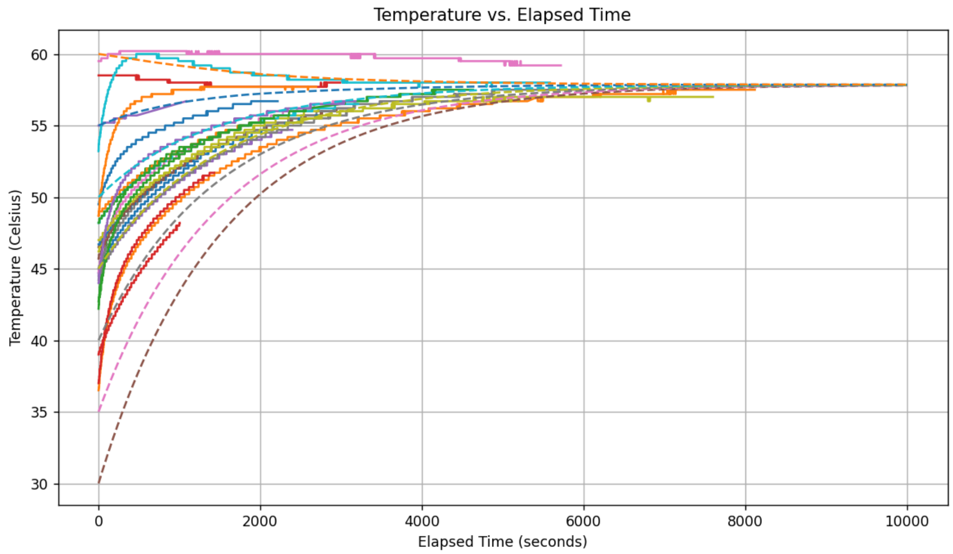


Figure 3: Plotted temperature data versus solutions for long data collections

parameter assumption may not be completely accurate. However, the negligence of the conductance through the glass is reasonable, supported by simple calculations for isolated glass of given thickness between air and water.

In conclusion, this fitting support the previously presented thermodynamic modeling of the housing but does not prove that the model is accurate. The model relies on several layers of extensive assumptions. If those assumptions are incorrect, the model would be invalid. Expansion to other data sets will help validate or disprove this signnificantly simplified model.