IMPACT MECHANICS – CASE STUDY 1:

CALIBRATION OF CONSTITUTIVE RELATION AND FAILURE CRITERION FOR IMPACT PROBLEMS

Calibration steps:

1) From the quasi-static tensile tests we will get a spreadsheet from the lab containing the time t (s), force F (kN), position P (mm), diameter D_x (measured in the thickness direction of the plate) and diameter D_y (measured in the transverse direction of the specimen). In these tests we assume $\dot{p}^* = 1$ and $T^* = 0$.

2) Based on these measurements, calculate the diameter reduction $\Delta D_x = D_{x0} - D_x$ and $\Delta D_y = D_{y0} - D_y$. Plot the $F - \Delta D_x$ and $F - \Delta D_y$ curves in the same diagram and confirm (as well as possible) that the material behaviour is isotropic.

3) Calculate the true cross-section area of the specimen as $A = \pi D_x D_y / 4$, where it is assumed that the deformed shape of the cross-section is an ellipse.

4) Calculate the true stress as $\sigma_t = F / A$ and the true strain as $\varepsilon_l = \ln(A_0 / A)$, where $A_0 = \pi D_{x0} D_{y0} / 4$. Plot the $\sigma_t - \varepsilon_l$ curve all the way to failure.

5) Zoom in on the elastic domain, and find the value of Young's modulus. If this value is different from the typical value of $E = 210\,000$ MPa for steel, correct the initial stiffness of the curve. This can be done through the relation

$$\Delta\varepsilon = \varepsilon_c - \varepsilon_m \quad \Rightarrow \quad \varepsilon_c = \varepsilon_m + \Delta\varepsilon = \varepsilon_m + \left(\frac{E_m - E_c}{E_m E_c}\right)\sigma$$

   where subscript c means "correct" and subscript m means "measured". It may also be necessary to adjust the zero-point for the strain, but this effect is normally minor.

6) From the $\sigma_t - \varepsilon_l$ curve, find and plot the $\sigma_t - \varepsilon_l^p$ curve. Here the plastic true strain is defined as $\varepsilon_l^p = \varepsilon_l - \sigma_t / E$. Remove all "negative" strains, and start the plot from where the strains show a steady increase with stress. This stress could in fact be defined as the yield stress.

7) Next we have to determine the necking point. Necking occurs at maximum load, i.e., when $dF = 0$ (or $d\sigma_e = 0$). From the spreadsheet, find $F_{\max}$ and the corresponding value for the true plastic strain. This strain is defined as ε_{lu}^p.

8) The formation of a neck in the tensile specimen introduces a complex triaxial stress state giving radial and transverse stresses which raise the value of the longitudinal stress required to cause plastic flow. In other words, the measured true stress needs to be corrected for triaxiality effects, since this stress is not equal to the equivalent stress after necking. The equivalent stress after necking can be obtained using the Bridgman-LeRoy correction given as

$$\sigma_{eq} = \frac{\sigma_t}{(1+2R/a)\ln(1+a/2R)}$$

where

$$a/R = 1.1\left(\varepsilon_l^p - \varepsilon_{lu}^p\right)$$

Plot the $\sigma_{eq} - p$ curve to failure and compare it to the $\sigma_t - \varepsilon_l^p$ curve. Note that in the uniaxial tension test $\sigma_t = \sigma_{eq}$ before necking, and that $\varepsilon_l^p = p$.

9) Fit the material parameters in the constitutive relation (JC or MJC) to the $\sigma_{eq} - p$ curve. Use the solver function in MS Excel (or MATLAB) and the method of least squares. Plot and compare the fitted constitutive relation with the measured $\sigma_{eq} - p$ curve. Run the fit with several starting values to see if the fit improves and to avoid local minima. Remember that $\dot{p}^* = 1$ and $T^* = 0$ in this step. This means that the strain rate term in the MJC constitutive relation will be $\left(1+\dot{p}^*\right)^C = 2^C$. However, since C is small for most metallic materials, it is reasonable to assume that $2^C \approx 1$.

10) The next to determine is the critical CL constant, i.e., the "plastic" work W_c to failure. In uniaxial tension $W_C = \int_0^{p_f} \sigma_t d\varepsilon_l^p$ so the same true stress-plastic strain curve as used in Step 6) without Bridgman-LeRoy corrections can be used to find the critical CL constant by numerical integration. Note that p_f is defined as the true plastic strain at failure, i.e., the true strain at maximum true stress. We do not have sufficient data in this case study to calibrate the JC fracture criterion, so these results are taken from the literature. It is also possible to estimate some of the JC fracture parameters from W_C.

11) Knowing the material constants from the quasi-static tests, the strain-rate sensitivity constant C can be determined from the dynamic (SHTB) tensile tests. In these tests we assume $\dot{p}^* \gg 1$ and $T^* = 0$. From the spreadsheet we have the engineering stress σ_e, engineering strain ε_e and the engineering strain rate $\dot{\varepsilon}_e$ for the various tests. The true plastic strain rate $\dot{\varepsilon}_l^p = \dot{p}$ can be found from the engineering strain rate $\dot{\varepsilon}_e$ as

$$\dot{\varepsilon}_l = \frac{d\varepsilon_l}{dt} = \frac{d\left(\ln(1+\varepsilon_e)\right)}{dt} = \frac{1}{1+\varepsilon_e}\dot{\varepsilon}_e \quad \Rightarrow \quad \dot{\varepsilon}_l^p = \dot{\varepsilon}_l - \dot{\varepsilon}_l^e = \dot{\varepsilon}_l - \frac{\dot{\sigma}_t}{E} \approx \dot{\varepsilon}_l$$

Establish and plot $\sigma_e - \varepsilon_e^p$ and $\sigma_t - \varepsilon_l^p$ curves for the different strain rates until necking and compare the latter results with the $\sigma_t - \varepsilon_l^p$ curves from the quasi-static test in Step 6).

12) Take the true stress for some true plastic strains in the pre-necking phase, e.g. at 2%, 4% and 6% plastic strain, and plot the true stress σ_t - logarithmic plastic strain rate $\dot{\varepsilon}_l^p$ curves for each specific plastic strain. You must also include the same data from the quasi-static test in Step 6). Normalize the constitutive relation (here shown for the MJC model) as

$$\frac{\sigma_{eq}}{\left(\sigma_0 + \sum_{i=1}^{2} Q_{Ri}\left(1 - \exp\left(-C_{Ri}\,p\right)\right)\right)} = (1 + \frac{\dot{p}}{\dot{p}_0})^C$$

and fit the strain-rate parameter c using the solver function in MS Excel (or MATLAB) and the method of least squares (in a similar way as in step 9)). The used-defined strain rate is taken as $5 \cdot 10^{-4}\ \mathrm{s}^{-1}$, i.e., the strain rate used in the quasi-static test.

13) From the curves and fits above, fill in the numbers for the material data and the fitted material constants in the tables below. The given data are from earlier fits of the same material, so you should get something similar. Physical constants and model parameters required for numerical simulation of impact problems are taken from the literature.

Material data				
σ_0 (MPa)	σ_u (MPa)	ε_u (-)	σ_f (MPa)	ε_f (-)
833.5	849	0.08	1491	1.26

Fitted material constants - JC									
A (MPa)	B (MPa)	n (-)	C (-)	m (-)	D_1 (-)	D_2 (-)	D_3 (-)	D_4 (-)	D_5 (-)
824.6	295.6	0.57	0.01	1.0	0.361	4.768	5.107	-0.0013	1.333

Fitted material constants - MJC							
σ_0 (MPa)	Q_{R1} (MPa)	C_{R1} (-)	Q_{R2} (MPa)	C_{R2} (-)	C (-)	m (-)	W_C (MPa)
777.0	125.2	29.5	2670.5	0.09	0.01	1.0	1492

Physical constants and model parameters							
E (MPa)	ν (-)	ρ (kg/m^3)	$\dot{p}_0$ (s^{-1})	c_ε (J/kg K)	T_0 (K)	T_m (K)	β (-)
210000	0.33	7850	$5 \cdot 10^{-4}$	452	293	1800	0.9