# New Programming Tools and Interfaces for Deploying AMPL Models

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# 3 Ways to Program in an Optimization Modeling System

### Work inside the modeling system

Write scripts using modeling language constructs

### Call the modeling system from a programming language

 Use modeling system APIs created for various general-purpose programming languages

### Mix modeling and programming constructs

- Embed programming language statements within model definitions and scripts
- Write programs in general-purpose languages that modify or extend model definitions



#### **Features**

- Algebraic modeling language
- Built specially for optimization
- Designed to support many solvers

#### Programming options

- Scripting based on modeling language extensions
- ❖ *APIs* for C++, C#, Java, MATLAB, Python, R
- *❖ Embedded Python* processed by the Python API
  - \* (available soon)

### Application-building toolkits (not covered here)

- QuanDec / built on Java API
- Opalytics (Accenture) / connected via Python API

### **Outline**

#### AMPL model

Optimal roll cutting

#### AMPL script

Trading off waste versus overruns

#### AMPL API programs

- ❖ Pattern enumeration in Python / R
- Pattern generation in Python

### Embedded Python (a preview)

- Specifying Python data in an AMPL model
- Executing Python sttements inside AMPL
- Handling callbacks

#### **AMPL Command Environment**

#### Roll-cutting problem

- \* Fill orders for rolls of various widths
  - \* by cutting raw rolls of one (large) fixed width
  - \* using a variety of cutting patterns

#### Optimization model

- Decision variables
  - \* number of raw rolls to cut according to each pattern
- Objective
  - \* minimize number of raw rolls used
- Constraints
  - \* meet demands for each ordered width
  - \* don't exceed demands too much

### **Mathematical Formulation**

#### Given

```
w width of "raw" rolls
```

W set of (smaller) ordered widths

*n* number of cutting patterns considered

#### and

```
a_{ij} occurrences of width i in pattern j, for each i \in W and j = 1, ..., n
```

- $b_i$  orders for width i, for each  $i \in W$
- o limit on overruns

# Mathematical Formulation (cont'd)

#### **Determine**

 $X_j$  number of rolls to cut using pattern j, for each j = 1, ..., n

#### to minimize

$$\sum_{j=1}^{n} X_{j}$$

total number of rolls cut

### subject to

$$b_i \leq \sum_{j=1}^n a_{ij} X_j \leq b_i + o$$
, for all  $i \in W$ 

number of rolls of width *i* cut must be at least the number ordered, and must be within the overrun limit

### **AMPL Formulation**

#### Symbolic model

```
param rawWidth;
set WIDTHS;

param nPatterns integer > 0;
set PATTERNS = 1..nPatterns;

param rolls {WIDTHS,PATTERNS} >= 0, default 0;

param order {WIDTHS} >= 0;
param overrun;

var Cut {PATTERNS} integer >= 0;

minimize TotalCut: sum {p in PATTERNS} Cut[p];

subject to OrderLimits {w in WIDTHS}:
    order[w] <= sum {p in PATTERNS} rolls[w,p] * Cut[p] <= order[w] + overrun;</pre>
```

$$b_i \le \sum_{j=1}^n a_{ij} X_j \le b_i + o$$

# AMPL Formulation (cont'd)

#### Explicit data (independent of model)

```
param rawWidth := 64.5 ;

param: WIDTHS: order :=
     6.77    10
     7.56    40
     17.46    33
     18.76    10 ;

param nPatterns := 9 ;

param rolls: 1 2 3 4 5 6 7 8 9 :=
     6.77    0 1 1 0 3 2 0 1 4
     7.56 1 0 2 1 1 4 6 5 2
     17.46 0 1 0 2 1 0 1 1 1
     18.76 3 2 2 1 1 1 0 0 0 ;

param overrun := 6 ;
```

# **AMPL Command Language**

Model + data = problem instance to be solved

```
ampl: model cut.mod;
ampl: data cut.dat;
ampl: option solver cplex;
ampl: solve;
CPLEX 12.8.0.0: optimal integer solution; objective 20
3 MIP simplex iterations
0 branch-and-bound nodes
ampl: option omit_zero_rows 1;
ampl: option display_1col 0;
ampl: display Cut;
4 13 8 5 9 2
```

# Command Language (cont'd)

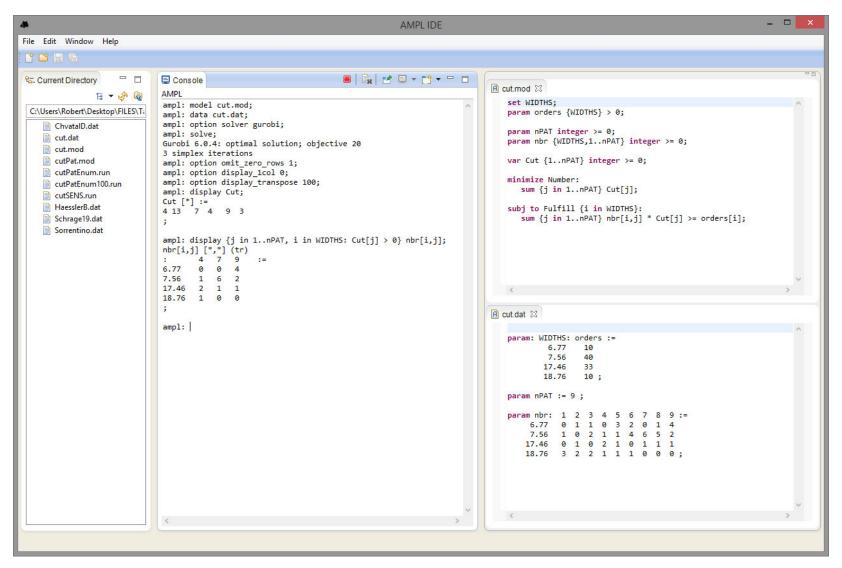
#### Solver choice independent of model and data

```
ampl: model cut.mod;
ampl: data cut.dat;
ampl: option solver gurobi;
ampl: solve;
Gurobi 8.0.0: optimal solution; objective 20
7 simplex iterations
1 branch-and-cut nodes
ampl: option omit_zero_rows 1;
ampl: option display_1col 0;
ampl: display Cut;
2 1 4 13 8 5 9 1
```

# Command Language (cont'd)

#### Results available for browsing

# **IDE for Command Language**



# **AMPL Script**

#### Trade off two objectives

- ❖ Minimize rolls cut
  - \* Fewer rolls, fewer overruns but less efficient patterns
- \* Minimize waste
  - \* More efficient patterns but more rolls, more overruns

```
minimize TotalCut:
    sum {p in PATTERNS} Cut[p];
minimize TotalWaste:
    sum {p in PATTERNS}
    Cut[p] * (rawWidth - sum {w in WIDTHS} w * rolls[w,p]);
```

#### AMPL Script

# Parametric Analysis of Tradeoff

#### Minimize rolls cut

Set large overrun limit in data

#### Minimize waste

- \* Reduce overrun limit 1 roll at a time
- If there is a change in number of rolls cut
  - \* record total waste (increasing)
  - \* record total rolls cut (decreasing)
- Stop when no further progress possible
  - \* problem becomes infeasible *or*
  - \* total rolls cut falls to the minimum
- Report table of results

# Parametric Analysis (cont'd)

Script (setup and initial solve)

```
model cutTradeoff.mod:
data cutTradeoff.dat;
set OVER default {} ordered by reversed Integers;
param minCut;
param minCutWaste;
param minWaste {OVER};
param minWasteCut {OVER};
param prev_cut default Infinity;
option solver gurobi;
option solver_msg 0;
objective TotalCut;
solve >Nul;
let minCut := TotalCut;
let minCutWaste := TotalWaste;
objective TotalWaste;
```

# Parametric Analysis (cont'd)

### Script (looping and reporting)

```
for \{k \text{ in overrun } ... 0 \text{ by } -1\}
   let overrun := k;
   solve >Nul;
   if solve_result = 'infeasible' then break;
   if TotalCut < prev_cut then {</pre>
      let OVER := OVER union {k};
      let minWaste[k] := TotalWaste;
      let minWasteCut[k] := TotalCut;
      let prev_cut := TotalCut;
   if TotalCut = minCut then break;
}
printf 'Min%3d rolls with waste%6.2f\n\n', minCut, minCutWaste;
printf ' Over Waste Cut\n';
printf {k in OVER}: '%4d%8.2f%5d\n', k, minWaste[k], minWasteCut[k];
```

#### AMPL Script

# Parametric Analysis (cont'd)

#### Script run

# **AMPL API Program**

#### Solve by pattern enumeration

- Set up a cutting-stock model
- \* Read data
  - \* demands, raw width
  - \* orders, overrun limit
- Compute data: all "good" patterns
  - \* extract widths from demand list
  - \* enumerate all patterns having waste < smallest width
- Solve for the cutting plan

### Hybrid approach

- Control & pattern enumeration in a programming language
- Model & modeling expressions in AMPL
- Visualization of results in a programming language

### **Preface**

#### AMPL APIS

- APIs for "all" popular languages
   C++, C#, Java, MATLAB, Python, R
- Common overall design
- ❖ Common implementation core in C++
- Customizations for each language and its data structures

### Key to examples: Python and R

- AMPL entities
- ❖ AMPL API Python/R objects
- ❖ AMPL API Python/R methods
- Python/R functions etc.

#### **AMPL Model File**

### Same pattern-cutting model

```
param nPatterns integer > 0;
set PATTERNS = 1..nPatterns; # patterns
set WIDTHS;
                             # finished widths
param order {WIDTHS} >= 0;  # rolls of width j ordered
                    # permitted overrun on any width
param overrun;
                                  # width of raw rolls to be cut
param rawWidth;
param rolls {WIDTHS,PATTERNS} >= 0, default 0;
                                  # rolls of width i in pattern j
var Cut {PATTERNS} integer >= 0;  # raw rolls to cut in each pattern
minimize TotalRawRolls: sum {p in PATTERNS} Cut[p];
subject to FinishedRollLimits {w in WIDTHS}:
   order[w] <= sum {p in PATTERNS} rolls[w,p] * Cut[p] <= order[w] + overrun;
```

# Some Python Data

A float, an integer, and a dictionary

```
roll_width = 64.5
overrun = 6
Orders = {
    6.77: 10,
    7.56: 40,
    17.46: 33,
    18.76: 10
}
```

... can also work with lists and Pandas dataframes

### Some R Data

### A float, an integer, and a dataframe

```
roll_width <- 64.5
overrun <- 6
orders <- data.frame(
  width = c( 6.77, 7.56, 17.46, 18.76 ),
  demand = c( 10, 40, 33, 10 )
)</pre>
```

# Pattern Enumeration in Python

Load & generate data, set up AMPL model

```
def cuttingEnum(dataset):
    from amplpy import AMPL

# Read orders, roll_width, overrum

exec(open(dataset+'.py').read(), globals())

# Enumerate patterns

widths = list(sorted(orders.keys(), reverse=True))
patmat = patternEnum(roll_width, widths)

# Set up model

ampl = AMPL()
ampl.option['ampl_include'] = 'models'
ampl.read('cut.mod')
```

#### Pattern Enumeration in R

#### Load & generate data, set up AMPL model

```
cuttingEnum <- function(dataset) {
   library(rAMPL)

# Read orders, roll_width, overrun

source(paste(dataset, ".R", sep=""))

# Enumerate patterns

patmat <- patternEnum(roll_width, orders$width)
   cat(sprintf("\n%d patterns enumerated\n\n", ncol(patmat)))

# Set up model

ampl <- new(AMPL)
   ampl$setOption("ampl_include", "models")
   ampl$read("cut.mod")</pre>
```

# Pattern Enumeration in Python

#### Send data to AMPL

```
# Send scalar values
ampl.param['nPatterns'] = len(patmat)
ampl.param['overrun'] = overrun
ampl.param['rawWidth'] = roll_width

# Send order vector
ampl.set['WIDTHS'] = widths
ampl.param['order'] = orders

# Send pattern matrix
ampl.param['rolls'] = {
    (widths[i], 1+p): patmat[p][i]
    for i in range(len(widths))
    for p in range(len(patmat))
}
```

#### Pattern Enumeration in R

#### Send data to AMPL

```
# Send scalar values
ampl$getParameter("nPatterns")$set(ncol(patmat))
ampl$getParameter("overrun")$set(overrun)
ampl$getParameter("rawWidth")$set(roll_width)

# Send order vector
ampl$getSet("WIDTHS")$setValues(orders$width)
ampl$getParameter("order")$setValues(orders$demand)

# Send pattern matrix
df <- as.data.frame(as.table(patmat))
df[,1] <- orders$width[df[,1]]
df[,2] <- as.numeric(df[,2])
ampl$getParameter("rolls")$setValues(df)</pre>
```

# Pattern Enumeration in Python

### Solve and get results

```
# Solve
ampl.option['solver'] = 'gurobi'
ampl.solve()

# Retrieve solution
CuttingPlan = ampl.var['Cut'].getValues()
cutvec = list(CuttingPlan.getColumn('Cut.val'))
```

#### Pattern Enumeration in R

### Solve and get results

```
# Solve
ampl$setOption("solver", "gurobi")
ampl$solve()

# Retrieve solution
CuttingPlan <- ampl$getVariable("Cut")$getValues()
solution <- CuttingPlan[CuttingPlan[,-1] != 0,]</pre>
```

# Pattern Enumeration in Python

#### Display solution

```
# Prepare solution data
summary = {
    'Data': dataset,
    'Obj': int(ampl.obj['TotalRawRolls'].value()),
    'Waste': ampl.getValue(
                 'sum {p in PATTERNS} Cut[p] * \
                    (rawWidth - sum {w in WIDTHS} w*rolls[w,p])'
solution = [
    (patmat[p], cutvec[p])
    for p in range(len(patmat))
    if cutvec[p] > 0
# Create plot of solution
cuttingPlot(roll_width, widths, summary, solution)
```

#### Pattern Enumeration in R

#### Display solution

```
# Prepare solution data
data <- dataset
obj <- ampl$getObjective("TotalRawRolls")$value()
waste <- ampl$getValue(
    "sum {p in PATTERNS} Cut[p] * (rawWidth - sum {w in WIDTHS} w*rolls[w,p])"
)
summary <- list(data=dataset, obj=obj, waste=waste)
# Create plot of solution
cuttingPlot(roll_width, orders$width, patmat, summary, solution)
}</pre>
```

# Pattern Enumeration in Python

#### Enumeration routine

```
def patternEnum(roll_width, widths, prefix=[]):
    from math import floor

max_rep = int(floor(roll_width/widths[0]))

if len(widths) == 1:
    patmat = [prefix+[max_rep]]

else:
    patmat = []
    for n in reversed(range(max_rep+1)):
        patmat += patternEnum(roll_width-n*widths[0], widths[1:], prefix+[n])

return patmat
```

### Pattern Enumeration in R

#### Enumeration routine

```
patternEnum <- function(roll_width, widths, prefix=c()) {</pre>
  cur_width <- widths[length(prefix)+1]</pre>
  max_rep <- floor(roll_width/cur_width)</pre>
  if (length(prefix)+1 == length(widths)) {
      return (c(prefix, max_rep))
  } else {
      patterns <- matrix(nrow=length(widths), ncol=0)</pre>
      for (n in 0:max_rep) {
          patterns <- cbind(</pre>
               patterns,
               patternEnum(roll_width-n*cur_width, widths, c(prefix, n))
      return (patterns)
```

# Pattern Enumeration in Python

#### Plotting routine

#### Pattern Enumeration in R

#### Plotting routine

```
cuttingPlot <- function(roll_width, widths, patmat, summary, solution) {
  pal <- rainbow(length(widths))
  par(mar=c(1,1,1,1))
  par(mfrow=c(1,nrow(solution)))

for(i in 1:nrow(solution)) {
    pattern <- patmat[, solution[i, 1]]
    data <- c()
    color <- c()}</pre>
```

# Pattern Enumeration in Python

### Plotting routine (cont'd)

```
for p, (patt, rep) in enumerate(solution):
   for i in range(len(widths)):
      for j in range(patt[i]):
         vec = [0]*len(solution)
         vec[p] = widths[i]
         plt.barh(ind, vec, 0.6, acc,
                   color=colorlist[i%len(colorlist)], edgecolor='black')
         acc[p] += widths[i]
plt.title(summ['Data'] + ": " +
   str(summ['Obj']) + " rolls" + ", " +
   str(round(100*summ['Waste']/(roll_width*summ['Obj']),2)) + "% waste"
plt.xlim(0, roll_width)
plt.xticks(np.arange(0, roll_width, 10))
plt.yticks(ind, tuple("x {:}".format(rep) for patt, rep in solution))
plt.show()
```

AMPL API

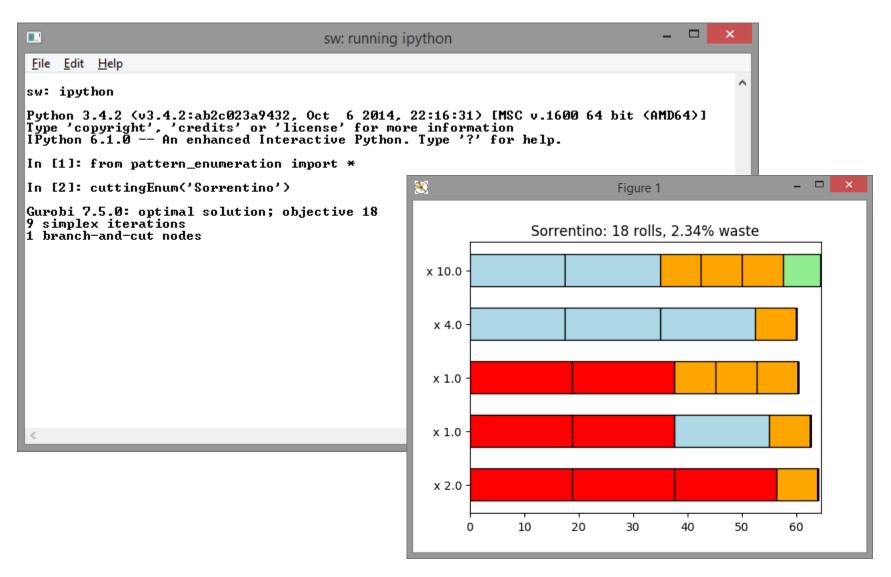
### Pattern Enumeration in R

### Plotting routine (cont'd)

```
for(j in 1:length(pattern)) {
    if(pattern[j] >= 1) {
        for(k in 1:pattern[j]) {
            data <- rbind(data, widths[j])
            color <- c(color, pal[j])
        }
    }
    }
    label <- sprintf("x %d", solution[i, -1])
    barplot(data, main=label, col=color,
            border="white", space=0.04, axes=FALSE, ylim=c(0, roll_width))
    }
    print(summary)
}</pre>
```

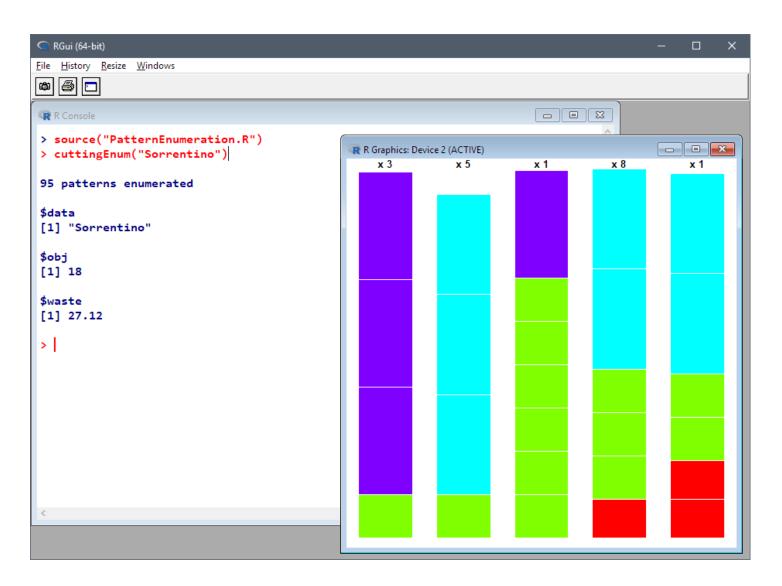
#### AMPL API

### Pattern Enumeration in Python



#### AMPL API

### Pattern Enumeration in R



# Embedded Python (a preview)

### Sending Python data to an AMPL model

- via AMPL API for Python
- via Python references in the AMPL model

#### Executing Python statements inside AMPL scripts

Generate specialized constraints for lot sizing

### Handling callbacks

- Write callback function in Python
- ❖ Export problem + callback, solve, import results

#### AMPL Model

Symbolic sets, parameters, variables, objective, constraints

```
diet.mod
# DATA
set FOOD;
set NUTR;
param cost {FOOD} > 0;
param f_min {FOOD} >= 0;
param f_max {j in FOOD} >= f_min[j];
param n_{\min} \{NUTR\} >= 0;
param n_max {i in NUTR} >= n_min[i];
param amt {NUTR,FOOD} >= 0;
# MODEL
var Buy {j in FOOD} >= f_min[j], <= f_max[j];</pre>
minimize Total_Cost:
    sum {j in FOOD} cost[j] * Buy[j];
subject to Diet {i in NUTR }:
    n_min[i] <= sum {j in FOOD} amt[i,j] * Buy[j] <= n_max[i];</pre>
```

# **Python Data**

#### Lists, dictionaries

## Sending Data to AMPL (API)

Call ampl methods to read model, send data

```
from amplpy import AMPL
ampl = AMPL()
ampl.read('diet.mod')
ampl.set['FOOD'] = food
ampl.param['cost'] = cost
ampl.param['f_min'] = f_min
ampl.param['f_max'] = f_max
ampl.set['NUTR'] = nutr
ampl.param['n_min'] = n_min
ampl.param['n_max'] = n_max
ampl.param['amt'] = {
    (n, f): amt[i][j]
    for i, n in enumerate(nutr)
    for j, f in enumerate(food)
}
ampl.solve()
```

# Sending Data to AMPL (Embedded)

### Move data correspondences into the model

```
dietpy.mod
# SYMBOLIC DATA WITH PYTHON LINKS
$SET[FOOD] { food };
$PARAM[cost{^FOOD}]{ cost };
$PARAM[f min{^FOOD}]{ f min }:
$PARAM[f_max{^FOOD}]{ f_max };
$SET[NUTR]{ nutr };
$PARAM[n_min{^NUTR}]{ n_min };
$PARAM[n_max{^NUTR}]{ n_max };
$PARAM[amt]{{
      (n, f): amt[i][j]
      for i, n in enumerate(nutr)
      for j, f in enumerate(food)
    }};
# MODEL
var Buy {j in FOOD } >= f_min [j], <= f_max [j];</pre>
. . . . . . .
```

## Sending Data to AMPL (Embedded)

### Process with PyMPL language extension

```
from amplpy import AMPL
from pympl import PyMPL
ampl = AMPL(langext=PyMPL())
ampl.read('dietpy.mod')
ampl.solve()
```

### **Executing Python inside AMPL**

### Fix AMPL variables according to Python variable

## **Executing Python inside AMPL**

Invoke Python generators for special lot-sizing constraints

```
$EXEC{
def mrange(a, b):
    return range(a, b+1)

s = ['s[{}]'.format(t) for t in mrange(0, NT)]
y = ['y[{}]'.format(t) for t in mrange(1, NT)]
d = [demand[t] for t in mrange(1, NT)]
if BACKLOG is False:
    WW_U_AMPL(s, y, d, NT, prefix='w')
else:
    r = ['r[{}]'.format(t) for t in mrange(1, NT)]
    WW_U_B_AMPL(s, r, y, d, NT, prefix='w')
};
```

```
ampl = AMPL(langext=PyMPL())
ampl.read('lotsize.mod')
ampl.solve()
```

## **Executing Python inside AMPL**

### Optional listing of generated constraints

```
var ws \{wi in 0..8\} = s[wi]:
var wr \{wi in 1..8\} = r[wi];
var wy \{wi in 1..8\} = y[wi];
param wD {1..8, 1..8};
data:
param wD :=
[1,1]400 [1,2]800 [1,3]1600 [1,4]2400 [1,5]3600 [1,6]4800 [1,7]6000 [1,8]7200
[2,1]0 [2,2]400 [2,3]1200 [2,4]2000 [2,5]3200 [2,6]4400 [2,7]5600 [2,8]6800
[3,1]0
       [3,2]0 [3,3]800 [3,4]1600 [3,5]2800 [3,6]4000 [3,7]5200 [3,8]6400
[4,1]0
       [4,2]0 [4,3]0 [4,4]800 [4,5]2000 [4,6]3200 [4,7]4400 [4,8]5600
[5,1]0
      [5,2]0 [5,3]0 [5,4]0 [5,5]1200 [5,6]2400 [5,7]3600 [5,8]4800
[6,1]0
      [6,2]0 [6,3]0 [6,4]0 [6,5]0 [6,6]1200 [6,7]2400 [6,8]3600
[7,1]0
       [7,2]0 [7,3]0 [7,4]0 [7,5]0 [7,6]0 [7,7]1200 [7,8]2400
[8,1]0
       [8,2]0 [8,3]0
                        [8,4]0 [8,5]0 [8,6]0 [8,7]0
                                                              [8,8]1200
model;
```

### **Executing Python inside AMPL**

Optional listing of generated constraints (cont'd)

```
var wa {1..8};
var wb {1..8};
subject to wXY {wt in 1..8}: wa[wt] + wb[wt] + wy[wt] >= 1;
subject to wXA {wk in 1..8, wt in wk..min(8, wk+8-1): wD[wt,wt]>0}:
    ws[wk-1] >=
        sum {wi in wk..wt} wD[wi,wi] * wa[wi]
        - sum {wi in wk..wt-1} wD[wi+1,wt] * wy[wi];
subject to wXB {wk in 1..8, wt in max(1, wk-8+1)..wk: wD[wt,wt]>0}:
    wr[wk] >=
        sum {wi in wt..wk} wD[wi,wi] * wb[wi]
        - sum {wi in wt+1..wk} wD[wt,wi-1] * wy[wi];
```

### **Callbacks**

#### AMPL model with embedded Python

```
$SET[OBJECTS]{list(range(n))};
$SET[RESOURCES]{list(range(m))};
$PARAM[value] {value, i0=0};
$PARAM[weight] {{
    (i, j): weight[i][j]
    for i in range(n)
    for j in range(m)
}};
$PARAM[capacity]{capacity, i0=0};
var x {OBJECTS} >= 0 <= 1 integer;</pre>
subject to Limits {r in RESOURCES}:
  sum {i in OBJECTS} weight[i, r] * x[i] <= capacity[r];</pre>
maximize Profit:
  sum {i in OBJECTS} value[i] * x[i];
```

### **Callbacks**

### Callback function

```
def callback(model, where):
    global solinfo
    if where == gpy.GRB.Callback.MIPSOL: # new MIP solution found
        nodecnt = model.cbGet(gpy.GRB.Callback.MIPSOL_NODCNT)
        obj = model.cbGet(gpy.GRB.Callback.MIPSOL_OBJ)
        solinfo.append((nodecnt, obj))  # append to solution list
        solcnt = model.cbGet(gpy.GRB.Callback.MIPSOL_SOLCNT)
        print(
            '** New solution at node {:.0f}, obj {:g}, sol {:d} **'.format(
                nodecnt, obj, solcnt
            ),
            file=log # write to log.txt
        if time()-t0 \geq= 10 and solcnt \geq= 2:
            model.terminate()
                                          # stop solution process and return
```

### **Callbacks**

### AMPL Python API: Export problem, solve, import solution

```
from pympl import PyMPL
from amplpy import AMPL
import gurobipy as gpy
ampl = AMPL(langext=PyMPL())
ampl.read('multiknapsack.mod')
grb_model = ampl.exportGurobiModel()
grb_model.params.threads = 1
grb_model.params.timelimit = 10
t0 = time()
solinfo = [] # list to store objective values and node counts
log = open('log.txt', 'w')
grb_model.optimize(callback)
ampl.importGurobiSolution(grb_model)
ampl.display('{i in OBJECTS: x[i] != 0} x[i]')
print(solinfo) # print stored objective values and node counts
```