SEASONALITY AND HETEROSCEDASTICITY IN CONSUMPTION-BASED ASSET PRICING:

AN ANALYSIS OF LINEAR MODELS

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ABSTRACT

This paper examines linear asset pricing models using aggregate consumption data which are not seasonally adjusted. Portfolio returns exhibit more diverse and often stronger consumption correlations, and the parameters of representative-agent utility functions seem to be estimated more precisely using these data. We incorporate seasonality in the form of "taste shift" parameters and seasonally varying heteroscedasticity. The pricing relations can be rejected using a short-term bill or multiple asset returns. The data indicate that the "equity premium puzzle" is not an artifact of seasonal adjustment. The evidence includes long-term bonds as well as bills and stocks, and seems to involve not only the relation of average returns to consumption covariances but also the patterns of variation in expected returns over time.

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I. INTRODUCTION

Studies of consumption-based asset pricing models typically have used seasonally adjusted consumption data. This paper examines linear models using aggregate data which are not seasonally adjusted. The effect of seasonal adjustment on asset pricing evidence has received little previous attention. One exception is the recent work of Miron (1986), who finds no evidence against models that incorporate seasonality, using data for a single asset. Dunn and Singleton (1986), using seasonally adjusted data, find that much of the evidence against the model is not evident unless multiple assets are examined. This paper extends the investigation of seasonality to models with multiple assets. Unlike Miron, we find strong evidence against models with seasonality.

Using seasonally adjusted consumption, studies have concluded that consumption volatility is too low to "fit" equity and bill returns in simple models without using implausibly large values of aggregate risk aversion. Mehra and Prescott (1985) deem this an "equity premium puzzle." Dunn and Singleton (1986) observe that consumption covariances of assets differ too little relative to average return differences in the Treasury bill market. Unadjusted consumption data display some potentially interesting differences. Consumption growth measures are much more volatile. Portfolio returns exhibit more diverse and often stronger correlations with unadjusted consumption measures. Both these differences are consistent with the possibility that the "equity premium puzzle" described by Mehra and Prescott (1985) and others is an artifact of seasonal adjustment. Furthermore, point estimates of aggregate risk aversion are typically closer to zero, and utility function parameters seem to be estimated more precisely using unadjusted consumption growth rates.

Despite these differences, our evidence indicates that the equity premium puzzle is not an artifact of seasonal adjustment. Furthermore, the movement of expected returns around their unconditional means reveals that a similar puzzle exists at the level of conditional moments, and extends to the long-term bond markets.

We extend previous investigations of linear, consumption-based pricing models in several ways. We follow Miron (1986) by incorporating seasonality in the form of seasonal "taste shifters." These parameters allow seasonal variation in the relation of expected real returns to expected consumption growth. We examine models incorporating seasonally varying heteroscedasticity, and we formulate a model which focuses on conditional expected excess returns, or risk premiums, and allows fairly general movements of conditional covariances over time.

The second section presents an overview of the model. Section III describes the data, focusing on the differences between seasonally adjusted and not seasonally adjusted consumption. Section IV presents the empirical results. The final section summarizes our conclusions.

II. LINEAR, CONSUMPTION-BASED PRICING MODELS

The consumption-based model implies that, in equilibrium, the price of an asset equals the expected discounted value of its future payoffs, weighted by marginal utilities of consumption. In a simple version of the model, assuming a representative agent with a time-additive utility function $\Sigma_t \beta^t u(C_t)$, this equilibrium condition implies Equation (1) (see, e.g., Lucas, 1978):

$$\beta^{-1} = E\{\frac{u'(C_{t+1})}{u'(C_t)} R_{t+1} \mid \Omega_t\}. \tag{1}$$

Equation (1) says that the expected value, given public information Ω_t, of one plus a real rate of return, R_{t+1}, multiplied by a ratio of marginal utilities, $u'(C)$, equals one plus a rate of time preference. If the utility function is concave, marginal utility declines as consumption C increases and the marginal utility ratio is negatively related to the growth rate of consumption. Therefore, the model implies that consumption growth and real returns should be positively related.[1]

Recent empirical papers examine versions of the model which assume that the utility function is isoelastic:

$$u(C) = \frac{C^{1-\alpha} - 1}{1 - \alpha}; \tag{2}$$

where α is a parameter interpreted either as the coefficient of relative risk aversion (e.g., Pratt, 1964) or as the inverse of an intertemporal elasticity of substitution (e.g., Hall, 1987a). If a normal distribution is assumed, a simple linear model results. Assume that the conditional distribution of $\ln(C_{t+1}/C_t)$ and $\ln(R_{t+1})$ given Z_t is normal, where Z_t is a vector of instrumental variables which are elements of Ω_t and $\ln(.)$ denotes the natural logarithm. Substituting the derivative of the utility function from Equation (2) into Equation (1), applying the law of iterated expectations and then the normal moment generator, implies

$$E\{r_{t+1} \mid Z_t\} = \psi_t + \alpha E\{c_{t+1} \mid Z_t\}, \tag{3}$$

where

$$\psi_t = (-\frac{1}{2}) \operatorname{var}\{r_{t+1} - \alpha c_{t+1} \mid Z_t\} - \ln\beta,$$

$$c_{t+1} = \ln(C_{t+1}) - \ln(C_t),$$

and

$$r_{t+1} = \ln(R_{t+1}).$$

Equation (3) is discussed in several theoretical papers [see Breeden (1986) for a recent lucid discussion] and is examined empirically by Hall (1987a), Ferson

(1983), Rotemberg (1984), Hansen and Singleton (1985, 1987), Wheatley (1986), Ferson and Merrick (1987), Grossman, Melino, and Shiller (1987), Harvey (1988), and others.

There are various ways to motivate the linearization of the model in Equation (3). One is to assume stationary joint normality of the distribution of consumption growth, real returns, and the instruments Z_t. In this case, the homoscedasticity of the normal distribution implies that the intercept ψ_t in Equation (3) is constant over time ($\psi_t = \psi$). We refer to the constant ψ case as the *homoscedastic form* of the model. A second approach is to motivate the linearization as an approximation, using a Taylor expansion (see, e.g., Rotemberg, 1984; or Breeden, 1986). Because ψ_t includes a conditional expectation given Z_t, it will in general vary with Z_t over time. We also examine conditionally *heteroscedastic models* which allow the intercept to vary as a function of the instruments [$\psi_t = \psi(Z_t)$].

Equation (3) relates the levels of expected real returns to expected consumption growth rate levels and to risk. Consumption-based pricing models imply that expected returns differ across assets depending on their covariances with consumption. Subtracting the expressions for the expected returns for two assets (subscripted i and f) implies

$$E\{r_{i,t+1} \mid Z_t\} - E\{r_{f,t+1} \mid Z_t\} + \frac{1}{2}(\sigma_{it}^2 - \sigma_{ft}^2) = \alpha \operatorname{cov}\{r_{i,t+1} - r_{f,t+1}; c_{t+1} \mid Z_t\}\,, \tag{4}$$

where σ_{it}^2 is the conditional variance of asset i. The left-hand side of Equation (4) corresponds to the conditional expected risk premium on asset i relative to asset f, given Z_t. (The difference in the variances arises as a "Jensen's inequality" term because the returns are continuously compounded.) Our empirical work will focus on both the ability of consumption-based models to "explain" the levels of real returns, using Equation (3), and their ability to explain expected risk premiums as in Equation (4).

If Z_t is chosen to be the null information set, then Equations (3) and (4) can be stated at the level of unconditional, or "average" expected returns. We examine evidence on consumption-based pricing at the level of both conditional and unconditional moments.

III. THE DATA

Previous studies have typically relied on seasonally adjusted aggregate consumption data, smoothed with the X-11 program [see Shiskin, Young, and Musgrave, 1967]. Essentially, this program subtracts from the original data a sequence of weighted averages of past and (in revised data) future values. Such data smoothing may be useful for some applications, but it can create problems if the objective is to study security returns using consumption-based pricing models.

Investors' utility functions are unlikely to be simple functions of seasonally adjusted consumption. It seems implausible that the future expenditures which X-11 averages into the current period's data can provide utility in the current period. Obviously, it is not possible to purchase goods at seasonally adjusted prices. Seasonal adjustment of the data can, in principle, lead to spurious rejections of the orthogonality conditions implied by the model. These conditions derive from the implication of Equation (3) and rational expectations that $u_{t+1} \equiv r_{t+1} - \psi_t - \alpha c_{t+1}$ is an error term with conditional mean zero given the instruments Z_t. Most empirical studies have included lagged values of the consumption variables as instruments. Because the seasonal adjustment program uses a two-sided filter, the adjustment can induce a spurious correlation between the error term and the instruments. (See, e.g., Miron, 1986; Singleton, 1987; and Wallis, 1974).

While this paper is motivated by seasonal adjustment, we recognize that there are other potential problems using any aggregate consumption series as a measure of marginal utility for security pricing. For example, since only a subset of the "goods" that provide utility is measured, a "missing variables" problem is created if aggregate preferences are not separable across goods. This is analogous to the problem of identifying the "true" market portfolio in tests of the capital asset pricing model. The durability of commodities implies that expenditures at time t can result in actual consumption in the future that could influence marginal utility and therefore security prices (e.g., Dunn and Singleton, 1986). Consumption models may be more sensitive to the form of preferences or to market imperfections than are models based on financial market aggregates (e.g., Bergman, 1985; Brown, 1986; and Grossman and Laroque, 1990). Other potential problems include infrequent and nonsynchronous sampling, various measurement errors, publication lag, and tax treatment of the returns to "investment" in consumer goods that differs from taxation of measured security returns.

We obtain the commerce department's quarterly nominal, not seasonally adjusted data for nondurables, services, and consumer durable goods expenditures for 1946 through 1985. Price deflators for personal consumption expenditures are available only in seasonally adjusted form, but unadjusted consumer price indices are available. We use these to construct series of real, per capita consumption growth rates that are similar to the data employed in previous studies, except for the seasonal adjustment.[2]

Tables 1 and 2 compare the quarterly unadjusted consumption data with seasonally adjusted data and quarterly real rates of return to five portfolios. (Annual data are presented in the appendix.) The portfolios include a value-weighted index of the smallest decile of common stocks (based on market value of equity outstanding at the beginning of each year) on the New York Stock Exchange (NYSE), a value-weighted index of all NYSE common stocks, a long-term government and a long- term corporate bond, and the three-month real return to a strategy of rolling over one-month Treasury bills. Real returns are nominal returns deflated by the consumer price index (CPI). The security return and CPI data are from Ibbottson

Table 1. Summary Statistics, Quarterly Data: 1947:2–1985:4 (155 Observations)

Variable	*Overall Mean*	*Overall Standard Deviation*	*Q1 Mean*	*Q2 Mean*	*Q3 Mean*	*Q4 Mean*
Consumption growth						
Nondurables (NSA)	0.00248	0.11657	−0.18327	0.07558	−0.00272	0.11558
Nondurables (SA)	0.00272	0.00859	0.00329	0.00308	0.00254	0.00198
Durables (NSA)	0.00656	0.15487	−0.21077	0.13267	−0.04002	0.13882
Durables (SA)	0.00862	0.04180	0.01350	0.00387	0.01552	0.00170
Services (NSA)	0.00742	0.01428	0.02368	−0.00298	0.00516	0.00426
Services (SA)	0.00588	0.00582	0.00584	0.00755	0.00446	0.00568
Real asset return						
Treasury bill	0.00123	0.00890	0.00220	−0.00116	−0.00017	0.00408
Government bond	−0.00083	0.04814	−0.00559	−0.00342	−0.00467	0.01022
Corporate bond	0.00064	0.04914	−0.00155	0.00654	−0.00251	0.01315
Value-weighted stock	0.01725	0.07839	0.01812	0.00303	0.00434	0.04353
Small stocks	0.02882	0.12518	0.09468	−0.01518	0.01587	0.02162
Price index						
CPI	0.01036	0.01034	0.00923	0.01262	0.01174	0.00782

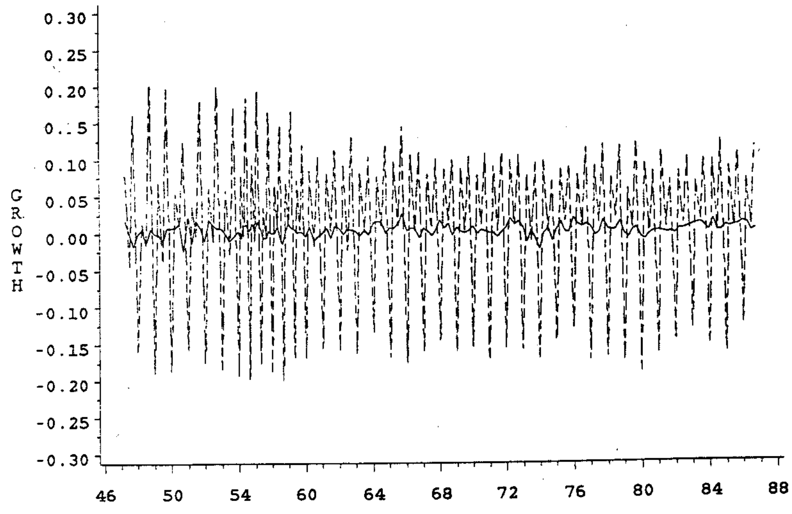
Notes: Real growth of consumption of nondurables, durables, and services at time t is the logarithm of period t, divided by period $t-1$ consumption per capita. Seasonally adjusted (SA) consumption data are deflated by seasonally adjusted consumption price deflators. Not seasonally adjusted (NSA) consumption and returns are deflated by the (not seasonally adjusted) consumer price index (CPI). All returns are continuously compounded.

and Sinquefield (1982) and the Center for Research in Security Prices at the University of Chicago (CRSP). The sample period for returns and growth rates is 1947:2–1985:4.

Table 1 presents means and standard deviations of the variables. Part of the evidence on consumption-based pricing models involves the low variability of (seasonally adjusted) consumption relative to asset returns (e.g., Grossman, Melino, and Shiller, 1987; Grossman and Shiller, 1981; and Mehra and Prescott, 1985). Note that the unadjusted consumption growth rates have sample standard deviations at least an order of magnitude larger than seasonally adjusted data. The X-11 program removes a substantial fraction of the variability. Comparing the sample standard deviations of the three components indicates that durable goods expenditures are the most volatile, followed by nondurables and then services.

Figure 1 illustrates the time series behavior of the seasonally adjusted and unadjusted real consumption growth rates. The time series plots indicate no apparent trends in any of the growth rates. The higher volatility and strong seasonality of the unadjusted data are readily apparent.

Real Consumption Growth (Nondurables)



Real Consumption Growth (Services)

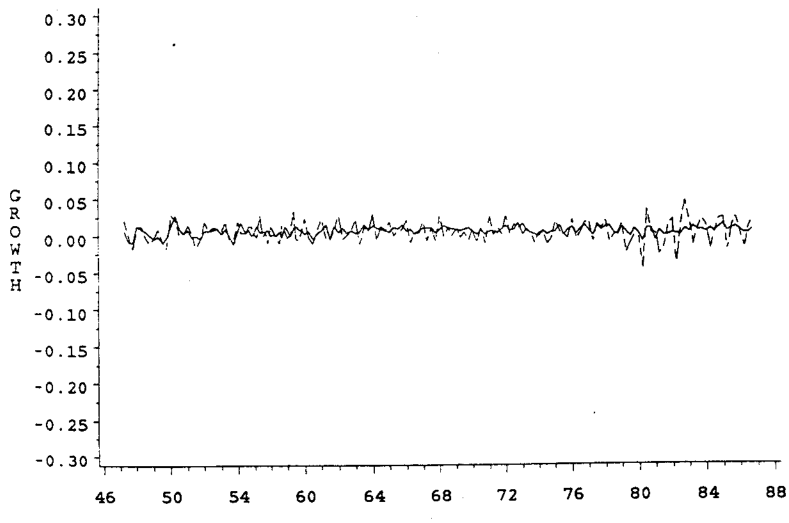


Figure 1. Time Series Plots

Note: Real per capital growth of consumption of nondurables, durables, and services. The growth rate is the logarithm of period t consumption divided by period $t-1$ consumption. The span of the data is from 1947:1 to 1985:4. Seasonally adjusted data are represented by the line and the not seasonally adjusted data are represented by the dash. The seasonally adjusted consumption data are deflated by the its own (seasonally adjusted) price deflator while the not seasonally adjusted data are deflated by the (not seasonally adjusted) consumer price index. All returns are continuously compounded.

(continued)

Real Consumption Growth (Durables)

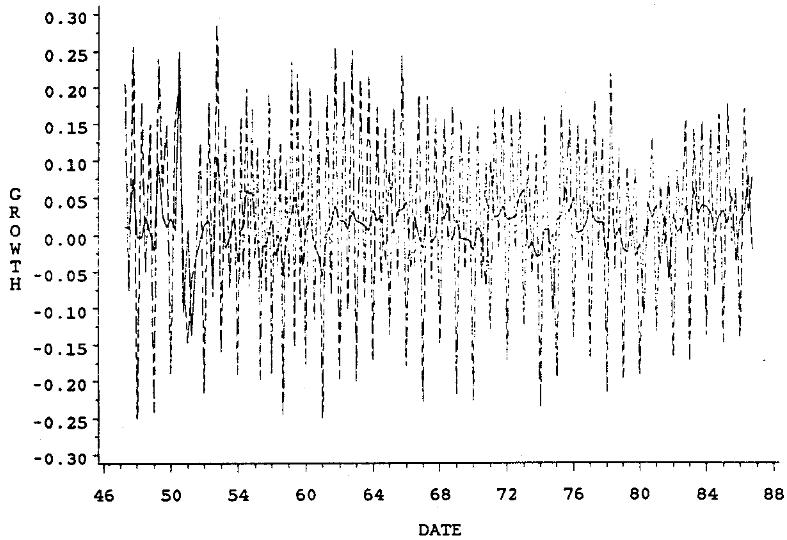


Figure 1. Continued

The right-hand columns of Table 1 display sample means of the variables, conditional on a given quarter. The means of the unadjusted consumption growth rates differ markedly across the quarters.[3] As Miron (1986) observes, the data indicate a large average increase in expenditures for durable and nondurable goods in the fourth quarter. The first quarter mean growth rates of these variables are negative. The growth of services displays a different pattern, with the highest mean growth rate in the first quarter. An analysis of variance attributes 48% of the variance of the growth rate of real, unadjusted services expenditures to the differences in quarterly means; for durable goods the figure is nearly 87%.

The real security returns also display seasonal-mean differences. The well-known small-firm, turn-of-the-year effect is evident in the real returns of the smallest decile of common stocks. The average first quarter return is 9.5%; thus, most of the 11.5% continuously compounded annual return is earned in the first quarter. The other securities display less dramatic, but often statistically significant seasonal-mean differences, earning their largest real returns in the fourth quarter on average. In the case of the Treasury bill, this reflects primarily a lower average inflation rate in the fourth quarter. (The standard errors of the quarterly means in Table 1 are approximately 0.16 times the overall sample standard deviations.[4]) Equation (3) posits a linear relation between expected consumption growth and expected real returns. The intercept in this linear relation is a security-specific constant ($\psi_t = \psi$) in the homoscedastic model. This implies that changes over time in the conditional expected return of each asset should be proportional to changes in expected consumption growth. Different patterns of time variation in the expected returns of different assets is not consistent with the model, unless the intercepts ψ_t in Equation (3) are allowed to vary over time. The seasonal patterns in Table 1 suggest that a simple consumption model with no seasonally varying parameters will be unable to capture a cross section of time-varying expected returns.

If the model is to explain a cross section of expected returns, Equation (4) implies it is important to be able to measure reliable differences in consumption covariances across assets. Dunn and Singleton (1986) examine consumption-based models using seasonally adjusted data. They conclude (p. 53) that "the large test statistics can be attributed to the fact that the sample covariances of differences in returns and marginal utilities are too small relative to the differences in sample mean returns. . . . What seems to be needed . . . is a model in which the marginal utilities have very different covariance properties with returns than those that have been studied to date."

Table 2 presents some sample correlations. The correlations of the real returns with the unadjusted consumption measures are stronger and more different across assets than are correlations with seasonally adjusted consumption. The first panel (LAG = 0) indicates that four of the fifteen correlations with unadjusted consumption exceed three standard errors. For a given consumption measure, there are significant differences in the correlations across assets. (The approximate standard error of the sample correlations in Table 2 is 0.08.) None of the return correlations with seasonally adjusted consumption is larger than two standard errors.[5]

Mehra and Prescott (1985) examine the equity premium, measured as the difference between NYSE stocks and Treasury bill returns. They conclude that the variability of aggregate consumption growth is too small to explain the average premium, unless risk aversion is very high. The equity premium depends on the covariance of returns with consumption, as indicated by Equation (4). In the model of Mehra and Prescott consumption is the payoff to "equity," so the covariance of equity with consumption is replaced by the variance of consumption.

Table 3 shows the results of using unconditional sample moments to obtain estimates of the risk aversion coefficient α, implied by Equation (4). Z_t is chosen to be the null information set, and a method of moments approach (described below) is used. The table illustrates that the equity premium puzzle extends to consumption covariances and to other assets. Using seasonally adjusted data, the estimates of α are large. Each exceeds 25.0 in absolute magnitude. The estimates based on equity premiums (the value-weighted index or small stocks) exceed 190.0. The $\hat{\alpha}$ are also very large, using systems with four excess returns.[6]

Intuition might suggest that applying not seasonally adjusted data should result in smaller estimates of α because of the higher volatility and the fact that the unconditional version of (4) uses all the sample covariation with consumption, including the seasonal component. Table 3 seems to confirm this intuition, at least for equity premiums and systems with multiple assets. Not only are the estimates

Table 2. Correlation Matrices, Quarterly Data: 1947:2–1985:4 (155 Observations)

	Non-durables	*Durables*	*Services*	*T-Bill*	*Government Bond*	*Corporate Bond*	*Value Weight*	*Small Decile*
			LAG = 0					
Nondurables	1.000							
Durables	0.913	1.000						
Services	–0.606	–0.526	1.000					
T. bill	0.011	0.004	0.408	1.000				
Government bond	0.072	0.020	0.083	0.392	1.000			
Corporate bond	0.050	–0.010	0.131	0.414	0.954	1.000		
Value weighted	0.056	0.047	0.153	0.172	0.360	0.430	1.000	
Smallest decile	–0.255	–0.240	0.297	0.113	0.208	0.280	0.798	1.000
			LAG = 1					
Nondurables (–1)	–0.662	–0.725	0.531	0.124	–0.018	0.029	0.027	0.227
Durables (–1)	0.106	0.074	–0.018	0.066	0.077	0.103	0.003	–0.014
Services (–1)	0.206	0.359	–0.276	–0.026	0.053	0.010	0.003	–0.059
T. bill (–1)	–0.166	–0.061	0.192	0.523	0.152	0.141	0.065	0.102
Government bond (–1)	–0.099	–0.010	0.280	0.197	–0.068	–0.000	0.131	0.197
Corporate bond (–1)	–0.093	0.011	0.293	0.211	–0.037	0.013	0.203	0.266
Value weighted (–1)	–0.132	–0.040	0.209	0.100	–0.152	–0.119	0.136	0.200
Smallest decile (–1)	0.114	0.220	–0.045	0.006	–0.225	–0.214	0.018	0.012
			LAG = 2					
Nondurables (–2)	0.335	0.471	–0.133	0.032	0.064	0.033	0.073	–0.065
Durables (–2)	0.486	0.604	–0.208	0.052	0.045	0.022	0.076	–0.110
Services (–2)	–0.122	–0.206	–0.021	–0.021	–0.031	0.018	–0.090	0.022
T. bill (–2)	0.030	0.067	–0.049	0.363	0.222	0.222	0.047	0.065
Government bond (–2)	0.036	0.048	–0.072	0.087	0.139	0.150	0.195	0.114
Corporate bond (–2)	0.013	0.046	–0.065	0.059	0.096	0.100	0.156	0.089
Value weighted (–2)	0.069	0.128	–0.071	–0.011	–0.138	–0.151	–0.068	–0.127
Smallest decile (–2)	–0.063	–0.055	0.013	–0.019	–0.149	–0.166	–0.151	–0.078
			LAG = 3					
Nondurables (–3)	–0.657	–0.646	0.217	–0.153	–0.136	–0.126	–0.171	0.089
Durables (–3)	–0.733	0.747	0.339	–0.122	–0.099	–0.081	–0.149	0.132
Services (–3)	0.479	0.425	–0.140	0.313	0.183	0.197	0.228	0.032
T. bill (–3)	0.072	0.051	0.051	0.498	0.178	0.203	0.116	0.129
Government bond (–3)	0.038	0.050	0.078	0.134	0.119	0.154	0.105	0.091
Corporate bond (–3)	0.086	0.081	0.058	0.142	0.112	0.142	0.050	0.021
Value weighted (–3)	0.044	–0.010	0.007	0.066	0.037	0.039	0.004	–0.022
Smallest decile (–3)	0.221	0.174	–0.046	0.109	0.102	0.108	0.079	–0.024

(continued)

Table 2. Continued

	Non-durables	*Durables*	*Services*	*T-Bill*	*Government Bond*	*Corporate Bond*	*Value Weight*	*Small Decile*
			LAG = 4					
Nondurables (–4)	0.986	0.892	–0.616	0.018	0.065	0.040	0.044	–0.277
Durables (–4)	0.902	0.892	–0.564	–0.037	0.061	0.034	0.047	–0.280
Services (–4)	–0.580	–0.506	0.649	0.226	–0.027	–0.029	0.009	0.143
T. bill (–4)	–0.013	–0.010	0.213	0.546	0.197	0.201	0.111	0.087
Government bond (–4)	0.056	0.079	0.048	0.174	0.092	0.112	0.084	0.076
Corporate bond (–4)	0.035	0.055	0.071	0.193	0.079	0.093	0.072	0.089
Value weighted (–4)	0.057	0.090	0.118	0.181	–0.003	0.006	–0.008	–0.004
Smallest decile (–4)	–0.236	–0.201	0.286	0.168	–0.076	–0.048	–0.047	0.107

Notes: Real per capita growth of nondurables, durables, and services expenditures is the logarithm of period t divided by period $t - 1$ *per capita* consumption expenditures. All the data are not seasonally adjusted. The consumption data and the returns are deflated by the (not seasonally adjusted) consumer price index.

of α much smaller, the standard errors of $\hat{\alpha}$ are markedly smaller. (This pattern does not extend to the bond returns combined with consumption of services.) However, two of the four estimates of α using stocks are still above 50.0 and one is more than two standard errors below zero, indicating "risk-loving" behavior.

Table A2 in the appendix shows results of repeating the experiment in Table 3 using observations on the individual quarters. These estimates rely on expected

Table 3. Estimates of the Coefficient of Relative Risk Aversion Based on Unconditional Mean Excess Returns and Covariances with Consumption Measures Quarterly Data, 1948:2–1985:4 (151 Observations)

Excess Return	*Services NSA*	*Services SA*	*Nondurables NSA*	*Nondurables SA*
Government bond	–173.126	26.502	–1.782	28.345
	(1819.98)	(135.39)	(8.52)	(152.07)
Corporate bond	23.367	–25.769	2.264	–36.033
	(150.11)	(129.08)	(12.02)	(171.48)
Value weighted stocks	147.493	318.265	39.828	193.748
	(112.03)	(213.45)	(55.89)	(113.74)
Small stocks	68.362	323.036	–9.747	198.793
	(24.46)	(163.78)	(3.64)	(116.39)
Four assets	42.214	229.75	–2.071	166.969
	(16.18)	(98.83)	(1.38)	(76.95)

Notes: NSA stands for not seasonally adjusted and SA denotes seasonally adjusted. The point estimates are based on generalized method of moments estimation using only a constant as an instrument.

excess returns and consumption covariances conditional on a given quarter, allowing all the parameters of the model to vary seasonally. The estimates of α are again often large in absolute magnitude and imprecise. Only 6 of the 64 $\hat{\alpha}$ exceed 2 approximate standard errors. These 6 are cases of common stock premiums (small stocks in the first quarter and the value-weighted index in the second quarter). There is no obvious pattern in the estimates, either across the quarters or between seasonally adjusted and unadjusted data.[7]

One motivation for studying conditional expectations is to bring more information to bear in parameter estimation and tests. We require instrumental variables Z_t that are correlated with expected consumption growth and real returns. Previous studies have used past returns and consumption as instruments.

The bottom four panels of Table 2 provide evidence of the predictability of returns, using the information in past returns and consumption. The panels show cross correlations at lags 1–4, with the autocorrelations on the diagonals. Assuming rational expectations, correlation of returns with past data indicates changes in the expected returns. As previous studies (e.g., Huizinga and Mishkin, 1984) suggest, the expected real returns of Treasury bills seem to be correlated with many of the lagged variables. Lagged values of consumption growth appear to have some predictive power for the small stocks and the value-weighted stock index returns, but only weak correlations with the government or corporate bonds. The bond returns do appear to be correlated with other lagged returns, notably with the Treasury bill.

Among the assets, Treasury bill returns have the most persistent autocorrelation structure. All four lags are several standard errors from zero and the large fourth-order autocorrelation (0.55) reflects a seasonal pattern. The autocorrelations of the small stock return also suggest a seasonal pattern but are smaller in magnitude.

The autocorrelation structure of the unadjusted consumption data is, not surprisingly, very different from that of seasonally adjusted data. There is very high autocorrelation at the seasonal lag. We compute (but do not report) autocorrelations of the variables out to sixty lags. All the series show autocorrelations that appear to decay toward zero. The autocorrelations of not seasonally adjusted consumption growth display the systematic sign reversals and slow decay typical of a stationary, but strongly seasonal time series. These patterns and the lack of trends in Figure 1 suggest that stationarity of the growth rates may be a reasonable assumption.

A homoscedastic, linear consumption model and market efficiency imply that a linear combination of the growth rate of consumption and a real return, $u_t = r_t - \psi - \alpha c_t$, has expected value equal to zero conditional on public information Ω_{t-1}. If $E\{u_t \mid \Omega_{t-1}\} = 0$; then $\operatorname{cov}(u_t, u_{t-j}) = 0$, for all nonzero j. The covariances of lagged real returns and consumption growth should be related in a simple way. Expanding the expression for $\operatorname{cov}(u_t, u_{t-j}) = 0$ implies that $\operatorname{cov}(r_t, r_{t-j}) + \alpha^2 \operatorname{cov}(c_t, c_{t-j}) = \alpha[\operatorname{cov}(c_t, r_{t-j}) + \operatorname{cov}(r_t, c_{t-j})]$. Because the autocovariances of unadjusted consumption are relatively large and are multiplied by α^2 in the above expression, one expects that large values of α will not "fit" the conditional moment restrictions on the levels of expected returns and consumption growth. This intuition is consistent with the estimates of α that we present below, based on Equation (3).

For comparability with previous studies (and given the correlations in Table 2), we use a constant plus four lagged values each of a consumption growth and a real return as instruments in most of our empirical exercises. We use four lags in order to include the seasonal lag. Larger numbers of lagged values are excluded in order to avoid extreme multicollinearity.[8]

IV. EMPIRICAL RESULTS

A. Results for the Homoscedastic Model

As a first step we replicate tests of linear, homoscedastic models similar to those in previous studies, using not seasonally adjusted consumption data. The homoscedastic model can be derived by assuming that the joint distribution of $\{c_{t+1}, r_{t+1}, Z_t\}$ is a stationary normal in Equation (3). In this case $\psi_t = \psi$ and the parameters of the following regression system are constant over time:

$$c_{t+1} = \beta_{co} + \beta'_c Z_t + \varepsilon_{c,t+1}$$

$$r_{t+1} = \beta_{ro} + \beta'_r Z_t + \varepsilon_{r,t+1}\ . \tag{5}$$

In Equation (5) β_c and β_r are regression slope vectors, β_{co} and β_{ro} are scalars, and the ε are normally distributed error terms.

The theoretical model (3) places nonlinear cross-equation restrictions on the parameters of the regression model (5):

$$\beta_r = \alpha\, \beta_c \quad ; \quad \beta_{ro} = \psi + \alpha\, \beta_{co}\ . \tag{6}$$

Studies by Hansen and Singleton (1983), Ferson (1983), Grossman et al. (1985), Ferson and Merrick (1987), and others examine the linear model, estimating system (5) via maximum likelihood and testing the restrictions (6). This approach assumes that the error terms in Equation (5) are normal and have a constant covariance matrix. An alternative approach is to note that Equation (3) and rational expectations implies

$$u_{t+1} = r_{t+1} - \alpha c_{t+1} - \psi\ ; \quad E(u_{t+1} \,|\, Z_t) = 0\ . \tag{7}$$

Equation (7) can be estimated and tested by generalized method of moments (GMM) [see Hansen (1982)] without assuming normality, a constant covariance matrix, or constant regression coefficients.

We use both these testing strategies. When we use maximum likelihood we examine both the likelihood ratio and the Lagrange multiplier test statistics. When we estimate Equation (7) using generalized method of moments, the minimized

Table 4. Single Asset Tests of the Linear, Homoscedastic Consumption-Based Asset Pricing Model, Equation (3): Quarterly Data, 1948:2–1985:4 (151 Observations)

Asset	*Consumption*	*Measure*	$\hat{\alpha}$	$\hat{\sigma}(\hat{\alpha})$	χ^2	*p value*	R_c^2	R_r^2
Treasury Bill	Nondurables	(NSA)	−0.004	0.006	32.08	0.000	0.980	0.479
		(SA)	0.718	0.228	17.48	0.015	0.141	0.420
	Services	(NSA)	0.112	0.060	27.10	0.000	0.551	0.460
		(SA)	0.571	0.817	23.02	0.002	0.032	0.465
Government bond	Nondurables	(NSA)	0.028	0.029	9.86	0.197	0.980	0.106
		(SA)	3.185	2.198	5.60	0.588	0.062	0.067
	Services	(NSA)	−0.243	0.463	6.17	0.520	0.500	0.069
		(SA)	−6.670	3.837	8.84	0.264	0.059	0.112
Corporate bond	Nondurables	(NSA)	0.014	0.029	10.94	0.141	0.981	0.098
		(SA)	3.917	2.456	5.64	0.583	0.074	0.063
	Services	(NSA)	−0.248	0.445	6.96	0.433	0.508	0.059
		(SA)	−3.982	3.457	9.72	0.205	0.063	0.087
Value weighted stocks	Nondurables	(NSA)	0.060	0.048	11.76	0.109	0.981	0.091
		(SA)	3.464	2.276	7.19	0.409	0.129	0.088
	Services	(NSA)	−0.201	0.629	9.32	0.231	0.463	0.097
		(SA)	0.046	4.900	8.71	0.274	0.056	0.067
Small stocks	Nondurables	(NSA)	−0.255	0.077	8.04	0.329	0.980	0.128
		(SA)	5.741	3.281	4.00	0.780	0.132	0.093
	Services	(NSA)	2.177	1.026	2.45	0.931	0.452	0.043
		(SA)	5.907	6.884	8.95	0.256	0.070	0.091

Notes: α is the coefficient of relative risk aversion and $\hat{\sigma}(\alpha)$ is the asymptotic standard error, based on generalized method of moments estimation. The instruments are 4 lagged values each of the consumption and return variables. χ^2 is the minimized value of the GMM criterion function. *p* Value is the probability that a χ^2 variate exceeds the sample value of the statistic. R_c^2 and R_r^2 are the coefficients of determination in ordinary least squares regressions for consumption growth and real returns, respectively. These are regressions on 4 lagged values of the consumption measure and the real return.

value of the GMM objective function provides a test statistic. This is a quadratic form with an asymptotic χ^2 distribution, under the null hypothesis that the error u_{t+1} has conditional mean zero given the instruments Z_t.

Table 4 summarizes results for the homoscedastic model, reporting the GMM estimates and χ^2 statistics. The results are very similar when maximum likelihood methods are used. Tests are conducted separately for each asset return and four measures of consumption: services and nondurables, seasonally adjusted (SA) and not seasonally adjusted (NSA).[9] The instruments Z_t are a constant and four lagged values of the consumption growth and real return variable, a total of nine instruments.

The R^2's of the consumption growth regressions from system (5) are summarized in Table 4 (denoted R_c^2). These are much higher in the unadjusted than in seasonally adjusted data. Much of the explanatory power is due to the seasonal lag. This is not surprising given the evidence in Tables 1 and 2. The regressions typically explain a larger fraction of the variance of consumption growth than analysis of variance attributes to seasonal mean shifts (i.e., dummy variables indicating the quarters). The residual autocorrelations of the regressions are not large.[10] The lagged predictors seem to do a reasonable job of capturing the seasonal variation in expected consumption growth.

The R^2's of the predictive equations for real returns (denoted R_r^2) are the highest for Treasury bills (exceeding 40%; all the other assets are between 6 and 12%). When services is the consumption instrument, the R^2's are generally lower than when nondurables is the instrument. The R^2's are the largest using unadjusted nondurables. The residual autocorrelations of the real return regressions are generally insignificant.

The test results in Table 4 using seasonally adjusted consumption provide a review of the similar results in previous studies. For example, Hansen and Singleton (1983) estimate Equation (3) using quarterly data on nondurables (1954–1978) and a value-weighted stock index. They report $\hat{\alpha} = 2.7$ and $\chi^2 = 10.03$, with 7 degrees of freedom. In Table 4 we obtain $\hat{\alpha} = 3.5$ and $\chi^2 = 7.2$ (using maximum likelihood, $\hat{\alpha} = 5.3$ and $\chi^2 = 10.55$). The estimates are smaller in absolute magnitude and seem more precise than in Table 3. However, they typically are not reliably different from the values implied by risk neutrality ($\alpha = 0$) or the logarithmic utility function ($\alpha = 1$), and values larger than $\alpha = 9.5$ are within three standard errors of the point estimates in more than half the cases.[11]

Ferson (1983) and Ferson and Merrick (1987) estimate Equation (3) using quarterly Treasury bill returns and nondurables plus services consumption, seasonally adjusted. They report χ^2 statistics that reject the cross-equation restriction for bills at conventional significance levels; a similar result is observed in Table 4. The tests do not produce large values of the χ^2 statistic for individual assets other than the Treasury bill.[12]

Test results using not seasonally adjusted consumption data reveal some interesting differences. The parameter estimates seem to be much more precise. The standard errors of $\hat{\alpha}$ are frequently on the order of 100 times smaller than with seasonally adjusted data. The point estimates of α are typically closer to zero by a factor of 10. These combined differences mean that values of α larger than 2.0 are included within a three-standard-error confidence interval in only one case. In that case, the point estimate is 2.18. Estimates of the risk aversion parameter close to zero are not surprising, given the patterns in the data discussed earlier. The χ^2 statistics are larger using unadjusted nondurables, but only for the bill do they attain conventional significance levels. Using services, the goodness of fit seems to be improved for the small stocks. This may be a result of the common seasonal components in these series.

Table 5. Multiple Asset Tests of the Linear, Homoscedastic Consumption-Based Asset Pricing Model, Equation (3): Quarterly Data, 1948:2–1985:4 (151 Observations)

Consumption	*Measure*	$\hat{a}$	$\hat{\sigma}(\hat{\alpha})$	χ^2	*p value*
Full system (5 assets, df = 39)					
Nondurables	(NSA)	–0.002	0.005	80.38	0.000
Nondurables	(SA)	0.336	0.162	56.88	0.032
Services	(NSA)	0.139	0.048	62.18	0.011
Services	(SA)	2.581	1.130	42.18	0.335
System without bills (4 assets, df = 31)					
Nondurables	(NSA)	0.074	0.024	66.38	0.000
Nondurables	(SA)	–0.580	1.034	40.69	0.114
Services	(NSA)	–0.625	0.377	47.95	0.027
Services	(SA)	–1.436	2.611	39.36	0.144
System without consumption equation (5 assets, df = 32)					
Nondurables	(NSA)			68.12	0.000
Nondurables	(SA)			44.83	0.065
Services	(NSA)			48.91	0.028
Services	(SA)			38.22	0.210

Notes: α is the coefficient of relative risk aversion and $\hat{\sigma}(\alpha)$ is the asymptotic standard error, based on generalized method of moments estimation. The instruments are 4 lagged values each of the consumption and return variables. χ^2 is the minimized value of the GMM criterion function. *p* Value is the probability that a χ^2 variate exceeds the sample value of the statistic.

Table 5 summarizes the results of estimating the model on multiple-asset systems. The first panel reports results for the five asset returns. There are four cases; one for each measure of consumption growth. The instruments are four lags of the consumption measure and four lags of the real Treasury bill return. The second panel omits the Treasury bill from the system because the bill produced the only rejections in single-asset tests.

As in Table 4, the estimates of α in Table 5 are closer to zero and the standard errors are much smaller with unadjusted data. Only in one case (nondurables, all five assets) do seasonally adjusted data produce a large χ^2 value. Unadjusted consumption data and multiple assets produce strong rejections in every case. (Maximum likelihood methods produce qualitatively similar results.[13])

The rejections in Table 5 can not be attributed to a misspecification of expected consumption growth or to a poor "fit" between expected consumption growth and asset returns. Rejections are confirmed in the third panel of Table 5, where consumption is not included. Without the consumption growth equation the coefficient of relative risk aversion is not identified. The testable implication is that the slope coefficients for the asset return regressions are identical, while the intercepts can differ because securities' ψ coefficients can differ.

The tests in Tables 4 and 5 examine restrictions on the variation of conditional expected returns and growth rates about their unconditional means. The levels of the unconditional mean returns and growth rates are unrestricted because the intercepts ψ in Equation (3) are unrestricted constants. The tests can detect variation of conditional expected returns, related to the seasonally varying instruments, that is sufficiently different across assets to reject the homoscedastic model.

B. Incorporating Seasonality and Heteroscedasticity

Miron (1986) examines consumption models which incorporate seasonality. He hypothesizes that there are seasonal "shocks" to preferences, replacing the utility function in Equation (2) with

$$u(C,s,t) = \{[C\exp(\delta_{s(t)})]^{1-\alpha}\}/(1-\alpha)\,,$$

where $\delta_{s(t)}$ is a taste shift parameter, indicating that the utility obtained from the level of consumption C at time t varies according to the season $s(t)$ prevailing at time t. Alternatively, the δ_s' could be interpreted as parameters of the consumer's household production function.[14] With this utility function, Equation (3) is modified as follows:

$$E\{r_{t+1} \mid Z_t\} = \psi_t^* + \alpha E\{c_{t+1} \mid Z_t\} \tag{8}$$

where

$$\psi_t^* = -\frac{1}{2}\operatorname{var}\{r_{t+1} - \alpha c_{t+1} \mid Z_t\} - \ln\beta - (1-\alpha)[\delta_{s(t+1)} - \delta_{s(t)}]\,.$$

Even when homoscedasticity is assumed [the conditional variance and ψ in Equation (3) is constant] Equation (8) allows the relation between the conditional expected returns of assets and expected consumption growth to include a seasonally varying intercept. Equation (8) is a linearization of the model examined by Miron (1986). Miron assumes homoscedasticity and finds no evidence against the model.[15] However, he examines only a single asset and uses two lagged values of his variables as instruments.[16]

With homoscedasticity, the seasonally varying part of Equation (8) is common across assets and will not affect expected return differences. Expected excess returns will not be affected by multiplicative taste shifters that are contained in the information set Z_t or by seasonal variation in the rate of time discount β. If seasonal variation is reflected in expected risk premiums, it affects the covariances of return differences with the marginal utility of consumption. Such effects can be accommodated through seasonal heteroscedasticity. Cross-asset differences in the intercepts of Equation (8) determine expected risk premiums in the model. Heteroscedasticity in the conditional distributions can result in variation in ψ_t^* as a function of the instruments. Since the intercepts are asset specific, seasonally varying conditional heteroscedasticity can be consistent with seasonal variation in expected risk premiums.

Other empirical evidence motivates a study of heteroscedastic models. Hansen and Singleton (1983) and Ferson (1983) suggest that rejections of Equation (3) may be the result of changing conditional variances and covariances. Several studies present direct evidence of conditional heteroscedasticity in predictive equations for security returns (e.g., Huizinga and Mishkin (1984), Bollerslev, Engle and Wooldridge (1988), Campbell (1987), French, Schwert, and Stambaugh (1987)). We conduct Breusch–Pagan (1979) tests for heteroscedasticity in the error terms of the regressions in system (5) and find strong statistical evidence of conditional heteroscedasticity. In further diagnostics we find "economic significance" in the sense that a linear, homoscedastic model captures only a small portion of the variation through time of the conditional expected returns of the assets.[17]

We examine two types of models with seasonality and conditional heteroscedasticity. The first specifies the functional form of the heteroscedasticity, the second does not. The first model assumes that the conditional variance is a positive linear function of the squared instruments Z_t: $\text{var}\{r_{t+1} - \alpha c_{t+1} \mid Z_t\} = (B'Z_t)^{\wedge 2}$, where B is an asset-specific coefficient vector and $x^{\wedge 2}$ stands for the element-by-element squares of x. Let $\zeta_{t+1} \equiv r_{t+1} - \alpha c_{t+1} + \frac{1}{2}(B'Z_t)^{\wedge 2} + \ln\beta + (1-\alpha)[\delta_{s(t+1)} - \delta_{s(t)}]$. Equation (8) and the model of the conditional variance imply that $E\{\zeta_{t+1} \mid Z_t\} = 0$ and $E\{\zeta^2_{t+1} \mid Z_t\} = \text{var}\{r_{t+1} - \alpha c_{t+1} \mid Z_t\} = (B'Z_t)^{\wedge 2}$. If we define $\eta_{t+1} = \zeta^2_{t+1} - (B'Z_t)^{\wedge 2}$, the model implies that $E\{\eta_{t+1} \mid Z_t\} = 0$. Stacking the error terms for the conditional mean and the conditional variance results in the following econometric model:

$$\omega_{t+1} = \begin{bmatrix} \zeta_{t+1} \\ \eta_{t+1} \end{bmatrix}; \quad E\{\omega_{t+1} \mid Z_t\} = 0. \tag{9}$$

One can think of this model as using the second equation of (9) to identify the conditional variance. This equation contributes enough orthogonality conditions to identify B, the parameters of the conditional variance function. The first equation imposes structure on the intercept of the linear relation of an expected real return and consumption growth to identify the seasonal shift and time preference parameters. The first quarter of the year is chosen as a reference quarter ($\delta_1 = 0$), so the shift coefficients δ_2, δ_3, δ_4 indicate how the model "seasonally adjusts" consumption levels "endogenously," relative to the first quarter. We estimate the parameters of Equation (9) (α, $\ln\beta$, B, δ_2, δ_3, δ_4) by GMM using the same instruments as before. Unadjusted consumption growth rates are used in order to capture stochastic seasonal variation of the conditional variance.[18] For a given asset there are 9 instruments in Z_t and therefore 14 parameters to estimate. The number of sample orthogonality conditions is $2 \times 9 = 18$ [i.e., $E(\omega_{t+1}Z_t) = 0$], so the degrees of freedom of the χ^2 statistic is 4.

Table 6 presents the results. Miron reports t-ratios of his δ coefficients that are larger than 2.0 for one of the six consumption components he examines (fuel oil

Table 6. Tests of a Linear, Heteroscedastic Asset Pricing Model: 1948:2–1985:4[a]

Asset	Consumption	$\hat{\alpha}$	$\widehat{\ln\beta}$	$\hat{\delta}_2$	$\hat{\delta}_3$	$\hat{\delta}_4$	$\overline{(B'Z)^{\wedge 2}}$[b]	χ^2[c]	p value
Treasury bill	Nondurables	0.190	–0.002	0.022	0.023	0.047	0.0048	30.52	0.000
		(0.157)[d]	(0.001)	(0.019)	(0.019)	(0.046)			
	Services	0.479	0.002	–0.004	–0.007	–0.016	0.0032	29.16	0.000
		(0.206)	(0.002)	(0.007)	(0.011)	(0.015)			
Government bond	Nondurables	0.015	0.001	0.006	0.010	–0.000	0.1722	8.77	0.067
		(0.777)	(0.003)	(0.065)	(0.068)	(0.151)			
	Services	0.567	0.003	–0.010	–0.025	–0.062	0.2067	2.26	0.689
		(1.494)	(0.013)	(0.086)	(0.145)	(0.275)			
Corporate bond	Nondurables	0.229	–0.001	0.026	0.027	0.057	0.0080	4.17	0.384
		(0.144)	(0.001)	(0.019)	(0.019)	(0.046)			
	Services	1.046	0.005	0.237	0.542	0.872	0.2435	1.21	0.876
		(1.449)	(0.012)	(7.153)	(16.756)	(27.263)			
Value-weighted stocks	Nondurables	2.983	–0.015	–0.118	–0.125	–0.286	0.9683	2.87	0.580
		(2.237)	(0.008)	(0.047)	(0.54)	(0.105)			
	Services	4.445	0.015	0.016	0.015	0.029	0.6171	2.38	0.665
		(3.377)	(0.028)	(0.004)	(0.006)	(0.010)			
Small stocks	Nondurables	2.046	–0.032	–0.179	–0.192	–0.425	1.4493	5.82	0.213
		(1.991)	(0.013)	(0.192)	(0.212)	(0.445)			
	Services	3.317	–0.008	0.008	0.008	0.010	1.3983	2.51	0.643
		(5.888)	(0.049)	(0.020)	(0.026)	(0.025)			
Five assets	Nondurables	5.513	0.002	–0.090	–0.086	–0.210		111.10	0.000
		(1.067)	(0.008)	(0.005)	(0.005)	(0.007)			
	Services	3.905	–0.034	0.007	0.006	0.007		147.52	0.000
		(1.138)	(0.008)	(0.004)	(0.005)	(0.005)			

Notes: Generalized method of moments estimation of a linear, heteroscedastic consumption-based asset pricing model based on quarterly data (151 observations).

$$\omega_{t+1} = \begin{pmatrix} \zeta_{t+1} \\ \eta_{t+1} \end{pmatrix} = \begin{pmatrix} r_{t+1} - \alpha c_{t+1} + (0.5)(B'Z_t)^{\wedge 2} + \ln\beta + (1-\alpha)[\delta_{s(t+1)} - \delta_{s(t)}] \\ \zeta_{t+1}^2 - (B'Z_t)^{\wedge 2} \end{pmatrix},$$

[a]The coefficient of relative risk aversion is α, β is the time discount factor, and the δ are seasonal taste shift parameters The instruments **Z** are a constant and 4 lagged values of the consumption and return variables. In the multiple asse systems, the Treasury bill is used as the lagged return.
[b]The conditional variance is modeled as a positive linear combination of squares of the instruments. $\overline{(B'Z)^{\wedge 2}}$ denote the average conditional variance.
[c]χ^2 is the minimized value of the GMM criterion function based on Equation (6). *p* Value is the probability that a χ variate exceeds the sample value of the statistic.

and coal). We find large *t*-ratios with both services and nondurables, but only for the value-weighted stock index. In the full five-asset systems the coefficients appear highly significant.[19] When the *t*-ratios are larger than 2.0 the taste shift parameters enter the model with intuitively plausible signs: negative for nondurables (where first quarter growth rates are the lowest) and positive for services (with the highest growth rate in the first quarter.)

The estimates of α in Table 6 range from 0.02 to 5.51 (standard errors range from 0.16 to 5.89). Unlike the previous table, none of the point estimates is negative. The estimates of $\ln\beta$ (the negative of the rate of time preference) are typically close to zero. When reliably different from zero the point estimates are negative, as economic theory suggests. Figure 2 presents time-series plots of the fitted conditional variance function, $(B'Z_t)^{\wedge 2}$, and two-standard-error bands, for each asset. The plots suggest significant movement in the conditional variances for the bills and bonds, but the evidence is less clear for the stocks because of the larger standard error bounds.

There is evidence against models with seasonality. Tests of the restrictions produce strong rejections for the Treasury bill and the asset systems. Large test statistics for the other assets are rare. Given a more general model, the goodness-of-fit results for the individual long-term assets are as expected.[20] Table 6 also reports the average of the fitted conditional variances $(B'Z_t)^{\wedge 2}$ for each asset, based on the individual models.[21] The sample values are ranked in a similar order as the sample variances of the real returns; the average value is the smallest for bills and the largest for small stocks. The theory implies that the difference in the conditional variances should be –2.0 times the expected excess returns. (The conditional variances are negatively related to the conditional covariances with consumption.) Table 6 indicates that the differences across assets in the fitted variances appear on average to go in the "wrong direction." This extends to corporate bonds and stocks the similar findings of Dunn and Singleton (1986) for Treasury bills.

Perhaps the model of the conditional variances does not produce a good fit for the conditional covariances of returns with consumption. We examine a heteroscedastic model that attempts to leave the functional form of conditional second moments unspecified, focusing on the relation of expected risk premiums to conditional covariances. We approximate the conditional covariances using the products of deviations from a regression model of the conditional means. A regression system like (5) approximates the conditional expected values of consumption growth and returns given Z_t. The conditional mean of the product of the error terms $E(\varepsilon_{c,t+1}\varepsilon_{r,t+1} \mid Z_t)$ is used to model the conditional covariance of r_{t+1} with c_{t+1} given Z_t. Define the error term:

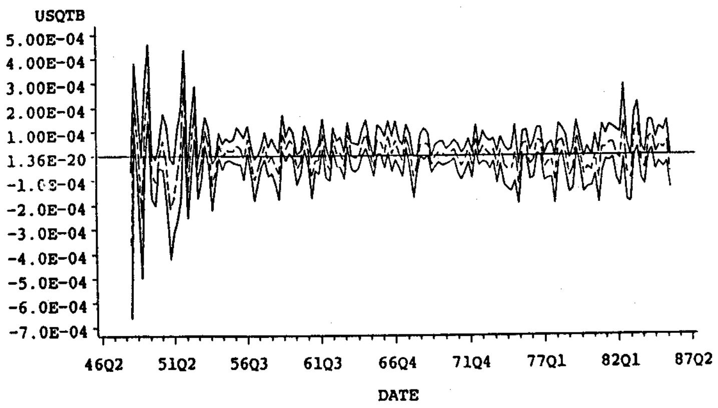
$$e_{t+1} = r_{t+1} - r_{f,t+1} - \alpha[(\varepsilon_{r,t+1} - \varepsilon_{f,t+1})\varepsilon_{c,t+1}] + \frac{1}{2}[\varepsilon^2_{r,t+1} - \varepsilon^2_{f,t+1}], \tag{10}$$

where $r_{f,t+1}$ is the real return of the treasury bill, $\varepsilon_{f,t+1}$ is the error term for the bill in Equation (5), and r_{t+1} is any other real return. Equation (4) implies that $E(e_{t+1} \mid Z_t) = 0$. Appending e_{t+1} to a regression system like (5), we obtain an econometric model that can be estimated and tested by GMM:

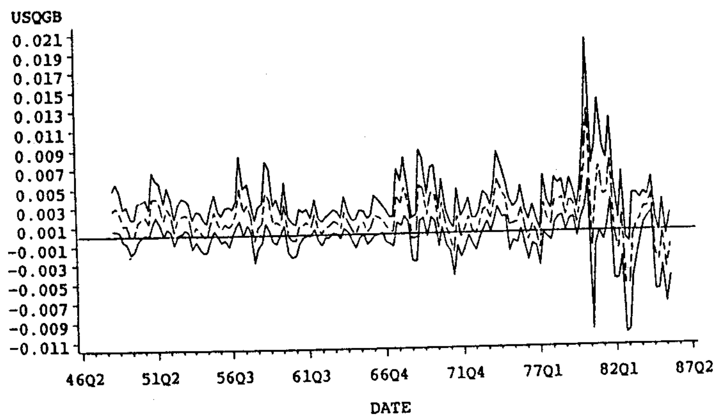
$$\xi_{t+1} = \begin{bmatrix} \varepsilon_{c,t+1} \\ \varepsilon_{f,t+1} \\ \varepsilon_{r,t+1} \\ e_{t+1} \end{bmatrix} \begin{matrix} = \\ = \\ = \\ = \end{matrix} \begin{bmatrix} c_{t+1} - B_c'Z_t \\ r_{f,t+1} - B_f'Z_t \\ r_{t+1} - B_r'Z_t \\ r_{t+1} - r_{f,t+1} - \alpha[(\varepsilon_{r,t+1} - \varepsilon_{f,t+1})\varepsilon_{c,t+1}] + \frac{1}{2}[\varepsilon^2_{r,t+1} - \varepsilon^2_{f,t+1}] \end{bmatrix} \tag{11}$$

$$E(\xi_{t+1} \mid Z_t) = 0.$$

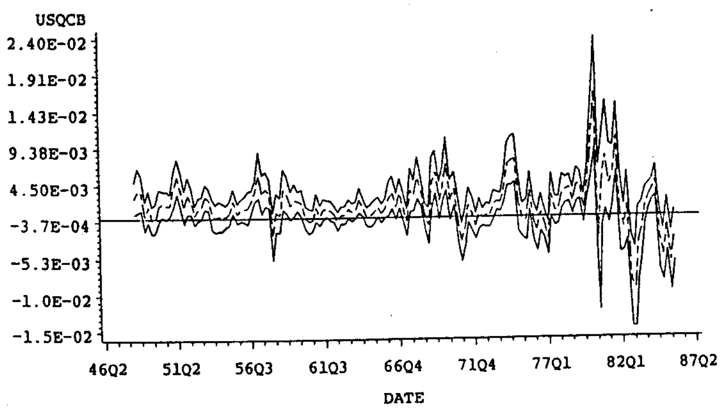
Fitted Conditional Variance and Two Standard Error Bounds:
Non-Durables (NSA) and Treasury Bills



Fitted Conditional Variance and Two Standard Error Bounds:
Non-Durables (NSA) and Government Bonds



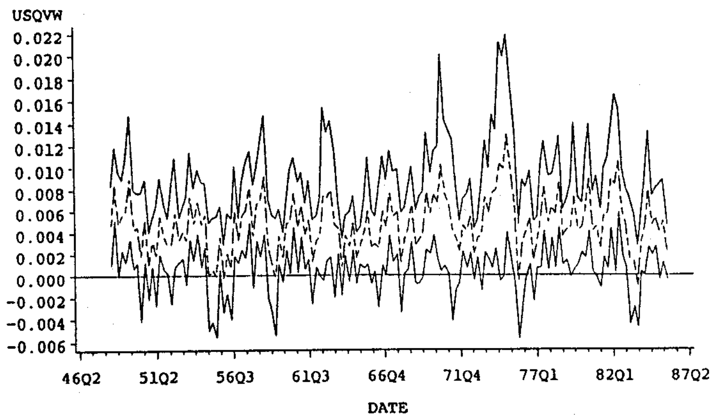
Fitted Conditional Variance and Two Standard Error Bounds:
Non-Durables (NSA) and Corporate Bonds



(continued)

Figure 2. Linear Models of Conditional Heteroscedasticity.

Fitted Conditional Variance and Two Standard Error Bounds:
Non-Durables (NSA) and Value Weighted Stocks



Fitted Conditional Variance and Two Standard Error Bounds:
Non-Durables (NSA) and Small Stocks

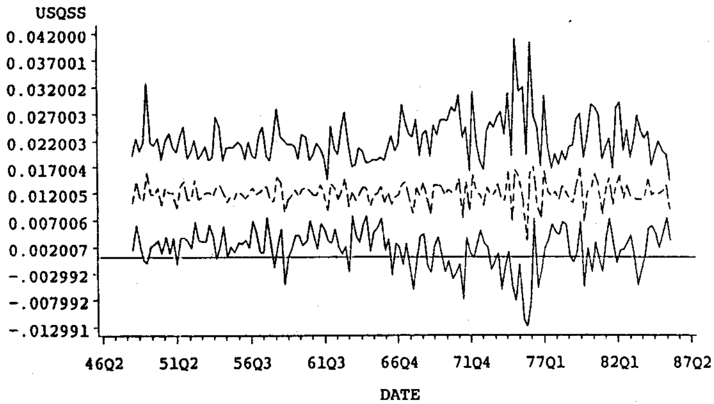


Figure 2. (continued)

The first three equations of system (11) comprise an exactly identified model for conditional expected returns and consumption growth. These equations do not produce an overidentifying restriction, and GMM estimation of these would give the same B estimates as equation-by-equation ordinary least squares. The overidentification in system (11) is the result of the last equation, which imposes the restriction that expected risk premiums are related to the consumption covariances according to Equation (4).

Table 7. Excess Return Tests of a Linear, Consumption-Based Asset Pricing Model with a General Form of Heteroscedasticity: Quarterly Data, 1948:2–1985:4 (151 Observations)

Return	*Consumption*	$\hat{\alpha}$	$\hat{\sigma}(\hat{\alpha})$	χ^2	*p value*	$\overline{PE}_{GB}$	$\overline{PE}_{CB}$	$\overline{PE}_{VW}$	$\overline{PE}_{SS}$
Government bond	Nondurables	−37.310	22.164	7.38	0.496	0.0015			
	Services	−70.046	19.109	12.31	0.138	−0.0033			
Corporate bond	Nondurables	−104.862	30.214	5.64	0.688		−0.0019		
	Services	−33.678	18.698	6.05	0.642		0.0038		
Value-weighted stocks	Nondurables	41.642	24.367	19.58	0.012			0.0187	
	Services	99.289	36.646	7.29	0.506			0.0033	
Small stocks	Nondurables	58.671	24.403	11.83	0.159				0.0288
	Services	85.893	28.640	1.99	0.981				0.0076
Four assets	Nondurables	−157.454	24.865	48.90	0.059	−0.0056	−0.0035	0.0129	0.0371
	Services	120.504	19.237	69.10	0.001	−0.0035	−0.0057	−0.0004	−0.0112
Four assets (with intercept)	Nondurables	−93.410	14.490	49.16	0.020	0.0010	0.0017	−0.0032	0.0094
	Services	69.790	17.736	49.84	0.017	0.0020	0.0011	0.0003	−0.0065

Notes: α is the coefficient of relative risk aversion, and $\hat{\sigma}(.)$ is asymptotic standard error based on generalized method of moments estimation. The instruments are 4 lagged values of the consumption and Treasury bill return variables. χ^2 is the minimized value of the GMM criterion function. *p* value is the probability that a χ^2 variate exceeds the sample value of the statistic. $\overline{PE}$ is the average pricing error over the year for government bonds (GB), corporate bonds (CB), value-weighted stocks (VW) and small stocks (SS).

Table 7 summarizes the results of estimating system (11). Using individual returns in excess of the Treasury bill, the χ^2 statistics are typically not large enough to reject the model at conventional significance levels. An exception is the case of value-weighted stocks and nondurables (right-tail p value 0.012). Using asset systems, the statistics indicate rejections at conventional levels (the largest p value is 0.059).

Dunn and Singleton (1986) conclude that the relation across assets of unconditional mean returns and covariances with consumption is an important factor in their rejections of a consumption-based model. In a nonlinear model like the one examined by Dunn and Singleton, both conditional and unconditional moment relations affect the test statistic. We found that the variation of expected returns about their unconditional means was sufficient to cause rejections of the homoscedastic models, but our results for the heteroscedastic models with seasonality involve unconditional as well as conditional moment restrictions.

The bottom panel of Table 7 summarizes results for a version of system (11) that includes an asset-specific intercept. This allows the unconditional mean risk premiums to be free parameters, so the estimates and tests should be driven only by the movement of the conditional expected risk premiums around their sample averages. The overall pricing errors are smaller in this model, but the results are otherwise similar to the model which does not include an intercept. We conclude that there is evidence against the model with seasonality when multiple assets are examined. The rejections are not driven primarily by unconditional moment relations.[22]

The estimates of α in Table 7 reflect the relation of conditional expected excess returns and conditional covariances. These are analogous to the estimates in Table 3, which use only unconditional moments. They differ from the estimates in Tables 4–6, which focus on the levels of the conditional expected values. The estimates of α in Table 7 are much larger in absolute magnitude and seem less precise than the estimates in Tables 4–6. Some "reliably" negative estimates of α are also observed in Table 7. Combining these observations with previous tables, we can provide a more complete characterization of the results.

Seasonally adjusted consumption data are "smoothed" dramatically and tend to imply larger estimates of the risk aversion coefficient α. However, the inability of the consumption models to fit excess returns without extreme values of the α coefficient does not seem to be an artifact of seasonal adjustment.[23] The poor fit between returns and consumption covariances is not confined to equity premiums but extends to long-term corporate and government bonds. The premium "puzzle" involves not only average excess returns but also the movement over time of conditional expected risk premiums. Extreme values of α are not obtained when the consumption models are fit to the levels of expected real returns and expected consumption growth rates, but such models do a poor job of capturing cross-asset return differences and the movements of expected returns through time.[24]

C. Results for Annual Data

This section summarizes results for annual real returns and consumption growth rates. Details are provided in the appendix. We examine annual data for information about the effects of seasonal adjustment versus seasonality in the data, and as a further sensitivity check on our results. It is possible that a consumption-based model is more appropriate for data measured at lower frequencies because consumers do not optimally adjust their consumption each quarter.

We find that the smoothing of data by the X-11 program affects the annual, as well as the quarterly data. Unadjusted annual consumption growth rates are more volatile than seasonally adjusted annual growth rates. Rejections of the homoscedastic model as well as a heteroscedastic model, which uses the levels of expected real returns and consumption growth, are confirmed in the annual data. Goodness-of-fit does not seem to be improved by moving to annual data. If anything, the rejections are more impressive. Models which use the levels of expected real returns and growth rates typically produce smaller point estimates and standard errors for the α parameter than do models like Equations (4) or (11), which focus on risk premiums. Many of the other comparisons of seasonally adjusted and not seasonally adjusted data are similar at the annual and the quarterly levels.

V. CONCLUSIONS

This paper examines linear, consumption-based asset pricing models using not seasonally adjusted aggregate consumption data. Unadjusted consumption data are more volatile and security returns often exhibit stronger correlations with unadjusted measures of consumption. Unadjusted data seem to produce more precise estimates of model parameters and may provide more powerful tests.

Previous evidence on consumption-based pricing using seasonally adjusted data indicates that unconditional moment relations provide important evidence against the model. Using unadjusted data we find that variation through time of conditional expected returns around the unconditional means is sufficiently rich to reject the models. Such evidence is especially evident in multiple-asset systems.

Similar to the evidence of seasonally adjusted consumption, individual long-term bonds or stocks seldom produce large test statistics, while Treasury bill returns do. Unlike the recent study of Miron (1986), we find evidence against models which incorporate seasonality. We examine models with seasonal "taste shifters" and heteroscedasticity. The fitted conditional covariances with consumption seem to be ordered across assets in the "wrong direction" on average, relative to the returns of long-term bonds and stocks. This is similar to observations of Dunn and Singleton (1986) for treasury bill returns and seasonally adjusted consumption. Even when stochastic, seasonal variation in consumption covariances is allowed, the model produces large pricing errors for excess returns.

Conditional heteroscedasticity can, of course, appear as a symptom of other misspecifications. To give one example, suppose that the "true" α coefficient varies as a function of the instruments. Then, even if the "true" error term is homoscedastic, when we estimate a model with a constant α the error term will appear to be conditionally heteroscedastic. Other potential misspecifications include more complex "shocks" to tastes or technology (e.g., Garber and King, 1983), complementarity and durability of consumer goods (e.g., Dunn and Singleton, 1986), "habit formation" in aggregate preferences (e.g., Constantinides, 1990), market incompleteness (Mankiw, Rottemberg, and Summers, 1986; Grossman and Laroque, 1990; Scheinkman and Weiss, 1986) or other imperfections.

We think that it would be useful to identify asset pricing models able to fully exploit the information about asset returns that seems to be reflected in not seasonally adjusted consumption data. Unfortunately, the simple models we examine fall short of this goal. It seems appropriate to end with a familiar call for further research in this area.

APPENDIX: RESULTS FOR ANNUAL DATA

Table A1 shows the sample means and standard deviations of annual consumption growth rates, observed quarterly. The observations are overlapping, so ordinary standard deviations are biased. Table Al presents sample standard deviations that are adjusted for bias, using alternative extreme assumptions about the underlying correlations. (The adjustments are explained in the notes to the table.) Table A1 indicates that unadjusted annual consumption growth rates are more volatile than seasonally adjusted annual growth rates. The differences are not as dramatic, of course, as in quarterly growth rates. Table A2 was discussed in the text.

Table A3 summarizes tests of the linear, homoscedastic model [Equation (7)]. Because of overlapping observations, the error term of (7) will follow a moving-average process. We estimate the model with GMM using the weighting matrix suggested by Hansen (1982), with three moving-average terms. The instrumental variables are a constant and four, nonoverlapping lagged values of the annual real return and consumption growth variable. The results reinforce a rejection of the homoscedastic model. The χ^2 statistics are large in most of the cases. (The largest right-tail p value in the table is 0.114; and 17 of the 20 p values are smaller than 0.023.) Although the point estimates of α show no consistent patterns, the standard errors of the $\hat{\alpha}$'s are typically smaller using unadjusted consumption. (An exception

Table A1. Summary Statistics Quarterly Data on Annual Growth Rates 1947:2–1985:4 (155 Observations)

Variable	*Measure*	*Mean*	*Unadjusted Standard Deviation*	*Adj (ρ = 0) Standard Deviation*	*Adj (ρ = ρ₁) Standard Deviation*
Consumption Growth					
Nondurables	(NSA)	0.006	0.022	0.0225	0.0306
Nondurables	(SA)	0.011	0.019	0.0195	0.0196
Durables	(NSA)	0.019	0.095	0.0977	0.1058
Durables	(SA)	0.034	0.083	0.0850	0.0852
Services	(NSA)	0.030	0.020	0.0208	0.0212
Services	(SA)	0.024	0.013	0.0129	0.0129

Notes: Nondurables, durables, and services are real, per capita growth rates of aggregate expenditures for consumer nondurables, durable goods, and services, respectively. SA is seasonally adjusted and NSA is not seasonally adjusted. Data are overlapping quarterly observations of annual, continuously compounded growth rates. The unadjusted standard deviation is the usual sample estimate (*N* weighing) using the overlapping data. The adjusted (Adj) sample variances are computed by multiplying the unadjusted variances by a bias adjustment factor equal to

$$\frac{N}{N-\frac{1'R1}{N}};$$

where N is the number of overlapping observations, **1** is a vector of ones, and R is the correlation matrix of the overlapping data (see Fama and French, 1988). The structure of R depends on the true autocorrelation structure of the underlying data. Adjustments are presented using two extreme assumptions. The first case [Adj ($\rho = 0$)] follows Fama and French, assuming the underlying data are serially uncorrelated. The second case [adj ($\rho = \rho_1$)] assumes a first-order autoregression, with autoregressive parameter equal to the largest of the first 8 sample autocorrelations of the quarterly data.

is when nondurables and common stocks are paired.) The standard errors of the intercepts ψ are always smaller using the unadjusted data.

We investigate (but do not report in the tables) a form of the linear model that uses the levels of real returns and consumption growth and assumes that the conditional variance is a linear function of the instruments. Consistent with the evidence from quarterly data, the χ^2 statistic indicates a strong rejection of the model for Treasury bills. Rejections of this model are also indicated for the common stocks, and marginal rejections for the long-term bonds. The standard errors of the estimates and the absolute magnitude of the $\hat{\alpha}$ are frequently larger than in quarterly data. Still, only in case is a value of α larger than 8.7 included within a three standard error confidence interval of the point estimate.

Time series plots of the fitted conditional variances using the annual data (not shown) look fairly reasonable in spite of the strong evidence against the restrictions. Although the conditional variance is not constrained to be positive, only in the case

Table A2. Estimates of the Coefficient of Relative Risk Aversion Based on Unconditional Mean Excess Returns and Covariances with Consumption Measures Estimates by Quarter

	Services NSA	Services SA	Nondurables NSA	Nondurables SA	Services NSA	Services SA	Nondurables NSA	Nondurables SA
	First Quarter				*Second Quarter*			
Government bond	–46.593	–230.713	610.714	–141.303	8.747	15.338	–228.087	29.621
	(46.36)	(221.66)	(593.54)	(136.98)	(47.84)	(83.89)	(1470.60)	(161.94)
Corporate bond	–19.762	–69.169	–51.913	–44.019	26.523	44.760	–147.624	104.358
	(40.02)	(138.96)	(104.31)	(88.56)	(44.01)	(74.26)	(269.95)	(172.41)
Small stocks	475.943	823.081	220.029	1409.479	20.175	–54.700	46.713	–71.642
	(175.80)	(255.55)	(71.48)	(457.02)	(57.17)	(154.97)	(132.16)	(203.16)
Value weighted stocks	153.904	196.535	–512.981	184.136	–308.347	167.233	–53.330	57.439
	(111.91)	(131.43)	(345.47)	(124.09)	(554.65)	(299.18)	(95.13)	(103.01)
	Third Quarter				*Fourth Quarter*			
Government bond	–757.864	62.611	50.709	190.643	54.702	7488.468	–20.535	–107.006
	(4180.32)	(160.36)	(129.85)	(488.16)	56.53	(22932.80)	(20.73)	(106.09)
Corporate bond	–18.702	51.111	62.449	–85.480	66.411	1546.524	34.437	–118.422
	(169.02)	(461.02)	(565.41)	(771.46)	(53.76)	(1234.30)	(27.11)	(90.47)
Small stocks	420.801	141.297	101.420	90.331	51.232	416.584	36.458	123.057
	(412.95)	(119.42)	(86.65)	(76.01)	(39.36)	(321.07)	(27.55)	(94.94)
Value weighted stocks	119.673	93.692	104.666	60.297	176.817	8990.786	185.193	1800.867
	(207.41)	(158.21)	(178.87)	(101.61)	(64.19)	(7109.88)	(55.11)	(1153.40)

Notes: Approximate standard errors in parentheses. NSA stands for not seasonally adjusted and SA denotes seasonally adjusted. The point estimates are derived from Equation (4) in the text and maximum likelihood estimates of the unconditional means and second moments.

Table A3. Tests of a Linear, Homoscedastic Consumption-Based Asset Pricing Model: Equation (3) with $\psi_t = \psi$, Annual Data Sampled Quarterly, 1948:2–1985:4 (136 Observations)

Asset	*Consumption*	*Measure*	$\hat{\alpha}$	$\hat{\sigma}(\hat{\alpha})$	$\hat{\psi}$	$\hat{\sigma}(\hat{\psi})$	χ^2	*p Value*
Treasury bill	Nondurables	(NSA)	0.776	0.301	−0.002	0.003	22.40	0.002
		(SA)	1.775	0.414	−0.013	0.006	13.82	0.055
	Services	(NSA)	0.926	0.181	−0.022	0.006	31.83	0.000
		(SA)	0.281	0.476	−0.004	0.012	22.06	0.002
Government bond	Nondurables	(NSA)	0.287	0.676	−0.012	0.010	16.82	0.019
		(SA)	1.558	0.858	−0.026	0.014	16.27	0.023
	Services	(NSA)	2.214	0.591	−0.080	0.020	11.61	0.114
		(SA)	−1.023	1.006	0.019	0.029	15.89	0.026
Corporate bond	Nondurables	(NSA)	0.673	0.706	−0.009	0.011	16.41	0.022
		(SA)	2.220	0.975	−0.028	0.016	19.12	0.008
	Services	(NSA)	2.296	0.665	−0.082	0.023	13.46	0.062
		(SA)	−0.818	1.032	0.020	0.012	16.36	0.022
Value-weighted stocks	Nondurables	(NSA)	−6.342	2.703	0.109	0.026	22.35	0.002
		(SA)	0.237	1.718	0.062	0.028	27.53	0.000
	Services	(NSA)	1.619	1.515	0.007	0.050	32.95	0.000
		(SA)	−0.007	2.327	0.059	0.060	36.87	0.000
Small stocks	Nondurables	(NSA)	2.329	2.510	0.075	0.026	23.61	0.001
		(SA)	1.249	2.216	0.091	0.036	16.61	0.020
	Services	(NSA)	3.335	1.790	0.002	0.058	19.22	0.008
		(SA)	3.704	3.094	0.010	0.076	17.70	0.013

Notes: α is the coefficient of relative risk aversion, ψ is the intercept in the linear relation of expected real returns and consumption growth, which is assumed constant over time, and $\hat{\sigma}(.)$ is asymptotic standard error, based on generalized method of moments estimation of Equations (7) and (8). The instruments are 4 lagged values each of the consumption and return variables. χ^2 is the minimized value of the GMM criterion function. *p* Value is the probability that a χ^2 variate exceeds the sample value of the statistic.

of the Treasury bill does an upper two-standard error confidence band for the variance dip below the zero line.

Table A4 reports the results of testing the heteroscedastic model of Equation (11) using annual data. The χ^2 statistics are not large for most of the individual excess returns. There are two cases where the p values are below 0.05. The estimates of α are frequently large in absolute magnitude, relatively imprecise, yet appear "reliably" negative for the government and corporate bond. The annual data confirm the result that smaller and more precise estimates of α are found when the model is fit to the levels of expected returns and growth rates. The goodness of fit of the models is not improved using annual data. This may reflect in part the greater predictability of annual real returns of bonds and stocks, relative to the quarterly returns.

Table A4. Single Excess Return Tests of a Linear, Consumption-Based Asset Pricing Model with a General Form of Heteroscedasticity: Annual Data Sampled Quarterly 1948:2–1985:4 (136 Observations)

Asset	*Consumption*	*Measure*	$\hat{\alpha}$	$\hat{\sigma}(\hat{\alpha})$	χ^2	*p Value*	$\overline{PE}$
Government bond	Nondurables	(NSA)	–66.140	18.544	10.621	0.224	–0.009
		(SA)	–250.100	37.774	24.848	0.002	0.191
	Services	(NSA)	–60.369	14.214	9.266	0.320	0.010
		(SA)	–58.406	20.781	9.883	0.273	–0.014
Corporate bond	Nondurables	(NSA)	–54.261	15.668	7.771	0.456	–0.002
		(SA)	–30.873	14.750	15.654	0.048	0.013
	Services	(NSA)	–37.566	9.529	4.788	0.780	0.022
		(SA)	–47.614	19.720	8.989	0.343	–0.003
Value-weighted stocks	Nondurables	(NSA)	48.832	22.309	7.719	0.461	0.035
		(SA)	168.690	61.517	6.790	0.559	–0.082
	Services	(NSA)	80.091	21.851	2.698	0.952	–0.039
		(SA)	183.985	71.200	6.704	0.569	–0.040
Small stocks	Nondurables	(NSA)	85.998	28.890	10.966	0.204	0.035
		(SA)	132.820	30.075	11.989	0.152	–0.095
	Services	(NSA)	81.696	25.589	2.828	0.945	0.003
		(SA)	133.568	31.265	12.369	0.135	–0.044

Notes: α is the coefficient of relative risk aversion, β is the time discount factor and $\hat{\sigma}(.)$ is asymptotic standard err based on generalized method of moments estimation. The instruments are fourth to sixteenth nonoverlappi lagged values of the consumption and return variables. χ^2 is the minimized value of the GMM criteri function. *p* Value is the probability that a χ^2 variate exceeds the sample value of the statistic. $\overline{PE}$ is the avera pricing error over the year.

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NOTES

1. Breeden (1986) illustrates that asset returns should also be related to intertemporal substitut in production. Hall (1987b) empirically examines production models.

2. Miron (1986) constructs not seasonally adjusted data on 13 components of quarterly, real personal consumption expenditures for 1946–1982. We study consumption data at the higher level of aggregation that most of the previous literature has employed.

3. The mean growth rates of the adjusted and unadjusted consumption data also differ, for two reasons. First, the X-11 program changes the means. Second, the growth rates are based on different price deflators. The mean real growth rate is the mean nominal growth rate less the mean rate of inflation: $g_r = g_n - i$. An example of how the difference in means is attributed to these two factors for the overall period follows:

$$0.0025 = 0.0129 - 0.0105 \text{ (nondurables - NSA)}$$

$$0.0027 = 0.0118 - 0.0091 \text{ (nondurables - SA).}$$

$$0.0074 = 0.0179 - 0.0105 \text{ (services - NSA)}$$

$$0.0059 = 0.0163 - 0.0104 \text{ (services - SA).}$$

4. Accounting for quarterly differences in the standard deviations of the real returns, the standard errors of the quarterly means range from 0.123 to 0.188 times the overall standard deviations of the returns reported in Table 1.

5. The correlations with seasonally adjusted data are not reported. To get a feel for conditional correlations, we examine correlations of returns with the residuals from regressions of the consumption growth variables on the predetermined instruments described in the next section. Using unadjusted consumption, correlations significantly different from zero occur more frequently than in Table 2. All the conditional correlations of returns with unadjusted services are in excess of 0.195, but the conditional correlations of small stock returns with nondurables and durables are close to zero. Using seasonally adjusted consumption data, only two of the conditional correlations are (slightly) larger than two approximate standard errors.

6. Pratt (1964) provides several interpretations for values of α. One case considers an agent who is indifferent between a gamble over h fraction of wealth, winning with probability p and losing with probability $1 - p$. Choosing $h = 1\%$, Pratt's analysis implies that (to a second order approximation) an agent with log utility ($\alpha = 1.0$) requires $p = 0.50025$. With $\alpha = 10$, $p = 0.5025$; $\alpha = 100$ implies $p = 0.750$, and Pratt's approximation breaks down at $\alpha = 200$ (where p approaches 1.0).

7. We also examine maximum-likelihood estimate of α, obtained by substituting the usual maximum likelihood estimates of the means and variances into Equation (4). These estimates and their approximate standard errors reveal very similar patterns and are not reported.

8. Tauchen (1986) conducts simulations which suggest the small sample properties of some GMM estimators may suffer if excessive numbers of lagged instruments are used. We conduct some experiments to check the sensitivity of our results to using fewer lagged values (e.g., 2 or 3 lags) and do not find that the results change qualitatively. See Hansen and Singleton (1987) for an analysis of optimal instrument selection in the context of linear models.

9. Given the high correlation of nondurables and durable goods in the not seasonally adjusted data, we do not examine durables.

10. We computed 24 residual autocorrelations for each consumption regression. Only 3 of the 480 autocorrelations exceeded three approximate standard errors. Most of the equations with unadjusted consumption had 1 to 3 of the 24 autocorrelations in excess of two standard errors; with seasonally adjusted consumption, autocorrelations in excess of two standard errors were rare.

11. Hansen and Singleton (1983) also note that estimates of α are smaller and seem more precise using conditional moments than when the estimators use only unconditional moments.

12. We replicate the tests in Table 4 using nondurables *plus* services over the 1954–1978 period examined by Hansen and Singleton (1983). The results were similar to the results reported in Table 4 for nondurables. This should be expected because nondurables is a much more volatile series than services, and tends to dominate the combined series. We also replicate the tests in Table 4 for Treasury

bills using each consumption variable and data for 1955–1985. Again, the results are qualitatively very similar. To check whether observing rejections for Treasury bills but not the other assets is caused by the use of the lagged Treasury bill as an instrument, we replicate the test for the value-weighted index and unadjusted nondurables consumption, using the Treasury bill as the instrument in place of the lagged stock returns. We obtain $\hat{\alpha} = 0.06$ ($\hat{\sigma} = 0.05$) and $\chi^2 = 13.41$ (p value = 0.063).

13. These results appear to differ from results reported by Hansen and Singleton (1983, Table 5), who conduct tests as in Table 5 using maximum likelihood and monthly data for 1960–1978 for a Treasury bill return and the value-weighted stock index. They use 4 lagged values of seasonally adjusted consumption and the two asset returns as predictors in the regressions; a total of 12 regressors. They report χ^2 (24) = 170.25, a strong rejection of the model. We replicate this case, using quarterly instead of monthly seasonally adjusted data and find χ^2 (24) = 47.74 (p value = 0.003). Using not seasonally adjusted nondurables we obtain χ^2 (24) = 69.30 (p value = 0.000) using maximum likelihood. We also replicate our tests in the first panel of Table 5 using maximum likelihood and data for 1955–1985. These results are similar, for the not seasonally adjusted data, to those in Table 5. With seasonally adjusted data, we observe larger χ^2 statistics [for nondurables, we obtain χ^2 (39) = 68.90; for services, χ^2 (39) = 61.55].

14. Miron models $\delta s(t)$ in two ways. In his first model, $\delta s(t) = \Sigma_{s=2,3,4}(G_s d_{st} + G_5\, t + G_6\, t^2)$, where d_{st} is a seasonal dummy indicator, t is time, and the G_i are parameters. In his second model, $\delta s(t) = G_7 \ln(X_t^7) + G_8 \ln(X_t^8)$, where X_t^7 is a measure of temperature and X_t^8 is a measure of precipitation. His evidence suggests that the seasonal dummies largely subsume the effects of his weather variables, so we focus on the model with seasonal dummies.

15. Miron (1986) does not assume directly that the conditional variance in Equation (8) is constant. He examines a nonlinear model but assumes homoscedasticity by estimating the model using two-stage, nonlinear least squares.

16. Miron's (1986) published paper does not indicate the number of lagged instruments or the choice of asset data. In private communication, he informed us that he used two lagged values as instruments. He also indicated that he studies 3-month Treasury bill data from the Federal Reserve Bulletin. This is the monthly average of 3-month bill discount rates and is not converted to a rate of return. Miron also indicated that there is an error in the reported degrees of freedom of the test statistics in his published paper.

17. If the homoscedastic model captures the predictable variation in a real return, the conditional mean of $u_{t+1} = r_{t+1} - \alpha c_{t+1} - \psi$ should be a constant equal to zero. (The GMM tests examine this condition.) We characterize how well the model captures time variation by forming a ratio. The denominator measures the variance of an expected real return as the sample variance of the fitted values from regressing the real return on the instruments. The numerator is the sample variance of the fitted values of u_{t+1}. The complement of this fraction is a measure of the portion of the variance of an expected real return that is captured by a model. We form similar ratios using the value-weighted return in place of c_{t+1} as the explanatory "factor." The variance ratios are small (less than 3.5%), except for the small stock portfolio using unadjusted consumption. This case produces the largest variance ratio of 54.5%.

18. We also examine a model, following Amemiya (1977) and Hasbrouck (1986), where the heteroscedasticity is assumed to be linear in the instruments and is not constrained to be positive. The results of this model are very similar (see the note 21 for details).

19. Miron finds that the shift coefficients are jointly significant in other cases; we find a similar result when we conduct a test of the hypothesis that $\delta_2 = \delta_3 = \delta_4 = 0$.

20. We also examine a model with seasonal taste shifters and homoscedasticity. The parameter estimates and test statistics were very similar to those reported in Table 6. We replicate these tests for the Treasury bill and for the asset system using nondurables consumption, omitting the first lagged values of the instruments. The results are very similar. To check whether rejections are sensitive to the use of the Treasury bill as an instrument, we replicate the test for the government bond and unadjusted nondurables consumption, using the Treasury bill as the instrument in place of the lagged government

bond returns. We obtain $\hat{\alpha} = -0.30$ ($\hat{\sigma} = 0.50$) and $\chi^2 = 8.85$ (p value = 0.065), a result fairly similar to that in the table.

21. We conducted tests similar to those in Table 6 using a linear model of conditional heteroscedasticity. We omitted the seasonal taste shifters from this model. All the other parameter estimates and test results were very similar to those reported in Table 6, except that the fitted variances appear more precise. We examine time series plots of the fitted conditional variance function. The plots for the value-weighted stock index capture an increase in volatility in the mid-1970s, during the Arab oil embargo. The upper, two-standard error confidence band never crosses below zero, although the fact that variances should be positive is not imposed in this model. The graphs for the long-term bonds indicate high volatility around the 1979–1982 shift in the implementation of monetary policy by the Federal Reserve, and low volatility before the late 1960s. Only in a few quarters does the upper confidence band dip below the zero line. In the case of the Treasury bill return, where the model is rejected, the plots look less reasonable. Changes in the methods for calculating the consumer price index, combined with relatively stable short-term nominal interest rates in the early part of the sample, may influence the real returns of the Treasury bills. We replicate the tests for Treasury bills using 1955–1985 data and the linear variance model. We find very similar results. The p values of the χ^2 statistics for the Treasury bill are less than 0.001, and the point estimates of the parameters are similar to those in Table 6.

22. We replicate the tests of system (11) using seasonally adjusted data, and the results are consistent with the observations of Dunn and Singleton. The χ^2 statistic indicates rejections without the intercept, but rejections are not indicated when the intercept is included.

23. When we replicate the tests in Table (7) using seasonally adjusted data we find that the point estimates and standard errors of α are frequently, but not always, larger in magnitude than with unadjusted data. This is similar to the findings in Table 3.

24. We estimated a model involving the level of real returns which incorporated a general approximation for the conditional variance. This model identified the time preference parameter, and it produced point estimates and standard errors of α that were fairly similar to the ones obtained in Tables 4–6 with seasonally adjusted data. Although the model did not produce large values of the test statistics, evidence that it did not fit the levels of expected returns very well was observed in the form of "reliably" negative estimated rates of time preference.

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