Strategic Rebalancing

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KEY FINDINGS

- Many investors do not realize that calendar rebalancing of a portfolio is an active strategy that essentially buys losers and sells winners.
- Calendar rebalancing induces negative convexity in portfolios and heightens drawdowns.
- We explore popular solutions including partial rebalancing as well as less frequent calendar rebalancing. We also consider a direct allocation to trend strategies.
- We present a dynamic "strategic rebalancing" based on trend information; notably, rebalancing is delayed if stock markets are in a negative trend.

ABSTRACT: *A mechanical rebalancing strategy, such as a monthly or quarterly reallocation toward fixed portfolio weights, is an active strategy. Winning asset classes are sold, and losers are bought. During crises, when markets are often trending, this can lead to substantially larger drawdowns than a buy-and-hold strategy. This article shows that the negative convexity induced by rebalancing can be substantially mitigated, taking the popular 60–40 stock–bond portfolio as the use case. One alternative is an allocation to a trend-following strategy. The positive convexity of this overlay tends to counter the impact on drawdowns of the mechanical rebalancing strategy. The second alternative is called* strategic rebalancing, *which uses smart rebalancing timing based on trend-following signals—without a direct allocation to a trend-following strategy. For example, if the trend-following model suggests that stock markets are in a negative trend, rebalancing is delayed.*

TOPICS: ***Portfolio construction, wealth management, pension funds, foundations & endowments****

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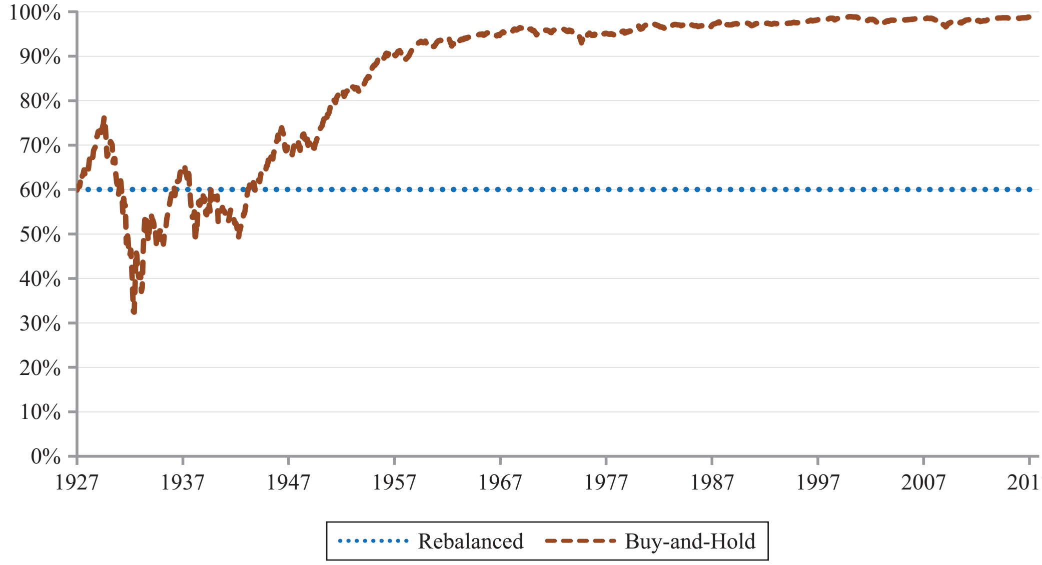
A pure buy-and-hold portfolio has the drawback that the asset mix tends to drift over time and, as such, is untenable for investors who seek diversification. As illustrated in Exhibit 1 for a US stock–bond portfolio, an initial 60% of capital allocated to stocks in 1927 drifts to a 76% allocation by 1929, a 32% allocation by 1932, and a level close to 100% over time because stocks tend to outperform bonds over the long run.

However, a stock–bond portfolio that regularly rebalances tends to underperform a buy-and-hold portfolio in times of continued outperformance of one of the assets. Using a simple two-period model, we explain the main intuition behind this effect: rebalancing means selling (relative) winners, and if winners continue to outperform, those sales detract from performance.

Because stocks typically have more volatile returns than bonds, relative returns

EXHIBIT 1

Allocation to Stocks for a Monthly Rebalanced and Buy-and-Hold Portfolio



Notes: Percentage allocated to stocks for a monthly rebalanced and a buy-and-hold portfolio. In both cases, at the start 60% of capital is allocated to stocks and 40% to bonds. We use monthly US data from January 1927 to December 2017. The stock data are from Kenneth French's website. The bond data are from the Federal Reserve, prepended with global financial data.

tend to be driven by stocks. Hence, of particular interest are episodes with continued negative (absolute and relative) stock performance, such as the 2007–2009 global financial crisis. In Exhibit 2, we contrast the monthly rebalanced and buy-and-hold cumulative performance over the financial crisis period; both have an initial 60–40 stock–bond capital allocation at the start of the evaluation period (at the end of December 2016). The maximum drawdown of the monthly rebalanced portfolio is 1.2 times (or 5 percentage points) worse than that of the buy-and-hold portfolio, right at the time when financial market turmoil is greatest.

In earlier work, Granger et al. (2014) formally showed that rebalancing is similar to starting with a buy-and-hold portfolio and adding a short straddle (selling both a call and a put option) on the relative value of the portfolio assets. The option-like payoff to rebalancing induces *negative* convexity by magnifying drawdowns when there are pronounced divergences in asset returns.[1]

We show that time-series momentum (or trend) strategies, applied to futures on the same stock and bond markets, are natural complements to a rebalanced portfolio. This is because the trend payoff tends to mimic that of a long straddle option position, or exhibits *positive* convexity; see, for example, Martin and Zou (2012) and Hamill, Rattray, and Van Hemert (2016).[2]

Our main analysis is for the 1960–2017 period, which includes the bond bear market of the 1960 and 1970s but omits the different bond regime before 1960.[3] We evaluate how 1-, 3-, and 12-month trend strategies perform during the five worst drawdowns for the 60–40 stock–bond portfolio. Allocating 10% to a trend strategy and 90% to a 60–40 monthly rebalanced portfolio improves the average drawdown by about 5 percentage points, compared to a 100% allocation to a 60–40 monthly rebalanced portfolio. The trend allocation

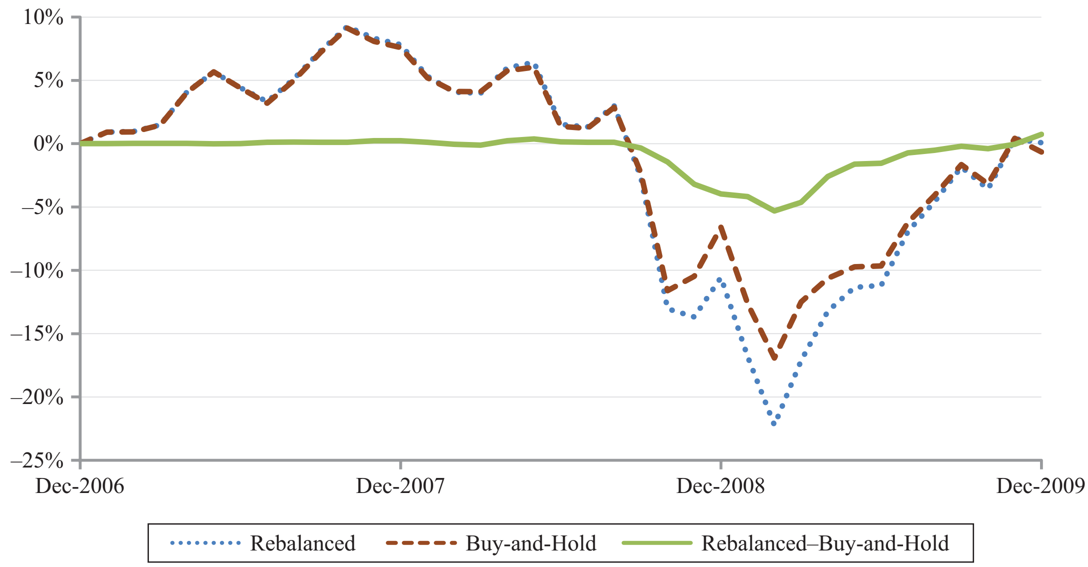
[1] Among practitioners, the term *negative convexity* rather than *concavity* is often used. This stems from reading position exposures on risk sheets and so preferring a measure that can be either positive or negative, like beta or delta, rather than the more cumbersome switching between convex and concave when the direction changes.

[2] Although time-series momentum applied to macro markets (e.g., a broad equity index or government bond) considered in this article tends to display positively convex returns, Daniel and Moskowitz (2016) argued that cross-sectional momentum applied to individual stocks is subject crash risk.

[3] See Harvey et al. (2018) for a discussion on the different US bond regimes.

EXHIBIT 2

Performance Monthly Rebalanced and Buy-and-Hold Portfolio (2007–2009)



Notes: Cumulative return for a monthly rebalanced and a buy-and-hold portfolio for the 2007–2009 financial crisis period, as well as the difference between them. Both portfolios start with an initial 60–40 stock–bond capital allocation in January 2007.

has no adverse impact on the average return over our sample period. That is, although one would normally expect a drag on the overall (long-term) performance when allocating to a defensive strategy, in our sample, the trend-following premium earned offsets the cost (or insurance premium) paid.[4]

An alternative to a trend allocation is strategic timing and sizing of rebalancing trades, which we label *strategic rebalancing*. We first consider a range of popular heuristic rules: varying the rebalancing frequency, using thresholds, and trading only partially back to the 60–40 asset mix. Such heuristic rules reduce the average maximum drawdown level for the five crises considered by up to 1 percentage point. However, using strategic rebalancing rules based on either the past stock or past stock–bond relative returns gives improvements of 2 to 3 percentage points.

The literature on rebalancing dates back to at least Perold and Sharpe (1988).[5] Our main contribution is that we show that the negative convexity induced by rebalancing is effectively countered with a trend exposure, which exhibits positive convexity and can be either implemented directly via an allocation to a trend product or alternatively with a strategic trend-based rebalancing rule. The five worst drawdowns for a 60–40 portfolio over the 1960–2017 period considered are materially reduced.

Our article is organized as follows. In the first section, we show that the return difference between a rebalanced and a buy-and-hold portfolio is concave in the relative stock–bond performance, both analytically in a stylized two-period model and empirically for the 1960–2017 period. In the second section, we show that the return to a trend strategy applied to stocks and bonds is convex in the relative stock–bond performance. We illustrate that a modest allocation to a trend strategy can effectively counter the negative convexity induced by rebalancing and as such reduce drawdowns. In the third section, we explore different heuristics and trend-based strategic rebalancing rules and show that the strategic rebalancing rules in particular are helpful for reducing drawdowns for a 60–40 stock–bond portfolio. In the fourth section, we compare a direct allocation to trend and an indirect trend exposure obtained with a trend-based strategic rebalancing rule. Some concluding remarks are offered in the last section.

[4] We find that the performance of trend strategies is consistent over time, not driven by any particular subperiod.

[5] See also Fernholz and Shay (1982), Booth and Fama (1992), Erb and Harvey (2006), and Brown (2015).

REBALANCED VERSUS BUY-AND-HOLD

The notion that 60–40 equity–bond is a good asset mix has been around for decades; see, for example, Ambachtsheer (1987) for an early reference.[6] From a general equilibrium point of view this makes sense: The ratio of equity and bond value in, for example, the United States has been around 60–40 over the past decades, even though this ratio is subject to considerable variability.[7] Large pension plans and sovereign wealth funds often explicitly target a fixed 60–40 asset mix. For example, in 2007 the Norwegian Government Pension Fund Global adopted a 60% target allocation to equities, with the remainder mostly invested in fixed income; see, for example, Chambers, Dimson, and Ilmanen (2012). In this section, we start by considering a two-period model to illustrate the difference between monthly rebalancing to a constant asset mix (Rebal) and buy-and-hold (Hold).

Writing the return for stocks and bonds in period $t = 1, 2$ as R_t^S and R_t^B, respectively, the return of a portfolio that rebalances between period 1 and 2 to allocations w^S and w^B for stocks and bonds is

$$1 + R^{REBAL} = (1 + w^S R_1^S + w^B R_1^B)(1 + w^S R_2^S + w^B R_2^B) \quad (1)$$

For a portfolio that starts with the same weights but does not rebalance after period 1, we get

$$\begin{aligned} 1 + R^{HOLD} &= w^S (1 + R_1^S)(1 + R_2^S) \\ &\quad + w^B (1 + R_1^B)(1 + R_2^B) \end{aligned} \quad (2)$$

We can rewrite the return for stocks and bonds in terms of the average returns $R_t^{Avg} = 0.5R_t^S + 0.5R_t^B$ and the stock–bond return difference, $\kappa_t = R_t^S - R_t^B$:

$$R_t^S = R_t^{Avg} + 0.5\kappa_t \quad (3)$$

$$R_t^B = R_t^{Avg} - 0.5\kappa_t \quad (4)$$

Substituting in these terms in the Rebal and Hold return expressions, taking the difference, using the fact that the allocations sum to one, and rearranging gives

$$R^{REBAL} - R^{HOLD} = -w^S w^B \kappa_1 \kappa_2 \quad (5)$$

Thus, if the relative performance is trending (κ_1,κ_2 are either both positive or both negative), the rebalanced portfolio underperforms. Intuitively, rebalancing means selling winners: if winners continue to outperform, that detracts from performance. Vice versa, if there is reversal of relative performance, the rebalanced portfolio outperforms.[8,9]

Note some special cases. For an equally weighted portfolio (50% of capital allocated to stocks and 50% to bonds), the Rebal–Hold return difference is $-0.25\kappa_1\kappa_2$, whereas for the 60–40 stock–bond portfolio it is slightly less: $-0.24\kappa_1\kappa_2$. In the case of a 100% allocation to either stocks or bonds, we have a zero return difference, which is intuitive because there can be no drift in the relative allocation for a one-asset portfolio.

Note also that the formula allows us to measure order of magnitude. If stocks underperform bonds by 40% in both periods 1 and 2 (i.e., $\kappa_1 = \kappa_2 = -40\%$), a 60–40 rebalanced portfolio has a 3.84 percentage point more negative return than the buy-and-hold portfolio. These numbers roughly correspond to what happened between October 2007 and February 2009, when stocks were down 50.3% (or two periods of −29.5% returns, taking into account compounding) and bonds were up 17.9% (or two periods of +8.6% returns). In Exhibit 1, the Rebal–Hold return difference is 5.3 percentage points per February 2009, which is slightly greater because compounding in a setting with more than two periods will exacerbate the rebalancing effect.

In a multiperiod setting, the return difference between a monthly rebalanced and buy-and-hold portfolio is similar to that of a short straddle written on

[6] This article focuses on 60–40 stock–bond in terms of capital allocation. More recently, volatility targeting has been gaining traction, and we defer to Harvey et al. (2018) for a discussion on a 60–40 stock–bond portfolio in terms of risk allocation.

[7] US government debt has averaged around 60% of gross domestic product (GDP) since 1966 (https://fred.stlouisfed.org/series/GFDEGDQ188S). The stock market capitalization-to-GDP ratio, also known as the Buffett indicator, has averaged around 90% since 1975 (https://fred.stlouisfed.org/series/DDDM01USA156NWDB). Note that both government debt and the stock market cap have trended upward similarly as fraction of GDP since the 1970s.

[8] Similarly, Perold and Sharpe (1988) noted that constant-mix portfolios have less downside protection and less upside than a buy-and-hold strategy, while performing better in relatively trendless but volatile environments.

[9] Goetzmann et al. (2007) showed that rebalancing strategies can make traditional performance metrics less reliable and suggested using manipulation-proof performance measures.

the relative performance of stocks and bonds.[10] Granger et al. (2014) made this point and provided both analytical expressions and simulation results.[11]

For our empirical analysis, we use monthly value-weighted returns of firms listed on the NYSE, AMEX, and NASDAQ from Kenneth French's website.[12] For bonds, we use US Treasury bond data from the Federal Reserve.[13]

In Exhibit 3, we plot the Rebal–Hold return difference when both have a 60–40 stock–bond mix at the start of the period (vertical axis) versus the stock–bond relative return (horizontal axis). Each dot in the exhibit corresponds to a one-year (rolling 12-month) window, in which data run from January 1960 to December 2017.[14] Indeed, the Rebal–Hold return difference looks a lot like the payoff of a short straddle on the relative performance—this illustrates the negative convexity.

ADDING A TREND STRATEGY ALLOCATION

We define a simple time-series momentum (trend) signal for asset k as the return over the past N months, divided by a volatility estimate, which we set equal to the standard deviation of the past 12 monthly returns and the square root of N to make it approximately unit standard deviation:[15]

$$mom_{t-1}^{k}(N) = \frac{\sum_{i=1}^{N} \tilde{R}_{t-i}^{k}}{\sigma_{t-1}^{k}\sqrt{N}} \tag{6}$$

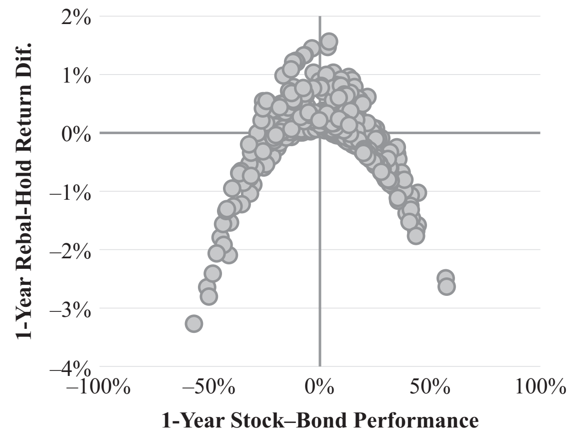
For the asset returns, we will use the stock and bond data used previously but in excess of one-month Treasury bill returns (denoted by a tilde), which is a proxy of the return on an unfunded futures contract on the stock or bond index. We cannot use stock and bond futures data directly because they are not available as far back as 1960.

We consider a number of trend strategies, combining trend signals for stocks, bonds, and a long stocks–short bonds spread position. Equation 7 illustrates how we determine the strategy return for the case of putting equal risk on the stocks and bonds trend (which we feature in Panel C of Exhibit 4). We set an ex ante reference volatility for monthly returns, σ^{Ref}, that leads to about 15%–20% annualized realized volatility for the strategy returns.[16] We conservatively assume that 20% of capital needs to be posted for margin and earns no interest, whereas the remaining 80% earns the T-bill return, R_t^F. Thus, we get the following expression for the strategy returns of an N-month trend strategy:

$$\begin{aligned} R_t^{mom(N)} = {} & 0.5mom_{t-1}^{S}(N)\frac{\sigma^{Ref}}{\sigma_{t-1}^{S}}\tilde{R}_t^{S} \\ & + 0.5mom_{t-1}^{B}(N)\frac{\sigma^{Ref}}{\sigma_{t-1}^{B}}\tilde{R}_t^{B} + 0.8R_t^{F} \end{aligned} \tag{7}$$

EXHIBIT 3

Rebal–Hold versus Stock–Bond 1-Year Returns



Notes: The return difference of a monthly rebalanced versus a buy-and-hold portfolio, when both have a 60–40 stock–bond mix at the start of the 1-year evaluation period (vertical axis) versus the stock–bond relative return over the same one-year period (horizontal axis). Each dot in the exhibit corresponds to a 1-year (rolling 12-month) window, in which data run from January 1960 to December 2017.

[10] Fung and Hsieh (2001) argued that trend-following strategies are theoretically more related to lookback straddles but found that empirically standard straddles explain trend-following returns as well as lookback straddles.

[11] Israelov and Tummala (2017) considered an option-selling overlay to augment portfolio rebalancing.

[12] See: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

[13] Federal Reserve Economic Data (FRED); see https://fred.stlouisfed.org.

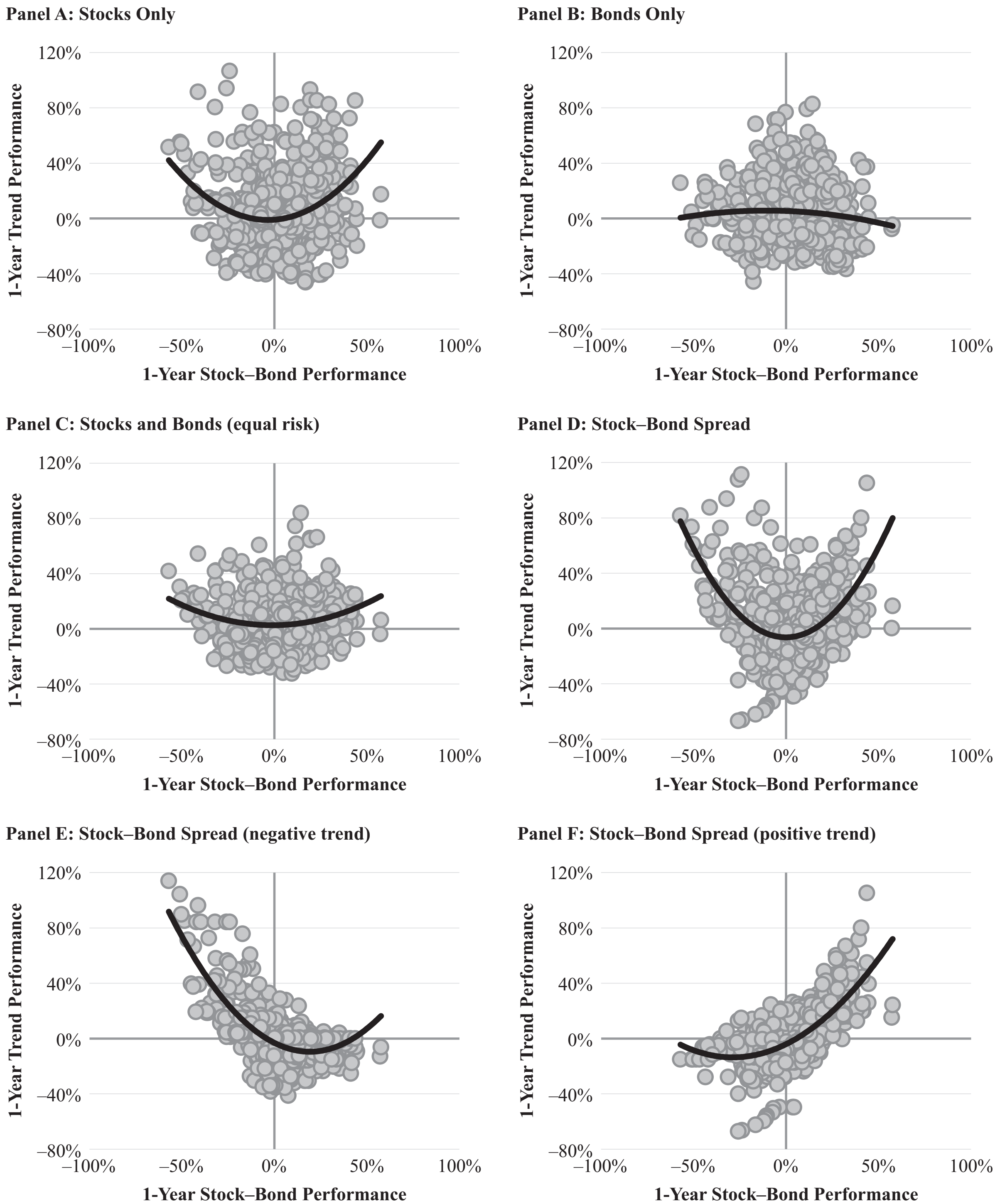
[14] Like Hamill, Rattray, and Van Hemert (2016), we argue that it is important to start as far back as 1960 so that the sample period includes a bond bear market environment (pre-1982).

[15] Moskowitz, Ooi, and Pedersen (2012) and Levine and Pedersen (2016) used similar formulations.

[16] There is no ex post scaling to hit a particular level of volatility.

Exhibit 4

Three-Month Trend versus Stock–Bond 1-Year Return



Notes: One-year (rolling 12-month) performance of various 3-month trend strategies (vertical axis) versus the 1-year relative stock–bond performance (horizontal axis) for 1960–2017. The solid line represents the best quadratic fit.

Other strategies that we feature (e.g., 100% equity) are defined analogously.

We introduce two more features so that the simple trend strategy has more bounded long and short positions in stock and bond futures. These features are consistent with how practitioners implement trend strategies. First, we put a floor and cap on the signal value of −1 and +1 respectively. Second, for the annualized security volatility used in the second equation, we use a floor of 10% in the case of stocks and the stock–bond spread asset and 5% in the case of bonds, which corresponds to about two-thirds of the full-sample realized value. This will limit the leverage at times of low realized asset volatilities. Neither of these additional features changes the return dynamics much, but they do achieve more bounded positions.[17]

For the trend strategy, implemented with futures, we follow Harvey et al. (2018) and assume a transaction cost of 1 bp (0.01%) of the traded notional for equities and 0.5 bp for bonds (an estimate for the current trading conditions). For rebalancing of the 60–40 target portfolio, implemented with cash equity and bond holdings, we follow a recent Norges Bank (2018) report and use 30 bps for equities and 13 bps for bonds (reflective of the 2015–2016 period).

In Exhibit 4, we plot the one-year return of various trend strategies (vertical axis) versus the one-year relative stock–bond return (horizontal axis), with the best quadratic fit added as a solid line. We use three-month trend signals, but we confirm that results are similar for 1- and 12-month formulations. We consider six trend formulations in Panels A–F, varying the traded asset in the trend program: stocks only (A); bonds only (B); stocks and bonds with equal risk weight (C); a long–short stock–bond spread position (D); a stock–bond spread, but only taking positions if the trend is negative (E); and a stock–bond spread, but only taking positions if the trend is positive (F).[18]

In case of a stock–bond spread trend strategy (Panel D), a very pronounced smile pattern is visible, which is expected given the known convexity property of trend strategy returns when evaluated against the returns of the traded asset (i.e., the horizontal axis in all panels is the stock–bond spread return). In addition, for stock only (Panel A) and for stock and bond equal risk (Panel C), which follows Equation 7, the trend strategy returns are convex; this is unsurprising, given that the stock and stock–bond excess returns are 0.9 correlated over the 1960–2017 period (stock return variance dominates the stock–bond return spread). As such, a trend strategy with a stock weighting looks like a natural complement to a rebalancing strategy, considering we have shown that the Rebal–Hold return difference displays negative convexity. A bond-only trend strategy (Panel B) does not show a clear relation against the stock–bond relative return. Finally, we consider a stock–bond spread trend strategy that trades only in the case of negative (Panel E) and positive (Panel F) trends. The payoff of these strategies mimics not so much a long straddle (put plus call) anymore but, rather, that of just a put (in the case of negative trend) and call (in the case of positive trend).

Given their seemingly complementary return profile in most cases considered, we next combine a monthly rebalanced portfolio (90%) and the various trend strategies (10%). In Appendix B, we also consider combing a 100% allocation to the monthly rebalanced portfolio with a 10% trend strategy, financed by borrowing at the short rate. We are particularly interested in whether the addition of a trend reduces drawdowns. In Exhibit 5, we depict the drawdown level for the monthly rebalanced 60–40 portfolio, computed as the return from the peak (highest cumulative return up to that point in time). On five occasions, the drawdown is worse than −15%, where the trough months are June 1970 ("back to earth" day for tech stocks), September 1974 (oil crisis/Yom Kippur War, Collapse of Bretton Woods, Watergate), November 1987 (Black Monday, program trading), June 2002 (tech bubble burst), and February 2009 (global financial crisis).[19] In subsequent analyses, we will report the change in the drawdown level at these five worst moments for the 60–40 portfolio.

In Exhibit 6, we report how a 90% monthly rebalanced 60–40 portfolio (Rebal) plus 10% allocation to a trend strategy performs when compared with the benchmark of 100% Rebal. We cover the same six trend

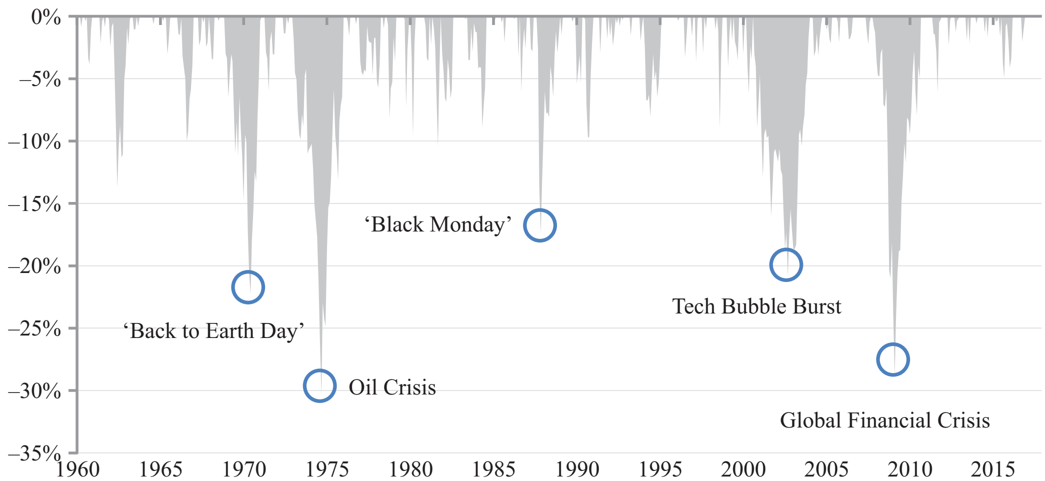
[17] This is consistent with how practitioners implement such trend strategies.

[18] Equal risk across asset classes (Panel C) is common in the managed futures space and will serve as our baseline case in subsequent analyses. Asvanunt, Nielsen, and Villalon (2015) also studied the impact of adding a two-asset (equity and bond) trend strategy, like we show in Exhibit 4 (Panel C), to a 60–40 stock–bond portfolio.

[19] Note that Black Monday refers to October 19, 1987 but that the market trough (using monthly data) is in November 1987.

Exhibit 5

Drawdown Level Monthly Rebalanced 60–40 Portfolio



Notes: Drawdown level for a monthly rebalanced 60–40 stock–bond allocation. Data are monthly from 1960 to 2017.

variations as before in Exhibit 4 and look at 1-, 3-, and 12-month trend windows. For trend systems applied to stocks only (Panel A) and bonds only (Panel B), the average return is similar to the 9.1% of the 100% Rebal portfolio—but with lower volatility—even though the trend system is applied to just one asset.[20] The drawdowns tend to be less severe, with an average improvement (ΔDD average) ranging from 3.6% to 5.7%. In the case of stocks (Panel A), the notable exception is the drawdown around Black Monday when a slow, 12-month trend strategy actually exacerbates the drawdown.

In the case of bonds (Panel B), the average stock allocation is much reduced at 54% (versus 60% stocks for the 100% Rebal portfolio); thus, the better performance during the stock market drawdowns is intuitive, although it is noteworthy that the average return is not reduced with the lower stock allocation (the bond trend strategy payoff fully compensates for the reduced equity-premium capture).

In the case of stock and bond trend, equal risk (Panel C), a 10% trend allocation again leads to average returns at least as good as the benchmark and an improved average drawdown level. Furthermore, in particular for the 12-month trend, the average allocation to stocks and bonds is close to that of the benchmark's 60% stocks and 40% bonds.

In Panel D, we consider the trend on the stock–bond spread asset. Note that this will lead to a 90% total stock + bond allocation by construction because the total (stock + bond) allocation from the 10% trend strategy is zero. Here, the average return is slightly lower than that of the benchmark. In addition, the drawdown around Black Monday is more greatly affected in the case of the 3- and 12-month trend compared to the specifications considered in Panels A, B, and C. In Panels E and F, we consider a version in which we only trade on the stock–bond signal if it is negative and positive, respectively. Just trading on negative signals helps in reducing drawdowns, but it leads to a much lower stock allocation and a lower average return compared to the benchmark.

The Sharpe ratios reported in Exhibit 6 for strategies with a 10% trend allocation range from 0.45 to 0.54, which is similar to the 0.47 Sharpe ratio for the 100% Rebal baseline case. However, the benefit of the trend allocation is not so much a higher Sharpe ratio but, rather, a more benign risk profile with shallower drawdowns. To illustrate this further, we use the Goetzman et al. (2007) performance measure that controls for higher moments. This measure is the annualized certainty-equivalent return (CEQ) in a power utility framework and so penalizes negative skewness and excess kurtosis relative to the Sharpe ratio metric. Appendix A considers

[20] Using data from 1960 (as in this article), Hamill, Rattray, and Van Hemert (2016) found that a simple trend strategy applied to 55 securities realizes a Sharpe ratio well above 1.

EXHIBIT 6
Rebal Plus Trend Performance Statistics

	Rebal (100%)	Rebal (90%) 1m Trend (10%)	Rebal (90%) 3m Trend (10%)	Rebal (90%) 12m Trend (10%)
Panel A: Trend Applied to Stocks Only				
Stock Allocation (avg)	60.0%	57.7%	60.1%	64.5%
Bond Allocation (avg)	40.0%	36.0%	36.0%	36.0%
Total Allocation (avg)	100.0%	93.7%	96.1%	100.5%
Rebal Trade (ann)	10.2%	9.1%	9.1%	9.1%
Stock Fut. Trade (ann)	0.0%	207.3%	106.7%	45.1%
Bond Fut. Trade (ann)	0.0%	0.0%	0.0%	0.0%
Cost Estimate (ann), bps	4.4	6.0	5.0	4.4
Return (ann)	9.1%	9.0%	9.1%	9.6%
Volatility (ann)	9.8%	8.7%	9.1%	9.7%
Ret./Vol. (ann)	0.92	1.03	1.00	0.99
Sharpe Ratio (ann)	0.47	0.52	0.51	0.53
ΔDD June 1970	0.0%	3.6%	1.9%	5.4%
ΔDD September 1974	0.0%	9.7%	8.9%	9.8%
ΔDD November 1987	0.0%	4.9%	–0.8%	–3.0%
ΔDD September 2002	0.0%	4.2%	4.2%	6.0%
ΔDD February 2009	0.0%	6.1%	7.9%	9.0%
ΔDD Average	0.0%	5.7%	4.4%	5.4%
Panel B: Trend Applied to Bonds Only				
Stock Allocation (avg)	60.0%	54.0%	54.0%	54.0%
Bond Allocation (avg)	40.0%	39.9%	41.7%	45.3%
Total Allocation (avg)	100.0%	93.9%	95.7%	99.3%
Rebal Trade (ann)	10.2%	9.1%	9.1%	9.1%
Stock Fut. Trade (ann)	0.0%	0.0%	0.0%	0.0%
Bond Fut. Trade (ann)	0.0%	438.0%	245.4%	111.6%
Cost Estimate (ann), bps	4.4	6.1	5.2	4.5
Return (ann)	9.1%	9.4%	9.1%	9.3%
Volatility (ann)	9.8%	9.2%	9.2%	9.2%
Ret./Vol. (ann)	0.92	1.02	0.99	1.01
Sharpe Ratio (ann)	0.47	0.54	0.50	0.53
ΔDD June 1970	0.0%	5.1%	3.5%	5.6%
ΔDD September 1974	0.0%	6.0%	5.0%	6.2%
ΔDD November 1987	0.0%	0.9%	0.8%	1.1%
ΔDD September 2002	0.0%	6.4%	5.2%	7.7%
ΔDD February 2009	0.0%	2.1%	3.8%	6.0%
ΔDD Average	0.0%	4.1%	3.6%	5.3%

(continued)

Exhibit 6 *(continued)*
Rebal Plus Trend Performance Statistics

	Rebal (100%)	Rebal (90%) 1m Trend (10%)	Rebal (90%) 3m Trend (10%)	Rebal (90%) 12m Trend (10%)
Panel C: Trend Applied to Stocks and Bonds (equal risk)				
Stock Allocation (avg)	60.0%	55.8%	57.0%	59.3%
Bond Allocation (avg)	40.0%	37.9%	38.8%	40.7%
Total Allocation (avg)	100.0%	93.8%	95.9%	99.9%
Rebal Trade (ann)	10.2%	9.1%	9.1%	9.1%
Stock Fut. Trade (ann)	0.0%	103.7%	53.4%	22.6%
Bond Fut. Trade (ann)	0.0%	219.0%	122.7%	55.8%
Cost Estimate (ann), bps	4.4	6.1	5.1	4.4
Return (ann)	9.1%	9.2%	9.1%	9.4%
Volatility (ann)	9.8%	8.8%	9.0%	9.3%
Ret./Vol. (ann)	0.92	1.04	1.01	1.02
Sharpe Ratio (ann)	0.47	0.54	0.51	0.54
ΔDD June 1970	0.0%	4.4%	2.7%	5.6%
ΔDD September 1974	0.0%	7.9%	7.0%	8.0%
ΔDD November 1987	0.0%	2.9%	0.0%	–1.0%
ΔDD September 2002	0.0%	5.4%	4.7%	6.8%
ΔDD February 2009	0.0%	4.4%	6.5%	7.6%
ΔDD Average	0.0%	5.0%	4.2%	5.4%
Panel D: Trend Applied to Stock–Bond Spread				
Stock Allocation (avg)	60.0%	57.2%	59.1%	62.6%
Bond Allocation (avg)	40.0%	32.8%	30.9%	27.4%
Total Allocation (avg)	100.0%	90.0%	90.0%	90.0%
Rebal Trade (ann)	10.2%	9.1%	9.1%	9.1%
Stock Fut. Trade (ann)	0.0%	200.1%	110.8%	49.7%
Bond Fut. Trade (ann)	0.0%	200.1%	110.8%	49.7%
Cost Estimate (ann), bps	4.4	6.9	5.6	4.7
Return (ann)	9.1%	9.0%	8.9%	9.1%
Volatility (ann)	9.8%	8.8%	9.0%	9.4%
Ret./Vol. (ann)	0.92	1.02	0.99	0.97
Sharpe Ratio (ann)	0.47	0.51	0.49	0.49
ΔDD June 1970	0.0%	3.9%	1.9%	4.9%
ΔDD September 1974	0.0%	7.9%	7.4%	6.9%
ΔDD November 1987	0.0%	0.7%	–3.9%	–4.5%
ΔDD September 2002	0.0%	5.1%	6.2%	7.5%
ΔDD February 2009	0.0%	7.1%	8.2%	8.9%
ΔDD Average	0.0%	4.9%	4.0%	4.7%

(continued)

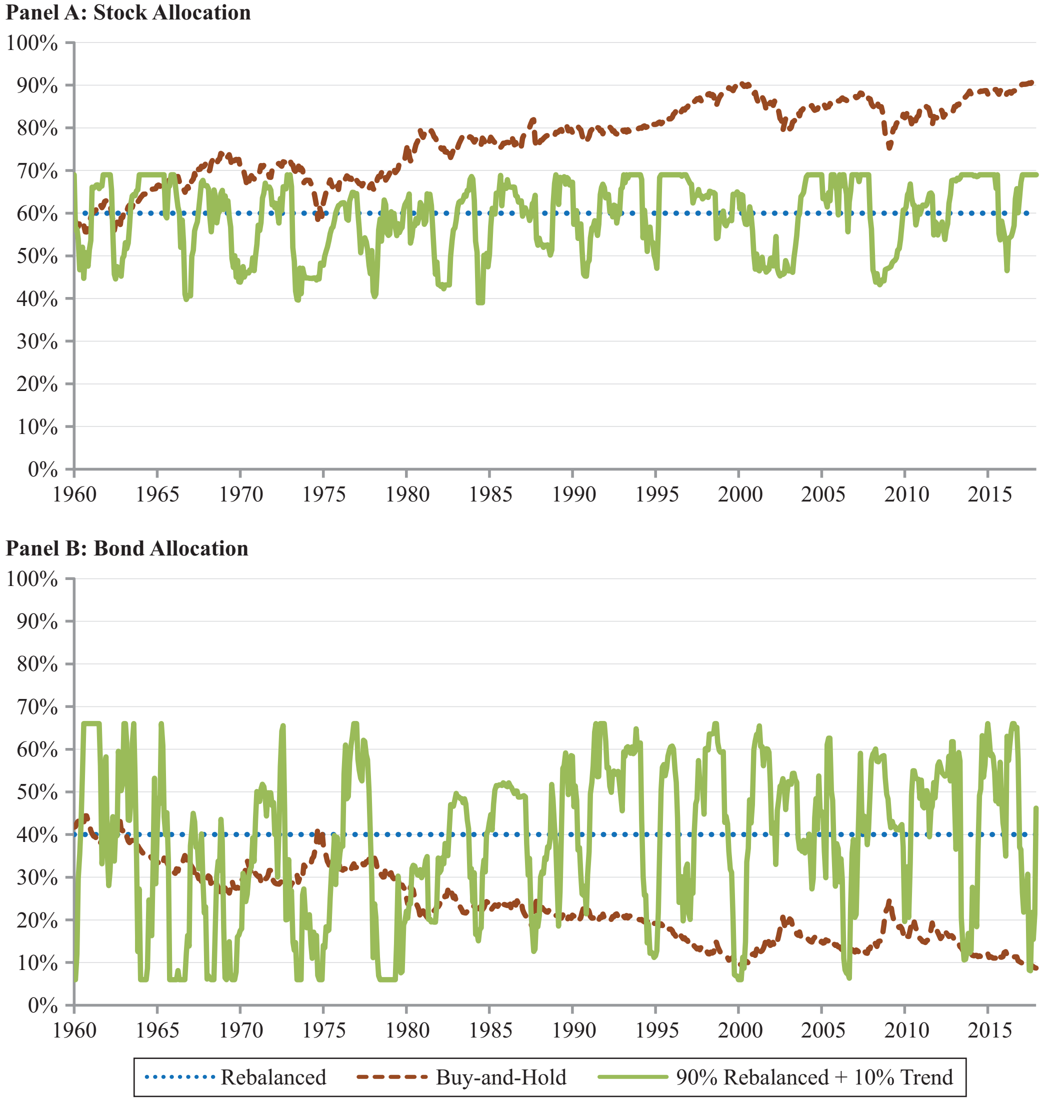
E X H I B I T 6 *(continued)*

Rebal Plus Trend Performance Statistics

	Rebal (100%)	Rebal (90%) 1m Trend (10%)	Rebal (90%) 3m Trend (10%)	Rebal (90%) 12m Trend (10%)
Panel E: Trend Applied to Stock–Bond, Negative Only				
Stock Allocation (avg)	60.0%	48.6%	49.2%	50.4%
Bond Allocation (avg)	40.0%	41.4%	40.8%	39.6%
Total Allocation (avg)	100.0%	90.0%	90.0%	90.0%
Rebal Trade (ann)	10.2%	9.1%	9.1%	9.1%
Stock Fut. Trade (ann)	0.0%	89.6%	45.0%	18.0%
Bond Fut. Trade (ann)	0.0%	89.6%	45.0%	18.0%
Cost Estimate (ann), bps	4.4	5.3	4.6	4.2
Return (ann)	9.1%	8.4%	8.5%	8.6%
Volatility (ann)	9.8%	8.1%	8.0%	8.2%
Ret./Vol. (ann)	0.92	1.04	1.06	1.04
Sharpe Ratio (ann)	0.47	0.49	0.51	0.50
ΔDD June 1970	0.0%	5.2%	4.6%	6.2%
ΔDD September 1974	0.0%	9.3%	9.6%	8.5%
ΔDD November 1987	0.0%	2.3%	2.3%	1.8%
ΔDD September 2002	0.0%	8.0%	7.9%	8.8%
ΔDD February 2009	0.0%	8.2%	10.7%	10.4%
ΔDD Average	0.0%	6.6%	7.0%	7.1%
Panel F: Trend Applied to Stock–Bond, Positive Only				
Stock Allocation (avg)	60.0%	62.6%	63.9%	66.2%
Bond Allocation (avg)	40.0%	27.4%	26.1%	23.8%
Total Allocation (avg)	100.0%	90.0%	90.0%	90.0%
Rebal Trade (ann)	10.2%	9.1%	9.1%	9.1%
Stock Fut. Trade (ann)	0.0%	110.5%	65.8%	31.7%
Bond Fut. Trade (ann)	0.0%	110.5%	65.8%	31.7%
Cost Estimate (ann), bps	4.4	5.6	4.9	4.4
Return (ann)	9.1%	9.1%	8.9%	9.1%
Volatility (ann)	9.8%	9.5%	9.8%	10.0%
Ret./Vol. (ann)	0.92	0.95	0.91	0.91
Sharpe Ratio (ann)	0.47	0.49	0.45	0.46
ΔDD June 1970	0.0%	1.5%	0.2%	1.5%
ΔDD September 1974	0.0%	2.0%	1.4%	1.8%
ΔDD November 1987	0.0%	0.1%	–4.4%	–4.5%
ΔDD September 2002	0.0%	–0.3%	0.8%	1.3%
ΔDD February 2009	0.0%	1.7%	0.4%	1.3%
ΔDD Average	0.0%	1.0%	–0.3%	0.3%

Notes: We contrast a 100% allocation to a monthly rebalanced portfolio with a 60–40 stock–bond capital allocation and one in which 10% of the portfolio is replaced with an allocation to various 1-, 3-, and 12-month trend specifications. Performance statistics reported on are the average stock, bond, and total allocation (block 1); the annualized notional trading (as a percentage of the total portfolio value) in the 60–40 rebalanced portfolio and stock and bond futures, as well as annualized trading costs (block 2); average return, standard deviation, the ratio of the two (not deducting the short rate), and the Sharpe ratio (which deducts the short rate) (block 3); and the change in the drawdown level for the five worst drawdowns (ΔDD) at the trough, compared to a 100% allocation to the monthly rebalanced 60–40 portfolio. The data are from 1960 to 2017.

Exhibit 7
Allocation to Stocks and Bonds for the Various Portfolios Considered



Notes: The (index plus futures) allocation to stocks (Panel A) and bonds (Panel B) for a monthly rebalanced 60–40 stock–bond portfolio, a buy-and-hold portfolio that starts with the same allocation mix, and a 90% monthly rebalanced and 10% 12-month (equity and bond, equal risk) trend strategy combination. Data run from 1960 to 2017.

four risk aversion parameters applied to the results in Panel C of Exhibit 6 (see Appendix Exhibit A1, Panels A and B). The results consistently show higher CEQs for the 10% to trend allocation. A similar insight can be gained by applying leverage to match the maximum drawdown size of the 100% rebalanced portfolio. Our results show that leveraged portfolios with 10% allocation to trend have significantly higher returns than the 100% rebalanced portfolio (Appendix A, Panel C of Exhibit A1).

We found that allocating 5% to a trend strategy (rather than the 10% considered in Exhibit 6) gives, not surprisingly, about half this reduction in the average drawdown. That is, the impact scales approximately proportionally with the trend allocation (for modest values).

One could argue that part of the improvement in the drawdown characteristics is due to the divestment of 10% of the rebalanced portfolio and part is due to the defensive nature of the 10% trend allocation. In Appendix B, we show that when keeping the full 100% rebalanced portfolio, the addition of a 10% trend investment (in this case financed by borrowing at the short rate) generally improves the drawdown of the overall portfolio.

In Appendix C, we show that the addition of trend has the same benefitical effect on drawdowns for the 30–70 stock–bond portfolio, which is much closer to equal risk to stocks and bonds than the 60–40 stock bond portfolio.

Finally, in Exhibit 7, we contrast the allocation to stocks and bonds (index plus futures) for a monthly rebalanced portfolio with a 60–40 stock–bond capital allocation, a buy-and-hold portfolio that starts with the same allocation mix, and an allocation of 90% to the rebalanced portfolio and 10% allocation to a 12-month stocks and bonds (equal-risk) trend strategy. Because of the signal caps and volatility floors used, the stock futures position is guaranteed to be between −15% and +15% and the bond futures position between −30% and +30%. The Rebal plus trend combination has a fluctuating stock and bond allocation but no long-term drift. It has a slight long bias with, on average, a 5.3% long stock and a 4.7% long bonds futures position, coincidentally almost exactly replacing the 10% reduction in the 60–40 rebalanced portfolio (see Exhibit 6, Panel C).

STRATEGIC REBALANCING

Rather than allocating to a trend strategy to counteract the tendency of rebalanced portfolios to underperform during equity market drawdowns, we now study whether one can get similar benefits by smartly timing and sizing rebalancing trades, which we call *strategic rebalancing*. We consider both commonly used heuristics and trend-based rules.

There is a large literature on evaluating heuristic rebalancing rules, in which one typically varies the rebalancing frequency, takes a threshold-based method, or combines these two approaches. In addition, rather than rebalancing fully toward the target asset mix, one can rebalance partially and so reduce turnover (and save

EXHIBIT 8
Rebalancing with Frequency and Threshold Rules

	Monthly			Quarterly			Annual			2% Threshold			4% Threshold		
	Full	Half	Quarter	Full	Half	Quarter	Full	Half	Quarter	Full	Half	Quarter	Full	Half	Quarter
Stock Allocation (avg)	60.0%	60.1%	60.2%	60.1%	60.3%	60.6%	60.4%	61.1%	62.2%	60.2%	60.4%	60.5%	60.5%	61.1%	61.4%
Stock Allocation (std)	0.0%	0.7%	1.4%	1.3%	1.7%	2.5%	2.9%	3.3%	4.1%	0.9%	1.2%	1.7%	1.7%	1.9%	2.3%
% Months Rebal	100.0%	100.0%	100.0%	33.3%	33.3%	33.3%	8.3%	8.3%	8.3%	15.8%	23.0%	34.5%	6.6%	9.3%	14.2%
Rebal Trade (ann)	10.2%	6.2%	4.2%	6.6%	3.9%	2.5%	3.6%	1.8%	1.1%	5.2%	3.9%	3.1%	3.7%	2.6%	2.1%
Cost (ann), bps	4.4	2.7	1.8	2.9	1.7	1.1	1.6	0.8	0.5	2.3	1.7	1.4	1.6	1.1	0.9
Return (ann)	9.1%	9.2%	9.2%	9.2%	9.2%	9.2%	9.3%	9.2%	9.1%	9.1%	9.2%	9.2%	9.2%	9.2%	9.2%
Volatility (ann)	9.8%	9.8%	9.8%	9.8%	9.8%	9.8%	9.8%	9.8%	10.0%	9.9%	9.8%	9.8%	9.8%	9.9%	9.9%
Ret./Vol. (ann)	0.93	0.94	0.94	0.94	0.95	0.94	0.95	0.94	0.92	0.93	0.94	0.94	0.93	0.94	0.94
DD June 1970	0.0%	0.1%	0.1%	0.1%	0.3%	0.2%	0.5%	0.2%	–0.7%	–0.2%	0.0%	0.1%	–0.2%	0.2%	–0.2%
DD September 1974	0.0%	0.2%	0.6%	0.5%	0.9%	1.3%	1.3%	1.7%	1.4%	0.0%	0.4%	0.7%	0.4%	0.4%	0.6%
DD November 1987	0.0%	0.1%	–0.3%	0.5%	–0.2%	–1.0%	–2.3%	–2.2%	–2.4%	–0.4%	–0.2%	–0.3%	–0.7%	–0.7%	–0.7%
DD September 2002	0.0%	0.4%	0.9%	1.2%	1.5%	1.9%	1.7%	1.9%	–0.1%	–0.2%	0.5%	1.2%	0.0%	1.1%	1.3%
DD February 2009	0.0%	0.8%	1.8%	0.9%	2.1%	3.0%	2.8%	3.2%	2.9%	0.2%	1.1%	2.0%	0.0%	1.1%	1.9%
DD Average	0.0%	0.3%	0.6%	0.6%	0.9%	1.1%	0.8%	0.9%	0.2%	–0.1%	0.4%	0.8%	–0.1%	0.4%	0.6%

Notes: We show results for frequency- (monthly, quarterly, annual) and threshold-based (60% ± 2% and 60% ± 4%) rebalancing rules. We consider a full, half, or quarter rebalancing toward the 60–40 capital allocation mix. We report the following in the different blocks: the average and standard deviation of the stock allocation, noting that the bond allocation is 100% minus the stock allocation (block 1); the fraction of months and annualized amount of rebalancing and annualized trading costs (block 2); the average and standard deviation of the return, as well as the ratio (block 3); and the change in the drawdown level for the five worst drawdowns (ΔDD) at the trough, compared to a 100% allocation to the monthly rebalanced 60–40 portfolio (block 4). Data are from 1960 to 2017.

Exhibit 9

Strategic Rebalancing with Stock–Bond Trend Rules

	Delay if 1m Trend			Delay if 3m Trend			Delay if 12m Trend		
	Negative	Positive	Continues	Negative	Positive	Continues	Negative	Positive	Continues
Stock Allocation (avg)	59.3%	61.0%	60.0%	58.9%	61.5%	60.2%	58.1%	61.6%	59.9%
Stock Allocation (std)	2.2%	1.6%	2.3%	2.6%	2.3%	2.8%	3.9%	2.5%	4.9%
% Months Rebal	49.3%	50.7%	48.3%	48.0%	52.0%	25.6%	42.2%	57.8%	12.6%
Rebal Trade (ann)	4.0%	4.5%	3.8%	3.5%	4.4%	2.7%	4.5%	4.3%	1.7%
Cost Estimate (ann), bps	1.7	1.9	1.7	1.5	1.9	1.2	1.9	1.9	0.7
Return (ann)	9.1%	9.2%	9.1%	9.1%	9.2%	9.1%	9.0%	9.2%	9.1%
Volatility (ann)	9.6%	9.9%	9.7%	9.5%	10.0%	9.7%	9.4%	10.0%	9.7%
Ret./Vol. (ann)	0.95	0.93	0.94	0.96	0.93	0.94	0.96	0.92	0.94
DD June 1970	0.5%	–0.1%	0.3%	0.7%	–0.1%	0.4%	0.7%	0.0%	0.3%
DD September 1974	1.6%	0.2%	1.2%	1.6%	0.1%	1.1%	3.0%	0.2%	2.5%
DD November 1987	0.3%	–2.2%	–1.8%	0.3%	–2.2%	–1.7%	0.5%	–1.9%	–1.7%
DD September 2002	1.8%	0.2%	1.3%	2.0%	0.2%	1.4%	5.2%	0.2%	4.8%
DD February 2009	4.2%	0.7%	3.5%	4.8%	0.7%	3.6%	5.6%	0.8%	5.6%
DD Average	1.7%	–0.2%	0.9%	1.9%	–0.2%	0.9%	3.0%	–0.1%	2.3%

Notes: We show results when rebalancing is delayed if the stock–bond trend is negative, positive, or continues to be of the same sign (in which case rebalancing only occurs if the trend just changed sign). In months with no delay, there is a rebalancing halfway toward the 60–40 asset mix. The trend direction is determined by comparing the return over the past 1, 3, and 12 months to the typical (average) return over 1-, 3-, and 12-month windows. We report the following in the different blocks: the average and standard deviation of the stock allocation, noting that the bond allocation is 100% minus the stock allocation (block 1); the fraction of months and annualized amount of rebalancing and annualized trading costs (block 2); the average and standard deviation of the return, as well as the ratio (block 3); and the change in the drawdown level for the five worst drawdowns (ΔDD) at the trough, compared to a 100% allocation to the monthly rebalanced 60–40 portfolio (block 4). Data are from 1960 to 2017.

on trading costs), providing yet another knob to turn. See, for example, Arnott and Lovell (1993) for early work on this topic.[21]

In Exhibit 8, we produce statistics for combinations of these oft-used rules, again with emphasis on whether drawdowns are less severe compared to rebalancing fully back to 60–40 every month (left-most column). Rebalancing monthly but only half or a quarter toward 60–40 mostly reduces drawdowns and moreover leads to lower turnover, with rebalancing trades per year of 6.2% and 4.2%, respectively, versus 10.2% with a full rebalance each month. The actual transaction cost savings will vary greatly across investors, but for the aforementioned transaction assumptions, the impact is arguably small at 0.4 to 4.4 bps per annum. Quarterly and annual rebalancing further improves drawdowns, except for the 1987 drawdown (Black Monday), which is a drawdown that quickly reversed.[22,23]

Threshold-based rules seem slightly less potent when we consider rebalancing if the fraction of stocks is outside of the 60% ± 2% and 60% ± 4% range (we considered other ranges, but they did not materially improve performance). In all cases, the correlation to the monthly, fully rebalanced strategy is near 1 (not reported in the exhibit) and the average return is barely affected.

Next, we turn our attention to trend-based strategic rebalancing rules. In Exhibit 9, we show results when rebalancing is delayed if the stock–bond spread trend is negative, positive, or continues in the same direction (i.e., rebalancing only when the current trend direction is opposite to the direction of a month ago,

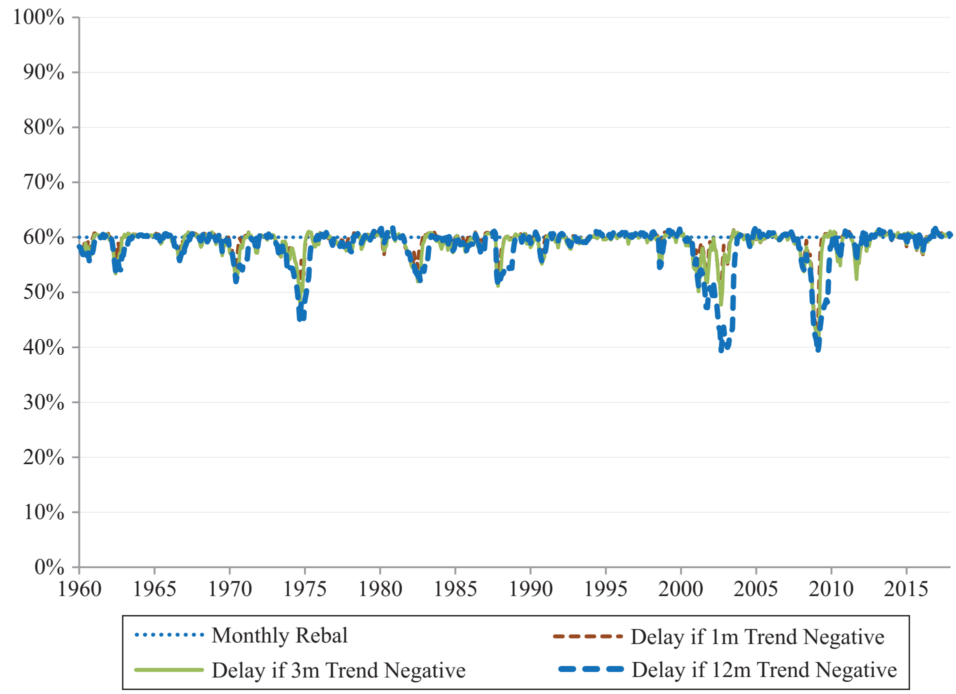
[21] Donohue and Yip (2003) and Masters (2003) linked rebalancing rules to considerations such as sensitivity to tracking error with the constant-mix portfolio and transaction costs.

[22] Driessen and Kuiper (2017), Ilmanen and Maloney (2015), and Huss and Maloney (2017) argued that rebalancing less frequently is a way to exploit predictability in asset returns.

[23] Here we use quarterly rebalancing per quarter ends (December, March, June, September) and annual rebalancing per year end (December). Using different months of the year leads to similar results.

EXHIBIT 10

Stock Allocation for Different Rebalancing Rules



Notes: The allocation to stocks for a monthly rebalanced 60–40 stock–bond portfolio as the baseline case. We also show the allocation to stocks when we apply a strategic rebalancing rule to delay rebalancing if the stock–bond trend is negative. In months with no delay, there is a rebalancing halfway toward the 60–40 asset mix. The trend direction is determined by comparing the return over the past 1, 3, and 12 months to the typical (average) return over 1-, 3-, and 12-month windows.

which likely corresponds to a less strong or inconsistent trending environment).[24] In months with no delay, there is a rebalancing halfway toward the 60–40 asset mix (mirroring the middle case considered Exhibit 8). The trend direction is determined by comparing the return over the past 1, 3, and 12 months to the typical (average) return over 1-, 3-, and 12-month windows.[25]

For all three trend windows, delay of the rebalancing when there is a negative trend in the stock–bond spread is most beneficial for reducing drawdowns. This is intuitive because drawdowns typically occur when stock returns are negative, and so a delay of rebalancing means not buying back stocks to bring it back in line with the 60–40 mix. This result is also consistent with Exhibit 4 (Panel E), which shows that the payoff of an explicit allocation to a stock–bond spread trend strategy that is constrained to be negative mimics that of a put option (on the stock–bond spread return), or with Exhibit 6 (Panel E), which shows that the same explicit trend allocation much reduces drawdowns. Delay of the rebalancing when the 12-month trend is negative leads to a reduction of more than 5 percentage points in the case of the tech bubble burst (September 2002 trough) and financial crisis (February 2009 trough), which is comparable to that of the 10% trend allocations considered before in Exhibit 6.

In Appendix C, we show that delaying the rebalancing when there is a negative trend in the stock–bond spread similarly reduces drawdowns for the 30–70 stock–bond portfolio.

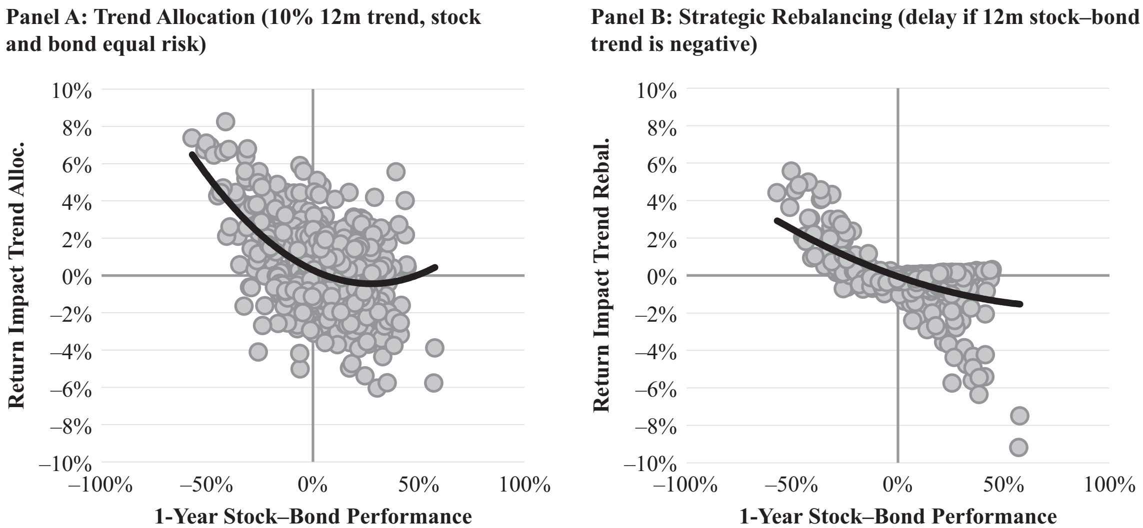
In Exhibit 10, we show the allocation to stocks for the strategic rebalancing rules wherein one delays rebalancing if the stock–bond return is in a negative trend, as considered before in Exhibit 9. The strategic rebalancing rule using 12-month trends leads to holding only around 40% stocks (20 percentage point underweight) at the

[24] We find that using a rule based on just the stock trend, rather than the stock–bond trend, leads to very similar results.

[25] We set this at 0.8%, 2.3%, and 9.1% for 1-, 3-, and 12-month trend horizons, corresponding to the empirical evidence over the 1960–2017 period.

Exhibit 11

Impact of Adding a Trend Exposure versus the Stock–Bond 1-Year Return



Notes: The 1-year (rolling 12-month) performance impact of adding a trend exposure to a monthly rebalanced 60–40 stock–bond strategy (vertical axis) versus the 1-year relative stock–bond performance (horizontal axis) for the period of 1960 to 2017. We consider a 10% allocation to a 12-month stocks and bonds (equal-risk) trend strategy (Panel B) and a strategic rebalancing rule to delay rebalancing when the 12-month stock–bond trend is negative (i.e., the return is below the average stock–bond 12-month return). Other statistics for these trend exposures can be found in Exhibit 6, Panel C, for the trend allocation and Exhibit 9 for the stock–bond trend rebalancing rule.

height of the two most severe stocks market drawdowns, after the tech bubble burst and during the global financial crisis. The stock allocation deviates less from 60% in case of one- and three-month trend, as shorter-term trend models tend to change sign more frequently.

The opposite case—delaying the rebalancing when there is a positive trend in the stock–bond spread return—does not tend to reduce drawdowns. The case of delaying if the trend continues to be of the same sign sits in between the negative and postive trend-based rule.

STRATEGIC REBALANCING VERSUS A DIRECT ALLOCATION TO TREND

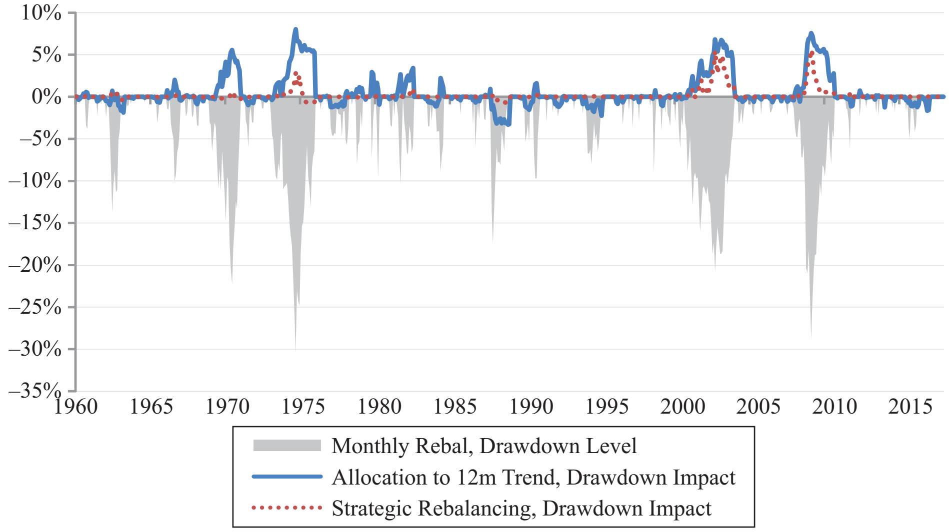
In Exhibit 11, we show the impact of adding the two different types of trend exposures considered in this article to a monthly rebalanced 60–40 stock–bond portfolio, as a function of the one-year stock-minus-bond return. In Panel A, we show the change in the one-year return with a 10% allocation to a 12-month stock-and-bond (equal-risk) trend strategy (reported in Exhibit 6, Panel C). In Panel B, we show the same output for a strategic rebalancing rule to delay rebalancing if the 12-month stock–bond spread trend is negative (reported in Exhibit 9).

Note that in both cases the trend exposure tends to be particularly helpful when stocks underperform bonds. Given that stocks are much more volatile than bonds, this usually means the exposure helps when stock returns are negative. In the case of a trend allocation (Panel A), the asymmetry of the impact contrasts with the much more symmetric effect for the standalone trend performance shown in Exhibit 4, Panel C. The reason is that, to allocate to the trend strategy, we reduced the allocation to the 60–40 stock–bond portfolio from 100% to 90%, which tends to help when the stock–bond performance is negative. In the case of the strategic rebalancing rule (Panel B), the asymmetric impact of the exposure comes directly from the rule itself being asymmetric (i.e., riding negative trends in the relative stock–bond return and not riding positive trends). The correlation between the two trend exposures in terms of their return difference with the 100% monthly rebalanced 60–40 stock–bond portfolio is 0.62, again suggesting they behave similarly.

In Exhibit 12, we show the impact on the drawdown level of the same two trend exposures that we considered before in Exhibit 11. Results at the trough of the five worst crises correspond to those reported before in Exhibit 6, Panel C and Exhibit 9. This exhibit compares

Exhibit 12

Impact of Adding a Trend Exposure on the Portfolio Drawdown Level



Notes: Drawdown level of a monthly rebalanced 60–40 stock–bond portfolio (blue line) and the impact (change in drawdown level) when a adding a trend exposure for the period of 1960 to 2017. We consider a 10% allocation to a 12-month stock-and-bond (equal-risk) trend strategy and a strategic rebalancing rule to delay rebalancing when the 12-month stock–bond trend is negative (i.e., the return is below the average stock–bond 12-month return). Other statistics for these trend exposures can be found in Exhibit 6, Panel C, for the trend allocation and Exhibit 9 for the stock–bond trend rebalancing rule.

the impact of the different trend exposures on the drawdown level alongside and over time. The main takeaway is that either a direct allocation to a trend strategy or using trend signals as a basis of a rebalancing rule tends to reduce the drawdown materially. The performance around Black Monday is the only exception here in case of an allocation to trend (less so for the strategic rebalancing rule).

CONCLUDING REMARKS

A pure buy-and-hold portfolio is untenable for most investors because it leads to a highly concentrated, undiversified portfolio. However, a 60–40 stock–bond portfolio (our use case) that rebalances every month to the 60–40 target ratio loses several percentage points more than a buy-and-hold portfolio during periods of continued stock market drawdowns. In essence, rebalancing to a constant asset mix means selling winners and buying losers, which hurts at times when the stock performance (relative to that of bonds) is trending. We show that the negative convexity induced by rebalancing is effectively countered with a trend exposure, which exhibits convexity and can be either implemented as a direct allocation to a trend strategy or with a strategic trend-based rebalancing rule.

Although our focus is on countering the negative convexity induced by rebalancing, other considerations matter in practice as well. For example, investors can also use monthly in- and outflows to move back toward the target asset mix. For example, Chambers, Dimson, and Ilmanen (2012) mentioned that the Norwegian Government Pension Fund Global directs monthly inflows into the asset class that is most underweight relative to the benchmark. For taxable investors, rebalancing using income has the added benefit that no assets need to be sold, which can be tax efficient; see Colleen, Kinniry, and Zilbering (2010).

Finally, we note that a stock–bond trend exposure is just one way to mitigate drawdowns in times of continued stock market losses. An investor has more arrows in her quiver. A good starting point is a more-diversified portfolio that includes more asset classes and has an international exposure. An allocation to a broader trend strategy that benefits from trends in other macro assets at times of equity market distress may further dampen equity market losses; see Hamill, Rattray, and Van Hemert (2016). In addition, Harvey et al. (2018) studied volatility targeting and showed that it can help manage the risk of a 60–40 stock–bond portfolio.

APPENDIX A

CERTAINTY EQUIVALENT PERFORMANCE GAIN

We use the Goetzman et al. (2007) manipulation-proof performance measure. This measure is the annualized CEQ in a power utility framework and so penalizes negative skewness and excess kurtosis relative to the Sharpe ratio metric. For a given risk aversion parameter γ, it is defined as

$$\text{CEQ}(\gamma)=\frac{1}{(1-\gamma)\Delta t}\ln\left(\frac{1}{T}\sum_{t=1}^{T}\left(\frac{1+R_t}{1+R_t^F}\right)^{1-\gamma}\right)$$

In Exhibit A1, we report the CEQ using monthly (Panel A) and annual (12-month overlapping) data (Panel B),

EXHIBIT A1

Alternative Performance Metrics (stocks and bonds trend, equal risk)

	100% Rebal	90% Rebal 10% TsMom1m	90% Rebal 10% TsMom3m	90% Rebal 10% TsMom12m
Panel A: CEQ, Using Monthly Returns				
CEQ				
= 0 (risk neutral)	4.6%	4.7%	4.6%	4.9%
= 2	3.7%	3.9%	3.8%	4.1%
= 5	2.2%	2.8%	2.6%	2.8%
= 10	–0.4%	0.8%	0.4%	0.5%
CEQ vs. 100% Rebal				
= 0 (risk neutral)	0.0%	0.1%	0.0%	0.3%
= 2	0.0%	0.3%	0.1%	0.4%
= 5	0.0%	0.6%	0.4%	0.6%
= 10	0.0%	1.2%	0.8%	0.9%
Panel B: CEQ, Using Annual Returns				
CEQ				
= 0 (risk neutral)	4.7%	4.8%	4.7%	5.0%
= 2	3.6%	3.9%	3.7%	4.1%
= 5	1.7%	2.4%	2.2%	2.6%
= 10	–2.0%	–0.2%	–0.6%	0.0%
CEQ vs. 100% Rebal				
= 0 (risk neutral)	0.0%	0.1%	0.0%	0.3%
= 2	0.0%	0.3%	0.1%	0.5%
= 5	0.0%	0.8%	0.5%	0.9%
= 10	0.0%	1.8%	1.4%	2.0%
Panel C: Unleveraged versus Leverage to Match 100% Rebal Average Drawdown				
Return Unleveraged and Leveraged (matched DD)				
Unleveraged	4.6%	4.7%	4.6%	5.0%
Lev., Matched DD	4.6%	5.8%	5.4%	6.2%
Return vs. 100% Rebal				
Unleveraged	0.0%	0.1%	0.0%	0.3%
Lev., Matched DD	0.0%	1.1%	0.8%	1.5%
***t*-Stat Return vs. 100% Rebal**				
Unleveraged	n.a.	0.3	–0.1	1.2
Lev., Matched DD	n.a.	4.0	2.7	4.2

Notes: CEQ using monthly (Panel A) and annual (12-month overlapping) returns (Panel B) for different levels of power-utility risk aversion, γ. In Panel C we also report unleveraged versus leveraged returns (taking into account the cost of borrowing) so that the average drawdown across the five episodes considered is equalized to that of the 100% Rebal benchmark. For all panels, the trend strategy is applied to stocks and bonds (equal risk), which is the case covered in Exhibit 6, Panel C. The data are from 1960 to 2017.

for the case of trend applied to stocks and bonds (equal risk), which corresponds to Panel C in Exhibit 6. The $\gamma = 0$ case corresponds to a risk-neutral setting, and the CEQ simply equals the annualized excess return. Using risk aversion parameter values of 2, 5, and 10, we see the benefit from a 10% trend allocation. For $\gamma = 10$, the CEQ is around 1% higher using monthly returns and 2% in case of annual returns. The higher value in case of annual returns shows that the benefits of a trend allocation are more pronounced at lower frequencies, where the intuition is that trend strategies require some time to start picking up on sustained market moves.

Note that the CEQ is still a measure that does not explicitly account for longer-term behavior, like the drawdown characteristic on which we have focused. Therefore, in Panel C we also report unleveraged versus leveraged returns (taking into account the cost of borrowing) so that the average drawdown across the five episodes considered is equalized to that of the 100% Rebal benchmark. The leverage applied is around 1.2 because drawdowns are around a factor of 1.2 lower without the leverage. A 10% trend allocation results in a 0.8% to 1.5% higher annualized return when leverage is applied to match the average drawdown depth. The *t*-stat on the return differential between the with-trend strategy and the 100% Rebal baseline case comes out as 2.7 to 4.2.

Appendix B

ADDING TREND TO A 100% REBALANCED PORTFOLIO

In Exhibit 6, we contrasted a 100% allocation to a monthly rebalanced portfolio with a 60–40 stock–bond capital allocation to one in which the investor holds 90% of the rebalanced portfolio and 10% in a trend strategy. One could argue that part of the improvement in the drawdown characteristics is due to the divestment of 10% of the rebalanced portfolio, and part is due to the defensive nature of the 10% trend allocation. To isolate the latter effect, in Exhibit B1, we contrast a 100% rebalanced portfolio to a 100% rebalance

Exhibit B1

Performance for Fully Invested Rebalanced Portfolio with Stocks and Bonds Trend

	Rebal (100%)	Rebal (100%) 1m Trend (10%)	Rebal (100%) 3m Trend (10%)	Rebal (100%) 12m Trend (10%)
Stock Allocation (avg)	60.0%	61.8%	63.0%	65.3%
Bond Allocation (avg)	40.0%	41.9%	42.8%	44.7%
Total Allocation (avg)	100.0%	103.8%	105.9%	109.9%
Rebal Trade (ann)	10.2%	10.2%	10.2%	10.2%
Stock Fut. Trade (ann)	0.0%	103.7%	53.4%	22.6%
Bond Fut. Trade (ann)	0.0%	219.0%	122.7%	55.8%
Cost Estimate (ann), bps	4.4	6.5	5.5	4.9
Return (ann)	9.1%	9.6%	9.5%	9.9%
Volatility (ann)	9.8%	9.8%	10.0%	10.2%
Ret./Vol. (ann)	0.92	0.99	0.96	0.97
Sharpe Ratio (ann)	0.47	0.53	0.51	0.53
ΔDD June 1970	0.0%	1.5%	–0.1%	2.6%
ΔDD September 1974	0.0%	4.2%	3.3%	4.3%
ΔDD November 1987	0.0%	1.1%	–1.8%	–2.7%
ΔDD September 2002	0.0%	2.8%	2.2%	4.2%
ΔDD February 2009	0.0%	1.6%	3.6%	4.7%
ΔDD Average	0.0%	2.3%	1.4%	2.6%

Notes: We contrast a 100% allocation to a monthly rebalanced portfolio with a 60–40 stock–bond capital allocation to one in which, in addition to a 100% rebalanced portfolio, the investor borrows money to also allocate 10% to a 1-, 3-, and 12-month stock-and-bond (equal-risk) trend strategy. Performance statistics reported on are the average stock, bond, and total allocation (block 1); the annualized notional trading (as percentage of the total portfolio value) in the 60–40 rebalanced portfolio and stock and bond futures, as well as annualized trading costs (block 2); average return, standard deviation, the ratio of these two, and the Sharpe ratio (block 3); and the change in the drawdown level for the five worst drawdowns (ΔDD) at the trough, compared to a 100% allocation to the monthly rebalanced 60–40 portfolio. The data are from 1960 to 2017.

portfolio plus 10% trend investment. As trend specification, we use 1-, 3-, and 12-month stock-and-bond (equal-risk) trends, which we used before in Exhibit 6, Panel C. We incorporate borrowing costs to allow for the effectively 110% investment. As can be seen in Exhibit B1, the addition of a trend strategy (without divestment of a portion of the rebalanced portfolio) still materially improves the average drawdown. Note also that the average return is increased versus the 90% rebalanced plus 10% trend case considered before in Exhibit 6 as a consequence of holding onto all of the rebalanced portfolio when adding trend.

Appendix C

THE 30–70 PORTFOLIO

Here we consider a 30–70 stock–bond portfolio instead of the 60–40 stock–bond portfolio considered before. The 30–70 stock–bond portfolio is closer to equal risk to stock and bonds and may be held by more conservative investors. The 30–70 stock–bond portfolio experiences some of its worst drawdowns on different dates. Focusing on the four drawdowns that exceeded 10%, we get the following results: May 1970 (−13.2%), September 1974 (−15.7%), March 1980 (−10.7%), and February 2009 (−10.3%). The March 1980 drawdown for the 30–70 portfolio did not show up in the 60–40 analysis before because it was bonds getting particularly hit when inflation spiked in 1980. Vice versa, the November 1987 and September 2002 drawdowns for the 60–40 portfolio do not appear here.

In Exhibit C1, one can see that when investing 10% in trend applied to stocks and bonds (equal risk), drawdowns are reduced for all four worst drawdowns for the 30–70 portfolio and for all three trend speeds.

We also repeat the analysis of the strategic rebalancing rules for the 30–70 stock–bond portfolio (see Exhibit C2). A rule to hold off rebalancing when the stock–bond trend is negative tends to improve drawdowns, as it did for the 60–40 stock–bond portfolio considered before in Exhibit 9.

Exhibit C1

Rebal Plus Stock-and-Bond Trend Performance Statistics for the 30–70 Portfolio

	Rebal (100%)	Rebal (90%) 1m Trend (10%)	Rebal (90%) 3m Trend (10%)	Rebal (90%) 12m Trend (10%)
Stock Allocation (avg)	30.0%	28.8%	30.0%	32.3%
Bond Allocation (avg)	70.0%	64.9%	65.8%	67.7%
Total Allocation (avg)	100.0%	93.8%	95.9%	99.9%
Rebal Trade (ann)	10.2%	9.1%	9.1%	9.1%
Stock Fut. Trade (ann)	0.0%	103.7%	53.4%	22.6%
Bond Fut. Trade (ann)	0.0%	219.0%	122.7%	55.8%
Cost Estimate (ann), bps	4.4	6.1	5.1	4.4
Return (ann)	7.8%	8.0%	7.9%	8.3%
Volatility (ann)	7.4%	6.7%	6.9%	7.1%
Ret./Vol. (ann)	1.06	1.19	1.15	1.16
Sharpe Ratio (ann)	0.46	0.53	0.50	0.54
DD May 1970	0.0%	3.4%	1.8%	4.8%
DD September 1974	0.0%	6.3%	6.4%	7.7%
DD March 1980	0.0%	4.7%	2.7%	3.5%
DD February 2009	0.0%	1.6%	2.1%	2.1%
DD Average	0.0%	4.0%	3.2%	4.5%

Notes: We contrast a 100% allocation to a monthly rebalanced portfolio with a 30–70 stock–bond capital allocation to one in which 10% of the portfolio is replaced with a 10% allocation to a 1-, 3-, and 12-month stock-and-bond (equal-risk) trend strategy. Performance statistics reported are the average stock, bond, and total allocation (block 1); the annualized notional trading (as percentage of the total portfolio value) in the 60–40 rebalanced portfolio; and stock and bond futures, as well as annualized trading costs (block 2); average return, standard deviation, the ratio of these two, and the Sharpe ratio (block 3); and the change in the drawdown level for the five worst drawdowns (ΔDD) at the trough, compared to a 100% allocation to the monthly rebalanced 60–40 portfolio. The data are from 1960 to 2017.

Exhibit C2

Strategic Rebalancing with Stock–Bond Trend Rules for the 30–70 Portfolio

	Delay if 1m Trend			Delay if 3m Trend			Delay if 12m Trend		
	Negative	Positive	Continues	Negative	Positive	Continues	Negative	Positive	Continues
Stock Allocation (avg)	29.4%	31.0%	30.1%	29.1%	31.5%	30.3%	28.6%	31.7%	30.4%
Stock Allocation (std)	1.7%	1.5%	1.9%	1.9%	2.1%	2.4%	2.9%	2.7%	4.2%
% Months Rebal	50.1%	49.9%	48.0%	50.0%	50.0%	27.0%	46.4%	53.6%	13.8%
Rebal Trade (ann)	3.5%	4.0%	3.3%	3.1%	3.8%	2.4%	4.1%	3.7%	1.6%
Cost Estimate (ann), bps	1.5	1.7	1.4	1.3	1.6	1.1	1.8	1.6	0.7
Return (ann)	7.9%	7.9%	7.9%	7.9%	8.0%	7.9%	7.8%	7.9%	7.9%
Volatility (ann)	7.3%	7.4%	7.3%	7.3%	7.4%	7.3%	7.3%	7.4%	7.4%
Ret./Vol. (ann)	1.08	1.07	1.08	1.08	1.07	1.07	1.07	1.07	1.07
DD May 1970	0.2%	0.1%	0.4%	0.8%	0.1%	0.5%	0.8%	0.1%	0.4%
DD September 1974	1.5%	0.1%	1.1%	1.5%	0.1%	1.0%	2.7%	0.2%	2.3%
DD March 1980	0.0%	0.2%	0.3%	1.7%	0.2%	0.4%	1.8%	0.3%	0.5%
DD February 2009	2.0%	0.6%	1.9%	1.9%	0.7%	1.9%	2.2%	0.7%	2.1%
DD Average	0.9%	0.3%	0.9%	1.5%	0.3%	0.9%	1.9%	0.3%	1.3%

Notes: We show results when rebalancing is delayed if the stock–bond trend is negative, positive, or continues to be of the same sign (in which case rebalancing only occurs if the trend just changed sign). In months with no delay, there is a rebalancing halfway toward the 30–70 asset mix. The trend direction is determined by comparing the return over the past 1, 3, and 12 months to the typical (average) return over 1-, 3-, and 12-month windows. We report in the different blocks the following: the average and standard deviation of the stock allocation, noting that the bond allocation is 100% minus the stock allocation (block 1); the fraction of months and annualized amount of rebalancing and annualized trading costs (block 2); the average and standard deviation of the return, as well as the ratio (block 3); and the change in the drawdown level for the five worst drawdowns (ΔDD) at the trough, compared to a 100% allocation to the monthly rebalanced 60–40 portfolio (block 4). Data are from 1960 to 2017.

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ADDITIONAL READING

Rebalancing

Seth J. Masters
The Journal of Portfolio Management
https://jpm.pm-research.com/content/29/3/52

ABSTRACT: *While the power of rebalancing to improve returns and reduce risk is generally acknowledged, there is relatively little work focused on the best way to implement a rebalancing policy. Most rebalancing policies use arbitrary "one size fits all" rules, and the more sophisticated approaches that have been proposed involve complex calculations. The author's simpler methodology allows investors to tailor their rebalancing policies to their risk tolerance, the cost of rebalancing, and the risk characteristics of each asset class in the portfolio. This approach addresses not only when to rebalance, but also how far back to rebalance. The result is a set of easily implemented rules for adhering to a rebalancing discipline.*

The Norway Model

David Chambers, Elroy Dimson, and Antti Ilmanen
The Journal of Portfolio Management
https://jpm.pm-research.com/content/38/2/67

ABSTRACT: *Norway's Government Pension Fund Global (GPFG) was recently ranked the largest fund on the planet. It is also highly rated for its professional, low-cost, transparent, and socially responsible approach to asset management. Investment professionals increasingly refer to Norway as being a model for managing financial assets. Chambers, Dimson, and Ilmanen present and evaluate the strategies followed by the GPFG, review its long-term performance, and describe how it responded to the financial crisis. They conclude the article with some lessons that investors can draw from Norway's approach to asset management, contrasting the Norway model with the Yale model of pension fund management.*

Optimal Portfolio Rebalancing with Transaction Costs

Christopher Donohue and Kenneth Yip
The Journal of Portfolio Management
https://jpm.pm-research.com/content/29/4/49

ABSTRACT: *Research has proven the optimality of a no-trade region around an investor's desired asset proportions to assure that trading occurs only when asset proportions drift outside this region, and then only to bring proportions back to the boundary of the no-trade region, not to the target proportions. Because current solution methods are complex, managers typically rely on ad hoc heuristics that are either calendar-based or volatility-based and whose performance against an optimal strategy is unknown. The authors characterize the size and shape of the no-trade region as a function of key problem parameters and compare the performance of different rebalancing strategies. The analysis suggests that extraction of key features associated with optimal rebalancing allows development of more tractable rebalancing heuristics that enhance the effectiveness of optimal rebalancing.*