



인공지능 실습

Chapter 5. Adversarial Search

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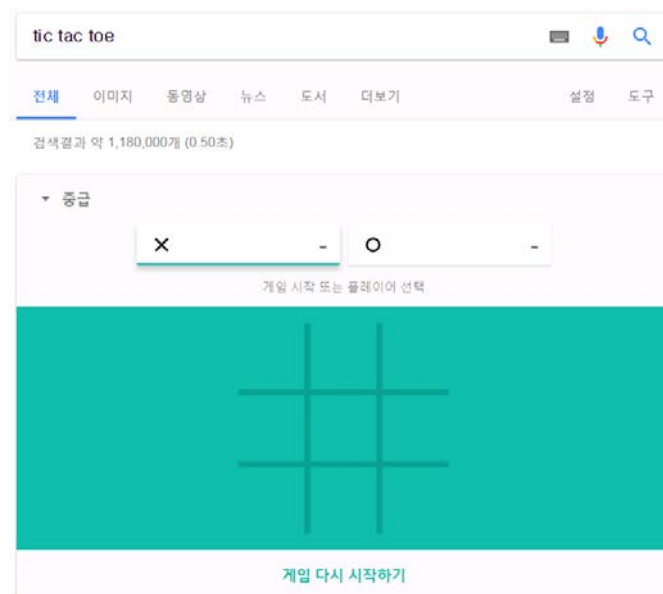
- 5.1. Adversarial Search 개요
- 5.2. Minimax & Expectimax Algorithm
- 5.3. Pruning
- 5.4. Imperfect Real-time Decisions
- 5.5. Multi-player Game

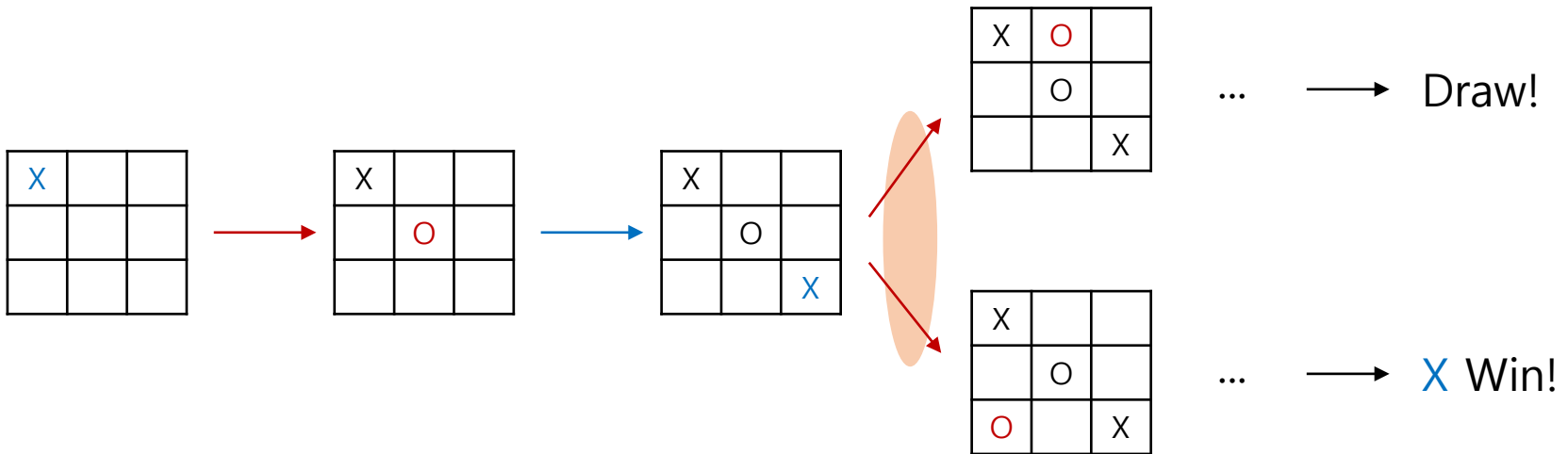
5.1. Adversarial Search 개요

- 두 명의 player: X(선수) and O(후수)
- 맵: 3 x 3 grid
- 승리조건: 연속된 세 칸에 본인의 돌을 놓으면 승리
- 맵에 빈 자리가 없는데도 승패가 결정이 나지 않으면 비김

구글에서 "tic tac toe" 검색
or

<https://playtictactoe.org/>





- 미래의 상대방의 action을 고려해 본인의 action을 결정해야 함

Tic-tac-toe의 Game Tree

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- Agent가 X(선수) 일 때

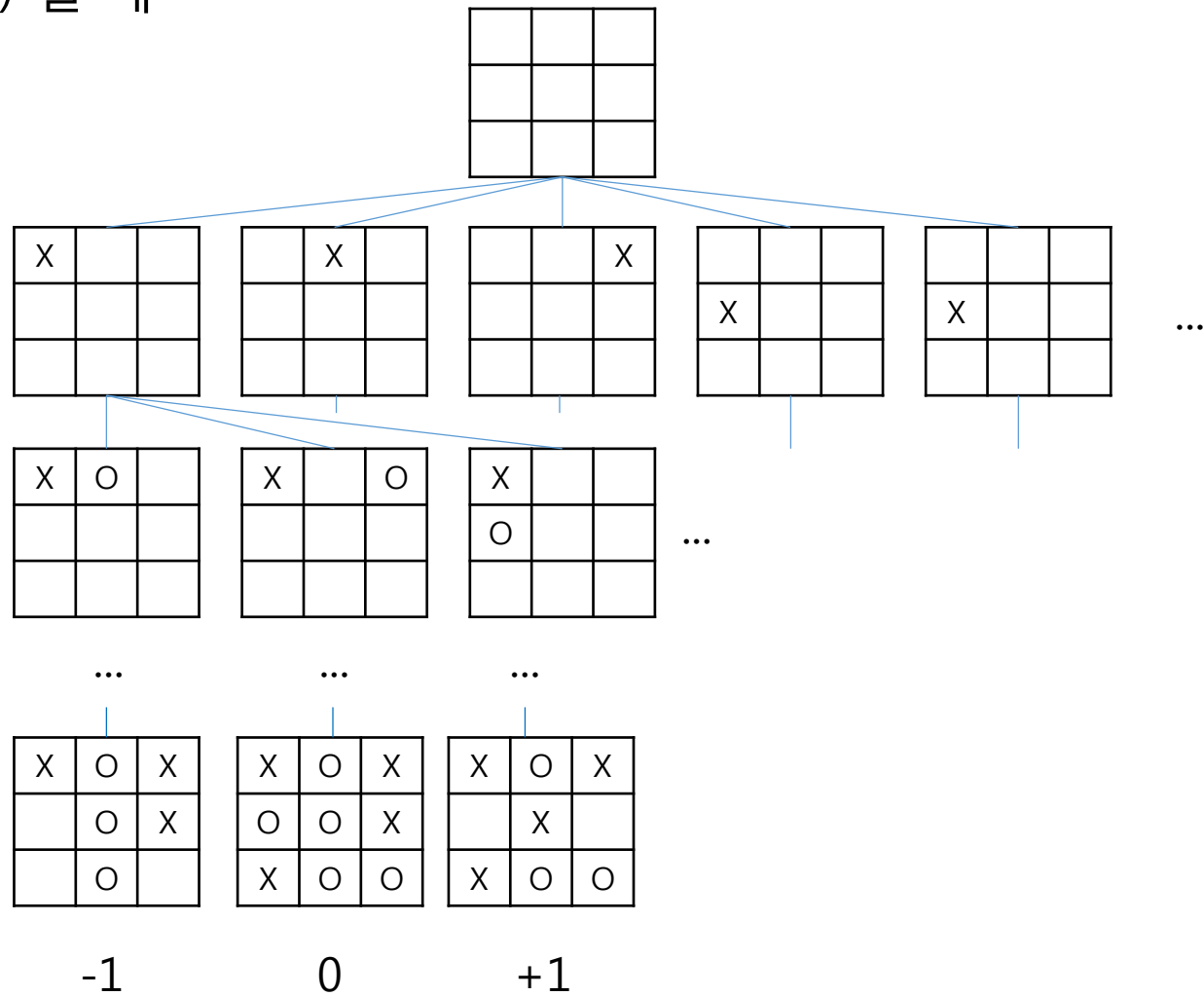
MAX (X)

MIN (O)

MAX (X)

종료 상태

Utility



- Deterministic action (v.s. MDP)
- Fully observable environment (v.s. POMDP)
- Zero-sum at the end (v.s. typical search, MDP)
 - 각 플레이어의 **utility**는 서로 동일하거나 반대
 - 중간 action에 대한 utility는 정의되지 않음
 - Chess의 목표는 많은 말을 잡는 것이 아닌 체크메이트(King)를 하는 것이 목적
 - Typical search 및 MDP에 비해 어려운 부분
- Turn-taking of players
 - Typical search에서는 goal state에 도달하기 위한 action sequence를 찾으면 됨
 - Opponent가 agent의 action에 맞춰 대응하므로 action sequence가 아닌 **policy**를 찾아야 함

- Deterministic policy: $\pi_p(s) = \text{Action}(s)$

action that player p takes in state s

- Stochastic policy: $\pi_p(s, a) \in [0, 1]$

probability of player p taking action a in state s

IBM DeepBlue (Chess) – 1996



Google AlphaGo – 2016



- 사람과의 대결을 통해 AI 기술수준을 보여 준 사례
- 학습의 명확한 기준(승패)이 존재

- s_{start} : 시작 상태
- $\text{Actions}(s)$: s 에서 취할 수 있는 모든 action
- $\text{Cost}(s, a)$: s 에서 a 를 취하는 cost(비용)
- $\text{Succ}(s, a)$: s 에서 a 를 취했을 때 다음 상태
- $\text{IsEnd}(s)$: s 의 종료 조건 만족 여부
- $\text{Utility}(s)$: 종료 상태 s 에서 agent의 utility
- $\text{Player}(s)$: s 에서 action을 취할 수 있는 player

Players = {**agent** (your program); **opp** (opponent)}

MAX

e.g. 알파고

MIN

e.g. 이세돌

- Agent 및 opponent가 차례로 1회씩 action을 선택하는 게임
 - 상대방의 선택을 알 수 없음

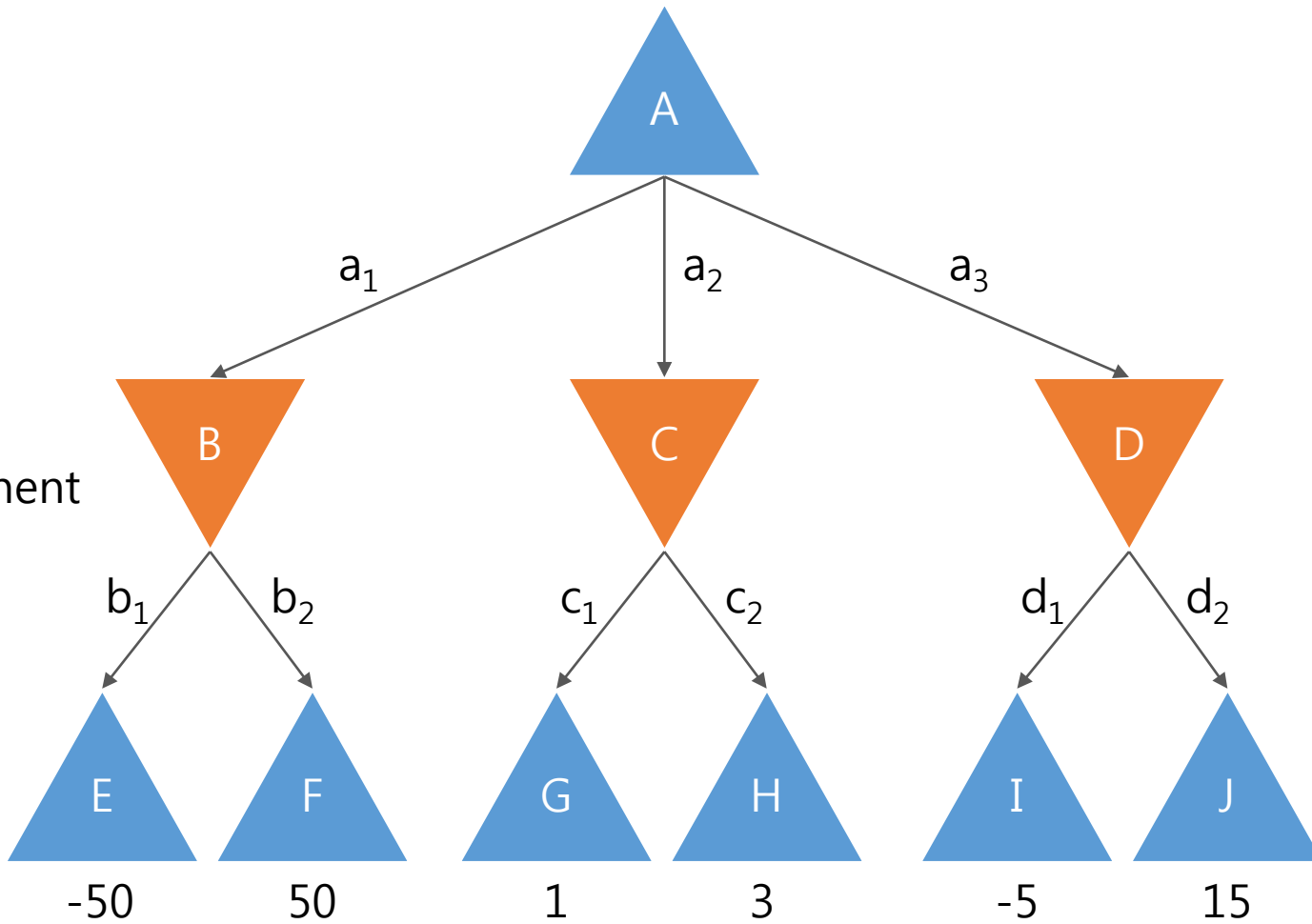
MAX
agent

선택

MIN
opponent

선택

결과



5.2. Minimax & Expectimax Algorithm

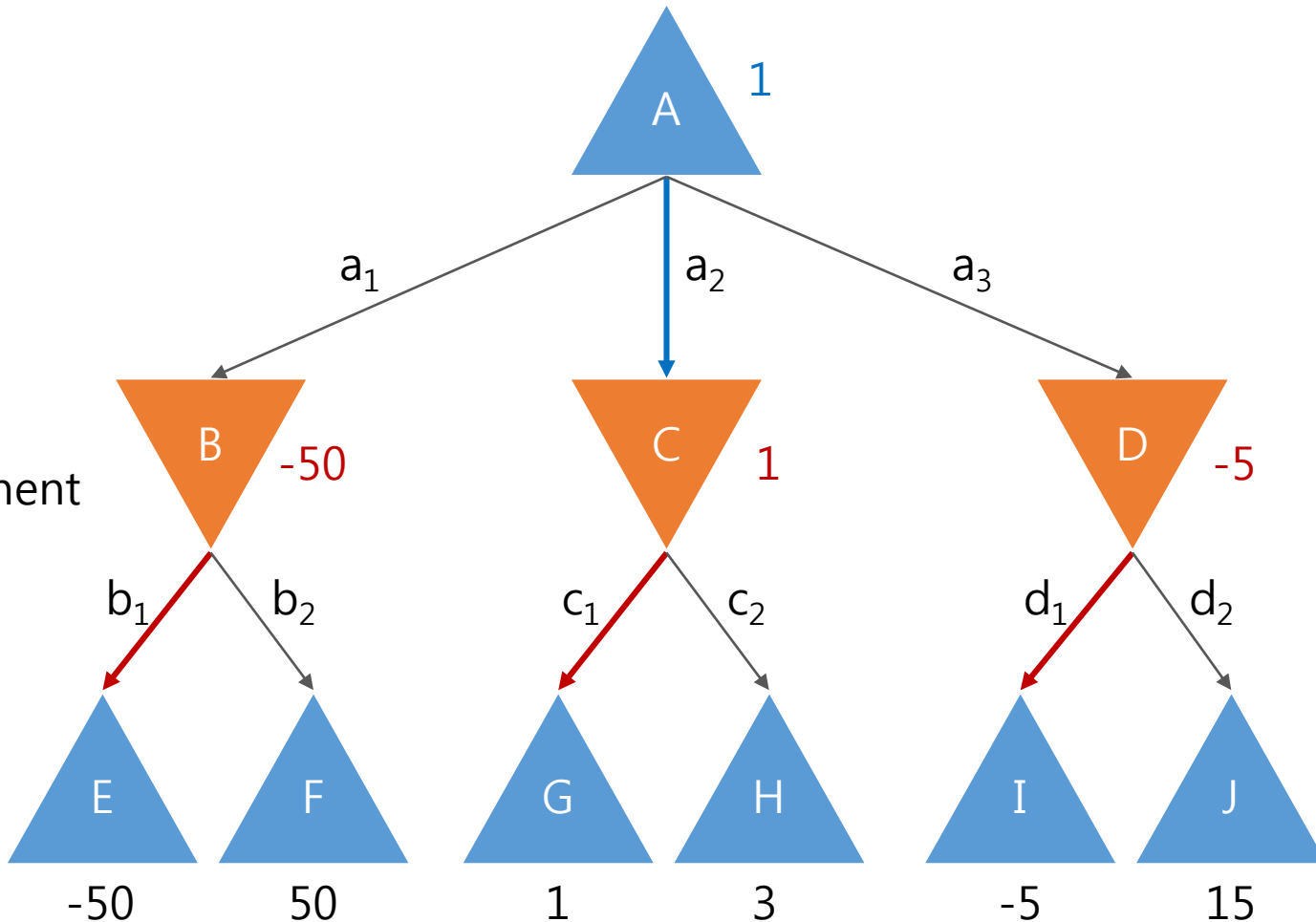
Minimax Algorithm 예

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- 최악의 경우를 가정
 - Opponent는 value를 낮추는 선택을 한다

MAX
agent

MIN
opponent



- Minimax policy

$$\pi_{\max}(s) = \arg \max_{a \in \text{Action}(s)} V_{\max, \min}(\text{Succ}(s, a))$$

$$\pi_{\min}(s) = \arg \min_{a \in \text{Action}(s)} V_{\max, \min}(\text{Succ}(s, a))$$

- Value function for minimax

$$V_{\max, \min}(s) = \begin{cases} \text{Utility}(s) & \text{IsEnd}(s) \\ \max_{a \in \text{Actions}(s)} V_{\max, \min}(\text{Succ}(s, a)) & \text{Player}(s) = \text{agent} \\ \min_{a \in \text{Actions}(s)} V_{\max, \min}(\text{Succ}(s, a)) & \text{Player}(s) = \text{opp} \end{cases}$$

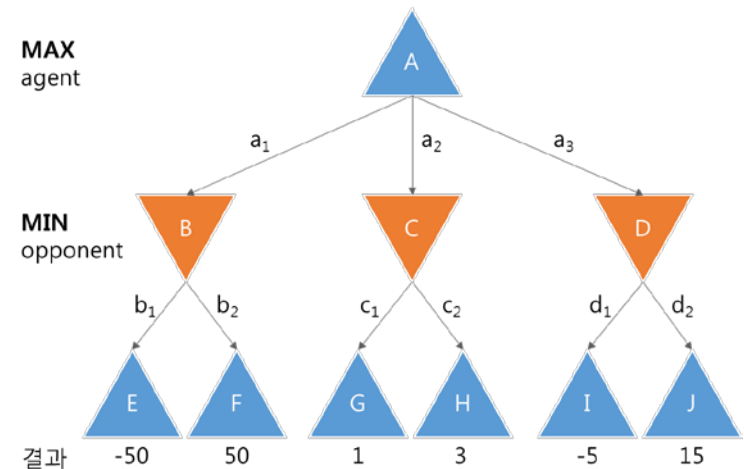
Minimax Algorithm Pseudocode

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```
def V(state)
    if is_end(state):
        return utility(state)
    if is_maxi_player(state):
        value =  $-\infty$ 
        for action in actions(state):
            value = max(value, V(next_state(state, action)))
    else:
        value =  $\infty$ 
        for action in actions(state):
            value = min(value, V(next_state(state, action)))
    return value
```

```
def policy(state)
    ...
```

- Simple Game을 대상으로 구현
 - 상태를 (플레이어, 상태 심볼)로 표현
 - 예. ('MIN', 'C')
- `simple_game/game.py`
 - `INT_INF = 1000000000000000`
 - `MAX_PLAYER = 'MAX'`
 - `MIN_PLAYER = 'MIN'`
 - `is_end(state)`
 - `utility(state)`
 - `get_initial_state()`
 - `get_next_state(state, action)`
 - `get_next_player(player)`
 - `get_possible_actions(state)`
 - `get_player_from_state(state)`



- `simple_game/play.py`
 - 실행 방법: `play.py minimax`
 - 두 상태에서의 value 및 action 확인
 - ('MAX', 'A')
 - ('MIN', 'B')
- `simple_game/ai.py`
 - **MinimaxAgent** 클래스의 **V** 및 **policy** 함수 구현
 - **V(state)**: 해당 state에서의 value 값 리턴
 - **policy(state)**: policy를 따라 해당 state에서의 action 결정

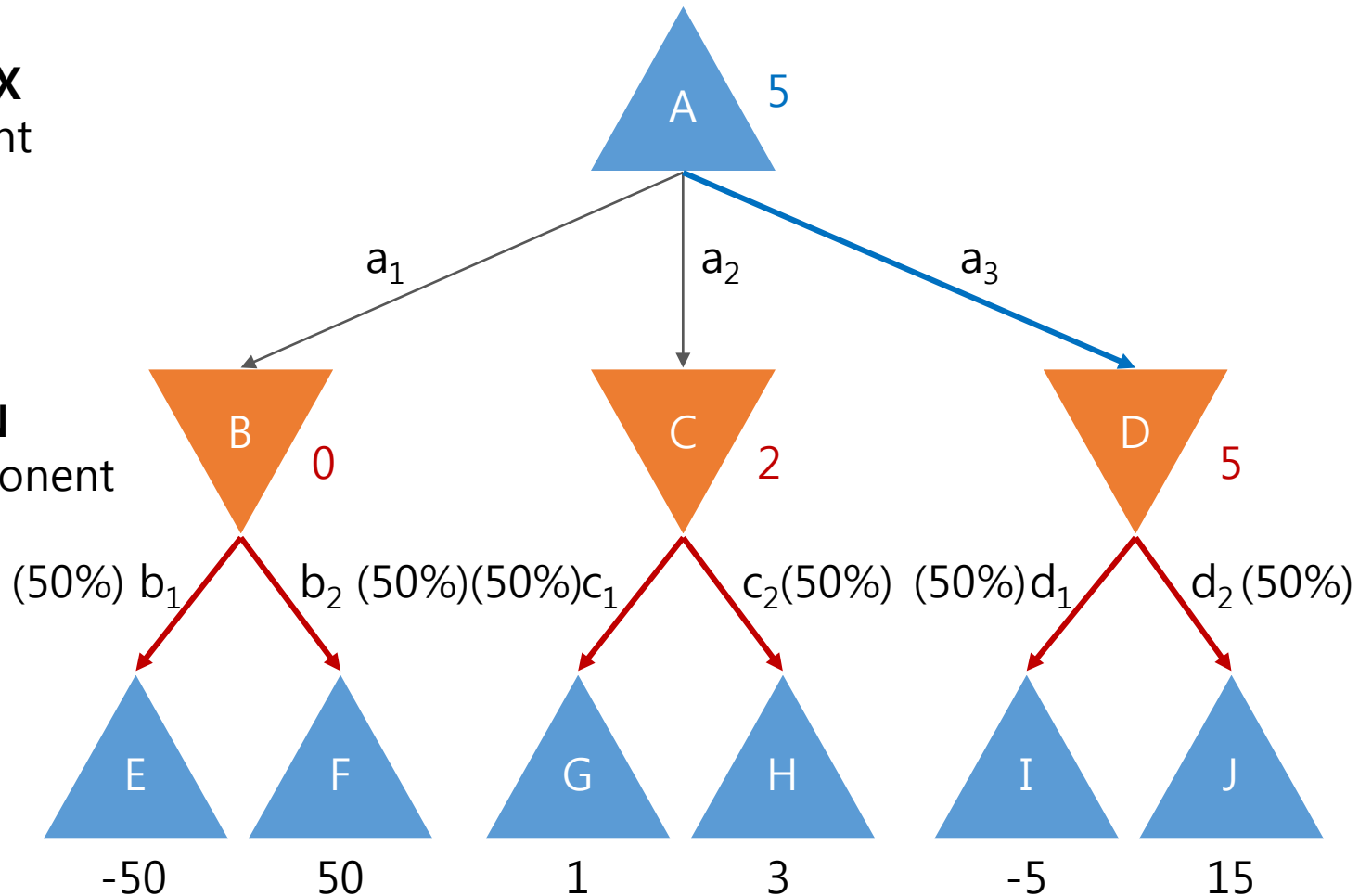
Expectimax Algorithm 예

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- 상대의 **random**한 선택을 가정

MAX
agent

MIN
opponent



- Expectimax policy

$$\pi_{\max}(s) = \arg \max_{a \in \text{Action}(s)} V_{\max, \text{opp}}(\text{Succ}(s, a))$$

$$\pi_{\text{opp}}(s, a) = \frac{1}{|\text{Action}(s)|} \text{ for } a \in \text{Actions}(s)$$

- Value function for expectimax

$$V_{\max, \text{opp}}(s) = \begin{cases} \text{Utility}(s) & \text{IsEnd}(s) \\ \max_{a \in \text{Actions}(s)} V_{\max, \text{min}}(\text{Succ}(s, a)) & \text{Player}(s) = \text{agent} \\ \sum_{a \in \text{Actions}(s)} \pi_{\text{opp}}(s, a) V_{\max, \text{opp}}(\text{Succ}(s, a)) & \text{Player}(s) = \text{opp} \end{cases}$$

Expectimax Algorithm Pseudocode

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```
def V(state)
    if is_end(state):
        return utility(state)
    if is_maxi_player(state):
        value =  $-\infty$ 
        for action in actions(state):
            value = max(value, V(next_state(state, action)))
    else:
        value = 0
        for action in actions(state):
            value += V(next_state(state, action)) / actions(state)
    return value
```

```
# Policy에 따라 결정한 action을 반환
def policy(state)
    ...
```

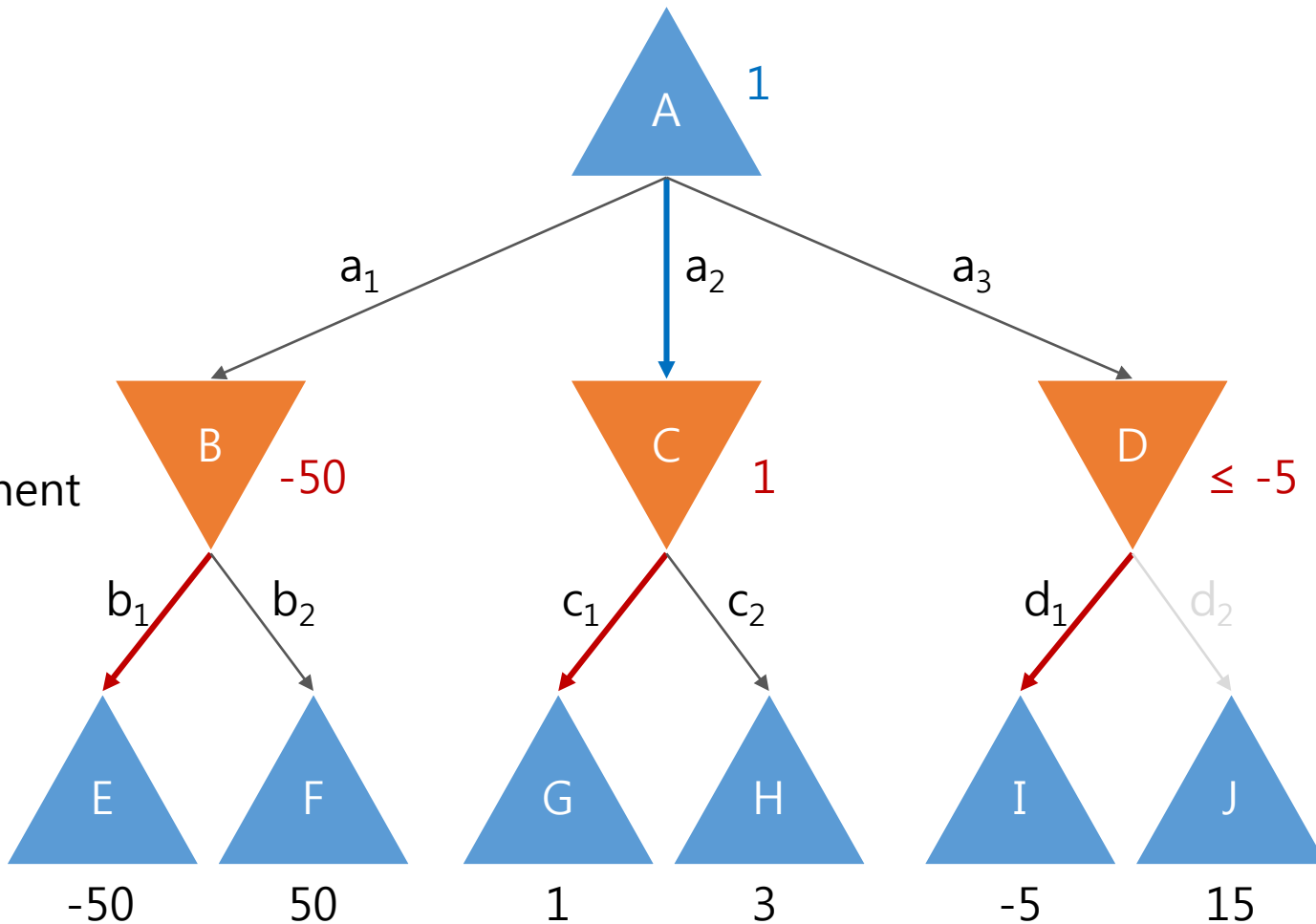
- `simple_game/play.py`
 - 실행 방법: `play.py expectimax`
 - 두 상태에서의 value 및 action 확인
 - ('MAX', 'A')
 - ('MIN', 'B')
- `simple_game/ai.py`
 - **ExpectimaxAgent** 클래스의 V 및 policy 함수 구현
 - `V(state)`: 해당 state에서의 value 값 리턴
 - `policy(state)`: policy를 따라 해당 state에서의 action 결정

5.3. Pruning

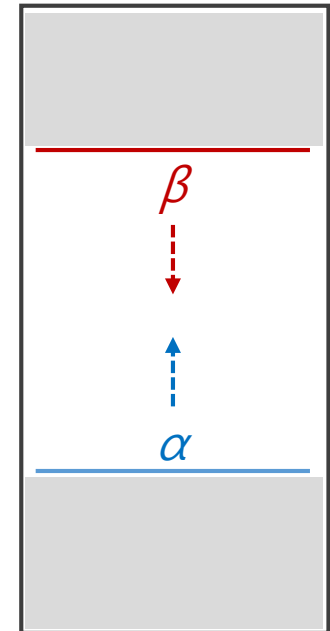
- d_2 는 고려하지 않아도 A에서의 선택은 바뀌지 않음

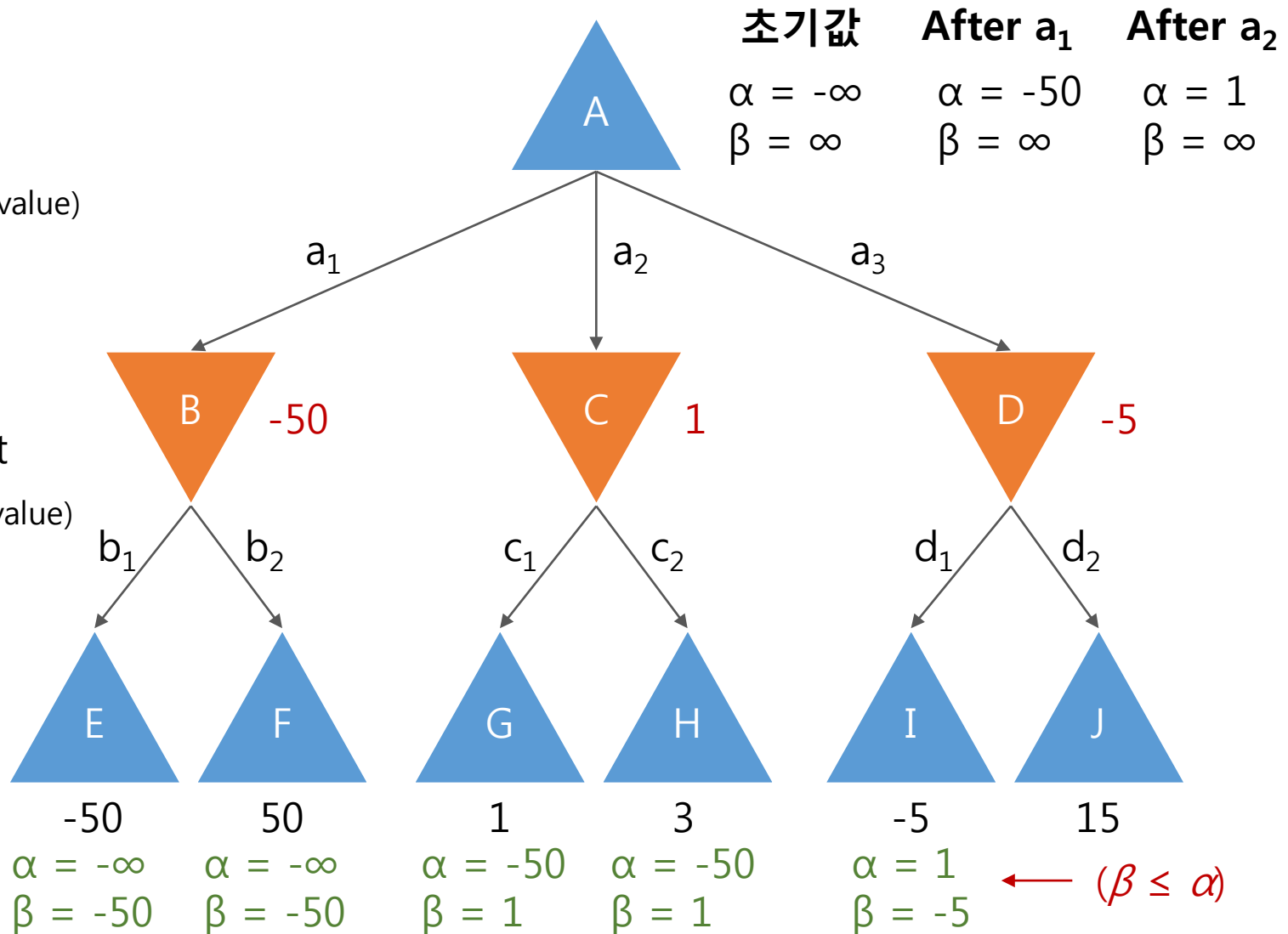
MAX
agent

MIN
opponent



- Search tree에서 최종 결정에 영향을 안 주는 부분은 무시함으로써 속도 향상
- α : lower bound on value of **max** node s
 - i.e. maximum value found so far
 - Initial $\alpha = -\infty$
 - $\alpha = \max(\alpha, \text{value})$
- β : upper bound on value of **min** node s
 - i.e. minimum value found so far
 - Initial $\beta = \infty$
 - $\beta = \min(\beta, \text{value})$
- Pruning: $\beta \leq \alpha$ 가 되면 해당 부분 search 중단



$$\alpha = \max(\alpha, \text{value})$$
$$\beta = \min(\beta, \text{value})$$


Alpha-beta Pruning Pseudocode

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```
def V(state,  $\alpha$ ,  $\beta$ )
    if is_end(state):
        return utility(state)
    if is_maxi_player(state):
        value =  $-\infty$ 
        for action in actions(state):
            value = max(value, V(next_state(state, action),  $\alpha$ ,  $\beta$ ))
             $\alpha$  = max( $\alpha$ , value)
            if  $\beta \leq \alpha$ : break
    else:
        value =  $\infty$ 
        for action in actions(state):
            value = min(value, V(next_state(state, action),  $\alpha$ ,  $\beta$ ))
             $\beta$  = min( $\beta$ , value)
            if  $\beta \leq \alpha$ : break
    return value
```

- `simple_game/play.py`
 - 실행 방법: `play.py pruning`
 - 두 상태에서의 value 및 action 확인
 - ('MAX', 'A')
 - ('MIN', 'B')
 - V 함수 호출 회수 minimax와 비교
- `simple_game/ai.py`
 - **PruningMinimaxAgent** 클래스의 V 및 policy 함수 구현
 - V(state): 해당 state에서의 value 값 리턴
 - policy(state): policy를 따라 해당 state에서의 action 결정

1	2	3
4	5	6
7	8	9

- Dictionary 형태의 상태표현
 - state['PLAYER']: player 저장
 - state[1]: 위치 1에 놓인 돌 \in ['X', 'O', '']
 - ...
 - state[9]: 위치 9에 놓인 돌 \in ['X', 'O', '']

- `tic_tac_toe/game.py`: Tic-tac-toe 게임 관련 유틸리티
- `tic_tac_toe/ai.py`
 - **`simple_game`의 AI 코드를 그대로 복사**
- `tic_tac_toe/play.py`: AI와 Tic-tac-toe 게임 플레이
 - `play.py` minimax
 - `play.py` pruning
- **AI의 quality 확인**
- **Minimax와 alpha-beta pruning의 탐색 시간 비교**
 - Minimax는 AI가 선타플레이어인 경우 굉장히 느림

Tic-tac-toe 최선의 전략

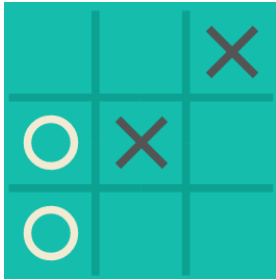
선수: 모서리나 중앙으로 공격

후수:

- 상대방이 모서리에 둔 경우 중앙으로 방어
- 상대방이 중앙에 둔 경우 모서리로 방어

- 플레이를 통해 AI가 위 전략을 구사하는지 확인
- 개발자가 이러한 전략을 모르더라도 최선의 전략을 구사하는 AI를 구현할 수 있다
 - 규칙(전문가) v.s. 계산 및 데이터

5.4. Imperfect Real-time Decisions



Tic-tac-toe

- Search tree 최대 노드 = $9!$



Chess

- 평균 action 수 = 35
- 평균 턴 수 = 50
- Search tree 평균 노드 수 = 35^{100}

복잡한 문제에서는 트리의 leaf를 보고 action을 결정하는 것이 불가능!

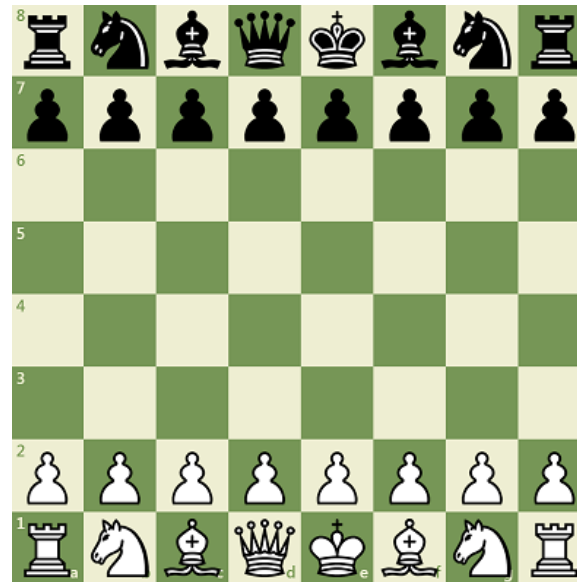


Imperfect real-time decisions

- Value function for depth-limited search

$$V_{\max, \min}(s, d) = \begin{cases} \text{Utility}(s) & \text{IsEnd}(s) \\ \text{Eval}(s) & d = 0 \\ \max_{a \in \text{Actions}(s)} V_{\max, \min}(\text{Succ}(s, a), d) & \text{Player}(s) = \text{agent} \\ \min_{a \in \text{Actions}(s)} V_{\max, \min}(\text{Succ}(s, a), d - 1) & \text{Player}(s) = \text{opp} \end{cases}$$

- Evaluation function $\text{Eval}(s)$
 - $V_{\max, \min}(s)$ 에 대한 추정(estimation)값
 - Search 문제에서의 $\text{FutureCost}(s)$ 와 유사한 개념
- Use: at state s , call $V_{\max, \min}(s, d_{\max})$



Chess pieces

	King (∞)
	Queen (9)
	Rook (5)
	Bishop (3)
	Knight (3)
	Pawn (1)

- $\text{Eval}(s) = \text{material} + \text{mobility} + \text{king-safety} + \text{center-control}$
 - $\text{material} = 10^{100}(K - K') + 9(Q - Q') + 5(R - R') + 3(B - B' + N - N') + 1(P - P')$
 - $\text{mobility} = 0.1 (\text{num-legal-moves} - \text{num-legal-moves}')$
- 체스에서는 이러한 hand-tuned evaluation function이 잘 작동함

Evaluation Function for Tic-tac-toe

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Board

1	2	3
4	5	6
7	8	9

Heuristic array

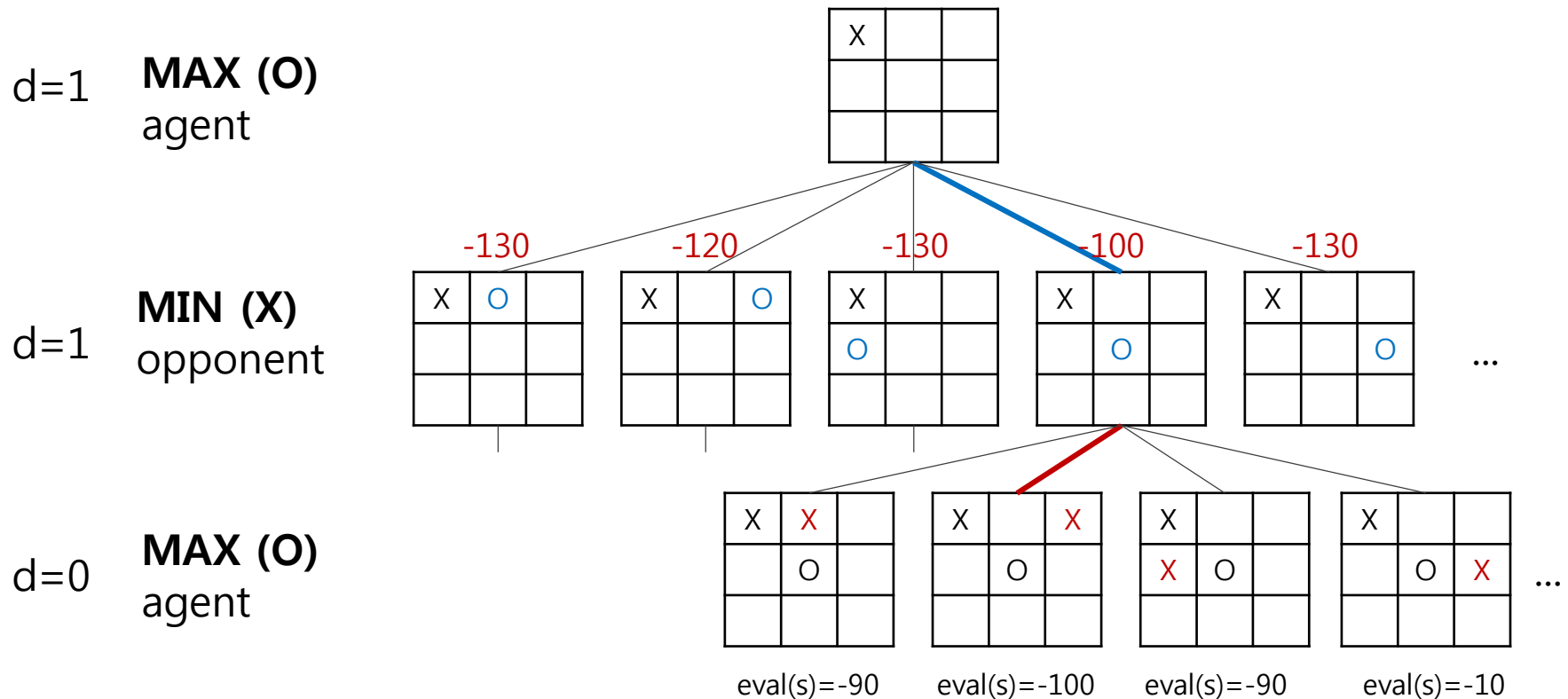
MAX	MIN			
	0	1	2	3
0	0	-10	-100	-1000
1	10	0	0	0
2	100	0	0	0
3	1000	0	0	0

```
def eval(state):  
    value = 0  
    for 승리조건 of board:  
        maxs = 승리조건에 놓여있는 MAX의 돌 수  
        mins = 승리조건에 놓여있는 MIN의 돌 수  
        value += heuristic[maxs][mins]  
    return value
```

Evaluation Function for Tic-tac-toe

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- Agent가 후수(O)



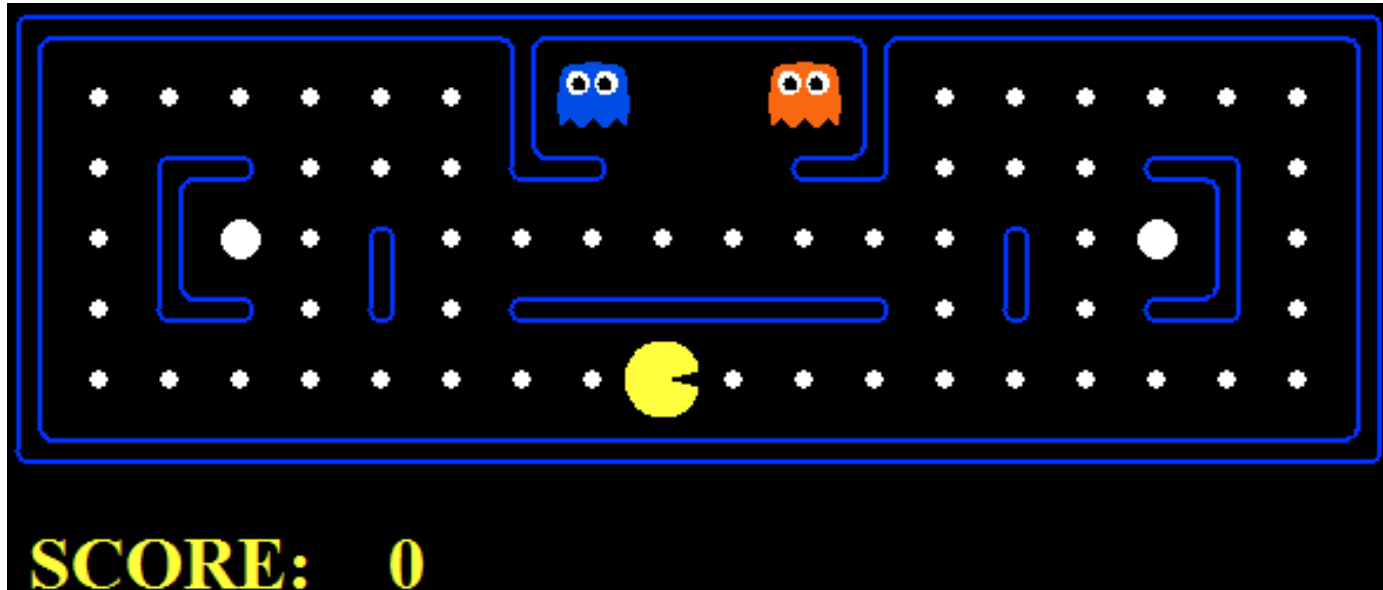
Agent는 O를 중앙(5)에 배치

- `tic_tac_toe/game.py`: Tic-tac-toe 게임 관련 유틸리티
- `tic_tac_toe/ai.py`: **DepthLimitedMinimaxAgent** 구현
 - Minimax를 복사해 수정
 - 위 evaluation function을 구현한 eval(s) 사용
- `tic_tac_toe/play.py`: AI와 Tic-tac-toe 게임 플레이
 - `play.py limited 1`
- 선택되는 action의 quality 및 탐색 속도 비교 (v.s. Minimax)
- Alpha-beta pruning에도 적용 (with higher depth)

Evaluation Function Design

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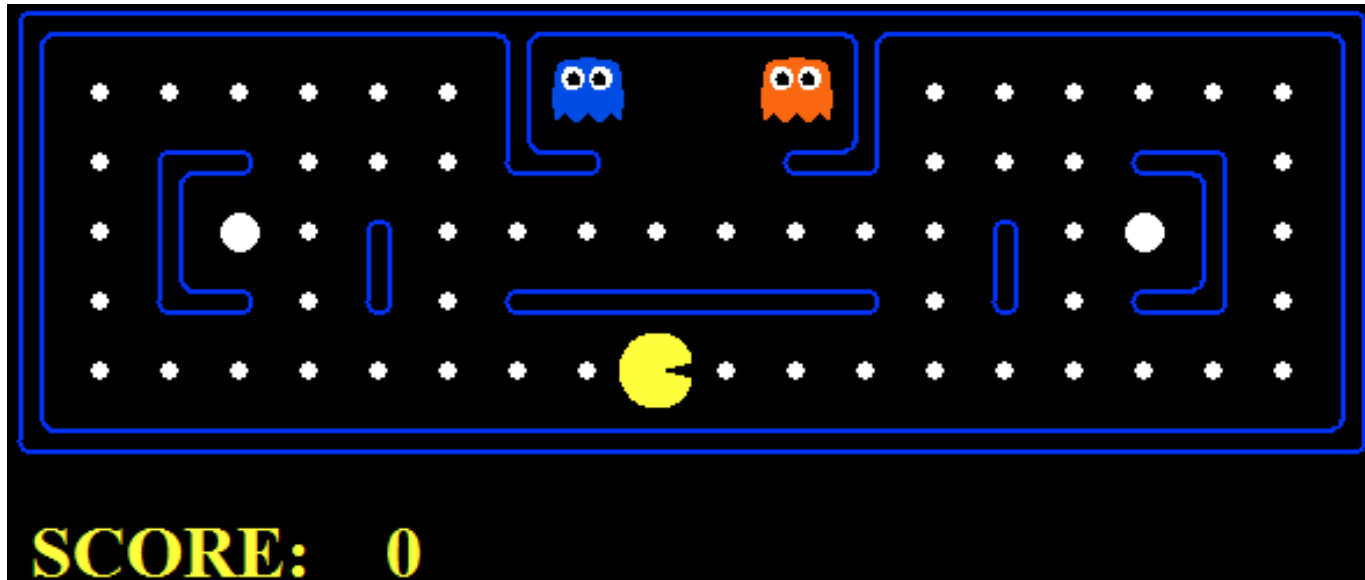
- Pacman (d = 2) with **simple** evaluation function
 - 두 칸 앞 까지 밖에 못 봄



Evaluation Function Design

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- Pacman (d = 2) with **better** evaluation function
 - 추가 feature: Ghost와의 거리, Food와의 거리, 등등...



- Feature vector: $\phi(s) \in \mathbb{R}^d$

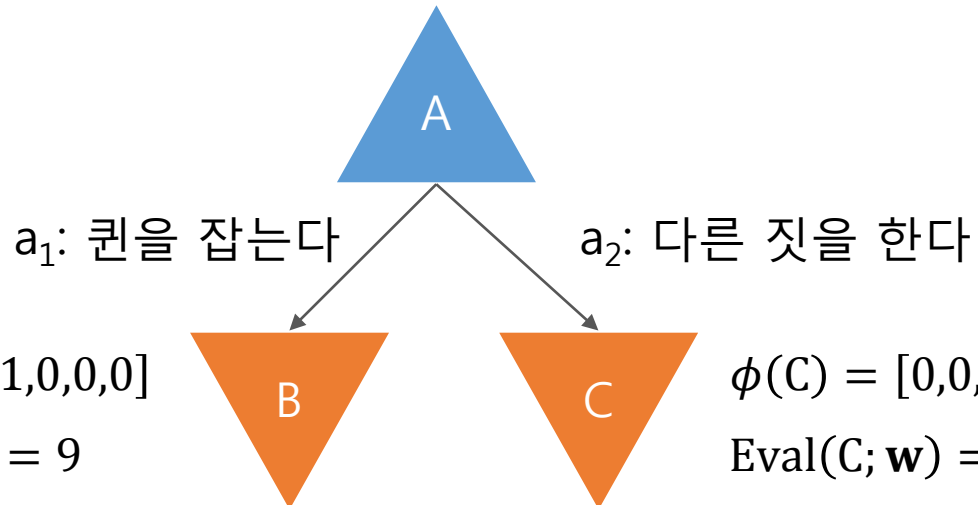
$$\phi_1(s) = K - K' \quad \phi_2(s) = Q - Q' \quad \dots$$

- Linear evaluation function

$$\text{Eval}(s; \mathbf{w}) = \mathbf{w} \cdot \phi(s)$$

예. $\mathbf{w} = [10^{100}, 9, 5, 3, 1]$

MAX
agent



MIN
opponent

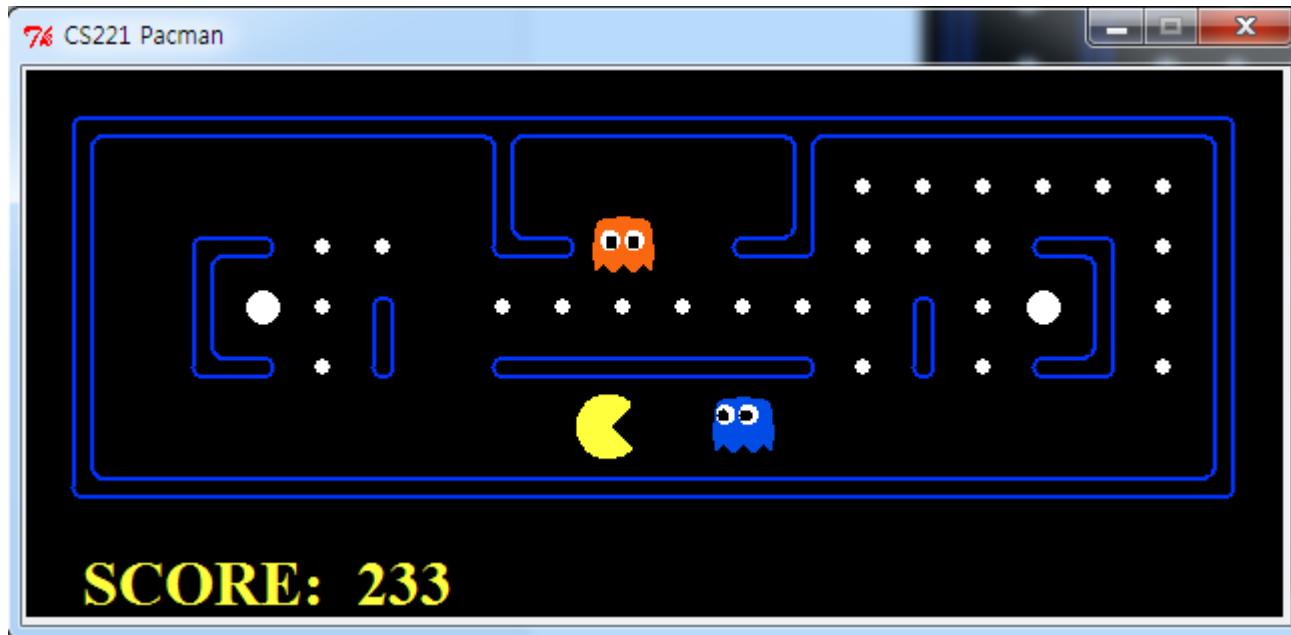
$$\begin{aligned} \phi(B) &= [0, 1, 0, 0, 0] \\ \text{Eval}(B; \mathbf{w}) &= 9 \end{aligned}$$

$$\begin{aligned} \phi(C) &= [0, 0, 0, 0, 0] \\ \text{Eval}(C; \mathbf{w}) &= 0 \end{aligned}$$

5.5. Multi-player Game

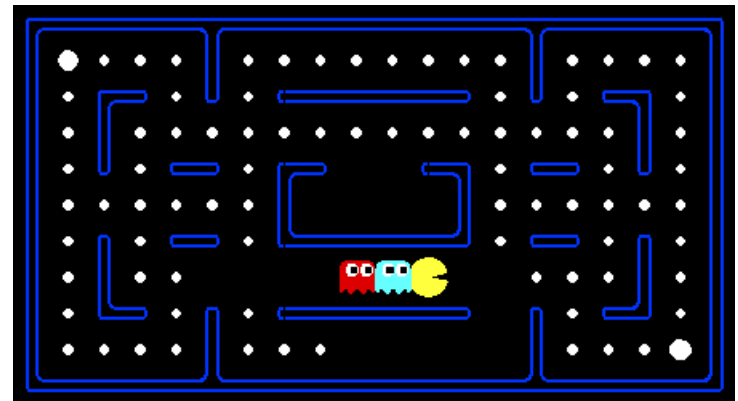
Play Pacman!

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- Play Pacman manually!
 - `python pacman.py -l smallClassic`
- 화살표키로 이동

- Pacman with multiple ghosts
 - a_0 : Agent
 - a_1, \dots, a_n : 각각의 ghost
 - Multiple MIN layers (one for each ghost)
- 승리조건: 모든 food를 먹음
- 점수
 - 1프레임에 -1점
 - Food를 먹으면 +10점
 - 승리하면 +500점
 - 패배하면 -500점
- 패배조건: Ghost와 만남
- Capsule을 먹으면 일정 시간 동안 Ghost에 면역 (+200점)



- Value function for multi-player **minimax** game

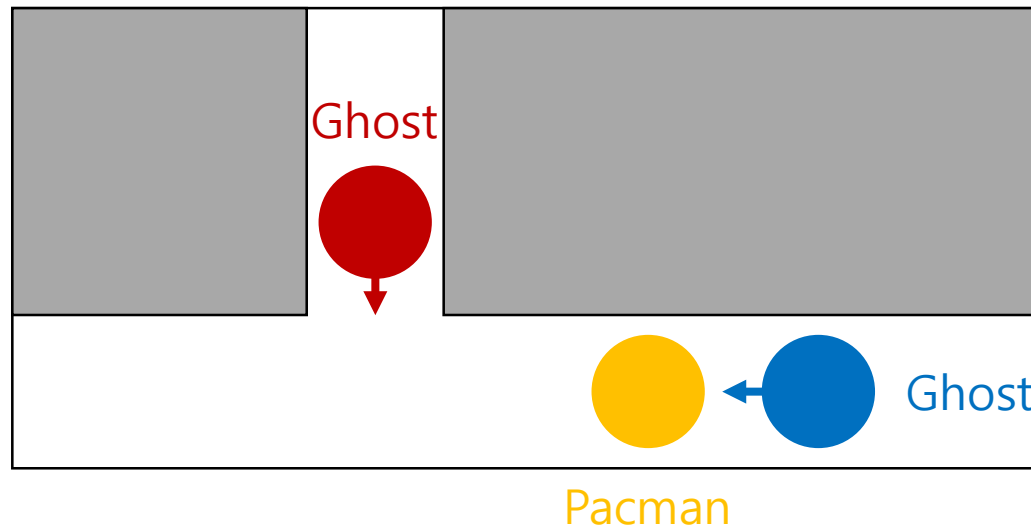
$$V_{\max, \min}(s, d) = \begin{cases} \text{Utility}(s) & \text{if IsEnd}(s) \\ \text{Eval}(s) & \text{if } d = 0 \\ \max_{a \in \text{Actions}(s)} V_{\max, \min}(\text{Succ}(s, a), d) & \text{if Player}(s) = a_0 \\ \min_{a \in \text{Actions}(s)} V_{\max, \min}(\text{Succ}(s, a), d) & \text{if Player}(s) = a_1 \dots, a_{n-1} \\ \min_{a \in \text{Actions}(s)} V_{\max, \min}(\text{Succ}(s, a), d - 1) & \text{if Player}(s) = a_n \end{cases}$$

- Value function for multi-player **expectimax** game

$$V_{\max, \text{opp}}(s, d) = \begin{cases} \text{Utility}(s) & \text{if IsEnd}(s) \\ \text{Eval}(s) & \text{if } d = 0 \\ \max_{a \in \text{Actions}(s)} V_{\max, \text{opp}}(\text{Succ}(s, a), d) & \text{if Player}(s) = a_0 \\ \sum_{a \in \text{Actions}(s)} \pi_{\text{opp}}(s, a) V_{\max, \text{opp}}(\text{Succ}(s, a), d) & \text{if Player}(s) = a_1 \dots, a_{n-1} \\ \sum_{a \in \text{Actions}(s)} \pi_{\text{opp}}(s, a) V_{\max, \text{opp}}(\text{Succ}(s, a), d - 1) & \text{if Player}(s) = a_n \end{cases}$$

Minimax and Expectimax

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- 다음 turn에서 Pacman의 action은? west or east?
 - Minimax case
 - Expectimax case
- `python pacman.py -p MinimaxAgent -l trappedClassic -a depth=3`

- `pacman.py`

- `-p <Agent 타입>`: ReflexAgent, MinimaxAgent, AlphaBetaAgent, ExpectimaxAgent
- `-l <Map 타입>`: mediumClassic, openClassic
- `-a depth=<숫자>`
- 예. `python pacman.py -p ReflexAgent -l mediumClassic`
- 예. `python pacman.py -p MinimaxAgent -l mediumClassic -a depth=2`

- `submission.py`

- Agents
 - ReflexAgent
 - MinimaxAgent: minimax
 - AlphaBetaAgent: minimax + alpha-beta pruning
 - ExpectimaxAgent: expectimax
- Agent의 `getAction(gameState)` 함수로 action 결정
- 모든 agent는 기본적으로 depth-limited search

- `gameState.getLegalActions()`
 - Returns the legal actions for the agent specified
 - Subset of [NORTH, SOUTH, EAST, WEST]
- `gameState.generateSuccessor(agentIndex, action)`
 - Returns the successor state after the specified agent takes the action
 - Pac-Man is always agent 0
- `gameState.isWin()` and `gameState.isLose()`
- `gameState.getNumAgents()`
 - Returns the total number of agents in the game
- `gameState.getScore()`
 - Returns the score corresponding to the current state of the game
- `self.evaluationFunction(gameState)`
 - Returns the game score of gameState
- `self.index`: Player index
- `self.depth`: Maximum depth

```
class ReflexAgent(Agent):
    def getAction(self, gameState):
        # Collect legal moves and successor states
        legalMoves = gameState.getLegalActions()

        # Choose one of the best actions
        scores = [self.evaluationFunction(gameState, action) for action in legalMoves]
        bestScore = max(scores)
        bestIndices = [index for index in range(len(scores)) if scores[index] == bestScore]
        chosenIndex = random.choice(bestIndices) # Pick randomly among the best

        return legalMoves[chosenIndex]
```

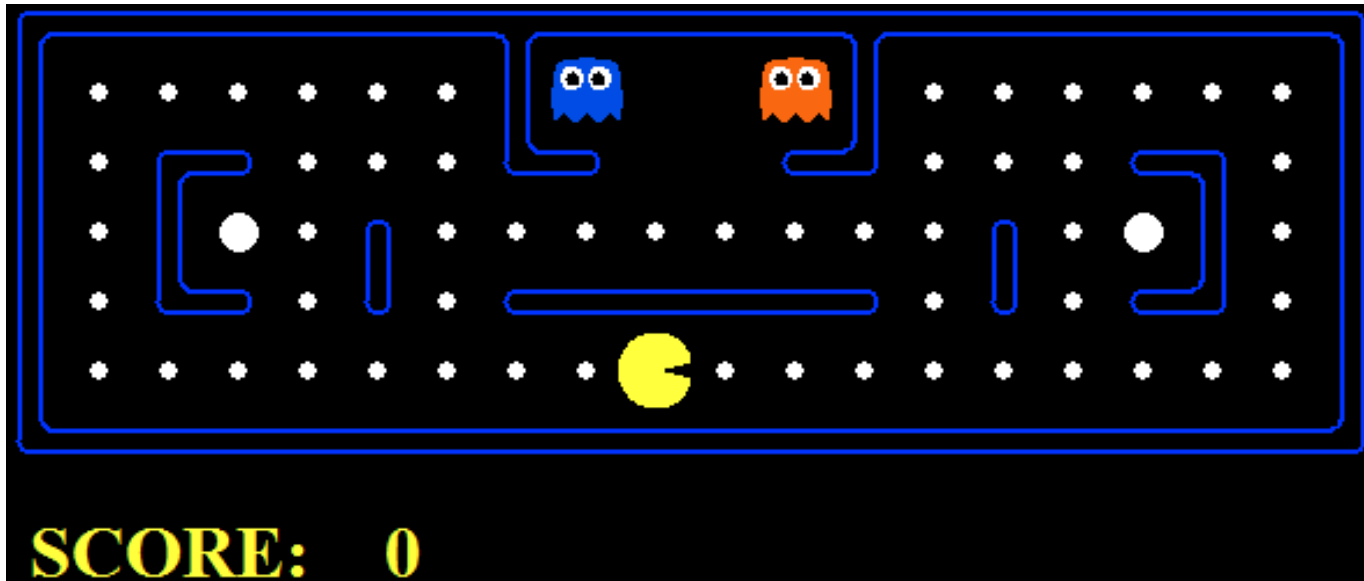
- getAction 함수: gameState에서 최선의 action을 return

- **MinimaxAgent의 getAction 함수 구현**
 - `python pacman.py -p MinimaxAgent -l smallClassic`
- **AlphaBetaAgent의 getAction 함수 구현**
 - `python pacman.py -p AlphaBetaAgent -l smallClassic -a depth=3`
- **ExpectimaxAgent의 getAction 함수 구현**
 - `python pacman.py -p ExpectimaxAgent -l smallClassic`

Pacman Competition

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- Pacman ($d = 2$) with **better** evaluation function
 - 추가 feature: Ghost와의 거리, Food와의 거리, 등등...



- `python pacman.py -l smallClassic -p ExpectimaxAgent -a evalFn=better -q -n 20`