# Assignment 6 - Viewing I & II

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1. Screen Space Description

   You can think of screen space as the 2D grid where everything gets drawn on your screen. It's kind of like a canvas that your computer uses to show stuff. The origin usually starts in a corner (like the top-left), and the x and y axes go across and down. It's often a left-handed system, which means the axes follow a certain direction convention. It's useful because it makes positioning things on screen a lot more predictable.

   To transform a 3D point to screen space using perspective projection:

   (a) **Model Transformation**: Convert object coordinates to world space.

   (b) **View Transformation**: Convert world coordinates to camera/view space.

   (c) **Projection Transformation**: Apply a perspective projection matrix to get clip space coordinates.

   (d) **Clipping**: Remove any parts of the geometry outside the view frustum.

   (e) **Perspective Divide**: Divide by $w$ to get normalized device coordinates (NDC).

   (f) **Viewport Transformation**: Convert NDC to screen space using screen width and height.

2. Perspective View Volume Properties

   **Given:** $z_{\text{near}} = -1$, distance to far $= 49 \Rightarrow z_{\text{far}} = -50$

   **Near clipping window:** width $= 200$, height $= 100$

   **a. Near and Far Planes:** $z_{\text{near}} = -1$, $z_{\text{far}} = -50$

   **b. Clipping Window:** $x_l = -100, \quad x_r = 100, \quad y_b = -50, \quad y_t = 50$

   **c. Perspective to Orthographic Matrix $M_{\text{persp}\to\text{ortho}}$:**

$$M_{\text{persp}\to\text{ortho}} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -51 & -50 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

   **d. Final Normalized Matrix $M_{\text{persp}\to\text{norm}}$** can be obtained by multiplying with the orthographic normalization matrix.

3. Viewport Transformation

**a. Viewport Matrix:**

$$M_{\text{viewport}} = \begin{bmatrix} 6 & 0 & 0 & 6 \\ 0 & 5 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (\text{width} = 12, \text{ height} = 10)$$

**b. Transform NDC (0.5, 0.5, 0.5):**

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 & 6 \\ 0 & 5 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 \\ 7.5 \\ 0.5 \\ 1 \end{bmatrix}$$

**c. Pixel Coordinates:** $(x_{\text{pixel}}, y_{\text{pixel}}) = (9, 7.5)$

4. Calculating Vertical Field of View (FOV)

   **Given:** Near plane at $z = -10$, Window size $= 10 \times 12$
   **Vertical FOV:**

$$\theta = 2 \cdot \tan^{-1}\left(\frac{H}{2 \cdot |z_{\text{near}}|}\right) = 2 \cdot \tan^{-1}\left(\frac{12}{20}\right) = 2 \cdot \tan^{-1}(0.6) \approx 61.9^\circ$$