Key Points to Remember

- Tensile stress is normal to the plane and pointing away from the plane.
- Compressive stress is normal to the plane and pointing towards the plane.
- Shear stress is parallel to the plane and positive shear stress tends to rotate the body in the clockwise direction.
- Negative shear stress tends to rotate the body in the anticlockwise direction.
- Material obeys Hooke's law within the elastic limit.
- Whenever a shear stress is applied on a body, a negative shear stress develops on a perpendicular plane, this shear stress is known as complementary shear stress.
- Young's modulus, $E = \dfrac{\text{Normal stress}}{\text{Longitudinal strain}}$.
- A specimen is being tested under the axial tensile or compressive load. If the load is removed from the specimen within the elastic limit, then all the strains are recovered. However, if the load is removed in the plastic stage, then a residual strain remains in the specimen.
- Tapered bar, change in length $= \dfrac{4PL}{\pi EDd}$.
- Tapered flat, change in length, $dl = \dfrac{PL}{Et(B-b)} \ln\left(\dfrac{B}{b}\right)$.
- Extension in bar due to self-weight, $\delta l = \dfrac{wl^2}{2E}$.
- Bar of uniform strength, $A_2 = A_1 e^{wl/\sigma}$.
- Bulk modulus, $K = \dfrac{p}{\varepsilon_v} = \dfrac{\text{Volumetric stress}}{\text{Volumetric strain}}$.
- To solve a statically indeterminate problem, an equation of compatibility is necessary.
- Strain energy, $U = \dfrac{1}{2} Pdl$.
- Strain energy per unit volume $= \dfrac{\sigma^2}{2E}$.
- Modulus of resilience $= \dfrac{\sigma_e^2}{2E}$.
- Stress due to sudden load, $\sigma_s = 2 \times \sigma_g$ (gradual stress) $= \dfrac{2W}{A}$.
- Instantaneous stress developed in a bar, $\sigma_i = \dfrac{W}{A}\left(1 + \sqrt{1 + \dfrac{2EAh}{WL}}\right)$.
- Mild steel is a ductile material. In the $\sigma - \varepsilon$ curve, there is an upper yield point and a lower yield point. This type of yielding is called discontinuous yielding.
- Barba's law $dl = bl + c\sqrt{A}$, where b and c are Barba's constants.

- Stress concentration factor at abrupt change in a dimension, $SCF = \frac{\sigma_{max}}{\sigma_{av}}$.
- Factor of safety, $FOS = \frac{\text{Ultimate strength}}{\text{Allowable stress}}$.

Review Questions

1. Explain the following:
   Hooke's law, elastic limit, resilience, toughness, ductility, index of cold working and Poisson's ratio.
2. With the help of sketches, explain the difference between positive shear stress and negative shear stress.
3. Explain how the stress due to sudden load is two times the stress when the same load is gradually applied.
4. Make a sketch of load extension diagram of mild steel and mark important points on it.
5. Explain the difference between the following:
   (i) resilience and toughness
   (ii) upper yield point and lower yield point
   (iii) elastic stage and plastic stage
6. Consider a bar subjected to an axial strain and show that volumetric strain is $\frac{\sigma}{E}(1-2\nu)$, where σ is axial stress, ν is Poisson's ratio and E is Young's modulus.
7. Explain why the stresses developed in a rope due to its own weight are negligible.
8. What is the effect of gauge length on percentage elongation of a bar?
9. Differentiate between sudden load and impact load.
10. Explain how complementary shear stresses are developed in a body.

Multiple Choice Questions

1. A bar is subjected to an axial tensile stress. If the volumetric strain in the bar is 0.44 times the axial strain, what is the Poisson's ratio of the material?
   (a) 0.44 (b) 0.30
   (c) 0.28 (d) None of these
2. On a plane, resultant stress is inclined at an angle of 30° with the plane. If the normal stress on the plane is 50 MPa, what is the shear stress on the plane?
   (a) 43.3 MPa (b) 86.6 MPa
   (c) 100 MPa (d) None of these
3. A bar is of square section $a \times a$ subjected to a tensile load P. On a plane inclined at 45° to the axis of the bar, normal stress will be
   (a) $2P/a^2$ (b) P/a^2
   (c) $P/2a^2$ (d) $P/4a^2$
4. A 100-mm-long steel bar is tested in tension, so that the change in the length is 0.05 mm. If E = 200 GPa, what is the stress developed in the bar?
   (a) 200 MPa (b) 100 MPa
   (c) 50 MPa (d) None of these
5. Two tie rods are connected through a pin of a cross-sectional area of 40 mm^2. If the tie rods carry a tensile load of 10 kN, the shear stress in the pin is
   (a) 125 MPa (b) 250 MPa
   (c) 500 MPa (d) None of these
6. A 10-m-long wire rope is suspended vertically from a pulley. The wire rope weighs 12 N/m in length. The cross-sectional area of the wire rope is 20 mm^2. What is the maximum stress developed in the wire?
   (a) 1.2 N/mm^2 (b) 2.4 N/mm^2
   (c) 3.0 N/mm^2 (d) 6 N/mm^2
7. A spherical ball of volume 1,000 cm^3 is subjected to a hydrostatic pressure of 90 N/mm^2, and bulk modulus of the material is 190 kN/mm^2 What is the change in volume of the ball?

(a) 473 mm^3 (b) 940 mm^3

(c) 502 mm^3 (d) None of these

8. A bar of a length of 100 mm and a cross-sectional area of 64 mm^2 is tested under tension. The Barba's constants for the material are $b = 0.21$ and $c = 0.5$. What is the percentage elongation in the bar?

(a) 30 (b) 25

(c) 21 (d) 20

9. Poisson's ratio of aluminium is

(a) 0.30 (b) 0.33

(c) 0.35 (d) None of these

10. In stress–strain curve, the area up to the elastic limit stress indicates which mechanical property?

(a) Ductility (b) Strength

(c) Resilience (d) None of these

11. In σ–ε curve for mild steel, load at which point considerable extension occurs with decrease in resistance is known as

(a) Upper yield point (b) Breaking load

(c) Ultimate load (d) None of these

12. A circular tapered bar tapering from a diameter of 20 mm to 10 mm over a length of 1,000 mm is subjected to an axial force of 10 kN. If $E = 10^5$ N/mm^2, what is the change in length of bar?

(a) $\frac{10}{\pi}$ mm (b) $\frac{21}{\pi}$ mm

(c) $\frac{2}{\pi}$ mm (d) None of these

Practice Problems

1. A hole of a diameterof 40 mm is to be punched in a 1-mm-thick mild stress sheet. If the ultimate shear stress of mild steel is 490 N/mm^2, determine the force required to punch the hole.
2. Two parts of a certain machine component are joined by a rivet of a diameter of 25 mm. Determine the shear and normal stresses in the rivet if $P = 20$ kN and the angle of joint is 30° to the axis of the load, as shown in Fig. 1.62.
3. A rigid square plate 1 × 1 m is supported over four legs A, B, C and D. A load W is applied at point K as shown in Fig. 1.63. Determine the compressive force in each leg. If the cross-sectional area of each leg is 20 mm^2 and $W = 1{,}000$ N, what are the stresses in each leg?

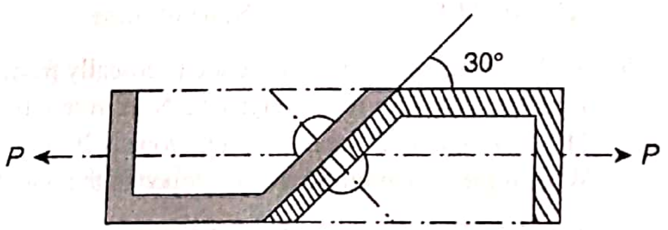

Figure 1.62

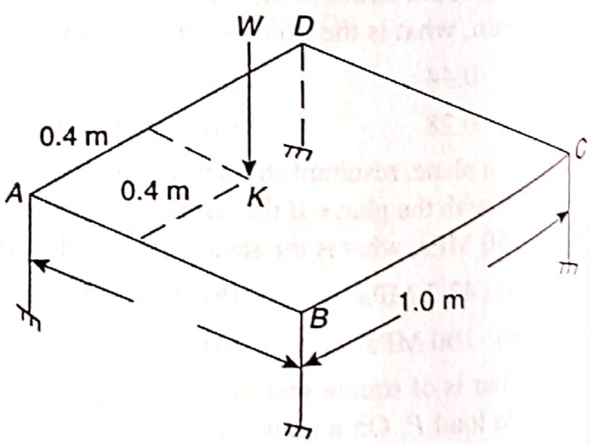

Figure 1.63

4. A rectangular tapered steel bar tapering from a section 30 × 20 mm to 12 × 10 mm area over an axial length of 100 mm is shown in Fig. 1.64. It is subjected to an axial force P. If the maximum stress is not to exceed 100 MPa, what is the value of P? If $E = 210$ GPa, what is the change in the length of the bar?

Answers to Exercises

Exercise 1.1: -8.835 kN, -0.476×10^{-3}
Exercise 1.2: $+20$ N/mm^2, 0.526×10^{-3}
Exercise 1.4: $\sigma = -19.88$ N/mm^2, $\tau = -19.88$ N/mm^2
Exercise 1.5: -0.1147×10^3, -0.2009×10^3, -0.459×10^3, -0.0449 mm
Exercise 1.6: $\delta l = +0.45$ mm, $\delta b = -6.75 \times 10^{-3}$ mm, where b is side, that is, $b = 30$ mm
Exercise 1.7: (a) 18.85 kN, (b) $+0.382$ mm
Exercise 1.8: -5.246 kN
Exercise 1.9: $+1.273 \times 10^{-2}$ mm, $+0.637 \times 10^{-2}$ mm, -4.138×10^{-2} mm, overall, -2.228×10^{-2} mm
Exercise 1.10: (7.644 N/mm^2, 0.191 mm)
Exercise 1.11: $d_m = 54.52$ mm
Exercise 1.12: 470 m, $\delta V = 128$ mm^3
Exercise 1.13: $\sigma_1 = +165$ N/mm^2, $\sigma_2 = -42.5$ N/mm^2
Exercise 1.14: 9.15 Nm, 42.297 Nm, 0.2244Nmm/mm^3
Exercise 1.15: 20.37 MPa, 0.097 mm
Exercise 1.16: 114.35 N/mm^2, 1.0937 mm, bar will not break
Exercise 1.17: (105 kN/mm^2, 310 N/mm^2).
Exercise 1.18: 1.59
Exercise 1.19: 37.27 mm

Answers to Multiple Choice Questions

1 (c)
2 (b)
3 (c)
4 (b)
5 (b)
6 (d)
7 (a)
8 (b)
9 (b)
10 (c)
11 (a)
12 (c)

Answers to Practice Problems

1. 61.575kN
2. 20.37N/mm^2, 35.284N/mm^2
3. $R_A = 350$ N, $R_B = 250$, $R_C = 150$, $R_D = 250$ N
   $\sigma_A = -17.5$ MPa, $\sigma_B = -12.5$ MPa,
   $\sigma_C = -7.5$ MPa, $\sigma_D = -12.5$ MPa
4. $P = 12$ kN, $\delta l = 0.021$ mm
5. 0.019 mm
6. 7°50′–69°44′
7. 12.373k/N
8. 20.6mm, L = 163.9 mm
9. 31.25 N/mm^2, 0.727 mm
10. -28.29 N/mm^2, -113.177 N/mm^2, $+56.59$ N/mm^2, $\delta l = -0.0485$ mm
11. $P = 16$ kN, $+60$ N/mm^2
12. 53.05, 106.1 MPa, 0.51mm, 0.255 mm
13. $\dfrac{U_A}{U_B} = 0.75$;
14. 14.551 + 1.82 = 16.371 mm
15. 461.56 N
16. 69.4 N/mm^2
17. 44.72 N/mm^2
18. 44.43 MPa, normal
19. 27.4 mm, 8.486 mm
20. $\delta_{AD} = \overline{0.096}$ mn, $\delta_{BE} = \overline{0.128}$ mm, $\delta_C = 0.80$ mm

Key Points to Remember

- A composite bar made of two bars of areas of cross-section A_1 and A_2 with E_1 and E_2 as Young's modulii of elasticity. If it is subjected to an external load W, then

$$W = W_1 + W_2$$

$$W = A_1\sigma_1 + A_2\sigma_2, \text{ where } \frac{\sigma_1}{\sigma_2} = \frac{E_1}{E_2}$$

- In a composite system, a rigid bar is supported by two wires/bars of different materials, with E_1 and E_2 being Young's modulii of elasticity. A load W is suspended on the rigid bar and the load can be applied at such a position that a rigid bar remains horizontal.
- If a load is applied on two bars placed co-axially, then a compatibility equation has to be formed involving the gap between the axial lengths of two bars.
- In a bolt and tube assembly, due to tightening of the nut on the bolt, tensile stress is developed in the bolt and compressive stress is developed in the tube. Then,

$$\sigma_b A_b = \sigma_t A_t \tag{2.19}$$

- Extension in length of bolt + contraction in length of tube = axial movement of nut on bolt. If a bar of length L, Young's modulus E and the coefficient of thermal expansion α, then during change of temperature ΔT of bar, stress is developed in bar if it is held between two rigid supports.

$$\sigma_T = \alpha E \Delta T$$

- A composite bar of length L is composed of two bars of area of cross-section and Young's modulus A_1, E_1 and A_2, E_2, with their coefficient of thermal expansion being α_1, α_2, respectively, and σ_{1T} and σ_{2T} are temperature stresses developed in two bars. Then,

$$\sigma_{1T} A_1 = \sigma_{2T} A_2$$

$$\frac{\sigma_{1T}}{E_1} + \frac{\sigma_{2T}}{E_2} = (\alpha_1 - \alpha_2)\Delta T$$

? Review Questions

1. What is a composite bar? Under the action of an external load on the bar, state how stresses are developed in different components of the composite bar.
2. What is the difference between a composite system and a composite bar. Explain with the help of a bimetallic strip and a bolt and tube assembly.
3. Consider a composite bar with its temperature changed. Explain how tensile and compressive stresses are induced in two components of the composite bar.
4. Consider a bar rigidly held between two supports. Explain why the bar bends if its temperature is increased.
5. Consider a composite bar with external load and change in temperature. Explain the development of resultant stresses due to the direct load and the temperature change in two components of the composite bar.

Multiple Choice Questions

1. A composite bar is made of steel and aluminium strips, with $A_a = 3A_s$, where A_a and A_s are areas of cross-section of aluminium and steel bars, respectively. $E_s/E_a = 3$. Due to an external load, if the stress developed in the aluminium is 30 MPa, then what is the stress developed in the steel bar?
   (a) 10 Mpa (b) 30 MPa
   (c) 90 MPa (d) None of these
2. A copper bar of area of cross-section 200 mm^2 is encased in a steel tube of area of cross-section 400 mm^2. Due to an external load, the stress in copper bar is 10 MPa and the load on composite bar is P. What is the load shared by the steel bar?

   $E_s/E_{cu} = 2.$

   (a) 0.5 P (b) 0. 6 P
   (c) 0.8 P (d) None of these
3. In a bolt and tube assembly, the pitch of thread on the bolt is 2.4 mm. If the nut is tightened by one-quarter of a turn and a reduction in length of the tube is 0.4 mm, what is the increase in length of the bolt?
   (a) 0.2 mm (b) 0.4 mm
   (c) 0.6 mm (d) None of these
4. In a composite bar, bars having same cross-sectional area, one bar shares one-third of the total load. What is the ratio of E_1/E_2?
   (a) 3 (b) 2
   (c) 1.5 (d) None of these
5. A bimetallic strip is made of two metals with equal areas of cross-section. Due to temperature change, the stress developed in one strip is –40 N/mm^2. What is the stress developed in another component of the composite bar?
   (a) –40 N/mm^2 (b) +20 N/mm^2
   (c) +40 N/mm^2 (d) None of these
6. A wire of diameter 1 mm is held between two rigid supports. The temperature of the wire drops by 10°C, $\acute{a} = 12 \times 10^{-6}$/°C, $E = 2{,}00{,}000$ N/mm^2. What is the stress developed in the wire?
   (a) +24 N/mm^2 (b) –24 N/mm^2
   (c) +12 N/mm^2 (d) None of these
7. What is the approximate ratio of $E_{steel}/E_{aluminium}$
   (a) 2.0 (b) 3.0
   (c) 2.5 (d) None of these
8. Three strips of same area of cross-section share a load of 5.5 kN. If their Young's modulus are in the ratio of $E_1 = 2E_2 = 3E_3$, then what is the load shared by the strip with Young's modulus E_1?
   (a) 3 kN (b) 3.5 kN
   (c) 2.5 kN (d) None of these
9. A steel bar of diameter 20 mm is encased in a copper tube of outside diameter 30 mm. An external load produces a stress of 30 N/mm^2 in the steel bar. What is the stress developed in the copper tube? Given $E_s/E_{cu} = 2$.
   (a) 30 N/mm^2 (b) 24 N/mm^2
   (c) 15 N/mm^2 (d) None of these
10. A steel rail track is laid by joining 30 m long rails end to end. At 30°C, there is no stress in the rails. At 50°C, what will be the stress in the rails if $\alpha = 11 \times 10^{-6}$/°C and $E = 200 \times 10^3$ N/mm^2.
   (a) 88 MPa (b) 44 MPa
   (c) 22 MPa (d) None of these

Practice Problems

1. A short hollow cast iron column of external diameter 25 cm and internal diameter 20 cm is filled with concrete. The column carries a total load of 350 kN. If $E_{CI} = 6\, E_{Con}$, calculate the stress in cast iron and concrete. What must be the internal diameter of the cast iron column if a load of 420 kN is to be carried and stresses and external diameter of the column remain unchanged?
2. Two brass strips are rigidly fixed to a steel strip of section 20 mm × 6 mm and length 1.2 m. The brass strips are 0.6 m long each with section 20 mm × 4 mm. The composite bar is subjected to a tensile force of 12 kN as shows in Fig. 2.21. Determine the deflection of point B if $E_S = 2\, E_B = 200$ GPa.

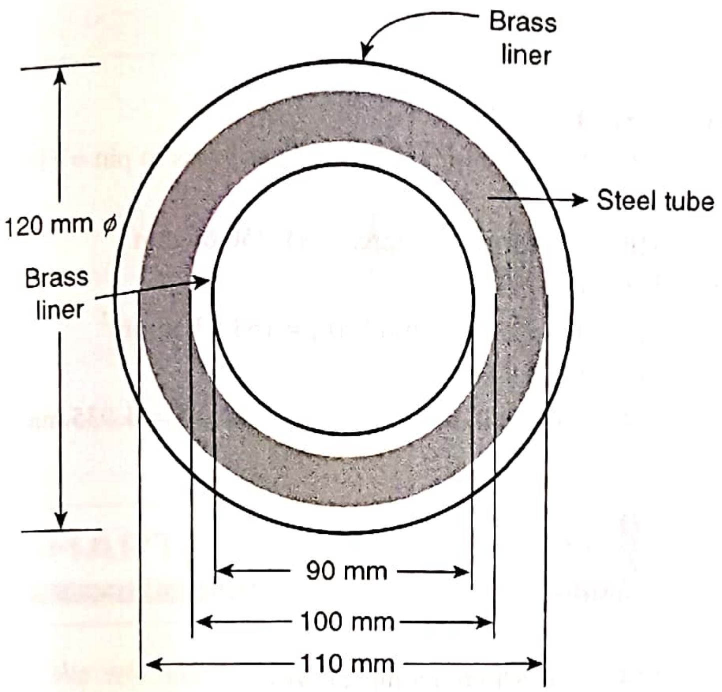

Figure 2.31

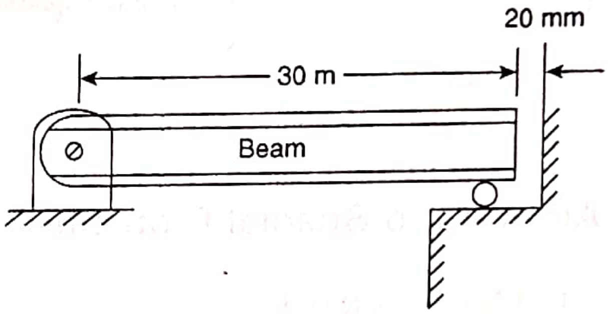

Figure 2.32

4. A tapered bar of steel is rigidly held between two vertical supports as shown in Fig. 2.33. If the temperature of the bar is increased by 20°C, what is the maximum stress developed in the bar? $E = 200$ GPa, $\alpha = 11.7 \times 10^{-6}$/°C.
5. A copper wire of diameter 2 mm is stretched tightly between two supports, 1 m apart under an initial tension of 200 N. If the temperature of the wire drops by 10°C, what is the maximum stress in the wire? $E = 105$ GPa, $\alpha = 18 \times 10^{-6}$/°C.

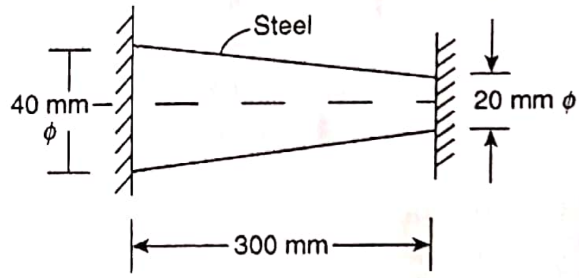

Figure 2.33

Answers to Exercises

Exercise 2.1: 69.45; 23.15 N/mm^2

Exercise 2.2: 37.5 mm from steel wire, 76.37 N/mm^2, 25.458 N/mm^2, downward movement, dL = 0.456 mm

Exercise 2.3: −33.45 MPa, −113.72 MPa

Exercise 2.4: 72.6 MPa, +148.13 MPa

Exercise 2.5: −84.52 N/mm^2, +28.98 N/mm^2

Exercise 2.6: +18.9 MPa in steel, −12.6 MPa in copper

Answers to Multiple Choice Questions

1. (c)
2. (c)
3. (a)
4. (b)
5. (c)
6. (a)
7. (b)
8. (a)
9. (c)
10. (b)

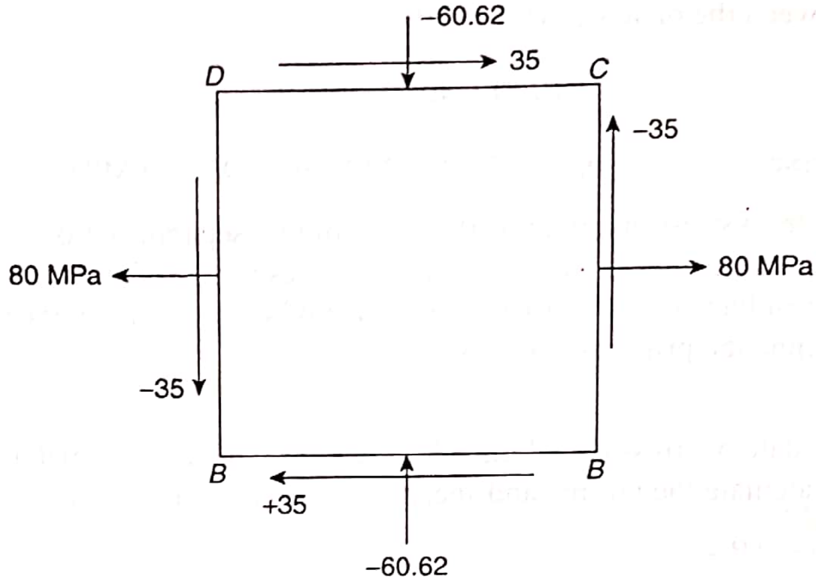

Figure 3.39 Resultant stress system

$$\frac{\sigma_1 - \sigma_2}{2} = -70.31 \text{ MPa}$$

$$\sqrt{\left(\frac{\sigma_1 + \sigma_2}{2}\right)^2 + \tau^2} = \sqrt{(-70.31)^2 + 35^2} = \sqrt{4943 + 5 + 1225}$$

$$= 78.54 \text{ MPa}$$

$$p_1 = 9.69 + 78.54 = 88.23 \text{ MPa}$$

$$p_2 = 9.69 - 78.54 = -68.85 \text{ MPa}$$

Key Points to Remember

- On each and every point of a stressed body, there exist a set of three planes, perpendicular to each other. On these planes only normal stresses act and shear stresses are absent on these plane.
- If on a reference plane normal stress is σ_1 and shear stress is $-\tau$ and on perpendicular plane normal stress is σ_2 and shear stress is $+\tau$, then the normal and shear stresses on an inclined plane are

  $\sigma_\theta = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta + \tau \sin 2\theta$ and $\tau_\theta = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta - \tau\cos 2\theta$.
- If on a reference plane normal stress is σ_1 and shear stress is $+\tau$ and on perpendicular plane normal stress is σ_2 and shear stress is $-\tau$, then the normal and shear stresses on an inclined plane are

  $\sigma_\theta = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta - \tau \sin \theta$ and $\tau_\theta = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta + \tau\cos 2\theta$.
- *Principal stresses*

$$p_1, p_2 = \frac{\sigma_1 + \sigma_2}{2} \pm \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

- Maximum shear stress

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

- *Principal angles*

$\tan 2\theta_1 = \dfrac{2\tau}{\sigma_1 - \sigma_2}$, if the shear stress on reference plane is negative.

$\tan 2\theta_1 = \dfrac{-2\tau}{\sigma_1 - \sigma_2}$, if the shear stress on reference plane is positive.

- *Stress invariant*

$$\sigma_1 + \sigma_2 = p_1 + p_2$$

- Radius of Mohr's stress circle gives the maximum shear stress τ_{max} at the point.
- On a reference plane if ε_{xx} is normal strain and $\dfrac{-\gamma_{xy}}{2}$ is shear strain and on another perpendicular plane ε_{yy} is normal strain and $+\dfrac{\gamma_{xy}}{2}$ is shear strain, then on inclined plane, the normal and shear strains are $\varepsilon_\theta = \dfrac{\varepsilon_{xx} + \varepsilon_{yy}}{2} + \dfrac{\varepsilon_{xx} - \varepsilon_{yy}}{2}\cos 2\theta + \dfrac{\gamma_{xy}}{2}\sin 2\theta$ and $\gamma_\theta = \dfrac{\varepsilon_{xx} + \varepsilon_{yy}}{2}\cos 2\theta - \dfrac{\gamma_{xy}}{2}\sin 2\theta$.
- In a three-dimensional stress system the principal stresses can be obtained from the following cubic equation:

$$p^3 - I_1 p^2 + I_2 p - I_3 = 0.$$

Where stress invariants are,

$$I_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz}$$

$$I_2 = \sigma_{xx}\sigma_{yy} + \sigma_{yy}\sigma_{zz} + \sigma_{zz}\sigma_{xx} - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2$$

$$I_3 = \sigma_{xx}\sigma_{yy}\sigma_{zz} - \sigma_{xx}\tau_{yz}^2 - \sigma_{yy}\tau_{zx}^2 - \sigma_{zz}\tau_{xy}^2 + 2\tau_{xy}\tau_{yz}\tau_{zx}.$$

- Strain gauge rosettes are used to determine surface strains in three directions so as to have three equations for the determination of three unknowns, that is, ε_{xx}, ε_{yy}, and γ_{xy}.

Review Questions

1. Define principal planes and principal stresses.
2. What is the utility of finding the principal planes at the critical section of a component?
3. What are complementary shear stresses?
4. Why three strain gauges are necessary to determine the complete state of stress at a point?
5. If the stresses on two planes are given which are not perpendicular to each other, describe the procedure to locate the centre of Mohr's stress circle.
6. What is the difference between three-element rectangular rosette and three-element delta rosette?
7. What do you mean by gauge factor of a strain gauge?
8. Enumerate the important information you can get from a Mohr's stress circle.
9. What is modified modulus of elasticity? What is its practical significance?
10. How to get volumetric strain from principal strains?
11. Differentiate between plane stress and plane strain conditions.

Multiple Choice Questions

1. On two perpendicular planes stresses are $\sigma_1 = 120$ MPa, $\sigma_2 = 60$ MPa and $\tau = \pm 40$ MPa, what is the maximum shear stress at the point?
   (a) 60 MPa (b) 50 MPa
   (c) 0 MPa (d) None of these
2. At a point two principal stresses are +120 MPa and −80 MPa. What is the shear stress on a plane inclined 45° to the plane of major principal stress?
   (a) 100 MPa (b) 50 MPa
   (c) 20 MPa (d) None of these
3. Major principal stress at a point is 220 MPa. The radius of Mohr's stress circle is 70 MPa. What is the minor principal stress at the point?
   (a) 150 MPa (b) 100 MPa
   (c) 80 MPa (d) None of these
4. On an element, on two perpendicular planes, the shear stresses are ±50MPa. There is no normal stress on these planes. What is the maximum principal stress at the point?
   a) 100 MPa (b) 50 MPa
   (c) 25 MPa (d) None of these
5. Principal stresses at a point are +120 MPa, −80 MPa and + 40 MPa. If the Poisson's ratio is 0.3 and E is 100 GPa, what is the maximum principal strain?
   (a) 1,320 μ strain (b) 1,200 μ strain
   (c) 1,080 μ strain (d) None of these
6. The major and minor principal stresses at a point are 120 MPa and 60 MPa, respectively. On the plane passing through the point, the shear stress on the plane is 15 MPa. What is the angle of this plane with the plane of major principal stress?
   (a) 45° (b) 30°
   (c) 15° (d) None of these
7. Principal stresses at a point are $p_1 = 200$ MPa and $p_2 = 100$ MPa. What is the maximum angle of obliquity θ?
   (a) $t = \sin^{-1}0.25$ (b) $\theta = \sin^{-1}0.333$
   (c) $\theta = \sin^{-1}0.5$ (d) None of these
8. In a rectangular strain gauge rosette, the strains recorded are $\varepsilon_0 = 400$ μ strain, $\varepsilon_{45°} = 300$ μ strain and$\varepsilon_{90°} = 200$ μ strain. What is the maximum principal strain at the point?
   (a) 500 μ strain (b) 400 μ strain
   (c) 300 μ strain (d) None of these
9. The major and minor principal stresses at a point are 120 MPa and 40 MPa, respectively. What is the normal stress on a plane of τ_{max} stress?
   (a) 120 MPa (b) 80 MPa
   (c) 40 MPa (d) None of these
10. In a strained material at a point, the strains are $\varepsilon_{xx} = 600$ μ strain, $\varepsilon_{yy} = 200$ μ strain and $\gamma_{xy} = 300$ μ strain. What is the maximum principal strain at the point?
   (a) 760 μ strain (b) 675 μ strain
   (c) 650 μ strain (d) None of these

Practice Problems

1. In a stressed body, on two perpendicular planes, the stresses are +200 MPa, τ(shear) and −100 MPa, τ(shear). If the maximum principal stress at the point is 300 MPa, determine the value of τ. Calculate the minor principal stress and τ_{max}.
2. On a certain plane, resultant stress is +150 MPa inclined at 20° to the normal on the plane. On a second plane perpendicular to the first plane, the resultant stress has normal component of −80 MPa. Determine the resultant stress on this plane. What are the principal stresses and directions of principal planes with respect to the first plane?
3. A circle of diameter 200 mm is inscribed on a steel plate before it is subjected to plane stresses as shown in Fig. 3.40. If $E = 200$ GPa and $v = 0.3$, determine the major and minor axes and their directions as the circle is deformed into an ellipse after application of stresses.
4. A piece of material is subjected to two tensile stresses 300 and 100 MPa at right angles. Find the position of the plane on which the resultant stress is most inclined to the normal and the magnitude of this resultant stress.

Answers to Exercises

Exercise 3.1: 121.07 MPa, +34.47 MPa

Exercise 3.2: 144.42 MPa, +5.11 MPa

Exercise 3.3: $p_1 = 120$ MPa, $p_2 = 40$ MPa; $\theta_1 = +45°$, $\theta_2 = 135°$ or $-45°$

Exercise 3.4: 130 MPa, 30 MPa; 18°26′, 108°26′; ±50 MPa; 63°26′, 153°26′; 121.87 MPa, 27.3 MPa

Exercise 3.5: 60 MPa, 17.32 MPa

Exercise 3.6: $\varepsilon_\theta = 275 \times 10^{-6}$, $\gamma_\theta = -250 \times 10^{-6}$

Exercise 3.7: ±250 μ strain; 16°, 106°; 92 μ strain, 155 μ strain

Exercise 3.8: 120 MPa, −80, +80 MPa; 26.6°, 63.40°, 90°; 33.33 MPa, 83.8 MPa

Exercise 3.9: $p_1 = 257.14$ MPa, $p_2 = 217.14$ MPa

Exercise 3.10: 188.57 MPa, 108.57 MPa

Exercise 3.11: 365.7 μ strain, −165.7 μ strain; 35.07 MPa, −4.296 MPa

Exercise 3.12: 230.55 μ strain, −130.55 μ strain, +42.07 MPa, −13.49 MPa

Answers to Multiple Choice Questions

1. (b)
2. (a)
3. (c)
4. (b)
5. (a)
6. (c)
7. (b)
8. (b)
9. (b)
10. (c)

Answers to Practice Problems

1. $\tau = 200$ MPa, $p_2 = -200$ MPa, $\tau_{max} = 250$ MPa
2. 95.035 MPa, 152.28 MPa, −91.325 MPa, 12°27′, 102°27′
3. $p_1 = 170$ MPa, $p_2 = 70$ MPa, $\varepsilon p_1 = 0.745 \times 10^{-3}$, $\varepsilon p_2 = 0.095 \times 10^{-3}$, major axis = 200.149 mm, minor axis = 200.019 mm
4. $\phi = 30°$, $\sigma_r = 173.2$ MPa
5. $\theta = 26.56°$, ($\sigma_2 = 100$); $p_1 = 220.7$ MPa, $p_2 = 79.3$ MPa
6. $\theta = 98°$; $p_1 = 130$ MPa, $p_2 = 20$ MPa
7. On AB, $\sigma_1 = 180.8$ MPa, $\tau = +59.95$; On BC, $\sigma_2 = 155.83$ MPa, $\tau = -59.95$ MPa; $p_1 = 229.545$ MPa, $p_2 = 107.085$ MPa

Answers to Special Problems

1. 100 MPa; $\sigma_\theta = 160$ MPa, $\tau_\theta = 60$ MPa
2. 161.8 MPa, −61.8 MPa; $\theta_1 = 13.28°$, $\theta_2 = 103.28°$
3. $\tau < \sqrt{\sigma_1\sigma_2}$
4. $p_1 = +111.8$ MPa, $p_2 = -111.8$ MPa; $\theta_1 = -13.280$, $\theta_2 = 76.720$
6. 225 MPa, 73 MPa

Hoop stress in tyre, $\sigma_c = \varepsilon_c \times E = \dfrac{1}{1{,}500} \times 20{,}000$

$$= 133.33\ \text{N/mm}^2 = \frac{pD}{2t}$$

where p is junction pressure between the tyre and the wheel

or,
$$\frac{pD}{2t} = 133.33$$

$$\frac{p \times 1{,}500}{2 \times 10} = 133.33$$

$$p = \frac{133.33}{75} = 1.77\ \text{N/mm}^2$$

Twisting moment

Total radial force on tyre, $R = b \times \pi D p$

$$= 80 \times \pi \times 1{,}500 \times 1.77$$
$$= 6{,}67{,}274\ \text{N}$$

Coefficient of friction, $\mu = 0.3$

Force of friction, $F = \mu R = 0.3 \times 667{,}274$

$$= 2{,}00{,}182\ \text{N}$$

Twisting moment, $T = F \times \text{radius of wheel}$

$$= 2{,}00{,}182 \times 750$$
$$= 1{,}50{,}136{,}500\ \text{N mm}$$
$$= 150.136\ \text{kN m}$$

Key Points to Remember

- For a shell, D/t ratio is greater than 20, it is classified as a thin shell.
- For a cylindrical thin shell is subjected to internal pressure, p.

Hoop stress, $\sigma_c = \dfrac{pD}{2t}$

Axial stress, $\sigma_a = \dfrac{pD}{4t}$

Hoop strain, $\varepsilon_c = \dfrac{pD}{4tE}(2 - \nu)$

Axial strain, $\varepsilon_a = \dfrac{pD}{4tE}(1 - \nu)$

Volumetric strain, $\varepsilon_v = \frac{pD}{4tE}(5-4v)$

- For a thin spherical shell subjected to internal pressure, p.

Hoop stress, $\sigma_c = \frac{pD}{4t}$

Diametral strain, $\varepsilon_c = \varepsilon_d = \frac{pD}{4tE}(1-v)$

Volumetric strain, $\varepsilon_v = \frac{3pD}{4tE}(1-v)$

- δV, addition of volume of liquid pumped inside the cylinder δV_1 (increase in the volume of cylinder) + δV_2 (decrease in the volume of liquid)
- Due to wire winding, initial compressive hoop stress is developed in cylinder. Pressure bearing capacity of cylinder is increased.
  - In double curved wall of a shell under pressure
  - $\frac{\sigma_1}{r_1} + \frac{\sigma_2}{r_2} = \frac{p}{t}$
- In a cylindrical shell, $\sigma_1 = \sigma_c, \sigma_2 = \sigma_a, r_2 = \infty$ (infinity)
- *In a conical water tank, stresses*

$$\sigma_1 = \frac{w\,(Hy - y^2)}{t} \times \frac{\tan\alpha}{\cos\alpha}$$

$$\sigma_2 = \times\frac{w\tan\alpha}{2t\cos\alpha}\left(Hy - \frac{2}{3}y^2\right)$$

Where α is semi-cone angle of tank, t is wall thickness, w is specific weight and H is depth of liquid.

? Review Questions

1. What is a thin cylindrical/spherical shell? On what D/t ratio it is classified as a thin shell?
2. Derive the expressions for hoop and axial stresses developed in a thin cylindrical shell subjected to internal pressure p.
3. Take a small element of a thin spherical shell and show the stresses acting on this element.
4. Derive the expression for the volumetric strain of a thin spherical shell subjected to internal pressure p.
5. A cylinder of diameter D and wall thickness t is wound with a single layer of wire under tension σ_w. Derive the expression for hoop stress developed in cylindrical shell.
6. Derive the following expression for a double curved surface under pressure p.

$$\frac{\sigma_1}{r_1} + \frac{\sigma_2}{r_2} = \frac{p}{t}$$

7. Derive the expression for maximum hoop stress developed in a conical water tank.
8. How the strain gauges mounted on an external surface of a thin shell subjected to an internal pressure can be used to measure E and v of the material of the shell.

Multiple Choice Questions

1. A thin cylindrical shell with $D/t = 30$ is subjected to an internal pressure of 3 N/mm^2. What is the hoop stress developed in shell?

   (a) 90 MPa (b) 45 MPa
   (c) 22.5 MPa (d) None of these

2. Athinsphericalshellofaninnerdiameterof400mm is subjected to an internal pressure of 2.5 N/mm^2. If the hoop stress is not to exceed 100 MPa, what is the thickness of shell?

   (a) 2.5 mm (b) 5 mm
   (c) 10 mm (d) None of these

3. A thin cylindrical shell is made of steel with $\nu = 0.3$. It is subjected to an internal pressure p. What is the ratio of hoop strain to axial strain?

   (a) 1.0 (b) 2.0
   (c) 3.0 (d) 4.25

4. A thin spherical shell is subjected to an internal pressure p. What is the ratio of volumetric strain to circumferential strain in shell?

   (a) 1.0 (b) 2.0
   (c) 3.0 (d) None of these

5. A cylindrical tank of an inside diameter of 1 m and a height of 20 m is filled with water of a specific weight of 10 kN/m^3. If the thickness of tank is 25 mm, then the maximum stress developed in wall of the tank is

   (a) 4 N/mm2 (b) 2 N/mm2
   (c) 1 N/mm2 (d) None of these

6. A thin cylindrical shell is made of steel with $\nu = 0.30$. It is subjected to an internal pressure. What is the ratio of volumetric strain to circumferential strain?

   (a) 2.0 (b) 2.235
   (c) 9.5 (d) None of these

7. A closed pressure vessel of a length of 400 mm, a wall thickness of 5 mm and an internal diameter of 100 mm is subjected to an internal pressure of 8 N/mm^2. The normal stress on an element of the cylinder on a plane 30° to the longitudinal axis will be

   (a) 140 MPa (b) 70 MPa
   (c) 77.32 MPa (d) None of these

8. A steam boiler of an internal diameter of 1.5 m is subjected to an internal pressure of 2 N/mm^2. If the efficiency of the longitudinal joint is 80 per cent and the maximum tensile stress in the plate section is not to exceed 125 MPa, what is the thickness of the plate?

   (a) 6 mm (b) 3 mm
   (c) 15 mm (d) None of these

Practice Problems

1. A steam boiler of an internal diameter of 1.5 m is subjected to an internal pressure of 1.2 N/mm^2. What is the tension per linear metre of the longitudinal joint in the boiler shell? Calculate the thickness of the plate if the maximum tensile stress in the plate section is not to exceed 90 N/mm^2, taking efficiency of longitudinal joint as 70 per cent.

2. A single strain gauge making an angle of 15° with the horizontal plane is used to determine the gauge pressure in a cylindrical tank with its axis vertical as shown in Fig. 5.17. The tank is 6 mm in thickness and 500 mm in diameter. It is made of steel with $E = 200$ GPa and $\nu = 0.29$. The strain gauge reading is 350 μ strain. Determine the pressure inside the tank.

$$\left[\text{Hint: } \varepsilon_\theta = \frac{\varepsilon_a + \varepsilon_c}{2} + \frac{\varepsilon_a - \varepsilon_c}{2}\cos 2\theta, \quad \theta = 105°\right]$$

3. The ends of a thin cylindrical shell are closed by flat plates. It is subjected to an internal fluid pressure of 3 N/mm^2, but the ends of the cylinder are rigidly stayed and no axial movement is permitted. The diameter

4. A thin spherical shell made of copper alloy is 300 mm in diameter and 1.5 mm in wall thickness. It is full of water at an atmospheric pressure. Find by how much the internal pressure will increase if 20 cc of water is pumped inside the shell. $E = 100$ GPa, $\nu = 0.29$
   For water, $K = 2{,}200$ N/mm^2
5. A pressurized steel cylinder tank has an inner radius of 600 mm and a thickness of 16 mm. The tank is subjected to a pressure $p = 1{,}750$ kPa and an axial force $P = 125$ kN. The butt weld seam forms an angle of 54° with the longitudinal axis of the tank. Determine (a) the normal stress perpendicular to weld and (b) the in-plane shear stress parallel to weld.
   $$\left[\text{Hint: } p = 1{,}750 \text{ kPa} = 1.75 \text{ MPa}, \sigma_c = \frac{pD}{2t}\right]$$
   $\sigma_a = \dfrac{pD}{4t}$, $\sigma_a' = \dfrac{p}{\pi D t}$, $\sigma_a - \sigma_a'$, net stress in axial direction
   $\theta = 54°$ with the plane of σ_c
6. A spherical steel vessel is made of two hemispherical portions fitted together at flanges. The inner diameter of the sphere is 600 mm and the wall thickness is 6 mm. Assuming that the vessel is a homogeneous sphere, what is the maximum working pressure for an allowable tensile stress in a shell of 150 MPa. If 20 bolts of a diameter of 16 mm are used to hold flanges together, what is the tensile stress in bolts when the sphere is under full pressure?
   $$\left[\text{Hint: } 150 = \frac{pD}{4t}, P_D = 20 \times \frac{\pi}{4} \times 16^2 \times \sigma\right]$$
7. For a hydraulic test, a steel tube of an internal diameter of 80 mm, a wall thickness of 2 mm and a length of 1.2 m is fitted with end plugs and filled with oil at a pressure of 2 MPa. Determine the volume of oil leakage which would cause the pressure to fall to 1.5 MPa.
   For oil, K for oil = 2.8 GN/m^2,
   For steel, $E = 208$ GPa, ν for steel = 0.29
   $$\left[\text{Hint: } \delta V = \delta V_1 + \delta V_2 = \varepsilon_v V + \frac{p}{K} V \text{ for } \Delta p\right]$$

Answers to Exercises

Exercise 5.1: 48.51 cc

Exercise 5.2: 3mm, $e_c = 0.3888 \times 10^{-3}$, $dD = 0.1166$ mm

Exercise 5.3: $t_2 = 2$ mm, 1.235

Exercise 5.4: $\sigma_{cr} = +37.864$ N/mm^2,
$\sigma_{wr} = 43.02$ N/mm^2

Exercise 5.5: 4.441 N/mm^2, at $y = \dfrac{H}{2}$;
3.331 N/mm^2 at $y = \dfrac{3H}{4}$

Answers to Multiple Choice Questions

1. (b)
2. (a)
3. (d)
4. (c)
5. (a)
6. (b)
7. (b)
8. (c)

Key Points to Remember

- In a thick shell, the wall thickness is significant and stresses vary along the thickness of the cylinder and cannot be assumed to be uniform.
- In a thick cylindrical shell subjected to internal pressure, Lame's equations can be used to determine radial and hoop stresses.

$$\sigma_r = \frac{B}{r} - A \text{ (compressive)}$$

$$\sigma_c = \frac{B}{r^2} + A \text{ (tensile)}$$

Constants A and B are determined by using boundary conditions.

- In a thick cylindrical shell with inner radius (R_1) and outer radius (R_2) subjected to internal pressure p

At inner radius, $$\sigma_{c\max} = p\frac{R_2^2 + R_1^2}{R_2^2 - R_1^2}$$

Axial stress, $$\sigma_a = \frac{pR_1^2}{R_2^2 - R_1^2} \text{ (constant tensile)}$$

- In a thick cylindrical shell with inner radius (R_1) and outer radius (R_2) subjected to external pressure p

At inner radius, $$\sigma_{c\max} = -p\frac{2R_2^2}{R_2^2 - R_1^2}$$

Constant axial stress, $$\sigma_a = p\frac{R_2^2}{R_2^2 - R_1^2} \text{ (compressive)}$$

- In a compound cylinder, the outer cylinder is shrink fitted over the inner cylinder. The junction pressure is developed which causes tensile hoop stress in the outer cylinder but compressive hoop stress in the inner cylinder.

R_1 = inner radius, R_2 = outer radius and R_3 = junction radius.

If both cylinders are made of same material, that is, same E

$$\delta R_3 = \text{shrinkage allowance} = \frac{p' \times R_3}{E}\left(\frac{R_2^2 + R_3^2}{R_2^2 - R_3^2} + \frac{R_3^2 + R_1^2}{R_3^2 - R_1^2}\right)$$

- In a hub and shaft assembly, junction pressure introduces tensile hoop stress in the hub but compressive hoop stress in the shaft.

In shaft, $\sigma_c = -p'$, $\sigma_r = p'$ (junction pressure)

- In a thick spherical shell subjected to internal pressure,

Hoop stress, $$\sigma_c = \frac{B}{r^3} + A \text{ (tensile)}$$

Radial stress, $$\sigma_r = \frac{2B}{r^3} - A \text{ (compressive)}$$

With the help of boundary conditions, the values of Lame's constants can be determined.

Review Questions

1. In a thick cylinder subjected to internal pressure, explain the assumption that plane sections remain plane after the application of internal pressure.
2. Show the variation of σ_r and σ_c in a thick cylinder along its thickness due to external pressure p.
3. In a compound cylinder, in which one cylinder is shrink fitted over another cylinder, show the variation of hoop stresses in outer and inner cylinders along the wall thickness.
4. Derive expressions of shrinkage allowance in a hub and shaft assembly.
5. Explain the purpose of compounding two cylinders.
6. In an assembly of bronze sleeve and steel shaft, if the temperature of the assembly is raised, at a particular temperature, the junction pressure becomes zero, why?

Multiple Choice Questions

1. In a thick cylindrical shell with $R_2 = 2R_1$ subjected to external pressure of 45 N/mm^2, what is the maximum hoop stress developed in the cylinder?
   (a) 120 N/mm^2 (b) 75 N/mm^2
   (c) 60 N/mm^2 (d) None of these
2. In a compound cylinder at junction, the sum of circumferential strains in outer and inner cylinders is 120×10^{-6}. If the junction's diameter is 200 mm, what is the shrinkage allowance?
   (a) 0.048 mm (b) 0.024 mm
   (c) 0.0024 mm (d) None of these
3. In a thick cylindrical shell, $\sigma_{c\,max} = 1.25p$, what is the ratio of R_2/R_1?
   (a) 1.5 (b) 2.0
   (c) 3.0 (d) None of these
4. In a shaft and hub assembly with shaft diameter 50 mm, hub diameter 100 mm and junction pressure 30 N/mm^2, what is the hoop stress in the shaft?
   (a) +50 N/mm^2 (b) +30 N/mm^2
   (c) −30 N/mm^2 (d) None of these
5. In a thick cylinder, $R_2/R_1 = 2$, the internal pressure is 60 N/mm^2. What is the maximum shear stress at inner radius?
   (a) 20 N/mm^2 (b) 30 N/mm^2
   (c) 80 N/mm^2 (d) None of these
6. In a thick cylindrical shell, $\sigma_{c\,max} = 120$, $p = 50$ N/mm^2, what is the value of Lame's constant A?
   (a) 30 MPa (b) 35 MPa
   (c) 70 MPa (d) None of these
7. In a compound cylinder, the hoop stresses developed at the junction in outer and inner cylinders are +84 MPa and −66 MPa. If E = 200 GPa, the junction radius is 100 mm, what is the shrinkage allowance as diameter?
   (a) 0.3 mm (b) 0.15 mm
   (c) 168 mm (d) None of these
8. The variation of hoop stress across the thickness of a thick cylinder is
   (a) Linear (b) Uniform
   (c) Parabolic (d) None of these
9. Purpose of compounding cylinder is
   (a) To increase pressure-bearing capacity of a single cylinder
   (b) To reduce the variation in hoop stress distribution
   (c) To increase the strength of the cylinder
   (d) All the above
10. A bronze sleeve of an outer diameter of 100 mm is forced over a solid steel shaft of a diameter of 60 mm. If the junction pressure is 32N/mm^2, the hoop strain at outer radius of sleeve is given by (if E = 100 GPa)
    (a) 320 mstrain (b) 360 mstrain
    (c) 720 mstrain (d) None of these

$$\left[\text{Hint: } \delta R_1 = p'R_1\left(\frac{2R_2^2}{R_2^2 - R_1^2}\right)\right]$$

4. A steel ring of internal radius r and external radius R is shrunk onto a solid steel shaft of radius $r + dr$. Prove that the intensity of pressure at the mating surface is equal to $\left(1 - \frac{r^2}{R^2}\right) E\frac{dr}{2r}$, where E is the modulus of elasticity of steel.
[Hint: use shrinkage formula for shaft + hub]
5. Two thick cylinders A and B are of same dimensions. The external diameter is double the internal diameter. Cylinder A is subjected to an internal pressure p_1, while cylinder B is subjected to an external pressure p_2. Find the ratio of pressure p_1 to p_2 if the greatest circumferential stress developed in both the cylinders is the same.
6. A steel cylinder of an internal diameter of 100 mm and an external diameter of 150 mm is strengthened by shrinking another cylinder onto it, the internal diameter of which before heating is 149.92 mm. Determine the outer diameter of the outer cylinder at the junction if the pressure at the junction after shrinking is 20 N/mm².

$$E = 210 \text{ GPa}.$$

[Hint: use expressions of shrinkage allowance are taking $R_1 = 50$ mm and $R_3 = 75$ mm]
7. A compound cylinder is made by shrinking a tube of an outer diameter of 150 mm over another tube of an inner diameter of 100 mm. Find the common diameter if the greatest hoop stress in the inner tube is numerically 0.7 times that of outer tube.

$$\left[\text{Hint: } p' \times \frac{2R_3^2}{R_3^2 - R_1^2} = 0.7 \times \frac{R_2^2 + R_3^2}{R_2^2 - R_3^2} \times p'\right]$$

8. A thick cylinder of an internal diameter of 120 mm and an external diameter of 180 mm is used for a working pressure of 15 N/mm². Because of external corrosion, the outer diameter of the cylinder is machined to 178 mm. Determine by how much the internal pressure is to be reduced so that the maximum hoop stress in the cylinder remains the same as before machining.

$$\left[\text{Hint: } R_2 = 90\text{mm}, R_2' = 89\text{mm } 15 \times \frac{R_2^2 + R_1^2}{R_2^2 - R_1^2} = p' \times \frac{R_2'^2 + R_1^2}{R_2'^2 - R_1^2}\right]$$

Answers to Exercises

Ecercise 6.1: 109.7, 56.9 μs

Exercise 6.2: $D = 169.7$ mm

Exercise 6.3: $p = 102.78$ N/mm²

Exercise 6.4: 20.485, −103.70, −83.22, +103.45, + 82.96 MPa.

Exercise 6.5: (a) 55.576 N/mm², (b) 144.5 N/mm²

Exercise 6.6: 7.46 MPa

Answers to Multiple Choice Questions

1. (a)
2. (b)
3. (c)
4. (c)
5. (c)
6. (b)
7. (b)
8. (c)
9. (d)
10. (b)

Key Points to Remember

Sign conventions

(a) On the left side of a section of a beam, upward force is positive shear force.

(b) On the left side of a section of a beam, downward force is a negative shear force.

(c) On the left side of a section, clockwise moment is a positive bending moment (tends to produce concavity upwards).

(d) On the left side of a section of a beam, anticlockwise moment is a negative bending moment (tends to produce convexity upwards).

(i) On a particular portion of a beam, with concentrated loads on both sides of the portion, shear force remains constant and bending moment changes linearly.

(ii) On a particular portion of a beam with udl, shear force changes linearly and bending moment curve is a parabolic curve.

(iii) Point of contraflexure in a beam is the point where bending moment changes sign.

(iv) If F is shear force, w is rate of loading and M is bending moment, then, in any portion of a beam,

$$\frac{dF}{dx} = -w, \quad \frac{dM}{dx} = F.$$

(v) In a beam, maximum bending occurs at a section where shear force changes sign or shear force is zero.

Review Questions

1. What do you understand by positive shear force and negative shear force?
2. What is the importance of drawing *SF* and *BM* diagrams?
3. What do you understand by bending moment producing concavity upwards and bending moment producing convexity upwards?
4. What is the point of contraflexure?
5. What is the relation between F and w (rate of loading) if F is shear force?
6. A beam is uniformly supported on a flat surface and concentrated loads are applied on the beam? What type of reaction is provided by the flat surface?
7. What is the purpose of making one support of beam as pin joint and the other end as roller support?
8. What is the purpose of fixing couple provided by the wall for fixed end of cantilever?

Multiple Choice Questions

1. A 5-m-long cantilever carries a load of 10 kN at free end and 10 kN at middle. What is the bending moment at the fixed end?

   (a) −25 kN m (b) −50 kN m

   (c) −75 kN m (d) None of these

2. A 10-m-long beam which is supported over 8 m span and having equal overhang on both the sides carries loads of 8 kN each at its ends and a load of 2 kN at its centre, the point of contraflexure lies at

   (a) the support (b) the section

   (c) 2 m from each end (d) None of these

3. An 8-m-long beam which is supported over a span of 6 m carries a point load of 20 kN at the centre of the span. What is M_{max} in beam?

   (a) 80 kN m (b) 60 kN m

   (c) 40 kN m (d) 30 kN m

4. In a beam, the point of contraflexure is a point where

(a) shear force is maximum
(b) shear force is zero
(c) bending moment changes sign
(d) bending moment is maximum

5. A beam carries transverse loads. *SF* and *BM* diagrams for the beam are drawn. In the portion of the beam, where SF remains zero, bending moment.
(a) maximum (b) minimum
(c) constant (d) none of these

6. A 10-m-long beam which is hinged at both the ends is subjected to a clockwise moment of 40 kN m at a distance of 3 m from one end. The *SF* at the centre of the beam is
(a) 6 kN (b) 2 kN
(c) 4 kN (d) None of these

7. A 10-m-long beam carries point loads. When *SF* diagram is drawn, there are two rectangles of 10 kN × 2 m side; one is starting from one end and above the base and the other stating from the other end and below the base line. The *BM* at the centre of the beam is
(a) 20 kN m (b) 30 kN m
(c) 40 kN m (d) None of these

8 A 10-m-long beam is supported over 6-m span with equal overhang on both the sides. It carries point loads of 40 kN each at its ends and a point load of 80 kN at the centre; if the points of contraflexure lie at a distance x from each end, the value of x is
(a) 4 m (b) 3 m
(c) 2 m (d) None of these

9 A 6-m-long cantilever carries a point load of 100 kN at the free end and another point load of *W* at the middle of its length. If the maximum *BM* in the cantilever is 900 kN m, the value of *W* is
(a) 50 kN (b) 100 kN
(c) 150 kN (d) 200 kN

10 An 8-m-long beam which is simply supported at its ends carries a point load of 800 N at distance of 3 m from one end. The B M under the load is
(a) 4 kN (b) 1.6 kN m
(c) 1.2 kN m (d) None of these

Practice Problems

1. A 6-m-long beam *AB* and an 8-m-long beam *BC* are hinged at end *B*. Loads on the beam are shown in Fig. 7.60. Draw *BM* diagram of beams *AB* and *BC*.

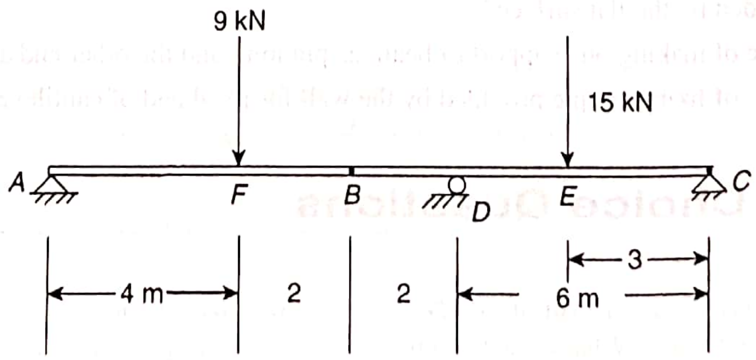

Figure 7.60

2. A 6-m-long beam *AB* which is hinged at *B* and roller supported at *C* as shown in Fig. 7.61 carries point loads of 4 kN at *A* and 8 kN at *D* and inclined load of 10 kN at *E*. Determine support reactions and draw *SF* and *BM* diagrams of the beam.

Answers to Multiple Choice Questions

1. (c)
2. (d)
3. (d)
4. (c)
5. (c)
6. (c)
7. (a)
8. (a)
9. (b)
10. (d)

Answers to Practice Problems

1. $M_A = 0$, $M_F = +12$ kN m, $M_B = 0$, $M_D = -12$ kN m, $M_E = +16.5$ kN m, $M_C = 0$
2. $R_c = 11.014$ kN, $R_{BV} = 8.056$ kN, $R_{BH} = 7.07$ kN, $F_{AC} = -4$ kN, $F_{CD} = 7.014$ kN, $F_{DE} = -0.986$ kN, $F_{EB} = -8.056$ kN, $M_A = 0$, $M_C = -4$ kN m, $M_D = 10.028$ kN m, $M_E = +8.056$ kN m
3. 2.5 kN m(cw), 11 kN ↑; $F_{AC} = 4$ kN, $F_{CB} = +4 - 5(x - 2)$ kN, $M_A = 0$, $M_C = +8$ kN, $M_4 = +6$ kN m, $M_5 = -2.5$ kN m
4. $R_A = 7.5$ kN ↑, $R_B = 7.5$ kN ↓, $M_A = M_B = 0$, $M_{1.5} = +5.625$ kN m, $M_{4.5} = -5.625$ kN m, the centre of the beam becomes point of contraflexure
5. $R_C = 6$ kN ↑, $R_D = 2$ kN ↑, $F_{AC} = -4$ kN, $F_{CD} = +2$ kN, $F_{DB} = -2$ kN, $M_A = 0$, $M_C = -4$ kN m, $M_E = 0$, $M_E' = -10$ kN m, $M_D = -4$ kN m, $M_B = 0$
6. $w = 0.75$ kN/m from O to A, $w = 1$ kN/m from A to B, point load 5 kN at D, $M_D = +4.125$ kN m
7. $w_2 = 12$ kN/m,
   $F_x(AC) = +w_2 x$ ($x - 0$ to 3 m)
   $F_x(CD) = +w_2 x - w_1$ ($x - 0.3$)
   ($x = 0.3$ to 0.7 m)
   $(AC)\ M_x = +w_2 \dfrac{x^2}{2} (x = 0 \text{ to } 0.3)$
   $(CB)\ M_x = +w_2 \dfrac{x^2}{2} - \dfrac{w_1}{2}(x - 0.3)^2$
   $M_{max} = 0.9$ kN m
8. $w = -0.25x^2 + 2x$; $R_A = R_B = 10.666$ kN. M_{max} at centre = 32.66 kN m
9. $R_A = 4.8\ 2$ kN, $w = 2.06$ kN/m, $M_c = +5.46$ kN m, $M_D = 1.28$ kN m

Answers to Special Problems

1. $R_c = R_E = 30$ kN; $F_{AC} = -10$ kN, $F_{CD} = +20$ kN, $M_c = -20$ kN; $M_D = +80$ kNm, Points of coutraflexure lie at 3 m from each end
2. $M_{max} = -12.5$ kN m at D
3. 3, 4.29 and 7 m from A, points of contraflexure
4. $F_x = \dfrac{kx^3}{3}, M_x = \dfrac{kx^4}{4}$
   $F_{max} = \dfrac{kL^3}{3}; M_{max} = \dfrac{kL^4}{4}$

Key Points to Remember

- Flexure formula $\frac{M}{I_{NA}} = \frac{E}{R} = \frac{\sigma}{y} = \frac{\sigma_c}{y_c} = \frac{\sigma_t}{y_t}$

  where M = bending moment at the section

  I_{NA} = moment of inertia of section about neutral axis

  E = Young's modulus of the material

  R = radius of curvature of beam at section

  σ = stress in a layer

  y = distance of layer from neutral axis

  σ_c = maximum stress in compression

  σ_t = maximum stress in tension

  y_c = distance of extreme layer in compression from neutral axis

  y_t = distance of extreme layer in tension from neutral axis

  $$M = \sigma_c Z_c = \sigma_t Z_t$$

  where, $$Z_c = \frac{I_{NA}}{y_c}, \; Z_t = \frac{I_{NA}}{y_t}$$

  where Z_c and Z_t are the section modulus in compression and in tension

- Modulus of rupture $= \frac{\sigma M_{ult}}{bd^2}$

- M_{ult} = ultimate bending moment a beam

  b, d = breadth and depth of a section, respectively

- In a beam of uniform strength ratio M/Z, that is, bending moment/section modulus is maintained constant

- In flitched beam

  $$M = M_1 + M_2$$

  resisting moment of beam of material (1) + resisting moment of beam of material (2)

  Equivalent section of a composite beam is made by considering modular ratio E_1/E_2

Review Questions

1. A section of a beam is subjected to bending moment, explain how stress distribution changes from negative to positive over depth of the section.
2. What is neutral layer? Why stress and strain are zero in the neutral layer?
3. Explain the formula $M = \sigma Z$, where σ is stress and Z is section modulus.
4. What is plane of bending? Why a section should be symmetrical about the plane of bending?
5. Take the case of channel section, explain symmetrical bending and unsymmetrical bending.
6. What is the most important assumption in a composite beam?
7. In simple bending, plane transverse sections remain plane after bending; explain this assumption with the help of simple sketch.

8. What is a beam of uniform strength, explain?
9. In RCC, why concrete section is assumed to carry zero tensile stress?
10. What do you understand by an equivalent section in a composite beam?

Multiple Choice Questions

1. A beam is of triangular section with base 10 mm and height 12 mm. what is its minimum section modulus?
   (a) 60 mm³ (b) 90 mm³
   (c) 120 mm³ (d) None of these
2. A beam of rectangular section is subjected to bending moment of 14.4 N m. If $b = 0.5d$, where b is breadth and d is depth and maximum stress developed is 100 MPa, what is the breadth of the section?
   (a) 18 mm (b) 12 mm
   (c) 6 mm (d) None of these
3. A steel strip of breadth 50 mm and depth 10 mm is bent around a drum of radius 10 m. If $E = 2 \times 10^5$ N/mm², what is the maximum stress developed in steel strip?
   (a) 200 MPa (b) 100 MPa
   (c) 50 MPa (d) None of these
4. T-section beam has a width of 40 mm and a depth of 100 mm. CG of section is located at a distance of 25 mm from outer edge of flange. Stress developed at upper edge of flange in 75 MPa, what is the stress developed at lower edge of web?
   (a) 150 MPa (b) 75 MPa
   (c) 25 MPa (d) None of these
5. A beam is of I-section with flanges 200 mm × 10 mm and web 180 mm × 10 mm. Due to the bending moment applied on the beam section, maximum stress developed in the beam section is 100 MPa, what is the stress developed at inner edge of the flange?
   (a) 110 MPa (b) 100 MPa
   (c) 90 MPa (d) None of these
6. An ms beam is subjected to a bending moment, such that a stress of 100 MPa is developed in a layer at a distance of 100 mm from the neutral layer. If $E = 200$ GPa, what is the radius of curvature of the beam?
   (a) 400 m (b) 200 m
   (c) 100 m (d) None of these
7. A beam of square section (with side of square horizontal and vertical) is subjected to a bending moment which produces 60 N/mm², maximum stress in the beam section. If the diagonals of the sections take vertical and horizontal directions, the bending moment remains the same, what is the maximum stress developed in the section?
   (a) 120 N/mm² (b) 90 N/mm²
   (c) 60 N/mm² (d) None of these
8. A cantilever of uniform strength σ, having rectangular section of constant breadth b but variable depth d is subjected to a udl throughout its length. If the depth of the section is 150 mm at the fixed and what is the depth at the middle of the length of cantilever?
   (a) 150 mm (b) 100 mm
   (c) 75 mm (d) None of these
9. A beam of rectangular section of breadth 100 mm and depth 200 mm is subjected to a bending moment of 20 kN m. Stress developed at a distance of 100 mm from top face is
   (a) 30 MPa (b) 15 MPa
   (c) 7.5 MPa (d) None of these
10. A cantilever of uniform strength σ having rectangular section of constant depth d but variable breadth b is subjected to a point load W at its free end. If the length of the cantilever is L, breadth of the cantilever at the middle of its length is
   (a) $\frac{WL}{\sigma d^2}$ (b) $\frac{3WL}{\sigma d^2}$
   (c) $\frac{2WL}{\sigma d^2}$ (d) None of these

Answers to Exercises

Exercise 8.1: ± 25 N/mm^2
Exercise 8.2: $b = 26.56$ mm, $d = 53.13$ mm
Exercise 8.3: 0.405 kN m
Exercise 8.4: $Z = 725.9 \times 10^3$ mm^3, $\sigma_{max} = 41.32$ N/mm^2
Exercise 8.5: -16.6 MPa, $+41.27$ MPa
Exercise 8.6: 0.845 kN m
Exercise 8.7: 41.484 kN m
Exercise 8.8: 25 N/mm^2
Exercise 8. 9: $1{,}444.6 \times 10^4$ mm^4
Exercise 8.10: 70.7 mm, 141.4 mm
Exercise 8.11: 52.85 kN m
Exercise 8.12: $\sigma_s = 17.93$ N/mm^2, $\sigma_w = 0.791$ N/mm^2
[Hint: maximum stress in steel at a, maximum stress in wood at b]
Exercise 8.13: 2,000 mm^2, 51.1 kNm
Exercise 8.14: $M = 711$ Nm

Answers to Multiple Choice Questions

1. (a)
2. (c)
3. (b)
4. (d)
5. (c)
6. (b)
7. (d)
8. (c)
9. (d)
10. (b)

Answers to Practice Problems

1. 8.32 N/mm^2
2. 167 N
3. 2,250 N
4. 270 mm, 9.72 kN
5. 384 mm
6. $b = 77.6$ mm
7. $y_1 = 108$ mm, $y_2 = 46$ mm, $I_{xx} = 534 \times 10^4$ mm^4
8. 28.548 kN m
9. 7.75 N/mm^2, 23.25 N/mm^2

Answers to Special Problems

1. 3.62 m, 6.02 kN
2. 2.657 kN/m, 95.38 N/mm^2
3. -4.32 N/mm^2, $+7.2$ N/mm^2 22.5 kN, 22.5 kN
4. 3.377 kN/m
5. 1.665 kN m
6. 4.586 N/mm^2

$$p_{cc} = \frac{\sigma_{cc}}{2} + \sqrt{\left(\frac{\sigma_{cc}}{2}\right)^2 + \tau_{cc}^2}$$

$$= \left(\frac{40.68}{2}\right) \pm \sqrt{\left(\frac{40.68}{2}\right)^2 + 46.8^2}$$

$$= 20.34 \pm \sqrt{413.716 + 2{,}190.24}$$

$$= 20.34 \pm 51.032$$

$$= 71.37 \text{ N/mm}^2, -30.69 \text{ N/mm}^2$$

Key Points to Remember

- Shear stress in any layer at a distance y from neutral layer

$$\tau = \frac{FA\bar{y}}{I_{NA}b}, \text{ where}$$

F = shear force at section

$A\bar{y}$ = first moment of area above the layer (or below the layer as the case may be) about neutral axis.

I_{NA} = second moment of area of the section about neutral axis.

- In a circular section, $\tau_{\max} = \frac{4}{3}\tau_{av}$
- In a rectangular section, $\tau_{\max} = 1.5\,\tau_{av}$
- In a thin circular section, $\tau_{\max} = 2\tau_{av}$, where τ_{av} is average shear stress
- In the case of I-section, most of the shear force F is shared by the web.
- In the case of I-section, most of the bending moment M is shared by the flanges.
- In the case of square, circular and rectangular, I-section maximum shear stress occurs at to the neutral layer.
- In the case of a square section with one diagonal becoming neutral layer, maximum shear stress is $\frac{9}{8}$ × average shear stress and occurs at a distance of $\frac{d}{8}$ from the neutral layer, while d is diagonal of the section.
- Near a free boundary, the shear stress on any section acts in a direction parallel to the boundary.

Review Questions

1. Explain how shear stress is developed in a section of a beam where there is change in bending moment?
2. Why the ratio of maximum shear stress/average shear stress is more in a thin circular section than in a solid circular section?
3. A rectangular section of a beam is subjected to a bending moment M and a shear force F. Why bending stresses are maximum at extreme layer while shear stress is zero at these layers?
4. In a triangular section of a beam, why shear stress due to shear force is not maximum at neutral axis?
5. In a circular section, why at the centre of section the direction of shear stress is perpendicular to boundary?

Multiple Choice Questions

1. In a rectangular section, τ_{av} = 50 MPa, what is shear stress at neutral axis?
   (a) 50 MPa (b) 75 MPa
   (c) 90 MPa (d) None of these
2. A thin circular tube is subjected to a transverse stress shear force F. If maximum shear stress developed in the section is 80 MPa, what is average shear stress in beam section?
   (a) 60 MPa (b) 50 MPa
   (c) 40 MPa (d) None of these
3. A I-section is subjected to transverse shear force. At which layer maximum shear stress is developed?
   (a) At neutral layer
   (b) At top edge of flange
   (c) At bottom edge of flange
   (d) None of these
4. A square section with side 'a' is used in a beam but the diagonals are placed in horizontal and vertical position. F is the shear force of a section, what is the maximum shear stress?
   (a) $1.5\ F/a^2$ (b) $1.25\ F/a^2$
   (c) $1.125\ F/a^2$ (d) None of these
5. A circular section of a beam with area 100 mm^2 is subjected to a transverse shear force of 750 N. Magnitude of maximum shear stress developed in the section is
   (a) 10 N/mm^2 (b) 8.75 N/mm^2
   (c) 7.5 N/mm^2 (d) None of these
6. A beam with a square section of 80 × 80 mm is simply supported at its ends. A load W is applied at the centre of the beam. If the maximum shear stress developed in beam section is 6 N/mm^2, what is the magnitude of W?
   (a) 2.56 kN
   (b) 25.6 kN
   (c) 51.2 kN
   (d) None of these
7. In a particular section of a beam, the maximum shear stress is double the average shear, what is the section of the beam?
   (a) Rectangular section
   (b) Solid circular section
   (c) Square section with one diagonal vertical
   (d) None of these
8. Match the section and ratio of τ_{max}/τ_{mean}

Section	Ratio
A Rectangular	I 1.33
B Circular	II 1.125
C Square with vertical diagonal	III 2.0
D Thin tubular section	IV 1.5

	A	B	C	D
(a)	IV	II	III	I
(b)	I	II	III	IV
(c)	IV	I	II	III
(d)	None of these			

Practice Problems

1. A 6-m-long beam of a rectangular section 40 mm wide × 60 mm deep is subjected to two loads of 2 kN each, at a distance of 2 m from each end. Calculate the principal stresses at a section under the load in layer 15 mm from the top.
2. A rolled steel section 60 mm × 40 mm is shown in Fig. 9.21. A transverse shear force of 50 kN is acting on this section. Determine shear stresses at points A, B and C.
3. A beam is made up by glueing four pieces of wood of size 50 mm × 80 mm to a 25 mm × 500 mm (deep) wooden beam. Allowable stress in the glued joint is 0.5 N/mm^2. Determine the maximum allowable shear force on the section (Fig. 9.22). [Hint: Maximum shear stress in glued joint will occur along aa]
4. Section of beam is circular of diameter D with a square hole of diagonal and $d' = 0.8D$ as shown in Fig. 9.23. Shear force on the section is F. Compare the magnitude of transverse shear stress at neutral axis NA with the average shear stress.

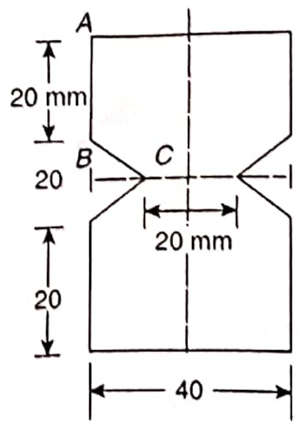

Figure 9.21

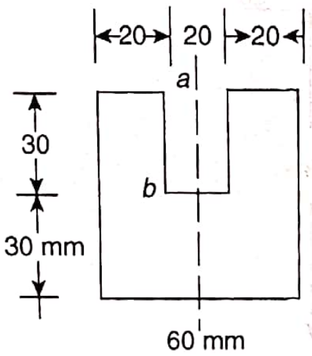

Figure 9.28 Special Problems 1

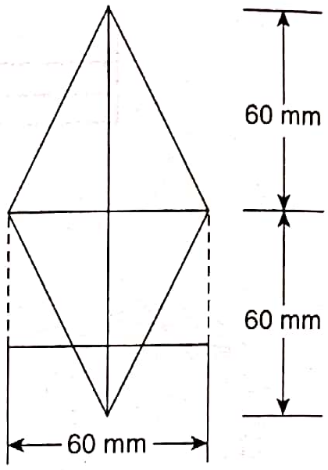

Figure 9.29 Special Problems 2

Answers to Exercises

Exercise 9.1: $F_{max} = 5$ kN

at $y = 0,\ \tau = 4.6875$ N/mm^2

$$y = \frac{a}{4},\ \tau = 3.515 \text{ N/mm}^2$$

$$y = \frac{a}{2},\ \tau = 0$$

Exercise 9.2: 33.64 mm

Answers to Multiple Choice Questions

1. (b)
2. (c)
3. (a)
4. (c)
5. (a)
6. (c)
7. (d)
8. (c)

Answers to Practice Problems

1. $\tau = 0.9375$ N/mm^2, $\sigma = 83.33$ N/mm^2, $p = 83.34$ N/mm^2
2. $\tau_A = 0, \tau_B = 27.9$ N/mm^2, $\tau_c = 61.63$ N/mm^2
3. $F = 5.8$ kN
4. $\tau_{NA} = 3.565\tau_{av}$
6. 93.37%
7. 48.6 kN
8. 88.9 mm spacing
9. 7.068 – 0.532, ±20.076, –35.64 N/mm^2

Answers to Special Problems

1. $\tau_a = 0$, $\tau_b = 6.185$ N/mm^2, $\tau_{NL} = 3.33$ N/mm^2, $I_{NA} = 87.3 \times 10^4$ mm^4
2. 0, 0.52, 0.74 and 0.66 N/mm^2
3. 1.67 MPa

Problem 10.9 A masonry pillar of diameter D in m is subjected to a horizontal wind pressure of intensity p kN/m^2. If the coefficient of wind resistance is k, prove that the maximum permissible height H of the pillar so that no tension is induced at the base is given by:

$$H = \frac{w\ D^2}{kp}, \text{ where } w = \text{weight density of masonry}$$

Solution

Weight density of masonry $= w$ kN/m^2
Say permissible height $= H$m
Direct stress due to self-weight, $\sigma_d = wH$ (compressive)
Intensity of wind pressure $= p$ kN/m^2

Bending moment, $$M = \frac{kp\,DH}{2} \times H = \frac{kp\,DH^2}{2}$$

Section modulus, $$Z = \frac{\pi D^3}{32}$$

Bending stress, $$\sigma_b = \pm\frac{M}{Z} = \pm\frac{16kPDH^2}{\pi D^3}$$
$$= \pm\frac{16kPH^2}{\pi D^2}$$

For no tension, $$\sigma_d > \sigma_b$$
$$wH > \frac{16\ kpH^2}{\pi D^2}$$

or, $$H < \frac{w\pi D^2}{16kp}.$$

Key Points to Remember

- A short column of rectangular section, $Z_x = \frac{BD^2}{6}$, $Z_y = \frac{DB^2}{6}$, carries eccentric load along x axis, with e_x as eccentricity
  Resultant stress, $\sigma_R = \frac{P}{BD} \pm \frac{Pe_x}{Z_y}$
- A short column of circular cross section of diameter D, supports an eccentric load P at an eccentricity e.
  Resultant stress, $\sigma_R = \frac{4P}{\pi D^2} \pm \frac{32Pe}{\pi D^3}$
- The core or the kernel of a section is a small area located around the centroid of the section of a column and if any vertical load is applied on the column within this area of core, there will not be any tensile stress developed anywhere in the section.
- Core of a rectangular section $B \times D$, is a rhombus of diagonals $B/3$ and $D/3$.
- Core or kernel of a circular section of diameter D is a circular area of diameter $D/4$.
- For a wall of rectangular section $B \times D$, wind pressure p acting on the face of breadth B and height H, stress due to bending moment created by the wind pressure is $\pm\frac{3pH^2}{D^2}$ at the base of the wall.

- For a chimney of outside diameter D, height H, coefficient of wind resistance k, wind pressure p, and inner diameter d.

Total wind force, $P_w = kpDH$

Moment, $M = \dfrac{P_w H}{2}$

Section modulus, $Z = \dfrac{\pi(D^4 - d^4)}{32D}$

Bending stress, $\sigma_b = \pm\dfrac{M}{Z}$

Direct compressive stress, $\sigma_d = \dfrac{4P}{\pi(D^2 - d^2)}$ (compressive)

Review Questions

1. What is the core or kernel of a section? What is its importance?
2. Mark the core of following sections.
   (a) A rectangular section (b) A circular section
   (c) A hollow circular section (d) I-section.
3. How the wind pressure on a wall produces an overturning moment on the wall?
4. What do you mean by coefficient of wind resistance for a chimney?
5. How the overturning moment due to wind load on a chimney is calculated?
6. Consider a C-clamp and show how the direct and bending stresses are developed in the critical section of the frame.
7. A cast iron column of hollow circular section is made by casting. What is the effect on stresses developed in column due to an axial compressive load if the core is offset during casting?

Multiple Choice Questions

1. A cast iron column is of circular section of a diameter of 200 mm. What is the diameter of core of the column?
   (a) 25 mm (b) 40 mm
   (c) 50 mm (d) None of these
2. A steel column has an outside diameter of 80 mm and an inside diameter of 60 mm. What is the diameter of core of the column?
   (a) 62 mm (b) 31.25 mm
   (c) 30 mm (d) None of these
3. A column with I-section has following properties: $Z_x = 90 \times 10^3$ mm^3, $Z_y = 54 \times 10^3$ mm^3, $A = 1{,}800$ mm^2. What is the allowable eccentricity e_y for no tension in column?
   (a) 30 mm (b) 40 mm
   (c) 50 mm (d) None of these
4. A short column is of hollow square section with outer side $2a$ and inner side a. A load acts at a distance of $0.25a$ from CG of the section, along one diagonal. The maximum and minimum stresses developed at corners of the section are 4.8 and -1.2 N/mm^2, respectively. The bending stress introduced at the extreme corners of the section by the eccentric load are
   (a) ± 3.6 N/mm^2 (b) ± 2.4 N/mm^2
   (c) ± 1.2 N/mm^2 (d) None of these
5. For a cylindrical chimney of hollow circular section, subjected to wind pressure, the coefficient of wind resistance is generally taken as
   (a) 0.3–0.5 (b) 0.45–0.55
   (c) 0.6–0.7 (d) None of these
6. A column is of hollow circular section. Due to an eccentric load, maximum and minimum stresses are

7 and 1 N/mm² (both compressive), respectively. Now, the eccentricity is doubled, what will be the maximum stress developed in column section?

(a) 14 N/mm² (b) 10 N/mm²

(c) 8 N/mm² (d) None of these

7. A short masonry square section of 1 m side is 10 m high. Wind pressure of intensity 2kN/m² acts on a vertical face of column. Weight density of masonry is 20 kN/m³ The maximum stress at the base of the column is

(a) 800 kN/m³ (b) 400 kN/m²

(c) 200 kN/m² (d) None of these

8. A chimney is of brick masonry of a weight density of 22 kN/m³. Height of chimney is 10 m. Outside diameter of chimney is 1 m while inside diameter is 0.5 m. What is the compressive stress developed at base, due to self-weight?

(a) 0.275 N/mm² (b) 0.22 N/mm²

(c) 0.20 N/mm² (d) None of these

9. A cast iron column of circular section carries an eccentric load due to which maximum and minimum stresses developed in column section are +420 kN/m² (compressive) and +120 kN/m² (compressive), respectively. If $Z = 1.2$ m³, what is the bending moment due to eccentric load?

(a) 270 kNm (b) 225 kNm

(c) 180 kNm (d) None of these

10. A cylindrical chimney of hollow circular cross section is subjected to wind pressure p. Density of masonry work is 20 kN/m³. The maximum and minimum stresses developed at the base of chimney are 650 and 150 kN/m², respectively. If intensity of wind pressure is increased by 50 per cent, what is the maximum stress developed in section of column?

(a) 850 kN/m² (b) 775 kN/m²

(c) 700 kN/m² (d) None of these

Practice Problems

1. A short column of hollow circular section of internal diameter d and external diameter D is subjected to a vertical load (eccentric). If $d = 0.8\,D$, determine the diameter of the core.
2. A short block has cross-sectional area of a triangle as shown in Fig. 10.23.
Determine the range along the axis yy over which the downward vertical force could be applied at the top of the block without causing any tension anywhere in the base. Neglect the weight of the block.
3. A flat plate of section of a thickness of 20 mm and a width of 60 mm, which is placed in a testing machine, is subjected to 60 kN of load acting along line AB as shown in Fig. 10.24, an extensometer adjusted along the line of the load recorded an extension of 0.078 mm on a gauge length of 150 mm. Determine

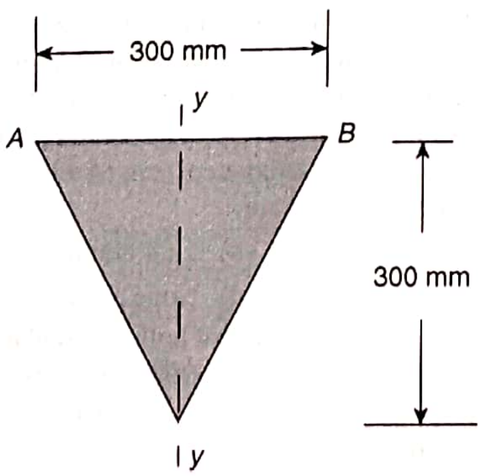

Figure 10.23 Practice Problem 2

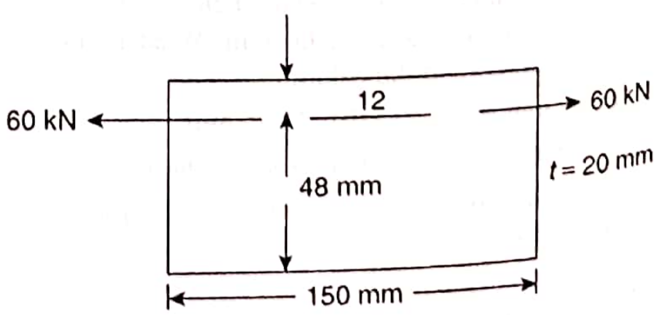

Figure 10.24 Practice Problem 3

Answers to Exercises

Exercise 10.1: 39.2 N/mm^2 (compressive), no tensile stress

Exercise 10.2: +4.063, 9.687, 2.187, −3.437 MPa (tensile)

Exercise 10.3: e_y < 60.87 mm, e_x < 7.83 mm, Rhombus with diagonals 15.66 mm and 121.74 mm

Exercise 10.4: σ_a = 196.1 kN/m^2 (compressive), σ_b = σ_d = 2546.6 kN/m^2, σ_c = 4897.1 kN/m^2 (compressive)

Exercise 10.5: (0.3875 kN/m^2)

Exercise 10.6: 607.26 kN/m^2 (compressive), 232.74 kN/m^2 (compressive)

Answers to Multiple Choice Questions

1. (c)
2. (b)
3. (c)
4. (a)
5. (c)
6. (b)
7. (a)
8. (b)
9. (c)
10. (b)

Answers to Practice Problems

1. (0.41 D)
2. yG = 100 mm, Gy = 200 mm, e_1 = 25 mm, e_2 = 50 m, range = 75 mm below AB to 150 mm below edge AB
3. −40 MPa, 140 MPa, 200 GPa
4. $\sigma_A = \sigma_B$ = +8.775 N/mm^2 (compressive), $\sigma_D = \sigma_C$ = 11.225 N/mm^2 (compressive)
5. 46.01, 26.78 MPa
6. 61.62 N/mm^2 at B, −31.36 N/mm^2 at A (tensile)
7. 6.2832 kN
8. 2,100 kNm; 1653.1 kN/m^2 (compressive), 29.1 kN/m^2 (compressive)
9. 4.05 m

Answers to Special Problems

1. Rhombus of diagonals 54.88 and 155.4 mm
2. 237.2 kN; 75 MPa (compressive), 17.4 MPa (compressive)
3. 48.50 mm
4. 27.952 N/mm^2

Key Points to Remember

- For any beam, relation between curvature $\frac{d^2y}{dx^2}$ and bending moment M is $EI\frac{d^2y}{dx^2} = M$
- If moment of inertia I is a variable, then $E\frac{d^2y}{dx^2} = \frac{M}{I}$
- For a beam, simply supported at ends, central load W, EI flexural rigidity and span length L

  Slope at supports $= \pm\frac{WL^2}{16EI}$, deflection at centre $= \frac{WL^3}{48EI}$
- For a beam, carrying a udl of w

  Slope at supports $= \pm\frac{wL^3}{24EI}$, deflection at centre $= \frac{5}{384}\times\frac{wL^4}{48EI}$
- For a cantilever with a point load at free end

  Slope at free end $= \frac{WL^2}{2EI}$, deflection at free end $= \frac{WL^3}{3EI}$
- For a cantilever carrying a udl of intensity w throughout its length

  Slope at free end $= \frac{WL^3}{6EI}$, deflection at free end $= \frac{WL^4}{8EI}$
- For a beam with a point load at a distance of a from one end and b from the other, hence, $a + b = L$, and point load is W

  Slope at ends $= -\frac{Wab(a+2b)}{6EIL}; +\frac{Wab(2a+b)}{6EIL}$

  Deflection under load $= -\frac{Wa^2b^2}{3EIL}$
- If a load W is allowed to fall through a height h onto a beam or a cantilever at a particular point, δ_i is the maximum instantaneous deflection produced under load, then

$$W(h+\delta_i) = \frac{1}{2}P\delta_i$$

  where P is the equivalent static load which when applied gradually produces deflection δ_i.
- In Macaulay's method, choose one end of the beam as origin and a section in the last portion of the beam, and make the equation of bending moment.
- If a moment M is applied at any section of the beam, while making bending moment equation, consider M and fix its position by taking distance of the point of application of moment on the section under consideration from this origin and write the distance with zero power multiplied by moment M.
- In area moment technique, two sections are considered at a distance of x_1 and x_2 from one end, $EI[i_2 - i_1]$ = area of Bending moment diagram between sections Y_2 and Y_1 at a distance of x_1 and x_2 from the origin.
- $EI[(x_2i_2 - y_2) - (x_1i_1 - y_1)] = a\bar{x}$

  where a is the area of BM diagram between x_1 and x_2

  $\bar{x}$ is the distance of CG of this area a, for the origin.

Review Questions

1. Derive the expression, $\frac{1}{R} = \frac{dy^2/dx^2}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{1.5}}$
2. While using Macaulay's method, explain how the location of a moment is specified in bending moment equation.
3. What is a conjugate beam? How reactions at ends give the slope at ends of the beam?
4. In moment area method, explain how the deflection between two sections is related with the first moment about origin of area of BM diagram between two sections.
5. Explain how falling load produces instantaneous deflection in beam and how instantaneous deflection is related to the stiffness of the beam.
6. In a composite beam, how flexural rigidity is determined by taking into account the flexural rigidity of two components of the beam.
7. In a beam with variable moment of inertia, explain how slope and deflection are determined?

Multiple Choice Questions

1. A simply supported beam of span length L, carries central load W, flexural rigidity EI, what is the slope at ends?
   (a) $\pm\frac{WL^2}{2EI}$ (b) $\pm\frac{WL^2}{12EI}$
   (c) $\pm\frac{WL^2}{16EI}$ (d) None of these
2. A simply supported beam of span length 6 m carries 6 kN central load, what is the deflection at centre if $EI = 2{,}700$ kNm^2?
   (a) 20 mm (b) 10 mm
   (c) 1 mm (d) None of these
3. A beam simply supported at ends over a span of 4 m, carries a udl of 15 kN/m throughout its length. If $EI = 25{,}000$ kN m^2, then the maximum deflection in beam is
   (a) 2 mm (b) 20 mm
   (c) 0.2 mm (d) None of these
4. A beam of length L, simply supported at its ends and carries a udl of w throughout its length. The centre of the beam is propped so that centre is brought to the level of ends. The reaction at the prop is
   (a) 0.33 wL (b) 0.5 wL
   (c) 0.675 wL (d) None of these
5. A cantilever of length L carries a load W at its middle. The slope at the middle of cantilever is q, what is the slope at free end of cantilever?
   (a) 2q (b) 1.6q
   (c) 1.2q (d) None of these
6. A cantilever of length 2 m carries a load of 2 kN at the middle of its length, If $EI = 100$ kN m^2 of the cantilever, what is the deflection at free end of cantilever?
   (a) 16.667 mm (b) 6.067 mm
   (c) 5.000 mm (d) None of these
7. A cantilever of length 4 m carries a udl of 2 kN/m throughout its length. The free end of the cantilever is propped such that the level of the free end is the same as that of the fixed one. Reaction from prop is
   (a) 8 kN (b) 6 kN
   (c) 3 kN (d) None of these
8. A beam of length 6 m carries a concentrated load W at its centre, such that BM at its centre of the beam is 6 kN m, if EI is the flexural rigidity of the beam, then deflection at the centre is
   (a) $\frac{9}{EI}$ (b) $\frac{18}{EI}$
   (c) $\frac{36}{EI}$ (d) None of these

9. A beam AB of 10 m long is supported over a span of 8 m with equal overhang on both the sides. A load W is applied at the centre of the beam. What is the slope at free ends of the beam?

   (a) $\pm 6.25\frac{W}{EI}$ (b) $\pm\frac{5W}{EI}$

   (c) $\pm\frac{4W}{EI}$ (d) None of these

10. A beam AB is 6 m long, simply supported at ends carries a load W at 2 m from A. What is the deflection made by the load?

   (a) $\frac{4.5W}{EI}$ (b) $\frac{32W}{9EI}$

   (c) $\frac{3W}{EI}$ (d) None of these

Practice Problems

1. A uniform beam of length 12 m is supported symmetrically over a span of 8 m. It is subjected to a uniformly distributed load of 4 kN/m over the supported length of 8 m as shown in Fig. 11.49. Determine the ratio of deflection at free-end and at the centre of the beam.
2. A propped cantilever of length L is fixed at one end and roller supported at the other end. Cantilever is subjected to a couple M as shown in Fig. 11.50. Determine the reaction at prop and deflection at C, if EI is the flexural rigidity of cantilever.

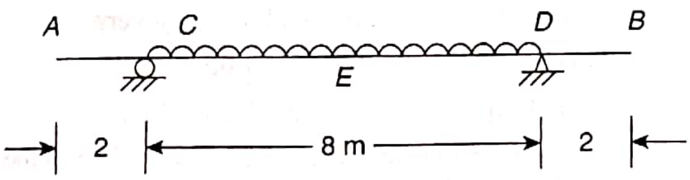

Figure 11.49

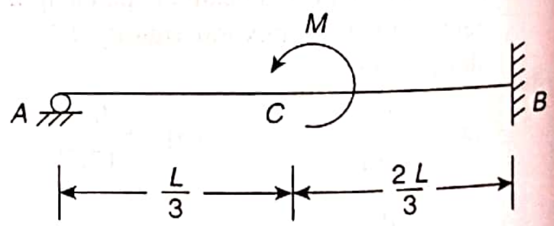

Figure 11.50

3. A long flat strip of 40 mm wide and 2.5 mm thick is lying on a flat horizontal plane. One end of the strip is lifted by 20 mm from the plane by applying a vertical force at the end. The strip is so long that at the other end remains undisturbed. Calculate: (a) the force required to lift the end and (b) maximum stress in the strip (Fig. 11.51). Weight density of steel = 76.44×10^{-6} N/mm^3.

$$E = 2 \times 10^5 \text{ N/mm}^2$$

Hint $\left[\frac{WL^2}{2} = PL\right]$

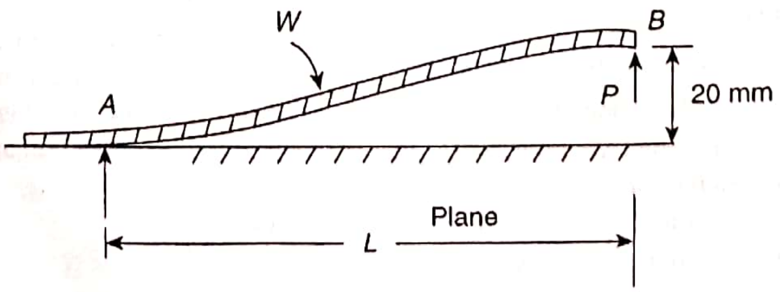

Figure 11.51

Solution

Length of splines, $L = 60$ mm
Outer diameter of splines = 60 mm
Inner diameter of splines = 48 mm

$$\frac{D-d}{2} = \frac{60-48}{2} = 6 \text{ mm}$$

Width of each spline, $w = 12$ mm (as shown in Fig. 12.28)
Area under shear = $w.L = 12 \times 60 = 720$ mm^2
Side pressure = 10 N/mm^2

$$\text{Compressive force per spline} = \left(\frac{D-d}{2}\right) \times L \times 10$$

$$= 6 \times 60 \times 10 = 3{,}600 \text{ N}$$

$$P = 3.6 \text{ kN}$$

$$\text{Angular speed, } \omega = \frac{2\pi \times 300}{600} = 31.416 \text{ rad/s}$$

Number of splines = 6

$$\text{Mean radius of spline,} \quad R_m = \frac{48+60}{2 \times 2} = 27 \text{ mm}$$

Torque transmitted = nPR_m

$$= 6 \times 3{,}600 \times 27$$

$$= 583.2 \text{ Nm}$$

$$\text{Power,} \quad P = \omega.T = 31.416 \times 589.2$$

$$= 18{,}322 \text{ Nm/s}$$

$$= 18.322 \text{ kW}$$

$$\text{Shear stress in splines, } \tau = \frac{P}{\text{area under shear}} = \frac{3{,}600}{720} = 5 \text{ N/mm}^2.$$

Key Points to Remember

- Torsion formula, $\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{R} = \frac{\tau_r}{r}$

  where

  T = torque on shaft
  J = polar moment of inertia of shaft section
  G = shear modulus
  θ = angular twist over length L

τ = maximum shear stress at radius R

τ_r = shear stress at any radius r.

- Modulus of rupture, $\tau' = \dfrac{10T_{max}}{D^3}$, for a solid shaft
- HP transmitted by a shaft = $\dfrac{2\pi NT}{746 \times 60}$

  where

  N = speed in rpm

  T = Torque in Nm
- If several shaft are in series, then same twisting moment acts on all the shafts, but angular twist will be different and total angular twist will be the sum of all the angular twists.
- It two shafts are in parallel, then angular twist in both shafts will be the same. Total torque T will be shared by the two, that is, $T_1 + T_2 = T$.
- Strain energy per unit volume = $\tau^2/4G$, where τ is the maximum shear stress (for a solid shaft).

  For hollow shaft shear strain energy per unit volume = $\tau^2/4G\,(D_2^2 + D_1^2/D_2^2)$, where D_2 and D_1 are outer and inner diameters of the shaft.
- If a key of breadth b, thickness t and length L connects a shaft and hub for power transmission, shear stress in key = $2T/DbL$
- Bearing stress in key = $4T/DtL$, where D is shaft diameter and T is torque transmitted by shaft.

Review Questions

1. A shaft is subjected to a twisting moment, show how shear strain varies along the radius of the shaft.
2. A hollow shaft of outer radius R_2 and inner radius R_1 is subjected to a twisting moment T. Derive expression for maximum shear stress in shaft.
3. A solid shaft is subjected to a twisting moment T, such that maximum shear stress developed on the surface of the shaft is τ, if G is shear modulus, prove that strain energy per unit is $\tau^2/4G$.
4. Take a hollow shaft subjected to twisting moment and with the help of a sketch show the variation of shear stress along radial thickness of shaft.
5. What are equivalent twisting moment and equivalent bending moment in a shaft? How these are obtained?
6. Make a simple sketch of a shaft subjected to twisting moment. Take a small element on the surface of the shaft and mark directions of principal stresses.
7. Show that volumetric strain for a shaft subjected to pure torsion is 0.

Multiple Choice Questions

1. A shaft of 20 mm diameter and length 1,000 mm is subjected to twisting moment such that $\theta = 0.1$ rad. What is the shear strain in the shaft at outer surface?

   (a) 0.001 rad (b) 0.0001 rad
   (c) 0.0005 rad (d) None of these
2. A hollow shaft of inner radius 30 mm and outer radius 50 mm is subjected to a twisting moment. If the shear stress developed at inner radius of shaft is 60 N/mm². What is the maximum shear stress in shaft?

   (a) 60 N/mm² (b) 75 N/mm²
   (c) 100 N/mm² (d) None of these
3. Torsional rigidity of a shaft is given by

   (a) T/G (b) T/J
   (c) JG (d) None of these

   where T is torque, G is shear modulus and J is polar moment of inertia.

4. A solid steel shaft is surrounded by a copper shaft, such that, $J_{steel} = \frac{1}{2} J_{copper}$. If $G_{copper} = \frac{1}{2} G_{steel}$, what is the ratio of T_S/T_C if compound shaft is subjected to a twisting moment?
   (a) 2 (b) 1
   (c) 0.5 (d) None of these
5. A shaft of diameter d and length L is subjected to twisting moment T, shear angle developed in shaft is 0.001 rad. Now, the length of the shaft is doubled, but the diameter and torque remain the same. What will be the shear angle?
   (a) 0.0005 rad (b) 0.001 rad
   (c) 0.002 rad (d) None of these
6. A shaft is transmitting 2 HP at 100 rpm, maximum shear stress in shaft is 50 MPa. Now the shear stress remains the same, how much HP will be transmitted at 200 rpm?
   (a) 1 HP (b) 2 HP
   (c) 3 HP (d) None of these
7. What is the shear modulus of steel?
   (a) 40–42 GPa (b) 60–63 GPa
   (c) 70–72 GPa (d) None of these
8. A solid shaft of diameter d is subjected to twisting moment T, such that maximum shear stress developed is τ. Now a hole of $0.5d$ diameter is drilled throughout the length of the shaft along the axis. How much torque T must be reduced so that shear stress remain the same?
   (a) $T/2$ (b) $T/4$
   (c) $T/8$ (d) $T/16$
9. A solid circular steel shaft is subjected to twisting moment T, such that maximum shear stress developed on shaft is 200 MPa. If $E = 200$ GPa and $\nu = 0.3$, what is the maximum principal strain developed on the shaft?
   (a) Zero (b) 1,000 micro strain
   (c) 1,300 micro strain (d) None of these
10. A shaft of 100 mm diameter is keyed to a pulley transmitting power. The dimension of key are $b = 20$ mm, $t = 20$ mm and $L = 100$ mm. Twisting moment of shaft is 1,000 Nm. What is the shear stress developed in key?
   (a) 10 N/mm^2 (b) 20 N/mm^2
   (c) 40 N/mm^2 (d) None of these

Practice Problems

1. A torsion bar 1.2 m long is to be designed. Shear modulus of bar is 82 kN/mm^2. Determine the required diameter of the shaft so that the resulting torsional spring constant (or torsional stiffness) of the bar is 40 Nm for 1° of angular twist.
2. A hollow circular steel shaft is required to transmit 200 HP at 360 rpm. The maximum torque developed is 1.3 times the mean torque. Determine the external diameter of the shaft if it is 1.6 times the internal diameter and the maximum shear stress in shaft is not to exceed 75 MPa. Given $G = 82$ kN/mm^2.
3. A vessel having a single propeller shaft 250 mm in diameter running at 200 rpm is re-engined to two propeller shafts of equal cross-section and producing 50 per cent more horse-power at 500 rpm. If the working shear stress in these shafts is 20 per cent more than that in the single shaft, determine diameter of these shafts.
4. A solid steel shaft of 50 mm diameter is made of a low carbon steel which is assumed to be elasto-plastic with $T_y = 150$ MPa, and $G = 84 \times 10^3$ N/mm^2. Determine the maximum shear stress due to an applied torque of (a) 2 kN m and (b) and 4 kN m.
5. A solid marine propeller shaft is transmitting power at 1,000 rpm. The vessel is being propelled at a speed of 20 km/h for the expenditure of 5,000 HP. If the efficiency of propeller is 70 per cent, and the greatest thrust is not to exceed 60 MPa, calculate the shaft diameter and maximum shearing stress developed in the shaft.

Answers to Multiple Choice Questions

1. (a)
2. (c)
3. (c)
4. (b)
5. (b)
6. (d)
7. (d)
8. (d)
9. (c)
10. (a)

Answers to Practice Problems

1. 23.1 mm
2. $D = 74.45$ mm
3. 157.5 mm
4. (a) 81.5 N/mm^2, and (b) $\tau' = 162.97$ N/mm^2 but shear stress is 150 MPa
5. 99.8 mm, 182.5 N/mm^2
6. 56.6 mm, 64.25 mm, 43.6 per cent
7. Internal diameter = 46.95 mm, N=750 rpm
8. 384.53 Nm
9. $\tau_{max} = 58.76$ in portion AB
10. 34.83 MPa, 0.0276 rad, 1.58°
11. $T_{AB} = 3.857$ kNm, $T_{BC} = 0.857$ kNm, $T_{CD} = -5.143$ kNm
12. 1.105 kN
13. 33.8 kN
14. $n = 6$

Answers to Special Problems

1. Internal diameter = 88.8 mm
2. 17.1 kNm, 41.87 MPa, 62.8 MPa
3. $T_0 = 9.887$ kNm, 116 N/mm^2, 45.35 N/mm^2
4. 43.14 mm
5. 280.5 mm
6. 2,262 Nm
7. 14.75 N/mm^2
8. $d = 27.76$ mm, $T_{AB} = 210$ Nm, $T_{BC} = 210 - 100 = 110$ Nm, $T_{CD} = 110$ Nm

$$19k_1 + 14 \times 4.32 = 138 \text{ N}$$

$$k_1 = \frac{138 - 14 \times 4.32}{19} = 4.08 \text{ N/mm}$$

$$k_1 = \frac{Gd_1^4}{8D_1^3 n_1} = \frac{80{,}000 \times d_1^4}{8 + 17^3 \times 10} = 4.08$$

$$d_1^4 = \frac{4.08 \times 80 \times 17^3}{80{,}000} = 20.045$$

$d_1 = 2.116$ mm (wire diameter of the inner spring).

Key Points to Remember

- For a close-coiled helical spring, W = axial load, R = mean radius, C = spring index and D/d = ratio of mean coil diameter and wire diameter.

$$\tau_{\max} = \frac{16WR}{\pi d^3}\left(\frac{4C-1}{4C-4} + \frac{0.615}{C}\right)$$

$$= \frac{16WR}{\pi d^3} \times \text{Wahl's factor}$$

- Stiffness of a close-coiled helical spring,

  $k = \dfrac{W}{\delta} = \dfrac{Gd^4}{64nR^3}$; G = shear modulus, n = number of coils, δ = axial change in length.

- In a close-coiled helical spring, angular rotation due to axial moment M,

$$\phi = \frac{128nRM}{Ed^4}$$

  where E = Young's modulus.

- For a open-coiled helical spring, W = axial load, R = mean coil radius, Twisting moment, $T' = WR\cos\alpha$ and Bending moments, $M' = WR\sin\alpha$, where α = helix angle.

- Axial deflection, $\delta = 2\pi nR^3 \sec\alpha\left(\dfrac{\cos^2\alpha}{GJ} + \dfrac{\sin^2\alpha}{EI}\right)$

  where $J = 2I$ = Polar moment of inertia = $\pi d^4/32$

  Angular rotation, $\phi = 2\pi nR^2 W \sin\alpha\left(\dfrac{1}{GJ} - \dfrac{1}{EI}\right)$

- Open-coiled helical string, subjected to an axial moment M,

  Twisting moment = $M\sin\alpha$, Bending moment = $M\cos\alpha$

  Angular rotation, $\phi = 2\pi nR \sec\alpha M\left(\dfrac{\cos^2\alpha}{EI} + \dfrac{\sin^2\alpha}{GJ}\right)$

  Axial deflection, $\delta = 2\pi nMR^2 \sin\alpha\left(\dfrac{1}{EI} - \dfrac{1}{GJ}\right)$

- Plane spiral spring, b = breadth, t = thickness, M = moment

  Maximum stress, $\sigma_{max} = 12M/bt^2$

  Energy stored $= \dfrac{M^2L}{2EI} = \dfrac{\sigma_{max}^{\ \ 2}}{24E} \times$ volume of trip

- Carriage spring, n = number of leaves, b = breadth, t = thickness, L = Length of longest leaf; R = radius of curvature.

  Initial central deflection, $y_0 = L^2/8R$

  Proof load, $W_0 = Enbt^3/3LR$

  $\sigma_{max} = 3WL/2nbt^2$.

Review Questions

1. What is the effect of spring index on the stresses developed in spring wire?
2. What is Wahl's factor and how it accounts for the curvature effect on stress in spring wire?
3. Show by a sketch, how resultant shear stress at inner coil radius is maximum.
4. Derive relation $\phi = ML/MI$, for a close-coiled helical spring subjected to axial moment M.
5. What are the twisting moment and bending moment components on an open-coiled helical spring subjected to axial load W?
6. What are the applications of a plane spiral spring? Make a simple sketch and mark the point where bending stress is maximum.
7. Where the conical springs are used? Derive the expression for maximum shear stress in a wire of conical spring.
8. In a leaf spring, explain the assumptions that a leaf touches the adjoining leaf only at ends.
9. What is proof load in a cantilever spring?
10. What is meant by resilience of a spring?

Multiple Choice Questions

1. A close-coiled spring absorbs 50 N mm energy is extending by 5 mm, what is the stiffness of spring?
   (a) 10 N/mm (b) 5 N/mm
   (c) 2 N/mm (d) None of these
2. A carriage spring is with longest leaf 800 mm long and radius of curvature is 2,000 mm. What is the central deflection?
   (a) 80 mm (b) 40 mm
   (c) 20 mm (d) None of these
3. A carriage spring subjected to a central load such that leaves become straight. What is this load called?
   (a) Safe load (b) Proof load
   (c) Ultimate load (d) None of these
4. A close-coiled helical spring of stiffness 4 N/mm is in series with another spring of stiffness 6 N/mm. What is the stiffness of composite spring?
   (a) 5 N/mm (b) 4 N/mm
   (c) 2.4 N/mm (d) None of these
5. An open-coiled helical spring with $\alpha = 45°$ is subjected to an axial load W such that shear stress due to twisting moment is 100 MPa. What is the bending stress due to bending moment?
   (a) 50 MPa (b) 100 MPa
   (c) 200 MPa (d) None of these
6. A flat spiral spring is subjected to winding couple producing σ_{m} at the point of greatest bending. What is the strain energy per unit volume?
   (a) $\dfrac{\sigma^2}{2E}$ (b) $\dfrac{\sigma^2}{12E}$

(c) $\frac{\sigma^2}{24E}$ (d) None of these

7. A closed-coil helical spring of wire, diameter 6 mm, is made by taking mean coil radius as 16 mm, what is its spring index?
(a) 2.66 (b) 5.33
(c) 8 (d) None of these

8. Stiffness of a close-coiled helical string of wire diameter d, modulus of rigidity G, number of coils n, mean coil radius R, is
(a) $\frac{Gd^4}{64nR^3}$ (b) $\frac{Gd^4}{16nR^3}$
(c) $\frac{Gd^4}{8nR^3}$ (d) None of these

9. A close-coiled helical spring is subjected to an axial moment M, producing an angle of rotation 90° at free end with respect to fixed end, the strain energy absorbed is 100π N mm, what is M?
(a) 100 Nm (b) 200 N mm
(c) 300 N mm (d) None of these

10. Wahl's factor takes into account
(a) Curvature of the helical wire
(b) Direct shear stress
(c) Both curvature and direct shear effect
(d) Neither (a) nor (b)

Practice Problems

1. Design a close-coiled helical spring to have a mean coil diameter of 120 mm and an axial deflection of 150 mm under an axial load of 4,050 N, so that the maximum shear stress developed in the spring is not to exceed 320 N/mm^2. Steel wires are available in the following diameters: 10, 12 and 16 mm. Determine the most suitable diameter of the wire and number of coils required. Also calculate the maximum shear stress developed in designed spring. Given, G = 84,000 N/mm^2.
2. A close-coiled helical spring is made of round steel wire. It carries an axial load of 150 N and is just to get over a rod of 36 mm. The deflection in the spring is not to exceed 25 mm. The maximum allowable shear stress developed in spring wire is 200 N/mm^2 (neglecting the effect of direct shear stress). G for steel = 80,000 N/mm^2. Find the mean coil diameter, wire diameter and number of turns.
3. A close-coiled helical spring is made of round wire having n turns and mean coil radius is five times the wire diameter. Show that stiffness of wire spring is $(R/n) \times$ constant. Determine constant, if G = 82 kN/mm^2. Such a spring is required to support a load of 1 kN with 100 mm compression and maximum shear stress is 245 N/mm^2, determine (a) mean coil radius and (b) number of turns.
4. A safety valve 80 mm diameter is to blow off at a pressure of 2 N/mm^2 gauge. The safety valve is held by a close-coiled helical spring of steel with a mean coil radius equal to 75 mm. Determine the diameter of the steel wire and the number of turns necessary, if the maximum shear stress in the wire is not to exceed 200 MPa and the spring is initially compressed by 25 mm. Given, G = 84 kN/mm^2. Take into account the effect of direct shear stress also.
5. A weight of 250 N is dropped onto a close-coiled helical spring through a height of 800 mm, which produces a maximum instantaneous stress of 200 N/mm^2 in the spring. If the mean radius of the coil is five times the wire diameter, determine (a) instantaneous compression in the spring and (b) number of coils in the spring. Given, wire diameter, d = 20 mm and G = 8,410 N/mm^2.
6. A laminated carriage spring made of 12 steel plates is 1 m long. The maximum central load is 6 kN. If the maximum allowable stress in steel is 200 MN/m^2 and the maximum deflection is 40 mm, determine the thickness and width of plates. Given, E = 200 GPa.
7. A rigid bar AB, weighing 100 N carries a load W = 300 N as shown in Fig. 13.19. The bar rests on three springs of stiffnesses: k_1 = 20 N/mm, k_2 = 8 N/mm and k_3 = 10 N/mm as shown in Fig. 13.19. If the unloaded springs are of the same length, determine the value of distance x such that the bar remains horizontal.
[Hint: Reaction $k_1\delta$, k_2d, $k_3\delta$]

Answers to Multiple Choice Questions

1. (d)
2. (b)
3. (b)
4. (c)
5. (c)
6. (c)
7. (b)
8. (a)
9. (d)
10. (c)

Answers to Practice Problems

1. 16 mm, 14.75 turns, 302.15 N/mm^2
2. 40.14 mm, 4.248 mm, 8.4 turns
3. constant 2.05, R = 51 mm, n = 10.46
4. d = 27.6 mm, n = 4.5
5. 151.4 mm; n = 10.12
6. t = 6.25 mm, b = 96 mm
7. x = 122.8 mm
8. x = 1.65 m, W_1 = −750 N, W_2 = 2,750 N
9. 19.08 mm, 36.7 turns

Answers to Special Problems

1. 0.2612
2. 1.025 N/mm
3. 592 N, 8.92 turns
4. W_1 = 351.63 N, W_2 = 148.37 N
   134.3 N/mm^2, 75.56 N/mm^2
5. t = 6.66 mm, n = 10, b = 85.54 mm
6. 6.56 mm
7. 6 springs

$$\sigma_0 = \frac{P}{A} = \frac{2{,}00{,}000}{1.31\times10^4} = 15.267 \text{ N/mm}^2$$

$$Z = \frac{I}{y} = \frac{0.4637\times10^8}{100} = 0.4637\times10^6 \text{ mm}^3$$

$$\sigma_b = \pm\frac{Pe}{Z}\sec\sqrt{\frac{P}{EI}}\times\frac{L_e}{2}$$

$$= \pm\frac{2{,}00{,}000\times30}{0.4637\times10^6}\times1.0323 = 13.357 \text{ N/mm}^2$$

$$\sigma_{max} = 15.267 + 13.357 = 28.624 \text{ N/mm}^2 \text{ (compressive)}$$

$$\sigma_{min} = 15.267 - 13.357$$

$$= 1.91 \text{ N/mm}^2 \text{ (compressive)}$$

For no tension

$$\sigma_0 = \sigma_b$$

$$15.267 = \frac{Pe'}{Z}\sec\sqrt{\frac{P}{EI}}\frac{L_e}{2}$$

$$= \frac{2{,}00{,}000\ e'}{0.4637\times10^6}\times1.0323$$

$$= 0.4637\times10^6$$

$$15.267 = 0.4313\times1.0323e'$$

Eccentricity,

$$e' = \frac{15.267}{0.4313\times1.0323}$$

$$= 34.3 \text{ mm}$$

Key Points to Remember

- Euler's buckling load, $P_e = \pi^2 EI_{min}/L_e^2$
  where I_{min} = minimum moment of inertia
  L_e = equivalent length of a column depending upon end conditions.
- Euler's formula is applicable for the slenderness ratio $L_e/K > \sqrt{\pi^2 E/\sigma_c}$, where σ_c is ultimate compressive strength of the material.
- Higher-order differential equation

$$\frac{d^4y}{dx^4} + k^2\frac{d^2y}{dx^2} = 0, \text{ where } k^2 = \frac{P}{EI}$$

- Solution, $y = C_1 \sin kx + C_2 \cos kx + C_3 x + C_4$
using different end conditions, the buckling load is determined.

- Rankine's load $= \dfrac{\sigma_c A}{1 + a\left(\dfrac{L_e}{k_{\min}}\right)^2}$

where

σ_c = crushing strength of column

L_e = equivalent length,

$k_{\min}$ = minimum radius of gyration

- Johnson's parabolic formula for working stress

$$\sigma_w = \sigma_{c'}\left(1 - b\frac{L_e^2}{k^2}\right)$$

where $\sigma_{c'}$ = allowable stress in compressive taking into account FOS

b = constant

- Eccentrically loaded column

$$\sigma_{\max} = \frac{P}{A} + \frac{Pe}{Z}\sec\frac{L_e}{2}\sqrt{\frac{P}{EI}}$$

where

P = axial applied load

Z = section modulus

e = eccentricity

I = moment of inertia

- Professor Perry's formula

$$\left(\frac{\sigma}{\sigma_0} - 1\right)\left(1 - \frac{\sigma_0}{\sigma_e}\right) = \frac{1.2\,ey_c}{k^2}$$

where

σ = maximum stress allowed

$\sigma_0 = P/A$, applied load/area

$\sigma_e = P_e/A$, Euler's load/area

e = eccentricity

y_c = distance of extreme layer in compression from neutral layer

- If the column has initial eccentricity e' along central section

$$\left(\frac{\sigma_{\min}}{\sigma_0} - 1\right)\left(1 - \frac{\sigma_0}{\sigma_e}\right) = \frac{e' y_c}{k^2}$$

- Energy approach, total potential, $K_p = 0$

$$\frac{1}{2}\int_0^L EI\left(\frac{d^2y}{dx^2}\right)dx = \frac{P_{cr}}{2}\int_0^L\left(\frac{dy}{dx}\right)^2 dx$$

to determine P_{cr}, critical load.

Review Questions

(1) What are the drawbacks of Euler's theory of buckling?
(2) What is limiting value of the slenderness ratio beyond which Euler's formula is applicable?
(3) What are the merits of Rankine's load over Euler's load in buckling?
(4) What do you mean by equivalent length of a column?
(5) Discuss the effect of the slenderness ratio of a column over buckling load?
(6) Why the value of 'a' Rankine's constant varies for different materials?
(7) What is straight line formula for working stress under buckling, where it is used?
(8) What approximation is taken by Professor Perry to modify the secant formula for eccentrically loaded column?
(9) What do you understand by total potential constant in energy approach?

Multiple Choice Questions

1. A column fixed at one end and free at the other end buckles at a load P. Now, both the ends of the column are fixed. What is the buckling load for these end conditions?
   (a) 16 P (b) 8 P
   (c) 4 P (d) 2 P
2. In Rankine's formula 'a' is used, what is its value for cast iron?
   (a) $\frac{1}{9,000}$ (b) $\frac{1}{7,500}$
   (c) $\frac{1}{1,600}$ (d) None of these
3. What is the approximate value of σ_c for cast iron/σ_c for mild steel?
   (a) 0.6 (b) 1.0
   (c) 1.7 (d) None of these
4. A hollow circular column, with D = 100 mm, d = 80 mm, what is radius of gyration?
   (a) 32 (b) 24
   (c) 19.4 (d) None of these
5. In Johnson's parabolic formula, allowable stress in compression for mild steel is
   (a) 270 N/mm^2 (b) 110 N/mm^2
   (c) 80 N/mm^2 (d) None of these
6. A 3-m-long column is hinged at one end and fixed at the other end, what is its equivalent length?
   (a) $3\sqrt{2}$ m (b) 3 m
   (c) $\frac{3}{\sqrt{2}}$ m (d) None of these
7. A column of a length of 2.4 m, an area of cross-section of 2,000 mm^2 and moment of inertia of $I_{xx} = 720 \times 10^4\,\text{mm}^4$ and $I_{yy} = 80 \times 10^4\,\text{mm}^4$ is subjected to buckling load. Both the ends of the column are fixed. What is the slenderness ratio of column?
   (a) 120
   (b) 80
   (c) 60
   (d) 40
8. The ratio of equivalent length of a column with one end fixed and the other end free to its own length is
   (a) 2 (b) 1.0
   (c) 0.5 (d) None of these
9. Euler's buckling theory is applicable for
   (a) Short columns
   (b) Long columns
   (c) Medium long columns
   (d) All of them
10. In Johnson's parabolic formula, what is constant b for hinged ends, for mild steel
   (a) 2×10^{-5} (b) 3×10^{-5}
   (c) 0.005 (d) None of these

prevent any expansion in its length. The pipe is unstressed at the normal temperature. Calculate the temperature stress in the pipe and the FOS against failure as a strut if the temperature rises by 40°C. Use Rankine's formula.

$$\left[\text{Hint: } \sigma_c = 330\text{MPa}, a = \frac{1}{7{,}500}, \alpha = 11.1\times10^{-6}/°\text{C} \quad E = 208 \text{ kN/mm}^2\right]$$

10. A 5-m-long steel strut, with $I = 50 \times 10^4$ mm^4, carries thrust load $P = 20$ kN, with eccentricities $e = 10$ mm on one side and $2e = 20$ mm on the other side. $E = 200$ GPa. Determine the distance from one end, where the bending moment on strut is maximum.

Answers to Exercises

Exercise 14.1: $\dfrac{\pi^2 EI}{L^2}$

Exercise 14.2: $\dfrac{4\pi^2 EI}{L^2}$

Exercise 14.3: 400 kN

Exercise 14.4: $L > 0.856$ m

Exercise 14.5: $\dfrac{\pi^2 EI}{L^2}$

Exercise 14.6: $\dfrac{\pi^2 EI}{4L^2}$

Exercise 14.7: 516.3kN

Exercise 14.8: 466 kN

Exercise 14.9: 12.07 mm

Exercise 14.10: 128 kN

Exercise 14.11: 8.05 N/mm^2

Exercise 14.12: 2,180 kN

Exercise 14.13: $W = 1.340$ kN

Exercise 14.14: 161.70 N/mm^2

Exercise 14.15: 9.88 EI/L^2

Answers to Multiple Choice Questions

1. (a)
2. (c)
3. (c)
4. (a)
5. (b)
6. (c)
7. (c)
8. (a)
9. (b)
10. (b)

Answers to Practice Problems

1. D = 55.47 mm; $d = 44.37$ mm
2. L = 769 mm
3. $a = \dfrac{1}{7{,}505}$, 61.67 kN
4. safe load = 11.50 kN
5. $\dfrac{\pi^2 I_C L_1^3}{48 I_B L_2^3}$
7. 5.29 kN
8. 32.3 mm
9. 92.35 MPa, 3.17
10. $x = 2.859$ m

Key Points to Remember

- If p_1, p_2 and p_3 are the principal stresses at a critical section of engineering component, if $p_1 > p_2 > p_3$ and σ_{yp} is the yield point stress of the material, when tested in simple tension or compression test, FOS is the factor of safety.
- As per the maximum principal stress theory $p_1 \le \dfrac{\sigma_{yp}}{\text{FOS}}$
- As per the maximum shear stress theory

  (i) $\dfrac{p_1 - p_3}{2} \le \dfrac{\sigma_{yp}}{2(\text{FOS})}$

  (ii) If $p_3 = 0$, then $\dfrac{p_1 - p_2}{2} \le \dfrac{\sigma_{yp}}{2\text{FOS}}$, if p_1 and p_2 of opposite signs

  (iii) If $p_3 = 0$, p_1 and p_2 of same signs

$$\frac{p_1}{2} \le \frac{\sigma_{yp}}{2(\text{FOS})}$$

- Maximum principal strain theory

$$[p_1 - \nu(p_2 + p_3)] \le \sigma_{yp},$$ where ν is the Poisson's ratio

- Strain energy theory (two dimensional)

$$\left(p_1^2 + p_2^2 - 2\nu p_1 p_2\right) \le \left(\frac{\sigma_{yp}}{\text{FOS}}\right)^2$$

- Shear strain energy theory

$$\left(p_1^2 + p_2^2 - p_1 p_2\right) \le 2\left(\frac{\sigma_{yp}}{\text{FOS}}\right)^2$$

Review Questions

1. What is the basic difference between ductile and brittle materials? Which theories of failure are used for brittle materials and which theories are used for ductile materials?
2. Why yield point stress is taken as failure stress for ductile materials?
3. Which theory of failure gives most conservative design?
4. Why is the maximum principal strain theory not used for general design purpose?
5. For brittle materials, such as rock and marble, which theory of failure is used and why?
6. Compare the maximum shear stress theory and the distortion energy theory for the design of a shaft made of ductile materials.

Multiple Choice Questions

1. Which of the following theories is used for brittle materials?
   (a) Maximum shear stress theory
   (b) Maximum principal strain theory
   (c) Distortion energy theory
   (d) None of these
2. Graphical representation of which one of the following theories is by an ellipse?
   (a) Maximum principal strain theory
   (b) Distortion energy theory
   (c) Maximum shear stress theory
   (d) None of these
3. A shaft is subjected to a torque and an axial compressive force. Shear stress due to torque is 30 MPa and axial compressive stress due to force is 80 MPa. If $\sigma_{yp} = 270$ MPa, what is FOS as per the maximum principal stress theory, Poisson's ratio = 0.3?
   (a) 3.0 (b) 2.90
   (c) 2.80 (d) None of these
4. Which one of the following theories gives conservative design of a component?
   (a) Maximum shear stress theory
   (b) Maximum principal strain theory
   (c) Distortion energy theory
   (d) None of these
5. A thin cylindrical shell with D/t ratio equal to 40 subjected to an internal pressure of 2 N/mm^2. The yield point stress of the material is 210 N/mm^2. Using the maximum shear stress theory for designing the thin shell, the FOS is:
   (a) 4.75 (b) 5.0
   (c) 5.25 (d) 5.50
6. A shaft subjected to pure torsion is to be designed. The yield point stress of the material is 270 MPa and the Poisson's ratio is 0.3, which of the following theories of failure gives the smallest diameter of the shaft?
   (a) Maximum principal stress theory
   (b) Maximum principal strain theory
   (c) Strain energy theory
   (d) None of these
7. A thick cylinder of an internal diameter of 100 mm and an external diameter of 200 mm is subjected to an internal pressure p. The yield strength of the material is 240 MPa. Taking an FOS of 2 and using the maximum principal stress theory of failure, the maximum value of the internal pressure p is
   (a) 120 N/mm^2 (b) 90 N/mm^2
   (c) 72 N/mm^2 (d) None of these
8. The principal stresses at a point are 70, 60 and −18 MPa; say σ_{yp} is the yield point stress of the material. Using the maximum principal stress theory, FOS is 4. What is the FOS if the maximum principal strain theory is used, Poisson's ratio of material is 1/3?
   (a) 4 (b) 4.5
   (c) 5.0 (d) 5.25
9. The principal stresses developed at a point are +80, −80 MPa and 0, 0. Using the shear strain energy theory, the FOS is $\sqrt{3}$. The yield point stress of the material is
   (a) $80 \times \sqrt{2}$ MPa (b) $80 \times \sqrt{3}$ MPa
   (c) 240 MPa (d) None of these
10. At a point in a strained material, the principal stresses are p_1, p_2 and 0.0. What combination of principal stresses will give same FOS by yielding according to the maximum shear stress theory and the distortion energy theory of failure?
    (a) $p_1 = -p_2$ (b) $p_1 = 0.5p_2$
    (c) $p_1 = p_2$ (d) None of these

Practice Problems

1. Considering the principal stresses in a steam boiler as p, $0.5p$, 0 and Poisson's ratio = 0.28, equivalent stress in a simple tensile test as σ, find p in each of the five theories of failure (except Mohr's theory).
2. What combination of principal stresses will give the same FOS for failure by

6. A shaft is subjected to a bending moment M and a twisting moment T. The value of $M = 0.5T$, show that strain energy per unit volume is equal to $\dfrac{512M^2}{E\pi^2 d^6}(3+v)$ when v is the Poisson's ratio, d is the diameter of shaft and E is the Young's modulus.

Answers to Exercises

Exercise 15.1: shaft diameter, $d = 19.375$ mm

Exercise 15.2: 72.47 mm

Exercise 15.3: 2.55

Exercise 15.4: 29.6 N/mm^2

Exercise 15.5: $t = 6$ mm

Exercise 15.6: $P_2 = -361$ MPa

Answers to Multiple Choice Questions

1. (d)
2. (b)
3. (a)
4. (a)
5. (b)
6. (a)
7. (c)
8. (c)
9. (c)
10. (c)

Answers to Practice Problems

1. σ, σ, 1.162σ, 1.015σ, 1.154σ
2. $p_1 > p_2$ both of same sign $p_1 = p_2\, p_1 > 0, p_2 < 0$, $p_1 \cdot p_2 = 0$
3. 3.44, 4.372, 4.10 Poisson's ratio, $v = 0.30$
3. 2.33
4. 2.524 kN m
5. (a) will not fail (b) will not fail

Answers to Special Problems

1. 27.45 N/mm^2
2. 2.36
3. (a) 6.293 mm, (b) 6.43 mm, (c) 5.50 mm
4. (a) $p = 66.67$ MPa, (b) $p = 76.98$ MPa
5. (a) 11 mm, (b) 11.31 mm

Multiple Choice Questions

1. A beam of length L, simply supported at the ends, carries a concentrated load W at its centre. If EI is the flexural rigidity of the beam, strain energy due to bending in beam is:

   (a) $\dfrac{W^2L^3}{96EI}$ (b) $\dfrac{WL^4}{48+EI}$

   (c) $\dfrac{W^2L^3}{24EI}$ (d) None of these

2. A beam of length L, simply supported at ends, carries a load W at a distance of a from one end and at a distance of b from other end such that $a + b = L$. If EI is the flexural rapidly of the beam, how much strain energy is absorbed in beam?

   (a) $\dfrac{W^2ab^3}{3EIL}$ (b) $\dfrac{W^2a^2b^3}{6EIL}$

   (c) $\dfrac{Wa^3b}{6EIL}$ (d) None of these

3. A shaft of length L and a polar moment of inertia J is subjected to a twisting moment T. If G is the shear modulus, the strain energy stored in shaft is

   (a) $\dfrac{TL^2}{2GJ}$ (b) $\dfrac{T^2L}{2GJ}$

   (c) $\dfrac{T^2L}{4GJ}$ (d) None of these

4. A body is subjected to a direct force F, twisting moment T and a bending moment M. The energy stored in the body is U. What is the displacement in the direction of force F?

   (a) $\dfrac{\partial U}{\partial M}+\dfrac{\partial U}{\partial T}+\dfrac{\partial U}{\partial F}$ (b) $\dfrac{\partial U}{\partial M}+\dfrac{\partial U}{\partial F}$

   (c) $\dfrac{\partial U}{\partial F}$ (d) None of these

5. A cantilever of length L is fixed at one end and a couple M is applied at the other end so as to bend the cantilever. If EI is the flexural rigidity of the cantilever, then slope at the free end of cantilever is

   (a) $\dfrac{ML}{2EI}$ (b) $\dfrac{ML}{EI}$

   (c) $\dfrac{2ML}{EI}$ (d) None of these

6. A cantilever is subjected to a load W at its free end, the deflection produced in centre of cantilever is δ. Now the load W is applied only at the centre of cantilever, what is the deflection at free end?

   (a) 2δ (b) 1.5δ
   (c) δ (d) 0.5δ

Practice Problems

1. A beam AB of length L, hinged at end A and roller supported at end B, is subjected to a couple M at point C, such that AC = $L/3$ as shown in Fig. 16.35. If EI is the flexural rigidity of the beam, determine the rotation in beam at point C.

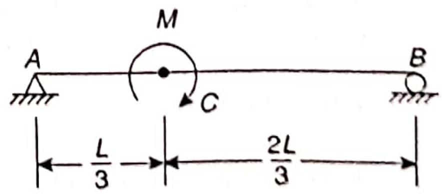

Figure 16.35

2. A beam AB of length L, simply supported at ends, carries a load W at C, such that AC = a, as shown in Fig. 16.36. If EI is the flexural rigidity of the beam, what is the deflection in the beam under the load?
3. A steel ring of 20 mm in diameter (in section) is bent into a quadrant of 1.5 m radius. One end of ring is rigidly fixed in the ground, and at the other end, a vertical load P is applied. Determine the value of P so that the vertical deflection at the point of loading is 16 mm (E = 208 kN/mm^2).
4. For a cantilever made of steel with length L, breadth b and depth d, show that if y_b and y_s are the deflections due to bending and shear at the free end due to a concentrated load W at the free end, then

$$\frac{y_s}{y_b} = k\left(\frac{d}{L}\right)^2,$$ where k is a constant

Answers to Exercises

Exercise 16.1: 1.914 mm

Exercise 16.2: 0.013 mm; 3.857 mm

Exercise 16.3: 1.44 mm, 0.1°

Exercise 16.4: 250 N

Exercise 16.5: 20.82 kN

Exercise 16.6: $\dfrac{WL^3}{32EI}$

Exercise 16.7: 19.3 mm

Answers to Multiple Choice Questions

1. (a)
2. (b)
3. (b)
4. (c)
5. (b)
6. (c)

Answers to Practice Problems

1. $\dfrac{ML}{9EI}$
2. $\dfrac{Wa^2b^2}{3EIL}$
3. 9.96 N
4. $k = 0.78,\ \dfrac{L}{d} = 7.21$
5. $\delta_H = \dfrac{10}{3}\dfrac{WL^3}{EI},\ \delta_V = \dfrac{17}{3}\dfrac{WL^3}{EI}$
6. $\dfrac{Pa}{AE}\left(7+4\sqrt{2}\right)$
7. $-\dfrac{WL}{8}$
9. $\dfrac{Pb(2a+b)}{2EI}, \dfrac{Pba^2}{2EI}$
10. $\dfrac{PR^3\pi}{2}\left(\dfrac{3}{GJ}+\dfrac{1}{EI}\right)$
11. $2.737 + 0.15 = 2.887$ mm.
12. $\dfrac{Pb^3}{6EI}+\dfrac{Pb^2a}{2EI}$
13. 0.294 mm

$$= \frac{20^2}{16}\left[1+\frac{1}{2}\left(\frac{200}{100}\right)^2+\frac{5}{16}\left(\frac{20}{100}\right)^4\right]$$

$$= 25[1+0.02+0.0005] = 25.5125$$

$$\frac{R_1^2}{h_1^2} = \frac{50^2}{25.5125} = 97.99$$

and $$h_2^2 = \frac{d^2}{16}\left[1+\frac{1}{2}\left(\frac{20}{140}\right)^2+\frac{5}{16}\left(\frac{20}{140}\right)^4\right]$$

$$= \frac{20^2}{16}[1+0.01+.0001] = 25.2525$$

$$\frac{R_2^2}{h_1^2} = \frac{70^2}{25.2525} = 194$$

Stress at A, $$\sigma_A = \frac{W\times R_1}{AR_1}\left(\frac{10}{50-10}\times\frac{R_1^2}{h_1^2}-1\right)+\frac{W}{A}$$

$$= \frac{W}{A}\times\frac{10}{40}\times 97.99$$

$$= \frac{1,000}{314.16}\times\frac{1}{4}\times 97.99 = +77.98\ \text{N/mm}^2$$

Stress at B, $$\sigma_B = \frac{W\times R_2}{A\times R_2}\left(\frac{10}{70-10}\times\frac{R_2^2}{h^2}-1\right)+\frac{W}{A}$$

$$= \frac{W}{A}\times\frac{1}{6}\times\frac{R_2^2}{h^2} = \frac{1,000}{314.16}\times\frac{1}{6}\times 194$$

$$= +102.92\ \text{N/mm}^2$$

Multiple Choice Questions

1. A bar of square section 6 cm × 6 cm is curved to a mean radius of 12 cm. A bending moment M is applied on the bar. The moment M tries to straighten the bar. If the stress at the innermost fibres is 60 N/mm² tensile, then the stress at the outermost fibres is
   (a) 60 N/mm² (compressive)
   (b) 60 N/mm² (tensile)
   (c) More than 60 N/mm² (compressive)
   (d) Less than 60 N/mm² (compressive).
2. The most suitable section of a crane hook is
   (a) Square (b) Round
   (c) Hollow round (d) Trapezoidal
3. A bar of square section 4 cm × 4 cm is curved to a mean radius of 80 m. A bending moment M, tending

to increase the curvature is applied on the bar. If the stress at the outermost fibres is 80 MPa tensile, then the stress at the innermost fibres will be approximately equal to

(a) 120 MPa (compressive)
(b) 100 MPa (compressive)
(c) 90 MPa (compressive)
(d) 80 MPa (compressive)

4. A ring is subjected to a diametral tensile load. The variation of the stress at the intrados surface from the point of loading up to the section of symmetry is

(a) Maximum tensile stress to maximum compressive stress.
(b) Throughout tensile stress.
(c) Maximum compressive stress to maximum tensile stress.
(d) Throughout compressive stress.

5. The distribution of stress along a section of a curved bar subjected to a bending moment tend to increase its curvature is

(a) Linear (b) Uniform
(c) Parabolic (d) Hyperbolic

Practice Problems

1. A sharply curved beam of rectangular section is 10 mm thick and 50 mm deep. If the radius of curvature, $R = 60$ mm, compute the stress in terms of the bending moment M at a point 20 mm from the outer surface.
2. Determine the diameter d of a round steel rod that is used as a hook to lift a 9-kN load acting through the centre of curvature of the centroidal axis of the hook. Assume that $\frac{R}{d} = 2$, and the maximum stress permitted is 125 N/mm^2.
3. The cross-section of a triangular hook has a base of 5 cm and altitude of 7.5 cm and a radius of curvature of 5 cm at the inner face of the shank. If the allowable stress in tension is 100 N/mm^2 and in compression is 80 N/mm^2, what load can be applied along a line 7.5 cm from the inner face of the shank?
4. Three plates are welded to form the curved beam of or, I-section shown in Fig. 17.26. If the moment $M = 1$ kN m, determine the stresses at points A and B and at the centre of the section (Fig. 17.26).
5. A steel link of rectangular section 24 mm × 8 mm is shown in Fig. 17.27. If the angle $\beta = 90°$ and the allowable stress in link is 100 MPa, determine the largest value of P which can be applied on the link.
6. A curved bar of rectangular section with breadth B and depth $D = 2B$, is bent to a radius of curvature equal to $1.2D$. It is subjected to a bending moment

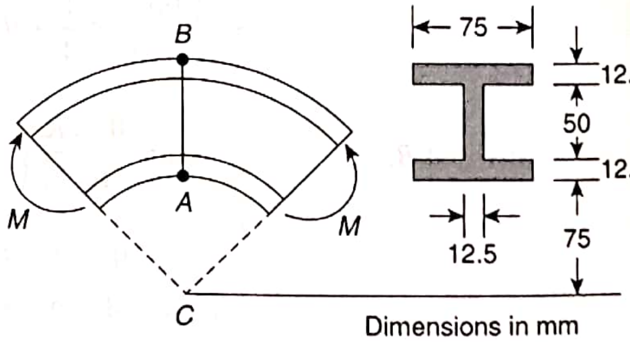

Figure 17.26

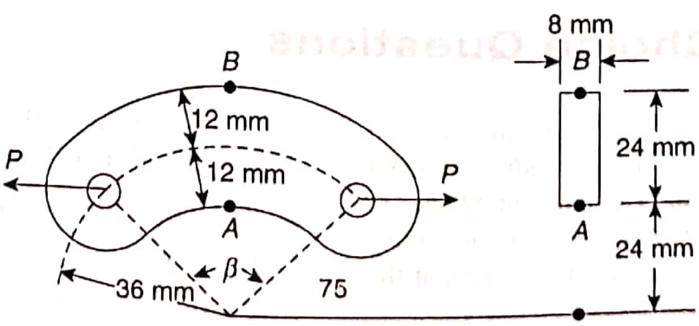

Figure 17.27

Answers to Exercises

Exercise 17.1: 2.16 kN m, +4.50 N/mm^2
Exercise 17.2: −93.8; +84.05 MPa
Exercise 17.3: 86.70, −37.873 MPa
Exercise 17.4: 30.60 kN
Exercise 17.5: −39.22, +99.83 MPa
Exercise 17.6: +26.1 N/mm^2, −30.33 N/mm^2
Exercise 17.7: 42.99, −30.26
Exercise 17.8: 0.055 mm
Exercise 17.9: 0.090 mm

Answers to Multiple Choice Questions

1. (d)
2. (d)
3. (d)
4. (c)
5. (d)

Answers to Practice Problems

1. 72.92 M
2. 43.52 mm
3. 15.7 kN
4. +22.755, −16.71, −3.55 MPa
5. 4.37 kN
6. 29.7, 59.4 mm
7. 93.6 MPa
8. 148.433 kN
9. 375 N, 496 N
10. 1.87
11. 79.56 to −17.37 MPa

Key Points to Remember

- Unsymmetrical bending occurs in a beam: (i) if the section is symmetrical but load-line is inclined to the principal axes or (ii) if section itself is unsymmetrical.
- Product of inertia, $I_{xy} = \int xy dA$

  Product of inertia of a section about its principal axes is zero.
- For a symmetrical section, principal axes are along the axes of symmetry.
- Parallel axes theorem for product of inertia,

$$I_{xy} = I_{\overline{xy}} + A_{\overline{xy}}$$

  where

  I_{xy} = Product of inertia about any co-ordinates axes X–Y

  $I_{\overline{xy}}$ = Product of inertia about centroidal axes $\overline{X}-\overline{Y}$

  $\overline{x}, \overline{y}$ = Coordinate of the centroid of the section about XY co-ordinates.
- If I_{xx}, I_{yy}, I_{xy} are moments of inertia about any co-ordinates axes X–Y passing through the centroid of the section. Inclination of principal axis with respect to X–X axes.

$$\theta = \frac{1}{2}\tan^{-1}\frac{2I_{xy}}{I_{yy} - I_{xx}}.$$

  Principal moments of inertia

$$I_{uu}, I_{vv} = \frac{1}{2}(I_{xx} + I_{yy}) \pm \sqrt{\left[\frac{1}{2}(I_{yy} - I_{xx})\right]^2 + I_{xy}^2}.$$

- If principal moments of inertia of a section are I_{uu}, I_{vv}, then moment of inertia about an axis X–X inclined at angle θ to U–V axis is

$$I_{xx} = I_{uu}\cos^2\theta + I_{vv}\sin^2\theta.$$

- Stresses due to unsymmetrical bending, if u, v are the co-ordinates of a point and M is the bending moment applied on the section and θ is the angle of inclination of axis of M, with respect to the principal axes UU. Resultant bending stress at the point

$$\sigma_b = M\left(\frac{v\cos\theta}{I_{uu}} + \frac{u\sin\theta}{I_{vv}}\right).$$

- Angle of inclination of neutral axis with respect to principal axis UU

$$\alpha = \tan^{-1}\left(\tan\theta \frac{I_{uu}}{I_{vv}}\right).$$

- Deflection of a beam under load W causing unsymmetrical bending.

$$\delta = \frac{KWl^3}{E}\sqrt{\frac{\sin^2\theta}{I_{vv}^2} + \frac{\cos^2\theta}{I_{uu}^2}}$$

where

K = Constant depending upon end conditions of the beam and position of the load
θ = Angle of inclination of load W with respect to VV principal axis.

- If the direction of the applied load on a beam passes through the shear centre of the section, no twisting takes place in the beam.
- For a section symmetrical about two axes, shear centre lies at the centroid of the section.
- For a section symmetrical about one axis only, shear centre lies along the axis of symmetry.
- About the shear centre, the moment due to the applied shear force is balanced by the moment of the shear forces obtained by summing the shear stresses over the various portions of the section.

Multiple Choice Questions

1. The product of inertia of a rectangular section of a breadth of 4 cm and a depth of 6 cm about its centroidal axis is
   (a) 144 cm^4 (b) 72 cm^4
   (c) 36 cm^4 (d) None of the above
2. The product of inertia of a rectangular section of a breadth of 3 cm and a depth of 6 cm about the co-ordinate axes passing at one corner of the section and parallel to the sides is
   (a) 81 cm^4 (b) 72 cm^4
   (c) 54 cm^4 (d) None of these
3. For an equal angle section, co-ordinate axes XX and YY passing through centroid are parallel to its length. The principal axes are inclined to XY axes at an angle
   (a) 22.5° (b) 45.0°
   (c) 67.5° (d) None of the above
4. For an equal angle section, moment of inertia I_{xx} and I_{yy} are both equal to 120 cm^4. If one principal moment of inertia is 180 cm^4, the magnitude of other principal moment of inertia is
   (a) 180 cm^4 (b) 120 cm^4
   (c) 60 cm^4 (d) 30 cm^4
5. For a section, principal moments of inertia are I_{uu} = 360 cm^4 and I_{vv} = 160 cm^4. Moment of inertia of the section about an axis inclined at 30° to the U–U axis, is
   (a) 310 cm^4 (b) 260 cm^4
   (c) 210 cm^4 (d) 120 cm^4
6. For an equal angle section, $I_{xx} = I_{yy}$ = 32 cm^4 and I_{xy} = −20 cm^4. The magnitude of one principal moment of inertia is
   (a) 52 cm^4 (b) 42 cm^4
   (c) 32 cm^4 (d) 16 cm^4
7. For a T-section, shear centre is located at
   (a) Centre of the vertical web
   (b) Centre of the horizontal flange

(c) At the centroid of the section
(d) None of the above.

8. For an I-section (symmetrical about X–X and Y–Y axis) shear centre lies at
   (a) Centroid of top flange
   (b) Centroid of bottom flange
   (c) Centroid of the web
   (d) None of the above.

9. For a channel section symmetrical about X–X axis, shear centre lies at
   (a) The centroid of the section
   (b) The centre of the vertical web
   (c) The centre of the top flange
   (d) None of the above.

10. If the applied load passes through the shear centre of the section of the beam, then there will be
    (a) No bending in the beam
    (b) No twisting in the beam
    (c) No deflection in the beam
    (d) None of these

Practice Problems

1. Figure 18.37 shows Z-section of a beam simply appalled over a span of 2 m. A vertical load of 2 kN acts at the centre of the beam and passes through the centroid of the section. Determine the resultant bending stress at points A and B.
2. Figure 18.38 shows a section of a beam subjected to shear force F. Locate the position of the shear centre as defined by e.

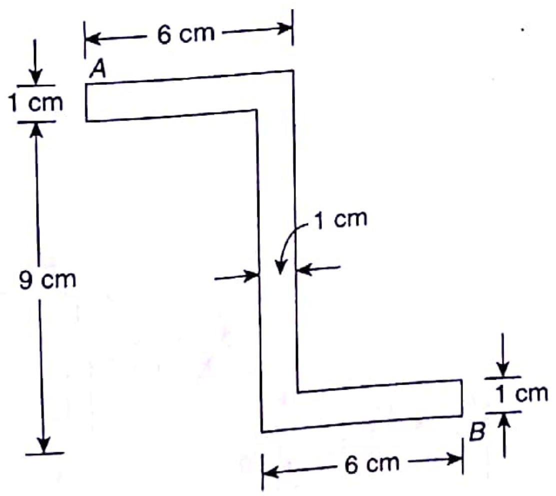

Figure 18.37 Practice Problem 1

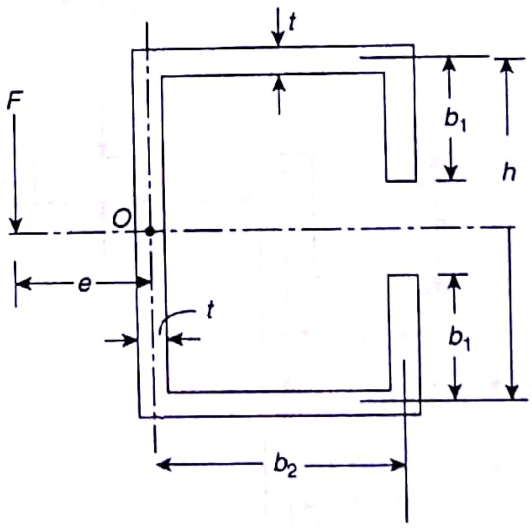

Figure 18.38 Practice Problem 2

3. Determine the location e of the shear centre point C for the thin-walled member having the cross-section shows in Fig. 18.39, where $b_2 > b_1$, the member segments have the same thickness t.
4. For an extruded beam having the cross-section shown in Fig. 18.40, determine (a) location of shear centre and (b) distribution of shear stresses caused by vertical shear force $F = 12$ kN.

7. Determine the location of shear centre of a thin-walled beam of uniform thickness having the cross-section shown in Fig. 18.43.

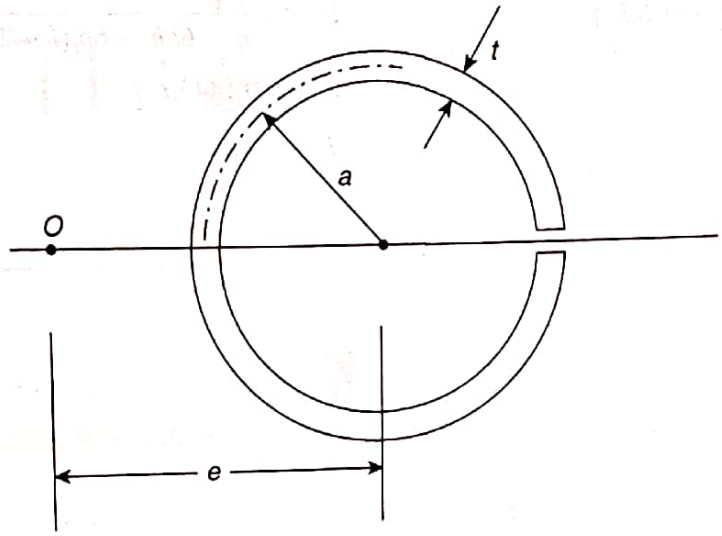

Figure 18.43 Practice Problem 7

Answers to Exercises

Exercise 18.2: $\dfrac{b^2h^2}{4}$

Exercise 18.3: 147.25×10^4 mm^4, $+32.30 \times 10^4$ mm^4.

Exercise 18.4: 540 cm^4, 540 cm^4, -324 cm^4

Exercise 18.5: 28° 31′, 118° 31′, 360 cm^4, 38.33 cm^4

Exercise 18.6: $\sigma_A = -491.67$ N/cm^2, $\sigma_B = +2{,}812.5$ N/cm^2

Exercise 18.7: 221.2 N, 15° 13′

Exercise 18.8: 2.7 cm

Answers to Multiple Choice Questions

1. (d)
2. (a)
3. (b)
4. (c)
5. (a)
6. (a)
7. (b)
8. (c)
9. (d)
10. (b)

Multiple Choice Questions

1. Principal stresses at a point are 120, –40 and –20 MPa. What is the maximum shear stress at the point?
   (a) 50 MPa (b) 70 MPa
   (c) 80 MPa (d) None of these

2. A bar is subjected to an axial load such that its length l is increased by $0.001l$. If Poisson's ratio is 0.3, what is the change in its diameter d?
   (a) $-3 \times 10^{-4}d$ (b) $+3 \times 10^{-3}d$
   (c) $+3 \times 10^{-4}d$ (d) None of these

3. Lame's coefficient for a material are λ and μ. What is Poisson's ratio?
   (a) $\dfrac{\lambda}{\lambda+\mu}$ (b) $\dfrac{\lambda}{2(\lambda+\mu)}$
   (c) $\dfrac{\lambda}{3(\lambda+\mu)}$ (d) None of these

4. Ratio of volumetric stress/volumetric strain is known as
   (a) Shear modulus (b) Bulk modulus
   (c) Young's modulus (d) None of these

5. Airy's stress function is $\phi = 50x^2 - 40xy + 80$. $80y^2$, what is the normal stress σ_{yy}?
   (a) 100 MPa (b) 40 MPa
   (c) +160 MPa (d) 80 MPa

6. A rectangular-section beam of breadth b, depth d is subjected to shear force F. At what depth y from top surface transverse shear stress is the maximum?
   (a) $y = 0$ (b) $y = \dfrac{d}{4}$
   (c) $y = \dfrac{d}{3}$ (d) $y = \dfrac{d}{2}$

7. A shaft is subjected to pure twisting moment M. Surface of the shaft represents
   (a) Plane strain condition
   (b) Plane stress condition
   (c) Both plane stress and plane strain conditions
   (d) Neither plane stress nor plane strain condition

8. A thin metallic sheet is subjected to a plane shear stress, what is the state of stress of thin sheet?
   (a) a plane strain state
   (b) a plane stress state
   (c) a hydrostatic state of stress
   (d) None of these

Practice Problems

1. Consider a beam of a rectangular section $B = 25$ mm, $D = 60$ mm subjected to a bending moment $+1.5 \times 10^6$ N mm. Write down (a) the stress tensor for an element located at top surface and (b) the stress tensor for an element located in a plane 15 mm below the top surface.
2. For a material Lame's coefficients are $\lambda = 1.2 \times 10^5$ MPa and $\mu = 0.8 \times 10^5$ MPa. Determine E, v and G for the material.
3. A cantilever of a rectangular section $B \times D$ is of length L as shown in Fig. 19.19. Write down stress tensor to determine state of stress at section XX at a distance of x from free end and at a layer at a distance of y from neutral layer.
4. A cylindrical bar of length L, area of cross-section A is fixed at top end. Write down Airy's stress function for stress due to self weight in bar, if w is the weight density of the bar (Fig. 19.20).
5. Consider the displacement field $S = (y^2 i + 3yxj) \times 10^{-2}$. Find whether, strain field is compatible. If yes, find strain components ε_x, ε_y and $\gamma_{xy}/2$ at point (1, –1).

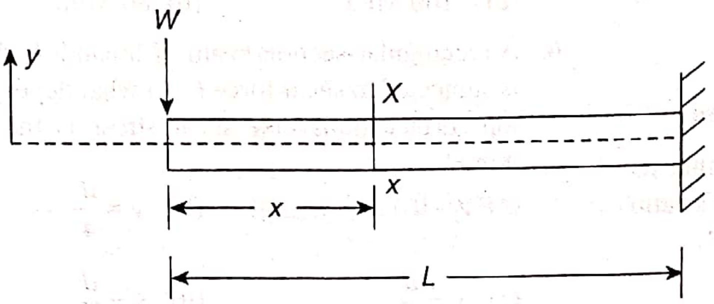

Figure 19.19

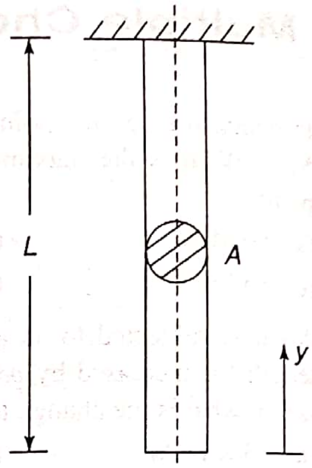

Figure 19.20

Answers to Exercises

Exercise 19.1: $\sigma_n = 120$ MPa, $\tau_n = 43.20$ MPa

Exercise 19.2: $\sigma_n = 117.77$ MPa, $\tau_n = 58.547$ MPa, angle 26°35

Exercise 19.3: +0.070 mm, –0.057 mm, +0.028 mm

Exercise 19.4: $3.011, 1.001, -1.993, \begin{bmatrix} 6 & 1 & 3 \\ 1 & 3 & 0.5 \\ 3 & 0.5 & 1 \end{bmatrix} \times 10^{-3}$

Exercise 19.5: $4k$, $-8k$, $+2k$

Exercise 19.6: 7639 N/mm², 11458 N/mm²

Exercise 19.7: λ = 111,120 MPa, μ = 77,220 MPa, K = 162,600 MPa

Exercise 19.8: strain field is possible $u = 8x + \dfrac{x^3}{3} + 2xy^2$, $v = 6y + 3yx^2 + \dfrac{y^3}{3}$

Exercise 19.9: $\sigma_{xx} = 160$ MPa, $\sigma_{yy} = 100$ MPa, $\tau_{xy} = +40$ MPa, a plane stress condition

Exercise 19.10: 432 Nm

Answers to Multiple Choice Questions

1. (c)
2. (a)
3. (b)
4. (b)
5. (a)
6. (d)
7. (c)
8. (b)

? Multiple Choice Questions

1. The most important reason for Bauschinger's effect in ductile materials is
   (a) Ductile materials weakness in shear
   (b) Compressive residual stress
   (c) Tensile residual stress
   (d) None of these
2. The notch angle in the Izod impact test specimen is
   (a) 25° (b) 30°
   (c) 45° (d) None of these
3. Length of specimen fixed in vice in Izod Impact test is
   (a) 40 mm (b) 45 mm
   (c) 47 mm (d) 50 mm
4. The angle between the opposite faces of the diamond pyramid in the case of Vicker's Pyramid. Hardness Test is
   (a) 118° (b) 120°
   (c) 136° (d) 140°
5. Which indentor is used for Microhardness test?
   (a) Hardened Brinell ball
   (b) Vicker's Diamond Pyramid
   (c) Conical shaped diamond
   (d) Knoop Indentor
6. The process which does not improve the fatigue strength of a material is
   (a) Shot peening of the surface
   (b) Cold rolling of surface
   (c) Electroplating of the surfaceg
   (d) None of these
7. The depth of penetration of the hardened steel ball in specimen is 0.140 mm. Rockwell Hardness of material is
   (a) 60 (b) 65
   (c) 70 (d) 75
8. What is Poisson's ratio of Nickel?
   (a) 0.33 (b) 0.31
   (c) 0.30 (d) 0.28
9. Which of the following materials is remarkably brittle?
   (a) Polymers (b) Ceramics
   (c) Bronzes (d) None of these
10. Which of the following materials fail in tension by making a necking to a point?
    (a) Mild steel (b) Wrought iron
    (c) Lead (d) Aluminium
11. Which of the following statements is incorrect?
    (a) Fatigue limit exists for most ferrous alloys
    (b) For many steels, fatigue limit ranges from 35 to 60 per cent of tensile strength
    (c) Increasing the mean stress level leads to increase in fatigue limit
    (d) None of these
12. Which of the following creep is of longest duration?
    (a) Primary Creep
    (b) Secondary Creep
    (c) Tertiary creep
    (d) All the above have equal duration

Answers to Multiple Choice Questions

1. (b)
2. (c)
3. (c)
4. (c)
5. (d)
6. (c)
7. (a)
8. (b)
9. (b)
10. (c)
11. (c)
12. (b)

7. Briefly describe the following:
   (a) Grey cast iron
   (b) Malleable cast iron
   (c) Nodular cast iron
8. What are the important applications of magnesium alloy?
9. What are the characteristics of titanium alloys? What are their applications?
10. Give the composition and uses of following brasses:
    (a) admiralty brass
    (b) cartridge brass
    (c) gilding metal
    (d) yellow brass
11. What are bronzes? What are the important applications of phosphor bronze?
12. What are the special properties of Inconels and Nimonics? Where are they used?
13. Name a few alloys used in heating elements in furnaces, hair dryers, toasters etc. What is the composition of these alloys?
14. What are Stellites? What are their salient properties? What are their applications?
15. What are tin Babbits and lead Babbits? Where these are used?
16. What are thermoplastic and thermosetting plastics? What are basic differences between them?
17. Name four thermosetting plastics? What are their applications?
18. Name at least eight thermoplastics along with their applications?
19. What is rubber? What is vulcanization of rubber? How the strength of rubber is increased by vulcanization?
20. What do you understand by ceramics? What is the difference between a refractory, a glass and an abrasive?
21. What do you mean by glass? Give composition and uses of
    (a) Sodalime glass
    (b) Lead glass or flint glass
    (c) Pyrex glass
    (d) Vycor glass
22. What do you understand by a polymorph of carbon, that is,?
    (a) diamond
    (b) graphite
    (c) fullerene

Multiple Choice Questions

1. What is the percentage of carbon in tool steel?
   (a) 0.35 (b) 0.65
   (c) 0.9 (d) None of these
2. Which one of following statements is incorrect for high-carbon steel?
   (a) these are wear resistant
   (b) always used under hardened and tempered conditions
   (c) they are tough and ductile
   (d) used for rail tracks, gears
3. What is the purpose of adding Nickel in steel?
   (a) Nickel stabilizes austenite
   (b) Nickel forms hard carbides
   (c) Nickel raises creep strength
   (d) None of above
4. For stone crushers which of the following steel is used:
   (a) Invar steel (b) High speed steel
   (c) Hadfield steel (d) None of these

5. Make proper combination

A Ferritic stainless steel I Ball and roller races

B Austenitic stainless II Turbine blades

C Martensitic stainless steel III Storage for acids

	A	B	C
(a)	I	II	III
(b)	III	I	II
(c)	III	II	I
(d)	II	III	I

6. What is the percentage of nickel in invar steel?
   (a) 30 (b) 36
   (c) 40 (d) None of these

7. Strength and ductility of SG iron are more than for grey cast iron, because of
   (a) Graphite is in nodular form
   (b) Graphite is in cluster form
   (c) Graphite is in flakes form
   (d) None of these

8. Titanium alloys are used in
   (a) Pump bodies
   (b) Crank shaft
   (c) Jet engine components
   (d) All the above

9. Brass with a composition of Cu 95%, zinc 5% is known as
   (a) Cartridge brass (b) Munz metal
   (c) Gilding metal (d) None of these

10. Which of following alloy is used in electrical resistance strain gauges?
    (a) Delta metal (b) Constantan
    (c) Nickel silver (d) None of these

11. Make proper combination of alloys and its applications:

A Munz metal I Pyrometer sheaths

B Inconel II Heating elements of furnaces

C Nichrome III Turbine blades

D Nimonics IV Pump fittings

	A	B	C	D
(a)	IV	II	I	III
(b)	IV	I	II	III
(c)	I	II	III	IV
(d)	II	I	IV	III

12. Identify the thermosetting plastics from the following:

I Nylon

II Melamine

III PMMA

IV Epoxy

V Teflon

(a) I, III, IV
(b) II, IV
(c) II, IV, V
(d) I, IV, V

13. Make proper combination of rubber and its applications:

A Neoprene I Shoe soles

B Butadiene II Expanded foams

C Polyurethanes III Tyres and tubes

D Butyl IV Gaskets

	A	B	C	D
(a)	I	II	III	IV
(b)	II	I	IV	III
(c)	IV	I	II	III
(d)	I	IV	III	II

14. In Vycor glass, what is the percentage of silica?
    (a) 35–58 (b) 70–75
    (c) 96–97 (d) None of these

Answers to Multiple Choice Questions

1. (c)
2. (c)
3. (a)
4. (c)
5. (b)
6. (b)
7. (a).
8. (c)
9. (c)
10. (b)
11. (b)
12. (b)
13. (c)
14. (c)

Answers to Multiple Choice Questions

1. (c)
2. (b)
3. (a)
4. (c)
5. (d)
6. (a)
7. (c)
8. (b)
9. (c)
10. (b)

Answers to Practice Problems

1. 0.8
2. $\frac{4}{3}\frac{M}{L}; \frac{2}{243} \times \frac{ML^2}{EI}$
3. $L = 900$ mm, $\sigma = 18.51$ MPa
5. $P = 22.5$ kN, reaction at ends = 6.75 kN, $i_A = -0.127°$, $i_B = +0.127°$
6. 2.4 kN
7. $0, -\frac{ML^2}{128EI}; 0, +\frac{ML^2}{128EI}; 0$
8. 10 mm
9. $\pm\frac{WL^2}{9EI}; \frac{23}{648}\frac{WL^3}{EI}$
10. 0.6°, −25 mm
11. 3.3 kN, 0.166 mm
12. 25.6 Nm
13. $-\frac{0.00652wL^4}{EI}$, $0.5196L$ from one end
14. 952.5 N

Answers to Special Problems

1. 14.3 N
2. $\pm\frac{WL^2}{16EI}; -\frac{WL^3}{48EI}$
3. 0.185°, 1.87 mm
4. $-\frac{5}{81} \times \frac{WL^2}{EI}, +\frac{4}{81} \times \frac{WL^2}{EI}, -\frac{4}{243} \times \frac{WL^3}{EI}$
5. 0.276°, −16.3 mm
6. 6.425 mm
7. 16.875 kN; 0.54°, −7.35 mm at 1.275 m from free end