Problem set 1

Numerical Methods for Data Science 2023/24

UC3M — *Master on Statistics for Data Science*

Due date: Monday October 9. Value: 50% of the final grade.

Note: This is an individual assignment. Evidence of plagiarism will be penalized. Hand in the assignment as a pdf file through the Assignment module in Aula Global, with Gurobi–Python code printouts and all required explanations.

Problem 1 (35 points). Consider the linear optimization problem, formulating the optimal allocation of two resources to four economic activities:

$$\begin{aligned}
&\text{maximize } 2x_1 + 3x_2 + 9x_3 + 6x_4 \\
&\text{subject to:} \\
&4x_1 + x_2 + 8x_3 + 2x_4 = 30 \\
&2x_1 + 3x_2 + 4x_3 + 6x_4 = 60 \\
&x_1 \geqslant 0, x_2 \geqslant 0, x_3 \geqslant 0, x_4 \geqslant 0.
\end{aligned}$$

(a, 7 points) Formulate the dual problem, and find all its optimal solutions using the graphical method.

(b, 7 points) Formulate the optimality conditions that must be satisfied by any optimal primal solution in relation with a dual optimal solution $\boldsymbol{\pi}^*$. Apply them, along with part (a), to find all optimal solutions to the primal problem.

(c, 7 points) Obtain all possible values for the reduced cost of each primal variable. Are reduced costs unique? Interpret the reduced costs obtained.

(d, 7 points) Carry out a sensitivity analysis with respect to simultaneous changes of contraint right-hand sides for the primal problem. Contrast the results with those obtained through Gurobi-Python.

(e, 7 points) Carry out a sensitivity analysis with respect to simultaneous changes of objective coefficients for the primal problem. Contrast the results with those obtained through Gurobi-Python.

Problem 2 (35 points). In a CSI investigation, the crime suspect left both his/her hand and shoe imprints in the crime scene. From that evidence the investigators want to infer the suspect's height. For that purpose, they plan to obtain a prediction equation for height based on hand and shoe size based on the following data:

Hand size (cm)	Shoe size (cm)	Height (cm)
16.9	29.7	175.3
17.1	30.9	177.8
19.3	33.8	185.4
16.8	31.8	175.3
15.3	27.6	172.7
25.2	35.9	198.5

(a, 20 points) Formulate the Linear Optimization moof seen in class for estimating the best prediction equation under the Mean Absolute Error (MAE) criterion, and implement it in Gurobi–Python.

(b, 7 points) Solve the model and give the optimal solution (prediction equation). Is it unique?

(c, 8 points) Obtain the optimal dual solution and discuss its possible interpretation.

Table 1: Stock information.

Stock	**Price Purchased**	**Current Price**	**Price Estimate Next Year**
A	15.68 €	31.80 €	29.50 €
B	22.10 €	24.28 €	26.31 €
C	30.39 €	32.50 €	34.55 €
D	8.93 €	14.16 €	15.23 €
E	40.55 €	50.99 €	62.43 €
F	18.58 €	24.17 €	26.68 €
G	22.54 €	23.67 €	23.85 €
H	24.84 €	28.77 €	31.66 €

Problem 3 (30 points). Suppose that, last year, you purchased 150 shares of eight different stocks, for a total of 1,200 shares. Table 1 lists the stocks that you purchased, the price you purchased them for last year, the current price, and the price estimate for next year.

If you sell any shares, you have to pay a transaction cost of 1% of the amount transacted. In addition, you must pay a capital-gains tax at the rate of 30% on any capital gains at the time of the sale. For example, suppose that you sell 100 shares of a stock today at 50 € per share, which you originally purchased for 30€per share. You would receive 5,000 €. However, you would have to pay capital-gains taxes of 0.30 × (5,000 - 3,000) = 600 €, and you would have to pay 50 € in transaction costs. Therefore, by selling 100 shares of this stock, you would have a net cash flow of 5,000 - 600 - 50 = 4,350 €.

You would like to sell enough shares of stock today to generate 10,000 € to use as part of a down payment on a new home. You need to decide how many shares of which stocks to sell to generate 10,000 €, after taxes and transaction costs, while maximizing the estimated value of your stock portfolio next year. We will assume that you cannot sell more shares of stock than you own, and you cannot buy additional shares.

(a, 10 puntos) Formulate the problem as a linear optimization model, identifying its elements.

(d, 10 puntos) Set up and solve this problem using Gurobi–Python. Give the optimal solution obtained.

(e, 10 puntos) Using Gurobi–Python, carry out a sensitivity analysis both on the objective function coefficients and on the right-hand sides, for changes in one coefficient only. Interpret the results obtained, and discuss any insights you can draw from them.