Problem set # 2

Optimization and Decision Analytics 2023/24

UC3M — *Master on Statistics for Data Science*

Due date: Monday October 23. Value: 50% of the final grade.

Note: This is an individual assignment. Evidence of plagiarism will be penalized. Hand in the assignment as a pdf file through the Assignment module in Aula Global, with Gurobi–Python code printouts and all required explanations.

Problem 1 (40 points). A company makes thee discrete products, labeled by $j = 1, \ldots, 3$, in quantities that must not exceed 60 units, respectively. Making each product incurs a fixed cost, given by 40 €, 50 €, and 45 €, respectively. Furthermore, if product 3 is produced then product 1 must also be produced. The marginal profit per unit for each product is given in the following table.

product	marginal profit per unit
1	4 € for the first 10 units, 3 € for the remaining units
2	6 € for the first 8 units, 4 € for the remaining units
3	5 € for the first 10 units, 2.5 € for the next 10 units, and 1 € for the remaining units

The company uses four resources, labeled by $i = 1, \ldots, 4$, and the per unit usage of each resource by each product, as well as the daily resource availability, are given in the following table:

resource	usage product 1	usage product 2	usage product 3	resource availability
1	12	15	10	1500
2	15	14	12	1900
3	11	13	9	1800
4	13	12	15	1200

(a, 20 points) Formulate the problem of finding an optimal production plan as an integer optimization problem, explaining its elements.

(b, 20 points) Implement the model in Gurobi–Python and solve it. Discuss the optimal solution and the solution statistics, such as the number of nodes explored, etc.

Problem 2 (25 points). Consider the integer optimization problem

$$
\begin{aligned}
(I) \qquad & \text{maximize } 55x_1 + 32x_2 + 84x_3 + 75x_4 \\
& \text{subject to: } 43x_1 + 27x_2 + 62x_3 + 81x_4 \leqslant 125 \\
& 0 \leqslant x_j \leqslant 1 \text{ and integer}, \quad j = 1, \ldots, 4.
\end{aligned}
$$

(a, 5 pts) Formulate the linear relaxation (L) of formulation (I).

(b, 5 pts) Obtain an optimal solution to the relaxation (L). What information can you infer from it about problem (I)?

(c, 5 pts) Is $(1, 0, 1, 0)$ a feasible solution for (I)? If so, give the best upper bounds that can be obtained with the information provided so far on the absolute and relative optimality gaps for $(1, 0, 1, 0)$.

(d, 5 pts) In light of the above, can you ensure that $(1,0,1,0)$ is optimal for problem (I)? And can you ensure that $(1,0,1,0)$ is not optimal for problem (I)? Contrast the answers with the optimal integer solution computed with Gurobi.

(e, 5 pts) Solve the problem by adding successive valid inequalities and solving the resulting linear relaxations, as explained in class.

Problem 3 (35 points). An optimization student wants to visit by car the following Spanish cities, starting and ending in Madrid: Alicante, Almería, Barcelona, Cuenca, Córdoba, Granada, La Coruña, and Valencia. The following table in shows the road distance between each pair of cities (assume symmetry for the blank entries):

	Alicante	Almería	Barcelona	Córdoba	Granada	La Coruña	Madrid	Cuenca
Alicante		295	508	505	348	867	419	305
Almería			790	315	166	883	542	520
Barcelona				860	844	894	619	539
Córdoba					166	976	298	333
Granada						799	361	345
La Coruña							510	636
Madrid								165
Cuenca								

(a, 10 pts) Formulate the problem of finding the shortest tour. How many constraints has in this case the integer optimization formulation seen in class?

(b, 25 pts) Apply the iterative procedure seen in class to try to find an optimal tour, carrying out at most three iterations using Gurobi–Python. Discuss the results.