Anthony Polloreno[1] and Graeme Smith[2]
[1] *Research House*
[2] *University of Waterloo,*
(Dated: May 19, 2025)

Recent works have demonstrated fundamental bounds on broadband sensing protocols. In particular, [**?**] established that the integrated quantum Fisher information is at most quadratic in time, while [**?**] established an upper bound on any algorithm solving a particular kind of AC sensing problem and considers the performance of this algorithm as a kind of Grover speedup. We establish that the first result implies the second, that the asymptotic bound can be achieved without Grover's algorithm and that any protocol that saturates the bound on IQFI gives an algorithm that saturates the bound on [**?**].

I. INTRODUCTION

II. BOUND ON IQFI

In [**?**] the authors prove a quadratic in time bound on integrated quantum Fisher information. The intuition, is generally that the IQFI for a sequence of N gates is bounded as $NT/\delta t$ with δt the gate time, with an error term. To illustrate how the Trotter-error bound fixes the step size δt, consider that the total error typically has two competing contributions:

$$f(\delta t) \;=\; \frac{C_1}{\delta t} \;+\; C_2\, B^2\, \delta t, \tag{1}$$

where C_1 and C_2 are constants that absorb all other factors, and B is the strength of the relevant Hamiltonian term (e.g. the magnetic field), and higher order terms are neglible. The first term grows large as $\delta t \to 0$, whereas the second term grows large as $\delta t \to \infty$. To find the optimal δt, we set the derivative of $f(\delta t)$ with respect to δt to zero:

$$\frac{d}{d(\delta t)}\left(\frac{C_1}{\delta t} + C_2 B^2 \delta t\right) \;=\; -\frac{C_1}{(\delta t)^2} \;+\; C_2 B^2 \;=\; 0. \tag{2}$$

Solving for δt yields

$$\delta t \;=\; \sqrt{\frac{C_1}{C_2}}\;\frac{1}{B}. \tag{3}$$

Thus, the optimal Trotter-step size δt is *inversely* proportional to B. Physically, as B increases, commutator errors in the Trotter expansion grow larger, and one must take smaller steps δt to maintain the same accuracy. This extends the result in [**?**] to a bound of BT^2 on the IQFI. To use a QFI based method to detect a magnetic field of size at least B_{min} requires QFI $\Omega(1/B_{min}^2)$, so that this bound implies

$$BT^2 = \Delta\omega / B_{min}^2. \tag{4}$$

Solving for T recovers the the bound in [**?**], for a single qubit. This bound, presented by Polloreno et al. and the Grover-Heisenberg bound, will simply be referred to as the PGH bound for brevity.

Furthermore, we show that our bound implies their bound, and hence the two results are the same. Suppose that we have an algorithm that solves AC. Then, consider the binary search estimation protocol. Choose a field strength B_0, and use AC to decide if there is a field larger than B_0. If there is, apply AC to sense $2B_0$. If there isn't, try to detect $B_0/2$. Repeat. For error ϵ this required $log(1/\epsilon)$ steps, performing binary search to detect the field.

III. QFI IS ASYMPTOTICALLY CONSTANT IN EACH BIN

A natural question to ask is if the QFI is constant in each bin - that is, can we use a fixed measurement basis to saturate the bound.

IV. EVERY IQFI PROTOCOL GIVES A SOLUTION TO AC

The QFI is measurable, and hence we can always produce an estimate of the IQFI simply by measuring the QFI across frequencies. We note that the misleading theorem in [**?**] looks at the QFI at zero, and concludes QFI is not sensitive to the broadband sensing problem. Indeed, the fact that the QFI is linear for all perturbatively small fields is precisely why the QFI is an excellent metric. For protocols which achieve quadratic IQFI for nonperturbative fields, this allows a form of hypothesis testing which lets us decide if a field is perturbative or not (without learning the field, and hence not violating the linear in time bound. That is, the problem AC is strictly an easier problem. Of course it isn't, we prove it's the same problem. In their email they say ours is easier. If both are easier, they're the same. Idiots.)

V. GX

VI. BROADBAND DETECTION DOES NOT REQUIRE A QUANTUM COMPUTER

At this point, we have seen that it is possible to perform broadband sensing without a quantum computer, for $\Delta\omega < \ B_{min}$. Intuitively for the gX protocol this is because we chosen to over integrate the signal in order to also acquire broadband sensitivity. Interestingly, while the square root factor in [?] is attributed to the space multiplexing performed by Grover search, we demonstrate time multiplexing across frequency bins. Consequently in our setting, the search can be performed in each bin in a classically distributed setting. However, we additionally note that the large field large bandwidth setting is not of particular interest. The algorithm designed in [?] is consequently of primary interest because, due to the equivalence between the theorems in [?] and [?], it is an estimation protocol that generates optimal IQFI. While the gX protocol does this by simply broadening the peak at gX, this protocol explicitly works to first sense for $T \sim 1/B_{min}$ in superposition, then searches the bins in parallel.

What this suggests is that the degree to which this is a quantum algorithm as opposed to a quantum sensing algorithm might be controlled by $\Delta\omega/B_{min}$. For the algorithm discussed in [?], they have constructed an algorithm that explicitly as bandwidth $\Delta\omega$, and hence must have QFI of $BT^2/\Delta\omega$ across the band.

VII. CONCLUSION