

TIME-VARYING CONDITIONAL COVARIANCES IN TESTS OF ASSET PRICING MODELS

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This paper proposes tests of asset pricing models that allow for time variation in conditional covariances. The evidence indicates that the conditional covariances do change through time. Estimates of the expected excess return on the market divided by the variance of the market (reward-to-risk ratio) are presented for the Sharpe–Lintner CAPM, as well as a number of tests of the model specification. The patterns of the pricing errors through time suggest the model's inability to capture the dynamic behavior of assets returns.

1. Introduction

Many asset pricing models (CAPMs) impose the restriction that expected returns are linearly related to asset risk. The conditional version of the Sharpe (1964) and Lintner (1965) CAPM imposes the restriction that conditionally expected returns on assets are linearly related to the conditionally expected return on a market-wide portfolio in excess of a risk-free return. The coefficient in the linear relation is the asset's *beta* or the ratio of the conditional covariance of the asset's return with the market to the conditional variance of the market.

Tests of the unconditional version of the CAPM such as Black, Jensen, and Scholes (1972), Fama and MacBeth (1973), Gibbons (1982), and Stambaugh (1982) assume that expected returns are constant, the market portfolio is observable, and assets' *betas* are stationary over a fixed period. Cross-sectional tests are conducted by regressing unconditional expected returns on unconditional *betas*. These tests are carried out on five- or ten-year subperiods because

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of concern that joint distribution of the returns is not constant over longer periods.

Gibbons and Ferson (1985), Rayner (1986), Ferson, Kandel, and Stambaugh (1987), and Ferson (1988) test the asset pricing models at the conditional level and allow for expected returns to vary through time. However, all of these studies assume that the conditional covariances are constant.

Time variation in conditional covariances has been modeled with the autoregressive-conditional heteroskedasticity in the mean model (ARCH-M) of Engle, Lilien, and Robbins (1987). Bollerslev, Engle, and Wooldridge (1988), Bodurtha and Mark (1988), and Ng (1988) carry out tests of the Sharpe–Lintner specification by modeling the conditional covariances as a function of past conditional covariances. A disadvantage of this approach is the strong assumption made about the functional form of the second moments. Another disadvantage is that ARCH processes do not, in general, aggregate. That is, if the asset returns follow a particular ARCH process, it does not necessarily follow that a portfolio of assets follows that ARCH process.¹

Following the instrumental variables approach of Campbell (1987), this paper proposes tests of the CAPM and a multifactor asset pricing model that allow for both time-varying expected returns and conditional covariances. A structure is imposed on the conditional first moments of the asset returns. The conditional covariances are approximated by the product of the innovations from linear projections of the asset returns and the factors onto the information set. This approach has a number of advantages over the ARCH model. First, the evolution of the instrumental variables need not be modeled. Second, the procedure is robust to nonnormality of the errors. Further, the ARCH models (to date) have considered only one covariance (*i.e.*, tests of the CAPM). Although my results consider only tests of the CAPM, I provide an econometric framework to implement tests of a multifactor model. In the multifactor model, the k covariances (associated with the k risk premiums) are allowed to vary through time.

The paper is organized as follows. In the second section, the methodology is outlined. Section 3 describes the data and presents estimates of what Merton (1980) calls the ‘reward-to-risk ratio’ for the Sharpe–Lintner model. Tests of the restrictions implied by the asset pricing model are also provided. Finally, a very general version of the model is tested in which the conditionally expected returns, covariances, and variances are allowed to change through time. Some concluding remarks are offered in the final section.

¹Pagan and Hong (1988) provide a critique of the ARCH modeling. Shanken (1989) provides another specification in which he assumes that the conditional betas are linear in a set of instrumental variables.

2. Methodology

2.1. Testing CAPM with constant reward to variability

Let Ω_{t-1} represent the information at $t-1$ that investors use to set prices. The conditional version of the Sharpe–Lintner capital asset pricing model implies

$$E[r_{jt}|\Omega_{t-1}] = \frac{E[r_{mt}|\Omega_{t-1}]}{\text{var}[r_{mt}|\Omega_{t-1}]} \text{cov}[r_{jt}, r_{mt}|\Omega_{t-1}], \quad (1)$$

where r_j is the return on asset j in excess of the Treasury bill rate and r_m is the excess return on the market portfolio. There are many ways to formulate a test of this model. It is not obvious which term – if any – should be held constant. Perhaps the most unlikely candidate for a constant is the asset's conditional *beta*. A firm's risk may change as new projects are undertaken and numerous studies have shown that unconditional estimates of the *beta* change through time.

Suppose the conditionally expected excess return on the asset is proportional to its conditional covariance with the market factor. Since the true information set, Ω_{t-1} , is not available, we must condition on the observed information, Z_{t-1} . The restriction implied by the CAPM² is

$$E[r_{jt}|Z_{t-1}] = \lambda \text{cov}[r_{jt}, r_{mt}|Z_{t-1}]. \quad (2)$$

The assumed constant, λ , is the ratio of the conditionally expected return on the market portfolio divided by the conditional variance of the market. The λ coefficient represents the compensation the representative investor must receive for a unit increase in the variance of the market return.³ This constant is in terms of a market-wide portfolio, not an individual asset.

To test the restrictions implied by the theory, we need models of the conditional moments. If we assume that the joint distribution of the asset returns and the instrumental variables falls into the general class of *spherically invariant distributions* [see Vershik (1964), Blake and Thomas (1968), and Bertsekas (1976)], then the conditional expectation of the asset return will be a

²This is only an approximation because (omitting the time subscripts) $E[\text{cov}(r_j, r_m|\Omega)|Z] = \text{cov}(r_j, r_m|Z) - \text{cov}(E[r_j|\Omega], E[r_m|\Omega]|Z)$. The covariance between the true conditional means is fundamentally unobservable. In the empirical work, it is assumed to be zero or a constant.

³This is often interpreted as the Arrow–Pratt relative risk aversion for wealth for the representative investor [see Merton (1980, p. 329)]. However, this interpretation is valid only under strong assumptions about the consumption process, e.g., i.i.d. consumption. Hall (1978) and Harvey (1988) provide evidence suggesting this assumption is not supported by the data.

linear function of the instrumental variables. The econometric relation implied is

$$u_t = r_t - Z_{t-1}\delta, \quad (3)$$

where u_t is a row vector of n forecast errors that is orthogonal to the vector of l instrumental variables, Z_{t-1} , r_t is a vector of n asset returns, and δ is a $l \times n$ coefficient matrix.

Given the constant reward-to-risk assumption, the CAPM implies that

$$E[r_t|Z_{t-1}] = \lambda E[(r_t - E[r_t|Z_{t-1}])(r_{mt} - E[r_{mt}|Z_{t-1}])|Z_{t-1}]. \quad (4)$$

Define the disturbance vector:

$$e_t = r_t - \lambda(r_t - E[r_t|Z_{t-1}])(r_{mt} - E[r_{mt}|Z_{t-1}]). \quad (5)$$

By substituting in (3), the disturbances in (5) are reexpressed as

$$e_t = r_t - \lambda(r_t - Z_{t-1}\delta)(r_{mt} - Z_{t-1}\delta_m). \quad (6)$$

The error terms in (3) and (6) are stacked into a system to form the following econometric model:

$$\varepsilon_t = (u_t \quad e_t) = \begin{pmatrix} [r_t - Z_{t-1}\delta]' \\ [r_t - \lambda(r_{mt} - Z_{t-1}\delta_m)(r_t - Z_{t-1}\delta)]' \end{pmatrix}. \quad (7)$$

This formulation is closely related to Campbell's (1987) specification. The model implies that $E[\varepsilon_t|Z_{t-1}] = 0$. With n assets, there are $n + 1$ columns in u (includes the market return) and n columns in e . If there are l instrumental variables, there are $[l \times (2n + 1)]$ orthogonality conditions and $[1 + l \times (n + 1)]$ parameters to estimate.

Hansen's (1982) generalized method of moments (GMM) provides a natural way to estimate the parameters of system (7). The GMM forms a vector of the orthogonality conditions:

$$g = \text{vec}(\varepsilon'Z), \quad (8)$$

where ε is the matrix of forecast errors (e, u) for T observations and $2n + 1$ equations, and Z is a $T \times l$ matrix of observations on the predetermined instrumental variables. Parameters (δ, λ) are chosen to make the orthogonality conditions as close to zero as possible by minimizing the quadratic form, $g'wg$, where the w is a symmetric weighting matrix that defines the metric used to

make g close to zero. Hansen (1982) outlines a form of the weighting matrix that guarantees that the estimates are consistent and asymptotically normal.

The minimized value of this quadratic form is distributed χ^2 under the null hypothesis with degrees of freedom equal to the number of orthogonality conditions minus the number of parameters to be estimated. This χ^2 statistic (known as the test of the *overidentifying* restrictions) provides a goodness-of-fit test for the model. A high χ^2 statistic means that the disturbances are correlated with the instrumental variables. This means, of course, that the model is misspecified.

A second possible test is to estimate the alternative system:

$$e_t = (u_t \quad e_t)' = \begin{pmatrix} [r_t - Z_{t-1}\delta]' \\ [r_t - (r_{mt} - Z_{t-1}\delta_m)(r_t - Z_{t-1}\delta)\text{diag}(\lambda)]' \end{pmatrix}, \quad (9)$$

where $\text{diag}(\lambda)$ is an $n \times n$ zero matrix with asset-specific λ coefficients in the main diagonal. The difference between (7) and (9) is the inclusion of asset-specific λ coefficients. The pricing model suggests that these coefficients should be equal across all assets. The null hypothesis that $\lambda_j = \lambda$ for all j can be tested by forming the following statistic:⁴

$$\chi_q^2 = g_r' w_{ur} g_r - g_{ur}' w_{ur} g_{ur}, \quad (10)$$

where r represents restricted, ur represents unrestricted, and q is the number of parameter restrictions (degrees of freedom).

A third test of specification is to estimate the following system:

$$e_t = (u_t \quad e_t)' = \begin{pmatrix} [r_t - Z_{t-1}\delta]' \\ [r_t - \alpha - \lambda(r_{mt} - Z_{t-1}\delta_m)(r_t - Z_{t-1}\delta)]' \end{pmatrix}, \quad (11)$$

where α is a $1 \times n$ coefficient vector. The difference between (7) and (11) is the inclusion of an asset-specific intercept. One could consider this test a multivariate instrumental variables-based analogue to Jensen's (1969) test where the CAPM implies $\alpha_j = 0$ for all j assets.

2.2. Testing multifactor model with constant reward to volatility

The work of Merton (1973), Long (1974), and Ross (1976) motivates the examination of a model that allows for more than one source of risk. The test of the CAPM can be viewed as a test of the conditional mean variance

⁴See Gallant and Jorgenson (1979), Eichenbaum, Hansen, and Singleton (1988), Eichenbaum and Hansen (1987), and Newey and West (1987) for discussions of this multivariate test.

efficiency of the market portfolio [see Fama (1976), Roll (1977), and Ross (1977)]. Similarly, with a multifactor asset pricing model, the test is equivalent to testing the conditional mean-variance efficiency of a combination of specified factors [see Chamberlain (1983), Grinblatt and Titman (1987), Huberman, Kandel, and Stambaugh (1987), Kandel and Stambaugh (1987), and Hansen and Richard (1987)]. Consider a k -factor pricing relation:

$$E[r_t|Z_{t-1}] = \lambda \text{cov}[r_t, f_t|Z_{t-1}], \quad (12)$$

where r represents the asset return in excess of a reference asset's return, λ is now a k -dimensional row vector of coefficients, and f are the k factors relevant for asset pricing. The f must be prespecified as in Chen, Roll, and Ross (1986). If the joint distribution of the factors and the instrumental variables falls into the class of spherically invariant distributions, the conditional expectation of these factors is linear in the instrument set. Then we can write the following econometric relation:

$$v_t = f_t - Z_{t-1}\gamma, \quad (13)$$

where v_t is a k -vector of errors and γ is $l \times k$ coefficient matrix.

As before, we can stack all the disturbances in one large system:

$$\eta_t = (u_t \quad v_t \quad w_t) = \begin{pmatrix} [r_t - Z_{t-1}\delta]' \\ [f_t - Z_{t-1}\gamma]' \\ [r_t - ((f_t - Z_{t-1}\gamma)\lambda')(r_t - Z_{t-1}\delta)]' \end{pmatrix}. \quad (14)$$

The restrictions implied by the model are $E[\eta_t|Z_{t-1}] = 0$. There are $(2n + k) \times l$ orthogonality conditions and $[k + (n + k) \times l]$ parameters to be estimated.

2.3. Asset pricing with time-varying covariances, returns, and volatilities

It is not necessary to assume that any quantity in (1) is constant. In a general form of the model, it is possible to allow for time variation in the conditionally expected returns on the asset, the conditional covariances, the conditional variances of the market, and the conditionally expected returns on the market. Even with all of these moments changing through time, the pricing model imposes cross-sectional restrictions on the asset returns.

Multiplying both sides of (1) by the conditional variance of the market excess return implies:

$$\begin{aligned} 0 = E[& (r_{mt} - E[r_{mt}|Z_{t-1}])^2 | Z_{t-1}] E[r_{jt}|Z_{t-1}] \\ & - E[(r_{mt} - E[r_{mt}|Z_{t-1}]) (r_{jt} - E[r_{jt}|Z_{t-1}]) | Z_{t-1}] E[r_{mt}|Z_{t-1}]. \end{aligned} \quad (15)$$

As in (3), let the conditionally expected returns be linear in the instrumental variables. Eq. (15) can be written as

$$0 = E[u_{mt}^2 Z_{t-1} \delta_j | Z_{t-1}] - E[u_{jt} u_{mt} Z_{t-1} \delta_m | Z_{t-1}], \quad (16)$$

where u_{mt} and u_{jt} are the innovations in the conditional mean of the market and asset j 's excess returns. Notice the conditionally expected returns on the asset and market are moved inside the expectation because they are known conditional on Z_{t-1} .

Define the disturbance:

$$h_t = u_{mt}^2 [Z_{t-1} \delta] - u_{mt} u_{jt} [Z_{t-1} \delta_m], \quad (17)$$

where u represents a vector of innovations in the conditional means of the asset excess returns. The conditional expectation of (17) delivers the capital asset pricing model in (1).⁵ The disturbances in (3) and (17) are combined into a system of equations and the parameters can be estimated using the method of moments:

$$\varepsilon_t = \begin{pmatrix} u_t & u_{mt} & h_t \end{pmatrix} = \begin{pmatrix} [r_t - Z_{t-1} \delta]' \\ [r_{mt} - Z_{t-1} \delta_m]' \\ [u_{mt}^2 Z_{t-1} \delta - u_{mt} u_{jt} Z_{t-1} \delta_m]' \end{pmatrix}. \quad (18)$$

In testing the model with constant betas or constant reward to volatility, it is difficult to interpret a rejection of the model. The model may be an adequate description of the data but the assumption of a constant parameter may be incorrect. In (18), everything is time-varying.⁶

3. Data and empirical results

3.1. Data

The asset return data are the market-value-sorted New York Stock Exchange (NYSE) monthly stock portfolio deciles compiled by the Center for Research in Security Prices (CRSP) at the University of Chicago. The portfolios are rebalanced each year on the basis of market value. The market portfolio is the value-weighted NYSE index. The excess return on a portfolio

⁵I am grateful to John Campbell for suggesting this test. Roger Huang (1989) has independently explored this specification.

⁶However, everything is time-varying in a particular way determined by the assumption of functional form on the conditional means. The issue of functional form for the conditional means and variances is explored in Harvey (1989).

in month j is calculated by subtracting the return on a Treasury bill that is closest to 30 days to maturity at the end of month $j - 1$. The bill data are drawn from the Government Bond File available from CRSP.

The information set includes the first lag of the excess return on the equally-weighted NYSE portfolio, the junk bond premium, a dividend yield measure, and a term premium. A constant and a dummy variable for January are also included. The junk bond premium is the difference in yields on a portfolio of Moody's BAA-rated bonds and a portfolio of Moody's AAA-rated bonds. The dividend yield spread is the difference between the yield on the Standard and Poor's 500 stock index and the yield on the one-month bill.⁷ Finally, the term premium is the difference in returns for holding a 90-day bill and a 30-day bill for one month. The data set contains 554 observations from September 1941 to December 1987.⁸

3.2. *Summary statistics and asset return regressions*

Summary statistics for the asset excess returns and the instruments are provided in table 1. Over the September 1941 to December 1987 period, there is a near-monotonic decrease in (unconditional) mean returns moving from the smallest decile portfolio to the largest portfolio ranked on total firm value in panel A. The same pattern is found in the standard deviations. The unconditional autocorrelations are also presented. Consistent with the evidence presented in other studies,⁹ the highest first-order autocorrelations are found with the mid to smallest deciles. The seasonal pattern for the smaller stocks documented by Keim (1983) is also evident in the twelfth-order autocorrelations.

Panel B of table 1 provides some summary statistics for the value-weighted excess return and the instrumental variables. The instruments have been chosen to reflect two considerations. First, the variables must be able to summarize expectations in the economy that are related to the prospects for stock returns (that is, they should be able to predict asset returns). For example, Keim and Stambaugh (1986) find that the junk bond has some explanatory power for excess stock returns. Fama and French (1988, 1989) document the explanatory power in the dividend yield, the long-term bond premium, and the junk bond premium. Campbell (1987) shows that term

⁷The early data for the bond yields are drawn from Moody's *Industrial Manual*. The dividend yields are from the Standard and Poor's *Current Statistics*. The recent data also appear in table 1.35 of the *Federal Reserve Bulletin*.

⁸The data set begins in September 1941 because this is the first month in which a three-month bill was consistently available at the end of the month. Also, there is a change in tax status for the bills beginning in 1941. Cecchetti (1988) discusses this tax change and provides a detailed analysis of the early fixed-income data.

⁹See, for example, Keim and Stambaugh (1986, table 1).

Table 1

Summary statistics for the asset excess returns^a and the instrumental variables^b based on monthly data from September 1941 to December 1987 (554 observations).

Variable	Mean	Std. dev.	Autocorrelation					
			ρ_1	ρ_2	ρ_3	ρ_4	ρ_{12}	ρ_{24}
<i>Panel A: All NYSE common stocks (ranked from smallest to largest total firm value)</i>								
Decile 1	0.01395	0.07514	0.123	−0.013	0.027	0.046	0.212	0.060
Decile 2	0.01189	0.06222	0.131	0.031	0.011	0.046	0.150	0.041
Decile 3	0.01106	0.05858	0.131	0.008	0.015	0.043	0.112	0.022
Decile 4	0.01040	0.05447	0.160	0.021	0.005	0.056	0.077	0.014
Decile 5	0.00945	0.05218	0.123	0.010	0.023	0.042	0.064	0.019
Decile 6	0.00955	0.05093	0.132	0.013	0.004	0.042	0.024	0.006
Decile 7	0.00913	0.04906	0.123	0.018	0.020	0.047	0.013	−0.014
Decile 8	0.00819	0.04714	0.098	−0.002	0.006	0.027	0.010	−0.003
Decile 9	0.00800	0.04493	0.097	−0.009	0.013	0.050	0.010	−0.011
Decile 10	0.00619	0.04031	0.045	−0.008	0.043	0.065	0.050	0.007
<i>Panel B: Instrumental variables</i>								
Value-weighted index	0.00697	0.04195	0.077	−0.005	0.035	0.057	0.030	0.000
Equally-weighted index	0.00977	0.05200	0.140	0.010	0.019	0.049	0.069	0.007
Term premium	0.00048	0.00100	0.287	0.095	0.086	0.082	0.011	0.096
Junk bond premium	0.00078	0.00038	0.977	0.945	0.921	0.902	0.713	0.511
Dividend yield spread	0.00024	0.00296	0.982	0.968	0.955	0.943	0.882	0.830

^a The decile returns are in excess of the holding-period return on the Treasury bill that is closest to 30 days to maturity.

^b The instrumental variables are: the excess return on the value-weighted NYSE index, the excess return on the equally-weighted NYSE index, the yield on Moody's BAA-rated bonds less the yield on Moody's AAA-rated bonds (junk bond premium), the return for holding a 90-day bill for one month less the return on a 30-day bill (term premium), and the dividend yield on the Standard and Poor's 500 stock index less the return on a 30-day bill (dividend yield spread).

premiums in bill returns are able to predict stock returns. Second, the number of instruments is kept small because the parameter space gets large when the cross-equation restrictions are tested.

The results of the ordinary least squares (OLS) regressions are reported in table 2. Tests of the residuals for heteroskedasticity (not reported) provide evidence against the null hypothesis of conditional homoskedasticity for some of the portfolios.¹⁰ As a result, all reported standard errors are corrected for conditional heteroskedasticity [White (1980)].

The explanatory power reported in table 2 is consistent with other studies. For example, using a number of term structure variables, Campbell (1987) reports an unadjusted R^2 of 11.2% for the excess return on the value-weighted index over the May 1959 to August 1979 period. Over the same sample, the instrumental variables used in table 2 deliver an unadjusted R^2 of 13.4%.

¹⁰ Schwert (1988a) explores the causes of conditional heteroskedasticity in stock returns.

Table 2

Regressions of excess returns^a for all NYSE common stocks (ranked by firm size) on the instrumental variables^b based on monthly data from September 1941 to December 1987 (554 observations).

$$r_{j,t} = \delta_0 + \delta_1 xew_{t-1} + \delta_2 jan_t + \delta_3 xh3_{t-1} + \delta_4 junk_{t-1} + \delta_5 xdiv_{t-1} + \epsilon_t.$$

Portfolio	δ_0	δ_1	δ_2	δ_3	δ_4	δ_5	In-sample \bar{R}^2	Out-of-sample \bar{R}^{2c}
Decile 1	-0.024 (0.009)	0.142 (0.081)	0.091 (0.018)	6.746 (2.742)	31.035 (12.167)	5.316 (1.539)	0.179	0.120
Decile 2	-0.018 (0.007)	0.114 (0.055)	0.064 (0.013)	7.692 (2.456)	24.429 (8.688)	4.333 (1.124)	0.149	0.108
Decile 3	-0.017 (0.006)	0.104 (0.050)	0.052 (0.011)	7.561 (2.393)	23.307 (7.971)	4.237 (1.031)	0.131	0.100
Decile 4	-0.014 (0.006)	0.100 (0.046)	0.038 (0.010)	8.251 (2.365)	19.880 (7.344)	3.926 (0.945)	0.110	0.079
Decile 5	-0.014 (0.006)	0.072 (0.043)	0.034 (0.009)	9.008 (2.338)	18.600 (6.802)	4.018 (0.898)	0.107	0.082
Decile 6	-0.014 (0.005)	0.059 (0.042)	0.027 (0.009)	(8.310) (2.289)	(20.041) (6.437)	(4.057) (0.876)	0.094	0.068
Decile 7	-0.013 (0.005)	0.046 (0.041)	0.018 (0.008)	8.554 (2.370)	18.696 (6.536)	4.086 (0.851)	0.084	0.059
Decile 8	-0.011 (0.005)	0.019 (0.040)	0.016 (0.008)	8.818 (2.324)	15.731 (6.143)	3.750 (0.798)	0.075	0.047
Decile 9	-0.009 (0.005)	0.010 (0.039)	0.010 (0.007)	9.059 (2.482)	14.339 (6.060)	3.997 (0.787)	0.080	0.052
Decile 10	-0.007 (0.004)	0.005 (0.039)	0.000 (0.007)	7.749 (2.534)	11.001 (5.487)	3.562 (0.665)	0.067	0.032
Val. weight	-0.009 (0.004)	0.017 (0.038)	0.007 (0.007)	8.234 (2.436)	12.797 (5.608)	3.690 (0.703)	0.075	0.044

^aAll asset returns are in excess of the holding-period return on the Treasury bill that is closest to 30 days to maturity. 'Val. weight' is the excess return on the value-weighted NYSE stock index.

^bThe instrumental variables are: the excess return on the equally-weighted NYSE index (*xew*), a dummy variable for January (*jan*), the return for holding a 90-day bill for one month less the return on a 30-day bill (term premium or *xh3*), the yield on Moody's BAA-rated bonds less the yield on Moody's AAA-rated bonds (junk bond premium or *junk*), and the dividend yield on the Standard and Poor's 500 stock index less the return on a 30-day bill (dividend yield spread or *xdiv*). Heteroskedasticity-consistent standard errors in parentheses [White (1980)].

^cOut-of-sample coefficient of determination calculated from one-step-ahead forecasts from January 1952 to December 1987. Parameters are reestimated at every step (431 forecasts).

Fama and French (1989) report an adjusted R^2 of 4% for the return on the value-weighted index in the 1941–1986 period using the dividend yield and a junk bond premium return as instruments. The difference between their 4% R^2 and the 7.5% R^2 reported in table 2 is primarily due to the inclusion of the term premium variable which Campbell shows has considerable power to predict stock returns.

I examined, but did not include in the final instrument set, two additional variables. The first was a dummy variable for the month of February. This entered the regressions with a small negative coefficient and was greater than one standard error from zero in six of the eleven regressions but never greater than 1.8 standard errors from zero. A second variable was a measure of the conditional volatility of the market. I forecast the value-weighted stock return using the five instrumental variables from September 1941 to December 1951 and obtained the fitted values. I then rolled the regression through December 1987, obtaining out-of-sample forecasts. Next, out-of-sample residuals were formed. These residuals were squared, and I followed the same rolling procedure to forecast the squared residuals. The out-of-sample forecasts are estimates of conditional volatility. I included this measure as an additional instrument. It did not enter any of the regressions more than two standard errors from zero. This is consistent with Schwert and Seguin (1989), who do not find a strong relation between conditional mean excess returns on size portfolios and a measure of conditional volatility derived from daily returns data.

Table 2 reports that the greatest explanatory power is found with the smallest decile (17.9%) and the least with decile 10 (6.7%). The value-weighted stock regression is able to explain 7.5% of the variation in the returns. There is no significant residual autocorrelation in any of the regression models. I checked for stability of the regression parameters using a Chow test (not reported) with a breakpoint of August 1979. For the eleven portfolios, there were no p -values less than 0.25. The degree of explanatory power was similar across the two subperiods. For example, the smallest stock portfolio has an R^2 of 17.9% in the full sample, while the first-subperiod R^2 is 18.6% and the second-subperiod R^2 is 15.9%.

To evaluate the out-of-sample forecasting performance of these models, one-step-ahead forecasts are obtained for January 1952 to December 1987, with the models' parameters being reestimated at every step in time. Interestingly, the out-of-sample forecast of the market excess return for October 1987 is negative. In the far right column, the out-of-sample R^2 s are reported for each of the portfolios. The similarity of the in sample and out-of-sample R^2 s suggests that the choice of instrumental variables is satisfactory.

3.3. *A test for time-varying conditional covariances*

The specification in (7) is well motivated only if the conditional covariances of the asset excess returns with the market excess returns change through time. Table 3 tests the null hypothesis that the conditional covariances are constant. The product of the regression residuals for the asset excess return and the market excess return is regressed on the instrumental variables. If the null hypothesis is true, only the intercept should be significantly different from zero.

Table 3

Regressions of the product of the return residuals^a for all NYSE common stocks (ranked by firm size) and the value-weighted NYSE index residuals on the instrumental variables^b from September 1941 to December 1987 (554 observations).

$$u_{jt}u_{mt} = \sum_{i=1}^I \alpha_i Z_{i,t-1} + \eta_{j,t}, \quad j = 1, \dots, 10.$$

Decile	\bar{R}^2	χ^2 ^c	P-value
1	0.026	13.894	0.016
2	0.038	23.389	< 0.001
3	0.037	24.646	< 0.001
4	0.039	25.364	< 0.001
5	0.040	29.697	< 0.001
6	0.038	29.100	< 0.001
7	0.048	33.873	< 0.001
8	0.051	34.623	< 0.001
9	0.052	37.309	< 0.001
10	0.054	33.955	< 0.001

^aAll asset returns are in excess of the holding-period return on the Treasury bill that is closest to 30 days to maturity.

^bThe instrumental variables are: a constant, the excess return on the equally-weighted NYSE index, a dummy variable for January, the return for holding a 90-day bill for one month less the return on a 30-day bill, the yield on Moody's BAA-rated bonds less the yield on Moody's AAA-rated bonds, and the dividend yield on the Standard and Poor's 500 stock index less the return on a 30-day bill.

^cHeteroskedasticity-consistent test of the restriction that all parameters (except the intercept) are equal to zero. There are five degrees of freedom. P-value is the probability that the variate exceeds the sample value of the statistic.

There is strong evidence against the null hypothesis. The R^2 s in table 3 indicate that the covariances are time-varying and predictable. A heteroskedasticity-consistent chi-square statistic tests the null hypothesis that all the parameters (except the intercept) are equal to zero, and the size of the statistic is evidence against the null. These results are consistent with the findings of Schwert and Seguin (1989), who also document strong evidence against the hypothesis of constant conditional covariances, using a single instrument based on the expected monthly sample standard deviations of daily stock returns.

3.4. The ratio of market return to volatility

Tests of (7) assume that the ratio of the conditionally expected return on the market divided by the conditional variance of the market is constant through time. This assumption can be tested by forming the system

$$\epsilon_t = \begin{pmatrix} u_{mt} & e_{mt} \end{pmatrix} = \begin{pmatrix} [r_{mt} - Z_{t-1}\delta_m]' \\ [r_{mt} - \lambda(r_{mt} - Z_{t-1}\delta_m)^2]' \end{pmatrix}. \quad (19)$$

This system has $l + 1$ parameters and $2 \times l$ orthogonality conditions. If the risk-to-reward parameter is constant, the forecast errors should be uncorrelated with the elements in the information set. Hence, it is possible to interpret the test of the overidentifying restrictions as a test of whether the λ is constant in (19). Another test is to include an intercept in the second part of (19). If the λ is constant, the intercept should be indistinguishable from zero. These two tests of (19) are also special cases of (7) and (11) and hence tests of the asset pricing model's restrictions. Campbell (1987, table 7) carries out these tests for the value-weighted index and finds evidence against the model's restrictions.

It is straightforward to modify (19) to test whether the 'cost of risk' (expected excess return on the market divided by the standard deviation of the market return), λ^* , is constant. An alternate system is

$$\varepsilon_t = (u_{mt} \quad e_{mt}) = \begin{pmatrix} [r_{mt} - Z_{t-1}\delta_m]' \\ [r_{mt} - \lambda^*|r_{mt} - Z_{t-1}\delta_m|] \end{pmatrix}. \quad (20)$$

This system has the same number of parameters and orthogonality conditions as (19).

Davidian and Carroll (1987) suggest an alternative 'robust' estimation of variance and standard deviation functions. First, the conditional mean of the market return is estimated. Second, the absolute residuals are regressed on the instruments. The fitted values from this regression are used as deflators in a weighted least squares regression for the conditional means. The WLS parameters are used to form residuals. Finally, the absolute values of these residuals are regressed on the information variables. The squares of the fitted values, $\hat{\sigma}_{mt}^2$, are the robust estimates of the conditional volatility.¹¹ The following equations are estimated:

$$\varepsilon_t = r_{mt} - \lambda \hat{\sigma}_{mt}^2 \quad (21)$$

and

$$\varepsilon_t = r_{mt} - \lambda^* \hat{\sigma}_{mt}. \quad (22)$$

Both equations have one parameter and imply six orthogonality conditions, leaving five overidentifying restrictions.

One can consider estimates of (19) and (20) as the instrumental-variables counterpart to a recent study by French, Schwert, and Stambaugh (1987). Using a monthly volatility measure calculated from daily returns on the Standard and Poor's 500 index, French, Schwert, and Stambaugh estimate a

¹¹Following Schwert (1988a,b), the absolute residuals are multiplied by $\sqrt{\pi/2}$ because the expected value of the absolute residual is less than the standard deviation from a normal distribution.

'reward-to-risk' parameter of 7.8 and a cost-of-risk parameter of 0.7 over the 1953–1984 period. Their estimates are slightly less than two standard errors from zero. The results in table 4 indicate that these parameters may be smaller than those reported by French, Schwert, and Stambaugh. The point estimate for the reward-to-risk measure is 5.27 and is greater than four standard errors from zero. The point estimate of the Sharpe measure or cost of risk is 0.27, which is three standard errors from zero. The point estimates from the robust estimation are slightly smaller but broadly consistent with estimates of (19) and (20).

The test of the overidentifying restrictions gives evidence against the constant reward to variance and the constant reward to standard deviation assumption. There is also evidence that an intercept is needed in the relation between expected return and volatility. Kandel and Stambaugh (1989) provide graphic evidence that the reward–risk ratio varies through time. Harvey (1989) extends the tests presented in table 4 by considering a nonparametric density estimation technique for obtaining conditional expectations. These results are consistent with those in table 4.

Unfortunately, it is difficult to tell from the χ^2 statistic whether a constant reward-to-risk parameter is an adequate working assumption. As a result, tests of (7) and (11), which assume constant reward to variability are carried out, as well as tests of (18), which do not assume any quantity is constant.

3.5. *Single portfolio tests with constant reward to variance*

Although the tests of the overidentifying restrictions presented for eq. (7) refer to a system of assets, similar tests can be carried out on individual assets. There are three equations. The first determines the asset excess return forecast error. The market excess return forecast error is the second equation, and the forecast error from the CAPM is the third. There are 13 parameters to be estimated and 18 orthogonality conditions, leaving five overidentifying restrictions to be tested.

Table 5 presents the estimates of the reward-to-risk parameter and the tests of the overidentifying restrictions. The estimates of the reward-to-risk coefficient range from 4.73 to 11.88 from the largest to smallest decile portfolios. The magnitude of the estimate for the larger size deciles is consistent with the results reported in table 4 for the value-weighted index of all NYSE stocks. Each estimate is at least four standard errors from zero.

One feature of these parameter estimates is inconsistent with the pricing model, however – there is a monotonic decline in the size of the reward–risk coefficients moving from smaller to larger firms. This coefficient should not be asset-specific. Given that we can roughly interpret the reward–risk coefficient as reflecting relative risk aversion, a clientele effect could explain differences in the coefficients across the different portfolios. But it is hard to explain why the most risk-averse investors would be choosing the riskiest stocks. A formal test

Table 4

Generalized method of moments test of whether the reward-to-risk or the cost-of-risk parameter is constant.

$$e_t = (u_{mt} \quad e_{mt})' = \begin{pmatrix} [r_{mt} - Z_{t-1}\delta_m]' \\ [r_{mt} - \lambda(r_{mt} - Z_{t-1}\delta_m)^2]' \end{pmatrix}, \quad (19)$$

$$e_t = (r_{mt} - \lambda\delta_{mt}^2), \quad (21)$$

$$e_t = (u_{mt} \quad e_{mt})' = \begin{pmatrix} [r_{mt} - Z_{t-1}\delta_m]' \\ [r_{mt} - \lambda^*|r_{mt} - Z_{t-1}\delta_m|]' \end{pmatrix}, \quad (20)$$

$$e_t = (r_{mt} - \lambda^*\delta_{mt}), \quad (22)$$

The reward-to-risk parameter is λ . r_m denotes the excess return on the value-weighted index of NYSE common stocks^a and Z represents the instrumental variables.^b δ^2 and δ are the robust estimates of the variance and standard deviation. The estimation period is September 1941 to December 1987 (554 observations).

Parameter held constant	λ	Test 1 ^c	Test 2 ^d
Reward-to-risk or excess return/variance (19)	5.2743 (1.1662)	< 0.001	< 0.001
Reward-to-risk or excess return/variance (21) [Robust variance estimation]	4.6869 (1.0867)	< 0.001	0.007
Cost-of-risk or excess return/standard deviation (20)	0.2697 (0.0559)	< 0.001	0.024
Cost-of-risk or excess return/standard deviation (22) [Robust std. dev. estimation]	0.2025 (0.0441)	< 0.001	0.077

^aThe value-weighted NYSE stock index, r_{mt} , is in excess of the holding-period return on the Treasury bill that is closest to 30 days to maturity.

^bThe instrumental variables are: a constant, the excess return on the equally-weighted NYSE index, a dummy variable for January, the return for holding a 90-day bill for one month less the return on a 30-day bill (term premium), the yield on Moody's BAA-rated bonds less the yield on Moody's AAA-rated bonds (junk bond premium), and the dividend yield on the Standard and Poor's 500 stock index less the return on a 30-day bill (dividend yield spread).

^cTest 1 is the probability that a χ^2 variate (minimized value of the GMM criterion function) exceeds the sample value of the statistic. In the first and third systems, there are seven parameters and twelve orthogonality conditions, which implies there are five overidentifying restrictions to be tested. The second and fourth models have only one equation. There is one parameter to estimate and six orthogonality conditions, leaving five overidentifying restrictions.

^dTest 2 is a heteroskedasticity-consistent test of whether a constant term is equal to zero.

of the null hypothesis that these parameters are equal is presented in the next section.

Table 5 also provides the average conditional covariance for each decile. These are not the unconditional covariances, because they are derived from conditioning information that includes variables in addition to a constant. Since there is a near-monotonic decline in the average excess returns on the

Table 5

The reward-to-risk parameter and the average pricing errors resulting from generalized method of moments estimation of a conditional version of the Sharpe–Lintner CAPM.

$$e_t = (u_{jt} \quad u_{mt} \quad e_{jt})' = \begin{pmatrix} [r_{jt} - Z_{t-1}\delta_j]' \\ [r_{mt} - Z_{t-1}\delta_m]' \\ [r_{jt} - \lambda(r_{mt} - Z_{t-1}\delta_m)(r_{jt} - Z_{t-1}\delta_j)]' \end{pmatrix}, \quad j = 1, \dots, 10. \quad (7)$$

This model allows for time-varying conditional covariances but holds the reward-to-risk parameter, λ , constant. r_j denotes the portfolio returns^a and Z represents the instrumental variables.^b Estimates are obtained for each portfolio of excess returns for NYSE common stocks (ranked by firm size) from September 1941 to December 1987 (554 observations).

Portfolio	Reward-to-risk λ	\bar{e}_j^c	$\overline{u_m \times u_j}^d$	χ^2	$P\text{-value}^e$	\bar{R}^{2f}
Decile 1	11.8820 (1.5480)	-0.018235	0.002709	63.50	< 0.001	0.055
Decile 2	8.1096 (1.3858)	-0.006359	0.002250	47.85	< 0.001	0.085
Decile 3	8.4663 (1.3936)	-0.007261	0.002164	44.11	< 0.001	0.079
Decile 4	7.6279 (1.3383)	-0.005174	0.002042	39.21	< 0.001	0.082
Decile 5	7.4621 (1.3281)	-0.005318	0.001979	35.13	< 0.001	0.080
Decile 6	7.2126 (1.3168)	-0.004498	0.001948	32.11	< 0.001	0.072
Decile 7	6.7168 (1.2590)	-0.003761	0.001918	29.62	< 0.001	0.074
Decile 8	5.9535 (1.2202)	-0.002908	0.001864	27.27	< 0.001	0.065
Decile 9	5.7702 (1.2071)	-0.002330	0.001790	27.01	< 0.001	0.070
Decile 10	4.7267 (1.1535)	-0.001526	0.001633	26.60	< 0.001	0.065

^aAll asset returns r_{jt} are in excess of the holding-period return on the Treasury bill that is closest to 30 days to maturity. The market return, r_{mt} , is the excess return on the value-weighted NYSE stock index.

^bThe instrumental variables are: a constant, the excess return on the equally-weighted NYSE index, a dummy variable for January, the return for holding a 90-day bill for one month less the return on 30-day bill (term premium), the yield on Moody's BAA-rated bonds less the yield on Moody's AAA-rated bonds (junk bond premium), and the dividend yield on the Standard and Poor's 500 stock index less the return on a 30-day bill (dividend yield spread).

^c \bar{e}_j represents the average pricing error.

^d $\overline{u_m \times u_j}$ is the average conditional covariance (\times denotes element-by-element multiplication).

^e χ^2 is the minimized value of the GMM criterion function. $P\text{-value}$ is the probability that a χ^2 variate exceeds the sample value of the statistic. There are 13 parameters and 18 orthogonality conditions, which implies there are five overidentifying restrictions to be tested.

^f \bar{R}^2 is the coefficient of determination from the regression of the model errors (e_{jt}) on the instrumental variables (Z_{t-1}).

portfolios from small to large firms, the same ordering is expected for the conditional covariances. As shown in the column denoted $\overline{u_{m_i} \times u_j}$, this is indeed the case.

Two goodness-of-fit statistics are also presented. The chi-square test of the overidentifying restrictions reported in table 5 provides evidence against the model specification for each of the deciles. In the far right column, the R^2 of a regression of the model errors (e_t) on the information variables (Z_{t-1}) is presented. In each of these regressions, at least one variable is statistically significant. Comparing the R^2 measures in table 5 and table 2, it seems that the asset pricing model is explaining only a fraction of the predictable variation in the asset returns. Since (7) contains a model for expected returns and the CAPM, all the tests presented are joint tests of the CAPM's restrictions and the model specified for conditional expectations.

Another formulation of (7) is also estimated:¹²

$$e_t = (u_t \quad e_t)' = \begin{pmatrix} [r_t - Z_{t-1}\delta]' \\ [Z_{t-1}\delta - \lambda(r_{mt} - Z_{t-1}\delta_m)(r_t - Z_{t-1}\delta)]' \end{pmatrix}. \quad (7a)$$

This system has the same number of parameters (and overidentifying restrictions). The difference is in the e_t part of the system where the expected excess returns are substituted for the actual returns. The results (not reported) are similar to those reported in table 5. The test of the overidentifying restrictions provides evidence against the specification for all ten portfolios. The same pattern in reward-risk ratios is also observed – highest ratios for the smaller firms and lowest ratios for the larger firms.

3.6. Multiple portfolio tests with constant reward to variance

Table 6 presents the multiple portfolio tests of the Sharpe–Lintner model. Estimates are presented for all ten portfolios. With this system, there are 67 parameters and 126 orthogonality conditions, leaving 59 overidentifying restrictions to be tested. The estimate of the reward-risk coefficient is 7.35 and is in the range of single-portfolio estimates. The coefficient is precisely estimated, being six standard errors from zero. Given the evidence from the single-portfolio estimation, it is not surprising that the test of the overidentifying restrictions suggests there is evidence against the specification at the 0.01% level.

These rejections are not sensitive to the inclusion of 1987 data. When the systems in tables 5 and 6 are reestimated over the September 1941 to December 1986 period (542 observations), the parameter estimates and the

¹²I thank Kenneth Singleton for suggesting this asymptotically-equivalent formulation.

Table 6

The reward-to-risk parameter and the average pricing errors resulting from generalized method of moments estimation of a conditional version of the Sharpe-Lintner CAPM.

$$e_t = (u_t, e_t)' = \begin{pmatrix} [r_t - Z_{t-1}\delta]' \\ [r_t - \lambda(r_{mt} - Z_{t-1}\delta_m)](r_t - Z_{t-1}\delta) \end{pmatrix}' \quad (7)$$

This model allows for time-varying conditional covariances but holds the reward-to-risk parameter, λ , constant. r denotes the portfolio returns^a and Z represents the instrumental variables.^b Estimates are obtained for ten portfolios of excess returns for NYSE common stocks (ranked by firm size) from September 1941 to December 1987 (554 observations).

Reward-to-risk λ	χ^2	P-value ^c	Portfolio	\bar{r}_j^d	\bar{e}_j	$\bar{r}_{j, \text{Jan}}^e$	$\bar{e}_{j, \text{Jan}}$	$\bar{R}^2 f$
7.3548 (1.1264)	109.09	< 0.001	Decile 1	0.013954	-0.003116	0.101391	0.061221	0.117
			Decile 2	0.011888	-0.004237	0.073506	0.040742	0.096
			Decile 3	0.011058	-0.004697	0.061839	0.031977	0.087
			Decile 4	0.010400	-0.004743	0.047931	0.021013	0.081
			Decile 5	0.009452	-0.005294	0.043258	0.018351	0.079
			Decile 6	0.009550	-0.005029	0.036921	0.012421	0.070
			Decile 7	0.009125	-0.005241	0.028291	0.005452	0.071
			Decile 8	0.008189	-0.005763	0.025334	0.002993	0.061
			Decile 9	0.008001	-0.005372	0.019839	0.000393	0.066
			Decile 10	0.006195	-0.005872	0.008538	-0.008694	0.063
$H_0: \lambda_j = \lambda$	χ^2	P-value ^g						
	30.36	< 0.001						

^aAll asset returns r_{jt} are in excess of the holding-period return on the Treasury bill that is closest to 30 days to maturity. The market return, r_{mt} , is the excess return on the value-weighted NYSE stock index.

^bThe instrumental variables are: a constant, the excess return on the equally-weighted NYSE index, a dummy variable for January, the return for holding a 90-day bill for one month less the return on a 30-day bill (term premium), the yield on Moody's BAA-rated bonds less the yield on Moody's AAA-rated bonds (junk bond premium), and the dividend yield on the Standard and Poor's 500 stock index less the return on a 30-day bill (dividend yield spread).

^c χ^2 is the minimized value of the GMM criterion function. P-value is the probability that a χ^2 variate exceeds the sample value of the statistic. There are 67 parameters and 126 orthogonality conditions, which implies there are 59 overidentifying restrictions to be tested.

^d \bar{r}_j^d is the average return and \bar{e}_j represents the average pricing error for decile j .

^e $\bar{r}_{j, \text{Jan}}$ is the average January returns and $\bar{e}_{j, \text{Jan}}$ represents the average January pricing error for decile j .

^f \bar{R}^2 is the coefficient of determination from a regression of the model errors (e_{jt}) on the instrumental variables.

^gThe test statistic (for the null hypothesis that all the λ coefficients are equal) has 9 degrees of freedom. P-value is the probability that the variate exceeds the sample value of the statistic.

chi-square statistics are very similar. Furthermore, the rejections are not being driven by the month of January. Tables 5 and 6 are reestimated using all months other than January (508 observations) and the January dummy variable is dropped from the instrument list. The test of the overidentifying restrictions provides strong evidence against the specification in every case.

The far right column reports the R^2 that results from regressing the model residuals for each portfolio on the instrumental variables. Although the test of the overidentifying restrictions and the R^2 are trying to measure the same type of information, the R^2 is revealing information about the fit of the model for each portfolio. Comparing the single-portfolio (table 5) and the multiple-portfolio R^2 measures, the R^2 s are higher for the lower valued portfolios (indicating poorer fit) and relatively smaller for the mid to larger capitalization portfolios. This is the result of forcing the same λ coefficient on all the portfolios. The λ in the multiple-portfolio estimation is closer to the λ obtained in the single-portfolio estimation for the mid to larger portfolios than for the smaller portfolios.

The use of a number of assets in the estimation allows for testing of cross-asset restrictions, such as the equality of the λ coefficients. The test statistic suggests strong evidence against the null hypothesis of equality of the coefficients (at the 0.1% level).

Another way to evaluate the model's performance is to examine how well the model predicts returns. Define the pricing error as

$$e_{jt} = r_{jt} - \hat{\lambda}(r_{mt} - Z_{t-1}\hat{\delta}_m)(r_{jt} - Z_{t-1}\hat{\delta}_j),$$

$$\bar{e}_j = \frac{1}{T} \sum_{t=1}^T e_{jt}.$$

The pricing error is the actual return minus the predicted return. The model is underpricing if the predicted return is less than the actual return. Table 6 presents the average excess returns over the sample period and the average pricing errors. The overall pricing errors are fairly small compared with the average excess returns. On average, the CAPM tends to overprice all size deciles. Interestingly, the largest average pricing errors are found in the larger deciles.

We can learn more about the model's pricing performance by evaluating the predictions in each month. Table 6 also presents average excess returns and pricing errors for January. The errors are large for the smaller deciles. For example, the average excess return on the smallest decile is 10.1% and the average pricing error is 6.1%. A comparison of performance in all deciles and months is provided in fig. 1, which shows that the most serious underpricing occurs in January. Underpricing changes to overpricing by the middle of the

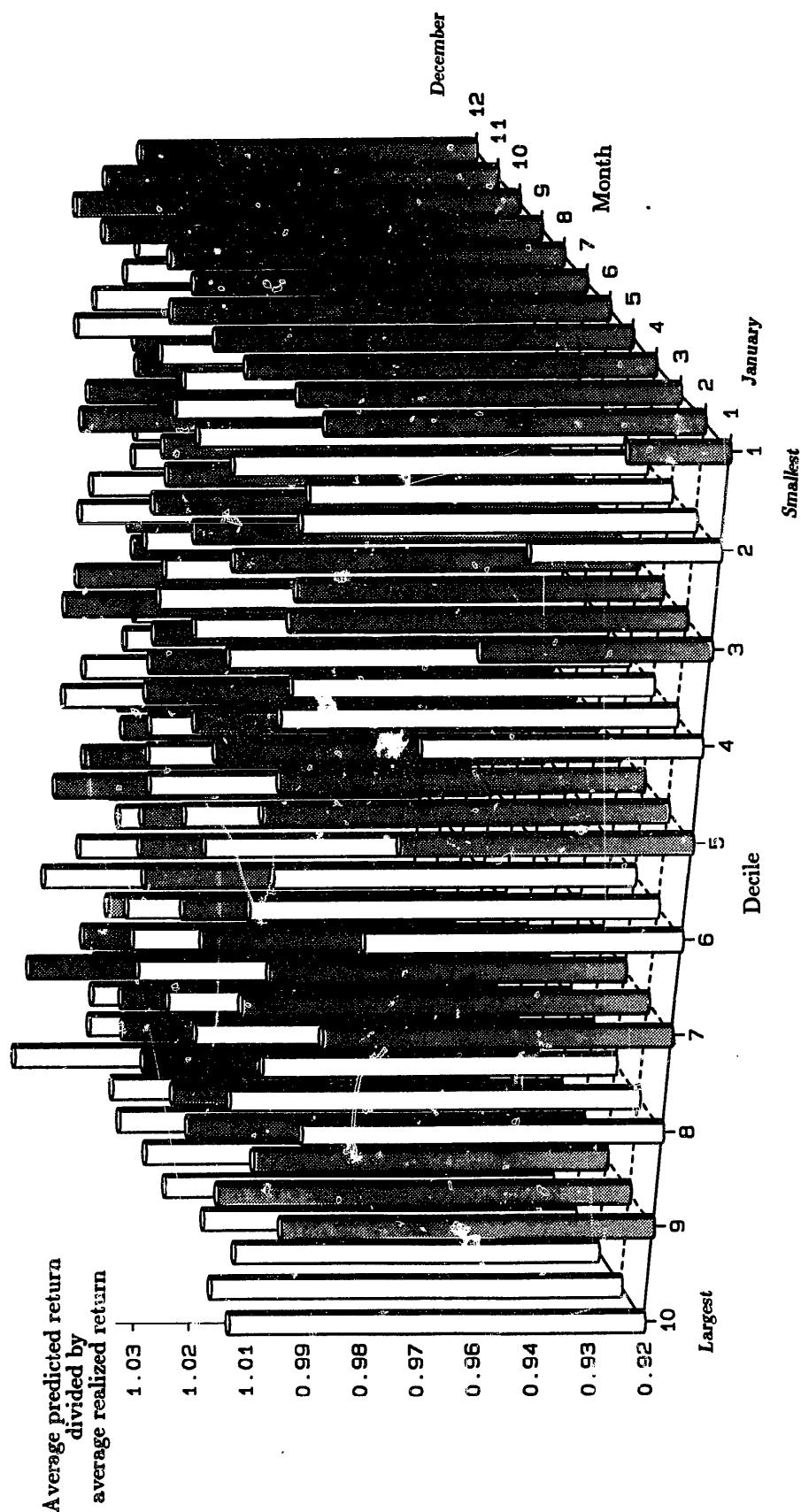


Fig. 1. Average predicted return from a conditional version of the Sharpe-Lintner CAPM as a proportion of the average portfolio excess return by month of the year.

The model allows for time-varying conditional covariances but holds the reward-to-risk parameter constant. The ratio is formed by taking one plus the average predicted excess return and dividing by one plus the average excess return for portfolios of NYSE common stocks (ranked by firm size). A ratio of unity indicates no average pricing error. If the ratio is less than one, then the model is underpricing. If the ratio exceeds unity, then the model is overpricing. The estimation period is September 1941 to December 1986 (542 observations).

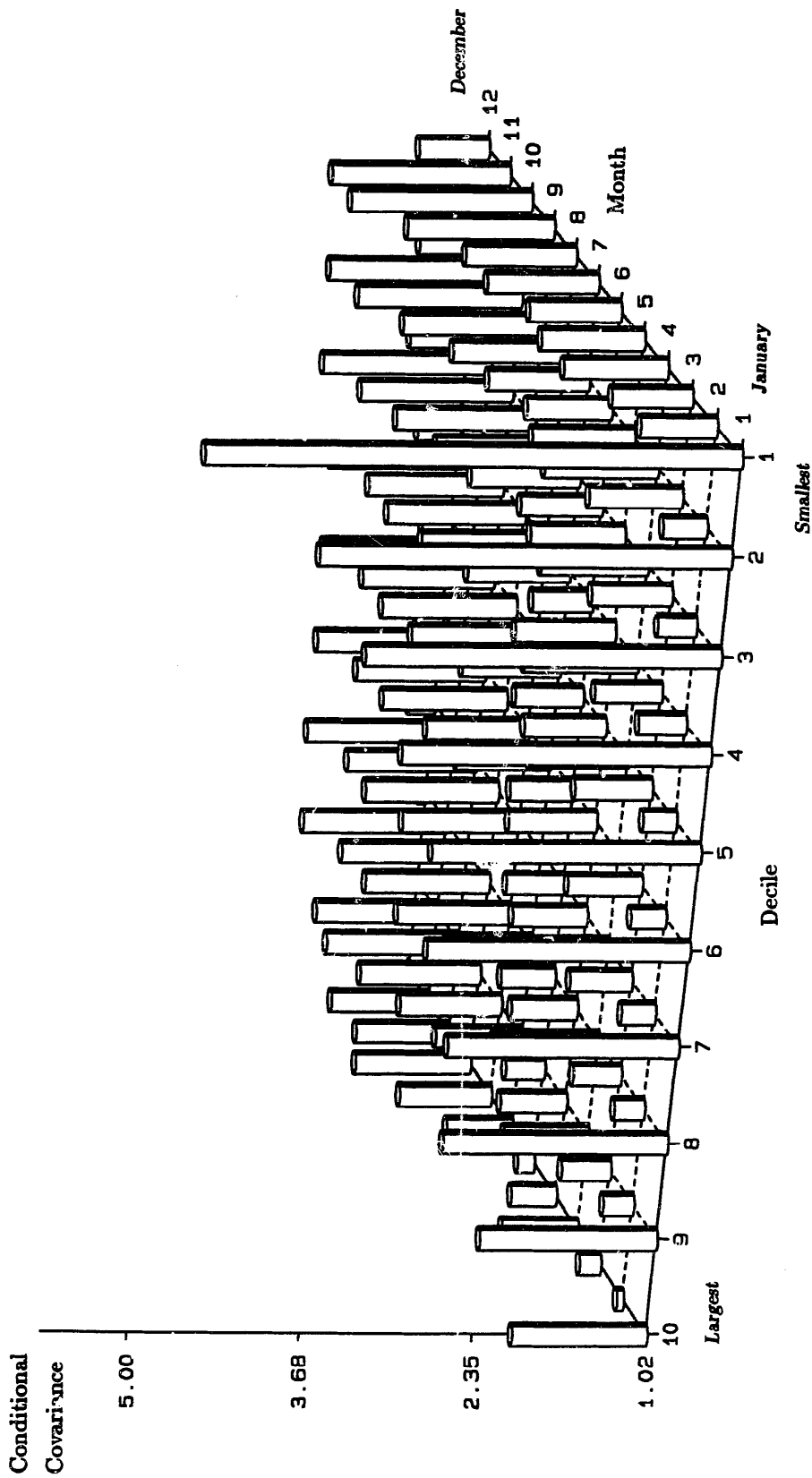


Fig. 2. Average conditional covariances of the portfolio return and the value-weighted NYSE index return based on a conditional version of the Sharpe-Lintner CAPM by month of the year.

The model allows for time-varying conditional covariances but holds the reward-to-risk parameter constant. The average conditional covariance is calculated by taking an average (by month) of the product of the excess returns innovations for portfolios of NYSE common stocks (ranked by firm size) and the market return innovations. The covariance is multiplied by 1,000. The estimation period is September 1941 to December 1986 (542 observations).

Table 7

The reward-to-risk parameter and the average pricing errors resulting from generalized method of moments estimation of a conditional version of the Sharpe-Lintner CAPM that includes portfolio-specific intercepts.

$$\varepsilon_t = (u_t \quad e_t)' = \begin{pmatrix} [r_t - \alpha - \lambda(r_{mt} - Z_{t-1}\delta_m)](r_t - Z_{t-1}\delta)' \\ [r_t - \alpha - \lambda(r_{mt} - Z_{t-1}\delta_m)](r_t - Z_{t-1}\delta) \end{pmatrix}'. \quad (11)$$

This model allows for time-varying conditional covariances but holds the reward-to-risk parameters, λ , constant. r denotes the portfolio returns^a and Z represents the instrumental variables.^b Estimates are obtained for ten portfolios of excess returns for NYSE common stocks (ranked by firm size) from September 1941 to December 1987 (554 observations).

Reward-to-risk λ	χ^2	P-value ^c	Portfolio	α_j	\bar{r}_j^d	\bar{e}_j	$\bar{r}_{j, jan}^e$	$\bar{e}_{j, jan}$	\bar{R}^{2f}
7.8123 (1.7838)	101.04	< 0.001	Decile 1	-0.002377 (0.003898)	0.013954	-0.001561	0.201391	0.064328	0.118
			Decile 2	-0.001439 (0.003454)	0.011888	-0.003640	0.073506	0.042338	0.096
			Decile 3	-0.001104 (0.003410)	0.011058	-0.004351	0.061839	0.033186	0.087
			Decile 4	-0.000863 (0.003275)	0.010400	-0.004729	0.047931	0.021671	0.081
			Decile 5	-0.001183 (0.003164)	0.009452	-0.004944	0.043258	0.019180	0.078

$H_0: \alpha_j = 0$	χ^2	$P\text{-value}^s$										
			Decile 6									
				-0.000803 (0.003172)	0.009550	-0.005062	0.036921	0.012800	0.069			
			Decile 7	-0.000453 (0.003063)	0.009125	-0.005628	0.028291	0.005450	0.071			
			Decile 8	-0.001264 (0.002990)	0.008189	-0.005314	0.025334	0.003655	0.061			
			Decile 9	-0.000893 (0.002831)	0.008001	-0.005270	0.019839	0.000735	0.066			
			Decile 10	-0.001542 (0.002608)	0.006195	-0.005072	0.008538	-0.008023	0.063			
	10.09	0.433										

^aAll asset returns r_{it} are in excess of the holding-period return on the Treasury bill that is closest to 30 days to maturity. The market return, r_{mt} , is the excess return to the value-weighted NYSE stock index.

^bThe instrumental variables are: a constant, the excess return on the equally-weighted NYSE index, a dummy variable for January, the return for holding a 90-day bill for one month less the return on a 30-day bill (term premium), the yield on Moody's BAA-rated bonds less the yield on Moody's AAA-rated bonds (junk bond premium), and the dividend yield on the Standard and Poor's 500 stock index less the return on a 30-day bill (dividend yield spread).

^c χ^2 is the minimized value of the GMM criterion function. P -value is the probability that a χ^2 variate exceeds the sample value of the statistic. There are 77 parameters and 126 orthogonality conditions, which implies there are 49 overidentifying conditions to be tested.

^d \bar{r}_j is the average excess return and \bar{e}_j represents the average pricing error for decile j .

^e $r_{j,jan}$ is the average January excess return and $\bar{e}_{j,jan}$ represents the average January pricing error for decile j .

^f R^2 is the coefficient of determination from a regression of the model errors on the instrumental variables.

^gThe test statistic (for the null hypothesis that all the α coefficients are equal to zero) has 10 degrees of freedom. P -value is the probability that the variate exceeds the sample value of the statistic.

year and peaks in September and October.¹³ This pattern is especially apparent in the smaller deciles.

Fig. 2 displays the average conditional covariances, which are always positive, but vary considerably through the year. The sharpest contrast in these covariances can be found looking across all deciles from January to February. If the pricing model is going to be able to explain asset returns, then high expected returns should be associated with high conditional covariances. This is indeed the case with these estimates, especially for January. Although the conditional covariance is large in January, however, it is not large enough to explain all of the January returns. For example, the average conditional covariance in January for the smallest decile is 0.0054617, which delivers a pricing error of 6.1%. To eliminate the pricing error, the conditional covariance would have to be 0.013787, or more than twice what is estimated in system (7). Comparing large stocks with small stocks, the January conditional covariance for the smallest decile must be seven times the size of the covariance for the largest decile.

3.7. *Tests with asset-specific intercepts*

The inclusion of an asset-specific intercept in the CAPM equation allows the unconditional mean excess returns to be unconstrained in the estimation procedure. The results for this test are reported in table 7. The reward-to-risk parameter has a point estimate of 7.81 with a *t*-ratio of 4.4. The test of the overidentifying restrictions suggests evidence against the model at the 0.01% level. The point estimates of the intercepts and standard errors are also reported. There is no clear pattern in the magnitude of the intercept across the size deciles. The intercepts are imprecisely estimated, with none being more than one standard error from zero, and a test of the null hypothesis that all the intercepts are equal to zero cannot be rejected. The R^2 s from regressing the model residuals on the information variables are similar to those reported in table 6, indicating that the fit has not been improved. Finally, although the overall pricing errors are similar to those reported in table 6, the January pricing errors are generally larger when an asset-specific intercept is included.

3.8. *Tests with time-varying reward to risk*

Table 8 presents results of estimating (18), which allows for time variation in expected asset returns, conditional covariances, and the conditional variance of the market. The system of equations is similar to (7) except there is one less parameter to estimate: the reward-to-variance ratio. As a result, there

¹³This overpricing in October is not a result of October 1987. To emphasize this, both figs. 1 and 2 are based on estimation of (7) over the September 1941 to December 1986 period.

Table 8

Tests of a conditional version of the Sharpe–Lintner CAPM that allow for time-varying expected returns, conditional covariances, and reward-to-risk.

$$e_t = (u_t \quad u_{mt} \quad e_t)' = \begin{pmatrix} [r_t - Z_{t-1}\delta]' \\ [r_{mt} - Z_{t-1}\delta_m]' \\ [u_{mt}^2 Z_{t-1}\delta - u_{mt} u_t Z_{t-1}\delta_m]' \end{pmatrix}. \quad (18)$$

r denotes the portfolio returns^a and Z represents the instrumental variables.^b Estimates are obtained with the generalized method of moments for ten portfolios of excess returns for NYSE common stocks (ranked by firm size) for September 1941 to December 1987 (554 observations).

χ^2	P -value ^c	Portfolio	$\overline{u_j \times u_m}$ ^d	$\overline{u_{j, jan} \times u_{m, jan}}$ ^e	$\overline{R^2}$ ^f
104.04	< 0.001	Decile 1	0.002011	0.003469	0.226
		Decile 2	0.001953	0.003130	0.216
		Decile 3	0.001925	0.002970	0.206
		Decile 4	0.001865	0.002784	0.168
		Decile 5	0.001823	0.002634	0.153
		Decile 6	0.001818	0.002674	0.121
		Decile 7	0.001797	0.002551	0.069
		Decile 8	0.001758	0.002552	0.073
		Decile 9	0.001686	0.002229	0.072
		Decile 10	0.001541	0.002161	0.142

^aAll asset returns r_{jt} are in excess of the holding-period return on the Treasury bill that is closest to 30 days to maturity. The market return, r_{mt} , is the excess return on the value-weighted NYSE stock index.

^bThe instrumental variables are: a constant, the excess return on the equally-weighted NYSE index, a dummy variable for January, the return for holding a 90-day bill for one month less the return on a 30-day bill (term premium), the yield on Moody's BAA-rated bonds less the yield on Moody's AAA-rated bonds (junk bond premium), and the dividend yield on the Standard and Poor's 500 stock index less the return on a 30-day bill (dividend yield spread).

^c χ^2 is the minimized value of the GMM criterion function. P -value is the probability that a χ^2 variate exceeds the sample value of the statistic. There are 66 parameters and 126 orthogonality conditions, which implies there are 60 overidentifying restrictions to be tested.

^d $\overline{u_j \times u_m}$ represents the average conditional covariance for decile j based on the multiple asset estimation.

^e $\overline{u_{j, jan} \times u_{m, jan}}$ represents the average conditional covariance for decile j in January.

^f $\overline{R^2}$ is the coefficient of determination from a regression of the model errors (e_{jt}) on the instrumental variables.

is one additional degree of freedom in the chi-square statistic that provides the test of the overidentifying restrictions.

The results in table 8 indicate a more dramatic rejection than the one documented in table 6. The chi-square statistic is much higher. Further, the regressions of the model residuals¹⁴ on the information variables show higher adjusted R^2 s in all but one portfolio. The model with a time-varying reward to risk appears to be doing worse than the model with a fixed parameter.

¹⁴The model residuals cannot be directly interpreted as 'pricing errors' as in (7). The residuals must be divided by the conditional variance of the market to be 'pricing errors'.

Table 9

Tests of a conditional version of the Sharpe–Lintner CAPM that allow for time-varying expected returns, conditional covariances, and reward-to-risk using data for all months except January.

$$e_t = (u_t \quad u_{mt} \quad e_t)' = \begin{pmatrix} [r_t - Z_{t-1}\delta]' \\ [r_{mt} - Z_{t-1}\delta_m]' \\ [u_{mt}^2 Z_{t-1}\delta - u_{mt}u_t Z_{t-1}\delta_m]' \end{pmatrix}. \quad (18)$$

r denotes the portfolio returns^a and Z represents the instrumental variables.^b Estimates are obtained with the generalized method of moments for ten portfolios of excess returns for NYSE common stocks (ranked by firm size) from September 1941 to December 1987 (508 observations).

χ^2	P-value ^c	Portfolio	$\overline{u_j \times u_m}$ ^d	Residuals e_{jt} on Z_{t-1} \bar{R}^2 ^e	Returns r_{jt} on Z_{t-1} \bar{R}^2 ^f
76.18	0.010	Decile 1	0.001889	0.019	0.072
		Decile 2	0.001852	0.028	0.067
		Decile 3	0.001835	0.023	0.069
		Decile 4	0.001785	0.012	0.069
		Decile 5	0.001752	0.016	0.069
		Decile 6	0.001741	0.017	0.065
		Decile 7	0.001729	0.010	0.067
		Decile 8	0.001685	0.003	0.059
		Decile 9	0.001636	0.000	0.068
		Decile 10	0.001484	0.012	0.068

^aAll asset returns r_{jt} are in excess of the holding-period return on the Treasury bill that is closest to 30 days to maturity. The market return, r_{mt} , is the excess return on the value-weighted NYSE stock index.

^bThe instrumental variables are: a constant, the excess return on the equally-weighted NYSE index, the return for holding a 90-day bill for one month less the return on a 30-day bill (term premium), the yield on Moody's BAA-rated bonds less the yield on Moody's AAA-rated bonds (junk bond premium), and the dividend yield on the Standard and Poor's 500 stock index less the return on a 30-day bill (dividend yield spread).

^c χ^2 is the minimized value of the GMM criterion function. P-value is the probability that a χ^2 variate exceeds the sample value of the statistic. There are 55 parameters and 105 orthogonality conditions, which implies there are 50 overidentifying restrictions to be tested.

^d $\overline{u_j \times u_m}$ represents the average conditional covariance for decile j based on the multiple asset estimation.

^e \bar{R}^2 is the coefficient of determination from a regression of the model errors (e_{jt}) on the instrumental variables.

^f \bar{R}^2 is the coefficient of determination from a regression of the asset returns (r_{jt}) on the instrumental variables.

Table 8 also provides estimates of the average conditional covariances. As in fig. 2, the January covariances are inversely related to size, but the covariances in table 8 are smaller for the small stocks than the ones presented in fig. 2. For example, the average conditional covariance for the smallest decile in table 8 is 0.003469. This is about 60% smaller than the same conditional covariance illustrated in fig. 2. Since it is unlikely that there is a large jump in the

(time-varying) reward-to-risk ratio in January,¹⁵ these small conditional covariances are causing large pricing errors in January.

The rejection documented in table 8 is not just being driven by the month of January, however. Table 9 replicates the estimation of (18) with a data set that does not include January. The dummy variable for January is dropped from the instrument list. There are now 105 orthogonality conditions and 55 parameters to estimate, leaving the 50 overidentifying restrictions. The model's restrictions can still be rejected at a probability value of less than 1%. The R^2 from a regression of the model residuals on the instruments is uniformly lower than the comparable measure in table 8. The non-January returns are less predictable, however, as evidenced in the final column of table 9.

4. Conclusions

This paper provides a way to test the CAPM that allows for both time-varying expected returns and time-varying conditional covariances. The results indicate that the conditional covariances do change through time. Models are estimated holding the expected excess return on the market divided by the variance of the market constant. Consistent with the asset pricing model, high returns are associated with high conditional covariances. There is evidence against the model's specification, however, because the pricing errors are correlated with elements in the investor's information set.

Although the estimates of the reward-to-risk ratio seem reasonable, evidence is presented that suggests this ratio is time-varying. A specification is estimated that allows for time variation in conditional covariances, conditionally expected returns, and the conditional variance of the market. The restrictions of the asset pricing model are rejected even in this general formulation. This finding adds to the evidence that the Sharpe–Lintner formulation of the CAPM is unable to capture the dynamic behavior of asset returns.

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¹⁵The January dummy variable is not significantly different from zero in the conditional mean regression for the market return in table 2. When the square of the residual from the conditional mean is regressed on the information set, the January variable is not significantly different from zero.

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