S&P 100 Index Option Volatility

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ABSTRACT

Using transaction data on the S&P 100 index options, we study the effect of valuation simplifications that are commonplace in previous research on the time-series properties of implied market volatility. Using an American-style algorithm that accounts for the discrete nature of the dividends on the S&P 100 index, we find that spurious negative serial correlation in implied volatility changes is induced by nonsimultaneously observing the option price and the index level. Negative serial correlation is also induced by a bid/ask price effect if a single option is used to estimate implied volatility. In addition, we find that these same effects induce spurious (and unreasonable) negative cross-correlations between the changes in call and put implied volatility.

AN IMPORTANT AREA OF current research is the estimation and behavior of market volatility implied by option pricing models. For example, Stein (1989) concludes that the S&P 100 index option market is inefficient because longer term options *overreact* to changes in the implied volatility of short maturity options. Day and Lewis (1988) document an increase in volatility around the quarterly expiration cycle. Schwert (1990) draws conclusions about the behavior of volatility around the stock market crash of 1987 by using implied volatility. All of these studies use a simplified option pricing framework to estimate implied market volatility.

Previous research generally assumes that the S&P 100 index option is European-style. This option is actually American-style. It is also common to assume that the dividend yield is constant. The dividends are not constant and exhibit distinct seasonal patterns. Harvey and Whaley (1991b) show that large pricing errors can be induced in the option prices if the American feature and the discrete dividends are ignored. Option pricing errors translate into errors in the implied volatility estimates.

This paper uses S&P 100 index call and put transactions and an American-style option pricing model with the exact discrete dividend series to calculate a time-series of implied volatilities. This valuation method,

however, is not sufficient to obtain accurate implied volatility estimates. A critical ingredient is the simultaneous observation of the option and the index value. Previous research uses closing option prices to calculate implied volatility. This is a mistake because the option market closes 15 minutes after the stock market. Our results show that spurious negative serial correlation is induced in the volatility changes if closing option prices are used.

Our results also indicate that some negative serial correlation is induced if only one option transaction is used—because of a bid/ask price effect. The implied volatility series we calculate uses multiple options transactions to alleviate this problem.

We also investigate the potential effect of infrequent trading on the time-series of implied volatility estimates. Using transaction data, we can simultaneously match the option with the index level. If there is infrequent trading in the underlying stocks that form the S&P 100 index, however, spurious patterns could be induced in the volatility estimates. To eliminate this possibility, we simultaneously imply both the volatility and the index level using multiple option transactions. We find that there is little difference between our implied volatility estimates using the implied index level and the implied volatility estimates using the closing stock index level. This evidence suggests that, in our sample, infrequent trading of S&P 100 stocks near the close is not that prevalent.

I. Implied Volatility Estimation

The S&P 100 index option contract (OEX) is traded on the Chicago Board Options Exchange (CBOE) and is by far the most actively traded index option contract in the world. In 1989, over fifty million contracts changed hands. Given that the index reflects a broad cross-section of stocks, many have considered the implied volatility of the OEX option as reflection of market volatility.[1] In many asset pricing models, changes in market volatility affect the expected returns on securities. As a result, it is economically important to accurately measure the market volatility.

A. Estimation Procedures

Implied volatilities are calculated using market parameters during the calendar year August 1, 1988 through July 31, 1989: the closing index level, the Treasury bill rate that matches the maturity of the option (or the 30-day rate, whichever has the longest maturity), and the actual dividends paid during the option's life. The volatility rate is the implied volatility for the

[1]The behavior of market volatility through time is an important area of current research. See Officer (1973), Black (1976), Merton (1980), Christie (1982), Pindyck (1984), Poterba and Summers (1986), French, Schwert, and Stambaugh (1987), Bollerslev, Engle, and Wooldridge (1988), and Schwert (1989).

at-the-money call and put options with the shortest maturity but with at least 15 days to expiration.

The at-the-money options are used to estimate implied volatility because they contain the most information about volatility; that is, they are the most sensitive to changes in the volatility rate. To illustrate this, consider the *vega* of a European option; that is, the partial derivative of the option price v with respect to the volatility:

$$\frac{\partial v}{\partial \sigma} = Sn(d_1)\sqrt{T}, \tag{1}$$

where

$$d_1 = \frac{\ln(S/X) + (r + 0.5\sigma^2)T}{\sigma\sqrt{T}},$$

where S is the index level, X is the exercise price of the option, r is the riskless rate of interest, σ is the volatility rate, T is the time to expiration of the option, and $n(d)$ is the unit normal density function evaluated at d. (This derivative happens to be the same for the European call and the European put options.) Since S and $\sqrt{T}$ are positive, the derivative is maximized where the probability value from the normal distributions is maximized. For the standard normal distribution, this occurs with the value of zero. The choice of at-the-money options ensures that S/X is close to one and $\ln(S/X)$ is close to zero.[2]

Following Harvey and Whaley (1991b), we use an American-style valuation method that accounts for the index paying multiple known discrete dividends during the option's life. The stock index grid is defined in terms of the index level net of the present value of the promised dividends. This procedure is computationally efficient and avoids the valuation simplifications used in previous research.[3]

B. Nonsimultaneous Price Problem

The stock market closes at 3:00 P.M. while the S&P 100 index option market closes at 3:15 P.M.. When closing prices are used to estimate implied volatility, this timing difference may induce negative first-order serial correlation in the implied volatility changes from day to day. What happens is that, in the interval from 3:00 to 3:15 P.M., new information enters the

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[2] To be more precise, the partial derivative (1) is maximized where $\ln(S/X) = -(r + 0.5\sigma^2)T$; that is, where the call option is slightly out of the money and the put option is slightly in the money. For example, if the interest rate is eight percent, the volatility rate is twenty percent, and the index level is 250.00, the 30-day call and put options that would maximize (1) would have an exercise price of 252.06. Given that S&P 100 index option exercise prices are in increments of five dollars, the nearest exercise price is 250—the at-the-money option.

[3] There is a common misconception that the binomial method is inapplicable, or at least impractical computationally, where the underlying index (or stock) pays discrete cash dividends. See, for example, Geske and Shastri (1985), p. 70.

market and causes index option prices to be revised. When the implied volatility is computed using closing prices, the implied volatility from the call is higher (lower) than it should be when market news was good (bad), and the implied volatility from the put is higher (lower) than it should be when news is bad (good). On the following day, the stock market prices adjust to the new information, and implied volatilities revert back to normal levels.

The nonsimultaneous price problem may also cause systematic patterns in the cross-correlations between call and put volatility. As argued above, good news causes an increase in call volatility and a decrease in put volatility thereby increasing the volatility spread. On the following day, this spread is reduced as implied volatilities revert to their normal levels. This effect should induce negative serial correlation in the changes in the call-put volatility spread and negative cross-correlation between call and put volatility changes.

To demonstrate the magnitude of the spurious behavior that may be induced by this problem, we compute the implied volatility for the at-the-money call and the at-the-money put using closing prices each day during the 253-day sample period. This estimator is denoted by the column heading "Closing Prices" in Tables I and II. In addition, we compute the implied volatility for the at-the-money call and the at-the-money put using the last transaction prices before 3:00 P.M. This estimator is denoted by "3:00 P.M. Prices." Comparing the results of the two estimators identifies the estimation error attributable to nonsimultaneous prices.

Descriptive statistics for the implied market volatility estimates during the sample period are reported in Table I. The mean and the standard deviation of the volatility estimates from the call and the put prices are very close. The means for the two estimators are only one and eight basis points apart, respectively, and the standard deviations indicate that the time-series of volatility estimates using the first estimator is slightly more noisy than the second.

An examination of the time-series properties of the implied volatility changes reveals a more dramatic contrast. Panel A of Table II shows that, high negative first-order serial correlation appears in the volatility change series. The first-order serial correlation for the calls is -0.44 and for the puts is -0.39. When simultaneous prices are used, both the serial correlations for the calls and puts drop to approximately -0.33. This reduction in the negative first-order serial correlation is exactly what is expected given the nonsimultaneous price problem.

The negative cross-correlation between call and put volatility changes is also evident from Panel B of Table II. The contemporaneous correlation between the call and the put volatility changes based on closing prices is -0.19. However, when 3:00 P.M. options prices are used, the contemporaneous correlation between the call and put volatility changes only slightly to -0.18.

The negative cross-correlation using the 3:00 P.M. option prices suggests that the simultaneous observation of the index level and the option price is not sufficient to guarantee a reliable volatility estimate. Indeed, one must

Table I
Summary Statistics for Implied Volatility Estimates[a] from S&P 100 Index Call and Put Option Prices August 1, 1988–July 31, 1989

The implied volatilities are based on an American option pricing approximation using discrete cash dividend payments on the underlying index. The volatility is computed for the closest at-the-money call option and put option each day during the sample period. The riskless rate of interest is the rate on the T-bill that matures nearest after the option expires and that has at least 30 days to maturity. The first implied volatility uses the closing option prices, 3:15 P.M. (CST), and the closing index level, 3:00 P.M. (CST). The second implied volatility uses the last option transaction price prior to 3:00 P.M. (CST) and the closing index level. The third implied volatility is based on a nonlinear least squares regression of option transaction prices during the 10-minute interval 2:55–3:05 P.M. (CST) on model prices using the contemporaneous index levels. The implied volatility measures for calls and puts are based on nonlinear least squares regressions of (a) call and put option transaction prices during the 10-minute interval 2:55–3:05 P.M. (CST) on model prices using the contemporaneous index levels and (b) call and put option transaction prices during the 10-minute interval 2:55–3:05 P.M. (CST) on model prices, estimating simultaneously the implied index levels.

	Call options			Put options			Call and put options 2:55–3:05 P.M. prices	
Estimator i	Closing prices	3:00 P.M. prices	2:55–3:05 P.M. prices	Closing prices	3:00 P.M. prices	2:55–3:05 P.M. prices	Observed index	Implied index
	(1)	(2)	(3)	(1)	(2)	(3)		
Mean σ_i	15.88	15.89	15.99	16.62	16.70	16.58	16.24	16.30
Std. dev. σ_i	2.53	2.44	2.18	2.44	2.33	2.30	2.16	2.13
Average no. of observations			28.21			26.83	55.04	55.04
Mean $(\sigma_i - \sigma_3)$	−0.11	−0.09		0.04	0.13			
Std. dev. $(\sigma_i - \sigma_3)$	1.19	0.77		0.78	0.58			

[a] $n = 253$.

consider that both the call and put option prices are implying volatility for the same underlying asset—the S&P 100 index portfolio. If the option model provides reasonable volatility forecasts, one would expect a positive correlation between the volatility changes. Panel B of Table II also shows significantly positive lead/lag one cross-correlations. Given the strong negative serial correlation in both the call and put options, the positive lead and lag cross-correlations imply that the call and the put volatility series are very different. This also suggests that the volatility estimates may not be that reliable.

C. Bid/Ask Price Effect

The bid/ask price effect refers to the fact that the closing price or, for that matter, any single transaction price takes place at a bid level or an ask level.

Table II
Diagnostics of Changes in Implied Volatility Estimates[a] from S&P 100 Index Call and Put Option Prices August 1, 1988–July 31, 1989

The implied volatilities are based on an American option pricing approximation using discrete cash dividend payments on the underlying index. The volatility is computed for the closest at-the-money call option and put option each day during the sample period. The riskless rate of interest is the rate on the T-bill that matures nearest after the option expires and that has at least 30 days to maturity. The first implied volatility uses the closing option prices, 3:15 P.M. (CST), and the closing index level, 3:00 P.M. (CST). The second implied volatility uses the last option transaction price prior to 3:00 P.M. (CST) and the closing index level. The third implied volatility is based on a nonlinear least squares regression of option transaction prices during the 10-minute interval 2:55–3:05 P.M. (CST) on model prices using the contemporaneous index levels. The implied volatility measures for call and puts are based on nonlinear least squares regressions of (a) call and put option transaction prices during the 10-minute interval 2:55–3:05 P.M. (CST) on model prices using the contemporaneous index levels and (b) call and put option transaction prices during the 10-minute interval 2:55–3:05 P.M. (CST) on model prices, estimating simultaneously the implied index levels.

Panel A. Serial Correlation of Implied Volatility Changes

	Call options			Put options			Call and put options 2:55–3:05 P.M.	
Lag k	Closing prices	3:00 P.M. prices	2:55–3:05 P.M. prices	Closing prices	3:00 P.M. prices	2:55–3:05 P.M. prices	Observed index	Implied index
1	−0.4414	−0.3274	−0.2810	−0.3934	−0.3321	−0.3075	−0.2562	−0.2460
2	−0.0368	−0.1328	−0.1280	−0.0388	−0.0864	−0.0180	−0.0864	−0.0731
3	−0.0168	−0.0414	0.0384	0.0316	0.0552	−0.0273	0.0230	0.0176
4	0.0351	0.0948	−0.0799	−0.0472	−0.1055	0.0116	−0.0763	−0.0754
5	0.0044	−0.0178	0.1207	−0.0031	0.0566	−0.0313	0.0515	0.0537
6	−0.0561	−0.0627	−0.1022	−0.0255	−0.0229	0.0069	−0.0627	−0.0505
7	0.0218	−0.0420	−0.0346	0.0576	−0.0377	−0.0376	0.0263	−0.0091
8	−0.0056	0.0805	0.0310	0.0362	0.1083	0.1280	0.0263	0.1042
9	0.0498	0.0248	0.0161	−0.0771	−0.0682	−0.1113	−0.0275	−0.0651
10	0.0367	−0.0250	0.0614	−0.0164	−0.0568	−0.0196	0.0389	0.0109

Panel B. Cross-Correlation of Implied Volatility Changes for Calls and Puts

Lag k	Closing prices	3:00 P.M. prices	2:55–3:05 P.M. prices
−3	0.0939	−0.0144	−0.0304
−2	−0.1082	−0.0054	−0.0107
−1	0.2109	0.1178	0.0009
0	−0.1858	−0.1788	0.2738
1	0.1265	0.1572	−0.0661
2	0.0039	−0.0049	0.0424
3	−0.0044	0.0362	−0.0046

[a] $n = 252$.

Roll (1984) demonstrates that the random movement between these price levels in successive transactions produces significant negative first-order serial covariance in price change series. Given that the daily volatility changes used in this study are derived from call and put option price changes, we should expect to see negative first-order serial correlation in the implied volatility changes due to the bid/ask price effect.[4,5]

The negative contemporaneous cross-correlation between call and put option implied volatilities reported in Panel B of Table II may also be due to the bid/ask price effect. As the index moves up (down), it is more likely that the call transactions are close to the ask (bid) price and the put transactions are close to the bid (ask) price.

To test these ideas regarding the bid/ask price effect, a third estimator was developed. In place of using a single closing price or a single option price prior to 3:00 P.M., a nonlinear regression of option transaction prices on model prices using all transactions during the 10-minute window 2:55–3:05 P.M. is used. The window reaches 5 minutes past the close of the stock market to allow for possible reporting delays. With a large number of transactions, the bid/ask price errors will offset one another. By comparing the properties of this least squares estimator with the 3:00 P.M. price estimator, we can identify the plausible magnitude of the bid/ask price effects.

The results reported in Table I indicate that the average level of implied volatility is unaffected by the bid/ask price effect. The average volatility implied by the 3:00 P.M. call (put) price is 15.89 (16.70) percent and the average volatility implied by the call (put) option transactions in the 10-minute window 2:55–3:05 P.M. is 15.99 (16.58) percent.[6] The standard deviations indicate that the nonlinear least squares estimator is much more precise, resulting from the use of multiple index option transactions. In fact, an average of 28.21 call option transactions and 26.83 put option transactions are included in the 10-minute window each day for call option and put option implied volatility estimation, respectively. A two-standard-deviation interval of the differences between the first estimate and the third and the second estimate and the third indicates a potential misstatement of ±238 and ±154 volatility basis points, respectively, for the call options.

The serial correlation of the implied volatility changes reported in Table II also favors the application of nonlinear least squares regression approach.

[4]Stephan and Whaley (1990) document very high negative first-order serial correlation in 5-minute stock option price changes.

[5]The same bid/ask price effect would generally appear in the price of the underlying asset, which is also used in the implied volatility estimation. Here the underlying asset is a stock index portfolio, however, and the index level is an average across one hundred stock prices, randomly distributed between bid and ask price levels.

[6]In a different sample, Harvey and Whaley (1991a) find that there is a significant difference in the average implied volatility from the put and call options. They also find that the ratio of average daily call open interest to average call volume (2.89) is much lower than the same ratio for put options (3.82). This is consistent with index puts being used as a convenient and inexpensive form of portfolio insurance and thus increases the average implied volatility.

The negative first-order serial correlation is further reduced from the -0.33 level for calls and puts using 3:00 P.M. prices to -0.28 and -0.31 for calls and puts, respectively, using the least squares procedure. Even after correcting for the bid/ask price effect, a high degree of negative first-order serial correlation remains in the volatility change series. Moreover, the remaining negative serial correlation is not due to any large outliers in the data. This is consistent with the volatility levels being a stationary mean-reverting time-series rather than an integrated time-series.[7]

The cross-correlation results reported in Panel B of Table II show that the bid/ask price effect is an important problem. When all the transactions in the 2:55–3:05 P.M. window are used, the contemporaneous cross-correlation increases from -0.18 using 3:00 P.M. prices to 0.27 using prices from the 10-minute window. Also note that there are no causal effects as evidenced by the insignificant lead and lag cross-correlations. Since the implied volatilities for the call and the put are supposed to represent the return volatility of the same underlying asset, the positive correlation of the two estimates makes the case for using all the transactions in the 10-minute window even stronger.

D. Infrequent Trading of Index Stocks

The remaining problem left to address is the infrequent trading of the stocks within the index portfolio. In the 3:00 P.M. volatility estimator as well as in the least squares volatility estimator, the contemporaneous observed index level is used. The observed index level is an average of last transaction prices of one hundred stocks, however, and many of these stocks may not have traded near the market close. The effect of infrequent trading would be to overstate (understate) true market volatility using call (put) option prices on days when the market advances and to understate (overstate) true market volatility using call (put) option prices on days the market declines (i.e., the true index level upon which the index options are priced leads the observed index level).

The magnitude of the error introduced by infrequent trading of stocks is not expected to be large. Stoll and Whaley (1990) show that almost all large capitalization stocks trade near the close, and all of the S&P 100 stocks are large capitalization stocks. In addition, the effects of infrequent trading are typically revealed by high positive serial correlation in the index return series, and, during the August 1, 1988 to July 31, 1989 sample period, the first-order serial correlation in the close-to-close S&P 100 returns is -0.068 and is insignificantly different from zero (i.e., the standard error is 0.063). Nevertheless, the observed index level, which is based on last transaction prices of the individual stocks, is always stale relative to the option price, and the magnitude of the infrequent trading problem must be assessed.

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[7]Our evidence suggests that it might be a mistake to implement an integrated GARCH model along the lines of Engle and Bollerslev (1986) to characterize the time-series of market volatility.

To investigate the possibility that infrequent trading of the index stocks is causing the remaining negative serial correlation in the volatility changes, two additional tests are devised. First, if the infrequent trading problem is as described, the call and put option volatilities should move in opposite directions. The contemporaneous cross-correlation estimate for the nonlinear least squares regression estimates of volatility using call and put option prices shows that this is not the case. To be certain, however, the call and put option transactions in the 10-minute window 2:55–3:05 P.M. are pooled each day, and one nonlinear regression model is estimated in an attempt to *average* the call and the put option volatilities and, hence, the volatility changes. (Recall that, under the infrequent trading effect, the call and put volatilities should move in opposite directions.) The second-to-last columns of Tables I and II contain summaries of the results. Interestingly, the results show that the first-order, negative serial correlation is reduced only slightly to -0.26 compared with -0.28 for the call and -0.31 for the put. Moreover, the slight reduction may result from a further reduction of the bid/ask price effect rather than a reduction due to controlling the infrequent trading effect.

The second test involves eliminating the use of the observed index level entirely. Using the pooled call and put option transactions each day, it is possible to simultaneously estimate the implied volatility rate as well as the *implied index level*. With this methodology, the infrequent trading problem vanishes because the index level is not used to solve for the volatility. An analysis of the serial correlation of the volatility changes reported in the last column of Table II reveals that the negative first-order serial correlation is virtually identical to the pooled call and put estimate.[8] Using the observed index level, the first-order serial correlation is -0.26, and, using the implied index level, the first-order serial correlation is -0.25. The mean and standard deviation of the daily estimates reported in Table I suggest the same conclusion.

Overall, the results of the infrequent trading tests combined with documentary evidence that large capitalization stocks trade near the close suggest that the infrequent trading of the stocks in the S&P 100 index is not a serious problem in estimating market volatility from index option transaction prices.

II. Conclusions

Using closing S&P 100 index option prices to estimate market volatility is problematic even if the correct index option valuation method is used. Much of the systematic pattern that appears in the changes of these volatili-

[8] It is interesting to note that, using our sample period, the daily price change series of the observed index level and the implied index level are virtually perfectly positively correlated. The estimated contemporaneous correlation was 0.96, and neither the lead nor lag cross-correlations were significantly different from zero.

ties through time is induced by nonsimultaneous prices and bid/ask price effects. Before attaching economic meaning to shifts in implied market volatility, these effects must be controlled for. The infrequent trading of index stocks does not appear to affect implied market volatility in a meaningful way, at least with respect to close of day stock transactions.

Some pricing simplifications are commonplace in research on implied volatility. For example, Stein (1989) uses closing price data and assumes a constant dividend yield when estimating implied volatility. On the basis of this data, he argues that the term structure of implied volatility contradicts the rational expectations hypothesis. Day and Lewis (1988) also use closing prices to form their volatility estimates. They find that there are unexpected increases in implied volatility around quarterly expiration dates. Both of these findings have important policy implications. Our results suggest, however, that the strength of these findings should be tempered with the knowledge of the errors induced in their volatility estimates by the valuation simplifications. It is important that future research on market volatility avoid these simplifications.

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