Market volatility prediction and the efficiency of the S&P 100 index option market*

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Most models of market volatility use either past returns or ex post volatility to forecast volatility. In this paper, the dynamic behavior of market volatility is assessed by forecasting the volatility implied in the transaction prices of Standard & Poor's 100 index options. We test and reject the hypothesis that volatility changes are unpredictable. However, while our statistical model delivers precise forecasts, abnormal returns are not possible in a trading strategy (based on daily out-of-sample volatility projections) which takes transaction costs into account, suggesting that predictable time-varying volatility is consistent with market efficiency.

1. Introduction

In many pricing models, risk is measured by market volatility, and changing market volatility affects the expected returns on all assets. Measuring the predictable variation in volatility may help explain why expected returns change through time. Many statistical models have been proposed to characterize the dynamic behavior of market volatility, including rolling variance

estimates [Officer (1973)], autoregressive conditional heteroskedasticity (ARCH) models [Engle (1982) and French, Schwert, and Stambaugh (1987)], and nonparametric methods [Pagan and Schwert (1990) and Harvey (1991)]. In practice, each of these approaches delivers a different volatility estimate.

Of course, true conditional volatility is unobservable. Consider the definition of conditional variance:

$$E\left[\left(r_t - E[r_t|\Omega_{t-1}]\right)^2|\Omega_{t-1}\right],\tag{1}$$

where r_t is the asset return realized at time t and $E[\cdot|\Omega_{t-1}]$ is the expectation conditioned upon the true information set, Ω, which is available at time $t-1$. Economic theory does not help us in specifying the true conditioning information. Furthermore, there is no guarantee that the volatility estimated with a subset of information is close to the true conditional volatility, because the standard law of iterated expectations does not apply to conditional variances.

In addition to specifying the correct information set, it is necessary to specify how information is transformed into expectations. It is possible that the true information set is large and true expectations are generated by a complicated nonlinear process. It is because of these difficulties that so many competing statistical models have been proposed.

We study the time variation in volatility implied by the Standard & Poor's 100 index (called 'S&P 100' or 'OEX') option prices.[1] The critical determinant of option price is the investors' assessment of the variance of stock return over the life of the option. In an efficient market, the market price of an option should reflect investors' expectations, conditioned on the true information, about future volatility. Given an option pricing model, we solve for the volatility implied by index option prices and characterize its time-series variation. In particular, we test the hypothesis that market volatility changes are unpredictable.

Our results indicate that market volatility changes are predictable in a statistical sense. We explore the economic implications of this predictability by assessing whether profits can be generated from an arbitrage trading strategy based on out-of-sample forecasts of volatility changes. While our forecasts are reasonably precise, the results of the trading simulations indi-

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[1]Implied volatility from option prices has been used in the finance literature in three ways. Some studies have examined how well implied volatility predicts future volatility [e.g., Latané and Rendleman (1976)]. Others have examined the contemporaneous association between changes in implied market volatility and changes in (a) certain macroeconomic variables [e.g., Schmalensee and Trippi (1978) and Franks and Schwartz (1988)] and (b) implied volatilities of individual stocks [e.g., Merville and Pieptea (1988)]. Finally, others have used changes in implied market volatility as a measure of abnormal activity in the marketplace [e.g., Poterba and Summers (1986), Day and Lewis (1988), and Schwert (1990)].

cate that, after transaction costs, arbitrage profits are not possible. These results support the notion that the S&P 100 index option market is efficient.

The paper is organized as follows. The second section discusses index option valuation and implied volatility estimation. The third section describes the data sources, primarily the S&P 100 index option transaction history from October 1985 through July 1989. An evaluation of the predictability of changes in market volatility is presented in section 4. Section 5 contains an economic analysis of whether the predicted changes in volatility can be used to generate abnormal rates of return in the S&P 100 index option market. Some concluding remarks are offered in the final section.

2. Conditional market volatility estimation

Conditional volatility is an ex ante measure, and there are many ways to estimate it. One approach measures ex post volatility and attempts to forecast ex post volatility using various instruments. For example, French, Schwert, and Stambaugh (1987) estimate the standard deviation of daily stock returns within a month and then fit a time-series model to the standard deviation estimates. The fitted values from the time-series model are the estimated conditional volatilities.

Another approach is to estimate conditional volatility from simultaneously observed stock and stock option prices. Index options, like stock options, are functions of six underlying parameters: the stock price (level) S, the exercise price of the option X, the time to expiration of the option T, the riskless rate of interest r, the standard deviation of the stock (index) return σ, and the amount and timing of any dividends paid during the option's life D:

$$c = f(S, X, T, r, \sigma, D), \tag{2}$$

where, for the sake of clarity, c represents the theoretical price of an index call option. Five of the six terms of the option pricing model are readily available. The exercise price and the time to expiration of the option are stated terms of the contract. The stock index level and the riskless rate of interest are easily-accessible market-determined values. Since firms tend to pay stable quarterly dividends at regular periodic intervals during the calendar year, little uncertainty exists about the dividend parameters for short-term index options. The unknown parameter in model (2) is the market volatility. If current market prices for the stock index and the index option reflect all available information, and if the option pricing model (2) is correctly specified, conditional market volatility can be estimated by equating the observed and model prices and inverting the option pricing equation.

In contrast to most previous approaches, the option-based approach does not require the specification of a time-series model that links ex post

volatility to ex ante volatility. This is a considerable advantage because the nature of the true forecasting model is unknown. The composition of the instrument set used in the prediction model, for example, might change through time. However, the use of implied volatility assumes that the option price reflects all available information and that the option pricing model is correctly specified.

The performance of a conditional volatility prediction model is usually measured by its ability to predict future ex post volatility. Day and Lewis (1990) find that implied volatility has incremental information regarding weekly S&P 100 index returns. They also compare the ability of implied volatilities and GARCH-based volatilities to provide out-of-sample forecasts of future volatility for the S&P 100 index. Lamoureux and Lastrapes (1990) conduct a similar analysis using ten stock options and find that the implied volatility measure outperforms the GARCH-based alternative for most of the stocks considered. Fleming (1991) shows that the implied volatility from S&P 100 index options generally provides an accurate forecast of future volatility over many different time horizons.

Our purpose, however, is not to compare the volatility predictions of various models. Indeed, since the true conditional volatility is unobservable, it is impossible to assess the accuracy of any particular model; forecasts can only be related to realized volatility. Our strategy is to assume that the implied volatility is a reasonable proxy for the conditional volatility, and our goal is to analyze the time variation in conditional volatility from the perspective of implied volatility.

The ability to use implied volatility as a proxy for conditional volatility depends on both the accuracy of the option pricing model and on the reliability of the information used in the estimation process. Harvey and Whaley (1991, 1992) examine a number of issues related to the estimation of implied volatility using S&P 100 index option prices. Two important issues addressed in their studies are that most of the previous empirical work on the S&P 100 index options uses (a) European-style formulas and/or (b) closing price data [e.g., Chance (1986), Day and Lewis (1988), and Franks and Schwartz (1988)]. European-style option pricing formulas are inappropriate because they fail to account for the early exercise premium of the American-style S&P 100 options.[2] Harvey and Whaley (1992) show that early exercise is commonplace for both calls and puts written on the S&P 100 index and that the early exercise premium can be quite large.

The use of closing price data is also inappropriate. Using closing price data poses two problems. First, the S&P 100 index option market closes at 3:15 PM CST, while the underlying stock market closes at 3:00 PM CST. Second,

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[2]The S&P 100 index options are the only American-style stock index options currently trading. All others have become European-style contracts.

the last transaction price for the option is a bid price or an ask price, depending on the motivation for the last transaction of the day.

Harvey and Whaley (1991, 1992) conclude that in order to ensure reliable estimation of implied volatility, (a) the option valuation method must be for American-style options and must account for the discrete cash dividend payments of the S&P 100 index, (b) simultaneous index option prices and stock index levels must be used, and (c) multiple option transactions rather than a single transaction should be used when estimating market volatility. We adopt all three recommendations in the estimation of implied market volatility in this paper.

Finally, there is an important caveat regarding the model underlying the implied volatility estimation. The American-style option valuation method that we propose in the next section maintains the Black–Scholes (1973) assumption that the variance rate of the stock index is constant over the life of the option. This assumption is violated in practice. In fact, investigating time variation in volatility is the very purpose of this study.

Nonetheless, there are two reasons for using a Black–Scholes partial differential equation framework. First, although the Black–Scholes constant variance rate assumption is violated in practice, the model's predictions are empirically indistinguishable from most stochastic volatility option pricing models when the options are at-the-money and have short times to expiration. Feinstein (1989) demonstrates that the Black–Scholes valuation framework can recover unbiased estimates of implied volatility from a stochastic volatility model such as Hull and White (1987). This result may not generalize, however, to other volatility specifications such as that examined by Wiggins (1987). Second, another objective is to examine S&P 100 option market efficiency. To do so, we forecast tomorrow's implied volatility, use the projected volatility to price options, and then execute a trading strategy based upon deviations of the market price from the model prices. If the trading strategy produces abnormal profits, it cannot be attributable to the option pricing model misspecification. The model can be viewed as a 'black box' that transforms price into implied volatility for forecasting purposes and then transforms the forecast of volatility back into price. On the other hand, if the trading strategy does not produce abnormal profits, model specification could be an issue. Given that implied volatility is a nonlinear transformation of option price, the best linear predictor of implied volatility is not necessarily the best predictor of option price.

3. Data

The S&P 100 index option market is the most active index option market in the world. Table 1 displays S&P 100 index option contract trading volume

Table 1

S&P 100 index option contract volume by year for the sample period March 11, 1983 through December 29, 1989.[a]

Year	Trading days	Call volume	Put volume	Total volume	Average daily volume
1983	205	5,145,533	5,450,131	10,595,664	51,686
1984	253	35,823,507	28,464,938	64,288,445	254,105
1985	252	55,795,875	35,009,055	90,804,930	360,337
1986	253	63,392,789	49,758,296	113,151,085	447,237
1987	253	55,158,010	46,669,067	101,827,077	402,479
1988	253	30,221,224	27,212,282	57,433,506	227,010
1989	252	25,642,938	26,798,563	52,441,501	208,101

[a]Source: *Market Statistics*, The Chicago Board Options Exchange.

from March 1983 through December 1989. These options were introduced in March 1983 and traded about 50,000 contracts per day during that year. Volume of trading increased substantially in the years following, peaking at a level of nearly 450,000 contracts per day in 1986. Index option trading dropped significantly after the October 1987 market crash. In 1989, the average daily trading volume was 208,101 contracts – high volume relative to other index option markets, but low relative to the pre-crash levels.

3.1. Option data sources

The data used in the estimation of implied market volatility come from several sources. First, transaction price data for the S&P 100 index options (calls and puts) for the period from October 1985 through July 1989 were obtained from the Chicago Board Options Exchange. Each transaction record contains (a) the option's identity (i.e., call/put indicator, exercise price, and expiration date), (b) the time of the transaction, (c) the transaction price, (d) the number of contracts traded, and (e) the S&P 100 index level at the time of the transaction. The sample is restricted to nearby, at-the-money call and put options with at least fifteen days to expiration.

Aside from option transaction information, the valuation model (2) requires information on the riskless rate of interest and expected dividend payments on the S&P 100 index. The proxy for the riskless rate of interest is the effective yield rate on the Treasury bill whose maturity is closest to the option expiration date or which has at least thirty days to maturity, whichever is greater. We use the actual cash dividend payments made during the life of

the option to proxy for the expected dividend payments. The cash dividend series for the S&P 100 index was obtained from Harvey and Whaley (1992).

3.2. S&P 100 index option valuation

Little has been written about the valuation of index options. The likely explanation for this is that stock index options are fundamentally the same as options on dividend-paying stocks, and published research in the theory of stock option pricing is deep. For example, Black and Scholes (1973) provide analytical solutions to the European call and put option pricing problems where the underlying stock pays no dividends. Merton (1973) proves that it is never optimal to exercise an American call on a non-dividend-paying stock early, so the Black–Scholes call option formula applies to American calls as well. Roll (1977), Geske (1979), and Whaley (1981) provide the valuation formula for an American call option on a dividend-paying stock. Although no analytical formulas have been derived for the American put option on either a non-dividend-paying or a dividend-paying stock, accurate approximations are possible using finite difference or binomial option pricing methods. [See, for example, Brennan and Schwartz (1977) or Cox, Ross, and Rubinstein (1979).]

There are some important differences, however, between stock option and stock index option valuation. Unlike stock options, exercising stock index options during the trading day is impractical. To illustrate, consider a put option on a stock. If the put is sufficiently far in-the-money during the trading day, the put option holder can initiate exercise by immediately buying the underlying stock. He then exercises his put option by calling his broker and instructing him to do so. At the end of the day, the exercise of the option is handled mechanically, and the put option holder delivers the share that he purchased earlier in the day and receives in cash the amount of the exercise price. In other words, by using this exercise procedure, the American option holder produces an early exercise cash flow equal to the exercise price of the option less the price paid for the stock during the day. Now, consider implementing the same procedure for an index put option. To exercise an in-the-money index put option early in the day, the put option holder must buy a stock portfolio early in the day and liquidate it at the end of the day because the index option contract has cash settlement. Obviously, the round-trip costs of such an exercise procedure are prohibitive.

In addition, the denomination of the option contract is so small, buying and selling the underlying stock portfolio is practically infeasible. For example, suppose we are considering exercising an S&P 100 put option with an exercise price of 250 when the underlying index level is 220. The denomination of the S&P 100 contract is 100 times the index level, so we need to buy

$22,000 worth of the S&P 100 portfolio for implicit delivery against the exercise of the put. The largest stock in the index is probably IBM, and, since IBM constitutes about 4% of the index value, we must purchase $880 worth of IBM. If IBM's share price is $100, then we must purchase 8.8 shares of IBM. Aside from the fact that we cannot purchase fractional shares of any stock, the transaction costs of odd lot trading are considerably higher than those of round lot trading. Hence, in general, exercise during the day will be avoided and index option holders will wait until the very end of the trading day to exercise an in-the-money option.

With early exercise opportunities occurring at discrete points in time, the valuation of S&P 100 index options is analytically tractable. Both the call and put options can be priced within the Geske–Johnson (1984) compound option valuation framework. The difficulty in using the analytical formulas, however, is that higher-order multivariate normal integrals must be evaluated numerically. The computational cost of such integral approximations may easily exceed the cost of using binomial and even finite difference option valuation techniques, so the implementation of the analytical formulas is of questionable value. In this study, a dividend-adjusted binomial method is used to compute the American-style S&P 100 index option prices.

3.2.1. Binomial method for pricing S&P 100 index options

The key to designing an efficient binomial method for pricing American-style index options where the index portfolio is allowed to pay discrete cash dividends is to define the stock index grid in terms of the index level net of the present value of the promised dividends. We begin by computing the current index level net of the present value of the promised dividends, that is,

$$S_0^x = S_0 - \sum_{i=1}^{n} D_i e^{-rt_i}, \tag{3}$$

where D_i (t_i) is the amount of (time to) the ith dividend paid during the option's life and S_0 is the current index level. Next, we set up the binomial lattice, beginning with S_0^x rather than S_0. That is, if the current index level net of dividends is S_0^x, the index level at the end of the next increment of time Δt is either uS_0^x or dS_0^x. If the number of time steps is defined as n (where $\Delta t = T/n$), there are $n + 1$ index levels at the option's expiration. The length of each interval or time step is Δt, and the factors u and d are defined as $u = e^{\sigma \sqrt{\Delta t}}$ and $d = 1/u$, with transition probabilities of up and down movements of $p = (r^* - d)/(u - d)$ and $1 - p$, respectively, where $r^* = e^{r\Delta t}$. In our application of the binomial method, the number of time steps is twice the number of days remaining in the option's life. Such a refined partitioning,

while computationally expensive, ensures very precise estimation of implied volatilites.

With the stock index level lattice (net of dividends) computed, the approximation method starts at the end of the option's life and works back to the present, one time increment, Δt, at a time. At the end of the option's life, the option value at each index level node is given by the intrinsic value of the option. In the case of a call, the option values are

$$C_{n,j}\left(S_{n,j}^x\right) = \begin{cases} S_{n,j}^x - X & \text{where} \quad S_{n,j}^x > X, \\ 0 & \text{where} \quad S_{n,j}^x \leq X. \end{cases} \tag{4}$$

The option values one step back in time (at time $n - 1$) are computed by taking the present value of the expected future value of the option. At time $n - 1$, the stock index level (denoted $S_{n-1,j}^x$) can move up with probability p or down with probability $1 - p$. The value of the option at time n if the index level moves up is $C_{n,j}$, and if the index level moves down it is $C_{n,j+1}$. The present value of the expected future value of the option is therefore

$$C_{n-1,j} = \frac{pC_{n,j} + (1-p)C_{n,j+1}}{r^*}. \tag{5}$$

Using this present value formulation, all of the option values at time $n - 1$ may be identified.

Before stepping back another time increment Δt in the valuation procedure, it is necessary to see if any of the option values are below their early exercise value. Here is where dividends may enter the picture again. If no dividends are paid at time $n - 1$, then the early exercise value is simply the grid index level less the exercise price. If a dividend is paid at time $n - 1$, however, the early exercise proceeds equal the grid index level plus the dividend less the exercise price. If any of the computed option values are below the exercise proceeds, they are replaced with the value of the exercise proceeds.

As we repeat the process and step back further in time, we must keep track of the sum of the present values of the dividends paid during the option's remaining life. At time $n - 1$, there was only one dividend and it was paid at time $n - 1$, so the sum equals the value of the single dividend paid at time $n - 1$. If we are at time $n - 2$ and there are dividends paid both at time $n - 2$ and time $n - 1$, however, the sum of the present values of the promised dividends that should be included in the early exercise boundary check at

time $n - 2$ is

$$PVD_{n-2} = D_{n-2} + \frac{D_{n-1}}{r^*}. \tag{6}$$

In other words, the early exercise boundary at time $n - 2$ is $S^x_{n-2,j} + PVD_{n-2} - X$. By the time the iterative procedure is complete, the early exercise boundary used to check the option price corresponding to the time 0 stock index level node will include the present value of all promised dividends as in eq. (3).

3.2.2. The wildcard option

The binomial method described above ignores the value of the wildcard option embedded in the S&P 100 index option contract. If an S&P 100 index call (put) option is exercised on a particular day, the call (put) option holder receives the difference between the closing index level established at 3:00 PM CST (the exercise price) and the exercise price (the closing index level). Since the index option market stays open until 3:15 PM, the option holder can wait until 3:15 PM to decide whether to exercise his option or not, and, if he does exercise, the cash proceeds are based upon the level of the stock market fifteen minutes earlier. The fact that stock prices can move during the fifteen-minute interval after the market close gives the option holder a 'wildcard option', that is, a put option providing the right to put the index option to the writer after 3:00 PM for cash proceeds established at 3:00 PM. [See Valerio (1989) for a treatment of the wildcard option embedded in the S&P 100 index option contract.]

The theoretical value of the wildcard option is very small for the options in our sample (at-the-money calls and puts with at least fifteen days to expiration). Since the stock market closes at 3:00 PM, no direct measure of stock market return for the interval 3:00 to 3:15 PM is available. As a proxy, the return of the most actively traded stock index futures contract – the S&P 500 index futures contract – is used. We computed the mean and the standard deviation of the logarithm of the ratio of the 3:15 PM price to the 3:00 PM price (the closing fifteen-minute return) of nearby S&P 500 futures contract during the time period April 21, 1982 through August 31, 1989. The mean return was 0.0237168% and the standard deviation was 0.180204%. Assuming that the S&P 100 index level is at 250.00 at 3:00 PM and that the mean fifteen-minute rate of return of the S&P 500 futures contract is a reasonable proxy for the S&P 100 index level, the expected 3:15 PM index level is $250.00 \times e^{0.000237168} = 250.06$. This figure is slightly understated since the expected rate of price appreciation in the stock index equals the expected

rate of price appreciation in the futures plus the difference between the riskless rate of interest and ihe dividend yield on the index.

The computations indicate that the expected change in the index level from 3:00 to 3:15 PM is only about 6¢. A 95% confidence interval for the 3:15 PM index level assuming the 3:00 PM price is 250.00 ranges from 249.16 to 250.96. But even these large implied price movements of nearly $1 pale in comparison to the time value of an option. An at-the-money call option with an exercise price of 250 and a time to expiration of fifteen days (the shortest maturity option used in the sample), with a riskless rate of interest of 8%, a dividend yield on the index portfolio of 4%, and a volatility rate of 20%, would be valued at $4.24. This time value would be forfeited if the wildcard option is exercised.

3.3. Implied volatility estimation

To mitigate the effects of nonsimultaneity of prices and bid/ask spreads, we estimate volatility each day using all nearby, at-the-money option transactions in a ten-minute interval surrounding the stock market close (at 3:00 PM CST). In the days immediately following the October 1987 crash, the index option market closed early. On such days, the ten-minute window of transactions is immediately prior to the market close, regardless of the market closing time. Separate volatility estimates are computed for the at-the-money call and the at-the-money put.[3] The median number of transactions in the ten-minute interval each day was 45 for the call and 35 for the put. Since the contemporaneous index level is recorded on each option transaction record, the nonsimultaneity problem is eliminated. In addition, since a large number of option transactions are used in a single nonlinear regression of observed option prices on model prices, the bid/ask price error should be diminished.

3.4. Time variation in the implied volatility

Changes in implied volatility should be orthogonal to any set of information variables if implied volatility follows a random walk. While the random walk model might appear naive, discussions with practitioners reveal that this model is widely used in trading index options. Given that we reject the orthogonality condition, we explore its economic significance by testing a trading strategy based on the predicted volatility changes.

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[3]Separate volatility estimates are computed because recent empirical evidence documents different volatility rates for the two types of options. Whaley (1986), for example, reports that put options on the S&P 500 index futures option tend to have higher volatilities than calls. Bates (1990) argues that the difference in implied volatilities between calls and puts might reflect investors' expectations about the probability of a market movement up or a market movement down.

As with any orthogonality conditions test, the investor's information set must be specified. Economic theory does not help us in choosing this information set. As such, the specification is ad hoc. Our strategy is to draw on other studies of time variation in expected returns and conditional volatility to prespecify a set of instrumental variables. The information used to determine the conditional mean may also be important for the conditional volatility since the conditional mean is part of the definition of conditional volatility, as in definition (1). These instrumental variables will be used to test the null hypothesis that volatility changes are unpredictable.

Little is known about the predictability of returns and volatility at the daily or intraday level. Most previous research has concentrated on longer horizon returns. For example, Fama and French (1988a,b, 1989) show that the dividend price ratio has the ability to track long-horizon expected returns. The dividend price ratio is the sum of the cash dividends paid over the past year divided by the current price level. It is unlikely that a 250-day moving summation would be meaningful in the prediction of daily returns or changes in daily implied volatility.

Our regression specification includes dummy variables for Monday and Friday. Since Monday (Friday) tends to be a day in which many traders open (close) positions for the week, excess buying (selling) pressure may result in higher (lower) implied volatility.

The lagged index return is also included in the regression. Many researchers, beginning with Fama (1965), have studied the autocorrelation patterns in returns. Others, such as Black (1976), Christie (1982), Schwert (1990), and Nelson (1991), have used a leverage argument to postulate a negative relation between price level and volatility. This implies a negative relation between returns and volatility changes.

A number of researchers, starting with Engle (1982), have proposed models of autoregressive conditional volatility, motivating the inclusion of lagged implied volatility in our forecasting model. Estimates of these models, such as the ones presented in French, Schwert, and Stambaugh (1987), indicate strong persistence in volatility levels. Unless the process is a random walk, persistence in volatility levels will induce autocorrelation in the volatility change series. We include two lags of both the call and put implied volatility changes to capture any autocorrelation. Using both calls and puts allows us to use a common information set to forecast both call and put volatility changes.

We also considered three interest rate variables. Keim and Stambaugh (1986) find a relation between the Moody's Baa–Aaa spread and stock returns. Campbell (1987) discovers that the slope of the term structure contains information about future returns. There is some evidence that interest rate measures affect conditional volatility. Schwert (1989a), Kandel and Stambaugh (1990), and Harvey (1991) show that the Baa–Aaa yield

spread has some ability to forecast market volatility. In addition, Shanken (1990) argues that the level of the short-term rate influences the level of volatility. Given this evidence, we consider the first differences of three interest rate variables: the Baa–Aaa yield spread (the junk bond spread), the Aaa–ninety-day Treasury bill spread (the slope of the term structure), and the yield on the Treasury bill that is closest to ninety days to maturity.

We also include the change in the time-adjusted relative basis of the nearby S&P 500 futures contract. The S&P 500 index, like the S&P 100 index, is a stock index based upon the transaction prices of many stocks. As such, the observed index level at any point in time is probably a stale reflection of the true index level. One way in which the degree of staleness can be evaluated is by examining changes in the basis between the S&P 500 futures price and the price of the underlying index. Indeed, Miller, Muthuswamy, and Whaley (1991) show that observed basis changes are consistent with infrequent trading of stocks. Since the futures contract represents a single security rather than a portfolio of securities, it is likely that new market information is incorporated in the futures price before it is incorporated in the prices of all of the index stocks. Since the implied volatilities estimated in this study are based on the index level (and not a futures contract), an increase in the basis on day $t-1$ may reflect an impending increase in the observed index level on day t. The measurement of the (annualized) time-adjusted relative basis is

$$TARB_t = \frac{(F_t - S_t)/S_t}{T},$$ (7)

where F_t (S_t) is the 3:00 PM CST price (level) of the nearby S&P 500 futures contract (S&P 500 index) on day t, and T is the time to expiration of the futures measured in years. These data were obtained from the *Chicago Mercantile Exchange*. The value of $TARB_t$ at any point in time should equal the short-term interest rate less the dividend yield of the S&P 500 index.

4. Empirical results

4.1. Summary statistics

Table 2 contains summary statistics for the variables under consideration. Two panels are presented, one showing the results from the full sample that includes all days during the period October 1, 1985 through July 31, 1989 and the other showing the results where the eleven trading days October 16–30, 1987 are excluded. In total, there are 938 observations for each variable in the period excluding the crash. The mean daily return of the S&P 100 index portfolio is 0.091% including dividends. Interestingly, the serial correlations

in the index returns are not significantly different from zero at any lag. Infrequent trading of portfolio stocks is usually revealed through significantly positive, but declining, serial correlation coefficients. No such pattern is evident in the data.

The average implied volatility of the calls, 19.02%, is less than the average implied volatility for the puts, 19.94%. Although not reported in the table, the unconditional mean of the difference in the volatilities is significantly different from zero,[4] indicating that the average implied volatility from the put options prices is significantly higher than the implied volatility from the call option prices. One possible explanation for this result is that purchase of index puts is a convenient and inexpensive form of portfolio insurance. Excess buying pressure of puts (relative to calls) may cause prices to increase, resulting in implied volatility estimates from put prices that are higher than those from calls. Although this explanation is difficult to prove, it is at least supported by open interest/trading volume data obtained from the Chicago Board of Options Exchange. During our sample period, the ratio of average daily call open interest to average call volume is 465,802/160,999 = 2.89, while the put ratio is 513,723/134,486 = 3.82. This indicates that, while call options trade more frequently, put options tend to be held open for longer periods of times, as is generally required in portfolio insurance programs.

The implied volatility series both have significant positive serial correlation, indicating persistence in the level of volatility. The autocorrelations eventually decline to zero at longer lags.[5] When the implied volatility series is differenced (i.e., when volatility changes are examined), negative serial correlation appears significant at lags 1 and 2. The estimated coefficients are -0.18 and -0.11 for calls and -0.15 and -0.12 for puts at lags 1 and 2, respectively. This negative serial correlation in the volatility changes is consistent with the results reported by French, Schwert, and Stambaugh (1987) using monthly standard deviations of daily S&P 500 returns.

The negative serial correlation in the volatility changes is evidence against the hypothesis that volatility changes are unpredictable. It is important to note that a much different implied volatility series emerges if a volatility measure is extracted from closing option prices. When closing prices are used for the same options during the sample period, the first-order serial correlation in the changes in the implied volatility is -0.41 for the calls and -0.48 for the puts (not reported in the tables). One might incorrectly conclude that the differenced series is highly predictable based on the autocorrelations. In

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[4]We also find that the conditional mean of the volatility spread exhibits significant predictable variation through time. These results are available on request.

[5]The declining autocorrelations are suggestive of stationarity. However, Schwert (1987) provides a number of counterexamples using macroeconomic time series. To assess the sensitivity of our empirical work to model specification, our models are separately estimated using the levels and the first differences of volatility.

Table 2

Means, standard deviations, and autocorrelations for the S&P 100 index returns, the option annualized percentage volatilities, and instrumental variables based on daily data for the sample period October 1, 1985 through July 31, 1989.[a]

Variable	Mean	Std. dev.	Autocorrelations							
			ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_{20}	ρ_{50}	ρ_{100}
Full sample (949 observations)										
S&P 100 return	0.00073	0.01433	0.00	-0.10	0.04	0.08	0.08	0.02	-0.05	0.01
Annualized call implied volatility (%)	19.53549	7.93276	0.94	0.89	0.87	0.85	0.81	0.53	0.33	0.09
Annualized put implied volatility (%)	20.80586	10.89608	0.90	0.80	0.77	0.74	0.70	0.37	0.21	0.07
Change in call volatility (%)	-0.00564	2.77830	-0.12	-0.18	-0.02	0.10	0.05	0.03	-0.02	0.00
Change in put volatility (%)	0.01482	4.92902	0.00	-0.36	0.02	0.03	0.11	0.00	-0.02	-0.01
Change in Baa–Aaa bond yields	-0.00041	0.02812	-0.12	0.00	-0.09	-0.06	0.01	-0.02	0.03	-0.01
Change in Aaa–90-day T-bill	-0.00311	0.07732	0.09	-0.01	-0.04	-0.06	0.01	0.02	-0.03	0.06
Change in 90-day T-bill	0.00120	0.07860	0.15	-0.01	-0.02	-0.03	-0.01	0.02	-0.01	0.02
Change in S&P 500 basis	0.00001	0.00834	-0.36	0.35	0.18	0.07	-0.06	0.03	-0.04	0.01
Sample excluding October 16–30, 1987 (938 observations)										
S&P 100 return	0.00091	0.01085	0.00	-0.04	-0.03	-0.05	-0.04	0.00	-0.02	-0.02
Annualized call implied volatility (%)	19.01600	6.09631	0.96	0.93	0.91	0.89	0.87	0.66	0.38	0.10
Annualized put implied volatility (%)	19.94464	6.39392	0.96	0.92	0.90	0.88	0.86	0.65	0.38	0.14
Change in call volatility (%)	-0.03174	1.62021	-0.18	-0.11	0.01	-0.04	-0.02	0.03	-0.04	-0.02
Change in put volatility (%)	-0.01758	1.46672	-0.15	-0.12	-0.01	-0.05	-0.06	0.01	0.03	-0.02
Change in Baa–Aaa bond yields	-0.00047	0.02766	-0.12	-0.01	-0.07	-0.05	0.00	-0.01	0.03	-0.01
Change in Aaa–90-day T-bill	-0.00491	0.07167	0.04	-0.06	-0.04	-0.03	0.00	0.06	-0.02	0.02
Change in 90-day T-bill	0.00345	0.07192	0.08	-0.07	-0.03	0.00	0.02	0.04	0.00	0.00
Change in S&P 500 basis	-0.00002	0.00469	-0.59	-0.10	0.04	-0.01	-0.04	-0.05	0.01	0.01

[a]The return on the S&P 100 index includes the daily dividend payments. Implied volatility is estimated using a nonlinear regression of all nearby, at-the-money option transactions on model prices in the 10-minute window 2:55 PM to 3:05 PM CST each day. The model used to price the options is a dividend-adjusted, binomial method. The change in volatility is the first difference of the implied volatility series. The change in the Baa–Aaa bond yields is based upon daily Moody's annualized percentage yields on Baa- and Aaa-rated bonds. The term structure variable is the first difference in the yield spread between Moody's Aaa-rated bonds and the Treasury bill that is closest to 90 days to maturity. The time-adjusted relative basis is the relative premium of the S&P 500 futures over the spot index divided by the time to expiration in years.

fact, much of this negative serial correlation is spuriously induced by the asychronous observation of the index and the option as well as the bid/ask price effect.

Summary statistics are also provided for the information variables. The three interest rate measures appear to be slow-moving near-integrated processes. There is little or no serial correlation in the first differences. In contrast to the other series, the first-order autocorrelation in the differenced time-adjusted, relative basis of the S&P 500 futures, -0.59, is significantly negative. This result is consistent with infrequent trading in that a widening (narrowing) of the basis on day $t-1$ probably reflects new market information being incorporated in the futures price before it is incorporated into the index level (holding the short-term interest rate and the dividend yield of the index constant). On the following day, the basis narrows (widens) when all of the stocks within the index have had an opportunity to trade in reaction to the new information.

Table 3 examines the data by the day of the week. The stock index return series displays some seasonality, although the nature of the seasonality differs from that reported for earlier periods and different indexes. French (1980) and Gibbons and Hess (1981), for example, report that stock index returns are abnormally high on Fridays and abnormally low on Mondays. The behavior in table 3 indicates that the Friday and Monday returns are not abnormal. Instead, in the sample that excludes the crash, the mean return on Wednesday is 50% higher than other days of the week, and the mean return on Thursday is negative.

For both calls and puts, the implied volatility on Fridays is lower than any other day of the week. This is consistent with a large number of traders closing out positions before the weekend. On average, volatility increases when markets reopen on Monday. For the call option, the decrease in volatility from Thursday to Friday averages 0.69, and the increase in volatility from Friday to Monday averages 0.73. Although the size of these changes in volatility looks important, it does not appear to be economically significant. For nearby, at-the-money S&P 100 index options, a typical 'vega' (i.e., the partial derivative of option price with respect to the volatility parameter) is about 0.35. An expected implied volatility change of 0.73%, therefore, implies an expected option price movement of 0.256. The bid/ask spread on an at-the-money index option is at least an eighth, and this cost would be incurred when the call is sold (purchased) on Thursday (Friday) and again when the call is purchased (sold) on Friday (Monday). The quarter point in transaction costs is sufficient to eliminate the apparent profit opportunity even before the costs of hedging the option position are considered.

4.2. Predicting changes in implied volatility

The autocorrelations in table 2 present some evidence against the null hypothesis that volatility changes are unpredictable. Table 4 confirms that

Table 3

Means of the S&P 100 index return and the option implied volatilities by the day of the week based on daily data for the sample period October 1, 1985 through July 31, 1989.[a]

Variable		Means				
	Overall	Monday	Tuesday	Wednesday	Thursday	Friday
Full sample						
S&P 100 return	0.00073	-0.00079	0.00180	0.00192	-0.00011	0.00077
Annualized call implied volatility (%)	19.53549	20.08136	19.75052	19.47370	19.57821	18.80317
Annualized put implied volatility (%)	20.80586	21.39082	21.13933	20.49562	20.61268	20.40344
Change in call volatility	-0.00564	1.06027	-0.13959	-0.25854	0.01679	-0.67148
Change in put volatility	0.01482	0.80337	0.01155	-0.65245	0.00542	-0.06222
Number of days	949	184	194	192	190	189
Sample excluding October 16–30, 1987						
S&P 100 return	0.00091	0.00096	0.00125	0.00148	-0.00014	0.00097
Annualized call implied volatility (%)	19.01600	19.42934	19.12984	19.08158	19.14415	18.29753
Annualized put implied volatility (%)	19.94464	20.20830	19.98687	19.84763	19.95229	19.73441
Change in call volatility (%)	-0.03174	0.73275	-0.14021	-0.02326	-0.02191	-0.68640
Change in put volatility (%)	-0.01758	0.13396	-0.01724	-0.13595	-0.00128	-0.06177
Number of days	938	182	192	190	188	186

[a]The return on the S&P 100 index includes the daily dividend payments. Implied volatility is estimated using a nonlinear regression of all nearby, at-the-money option transactions on model prices in the 10-minute window 2:55 PM to 3:05 PM CST each day. The model used to price the options is a dividend-adjusted, binomial method. The change in volatility is the first difference of the implied volatility series.

Table 4

Regressions of the change in implied volatility on the S&P 100 index option on information variables which include Monday and Friday dummy variables, lagged call and put option implied volatilities, interest rate measures, and the S&P 500 time-adjusted relative basis for the sample period October 1, 1985 through July 31, 1989.[a,b]

Information variables	Full sample (949 observations)				Excluding October 16–30, 1987 (938 observations)			
	Change in call volatility		Change in put volatility		Change in call volatility		Change in put volatility	
	Coefficient	t-ratio	Coefficient	t-ratio	Coefficient	t-ratio	Coefficient	t-ratio
Intercept	0.034	0.373	−0.001	−0.010	−0.028	−0.488	−0.092	−1.565
Monday dummy$_t$	0.776	2.845	0.668	1.188	0.661	4.639	0.249	1.776
Friday dummy$_t$	−0.774	−4.617	−0.294	−1.262	−0.646	−4.959	−0.027	0.237
S&P 100 return$_{t-1}$	−38.997	−1.178	−97.817	−1.460	4.993	0.629	8.930	1.223
Chg. annualized % call volatility$_{t-1}$	−0.471	−4.406	−0.203	−0.916	−0.237	−4.409	0.153	2.585
Chg. annualized % call volatility$_{t-2}$	−0.099	−1.314	0.334	1.888	−0.176	−4.812	0.146	3.530
Chg. annualized % put volatility$_{t-1}$	0.092	1.107	−0.116	0.713	0.238	4.589	−0.233	−3.277
Chg. annualized % put volatility$_{t-2}$	−0.052	−0.718	−0.441	−3.120	0.107	2.945	−0.210	−3.725
Chg. Baa–Aaa bond yields$_{t-1}$	−1.373	−0.397	−8.369	−1.735	−0.263	−0.131	−2.516	−1.269
Chg. Aaa–90-day T-bill$_{t-1}$	5.907	1.077	0.370	0.042	0.842	0.466	−1.397	−0.794
Chg. 90-day T-bill$_{t-1}$	5.742	1.043	2.347	0.274	0.767	0.406	−1.316	−0.703
Chg. S&P 500 basis$_{t-1}$	−86.303	−1.013	−52.745	−0.339	−21.678	−0.862	−52.145	−1.969
$\bar{R}^2$	0.228		0.207		0.187		0.121	
Out-of-sample $\bar{R}^2$	0.000		0.000		0.151		0.039	
Out-of-sample correct direction	60.4%		53.5%		62.2%		56.6%	

[a] The return on the S&P 100 index includes the daily dividend payments. Implied volatility is estimated using a nonlinear regression of all nearby, at-the-money option transactions on model prices in the 10-minute window 2:55 PM to 3:05 PM CST each day. The model used to price the options is a dividend-adjusted, binomial method. The change in volatility is the first difference of the implied volatility series. The change in the Baa–Aaa bond yields are based upon daily Moody's annualized percentage yields on Baa- and Aaa-rated bonds. The term structure variable is the first difference in the yield spread between Moody's Aaa-rated bonds and the Treasury bill that is closest to 90 days to maturity. The time-adjusted relative basis is the relative premium of the S&P 500 futures over the spot index divided by the time to expiration in years.

[b] All t-ratios are heteroskedasticity-consistent. The R-square measures are adjusted for degrees of freedom. In the out-of-sample analysis, the models are initially estimated using the first 100 observations. A forecast is obtained for $t = 101$ and then the model's parameters are re-estimated. This procedure is then repeated 850 times for the full sample and 839 for the sample that excludes the observations around the October 1987 stock market crash. The out-of-sample R-square is calculated in the usual way using the out-of-sample forecasts as the fitted values. Out-of-sample correct direction is the percent of times that the sign volatility change was correctly predicted.

there is significant time variation in the volatility change series when additional conditioning information is used. The left panel of table 4 presents results for the full sample and the right panel presents results excluding the market crash. A comparison of these panels reveals that the observations around October 19, 1987 have a dramatic influence on the estimated coefficients. For this reason, the discussion of the results focuses on the sample that excludes the crash.

As expected from table 3, the Monday and Friday dummy variables are important for the call volatility change regression. The Friday dummy variable has a significantly negative coefficient and the Monday dummy has a significantly positive coefficient. The effect is less pronounced for the put option regression. The Monday coefficient is positive at the 10% level of significance. The Friday coefficient is indistinguishable from zero, however. The magnitude of the coefficients mimics the mean shifts documented in table 3.

Motivated by the results of Day and Lewis (1988), we included an additional dummy variable for the quarterly expiration to test whether the significance of the Monday–Friday dummy variables was being driven by expiration effects. The expiration variable never attained a t-ratio greater than 1.0. Unlike Day and Lewis, however, we do not use options with times to expiration of less than fifteen days. Recall that implied volatility corresponds to the remaining life of the option. Thus, even if volatility is abnormally high on an expiration day, at best it constitutes one-fifteenth of the average volatility rate implied by the options in our sample.

The lagged S&P 100 return, included to proxy for a leverage effect, does not significantly enter the call or the put regressions. It is interesting to note that the correlation between contemporaneous returns and the change in call (put) implied volatility is -42% (-20%), which is consistent with the leverage interpretation. The effect of the market crash is particularly dramatic for this variable. The coefficient in the call regression changes from 5.0 to -39.0 when the crash observations are added to the sample. The coefficient in the put regression changes from -8.9 to -97.8. In the full sample, the regression is trying to fit the large increase in volatility that occurred during when the market declined in October 1987.

The regression results indicate that the changes in the default risk premium, term structure, and level of short-term interest rates on day $t-1$ have no significant power to predict the change in call or put volatility on day t. The high degree of persistence in the levels suggests that these variables pick up fairly long-term movements in expected returns. Given the limited ability of interest rate measures to predict monthly volatility [see, for example, the evidence in Schwert (1989a)], it is perhaps not surprising that predictability is not found in our four-year sample of daily data.

The bulk of the explanatory power in the regressions is being driven by the lagged volatility changes. If the volatility level is a mean-reverting time series,

then the volatility changes should show first-order serial correlation. This correlation structure was documented in table 2. Negative partial correlation also appears, as evidenced by the significantly negative coefficients on the lagged call volatility change in the call regression and the significantly negative coefficients on the lagged put volatility change in the put regression.

The negative relation may be influenced by the infrequent trading of index stocks. Suppose, for example, that good news about the prospects of the market unexpectedly arrives late in the trading day. Because the index option market is so active, it is likely that the information is quickly incorporated in option prices.[6] To incorporate the good news into the index level, 100 stocks must be traded. If the information is not fully impounded in the observed index level (or, alternatively, index return) by the close of trading, the observed index level is lower than the 'true' level of the index, and the implied volatility of the call is higher than it should be. On the next day, when all stocks in the index have finally traded in reaction to the previous day's news, the observed index level 'catches up' and the implied volatility of the call is reduced. On the other hand, if bad news about the market arrives late in the day, the price of index puts is quickly bid up. Not all stocks within the index are traded prior to the market close, so the observed closing index level is greater than it should be. The implied volatility of the put will therefore be higher than it should be, and foreshadows the impending decline in the observed index level and in the implied put volatility estimate on the following day. Under this scenario, the change in the implied volatility on day t will be negatively correlated with the change in the implied volatility on day $t - 1$.

Following the same line of reasoning, the change in the implied volatility for the call (put) option on day t should be positively related to the change in the implied volatility of the put (call) option on day $t - 1$. This is exactly what is found in table 4. For the call regression, the lagged put volatility changes have positive coefficients. In the put regression, the lagged call volatility changes also have positive coefficients. All of these coefficients are statistically significant. It is worthwhile to note that the pattern of the signs of the lagged volatilities is not being driven by contemporaneous correlation between the change in the implied volatility of the call and the change in the implied volatility of the put. In fact, for the sample excluding the October 1987 crash, the contemporaneous correlation is estimated to be 0.06.

The potential effect of infrequent trading can be examined by simultaneously estimating the implied volatility and the implied S&P 100 index level, and comparing the actual index level to the implied index level. Using weekly

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[6]Because buying options is considerably less expensive than selling options from a transaction cost standpoint (where the cost of margin money is considered), we observe that buying calls (puts) rather than selling puts (calls) is the preferred investment strategy when expectations are that the market will rise (fall).

data, Day and Lewis (1990) find little difference between the actual and implied return series. Using intraday data, Kleidon and Whaley (1991) find weak evidence that the implied index leads the actual index by as much as five minutes. These results, however, are probably driven by less activity in the middle of the trading day rather than the end of the day when almost all of the S&P 100 stocks trade. Kleidon and Whaley find that the S&P 500 futures leads both the actual and implied S&P 100 index. Given these results, we focus on the S&P 500 basis to proxy for infrequent trading.

If a widening of the basis reflects new information being incorporated in the futures price and only later being incorporated in the stock price, the expected change in the call implied volatility in day t should be negative (since the level of the implied volatility was 'too high' on day $t - 1$, only to be corrected on day t). By the same argument, a positive effect should be detected in the implied put volatility regression, because on day $t - 1$ the implied volatility is 'too low' and should increase on day t.

The results in table 4 provide no support for this explanation. The *TARB* variable enters the put regression with a significantly negative coefficient. The coefficient in the call regression is not distinguishable from zero. In the crash data set, both coefficients are insignificantly different from zero.

Overall, the regression results demonstrate that the change in both call and put volatility is partially predictable. Indeed, the R^2 in the call (put) option regression is 18.7% (12.1%). There is no significant autocorrelation in the regression residuals. In the sample that includes the crash, the explanatory power is higher, because the total variance of the dependent variables roughly doubles for calls and triples for puts and the regression coefficients try to mimic the dependent variable during this high variance period in 1987. As a result, the fit is good during the stock market crash, but poor in all other periods.

To check the regression specification, out-of-sample forecasts are calculated. The regression is initially estimated over the first 100 days. The model is then reestimated at every point in time, and out-of-sample forecasts are formed. Actually, only forecasts of volatility changes after August 1989 are pure out-of-sample forecasts, because the model has already been fit over the full sample. From another perspective, since our empirical strategy was to prespecify a set of instrumental variables, as opposed to conducting a search for a set of variables that provided maximum explanatory power, the forecasts may be interpreted as out-of-sample. Nevertheless, our choice of instruments is based on other researchers' data analysis.

Table 4 reports the summary statistics for the out-of-sample analysis. The degree of explanatory power is not substantially altered for calls. The adjusted R^2 falls from 18.7% to 15.1%. The explanatory power drops substantially in the put regression, from 12.1% to only 3.9%. The out-of-sample results show that the sign of the change in volatility is correctly predicted

62.2% of the time for calls and 56.6% of the time for puts. The out-of-sample R^2 for the sample that includes the crash period goes to zero for both the call and put regressions. Similar to the in-sample analysis, the variance of the dependent variable increases dramatically. In contrast to the in-sample analysis, however, the routine is not able to fit coefficients to mimic the crash. There are massive errors in the out-of-sample forecasts for October 19, 1987 in both the put and call regressions. These forecast errors increase the error sum of squares and drive the R^2 values to zero.

Although not reported in the tables, the regressions were also performed on levels rather than changes. In the levels regressions, the instruments were not differenced. Also, one additional lag of the call and put implied volatility level was included so that both the change and the level regressions use information back to $t - 3$. The R^2 values for the call and put level regressions were large, 94.5% and 95.7%, respectively. To link the results of the two specifications, an R^2 measure was constructed for the change in volatility implicit from the level regression. The R^2 value for the call was 22.7% and the value for the put was 17.5%. In both cases, however, the explanatory power is greater. In the out-of-sample analysis, the implicit change in volatility R^2 was 17.1%, compared to 15.1% in the differenced specification. The R^2 for the put regression was 6.2%, again higher than the 3.9% estimated for the differenced specification. The level regression correctly predicted the direction of the call (put) volatility changes 62.9% (60.3%) of the time.

In summary, the evidence suggests that there is predictable time variation in the implied volatility changes. The predetermined variables offer predictive power in both the implied call volatility and implied put volatility regressions, although both the in-sample and out-of-sample analyses indicate that it is easier to predict changes in the call volatility than those in put volatility.

5. Economic analysis

We have provided evidence that volatility changes are predictable in a statistical sense. We now examine whether this predictability is large and persistent enough to be economically meaningful. Specifically, we test whether the out-of-sample volatility change predictions can be used to generate abnormal rates of return in the S&P 100 index option market.

5.1. Delta-neutral hedges based on volatility change predictions

The regression results in table 4 indicate that there is predictable variation in the daily changes in implied volatility. On one hand, the predictable variation could be caused by investors rationally updating their assessments of the distribution of returns as new information arrives in the marketplace.

Alternatively, the variation could also be driven by market overreactions as well as other inefficiencies. In practice, it is difficult to distinguish between these two possible explanations. However, if volatility change predictions can be used to generate abnormal risk-adjusted profits net of transaction costs, the market inefficiency view is supported. If no economic profits are earned, the more likely explanation is rational price adjustment in the marketplace.

The trading strategy that we use to assess potential abnormal economic profits is based on out-of-sample forecasts of the change in volatility. At 3:05 PM on each day t, the implied volatility estimates up to and including day t are regressed on the information variables available up to and including day $t - 1$. The coefficient estimates are then applied to the information variables available on day t to form a forecast of the volatility change for day $t + 1$. If volatility is predicted to increase (decrease) from day t to day $t + 1$, the option is purchased (sold).

A 3:05 PM cutoff is used each day to permit option trading before the 3:15 PM market close. As such, all information variables must be available at 3:05 PM on day t in order to forecast day $t + 1$ volatility. In the regression specification reported in table 4, the values of the three interest rate variables are not known until after 3:05 PM. As a result, we drop these variables from the regression. Indeed, these variables did not make a significant contribution to any of the regressions, so that their exclusion should increase the out-of-sample forecasting performance of the model and thereby increase the power of our test.

The trading strategy also requires that the option positions are hedged. Conceivably, we may correctly forecast the direction of the implied volatility but still lose money if the market moves the wrong way. As a result, we 'delta hedge' the index option. The delta of an index option is the partial derivative of the option price with respect to a change in the index level. It gives us a predicted dollar change in the option price for a one dollar change in the index. Thus, if we buy a number of call (put) options equal to the reciprocal of the delta value (i.e., the 'hedge ratio') and sell (buy) one unit of the underlying index, the overall portfolio value is insensitive (approximately) to market movements. Since the transaction costs involved in buying and selling the S&P 100 stock portfolio to hedge the index option position on a daily basis are prohibitive, we delta hedge using the S&P 500 futures contract. The investment outlay in forming this delta-neutral hedge portfolio depends on the option price as well as the hedge ratio,[7] so the investment outlay is

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[7]Within our delta-neutral trading strategy, the investment outlay equals the option price O times the absolute number of options bought (sold) $|n_O|$. Thus, two assumptions are implicitly made. First, for options, an investment of $|n_O|O$ is required independent of whether options are purchased or sold. While this assumption conforms with the market practice concerning option purchases, it overstates the outlay from the standpoint of option sales. Second, for futures positions, the investment outlay is assumed to be zero, as is the convention in the marketplace. Any margin money required is assumed to be deposited in the form of interest-bearing securities.

standardized to 100 by scaling up the investment outlay by the factor

$$k \equiv \frac{100}{|n_O|O}, \tag{8}$$

where n_O is the number of options bought (sold) and O is the option price. The same procedure is applied for the puts, except that, if the deviation is negative (positive), the S&P 500 futures contract is bought (sold).

The trading strategy is applied over three different trading horizons: (a) 3:05 PM on day t to 10:00 AM on day $t + 1$, (b) 10:00 AM on day $t + 1$ to 3:05 PM on day $t + 1$, and (c) 3:05 PM on day t to 3:05 PM on day $t + 1$. Under the first trading horizon, theoretical option prices are computed at 3:05 PM on day $t + 1$ based on the predicted volatility for day $t + 1$. If the price deviation (i.e., observed price minus theoretical price) is negative (positive), we purchase (sell) the option and hedge with the futures at the first observed prices after 3:05 PM and hold the position until 10:00 AM the next day. Under the second trading horizon, theoretical option prices are computed at 10:00 AM on day $t + 1$. Price deviations are then computed, and trades are placed at the first observed prices after 10:00 AM. The position is closed at 3:05 PM the same day. Finally, the third trading horizon involves initiating a position just after 3:05 PM on day t and holding it until 3:05 on day $t + 1$.

Although the delta-neutral hedge positions are intended to be riskless, variability in hedge profits will undoubtedly arise. For one thing, we use S&P 500 futures to cross-hedge S&P 100 option positions; not only is there tracking risk between the S&P 500 and S&P 100 cash indexes, but there is also basis risk between the S&P 500 cash and futures. Second, a delta-neutral hedge is riskless for only an instant in time and for only infinitesimal movements in the index level. The strategies that we use involve hedge portfolios carried up to 24 hours, during which time the delta value may change not only from time erosion but also from a change in the index level. Finally, within our trading strategies, the delta values are calculated using our volatility forecast; inaccuracies in the volatility forecasts will induce noise in the estimates of the delta values, and hence uncertainty in the hedge portfolio profit. None of these considerations should systematically bias our results, however.

5.2. Before-transaction-cost trading profit simulation results

Table 5 contains a summary of the delta-neutral trading profit simulation results for the three time horizons: (a) 3:05 PM on day t to 10:00 AM on day $t + 1$, (b) 10:00 AM on day $t + 1$ to 3:05 PM on day $t + 1$, and (c) 3:05 PM on

day t to 3:05 PM on day $t + 1$. Trading profit is computed as

$$\pi_t = k \left[n_O (O_{\text{close}} - O_{\text{open}}) + n_F (F_{\text{close}} - F_{\text{open}}) \right], \tag{9}$$

where k is the scale factor (8), n_O is the number of options bought (sold) when the position is established at 3:05 PM on day t or at 10:00 AM on day $t + 1$, n_F is either 1 or -1 depending upon whether the futures contract is bought or sold, and the terms in parentheses are the option (O) and futures (F) price changes from the open to the close of the trading horizon. The values in the table are the means and the standard deviations of the daily trading profits per $100 invested and the t-ratio corresponding to the null hypothesis that the trading profit is zero. Also reported is the number of observations, that is, the number of days during the sample period that the deviation from prediction was large enough for a delta-neutral hedge position to be taken. Naturally, where the deviation filter equals zero, a hedge position is taken in each day of the 839-day sample period for both calls and puts (i.e., 1,678 hedge portfolios in total). Note that only the results of the sample that excludes the crash are reported. On October 19 and 20, 1987, the realized volatility changes had the same sign as the predicted changes, but were much larger in magnitude. Profits from our trading strategy were extraordinarily high on these days, not solely as a result of the ability of our prediction model, but rather as a result of the extraordinary market movements. To mitigate the effects of these influential observations, we concentrate on the non-crash sample.

Summary statistics for before-transaction-cost trading day profits during the sample period are presented in the first panel of table 5. The mean trading day profits using a zero price deviation filter is $1.346 for all options, $1.148 for call options, and $1.544 for put options over the 3:05 PM to 10:00 AM trading horizon. The implication is that a before-transaction-cost rate of return of 1.346% on average is earned using call and put options, 1.148% is earned using call options only, and 1.544% is earned using put options only. The standard deviations corresponding to these sample values, $6.897, $7.103, and $6.684, indicate that the hedge portfolio profits are far from certain. Nonetheless, mean profits are impressive. Apparently, trading on the basis of the out-of-sample volatility change predictions can produce abnormal profits before transaction costs, at least with respect to this overnight trading horizon.

The average trading profit results for the remaining two trading horizons using the zero price deviation filter provide an interesting contrast. Again, the before-transaction-cost profits are significantly greater than zero. The magnitudes of the average profits, however, are generally not as large as the 3:05 PM to 10:00 AM subperiod. Using both call and put options, the mean

Table 5

Summary of profits from an arbitrage trading strategy formed on the basis of out-of-sample daily volatility predictions for S&P 100 index options.

The prediction of volatility for day $t + 1$ is based on a linear regression of day t volatility change on predetermined information variables including dummy variables for Monday and Friday, the lagged return on the S&P 100 index, two lags of both the call and put implied volatility changes, and change in the S&P 500 basis. With the prediction of day $t + 1$ volatility, theoretical option prices are computed. If the observed option price is below (exceeds) the theoretical price, $100 of the options are purchased (sold). The position is hedged using the delta which is the partial derivative of the option price with respect to the underlying index level. The delta determines how many S&P 500 futures contracts should be used to hedge the purchase or sale of the index option. Three trading horizons are used: (a) 3:05 PM on day t to 10:00 AM on day $t + 1$, (b) 10:00 AM to 3:05 PM on day $t + 1$, and (c) 3:05 PM on day t to 3:05 PM on day $t + 1$. Three levels of filters are considered: trade every day, trade only if the price deviation is greater than $0.25, and trade if the price deviation is greater than $0.50. The second panel presents profits after transaction costs: 1/8 round trip per contract in the S&P 100 option market and $0.05 in the S&P 500 futures. The trading strategies are evaluated over the period March 7, 1986 through October 16, 1987 and November 2, 1987 through July 31, 1989 (839 days).

Filter	Option type	3:05 PM day, to 10:00 AM day $_{t+1}$				10:00 AM day $_{t+1}$ to 3:05 PM day $_{t+1}$				3:05 PM day, to 3:05 PM day $_{t+1}$			
		No. of obs.	Mean profit (%)	Std. dev.	t-ratio	No. of obs.	Mean profit (%)	Std. dev.	t-ratio	No. of obs.	Mean profit (%)	Std. dev.	t-ratio
						Before transaction costs							
0.00	Both	1,678	1.346	6.897	7.99	1,678	0.999	8.408	4.87	1,678	1.380	10.878	5.20
	Calls	839	1.148	7.103	4.68	839	1.118	8.623	3.75	839	1.283	11.254	3.30
	Puts	839	1.544	6.684	6.69	839	0.881	8.193	3.11	839	1.476	10.496	4.07
0.25	Both	586	2.551	8.433	7.32	688	1.743	8.260	5.53	586	2.339	12.491	4.53
	Calls	322	2.115	8.218	4.62	365	1.631	8.608	3.62	322	2.257	13.425	3.02
	Puts	264	3.082	8.674	5.77	323	1.869	7.860	4.27	264	2.439	11.274	3.51
0.50	Both	199	5.019	10.308	6.87	243	2.471	9.611	4.01	199	3.799	13.872	3.86
	Calls	112	4.141	9.229	4.75	138	1.658	10.000	1.95	112	3.330	13.135	2.68
	Puts	87	6.150	11.508	4.98	105	3.540	9.010	4.03	87	4.402	14.823	2.77

Table 5 (continued)

Filter	Option type	3:05 PM day$_t$ to 10:00 AM day$_{t+1}$				10:00 AM day$_{t+1}$ to 3:05 PM day$_{t+1}$				3:05 PM day$_t$ to 3:05 PM day$_{t+1}$			
		No. of obs.	Mean profit (%)	Std. dev.	t-ratio	No. of obs.	Mean profit (%)	Std. dev.	t-ratio	No. of obs.	Mean profit (%)	Std. dev.	t-ratio
						After transaction costs							
0.00	Both	1,678	−1.559	7.050	−9.06	1678	−1.914	8.471	−9.25	1678	−1.525	10.946	−5.71
	Calls	839	−1.792	7.253	−7.16	839	−1.780	8.580	−6.01	839	−1.657	11.180	−4.29
	Puts	839	−1.327	6.838	−5.62	839	−2.047	8.367	−7.09	839	−1.394	10.713	−3.77
0.25	Both	586	−0.040	8.537	−0.11	688	−0.858	8.180	−2.75	586	−0.252	12.484	−0.49
	Calls	322	−0.575	8.348	−1.24	365	−0.986	8.543	−2.21	322	−0.433	13.301	−0.58
	Puts	264	0.612	8.733	1.14	323	−0.714	7.761	−1.65	264	−0.031	11.432	−0.04
0.50	Both	199	2.745	10.192	3.80	243	0.254	9.394	0.42	199	1.525	13.670	1.57
	Calls	112	1.745	9.123	2.02	138	−0.580	9.838	−0.69	112	0.934	12.725	0.78
	Puts	87	4.032	11.348	3.31	105	1.348	8.702	1.59	87	2.284	14.839	1.44

before-transaction-cost profit is $1.346 during the 19-hour subperiod 3:05 PM to 10:00 AM, while the mean profit for the 5-hour trading horizon from 10:00 AM to 3:05 PM is $0.999 and for the 24-hour horizon from 3:05 on day t to 3:05 on day $t + 1$ is $1.380. Apparently, more of the volatility forecast precision arises from predicting what happens during the near-term, 19-hour subperiod from 3:05 PM on day t to 10:00 AM on day $t + 1$ than from predicting volatility changes during the 5-hour subperiod from 10:00 AM to 3:05 PM on the following day.

Two variations of the same trading strategy are also performed. In the results discussed thus far, only the zero price deviation filter reported is used. With a zero price deviation filter, call and put option positions are taken on each of the 839 days for which we have an out-of-sample volatility forecast. To test the possibility that none of the price deviations are large enough to cover the transaction costs necessary to capture the gains, a filter is applied to the price deviation. Under the filtering arrangement, option positions are undertaken only if the absolute value of the price deviation exceeds $0.25 or $0.50. The results using these two filters are reported at the bottom of the first panel in table 5. Note that as the filter is increased, the number of days in which option positions are taken drops dramatically. Option positions are taken in only 34.9% of total days using the $0.25 filter and in 11.8% of the days using the $0.50 filter for the trading horizons that begin at 3:05 PM on day t (i.e., the first and the third horizons), and option positions are taken in 41% of total days using the $0.25 filter and in less than 15% of the days using the $0.50 filter for the trading horizon that begins at 10:00 AM on day $t + 1$ (i.e., the second horizon). On the other hand, the profitability of the option positions increases dramatically. With a $0.50 filter, the call option positions produce an average profit of $4.141 per $100 investment over a 19-hour trading horizon, and the put option positions produce an average $6.150 profit. The average profits for the remaining two horizons are also increased as a result of using a price filter, although not to the same levels.

5.3. After-transaction-cost trading profit simulation results

None of the profits reported thus far have attempted to account for the effects of transaction costs. [Phillips and Smith (1980) assess the effect of transaction costs on previous studies of stock market option market efficiency.] In the bottom panel of table 5, the effects of plausible transaction costs are imposed. In the trading strategies, actual S&P 100 option and S&P 500 futures transaction prices are used. These transaction prices are equally likely to have been at bid and ask levels, so, on average, it is reasonable to assume that half the bid/ask spread is incurred in both the option and the futures market each time a transaction takes place. An at-the-money, short maturity S&P 100 option has a bid/ask spread of at least an eighth, and the

nearby stock index futures contract has a minimum bid/ask spread of 0.05. Thus, an estimate of the minimum transaction costs faced in entering and reversing the option and futures positions in our delta-neutral option strategy is

$$k\left[2 \times n_O \times \frac{\frac{1}{8}}{2} + 2 \times \frac{0.05}{2}\right] = k\left[\left(\tfrac{1}{8}\right) \times n_O + 0.05\right]. \tag{10}$$

Using this estimator for transaction costs, the profits are dramatically altered. Where abnormal profits were realized before transaction costs at all filter levels, table 5 shows that the after-transaction-cost profits are significantly greater than zero only when the $0.50 filter is used. The $0.25 filter also produces a positive, albeit insignificant, profit for put options during the 19-hour interval 3:05 PM to 10:00 AM, which is 0.612% on average. The call option profit is negative and insignificant. Although not reported in the table, after-transaction-cost profits are not significantly greater than zero when the bid/ask spreads in the option and futures markets are increased to $\frac{1}{4}$ and 0.10, respectively.

5.4. Summary of trading simulation results

To summarize, what appeared to be a reasonably powerful prediction model for implied volatility from a statistical standpoint appears to be fairly weak from an economic standpoint. While the regression model's predictions of volatility are reasonably precise from a statistical standpoint, the magnitudes of the predicted changes are not large enough to permit abnormal risk-adjusted profits to be earned after minimal transactions costs are imposed. On the basis of these results, the null hypothesis that the S&P 100 index option market is allocationally efficient cannot be rejected.

6. Conclusions

Asset pricing theory suggests that changes in market volatility should affect expected asset returns. Understanding the dynamics of market volatility, therefore, may help us understand time-varying expected returns. While most studies have used ex post volatility or functions of past returns to forecast volatility, our study characterizes market volatility with the implied volatility from S&P 100 index options. We reject the hypothesis that volatility changes are unpredictable on a daily basis.

There are many potential sources of predictable variation: investors could be rationally changing their assessments of the distribution of returns on the basis of new information, or some form of informational inefficiency could be generating predictable changes. While it is difficult to isolate the source of the variation, volatility change predictions that generate economic profits net of transaction costs would tend support the informational inefficiency expla-

nation. However, we find that, after transaction costs, a trading strategy based upon out-of-sample volatility changes does not generate economic profits. Our results support the notion that S&P 100 index option market is allocationally efficient.

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