



# Momentum turning points

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## ARTICLE INFO

### Article history:

Received 28 July 2021

Revised 22 May 2023

Accepted 22 May 2023

### JEL classification:

G12

G13

### Keywords:

Time-series momentum

Turning points

Volatility timing

Market timing

Trend following

Momentum speed

## ABSTRACT

We use slow and fast time-series momentum to characterize four stock market cycles—Bull, Correction, Bear, and Rebound. The steep market declines of Bears concentrate in high-risk states, yet predict negative expected returns, which is difficult to rationalize by most models of time-varying risk premia. Using a model to analyze slow and fast momentum strategies, we estimate both relatively high mean persistence and realization noise in U.S. stock market returns. Intermediate-speed momentum portfolios, formed by blending slow and fast momentum strategies, translate predictive information in market cycles into positive unconditional alpha, for which we propose a novel decomposition.

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## 1. Introduction

Time-series (TS) momentum strategies are based on two main premises. First, expected returns vary over time. Second, the *signs* of expected returns have some degree of persistence. If the expected return is positive (negative) this month, then it is more likely to remain positive (negative) next month than to flip sign. TS momentum strategies attempt to exploit such variation and persistence in expected returns by taking long positions in “uptrend” phases and short positions in “downtrend” phases, based

on the signs of some look-back horizon of trailing returns. In uptrend or downtrend phases, such strategies will tend to place good bets. However, at momentum turning points, which mark reversals in trend from uptrend to downtrend or vice versa, TS momentum strategies are prone to place bad bets because they rely on observations of realized returns, which reflect a mixture of different trend regimes and noise.

The speed (or sensitivity to recent data) of the momentum signal balances the tension between reducing the impact of noise and reacting quickly to turning points. This tension plays out differently for different speeds. Either the momentum signal attempts to reduce the influence of noise by having a relatively long look-back window (e.g., 12 months) but thus is *slow* to react to a turning point (Type II error of missed detection), or the momentum signal attempts to be *fast* to react to a turning point by

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having a relatively short look-back window (e.g., 1 month) and therefore is more influenced by noise (Type I error of false alarm).

We develop a model of expected returns to examine connections between the performance of different speeds and the following unobservable variables: trend (persistence in expected returns), turning points, and noise levels in realized returns.<sup>1</sup> The literature has documented that TS momentum strategies based on slow momentum (i.e., the standard 12-month signal) tend to perform better than strategies based on fast momentum (e.g., the 1-month signal). Our model indicates that when the slow strategy (SLOW) outperforms the fast strategy (FAST) in the long run, it is because expected returns are relatively persistent and realized returns are relatively noisy. Yet in the short run after turning points, higher persistence translates into short-run average gains for FAST, while both higher persistence and higher noise translate into higher losses for SLOW. Thus, the same conditions that make SLOW attractive overall relative to FAST can also make it prone to suffer more around turning points and, at these times, FAST can be more effective.

These connections suggest that the union of information embedded in SLOW and FAST positions can be useful in detecting market turning points. When bets indicated by SLOW and FAST disagree, the intuition is that the market is more likely to be at a turning point. The agreement of SLOW and FAST to go long (short) is more likely to indicate the market is in the midst of an uptrend (downtrend). We find this intuition is not only supported by our model, but also consistent with the returns behavior of both the U.S. and international stock markets after each of the four different phases, or market cycles, defined by the up or down directions of each of the slow and fast momentum strategies.

Figure 1 summarizes, over a recent 50-year period of the U.S. stock market, the conditional behavior of the average, volatility, and skewness of returns in months following four market cycles and the monthly relative frequency of such states.<sup>2</sup> When both slow and fast momentum agree on the direction of trend, we call it a “Bull” or “Bear” state, depending on whether the agreement is to take a long or short position, respectively. These labels loosely map to phases of uptrend and downtrend: Bull states are followed by relatively high average returns with low volatility, and Bear states are followed by negative average returns with the highest relative volatility. When slow and fast momentum disagree, we call it a “Correction” state if slow momentum indicates a long position and a “Rebound” state if slow momentum indicates a short position. Similarly, these labels loosely map to the potential occurrences of turning points from uptrend to downtrend and vice versa. Correction states are followed by deteriorat-

ing average returns, increased volatility, and severe downside outcomes—possibly a lead up to a Bear state. Rebound states are followed by average returns and skewness similar to Bull phases, but with higher volatility—possibly a lead up to a Bull state. Lastly, Corrections and Rebounds are significant in frequency and combined occur more than one-third of the time covered in our analysis.

Figure 1 (a) indicates that the combination of both negative slow and negative fast momentum signals captures a state with negative expected returns (Bear states). To understand this phenomenon, we map the empirical properties of SLOW and FAST applied to the U.S. stock market into our model. We find that both persistence in expected returns and noise in realized returns are relatively high. Higher persistence in expected returns increases the potential for momentum signals to define predictive states. Because expected returns are unobservable and the noise content of observable realized returns is high, the extra observations used in slow momentum are necessary to enhance our inference of the trend. Slow momentum alone, however, is not sufficient for a clean inference because it relies on stale information.<sup>3</sup> The additional confirmation from fast momentum makes our combined inference more reliable, which explains the positive and negative expected returns following Bull and Bear states, respectively. Conflicting signals, associated with Correction and Rebound states, make our trend inference less reliable and, therefore, expected returns following these states align with the unconditional inference that stock market excess returns are positive.

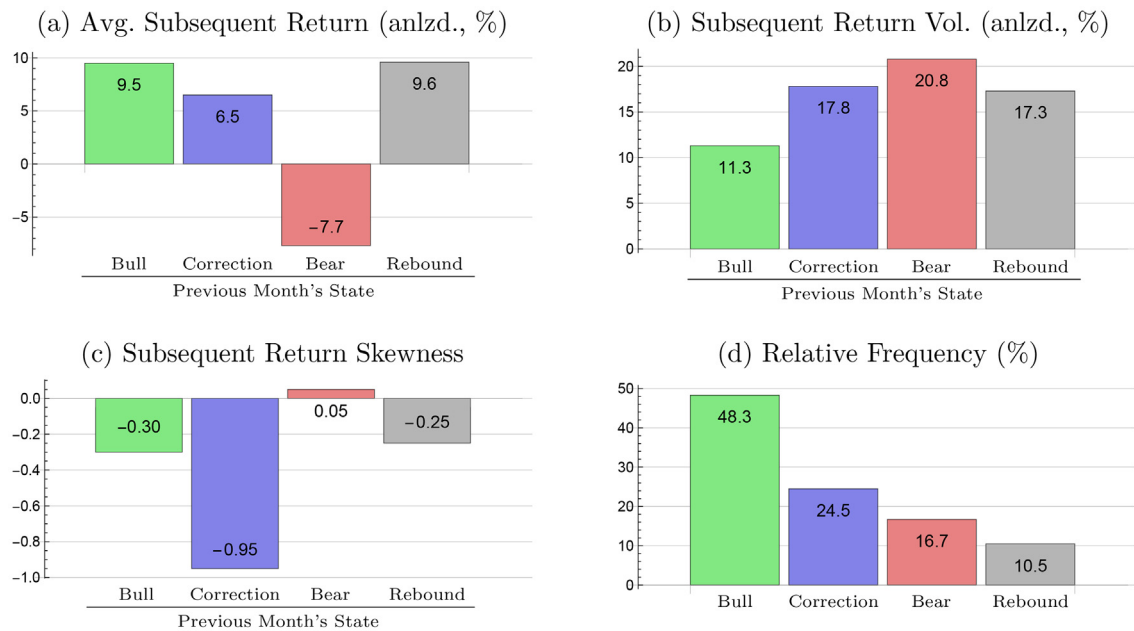
Our market cycles also have close connections to the macroeconomy and the business cycle; in particular, Bear states, which have negative expected returns, are closely linked with high macroeconomic risk states. First, we study the behavior of 15 major macroeconomic indicators from the three broad categories of economic activity, risk, and survey-based variables. We find that surprises in these variables exhibit distinct behavior across our market cycles: bad news during Bear states, good news during Bull states, and insignificant news during Correction and Rebound states. Second, we document that Bear states are the highest relative frequency state in the early months of recessions—occurring at twice the relative frequency of the other three states combined. Bull and Rebound states, which predict positive subsequent returns, increase to dominant relative frequencies late in periods of recession—over 90% combined relative frequency.

Momentum is one of the most pervasive factors—yet one of the most difficult to explain. In particular, negative expected returns after steep market declines in high-risk states are difficult to rationalize by most models of time-varying risk premia, which imply countercyclical expected market returns. The occurrence of Bear states early in recessions, when risk is arguably at its highest, makes the negative expected returns of these states even more puzz-

<sup>1</sup> We thank an anonymous referee for suggesting this approach. See Section 2.2 for model details and analysis.

<sup>2</sup> For most of the paper, we use a recent 50-year period for consistency with later analyses in which we use earlier data to warm up dynamic strategies for out-of-sample evaluation. In Internet Appendix F, we report results based on a longer evaluation period beginning in 1927, from which we draw similar inferences.

<sup>3</sup> The sign of the slow signal by itself or the fast signal by itself is not enough to detect a negative expected return state. Annualized average monthly excess U.S. stock market returns following a negative average 12-month trailing return or negative 1-month trailing return, respectively, are 0.2% and 3.2% during the period 1927–07 to 2018–12.



**Fig. 1.** U.S. stock market cycles.

*Notes:* This figure reports (a) the conditional average, (b) the conditional volatility and (c) the conditional skewness of monthly aggregate U.S. stock market excess returns, based on the market cycle state in the prior month, over the 50-year evaluation period from 1969-01 to 2018-12 (data before 1969-01 used to compute market states in first year). A month ending at date  $t$  is classified as Bull if both the trailing 12-month return (arithmetic average monthly return),  $r_{t-12,t}$ , is nonnegative and the trailing 1-month return,  $r_{t-1,t}$ , is nonnegative. A month is classified as Correction if  $r_{t-12,t} \geq 0$  but  $r_{t-1,t} < 0$ ; as Bear if  $r_{t-12,t} < 0$  and  $r_{t-1,t} < 0$ ; and as Rebound if  $r_{t-12,t} < 0$  but  $r_{t-1,t} \geq 0$ . The figure also reports (d) the relative frequency of these cycles over the same evaluation period. Market excess returns are U.S. excess value-weighted factor returns (Mkt-RF) from the Kenneth French Data Library.

zling. Behavioral models of slow information absorption have the potential to explain the predominance of negative expected return states, such as Bear states, early in recessions and the transition to dominance of positive expected return states, such as Bull and Rebound states, later in recessions.<sup>4</sup> None of these theories, however, tells us the optimal look-back horizon for a momentum strategy or how that horizon might change through time.

We analyze momentum strategies of various intermediate “speeds,” which are fixed-proportion blends of SLOW and FAST, and highlight three findings. First, we show analytically and empirically that intermediate-speed momentum strategies have higher Sharpe ratios than the average Sharpe ratios of SLOW or FAST and link this behavior to the volatility of returns following turning-point states (Corrections and Rebounds). Second, we shed new light on the drivers of TS momentum alpha. Recent studies have argued that the profitability of TS momentum strategies is predominantly attributed to their static (average) tilts.<sup>5</sup> Empirically, we document that TS momentum strategies of all speeds on the U.S. stock market have positive average exposures (i.e., going long more often than short), yet their market betas are lower than suggested by these exposures—beta estimates nearer to zero for slow-to-intermediate speeds and negative for faster speeds. Our analysis reveals that this seeming disparity arises from the ability of these TS momentum portfolios to time volatil-

ity states. We derive a novel model-free decomposition of alpha into the sum of two components: market timing and volatility timing. Empirically, we estimate that market timing drives about two-thirds of the alpha of TS momentum strategies on the U.S. stock market over a recent 50-year period and volatility timing drives the remaining one-third. Third, we show analytically and empirically that the skewness of an intermediate-speed momentum strategy is scaled and shifted relative to the average skewness of SLOW and FAST, with the shift in the same direction as its Sharpe ratio: toward higher (more positive, less negative) skewness whenever the intermediate-speed Sharpe ratio is positive.

We also study *dynamic* strategies with speeds that may vary each month based on observed market cycles.<sup>6</sup> We derive the state-dependent speed rule which yields the maximum Sharpe ratio. We estimate this strategy using historical returns and find its out-of-sample performance can efficiently track the best possible state-dependent performance.

Lastly, to test the external validity of our findings for the U.S. stock market, we examine the empirical performance of TS momentum strategies of various static and

<sup>4</sup> See Gómez-Cram (2022).

<sup>5</sup> See Goyal and Jegadeesh (2017) and Huang et al., 2020.

<sup>6</sup> Our approach is not to be confused with moving-average crossovers. Levine and Pedersen (2016) show that moving-average crossovers are essentially equivalent to static blends of time-series momentum strategies. Moreover, Hurst et al. (2013) show that the returns of trend-following strategies such as managed futures funds and CTAs can be explained by static blends of time-series momentum strategies.

dynamic speeds across 20 international equity markets. Consistent with our analytical results, the Sharpe ratios of intermediate-speed strategies are higher than the average of the Sharpe ratios of SLOW and FAST uniformly across all markets. In addition, the Sharpe ratio of the dynamic-speed strategy is higher than the highest static-speed strategy for most countries.

### 1.1. The setting

The premise of time-varying expected returns has extensive support in the literature (see, e.g., Fama and French, 1988; Cochrane, 2011, among many). The idea of persistent trends also has support in the literature, which documents that asset returns measured over the recent past are positively correlated with future returns (Jegadeesh and Titman, 1993, 2001; Asness, 1994; Conrad and Kaul, 1998; Lee and Swaminathan, 2000; Gutierrez and Kelley, 2008) and that these momentum effects are stable across assets and countries (Rouwenhorst, 1998; Griffin et al., 2003; Israel and Moskowitz, 2012; Asness et al., 2013).<sup>7</sup> Recent studies provide evidence that TS momentum strategies can successfully exploit these trends (Moskowitz et al., 2012; Georgopoulou and Wang, 2017; Ehsani and Linnainmaa, 2019).

Our examination of the economic linkages of market cycles contributes to the growing literature that suggests stronger connections between stock returns and fundamentals of the economy than previously understood. We document statistically positive and negative surprises across an array of macroeconomic indicators when conditioned on our Bull and Bear states, respectively. These macro news relationships are analogous to the findings of Albuquerquie et al. (2015) that macroeconomic variables are correlated with stock market returns across their “long bull” and “long bear” episodes of the stock market, which they report last about 14.8 and 3.2 years on average, respectively. However, our results arise in shorter stock market episodes based on 12-month and 1-month momentum agreement, a novel finding. Cujean and Hasler (2017) document that TS momentum strengthens in bad economic times. We show that our Bull and Bear states, which proxy for uptrend and downtrend regimes of the stock market, concentrate in recessions, but at different periods within: Bears are more frequent early in recessions and Bulls are more frequent late in recessions. This distinction of strong uptrend and downtrend periods within bad times is con-

sistent with the findings of Gómez-Cram (2022), which we discuss in detail in Section 3.

Huang et al. (2020) argue that predictability of 12-month TS momentum as captured by forecasting regressions does not appear to be statistically significant and, accordingly, the profitability of a diversified 12-month TS momentum portfolio can be largely attributed to its static tilt (net long positions). We arrive at a much different conclusion. Despite their positive static (average) tilts, TS momentum portfolios need not have positive betas to the underlying asset. Applying our decompositions of TS momentum beta and alpha, we document the role of innate volatility timing in TS momentum strategies, which can explain this disparity between static tilts and betas and contribute to meaningful alphas. Furthermore, appealing to static tilts as the primary explanation for TS momentum performance overlooks results from the conditional CAPM literature. Under the conditional CAPM, static tilts can be viewed as expected conditional betas, which differ from unconditional betas if any of expected returns, variances, or covariances change over time<sup>8</sup>—all active elements in TS momentum environments.

Goyal and Jegadeesh (2017) also argue that the net dollar exposure (i.e., static tilt) of TS momentum strategies is a key determinant of their profitability. Specifically, they propose that adding the net dollar exposure of a TS momentum portfolio to a cross-sectional (CS) momentum portfolio can reproduce TS momentum profits. This argument does not apply, however, to the case of TS momentum on a single asset, for which a CS momentum strategy is either not defined or trivial. In this paper, we focus on single-asset TS momentum.

Although our focus is on TS momentum, our approach shares some themes with the CS momentum literature. In a TS setting, we employ market cycles as Cooper et al. (2004); Daniel and Moskowitz (2016), and Daniel et al. (2019) do in CS settings. Cooper et al. use a slow trailing three-year return to define market states, whereas we use slow and fast trailing returns signals.<sup>9</sup> Daniel and Moskowitz study CS momentum crashes and recoveries and propose a dynamic CS weighting strategy. Daniel et al. use a two-state hidden Markov model of unobserved “turbulent” and “calm” states to predict crashes in CS momentum portfolios. We employ a finer cycle partition of observable states dictated by slow and fast momentum directions and develop a dynamic TS momentum strategy which blends slow and fast strategies using cycle-conditional information.

In a TS setting, we evaluate different horizons as Novy-Marx (2012) does in a CS setting. Novy-Marx points out that CS momentum works best at 7- to 12-month horizons. We demonstrate that shorter horizons are significant predictors of aggregate stock market returns and can augment slower signals. Liu and Zhang (2008) show in a CS mo-

<sup>7</sup> Explanations of momentum rely on either behavioral or rational foundations. Studies on behavioral foundations include Barberis et al. (1998); Daniel et al. (1998); Odean (1998); Hong and Stein (1999, 2007); Gervais and Odean (2001); Grinblatt and Han (2002); Barberis and Shleifer (2003); Chan (2003); Cross et al. (2005); Grinblatt and Han (2005); Cross et al. (2006); Frazzini (2006); Shefrin (2008); Chui et al. (2010); Haldane (2010); Dasgupta et al. (2011); Bandarchuk and Hilscher (2012); Antoniou et al. (2013); Avramov et al. (2013), and Chen and Lu (2013). Studies on rational foundations include Lo and MacKinlay (1990); Carhart (1997); Berk et al. (1999); Ahn et al. (2002); Johnson (2002); Lewellen (2002); Allen et al. (2006); Watanabe (2008); Banerjee et al. (2009); Verardo (2009); McLean (2010); Cespa and Vives (2012); Makarov and Rytchkov (2012); Vayanos and Woolley (2012, 2013); Aretz and Pope (2013); Jordan (2013); Liu and Zhang (2013), and Zhou and Zhu (2013).

<sup>8</sup> See Jagannathan and Wang (1996); Lewellen and Nagel (2006), and Boguth et al. (2011).

<sup>9</sup> Specifically, these authors rely on trailing three-year market returns to define two states—“up” and “down”—and forecast stock momentum consistent with the predictions of behavioral models (Daniel et al., 1998, and Hong and Stein, 1999).

momentum setting that recent winners load temporarily on the growth rate of industrial production and that macroeconomic risk can explain more than half of CS momentum profits. This work relates to our findings of distinct macroeconomic linkages to the market cycles determined by slow and fast TS momentum directions.

Our decompositions of beta and alpha for TS strategies have similarities in form to decompositions of the differences between unconditional and expected conditional versions of beta and alpha for CS strategies in the conditional CAPM literature.<sup>10</sup> For TS strategies, expected conditional alpha is not a useful benchmark because the conditional CAPM trivially holds for any TS strategy applied directly to excess market returns—conditional alpha is zero in every period. Moreover, decompositions in terms of conditional betas and conditional expected market returns are functions of an arbitrary conditioning information set, which is meant to represent what investors observe. In contrast, we decompose (unconditional) beta and alpha into direct functions of TS strategy weights and market returns, avoiding expressions dependent on conditioning information.

Our paper also bridges to the literature on volatility-managed (VOM) portfolios sparked by [Moreira and Muir \(2017\)](#) and further studied by [Harvey et al. \(2018\)](#). VOM portfolio alpha, by construction, is predominantly driven by volatility timing. Using our alpha decomposition, we show that TS momentum strategy alpha is driven by both market timing and innate volatility timing, without enhancements such as scaling by trailing volatility.

## 2. Momentum speed and market cycles

In this section, we define a collection of TS momentum strategies of different speeds by blending elementary TS momentum strategies, SLOW and FAST, in various proportions. We also define market cycles stemming from the intersection of SLOW and FAST.

### 2.1. Characterizing SLOW and FAST

We construct a simple framework. At date  $t$ , if the trailing 12-month excess return (arithmetic average monthly excess return),  $r_{t-12,t}$ , is nonnegative, then SLOW goes long one unit in the subsequent month, otherwise, it goes short one unit:

$$w_{\text{SLOW},t} := \begin{cases} +1 & \text{if } r_{t-12,t} \geq 0, \\ -1 & \text{if } r_{t-12,t} < 0. \end{cases} \quad (1)$$

If the prior 1-month return,  $r_{t-1,t}$ , is nonnegative then FAST goes long one unit in the subsequent month, otherwise, it goes short one unit:

$$w_{\text{FAST},t} := \begin{cases} +1 & \text{if } r_{t-1,t} \geq 0, \\ -1 & \text{if } r_{t-1,t} < 0. \end{cases} \quad (2)$$

<sup>10</sup> See [Jagannathan and Wang \(1996\)](#); [Lewellen and Nagel \(2006\)](#), and [Boguth et al. \(2011\)](#).

The realized returns of SLOW and FAST for month  $t+1$  are  $r_{\text{SLOW},t+1} = w_{\text{SLOW},t}r_{t+1}$  and  $r_{\text{FAST},t+1} = w_{\text{FAST},t}r_{t+1}$ , respectively, where  $r_{t+1}$  is the realized underlying market excess return from date  $t$  to date  $t+1$ .

This strategy design intentionally omits more complex features to be consistent with recent TS momentum studies ([Goyal and Jegadeesh, 2017](#), and [Huang et al., 2020](#)). We map trailing-return signals into binary weights rather than varying weights with signal magnitudes; the signal is not scaled by trailing volatility as in the work of [Moskowitz et al. \(2012\)](#), and the signal does not exponentially weight past returns.<sup>11</sup> Furthermore, we choose to operate on elementary strategies to facilitate our study in [Section 5](#) of dynamic speed selection, which is a challenging and central problem in trend following.

The 12-month trailing return is a relatively slow-moving signal from month to month since 11 of 12 months of returns (92%) overlap in subsequent signals.<sup>12</sup> In contrast, the 1-month trailing return is a relatively fast-moving signal, having no overlapping returns between subsequent signals. We could have used horizons longer than 12 months to define slow momentum, but the signal overlap for such horizons in subsequent months would not differ materially from the high 92% overlap of the 12-month signal. Moreover, 12 months is the standard horizon analyzed in the TS momentum literature ([Moskowitz et al., 2012](#), and [Huang et al., 2020](#), among others). We could have used horizons longer than one month (e.g., 2 or 3 months) to define fast momentum. However, longer horizons have significant signal overlap (50% for the 2-month signal and 67% for the 3-month signal), which materially slows changes in their strategy positions. Moreover, the 1-month horizon is the shortest at the monthly level of data. For these reasons, we chose 12-month and 1-month horizons.<sup>13</sup>

<sup>11</sup> Note that volatility targeting may constitute a phenomenon independent from TS momentum (e.g., [Kim et al., 2016](#), [Moreira and Muir, 2017](#), and [Harvey et al., 2018](#)). In Internet Appendix C, we further compare our approach with that of [Moskowitz et al. \(2012\)](#) and find that our medium-speed strategy (defined in [Section 2.3](#)) has a higher Sharpe ratio and generates alpha with respect to their 12-month strategy. In addition, trend following is often identified with moving-average crossover strategies, which exponentially weight past prices, whereas our SLOW and FAST portfolios average past returns. As shown by [Levine and Pedersen \(2016\)](#), moving-average crossovers and time-series momentum are close to equivalent. More generally, our focus is on blending strategies with different speeds rather than elaborating on a particular definition of a single speed. In Internet Appendix D, we further analyze TS momentum strategies with both binary and linear positions as functions of exponentially weighted moving averages of past returns.

<sup>12</sup> Note that we do not skip the immediately lagged month as is done in some momentum strategies on individual securities in order to avoid short-term idiosyncratic reversals. Our empirical application is stock indices, for which the immediately lagged returns tend not to exhibit such reversals.

<sup>13</sup> The empirical evidence pertaining to the U.S. stock market supports our rationale. Over our 50-year evaluation period, the 3-month TS momentum portfolio showed a higher correlation to 12-month momentum than to 1-month momentum (41% and 31%, respectively), whereas the correlation of the 24-month momentum portfolio to 12-month momentum was above 60%. In addition, the turnover of the 1-month signal was twice as large as that of the 3-month signal, another indication of the differences underlying two apparently similar strategies.



## 2.2. A model of time-varying trend

We can view TS momentum strategies as maps of indirect estimates of expected returns (based on lagged realized returns) to portfolio positions. SLOW and FAST map signs of average trailing realized returns (as estimates of signs of expected returns) to long or short positions when such estimates are positive or negative, respectively. The accuracy of such estimates depends on the persistence of expected returns as well as the influence of noise. To examine the roles of persistence and noise, we consider a first-order autoregressive process, AR(1), for expected returns. In this model, realized returns are noisy observations of the AR(1) process. We apply SLOW and FAST to these realized returns and explore their performance based on different parameterizations of the persistence parameter and the amount of noise.

Specifically, the unobservable monthly expected excess return process,  $\{\mu_t\}$ , follows

$$\mu_t = c + \varphi \mu_{t-1} + \varepsilon_t, \quad (3)$$

where  $c$  is a scalar,  $\varphi$  is a persistence scalar, and  $\{\varepsilon_t\}$  is a sequence of independent and identically distributed mean-zero normal random variables each having variance  $\sigma_\varepsilon^2 > 0$ . We assume that persistence is positive ( $\varphi > 0$ ) to capture the notion of trend, consistent with our description in Section 1. For example, if last month's underlying mean return were positive ( $\mu_{t-1} > 0$ ), then this month's mean return is more likely to also be positive ( $\mu_t > 0$ ) than negative. We also assume  $\varphi < 1$  so that the process is weak-sense stationary (i.e., the expected return process is time varying but its unconditional mean and autocovariances are time invariant). Under these assumptions, by standard AR(1) properties,  $\mu_t$  has unconditional expected value  $\mathbf{E}[\mu_t] = \frac{c}{1-\varphi}$  and unconditional variance  $\mathbf{Var}[\mu_t] = \frac{\sigma_\varepsilon^2}{1-\varphi^2} > 0$ . Let realized excess returns in each month,  $r_t$ , be random variables centered around  $\mu_t$ , but observed with noise:

$$r_t = \mu_t + z_t, \quad (4)$$

where  $\{z_t\}$  is a sequence of independent and identically distributed mean-zero normal random variables, and where each  $z_t$  has variance  $\sigma_z^2 > 0$  and is jointly normal with  $\varepsilon_t$  having correlation coefficient  $\lambda \in (-1, 1)$ . Then, the unconditional average return is  $\mathbf{E}[r_t] = \frac{c}{1-\varphi}$  and the unconditional return variance is  $\mathbf{Var}[r_t] = \sigma_r^2$ , where we define

$$\sigma_r = \sqrt{\frac{\sigma_\varepsilon^2}{1-\varphi^2} + \sigma_z^2 + 2\lambda\sigma_\varepsilon\sigma_z}.$$

We focus on the case in which  $c = 0$  (unconditional mean excess return is zero) to ensure that our analysis is nontrivial for all choices of persistence and noise parameters. In contrast, if unconditional mean excess returns were positive (negative), then for sufficiently small noise  $\sigma_z$ , realized returns would be predominantly positive (negative), and therefore, a profitable TS strategy would be trivially almost always long (short). Moreover, for  $\mathbf{E}[\mu_t] = 0$ , excess returns on TS momentum strategies are entirely alpha. Therefore, their performance is exclusively a measure of their ability to time return opportunities because their

average positions are unprofitable in expectation. Thus, expected returns are time varying but zero on average, deviating away from zero and remaining away with some persistence before eventually crossing zero to have the opposite sign, where they remain with some persistence until eventually crossing back, and so on.

Finally, define the noise ratio,  $\theta$ , as the fraction of variance in realized return  $r_t$  contributed by noise  $z_t$ :

$$\theta = \frac{\mathbf{Cov}[z_t, r_t]}{\mathbf{Var}[r_t]} = \frac{\sigma_z^2 + \lambda\sigma_\varepsilon\sigma_z}{\sigma_r^2} = \frac{\sigma_z^2 + \lambda\sigma_\varepsilon\sigma_z}{\frac{\sigma_\varepsilon^2}{1-\varphi^2} + \sigma_z^2 + 2\lambda\sigma_\varepsilon\sigma_z}. \quad (5)$$

If  $\theta$  is small, then realized and expected returns are close to each other and, therefore, realized returns are more informative about the underlying trend (i.e., the sign of expected returns). If  $\theta$  is large, then realized returns are dominated by noise and thereby less informative about the underlying trend.

**Proposition 1** characterizes in closed form the Sharpe ratios of our elementary trend-following strategies, SLOW and FAST (defined in Section 2.1), in terms of persistence  $\varphi$  and noise ratio  $\theta$ .

*Proposition 1. The (annualized) Sharpe ratios of FAST and SLOW, respectively, can be expressed in terms of persistence  $\varphi$  and noise ratio  $\theta$  as follows:*

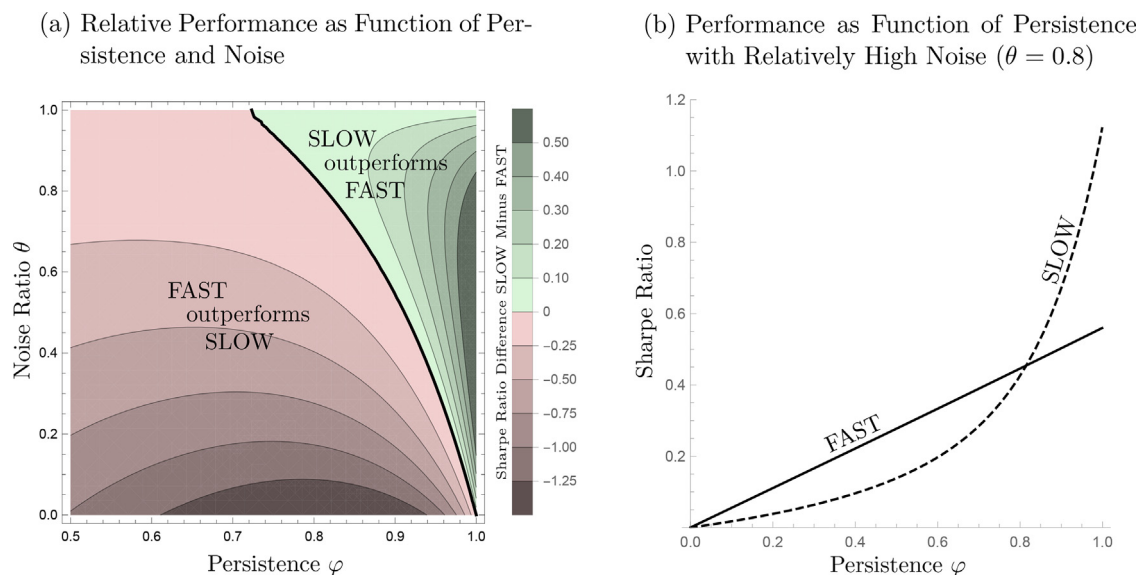
$$\text{Sharpe}[r_{\text{FAST},t}] = \frac{\sqrt{\frac{24}{\pi}}(1-\theta)\varphi}{\sqrt{1 - \frac{2}{\pi}(1-\theta)^2\varphi^2}}, \quad (6)$$

$$\begin{aligned} \text{Sharpe}[r_{\text{SLOW},t}] &= \frac{\sqrt{\frac{24}{\pi}}(1-\theta)\varphi}{\sqrt{\frac{(1-\theta)[12(1-\varphi^2)-2\varphi(1-\varphi^{12})]+12\theta(1-\varphi)^2}{(1-\varphi^{12})^2} - \frac{2}{\pi}(1-\theta)^2\varphi^2}}. \end{aligned} \quad (7)$$

Moreover, the denominators in (6) and (7) are proportional to the standard deviations of the respective strategies and, therefore, are positive.

*Proof.* See Internet Appendix A.  $\square$

**Proposition 1** indicates that both strategies have positive Sharpe ratios if there is some persistence ( $\varphi > 0$ ) and if  $\theta < 1$ . It is possible in general, however, that  $\theta$  is above one for certain values of model parameters ( $\varphi$ ,  $\sigma_\varepsilon$ ,  $\sigma_z$  and  $\lambda$ ) if realized return noise and expected return innovations are negatively correlated (i.e.,  $\lambda < 0$ ). Because we are studying momentum strategies, we are interested in those combinations of parameters for which conditional expected returns (given all past return observations) are positively related to past returns. Under this condition, **Proposition 2** shows that we can focus our attention on values of  $\theta < 1$ . Furthermore, if we impose the mild condition that the variance of noise in realized returns is at least half as large as the variance of innovations in expected returns ( $\sigma_z^2 \geq \frac{1}{2}\sigma_\varepsilon^2$ ), then **Proposition 2** indicates that we can focus our attention on values of  $\theta$  in the unit interval. In Section 4.1.3, we find that the inferred value of the noise ratio for U.S. stock market returns is consistent with this implication that  $\theta$  lies in the unit interval.



**Fig. 2.** SLOW and FAST absolute and relative performance.

*Notes:* Figure (a) plots contour levels of the difference between the Sharpe ratios of SLOW and FAST for various combinations of persistence levels and noise ratios. Combinations of persistence and noise ratios in the upper-right region above the bold curve correspond to a higher Sharpe ratio for SLOW than FAST. Figure (b) plots Sharpe ratios of SLOW (dashed) and FAST (solid) for various persistence levels and at noise ratio  $\theta = 0.8$ .

**Proposition 2.** If  $E[r_t | r_{t-1}, r_{t-2}, \dots]$  is positively related to past returns  $r_{t-1}, r_{t-2}, \dots$  (i.e.,  $\frac{\partial E[r_t | r_{t-1}, r_{t-2}, \dots]}{\partial r_{t-i}} > 0$  for all  $i = 1, 2, \dots$ ), then  $\theta < 1$ . If, in addition, we have  $\sigma_z^2 > \frac{1}{2}\sigma_\varepsilon^2$ , then  $\theta > 0$ .

*Proof.* See Internet Appendix A.  $\square$

Under the conditions of Proposition 2, the Sharpe ratios of SLOW and FAST are continuous functions over the bounded set of persistence ( $0 < \varphi < 1$ ) and noise ( $0 < \theta < 1$ ) parameters, so we can analyze these Sharpe ratios graphically on the unit square:  $(\varphi, \theta) \in [0, 1] \times [0, 1]$ . In Fig. 2, we compute and plot the absolute and relative levels of the Sharpe ratios expressed in Proposition 1 for a range of persistence and noise ratios. Figure 2(a) is a contour plot of the difference between the Sharpe ratios of SLOW and FAST for all noise levels and a range of persistence levels.<sup>14</sup> This figure reinforces the intuition that for sufficiently high persistence, SLOW outperforms FAST, and for sufficiently low noise, FAST outperforms SLOW. The literature has documented that SLOW tends to work better than FAST for many different asset classes when measured by overall risk-adjusted performance. From Fig. 2(a), we infer that such relative performance would be consistent with a market in which the excess returns exhibit both relatively high mean persistence and realized noise.<sup>15</sup> Figure 2(b) plots the Sharpe ratios of SLOW and FAST for various persistence

levels at a relatively high noise ratio. We infer from this figure that for noisy realized returns the outperformance of SLOW over FAST accelerates as mean persistence increases.

### 2.2.1. Model turning points

How do turning points affect the performance of SLOW and FAST? Although expected returns are unobservable in practice, we can use our model to analyze the hypothetical impact of (model) turning points (i.e., changes in signs of model conditional means). Uptrend corresponds to periods of positive expected returns,  $\{\mu_t : \mu_t > 0\}$ , and downtrend corresponds to periods of negative expected returns,  $\{\mu_t : \mu_t < 0\}$ . Turning points, therefore, occur whenever successive values of  $\{\mu_t\}$  change sign.<sup>16</sup> For developing intuition from this hypothetical exercise, we set  $\lambda = 0$ , which implies that innovations  $(\varepsilon_t, z_t)$  are uncorrelated and that  $\theta \in (0, 1)$ .

**Proposition 3.** Let  $\lambda = 0$ . The conditional expected returns of FAST and SLOW, respectively, given the most recent two (unobservable) consecutive return means are as follows:

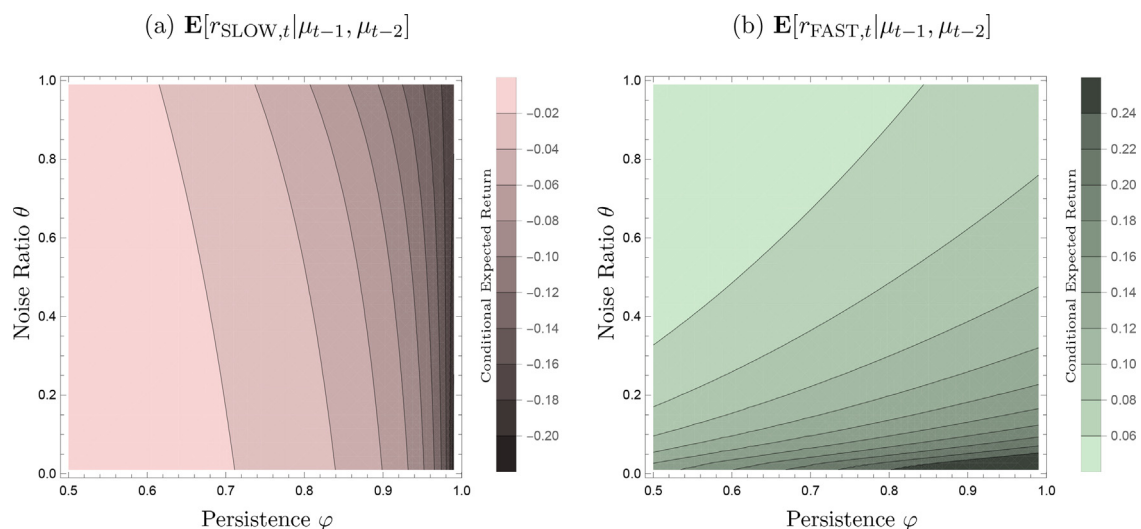
$$E[r_{\text{FAST},t} | \mu_{t-1}, \mu_{t-2}] = \varphi \mu_{t-1} \cdot \left[ 2\Phi\left(\frac{\mu_{t-1}}{\sigma_r \sqrt{\theta}}\right) - 1 \right], \quad (8)$$

$$\begin{aligned} E[r_{\text{SLOW},t} | \mu_{t-1}, \mu_{t-2}] \\ = \varphi \mu_{t-1} \cdot \left[ 2\Phi\left(\frac{\frac{1}{12}(\mu_{t-1} + \frac{1-\varphi^{11}}{1-\varphi} \mu_{t-2})}{\sigma_r \sqrt{\frac{(1-\theta)}{12} \left[ \frac{1+\varphi}{1-\varphi} + \frac{1}{12} - \frac{2(1-\varphi^{12}) + (1-\varphi^{11})^2}{12(1-\varphi)^2} \right] + \frac{\theta}{12}}} \right) - 1 \right], \end{aligned} \quad (9)$$

<sup>14</sup> We plot persistence above 0.5, however, the patterns evident in the plot extend continuously for lower persistence levels. We focus on the higher persistence levels to highlight the region where relative performance switches.

<sup>15</sup> In Section 4.1.3, we estimate the model-implied values of mean persistence and realized noise for U.S. stock market returns and find that both are relatively high.

<sup>16</sup> In subsequent sections, we develop a simple measure of turning points based on observable realized returns, consistent with our analysis in this section.



**Fig. 3.** Expected returns after model turning points.

*Notes:* This figure shows contour plots of the conditional expected returns of (a)  $E[r_{\text{SLOW},t}|\mu_{t-1}, \mu_{t-2}]$ , and (b)  $E[r_{\text{FAST},t}|\mu_{t-1}, \mu_{t-2}]$ , following a (model) turning point ( $\text{sign}(\mu_{t-1}) \neq \text{sign}(\mu_{t-2})$ ) over various values of mean persistence  $\varphi$  and noise ratios  $\theta$  for the parameters  $-\mu_{t-1} = \mu_{t-2} = 0.3\sigma_r$ , where  $\sigma_r = 1$ .

where  $\Phi(\cdot)$  is the standard normal cumulative distribution function. If  $\mu_{t-1} \neq 0$ , then  $E[r_{\text{FAST},t}|\mu_{t-1}, \mu_{t-2}] > 0$ . If  $\mu_{t-1}$  and  $\mu_{t-2}$  are each nonzero with different signs and  $|\mu_{t-1}| < \frac{1-\varphi^{11}}{1-\varphi}|\mu_{t-2}|$ , then

$$E[r_{\text{FAST},t}|\mu_{t-1}, \mu_{t-2}] > 0 > E[r_{\text{SLOW},t}|\mu_{t-1}, \mu_{t-2}]. \quad (10)$$

*Proof.* See Internet Appendix A.  $\square$

**Proposition 3** provides closed-form expressions for conditional expected excess returns of SLOW and FAST as functions of the expected returns of the previous two periods. The condition that  $\mu_{t-1}$  and  $\mu_{t-2}$  are each nonzero with different signs means we have encountered a (model) turning point. Moreover, (10) indicates that, after turning points, FAST tends to be profitable and SLOW tends to incur losses, on average. The condition  $|\mu_{t-1}| < \frac{1-\varphi^{11}}{1-\varphi}|\mu_{t-2}|$  derives from the conditional expected value of the 12-month signal,  $\frac{1}{12}\left(\mu_{t-1} + \frac{1-\varphi^{11}}{1-\varphi}\mu_{t-2}\right)$ , which appears in (9). This condition requires that the magnitude of the conditional expected value of the 1-month signal,  $\mu_{t-1}$ , was not excessively large compared to  $\mu_{t-2}$ , so that a single month's return was not expected to be large enough to flip the sign of the slow signal. For example, if  $\varphi = 0.8$ , and if two months ago the expected return was half-a-standard-deviation above zero,  $\mu_{t-2} = 0.5\sigma_r$ , then these conditions require that the most recent expected return turned negative but not more than about two-standard-deviations below zero,  $\mu_{t-1} \geq -2.3\sigma_r$ . Any turning point with opposite-sign expected returns of equal magnitude (e.g.,  $-\mu_{t-1} = \mu_{t-2} = 0.5\sigma_r$ ) satisfies this condition. Again, we can analyze the results of **Proposition 3** graphically.

**Figure 3** plots the conditional expected returns of (a) SLOW and (b) FAST after a turning point as given in **Proposition 3** for various values of mean persistence and

noise, with opposite-sign expected returns at 0.3-standard-deviation magnitude each,  $-\mu_{t-1} = \mu_{t-2} = 0.3\sigma_r$ , where  $\sigma_r = 1$ . After the turning point, expected returns are negative for SLOW and positive for FAST under all noise and persistence levels.<sup>17</sup> **Figure 3(a)** shows that the conditions where SLOW performs well overall correspond with the conditions where SLOW struggles most after turning points: relatively high mean persistence and realized noise. Therefore, the acceleration of overall performance of SLOW with higher persistence under relatively high noise (as shown in **Fig. 2(b)**) tends to come at the cost of exacerbating momentum crashes. **Figure 3(b)** shows that with lower noise levels or higher persistence levels, FAST delivers increasingly positive returns after turning points.

Opposite signs of expected returns of SLOW and FAST after a turning point imply that these strategies tend to take opposite positions after turning points. This observation indicates that the information in the union of SLOW and FAST strategy positions can be useful in detecting turning points. We further explore these connections in subsequent sections.

We fully recognize that the model assumptions we have employed do not hold in the real world. For example, returns are generally not normally distributed and expected returns need not follow AR(1). Throughout the remainder of the paper, we do not maintain the distributional assumptions introduced in this section unless explicitly referring back to this model. Instead, we employ the mild standing assumption that moments in analytical expressions and derivations exist and second moments are non-zero. In this sense, we characterize our subsequent results as being model free.

<sup>17</sup> We plot persistence above 0.5, however, the signs of expected returns in the plot remain the same for lower persistence levels.



### 2.3. Characterizing intermediate speeds and market cycles

We define a continuum of intermediate strategies with signal speeds between SLOW and FAST by

$$w_t(a) := (1 - a)w_{\text{SLOW},t} + a w_{\text{FAST},t}, \quad (11)$$

$$r_{t+1}(a) := w_t(a)r_{t+1} = (1 - a)r_{\text{SLOW},t+1} + a r_{\text{FAST},t+1}, \quad (12)$$

where the speed parameter  $a \in [0, 1]$  is a scalar. At  $a = 0$ , the speed is slow:  $w_t(0) = w_{\text{SLOW},t}$ . Here, if the preceding 12-month return is positive, the strategy is long one unit even when the most recent month's return is negative. At  $a = 1$ , the speed is fast:  $w_t(1) = w_{\text{FAST},t}$ . Here, any change of sign in the most recent month's return results in a strategy position change. For  $a \in (0, 1)$ , the speed is intermediate and, in particular, at  $a = \frac{1}{2}$  we call the speed "medium" (MED):

$$w_{\text{MED},t} := w_t\left(\frac{1}{2}\right) = \frac{1}{2}w_{\text{SLOW},t} + \frac{1}{2}w_{\text{FAST},t}, \quad (13)$$

with  $r_{\text{MED},t+1} := w_{\text{MED},t}r_{t+1}$ .

Blending slow and fast momentum portfolios to form intermediate-speed strategies is a key analytical choice. By doing so, we embed sensitivity to periods of disagreement between the slow and fast strategies, which potentially signal turning points. For example, when SLOW indicates a long position (+1) and FAST indicates a short position (−1), then the intermediate-speed strategy with speed  $a = \frac{3}{4}$  takes a lower magnitude short position ( $-\frac{1}{2} = \frac{1}{4}(+1) + \frac{3}{4}(-1)$ ). The MED portfolio is out of the market altogether ( $0 = \frac{1}{2}(+1) + \frac{1}{2}(-1)$ ). In contrast, strategies that go long or short based on the sign of some horizon of trailing returns,  $r_{t-k,t}$ , do not scale down their positions when signals of longer and shorter horizons disagree. For example, the sign of the six-month signal,  $r_{t-6,t}$ , could take several months to reflect a turning point in trend, during which time the strategy is fully long or short.<sup>18</sup>

Figure 4 maps the intersection of slow and fast signals to one of four observable market cycles or states. We use  $s_t$  to denote the state at date  $t$ . To capture the notion of a sustained upward trend, we label a month ending at date  $t$  as Bull if both the 12-month and 1-month trailing returns are nonnegative ( $r_{t-12,t} \geq 0$  and  $r_{t-1,t} \geq 0$ , on the upper-right quadrant of the diagram). We label the other three quadrants of the diagram (going clockwise) of trailing 12-month and 1-month returns as Correction, Bear, and Rebound, respectively. We will also use the abbreviations Bu, Co, Be, and Re to denote the respective market states (e.g.,  $s_t = \text{Re}$  for a Rebound). Admittedly, the labels assigned to these cycles have a relatively loose meaning. In particular, short-lived market gyrations could lead to brief and unintuitive correction or rebound classifications, which could be addressed by more sophisticated classification rules. Yet, because our simple cycle definitions are one

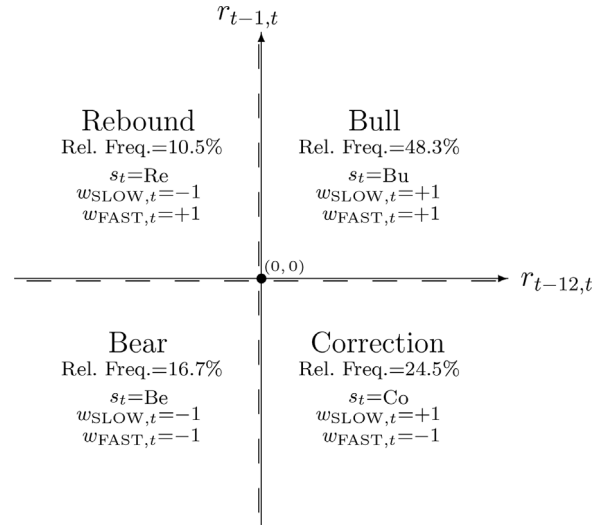


Fig. 4. Stock market cycles as a function of momentum.

Notes: This diagram defines the four cycles of an asset's price trajectory as a function of SLOW and FAST momentum positions. A month ending at date  $t$  is classified as Bull if both the trailing 12-month return (arithmetic average monthly return),  $r_{t-12,t}$ , is nonnegative, and the trailing 1-month return,  $r_{t-1,t}$ , is nonnegative. A month is classified as Correction if  $r_{t-12,t} \geq 0$  but  $r_{t-1,t} < 0$ ; as Bear if  $r_{t-12,t} < 0$  and  $r_{t-1,t} < 0$ ; and as Rebound if  $r_{t-12,t} < 0$  but  $r_{t-1,t} \geq 0$ . Also, the diagram reports the relative frequency of monthly aggregate U.S. stock market cycles over the 50-year period from 1969-01 to 2018-12. Market returns are U.S. excess value-weighted factor returns (Mkt-RF) from the Kenneth French Data Library.

to one with the four possible slow/fast strategy position pairs for the subsequent month, these cycles provide an exact ex ante specification of the slow and fast momentum strategies, and by extension, all intermediate speed strategies in (11) and (12).

Over our 50-year evaluation period, Bull months have been the most common with a relative monthly frequency of 48.3%, reflecting the average positive risk premium offered by the U.S. stock market. Bear months are relatively uncommon—approximately one-sixth of the time (16.7%)—whereas Correction and Rebound months amount to the remaining 35.0% of the months. In other words, about once every three months, on average, SLOW and FAST suggest a different position in the stock market. These phases will be the focus of much of our analysis.

Market cycles conveniently map to the properties of SLOW and FAST for analysis. For instance, consider the strategy in (13) that employs the medium-speed signal, which is the equally weighted average of the slow and fast strategies. Conditioning on the various market cycles as of date  $t$ , the equally weighted strategy return at date  $t + 1$ ,  $r_{\text{MED},t+1} = w_{\text{MED},t}r_{t+1}$ , has long-run expected value:

$$\begin{aligned} \mathbf{E}[r_{\text{MED},t+1}] &= \mathbf{E}\left[\frac{1}{2}(w_{\text{SLOW},t} + w_{\text{FAST},t})r_{t+1}\right] \\ &= \mathbf{E}\left[\frac{1}{2}(1 + 1)r_{t+1} | s_t = \text{Bu}\right] \mathbf{P}[s_t = \text{Bu}] \\ &\quad + \mathbf{E}\left[\frac{1}{2}(1 - 1)r_{t+1} | s_t = \text{Co}\right] \mathbf{P}[s_t = \text{Co}] \\ &\quad + \mathbf{E}\left[\frac{1}{2}(-1 - 1)r_{t+1} | s_t = \text{Be}\right] \mathbf{P}[s_t = \text{Be}] \\ &\quad + \mathbf{E}\left[\frac{1}{2}(-1 + 1)r_{t+1} | s_t = \text{Re}\right] \mathbf{P}[s_t = \text{Re}] \\ &= \mathbf{E}[r_{t+1} | \text{Bu}] \mathbf{P}[\text{Bu}] - \mathbf{E}[r_{t+1} | \text{Be}] \mathbf{P}[\text{Be}], \end{aligned} \quad (14)$$

<sup>18</sup> In Internet Appendix E, we compare the performance of the 6-month TS momentum strategy, MOM6, with our intermediate-speed strategies and find that across different metrics MOM6 performs similarly to the SLOW strategy (i.e., MOM12). This result is not surprising because 5 of 6 months (83%) in the 6-month signal,  $r_{t-6,t}$ , overlap in consecutive months. This overlap is close to that of the 12-month signal,  $r_{t-12,t}$  (92%).

where  $\mathbf{P}$  represents the probability measure. This framework can be extended beyond the calculation of expected returns. In particular, the nature of market cycles also has an impact on the Sharpe ratio of a trend strategy (see [Section 4.1](#)) and its skewness (see [Section 4.3](#)). Moreover, it opens the intriguing possibility of “speed timing” by strategically adjusting the speed parameter  $a$  across market states, a challenge so far not tackled, to the best of our knowledge, by any existing paper (see [Section 5](#)).

### 3. The economic linkages of market cycles

We further motivate the relevance of our cycles by evaluating their economic linkages.

#### 3.1. Market cycles and macroeconomic news

We study three groups of macro indicators. The first group includes four indices assembled by the Federal Reserve Bank of Chicago that cover major components of the U.S. economy—production, consumption, employment, and sales—and monetary policy shocks estimated by [Gertler and Karadi \(2015\)](#). The second group includes five risk-related indicators: (i) the National Financial Conditions Index (NFCI) by the Chicago Fed; (ii) the [Pastor and Staambaugh \(2003\)](#) liquidity innovation measure (PS Liquidity); (iii) the Treasury–Eurodollar (TED) spread; (iv) the liquidity metric for the Treasury bond market (Noise) developed by [Hu et al. \(2013\)](#); and (v) the high-volatility/low-volatility valuation spread (Vol Spread) of [Pflueger et al. \(2018\)](#). The last group of variables, which are based on media or survey data, includes: (i) the daily news-based Economic Policy Uncertainty Index (Policy) by [Baker et al. \(2016\)](#); (ii) the University of Michigan Consumer Sentiment Index; (iii) the Purchasing Manager’s Index (PMI) compiled by the Institute for Supply Management; (iv) the current-quarter recession probabilities by the Survey of Professional Forecasters (SPF); and (v) the next-quarter corporate profit expectations also from the SPF. In total, we collected 15 time series of macro indicators.

In [Table 1](#), we map the four market cycles contemporaneously to innovations in the 15 macro indicators. These innovations are the residuals from individual autoregressive models, whose order is determined by the Bayesian Information Criterion.<sup>19</sup>

Our evidence reveals a common theme. During Bull and Bear markets, when SLOW and FAST agree, we observe significant positive and negative macroeconomic shocks, respectively, for most economic variables. During Corrections and Rebounds, innovations are largely statistically insignificant. A potential shift or stall in the macro environment, captured by neither significant positive nor negative average innovations, tends to coincide with turning points in the cycles of the stock market. Only monetary policy shocks paint a somewhat different picture, with surprise

cuts associated with Rebound phases and hikes with Correction phases.

#### 3.2. Market cycles and business cycles

In [Fig. 5](#), we plot the relative frequencies of Bull, Correction, Bear, and Rebound months in early and late periods of NBER recessions and expansions over the most recent 50-year period of our sample.<sup>20</sup> Bear states are most frequent early in recessions. Their frequency plummets toward the end of recessions, stays low during early periods of expansions, and ramps up again at the tail end of expansions. Bull states are most infrequent early in recessions. Their frequency ramps up toward the end of recessions, stays high and continues to rise early in expansions, and gradually decreases toward the tail end of expansions. Correction states are rare during recessions. Their frequency increases during expansions through to the cusp of the next recession. Rebound states are second-most frequent early in recessions and ramp up toward the end of recessions. Rebound-state frequency sharply drops early in expansions and remains infrequent through to the cusp of the next recession.

This evidence connects the empirical fact we documented earlier in [Fig. 1](#), of predictably negative returns following Bear market states and predictably positive returns following Bull (and Rebound) market states, to the central finding of [Gómez-Cram \(2022\)](#) that returns are predictably negative early in recessions and rise to relatively high positive levels late in recessions. First, consider [Fig. 5\(a\)](#), which shows that Bear market states are most frequent early in recessions, whereas Bull market states are least frequent. In particular, Bears account for two-thirds of market states in the first three months of recessions, 14 times the relative frequency of Bulls during the same early recession period. Because Bear states, by definition, mean that the most recent month’s return is negative, a high relative frequency of Bear states concentrated early in recession periods implies sequences of predictably negative returns during these early periods. Second, consider [Fig. 5\(b\)](#), which shows that toward the end of recessions the relative frequency of Bear market states plummets while the relative frequencies of Bull and Rebound states ramp up to dominate the other states—over 90% relative frequency in the last three months. Because both Bull and Rebound states, by definition, mean the most recent month’s return is non-negative, the dominance of these states concentrated late in recessions implies the strongest periods of predictably positive returns occur late in recessions.

This evidence is consistent with the findings of [Cujean and Hasler \(2017\)](#) and [Gómez-Cram \(2022\)](#) that time-series momentum is concentrated in bad economic times. Consider the relative frequencies of negative- and positive-return months. Negative-return months correspond to the occurrence of Bear or Correction states and positive-return months correspond to the occurrence of Bull or Rebound states. In recessions, Bears and Corrections

<sup>19</sup> The only two exceptions to this approach consist of the liquidity innovation metric (PS Liquidity) constructed by [Pastor and Staambaugh \(2003\)](#) and the monetary policy shocks (MP Shock) estimated by [Gertler and Karadi \(2015\)](#).

<sup>20</sup> We obtain qualitatively similar results (unreported) over the longer sample based on data beginning in 1926.

**Table 1**  
Macro innovations  $t$ -statistics by market cycle.

Economy News					
Market Cycle	Production	Consumption	Employment	Sales	MP Shock
Bull	1.26	1.93	1.93	2.52	0.16
Correction	1.73	-0.32	1.15	-0.04	1.74
Bear	-3.60	-3.19	-4.25	-3.89	0.50
Rebound	-0.89	0.31	-0.63	-0.53	-3.60
Risk News					
Market Cycle	NFCI*	PS Liquidity	TED*	Noise*	Vol Spread
Bull	1.86	4.34	1.57	1.62	3.41
Correction	-1.23	-1.64	-2.33	-0.99	-0.93
Bear	-3.32	-6.32	-2.50	-3.72	-4.34
Rebound	2.01	0.92	2.89	2.03	-0.43
Survey News					
Market Cycle	Policy*	Consumer Sentiment	PMI	SPF Recession*	SPF Corp. Profits
Bull	1.72	1.64	3.30	2.17	1.94
Correction	-0.05	-0.05	-1.11	-1.28	-0.61
Bear	-4.27	-3.45	-2.88	-2.90	-2.78
Rebound	0.61	0.22	-2.13	0.90	0.23

Notes: This table reports the  $t$ -statistics of the innovations in 15 macro series conditional on four contemporaneous states of the U.S. stock market (Bull, Correction, Bear, and Rebound). These innovations are the residuals from individual autoregressive models, whose order is determined by the Bayesian Information Criterion. The only two exceptions to this approach consist of the Pastor and Stambaugh (2003) liquidity innovation metric (PS Liquidity) and the monetary policy shocks (MP Shock) estimated by Gertler and Karadi (2015). The symbol \* indicates that for the selected series the sign is reversed, so that a negative shock can be interpreted as a negative outcome. A month ending at date  $t$  is classified as Bull if both the trailing 12-month return (arithmetic average monthly return),  $r_{t-12,t}$ , is nonnegative, and the trailing one-month return,  $r_{t-1,t}$ , is nonnegative. A month is classified as Correction if  $r_{t-12,t} \geq 0$  but  $r_{t-1,t} < 0$ ; as Bear if  $r_{t-12,t} < 0$  and  $r_{t-1,t} < 0$ ; and as Rebound if  $r_{t-12,t} < 0$  but  $r_{t-1,t} \geq 0$ . The starting and ending dates of the macro series vary according to availability from the original sources, and they are listed in Internet Appendix B. The stock-market returns are U.S. excess value-weighted factor returns (Mkt-RF) from the Kenneth French Data Library.

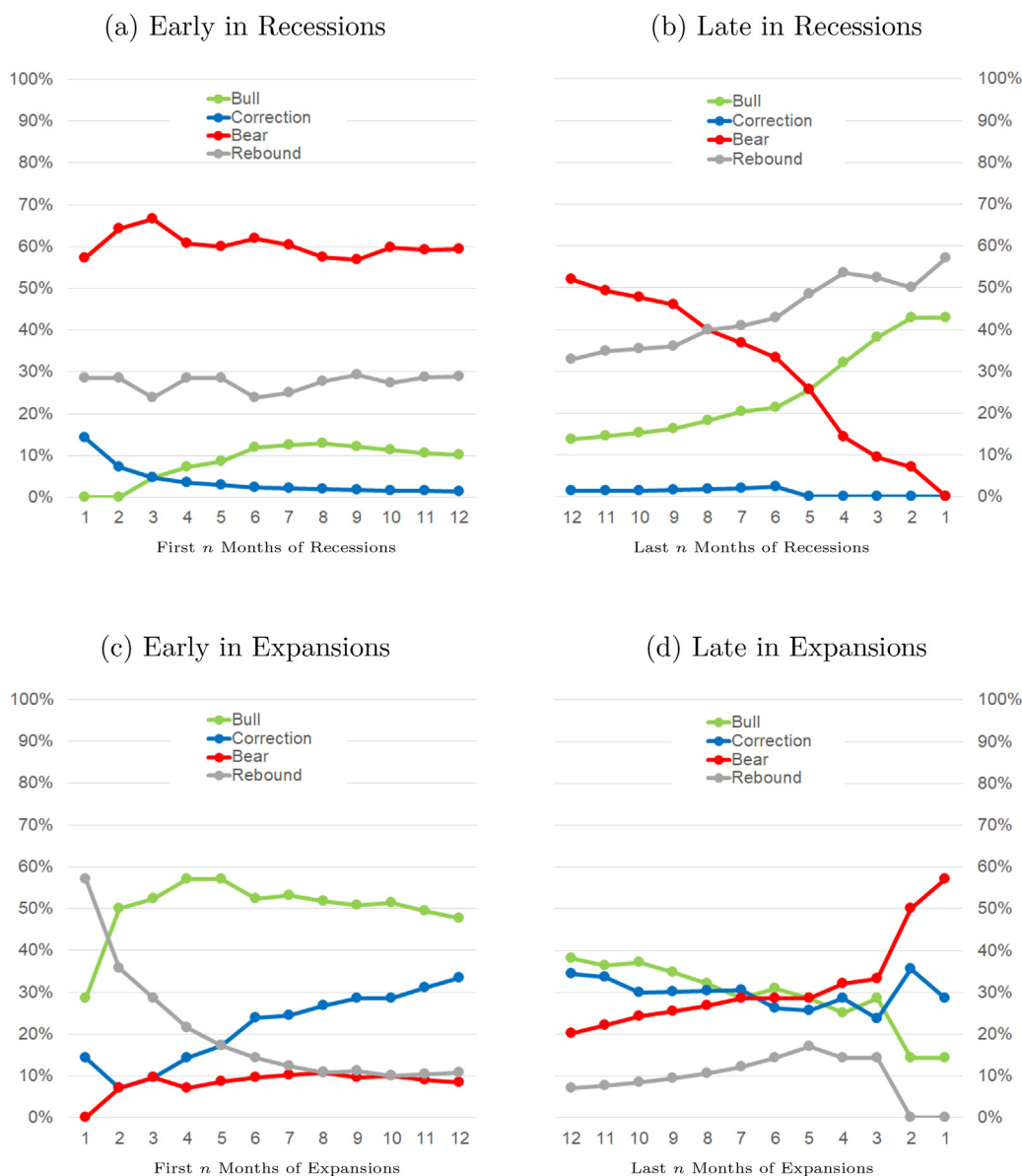
are at their highest combined frequency (over 70% early in recessions) and Bulls and Rebounds are at their highest combined frequency (over 90% late in recessions). The concentrated sequences of these pairs of states occurring in recessions suggest that negative returns cluster the most early in recessions and positive returns cluster the most late in recessions. This evidence suggests that recessions tend to possess the strongest periods of both negative momentum (early) and positive momentum (late).

### 3.3. Economic implications

Given the close connections between market cycles and macroeconomic risk states, predictably negative returns after Bear states pose a puzzle for traditional macrofinance asset-pricing models. These models generally predict countercyclical risk premia: compensation for risk taking is positive and relatively high in bad times. As documented in Section 3.1, macro news is generally negative in Bear states, indicating the tendency of Bear states to coincide with bad times. Moreover, as discussed in Section 3.2, the earliest periods in recessions, in which the depth of the recession has not yet been realized, are arguably the periods of greatest economic uncertainty and, therefore, should be the periods of highest expected returns under these models. Thus, the high relative frequency of Bear states in early recession periods is especially difficult for these models to rationalize.

These findings parallel those of Gómez-Cram (2022), who constructs a state-space model estimated from a collection of macroeconomic variables to show that returns are predictably negative in the early periods of recessions and rise to high positive expectations in later periods of recessions. We arrive at these puzzling facts in a simpler way using only trailing stock market returns, but the central implications for models of time-varying risk premia are shared with those explored by Gómez-Cram. Specifically, Gómez-Cram analyzes rational models including the conditional CAPM, several popular macrofinance models that relate economic state variables to the equity premium through the main asset-pricing equation, and several models that impose economic restrictions based on the present-value identity.<sup>21</sup> Implications of these models include: stock market returns are countercyclical, early periods of recessions support uptrend rather than downtrend, and strategies that reduce exposure to risk during bad times do not generate positive unconditional alpha. These implications are incompatible with our evidence. In addition, in subsequent sections we develop momentum strategies formed using our market cycles that generate positive unconditional alpha, which such models cannot account for.

<sup>21</sup> See Gómez-Cram (2022) and the references therein.



**Fig. 5.** Relative frequencies of U.S. market cycles in early and late stages of nber recessions and expansions.

*Notes:* This figure plots the relative frequencies of market-cycle months during the (a) first and (b) last  $n$  months of NBER recessions and (c) the first and (d) last  $n$  months of NBER expansions from 1969-01 to 2018-12. A month ending at date  $t$  is classified as either an NBER recession or expansion month based on the NBER recession-indicator time series from FRED. A month ending at date  $t$  is classified as Bull if both the trailing 12-month return (arithmetic average monthly return),  $r_{t-12,t}$ , is nonnegative and the trailing 1-month return,  $r_{t-1,t}$ , is nonnegative. A month is classified as Correction if  $r_{t-12,t} \geq 0$  but  $r_{t-1,t} < 0$ ; as Bear if  $r_{t-12,t} < 0$  and  $r_{t-1,t} < 0$ ; and as Rebound if  $r_{t-12,t} < 0$  but  $r_{t-1,t} \geq 0$ . Market excess returns are U.S. excess value-weighted factor returns (Mkt-RF) from the Kenneth French Data Library.

Behavioral models of slow information updating, however, have the potential to explain the evidence. In particular, Gómez-Cram (2022) develops a model in which investors use the present-value identity to value stocks but discount future cash flows using subjective expectations that are slow to reflect new information, such as worsening economic conditions. In this behavioral framework, the subjective cash-flow component of expected returns can offset the rational risk-premia component to induce neg-

ative return predictability when the economy is transitioning to recession. Moreover, later in recessions as economic conditions are more fully reflected in beliefs, the impact of the subjective component reduces and expected returns revert to the countercyclical behavior inherent in rational frameworks.

Furthermore, our results suggest a role for the sense of cyclicity itself to vary over time in models of time-varying risk premia. Bull, Correction, Bear, and Rebound

**Table 2**  
Performance summary by speed.

	Market	$a = 0$ SLOW	$a = \frac{1}{4}$	$a = \frac{1}{2}$ MED	$a = \frac{3}{4}$	$a = 1$ FAST
<i>Return and Risk</i>						
Average (%) (anlzd.)	5.91	6.46	6.17	5.88	5.59	5.30
Volatility (%) (anlzd.)	15.64	15.62	12.72	11.60	12.74	15.66
Sharpe Ratio (anlzd.)	0.38	0.41	0.48	0.51	0.44	0.34
<i>Market Timing</i>						
Average Position	1.00	0.46	0.39	0.32	0.25	0.18
Market Beta	1.00	0.15	0.05	−0.04	−0.13	−0.23
Alpha (%) (anlzd.)	0.00	5.58	5.85	6.12	6.39	6.66
Alpha $t$ -statistic	—	2.55	3.26	3.73	3.59	3.09
<i>Tail Behavior</i>						
Skewness	−0.55	−0.43	−0.13	0.02	0.03	0.15
Max. Drawdown (%)	−54.36	−43.43	−37.96	−34.43	−34.07	−44.53
Average (anlzd.)/ Max. DD	0.11	0.14	0.16	0.17	0.17	0.12

Notes: This table reports the sample average, volatility, Sharpe ratio, average position, market beta and alpha, alpha  $t$ -statistic, skewness, maximum drawdown, and ratio of average return to absolute maximum drawdown for monthly returns of the long-only market strategy (Market) and of TS momentum strategies of various speeds. The slow strategy weight applied to the market return in month  $t + 1$ ,  $w_{\text{SLOW},t}$ , equals +1 if the trailing 12-month return (arithmetic average monthly return) is nonnegative, and otherwise equals −1. The fast strategy weight,  $w_{\text{FAST},t}$ , equals +1 if the trailing 1-month return is nonnegative, and otherwise equals −1. Intermediate-speed strategy weights,  $w_t(a)$  are formed by mixing slow and fast strategies with mixing parameter  $a$ :  $w_t(a) = (1 - a)w_{\text{SLOW},t} + aw_{\text{FAST},t}$ , for  $a \in [0, 1]$ . Strategy returns are formed as  $r_{t+1}(a) = w_t(a)r_{t+1}$ , where  $r_{t+1}$  is the U.S. excess value-weighted factor return (Mkt-RF) from the Kenneth French Data Library. The evaluation period is 1969-01 to 2018-12.

**Table 3**  
Market-cycle decomposition of returns by speed.

Panel A: Average Returns						
Average (%) (anlzd.)	Market	$a = 0$ SLOW	$a = \frac{1}{4}$	$a = \frac{1}{2}$ MED	$a = \frac{3}{4}$	$a = 1$ FAST
Unconditional	5.91	6.46	6.17	5.88	5.59	5.30
Cycle-Conditional Decomposition						
Bull	4.59	4.59	4.59	4.59	4.59	4.59
Correction	1.59	1.59	0.79	0.00	−0.79	−1.59
Bear	−1.29	1.29	1.29	1.29	1.29	1.29
Rebound	1.01	−1.01	−0.51	0.00	0.51	1.01
Bull + Bear	3.31	5.88	5.88	5.88	5.88	5.88
Correction + Rebound	2.60	0.58	0.29	0.00	−0.29	−0.58
Panel B: Variance of Returns						
Variance (%) (anlzd.)	Market	$a = 0$ SLOW	$a = \frac{1}{4}$	$a = \frac{1}{2}$ MED	$a = \frac{3}{4}$	$a = 1$ FAST
Unconditional	2.44	2.44	1.61	1.34	1.62	2.45
Cycle-Conditional Decomposition						
Bull	0.63	0.63	0.63	0.63	0.63	0.63
Correction	0.77	0.77	0.19	0.00	0.20	0.79
Bear	0.73	0.71	0.71	0.71	0.71	0.71
Rebound	0.31	0.32	0.08	0.00	0.08	0.31
Bull + Bear	1.36	1.34	1.34	1.34	1.34	1.34
Correction + Rebound	1.09	1.10	0.27	0.00	0.28	1.10

Notes: This table reports the unconditional sample average (Panel A) and sample variance (Panel B) of monthly returns of the long-only market strategy and of TS momentum strategies of various speeds and their contribution in months immediately following various market states. Contributions in each column sum to their corresponding unconditional values in the first row of each panel (subject to rounding) either across the four individual states in the middle rows or across the two state pairs in the last two rows. The long-only market strategy (Market) and TS momentum strategies of various speeds are the same as those described in Table 2. Market states,  $s_t$ , are defined as follows. Bull:  $w_{\text{SLOW},t} = w_{\text{FAST},t} = +1$ ; Correction:  $w_{\text{SLOW},t} = +1$ ,  $w_{\text{FAST},t} = -1$ ; Bear:  $w_{\text{SLOW},t} = w_{\text{FAST},t} = -1$ ; and Rebound:  $w_{\text{SLOW},t} = -1$ ,  $w_{\text{FAST},t} = +1$ . The evaluation period is 1969-01 to 2018-12.

states can each occur in any period of the business cycle. As shown in Fig. 5, there are periods within both the high-risk state of recessions and the low-risk state of expansions in which Bear states are most likely or Bull and Rebound states are most likely. For example, we see a ramp up in the relative frequency of Bear states late in expansions, which is not inconsistent with countercyclical risk premia.

In addition, the spread between the predicted expected returns of Bull and Bear states can be economically meaningful in any period of the business cycle.<sup>22</sup> We conjecture that a model of subjective beliefs with reaction speeds to

<sup>22</sup> Over the period 1927-07 to 2018-12, the annualized Bull–Bear spread is 16.0% unconditionally, 23.7% in recessions, and 7.5% in expansions.



macro news that vary as a function of the market cycle could help reconcile variation in cyclicity with the prevailing view of wholly countercyclical risk premia.

#### 4. Static speed selection

In this section, we analyze momentum strategies of various intermediate speeds defined as in (11), (12), and (13). Our empirical analyses focus on U.S. aggregate stock market returns over a recent 50-year period.<sup>23</sup> For most of the paper, we evaluate performance over a recent 50-year period for consistency with later analyses in which we use earlier data to warm up our dynamic strategies for out-of-sample evaluation. In Internet Appendix F, we report results for U.S. equities based on a longer evaluation period beginning in 1927, from which we draw similar inferences as in our main analyses.

Table 2 summarizes estimates of basic unconditional moments for TS momentum strategies of various speeds alongside the long-only market strategy, whose strategy weights equal to one every month. Results are gross of transaction costs.<sup>24</sup> We highlight these results in three parts: return and risk, market timing, and tail behavior.

*Return and risk.* First, momentum strategies of every speed exhibit positive average historical returns in our backtest. Although they all report average returns similar in magnitude to the long-only market strategy, intermediate-speed strategies achieve their returns with lower volatilities, and therefore report the highest average return per unit of risk as measured by the Sharpe ratio (see Section 4.1).

*Market timing.* Second, momentum strategies of every speed have positive average positions in the underlying market (ranging from 0.18 to 0.46 long, on average), yet their market betas are relatively small in magnitude with negative point estimates for intermediate-to-fast speeds. Intermediate-speed strategies are approximately market-beta neutral and deliver alphas having the largest *t*-statistics (see Section 4.2).

*Tail behavior.* Third, momentum strategies of every speed exhibit more desirable skewness than the market strategy. Higher positive (or less negative) skewness is a desirable property for most investors, and skewness of intermediate-to-fast strategies performs best in this regard. Moreover, intermediate-speed strategies have less severe maximum drawdowns and record the highest average returns per unit of drawdown risk (see Section 4.3).

These three categories of results emphasize several merits of intermediate-speed momentum strategies. In par-

ticular, MED delivers the highest Sharpe ratio (0.51) and it does so with approximate market beta neutrality (beta of  $-0.04$ ), delivering the most statistically significant alpha (*t*-statistic of 3.73), without exhibiting the negative skewness behavior of the underlying market nor the severe drawdown behavior of the market and the slow and fast strategies. In the following three subsections, we identify properties of the stock market's return distribution and develop related theory to help explain these findings.

##### 4.1. Return and risk

###### 4.1.1. Conditional return contributions

Table 3 (Panel A) decomposes the unconditional average returns in the first row of Table 2 into their conditional contributions following each market cycle. Specifically, we employ the following decomposition of returns using zero-one indicators of the market cycle  $s_t$  at date  $t$ :

$$r_{t+1} = r_{t+1} 1_{\{s_t=Bu\}} + r_{t+1} 1_{\{s_t=Be\}} + r_{t+1} 1_{\{s_t=Co\}} + r_{t+1} 1_{\{s_t=Re\}}. \quad (15)$$

For example, the conditional contribution to speed-*a* TS momentum strategy after Bull cycles is the sample estimate of  $E[r_{t+1}(a)1_{\{s_t=Bu\}}]$ . For the market excess return, which has unconditional expectation 5.91%, its conditional contribution after Bull cycles is 4.59%, which is the product of its conditional expected value after Bull cycles, 9.5%, and the relative frequency of Bull cycles, 0.483 (cf. Fig. 1). The first row of Panel A repeats the unconditional values reported in Table 2 for ease of reference. The middle four values in each column (and the last two values in each column) sum to the value in the first row (subject to rounding).

First, consider the decomposition of average returns to the slow strategy as compared with the long-only market strategy. After Bulls and Corrections, SLOW is long one unit, such as the market strategy, and so gets the same contribution as the market after these phases, namely, 4.59% and 1.59%, respectively. After Bears and Rebounds, SLOW shorts the market and flips the sign of the market contribution from  $-1.29\%$  to  $1.29\%$  and from  $1.01\%$  to  $-1.01\%$  after Bears and Rebounds, respectively.

Second, consider the average return contributions across different speeds. TS momentum strategies of all speeds get the same contribution after Bull and Bear cycles because each is long after Bulls and short after Bears. These states account for the bulk of the average returns to all strategies. Contributions after Corrections and Rebounds explain the differences in average returns by speed. SLOW's loss after Rebounds is similar in magnitude to FAST's loss after Corrections. The net contribution to SLOW after Corrections and Rebounds is 0.58%. In contrast, FAST flips the sign relative to SLOW after Correction and Rebound cycles, yielding a net contribution of  $-0.58\%$ . MED is simply out of the market after Corrections and Rebounds. These facts explain why the unconditional average returns are close to each other and to that of the long-only market strategy.

Table 3 (Panel B) decomposes the unconditional variance of returns (squares of volatilities reported in second row of Table 2) into their cycle-conditional contributions following each observed market cycle based on

<sup>23</sup> Recent studies focus on applications of momentum to aggregate factors. Ehsani and Linnainmaa (2019) show evidence of TS momentum across equity factors that appears to subsume the CS momentum risk factor. A companion paper by Arnott et al. (2023) shows that CS factor momentum is a pervasive phenomenon that, moreover, drives industry momentum. Gupta and Kelly (2019) show that portfolios of TS factor momentum can add value to a wide array of investment strategies.

<sup>24</sup> We estimate transaction costs to be modest under conservative assumptions. Let the bid-ask spread on S&P 500 Index futures be 2 basis points (bps), roll costs 0.5 bps (with rolls happening every quarter), and commissions 0.1 bps; then the ratio of alpha to transaction costs does not surpass 3% for the strategies reported in Table 2.

the same cycle decomposition of returns as in (15). The cycle-conditional variance contributions are computed as the covariances of each of the terms on the right-hand side of (15) with the overall return. For example, the conditional variance contribution to speed- $a$  TS momentum strategy after Bull cycles is the sample estimate of  $\text{Cov}[r_{t+1}(a)1_{\{\text{Bu}\}}, r_{t+1}(a)]$ . The middle four values in each column (and the last two values in each column) sum to the value in the first row (subject to rounding).

The majority of return variance for all speeds of TS momentum strategies and for the long-only market strategy are generated after Bull and Bear markets. However, intermediate-speed strategies scale down their market exposure relative to SLOW and FAST after Corrections and Rebounds and, therefore, experience lower exposure to variance risk emanating from these states. In particular, after these periods of disagreement, MED exits the market altogether and avoids any exposure to the variance risk of these states. These facts explain why the unconditional return variances (and volatilities) are lower for intermediate-speed strategies compared to SLOW and FAST and compared to the long-only market strategy.

#### 4.1.2. Mapping SLOW/FAST disagreement to Sharpe ratios

When one blends two strategies that are relatively uncorrelated, such as SLOW and FAST, it is not that surprising to see the variance decrease and the Sharpe ratio increase. In this subsection, however, we provide an alternative characterization of the Sharpe ratio in terms of a disagreement multiplier, which isolates the volatility contribution from states of disagreement between SLOW and FAST after Corrections and Rebounds. This multiplier is also useful for understanding the skewness of a blend, a topic we discuss in Section 4.3.

The role of such disagreement in determining Sharpe ratios manifests analytically as a *disagreement multiplier*,  $D(a)$ , which appears in (19) of Proposition 4.  $D(a)$  captures the ratio of the volatility of the unconditional market return to the volatility of the momentum strategy, in terms of volatility contributions from Bull or Bear states and from Correction or Rebound states, respectively.<sup>25</sup>

**Proposition 4** (Sharpe ratio decomposition). *The Sharpe ratio of  $r_{t+1}(a)$  can be expressed in terms of the Sharpe ratios of  $r_{\text{SLOW},t+1}$  and  $r_{\text{FAST},t+1}$ , respectively, and the market cycles, as follows:*

$$\begin{aligned} \text{Sharpe}[r_{t+1}(a)] = & (1-a) \text{Sharpe}[r_{\text{SLOW},t+1}]D(a, \mathbf{E}[r_{\text{SLOW},t+1}]) \\ & + a \text{Sharpe}[r_{\text{FAST},t+1}]D(a, \mathbf{E}[r_{\text{FAST},t+1}]), \end{aligned} \quad (16)$$

<sup>25</sup> As before, let  $r_{t+1}$  denote the return on an underlying security and recall that the speed- $a$  strategy return,  $r_{t+1}(a)$ , is just the weighted average of SLOW and FAST with weight  $a$  applied to FAST. Also, let  $\{\text{Bu}\}_{\text{Be}}$  denote

the union of events Bull or Bear, and  $\{\text{Co}\}_{\text{Re}}$  the union of events Correction or Rebound. We apply similar notation throughout the paper for the union of any of the four market states.

where

$$D(a, \mu) := \sqrt{\frac{\mathbf{E}[r_{t+1}^2] - \mu^2}{\mathbf{E}[r_{t+1}^2 | \text{Bu}] \mathbf{P}[\text{Bu}] + (2a-1)^2 \mathbf{E}[r_{t+1}^2 | \text{Co}] \mathbf{P}[\text{Co}] - (\mathbf{E}[r_{t+1}(a)])^2}}. \quad (17)$$

Approximating squared average strategy returns by  $(\mathbf{E}[r_{t+1}(a)])^2 \approx 0$  for  $a \in [0, 1]$ , we have

$$\text{Sharpe}[r_{t+1}(a)] \approx ((1-a) \text{Sharpe}[r_{\text{SLOW},t+1}] + a \text{Sharpe}[r_{\text{FAST},t+1}])D(a), \quad (18)$$

where the term  $D(a)$ , multiplying the weighted average of Sharpe ratios in (18), is

$$D(a) := D(a, 0) = \sqrt{\frac{\mathbf{E}[r_{t+1}^2]}{\mathbf{E}[r_{t+1}^2 | \text{Bu}] \mathbf{P}[\text{Bu}] + (2a-1)^2 \mathbf{E}[r_{t+1}^2 | \text{Co}] \mathbf{P}[\text{Co}]}}. \quad (19)$$

$D(a)$  is greater than or equal to one and is maximized at  $a = \frac{1}{2}$  on  $a \in [0, 1]$ :

$$D(\frac{1}{2}) \geq D(a) \geq 1, \quad a \in [0, 1]. \quad (20)$$

Moreover,  $a = \frac{1}{2}$  is the unique maximizer with  $D(\frac{1}{2}) > 1$  if the relative frequency of such states is not zero so that  $\mathbf{E}[r_{t+1}^2 | \text{Co}] \mathbf{P}[\text{Co}] > 0$ .

*Proof.* See Internet Appendix A.  $\square$

Proposition 4 indicates that the risk-adjusted performance of intermediate-speed momentum strategies ( $0 < a < 1$ ) is greater than the  $a$ -weighted average risk-adjusted performances of the slow and fast momentum strategies, taken separately, as long as squared expected returns are relatively small and can be approximated by zero.<sup>26</sup> In particular, (18) states that the Sharpe ratio of a strategy with speed  $a$  is approximated by the product of two components: (A) the  $a$ -weighted blend of the separate Sharpe ratios of SLOW and FAST, and (B) the disagreement multiplier,  $D(a)$ . In general, the speed  $a^*$  of the strategy that maximizes the Sharpe ratio depends on both of these components.

Proposition 4 states that component (B) is maximized at  $a = \frac{1}{2}$ . Intermediate-speed strategies reduce volatility originating from Correction and Rebound states. Such volatility exposure is largely captured by the conditional average squared returns following these states,  $\mathbf{E}[r_{t+1}^2 | \text{Co}]$ , times the relative frequency of these states,  $\mathbf{P}[\text{Co}]$ . Hence,

$\mathbf{E}[r_{t+1}^2 | \text{Co}] \mathbf{P}[\text{Co}]$  represents the frequency-weighted contribution of these states to the unconditional volatility of the strategy. Intermediate-speed strategies with parameter  $a$

<sup>26</sup> For example, if the average return is 5%, then its square is only 0.25%. Note that the approximation is exact at the endpoints of the interval  $[0, 1]$ .

in the vicinity of  $\frac{1}{2}$  tend to scale down positions following such periods, boosting the overall risk-adjusted performance by reducing exposure to risk. The factor  $(2a - 1)^2$  multiplying  $\mathbf{E}[r_{t+1}^2 | \text{Co}] \mathbf{P}[\text{Co}]$  captures this scaling-down, and this reduction in the denominator boosts the disagreement multiplier  $D(a)$  above one. At  $a = \frac{1}{2}$ , volatility contributions from Correction and Rebound states are completely eliminated ( $(2(\frac{1}{2}) - 1)^2 = 0$ ).

If the Sharpe ratios of SLOW and FAST are sufficiently different from each other, then component (A) can play a dominant role. For example, suppose the Sharpe ratio of SLOW ( $a = 0$ ) is much larger than that of FAST. Although component (B) is maximized at  $a = \frac{1}{2}$ , it might not be large enough to overcome component (A)'s reduced contribution at  $a = \frac{1}{2}$  to the product in (18). In this example,  $a^*$  could be 0.

However, as a corollary, if the Sharpe ratios of SLOW and FAST are sufficiently close to each other, then component (A) is relatively insensitive to  $a$ , and so  $a^*$  depends primarily on the disagreement multiplier. In this case,  $a^* \approx \frac{1}{2}$ . This corollary helps explain MED's relatively high Sharpe ratio in Table 2 because average returns are roughly the same magnitude across all speeds. Moreover, we can apply this corollary to the situation in which one does not have conviction about which of SLOW and FAST will have the higher Sharpe ratio ex ante. Compared to a randomized experiment of choosing to invest in either SLOW or FAST, this corollary informs us that MED has a higher ex ante Sharpe ratio than the expected Sharpe ratio of the experiment.

Furthermore, because Proposition 4 is a model-free result, it extends beyond our running empirical example of the U.S. stock market and beyond our model of Section 2.2 to momentum strategies of various speeds applied in any market. We examine the performance of international equity markets in Section 6 and of equity factors in Internet Appendix G. Because equity factors take offsetting long and short positions in equity subportfolios, the economic linkages to equity returns discussed in Section 3 may not apply to factor returns. However, the general statistical properties outlined in Proposition 4 still apply to equity factors: under mild assumptions, the Sharpe ratio of MED is higher than the average of the Sharpe ratios of SLOW and FAST. In addition, for each of the equity factors in Internet Appendix G, there is an intermediate-speed strategy that has both Sharpe ratio and alpha  $t$ -statistic higher than those of both SLOW and FAST.

#### 4.1.3. Mapping SLOW/FAST Sharpe ratios to mean persistence and realized noise levels in market returns

We now use the empirical results of the momentum strategies reported in Table 2 together with the model of Section 2.2 to compute implications for the persistence behavior of expected returns and the noise-level behavior of realized returns. We also discuss the optimal static blend under the model.

Using (6) and (7) of Proposition 1, we map our empirical evidence into Fig. 2(a). Specifically, we have two equations of the model Sharpe ratios of FAST and SLOW (respectively, (6) and (7)) in terms of two unknowns: per-

sistence in expected returns,  $\varphi$ , and the noise ratio  $\theta$ . We jointly estimate  $(\hat{\varphi}, \hat{\theta})$  as the root of these two Sharpe ratio equations each set equal to their corresponding empirical Sharpe ratio values:

$$0.34 = \frac{\sqrt{\frac{24}{\pi}}(1 - \hat{\theta})\hat{\varphi}}{\sqrt{1 - \frac{2}{\pi}(1 - \hat{\theta})^2(\hat{\varphi})^2}}, \quad (21)$$

$$0.41 = \frac{\sqrt{\frac{24}{\pi}}(1 - \hat{\theta})\hat{\varphi}}{\sqrt{\frac{(1 - \hat{\theta})[12(1 - \hat{\varphi})^2 - 2\hat{\varphi}(1 - \hat{\varphi})^{12}] + 12\hat{\theta}(1 - \hat{\varphi})^2}{(1 - \hat{\varphi})^{12}} - \frac{2}{\pi}(1 - \hat{\theta})^2(\hat{\varphi})^2}}, \quad (22)$$

where 0.34 and 0.41 are the empirical Sharpe ratios of FAST and SLOW, respectively, reported in Table 2. We obtain  $(\hat{\varphi}, \hat{\theta}) = (0.85, 0.86)$ , rounded to the nearest percent. As a robustness check to potential estimation error, we perform 1000 block-bootstrap estimates of  $(\hat{\varphi}, \hat{\theta})$  based on FAST/SLOW Sharpe ratio pairs computed from 10 non-overlapping 5-year blocks from our 50-year history, sampled with replacement, plugged into (21) and (22), respectively.

Figure 6(a) plots these parameter pair estimates on the unit square. Figure 6(a) also plots the 5th percentiles (dashed lines) for the bootstrapped estimates of each of  $\hat{\varphi}$  and  $\hat{\theta}$ . These results suggest that expected returns have relatively high persistence (0.85 point estimate, above 0.68 in 95% of bootstrap estimates) and that realized returns have relatively high noise (0.86 point estimate, above 0.72 in 95% of bootstrap estimates).

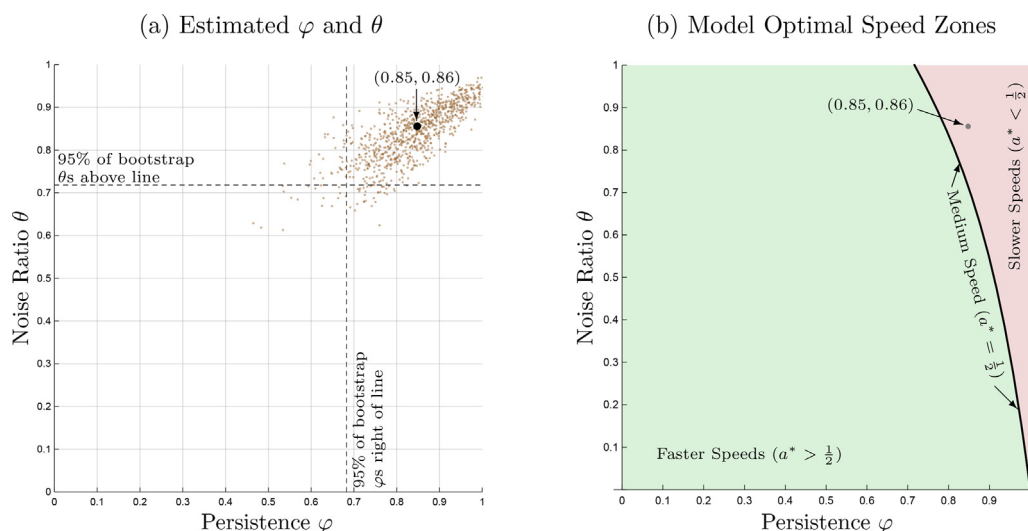
The unconditional Sharpe ratio of a static blend (12) of FAST and SLOW under the model of Section 2.2 cannot be analyzed in general in closed form, so we do not provide a general solution for the speed parameter  $a^* \in [0, 1]$  that maximizes the unconditional Sharpe ratio.<sup>27</sup> However, when the Sharpe ratios of FAST and SLOW are equal, Proposition 5 indicates that the equal-blend MED generates the maximum Sharpe ratio among all static blends under the model.

**Proposition 5** (Maximization when Sharpe ratios are equal). *Under the model of Section 2.2, if SLOW and FAST have the same Sharpe ratio, then  $a^* = \frac{1}{2}$  gives the optimal Sharpe ratio among all static blends,  $a \in [0, 1]$ , of SLOW and FAST.*

*Proof.* See Internet Appendix A.  $\square$

By Proposition 5, for all parameter pairs  $(\varphi, \theta)$  on the solid curve in Fig. 6(b), the optimal blend that maximizes the unconditional Sharpe ratio is  $a^* = \frac{1}{2}$ . Our estimate of  $(\hat{\varphi}, \hat{\theta}) = (0.85, 0.86)$  lies close to this curve, consistent with  $a = \frac{1}{2}$  generating a higher Sharpe ratio than other static speeds reported in Table 2 ( $a = 0, \frac{1}{4}, \frac{3}{4}, 1$ ). Because FAST has a higher Sharpe ratio than SLOW for combinations of persistence and noise below the curve in Fig. 6(b),

<sup>27</sup> The expected return of a static blend is available in closed form. To compute the volatility, however, we need the expected value of the square of a normal random variable times the signs of two jointly normal correlated random variables:  $\mathbf{E}[r_t^2 \cdot \text{sign}(r_{t-1}) \cdot \text{sign}(\frac{1}{2}(r_{t-1} + \dots + r_{t-12}))]$ . This expectation does not admit a closed-form expression.



**Fig. 6.** Implied mean persistence and realized noise level.

*Notes:* Figure (a) plots the model-implied mean persistence and realized noise ratio pair  $(\hat{\varphi}, \hat{\theta}) = (0.85, 0.86)$  inferred from the empirical Sharpe ratios of FAST and SLOW and model Eqs. (6) and (7). The small brown dots represent model-implied estimates of  $(\hat{\varphi}, \hat{\theta})$  corresponding to 1,000 block-bootstrap estimates of Sharpe ratios of FAST and SLOW. The vertical dashed line ( $\varphi = 0.68$ ) represents the 5th percentile of bootstrap  $\hat{\varphi}$  estimates. The horizontal dashed line ( $\theta = 0.72$ ) represents the 5th percentile of bootstrap  $\hat{\theta}$  estimates. Figure (b) plots the model-implied Sharpe-ratio-maximizing speeds  $a^*$  for various persistence and noise ratios. The solid curve represents combinations of persistence and noise ratios such that the Sharpe-ratio-maximizing speed is the equal blend  $a^* = \frac{1}{2}$ . Combinations in the zone below this curve (green zone) are associated with faster optimal speeds,  $a^* > \frac{1}{2}$ . Combinations above (red zone) are associated with slower optimal speeds,  $a^* < \frac{1}{2}$ . (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

we can infer from (18) that such combinations are associated with faster optimal speeds ( $a^* > \frac{1}{2}$ ). Similarly, combinations of persistence and noise above this curve are associated with slower optimal speeds ( $a^* < \frac{1}{2}$ ). Note that Proposition 5 highlights a nontrivial property of our static-speed momentum strategies; it does not generally hold for any two portfolios that if those portfolios have the same Sharpe ratios then an equal blend of those portfolios has the maximum Sharpe ratio of all blends.

## 4.2. Market timing

### 4.2.1. The determinants of market beta and alpha

Given the preponderance of Bull cycles—around half of all months—as reported in Fig. 4, it should not come as a surprise that trend following of the U.S. stock market implies a positive static tilt. Indeed, as detailed in Table 2, the average position of the momentum strategies ranged from 18% to 46% from fast to slow speeds. As also detailed in Table 2, however, the betas of the momentum strategy returns are relatively low in magnitude and range from  $-0.23$  to  $0.15$  from fast to slow speeds, with negative point estimates for intermediate-to-fast speeds—something we might not expect given their positive tilts.

To understand this evidence, we first disentangle the timing bets and static bets in expected returns with a widely used decomposition:

$$\begin{aligned} r_{t+1}(a) &= w_t(a)r_{t+1} \\ &= \underbrace{(w_t(a) - \mathbf{E}[w_t(a)])}_{\text{dynamic}} r_{t+1} + \underbrace{\mathbf{E}[w_t(a)]}_{\text{static}} r_{t+1}, \end{aligned} \quad (23)$$

where the first equality above matches the first equality in (12). Taking expectations, we have

$$\mathbf{E}[r_{t+1}(a)] = \underbrace{\mathbf{Cov}[w_t(a), r_{t+1}]}_{\text{market timing}} + \underbrace{\mathbf{E}[w_t(a)] \mathbf{E}[r_{t+1}]}_{\substack{\text{static} \\ \text{dollar} \\ \text{exposure}}}, \quad (24)$$

because  $(w_t(a) - \mathbf{E}[w_t(a)])$  is mean zero. The covariance term is also known as the market-timing component, and it represents the share of expected returns generated by the dynamic bets of the signal as reflected by the strategy weights  $w_t(a)$ . In contrast, the average strategy weight summarizes the static dollar exposure of the strategy.

Huang et al., 2020 argue that static average tilts are the primary source of TS momentum performance. To support their argument, they construct two portfolios: (1) a TS momentum portfolio that goes long or short based on the sign of trailing 12-month returns, similar to our SLOW, and (2) a portfolio that goes long or short based on the sign of the entire history of returns. This second strategy is essentially a buy-and-hold market strategy when inception-to-date market returns are positive. They show that these two strategies perform similarly with respect to average and risk-adjusted returns, thereby calling into question the role of timing in the 12-month signal.

However, if their argument were correct, we would have a beta puzzle. To see the problem, consider the first two columns of Table 2. The “Market” column reflects a buy-and-hold portfolio and the “SLOW” column reflects a 12-month TS momentum portfolio. Indeed, both portfolios exhibit similar performance in average returns (approximately 6%) and Sharpe ratios (approximately 0.40). Yet, if SLOW is inheriting its performance from its static tilt, then



why is its market beta so low—only 0.15? Small betas imply meaningful alphas, which map to the alphas estimated in Table 2. Moreover, small betas imply low correlations, so we could blend SLOW with the market buy-and-hold strategy to achieve a better Sharpe ratio—a contradiction to their argument.

What explains this apparent beta puzzle? The market-timing component must be adding negative beta with respect to the underlying market, which offsets the beta of the static allocation. As reported in Panel A of Table H.1 in Internet Appendix H, the timing share of the portfolios displays insignificant or only marginally significant excess returns, a result that echoes the results of Huang et al., 2020. Yet, as reported in Panel B of the same table, these portfolios recorded negative betas with respect to the underlying market. To better understand this mechanism, we next disentangle the timing and static components in the market covariance, beta, and alpha:

**Proposition 6** (Covariance with underlying market). *The (contemporaneous) covariance between the speed strategy returns and the long-only market strategy returns can be decomposed as follows:*<sup>28</sup>

$$\begin{aligned} \text{Cov}[r_{t+1}(a), r_{t+1}] &= \mathbf{E}[w_t(a)]\text{Var}[r_{t+1}] \\ &+ \text{Cov}[w_t(a), r_{t+1}]\mathbf{E}[r_{t+1}] \\ &+ \text{Cov}[w_t(a), (r_{t+1} - \mathbf{E}[r_{t+1}])^2], \end{aligned} \quad (25)$$

for  $a \in [0, 1]$ . The market beta and alpha, respectively, can be decomposed as follows:

$$\begin{aligned} \text{Beta}[r_{t+1}(a)] &= \underbrace{\mathbf{E}[w_t(a)]}_{\text{static component}} + \underbrace{\frac{\text{Cov}[w_t(a), r_{t+1}]\mathbf{E}[r_{t+1}]}{\text{Var}[r_{t+1}]}}_{\text{market timing component}} \\ &+ \underbrace{\frac{\text{Cov}[w_t(a), (r_{t+1} - \mathbf{E}[r_{t+1}])^2]}{\text{Var}[r_{t+1}]}}_{\text{volatility timing component}}, \end{aligned} \quad (26)$$

$$\text{Alpha}[r_{t+1}(a)] = \text{Cov}[w_t(a), r_{t+1}] \left( 1 - \frac{(\mathbf{E}[r_{t+1}])^2}{\text{Var}[r_{t+1}]} \right)$$

<sup>28</sup> Our decompositions have similarities in form to the relationships established between unconditional beta (alpha) and expected conditional beta (alpha) in the conditional CAPM literature. Jagannathan and Wang (1996) were the first to establish the exact relation between unconditional beta and expected conditional beta (equations A10–A14 in the appendix of their paper). This relationship is also developed and examined by Lewellen and Nagel (2006) and Boguth et al. (2011). Conditional beta and expected conditional beta in these studies can be mapped to our strategy weight and static beta component, respectively. However, the other components in these studies include covariances between conditional beta and each of conditional market returns, conditional squared market returns, and conditional market variance, given a conditioning information set. Boguth et al. (2011) organize these terms into two groups and use the labels “market timing” and “volatility timing” (Eqs. (5) and (6)). Because our expressions use market excess returns directly rather than conditional expected returns given a conditioning information set, there is no direct counterpart to the market and volatility timing components in our decompositions.

$$- \frac{\text{Cov}[w_t(a), (r_{t+1} - \mathbf{E}[r_{t+1}])^2]}{\text{Var}[r_{t+1}]} \mathbf{E}[r_{t+1}], \quad (27)$$

for  $a \in [0, 1]$ . Approximating squared average market returns by  $(\mathbf{E}[r_{t+1}])^2 \approx 0$ , we have

$$\begin{aligned} \text{Alpha}[r_{t+1}(a)] &\approx \underbrace{\text{Cov}[w_t(a), r_{t+1}]}_{\text{market timing component}} \\ &+ \underbrace{\left[ - \frac{\text{Cov}[w_t(a), (r_{t+1} - \mathbf{E}[r_{t+1}])^2]}{\text{Var}[r_{t+1}]} \mathbf{E}[r_{t+1}] \right]}_{\text{volatility timing component}}. \end{aligned} \quad (28)$$

*Proof.* See Internet Appendix A.  $\square$

Equation (26) of Proposition 6 indicates that the market beta of any of our momentum strategies can be decomposed into a static component and a market-timing component, similar to the decomposition of the expected return in (24); however, an important additional volatility-timing component arises, which reflects the predictability of strategy weights for subsequent return volatility.<sup>29</sup> Therefore, if the momentum weights significantly covary with the subsequent squared return deviations from their mean, then the beta of the momentum portfolio is not well approximated by the beta of the average momentum position. Thus, even if the market-timing component is relatively small compared to a larger positive static component, the volatility-timing component could be relatively large, but of opposite sign, and enough to offset the static component of the market beta. As indicated in Table 4, this is indeed the case for the U.S. stock market. Such TS momentum predictability is not taken into account by Huang et al., 2020.

Table 4 repeats the market beta, alpha, and alpha  $t$ -statistic estimates reported in Table 2 for ease of reference and also shows the market beta breakdown into the three (additive) components corresponding to the terms in (26). The static beta component equals the average position as reported in Table 2. The market-timing component of beta is small and near zero for all speeds. The volatility-timing component of beta is roughly as large in magnitude as the static component, but of opposite sign. Together, these terms sum to the market betas in the top row.

Table 4 also reports the market alpha breakdown into its two (additive) components corresponding to the terms in (27). The market-timing component in the alpha approximation of (28) is identical to the market-timing component in the widely used expected return decomposition of (24). The alpha decomposition reveals, however, that volatility timing, in addition to market timing, can be a

<sup>29</sup> Previous studies focus almost exclusively on market timing. An exception is Boguth et al. (2011), who argue that volatility timing can be an important component, if not the dominant driver, of unconditional alpha. Our results are consistent and we show that volatility timing comprises approximately one- to two-thirds of the alpha of our TS momentum strategies.



**Table 4**  
Beta and alpha decompositions by speed.

Market beta and alpha	$a = 0$ SLOW	$a = \frac{1}{4}$	$a = \frac{1}{2}$ MED	$a = \frac{3}{4}$	$a = 1$ FAST
Beta	0.15	0.05	−0.04	−0.13	−0.23
Alpha (%) (anlzd.)	5.58	5.85	6.12	6.39	6.66
Alpha <i>t</i> -statistic	2.55	3.26	3.73	3.59	3.09
Beta Components					
Static	0.457	0.387	0.317	0.247	0.177
Market Timing	0.008	0.008	0.008	0.008	0.009
Volatility Timing	−0.315	−0.339	−0.364	−0.389	−0.414
Alpha Components (%) (anlzd.)					
Market Timing	3.72	3.84	3.96	4.09	4.21
Volatility Timing	1.86	2.00	2.15	2.30	2.44

Notes: This table reports the sample market beta, alpha, and alpha *t*-statistic of monthly returns of momentum strategies of various speeds repeated from Table 2. The table also reports the (additive) decomposition of beta into static, market-timing, and volatility-timing components according to estimates of the terms in (26) and the (additive) decomposition of alpha into market timing and volatility timing according to estimates of the terms in (27). TS momentum strategies of various speeds are the same as those described in Table 2. The evaluation period is 1969–01 to 2018–12.

driver of alpha. Indeed, Table 4 reports that the volatility-timing component composes a large portion—over 33%—of each strategy's overall alpha estimated over the evaluation period. Volatility timing accounts for over 65% of alpha over the last 15 years (see Internet Appendix I).

Why do we find near-zero market timing in beta yet a large relative market-timing component in alpha? The answer is fairly straightforward. When beta is zero, alpha is just the expected strategy excess return, which is composed of market timing and static components only, as in (24). However, the static part of beta (which can be non-zero even when beta is zero) perfectly offsets any static component in alpha coming from expected strategy returns. Therefore, there is no static component in the decomposition of alpha in (27). If there were no volatility timing, then we would expect 100% of alpha to come from market timing—via expected strategy returns or market timing in beta. The reason that alpha market timing is only about 65% of alpha rather than 100% is that volatility timing explains the remaining portion.

To summarize, the average position of a TS momentum portfolio does not need to reflect the beta exposure of the strategy. If weights predict volatility, then the market timing and static dollar exposure terms offer an incomplete picture of the alpha generated by TS momentum.

#### 4.2.2. Relation to volatility-managed portfolios

A potential overlap between TS momentum strategies and the volatility-managed (VOM) portfolios of Moreira and Muir (2017) should not be surprising. VOM portfolios increase exposure to an underlying portfolio following low-volatility states and decrease exposure following high-volatility states. Moreira and Muir document that managing the leverage of a strategy in this manner can increase Sharpe ratios and deliver alpha with respect to the underlying portfolio. The contemporaneous correlation between stock market returns and their monthly volatility has been about −28% over our 50-year evaluation period.<sup>30</sup>

<sup>30</sup> This negative association is often referred to as the leverage effect in reference to an economic explanation of the phenomenon offered by

This negative correlation implies that trailing volatility tends to be lower (higher) when trailing returns are positive (negative). Therefore, we expect some positive comovement between the weights of a TS momentum strategy and a VOM strategy applied to a common underlying portfolio.

Is TS momentum alpha primarily a repackaging of volatility management alpha? Pertaining to the U.S. stock market, Moreira and Muir highlight two key ingredients in VOM portfolios: (1) trailing volatility tends to be uncorrelated to subsequent returns, and (2) volatility tends to be persistent at short horizons. These ingredients entail little-to-no market timing and mostly volatility timing in VOM portfolio alpha, by construction. Moreira and Muir's theoretical results predict positive VOM alpha when there is no correlation between volatility and expected returns, highlighting that the positive empirical VOM alpha they report can be explained solely by volatility timing. In contrast, TS momentum alpha can have significant market timing in addition to volatility timing, as illustrated in Table 4. Therefore, TS momentum strategies appear to extract a distinct source of alpha relative to VOM strategies. To get a broader answer, one can apply our alpha decomposition to any collection of TS strategies on a common underlying portfolio—including more complex versions of momentum and VOM strategies—to measure the degree of overlap in their market timing and volatility timing. See also Internet Appendix C, in which we compare MED with the TS momentum strategy of Moskowitz et al. (2012), which explicitly uses volatility targeting.

#### 4.3. Tail behavior

##### 4.3.1. Market-cycle decomposition

Table 5 reports various percentiles of monthly market returns in months following each of the four market cy-

Black (1976) and Christie (1982). According to this explanation, negative equity returns lead to higher firm leverage, which in turn should translate into higher equity volatility as firms become riskier. See also Harvey et al. (2018).

**Table 5**

Cycle-conditional market return distributions.

Return percentiles (%)	Bull	Correction	Bear	Rebound
MIN	−9.55	−23.24	−17.23	−10.35
P01	−7.85	−14.62	−12.79	−10.16
P05	−4.64	−7.14	−10.10	−8.41
P10	−3.37	−5.64	−8.06	−5.51
P25	−1.51	−2.08	−4.83	−2.44
P50	1.05	1.07	−0.89	1.15
P75	3.07	3.82	3.98	4.59
P90	4.68	5.84	6.82	7.24
P95	6.13	7.15	7.99	7.98
P99	7.21	11.79	13.68	10.61
MAX	9.59	12.47	16.10	11.30

Notes: This table reports various percentiles of monthly market excess returns in months immediately following each of four market states—Bull, Correction, Bear, and Rebound—defined in terms of slow and fast momentum strategy positions as in Table 3. PX is the Xth percentile. For example, P10 is the 10th percentile of monthly returns. The evaluation period is 1969–01 to 2018–12.

cles. Corrections introduce extreme outcomes and volatility despite most outcomes being positive (median monthly return of 1.07%).<sup>31</sup> Yet, extreme outcomes tend to be more severe on the downside than the upside. The fast strategy tends to flip deeper Correction losses into gains by going short after Corrections, which can help explain its slightly positive point estimate for skewness in Table 2. However, FAST also has full exposure to volatility from both Correction and Rebound states, in which the spread of returns is larger on both the positive and negative sides relative to Bull states. Intermediate-speed strategies reduce exposure to both volatility and extreme events associated with these states. In particular, MED avoided this exposure altogether with a zero position in months after these states, which can help explain its approximately zero point estimate for skewness in Table 2.

Downside risk exposure after each market cycle can help explain the pattern of maximum drawdowns across strategies reported in Table 2. After Bear states, the magnitudes of lower percentile (negative) returns are higher in most cases than the magnitudes of symmetrically higher percentile (positive) returns. Momentum strategies of all speeds are short after Bear markets, turning this downside into upside. Consistent with this evidence, Table 2 reports lower maximum drawdowns than the long-only market strategy for all speeds. Intermediate speeds further reduce exposure to extreme downside events by scaling down after Corrections and Rebounds. Accordingly, intermediate speeds report lower maximum drawdowns and higher average returns per unit of absolute maximum drawdown in Table 2.

#### 4.3.2. Skewness

We formalize the relationship between skewness of MED relative to the skewness of SLOW and FAST in terms of MED's Sharpe ratio and the disagreement multiplier,  $D(a)$ , introduced in Section 4.1.2. The connection between

the Sharpe ratio and the skewness of a random variable is illuminated in Lemma 7, which we apply to obtain Proposition 8.

**Lemma 7.** For any  $Y$  such that its first three moments are defined and  $\text{SD}[Y] > 0$ , then  $\text{Skew}[Y] = \frac{\mathbb{E}[Y^3]}{(\text{SD}[Y])^3} - \text{Sharpe}[Y](3 + (\text{Sharpe}[Y])^2)$ , where  $\text{Sharpe}[Y] = \frac{\mathbb{E}[Y]}{\text{SD}[Y]}$ .

*Proof.* See Internet Appendix A.  $\square$

**Proposition 8** (Skewness decomposition). The skewness of  $r_{t+1}(a)$  can be expressed in terms of the skewness of  $r_{\text{SLOW},t+1}$  and  $r_{\text{FAST},t+1}$ , respectively, and a disagreement multiplier. An exact expression as well as an approximation for all  $a \in [0, 1]$  based on  $(\mathbb{E}[r_{t+1}(a)])^2 \approx 0$  for  $a \in [0, 1]$  are given in (A49) and (A50), respectively, in Internet Appendix A. In the special case of  $a = \frac{1}{2}$ , we have

$$\begin{aligned} & \text{Skew}[r_{t+1}(\tfrac{1}{2})] \\ & \approx \tfrac{1}{2}(\text{Skew}[r_{\text{SLOW},t+1}] + \text{Skew}[r_{\text{FAST},t+1}])D(\tfrac{1}{2})^3 \\ & \quad + 3\text{Sharpe}[r_{t+1}(\tfrac{1}{2})] \\ & \quad \times \left( (D(\tfrac{1}{2}))^2 \left[ 1 + \left( \frac{\text{Sharpe}[r_{\text{FAST},t+1}] - \text{Sharpe}[r_{\text{SLOW},t+1}]}{2} \right)^2 \right] - 1 \right), \end{aligned} \quad (29)$$

where  $D(\frac{1}{2})$  is as defined in (19) for  $a = \frac{1}{2}$ .

*Proof.* See Internet Appendix A.  $\square$

Proposition 8 indicates that skewness of MED is scaled up relative to the average skewness of SLOW and FAST—since  $(D(\frac{1}{2}))^3 > 1$  per Proposition 4—and shifted to the right when MED has a positive Sharpe ratio—since  $(D(\frac{1}{2}))^2 > 1$  and because the term in the outer parentheses is always positive. The multiplier  $D(\frac{1}{2})$  is the same disagreement multiplier which appears in Proposition 4,  $D(a)$ , evaluated at  $a = \frac{1}{2}$ . As reported in Table 2, the slow strategy has negative skewness of −0.43, but the fast strategy has positive skewness of 0.15. Their average skewness is negative at −0.14. The disagreement multiplier  $D(\frac{1}{2}) = 1.34$  amplifies the first term in (29) by a factor of  $(D(\frac{1}{2}))^3 = 2.42$ , bringing its contribution to −0.34. The second term in (29) shifts this value to the right by 0.36, yielding the slight positive skewness of 0.02 for MED as reported in Table 2.

As before, because Proposition 8 is a model-free result, it extends beyond our running empirical example and beyond the model of Section 2.2 to momentum strategies of various speeds applied in any market. Moreover, as a corollary to this result, if SLOW and FAST both have nonnegative skewness and Sharpe ratios when applied in some market, then the skewness of MED is going to be positive and higher than the maximum skewness of both SLOW and FAST.

## 5. Dynamic speed selection

The differences in the conditional return distributions following Correction and Rebound cycles (as reported in Table 5 and discussed in Section 4.3.1) raise the question

<sup>31</sup> Five of the 10 worst months in our 50-year evaluation period were after Correction phases: −23.24% (1987–10); −16.08% (1998–08); −12.90% (1980–03); −11.91% (1978–10); and −10.72% (2000–11). Four of the 10 best months in the evaluation period were after Correction phases: 12.47% (1987–01); 12.16% (1976–01); 11.35% (2011–10); and 10.84% (1991–12).

of whether a dynamic momentum strategy could offer improved performance over a static strategy. For example, how might a strategy perform if its weights following Corrections and Rebounds were individually specified instead of implied by the weighted average of slow and fast strategies? Would strategy weights of 0 and  $\frac{1}{2}$  following Corrections and Rebounds perform better than the zero weights implied by the MED strategy, or the  $-\frac{1}{2}$  and  $\frac{1}{2}$  weights of the  $a = \frac{3}{4}$  strategy? We address questions of this nature in this section.

Instead of using a static speed parameter  $a$ , we let it vary dynamically with the four market cycles. The dynamic speed  $a_{s(t)}$  at date  $t$  is a function of the observable market state  $s(t)$  at that date, which is one of  $\{\text{Bu}, \text{Co}, \text{Be}, \text{Re}\}$ . For example, if in a Correction cycle at date  $t$  ( $s(t) = \text{Co}$ ), then  $a_{\text{Co}}$  is the parameter which will govern the blending between slow and fast strategy weights in the subsequent month. If the cycle remains in Correction at date  $t + 1$ , then we apply the same  $a_{\text{Co}}$  for the next month. If the cycle shifts to Bear at date  $t + 2$ , then we apply  $a_{\text{Be}}$  for the next month, and so on.

The corresponding dynamic strategy return is

$$r_{t+1}(a_{s(t)}) = w_t(a_{s(t)})r_{t+1} \\ = [(1 - a_{s(t)})w_{\text{SLOW},t} + a_{s(t)}w_{\text{FAST},t}]r_{t+1}. \quad (30)$$

Note that since  $w_{\text{SLOW},t} = w_{\text{FAST},t}$  with magnitude one after Bull or Bear cycles, the dynamic weight in (30) is invariant to the values of  $a_{\text{Bu}}$  and  $a_{\text{Be}}$ . That is,  $w_t(a_{\text{Bu}}) = 1$  after Bull for all  $a_{\text{Bu}}$  and  $w_t(a_{\text{Be}}) = -1$  after Bear for all  $a_{\text{Be}}$ . Nevertheless, the dynamic weight is sensitive to the values of  $a_{\text{Co}}$  and  $a_{\text{Re}}$  following Correction and Rebound cycles, respectively. In Proposition 9, we establish the values of these state-conditional speed parameters that maximize the steady-state Sharpe ratio of the dynamic returns.

**Proposition 9** (Sharpe-ratio-maximizing dynamic speed rule). *Consider the problem of choosing the state-conditional speeds,  $a_{s(t)}$ , that will be applied following every occurrence of state  $s(t)$  in order to achieve the highest steady-state Sharpe ratio:*

$$\max_{a_{s(t)}: s(t) \in \{\text{Co}, \text{Re}\}} \text{Sharpe}[r_{t+1}(a_{s(t)})]. \quad (31)$$

If  $\mathbf{E}[r_{t+1}|\text{Bu}]\mathbf{P}[\text{Bu}] > \mathbf{E}[r_{t+1}|\text{Be}]\mathbf{P}[\text{Be}]$ , then

$$a_{\text{Co}} = \frac{1}{2} \left( 1 - \frac{\mathbf{E}[r_{t+1}^2|\text{Bu}]\mathbf{P}[\text{Bu}]}{\mathbf{E}[r_{t+1}|\text{Bu}]\mathbf{P}[\text{Bu}] - \mathbf{E}[r_{t+1}|\text{Be}]\mathbf{P}[\text{Be}]} \frac{\mathbf{E}[r_{t+1}|\text{Co}]}{\mathbf{E}[r_{t+1}^2|\text{Co}]} \right), \quad (32)$$

$$a_{\text{Re}} = \frac{1}{2} \left( 1 + \frac{\mathbf{E}[r_{t+1}^2|\text{Be}]\mathbf{P}[\text{Be}]}{\mathbf{E}[r_{t+1}|\text{Bu}]\mathbf{P}[\text{Bu}] - \mathbf{E}[r_{t+1}|\text{Be}]\mathbf{P}[\text{Be}]} \frac{\mathbf{E}[r_{t+1}|\text{Re}]}{\mathbf{E}[r_{t+1}^2|\text{Re}]} \right), \quad (33)$$

is the unique state-conditional speed pair that maximizes (31).

*Proof.* See Internet Appendix A.  $\square$

Proposition 9 specifies the dynamic speed selections that maximize the steady-state Sharpe ratio in terms of cycle-conditional first- and second-population moments of market returns. The condition  $\mathbf{E}[r_{t+1}|\text{Bu}]\mathbf{P}[\text{Bu}] >$

$\mathbf{E}[r_{t+1}|\text{Be}]\mathbf{P}[\text{Be}]$  ensures that the weights are maximizers and not minimizers. This condition is typically satisfied because expected returns are typically positive following Bull cycles and negative following Bear cycles.

Because population values of the first and second moments in (32) and (33) are not observable, we use historical estimates of these moments to approximate their values. We use the label “DYN” to denote the investable strategy that employs state-dependent speeds based on estimated versions of (32) and (33), using only data prior to strategy implementation (i.e., no look-ahead bias). Specifically, DYN weights are defined as follows:  $w_{\text{DYN},t} := w_t(\hat{a}_{s(t)}) = (1 - \hat{a}_{s(t)})w_{\text{SLOW},t} + \hat{a}_{s(t)}w_{\text{FAST},t}$ , where  $s(t) \in \{\text{Bu}, \text{Co}, \text{Be}, \text{Re}\}$ ;  $\hat{a}_{\text{Bu}} = \hat{a}_{\text{Be}} = \frac{1}{2}$ ; and  $\hat{a}_{\text{Co}}$  and  $\hat{a}_{\text{Re}}$  are estimated using sample averages  $\hat{\mathbf{E}}[\cdot]$  and  $\hat{\mathbf{P}}[\cdot]$ , which denote estimates of the average and frequency of their given argument, respectively, based on data prior to strategy implementation.<sup>32</sup> For example,  $\hat{\mathbf{E}}[r_{t+1}^2|\text{Bu}]$  is the historical sample average of squared returns in months after Bull or Bear states within the historical estimation window, and  $\hat{\mathbf{P}}[\text{Bu}]$  is the frequency of Bull or Bear states within that historical sample. If these cycle-conditional return moments are relatively stable over time, then DYN may offer improved performance out of sample.

### 5.1. Performance of dynamic strategy

Table 6 reports the out-of-sample performance of DYN over various evaluation windows in our 50-year evaluation period—from 50 years ago forward in 5-year increments to the most recent 15 years. State-dependent speeds  $\hat{a}_{\text{Co}}$  and  $\hat{a}_{\text{Re}}$  of DYN are fixed over each evaluation window using data from the corresponding fixed training window from 1926–07 to the month before the evaluation start month.<sup>33</sup> We define the efficiency of the DYN strategy over each evaluation window as the ratio of its Sharpe ratio to that of the highest performing state-conditional speed strategy ex post, which we label the “OPT” strategy. OPT uses fixed speeds  $a_{\text{Co}}^*$  and  $a_{\text{Re}}^*$  over each evaluation window that give the highest in-sample Sharpe ratio rather than speeds estimated from trailing data. OPT’s Sharpe ratios vary over different evaluation windows from 0.570 to 0.721, indicating that different periods expose the strategies to different performance opportunities. We can measure the performance of any state-conditional-speed strategy relative to the ceiling set by OPT, including DYN and the special cases of static speeds. DYN consistently exhibits efficiency of 92% or higher across different evaluation windows, reflecting its capacity to exploit performance opportunities out of sample.

The out-of-sample performance of DYN also exhibits modest gains compared to static-speed strategies. Figure 7 compares the Sharpe ratios of DYN and OPT with those of various static-speed strategies over evaluation windows

<sup>32</sup> If either of the estimates  $\hat{a}_{\text{Co}}$  or  $\hat{a}_{\text{Re}}$  fall outside the unit interval  $[0, 1]$ , then we set its value to the nearest endpoint, 0 or 1.

<sup>33</sup> Results are similar if we update speed estimates  $\hat{a}_{\text{Co}}$  and  $\hat{a}_{\text{Re}}$  at end of each month  $t - 1$  during the evaluation period using data from expanding rather than fixed training windows from 1926–07 to month  $t - 1$  to form strategy weights for each month  $t$ .

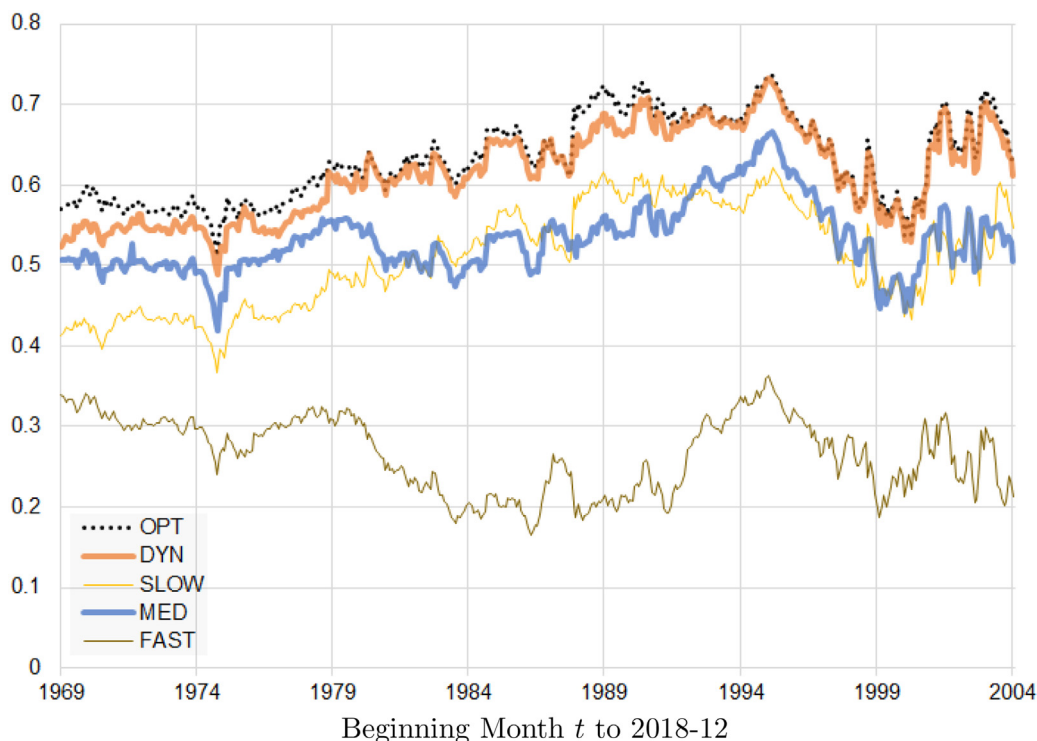


Fig. 7. Sharpe ratios (month  $t$  to end of sample): DYN vs. static strategies.

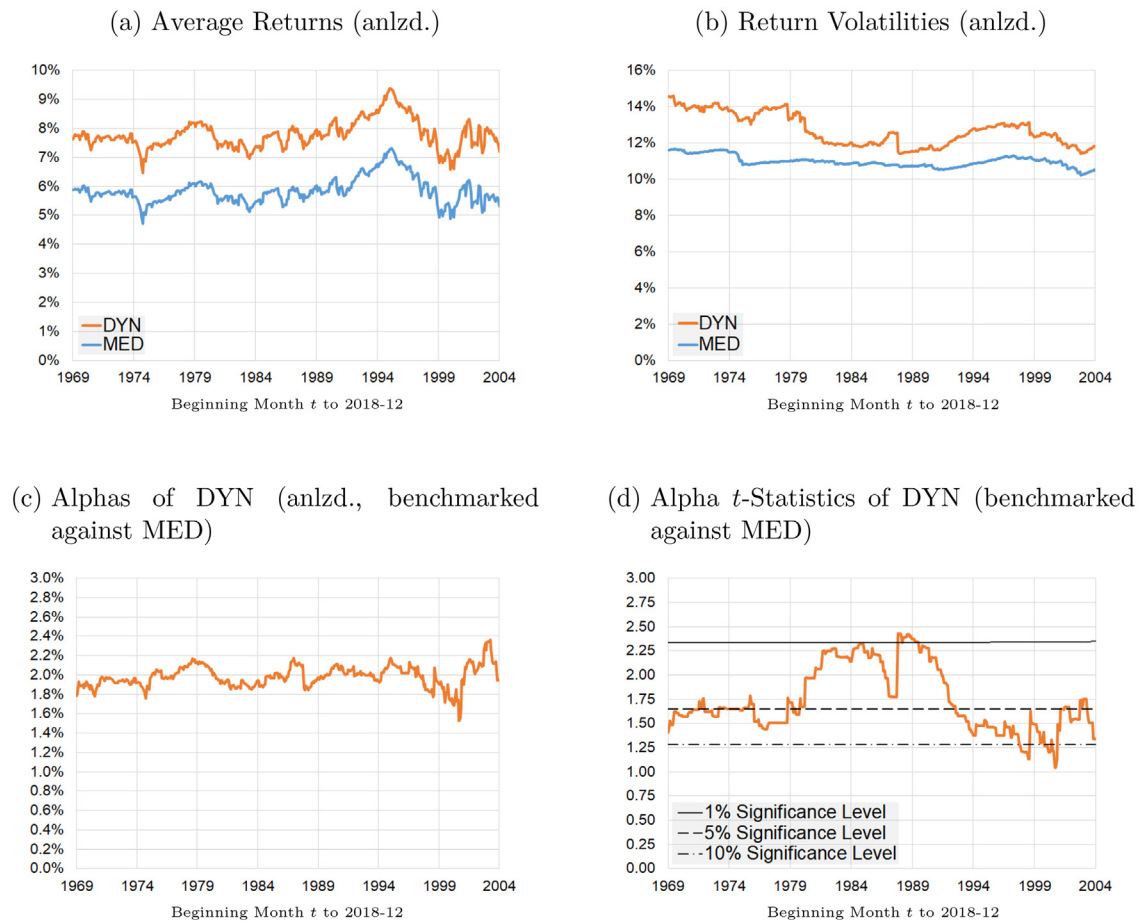
Notes: This figure plots, for each month  $t$  in 1969-01 to 2004-01, the Sharpe ratio of various strategies over the evaluation period beginning month  $t$  and ending December 2018. Strategies OPT and DYN are the same as those described in Table 6. Static-speed strategies are the same as those described in Table 2.

Table 6

DYN strategy performance over a 50-year period.

DYN Strategy			Evaluation					
Training window			Evaluation window			Sharpe ratio (anlzd.)		
From (yr-mo)	To (yr-mo)	Length (yrs)	From (yr-mo)	To (yr-mo)	Length (yrs)	DYN ( $\hat{a}_{Co}$ , $\hat{a}_{Re}$ )	OPT	Efficiency DYN/OPT
1926-07	1968-12	42.5	1969-01	2018-12	50.0	0.524 (0.00,0.58)	0.570	0.920
1926-07	1973-12	47.5	1974-01	2018-12	45.0	0.547 (0.07,0.59)	0.572	0.956
1926-07	1978-12	52.5	1979-01	2018-12	40.0	0.611 (0.08,0.65)	0.626	0.977
1926-07	1983-12	57.5	1984-01	2018-12	35.0	0.614 (0.22,0.67)	0.623	0.985
1926-07	1988-12	62.5	1989-01	2018-12	30.0	0.688 (0.26,0.69)	0.721	0.954
1926-07	1993-12	67.5	1994-01	2018-12	25.0	0.675 (0.11,0.71)	0.684	0.988
1926-07	1998-12	72.5	1999-01	2018-12	20.0	0.564 (0.17,0.69)	0.579	0.975
1926-07	2003-12	77.5	2004-01	2018-12	15.0	0.611 (0.16,0.69)	0.621	0.984

Notes: This table reports the DYN momentum strategy's Sharpe ratio and its efficiency for various evaluation windows within the 50-year evaluation period. DYN is the dynamic (state-dependent) speeds strategy whose weights,  $w_t(a_{s(t)}) = (1 - a_{s(t)})w_{SLOW,t} + a_{s(t)}w_{FAST,t}$ , are based on point estimates of the mean-variance optimal state-dependent speeds,  $a_{s(t)}$ , using  $\hat{a}_{Bu} = \hat{a}_{Be} = \frac{1}{2}$  and sample versions of (32) and (33) for  $\hat{a}_{Co}$  and  $\hat{a}_{Re}$ , respectively, based on data in the training window;  $w_{SLOW,t}$ , equals +1 if the trailing 12-month return (arithmetic average monthly return) is nonnegative and otherwise equals -1;  $w_{FAST,t}$ , equals +1 if the trailing 1-month return is nonnegative and otherwise equals -1; and the four observable market states at date  $t$  are defined as: Bu:  $w_{SLOW,t} = w_{FAST,t} = +1$ ; Be:  $w_{SLOW,t} = w_{FAST,t} = -1$ ; Co:  $w_{SLOW,t} = +1$ ,  $w_{FAST,t} = -1$ ; and Re:  $w_{SLOW,t} = -1$ ,  $w_{FAST,t} = +1$ . If either of the estimates  $\hat{a}_{Co}$  or  $\hat{a}_{Re}$  falls outside the unit interval [0,1], then we set its value to the nearest endpoint, 0 or 1. Strategy returns are formed as  $r_{t+1}(a) = w_t(a)r_{t+1}$ , where  $r_{t+1}$  is the U.S. excess value-weighted market factor return (Mkt-RF) from the Kenneth French Data Library. OPT is the dynamic strategy that would have achieved the maximum Sharpe ratio, ex post. State-dependent speeds of both strategies are fixed over the evaluation window. For example, using 57.5 years of data from 1926-07 to 1983-12 to estimate speeds for DYN, its Sharpe ratio over the subsequent 35-year evaluation period from 1981-07 to 2018-12 was 0.614. Relative to the best possible ex post state-dependent strategy, OPT, which had a Sharpe ratio of 0.623, DYN was 98.5% = 0.614/0.623 efficient.



**Fig. 8.** Performance of DYN vs. MED (month  $t$  to end of sample).

Notes: This figure plots, for each month  $t$  in 1969–01 to 2004–01, (a) the average returns and (b) volatilities of DYN and MED, (c) the alphas of DYN with respect to MED, and (d) the alpha  $t$ -statistics of DYN with respect to MED with one-sided significance reference levels over the evaluation period beginning month  $t$  and ending December 2018. Strategy MED is the same as in Table 2 and strategy DYN is the same as in Table 6.

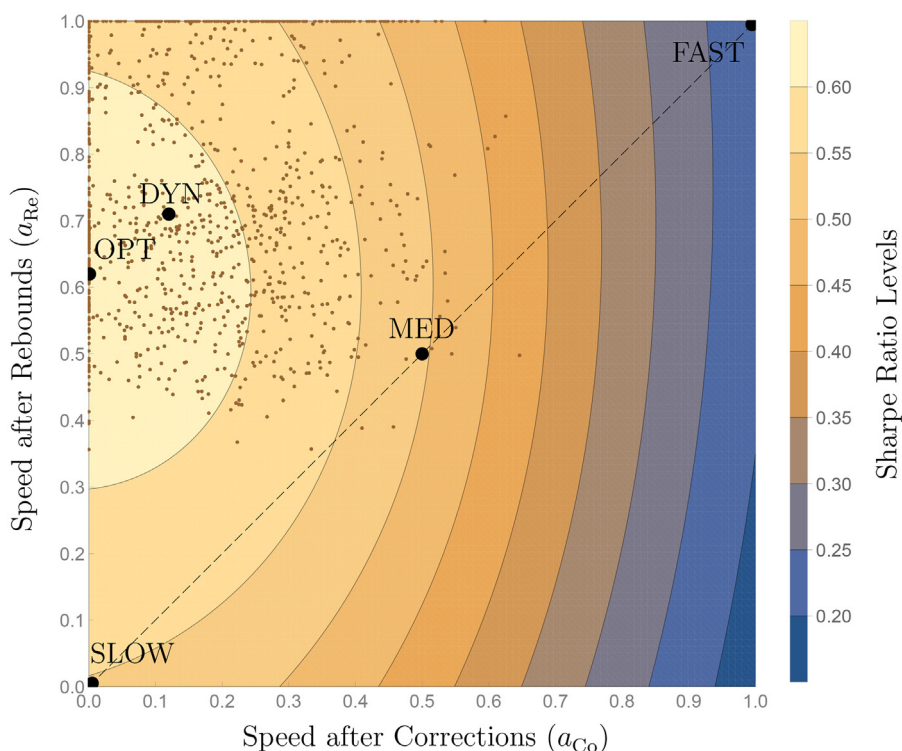
beginning from each date on the horizontal axis through the end of the sample period. The time-series values of DYN and OPT at 5-year-interval timestamps correspond to the values reported in Table 6. Note that the static-speed strategy with the highest Sharpe ratio is not always MED.<sup>34</sup> Regardless of which static-speed strategy performs best, DYN edges out the best static-speed strategy, including MED, uniformly across all evaluation periods. Although its Sharpe ratio outperformance on an absolute scale is marginal in some periods (e.g., DYN's 0.52 to MED's 0.51 over the 50-year evaluation window beginning 1969), DYN consistently attains nearly all of the performance opportunities available (captured by OPT). For example, the gap between DYN and OPT is consistently slim—efficiency never below 92%. In contrast, the gap between MED and OPT is sometimes relatively large—MED as low as 75% efficiency. Therefore, from an ex ante point of view not know-

ing the best static speed, DYN offers a systematic approach to adjust the speed.

Furthermore, DYN generates alpha with respect to MED. Figure 8 reports several time series of performance metrics for DYN and MED over evaluation windows beginning from each date on the horizontal axis through the end of the sample period. DYN uniformly generates higher excess returns than MED (Fig. 8(a)), but also uniformly generates higher volatility (Fig. 8(b)). Recall that MED completely exits the market after Corrections and Rebounds, which eliminates its exposure to volatility as well as to return opportunities after these states. In contrast, DYN takes (scaled-down) positions after these states, allowing DYN to take advantage of potentially favorable return/volatility trade-offs reflected in past market-cycle return patterns. Figure 8(c) plots the time series of alphas of DYN with respect to MED, showing an average of about 2.0% per annum across evaluation windows. Figure 8(d) plots the  $t$ -statistics corresponding to these alphas and shows an average one-sided significance level of about 5%. Despite the fact that their Sharpe ratios are relatively close over the

<sup>34</sup> Figure J.1 in Internet Appendix J is the same as Fig. 7 but also includes Sharpe ratios for static speeds  $a = \frac{1}{4}$  and  $a = \frac{3}{4}$ , highlighting that other static speeds can generate higher Sharpe ratios than MED.





**Fig. 9.** Sharpe ratio performance of different state-conditional speed pairs over the most recent 15 years.

*Notes:* This figure reports Sharpe ratio levels of dynamic-speed momentum strategies over the evaluation period 2004-01 to 2018-12 for speed pairs  $(a_{co}, a_{re})$  where strategy weights take the form  $(1 - a_{s(t)})w_{SLOW,t} + a_{s(t)}w_{FAST,t}$  and DYN is the dynamic (state-dependent) speeds strategy as described in the notes of Table 6. The dotted diagonal highlights the continuum of static strategies, which apply the same speeds after all states—SLOW ( $a_{s(t)} = 0$ ), MED ( $a_{s(t)} = 0.5$ ), and FAST ( $a_{s(t)} = 1$ )—are labeled. OPT represents the dynamic strategy pair that would have achieved the best Sharpe ratio ex post. Strategy returns are formed as  $r_{t+1}(a) = w_t(a)r_{t+1}$ , where  $r_{t+1}$  is the U.S. excess value-weighted market factor return (Mkt-RF) from the Kenneth French Data Library. The small dots represent block bootstrap point estimates for DYN using monthly data from 1926-07 to 2003-12. Note that the intermediate-speed static strategy with  $a = \frac{1}{4}$  achieves approximately the highest Sharpe ratio of all static strategies over the measurement period.

50-year evaluation period beginning 1969, DYN exhibits statistically significant alpha relative to MED at the one-sided 10% significance level.

### 5.2. DYN sensitivity to estimation error

We now analyze the sensitivity of the state-conditional speed estimates of DYN to estimation error via block-bootstrap resampling of historical returns. We find that state-dependent speed pairs that yield the highest performance for DYN tend to be those in the region having relatively slow speed after Corrections and relatively fast speed after Rebounds. Moreover, Sharpe ratio performance is relatively insensitive to variation of speeds within this region.

Figure 9 graphs Sharpe ratio contour levels for momentum strategies with state-dependent speed pairs  $(a_{co}, a_{re})$  on the unit square over the most recent 15-year evaluation period 2004-01 to 2018-12. Each speed pair is set at the beginning of the evaluation period. The concentric curves represent speed pairs with equal Sharpe ratio levels. Lighter-shaded regions reflect higher Sharpe ratios and darker-shaded regions reflect lower Sharpe ratios. Static strategies—strategies whose speeds do not vary by state—are represented by the points on the dotted diagonal line.

OPT is the state-dependent speed pair that would have achieved the highest Sharpe ratio ex post, and DYN indicates the state-dependent pair based on estimates of state-conditional first and second moments using monthly data from 1926-07 to 2003-12 prior to the evaluation period.

The figure illustrates that the Sharpe ratio is less sensitive to deviations in speed-pair values near the in-sample mean-variance optimal speed pair, OPT, than it is farther away. The DYN speed pair based on the historical sample is close to OPT and is approximately as good a performer. The small dots represent the state-dependent speed pairs from 1,000 block-bootstrap estimates of state-conditional first and second moments using monthly data from 1926-07 to 2003-12.<sup>35</sup> The bootstrap DYN speed pairs tend to fall into the upper-left quadrant of the graph in Fig. 9,

<sup>35</sup> The block bootstrap is performed as follows. We partition the 930 historical monthly returns on the U.S. stock market from 1926-07 to 2003-12 into 10 equal-sized nonoverlapping blocks of 93 consecutive months. We randomly sample 10 of these blocks with replacement to form one bootstrap historical sample. For each bootstrap sample, we determine cycle states and estimate state-conditional first and second moments. We plug these estimates into (32) and (33) to generate a speed pair for the sample.

**Table 7**  
Market cycle transitions.

	Monthly Transition Probability (%)					
	Bull <sub>t+1</sub>	Correction <sub>t+1</sub>	Bear <sub>t+1</sub>	Rebound <sub>t+1</sub>	Up <sub>t+1</sub>	Down <sub>t+1</sub>
Bull <sub>t</sub>	63.0	34.9	1.7	0.3	63.3	36.7
Correction <sub>t</sub>	61.2	29.9	8.8	0.0	61.2	38.8
Bear <sub>t</sub>	9.0	0.0	55.0	36.0	45.0	55.0
Rebound <sub>t</sub>	14.3	1.6	42.9	41.3	55.6	44.4

Notes: This table reports the relative frequency of transitions from one market state (row) in month  $t$  to another market state (column) in month  $t + 1$  (first four columns) and the relative frequency of positive (Up) and negative (Down) returns next month (last two columns) over the 50-year period from 1969-01 to 2018-12. Market-state classifications are the same as those described in Table 3.

**Table 8**  
Sharpe ratios of static-speed strategies in international markets.

Country	Sharpe Ratio (anlzd.)			
	$a = 0.00$ SLOW	$a = 1.00$ FAST	Average of SLOW, FAST	$a = 0.50$ MED
AUS	0.121	0.076	0.098	0.123
AUT	0.435	0.600	0.518	0.627
BEL	0.476	0.460	0.468	0.619
CAN	0.234	0.498	0.366	0.486
CHE	0.619	0.468	0.543	0.687
DEU	0.498	0.211	0.354	0.436
DNK	0.454	0.410	0.432	0.569
ESP	0.281	0.169	0.225	0.280
FIN	0.720	0.649	0.685	0.869
FRA	0.382	0.355	0.369	0.466
GBR	0.227	−0.111	0.058	0.072
HKG	0.274	0.451	0.363	0.443
IRL	0.339	0.309	0.324	0.413
ITA	0.539	0.016	0.277	0.351
JPN	0.411	0.127	0.269	0.333
NLD	0.488	0.090	0.289	0.391
NOR	0.376	0.621	0.498	0.669
SGP	0.184	0.502	0.343	0.462
SWE	0.440	0.303	0.371	0.475
USA	0.511	0.155	0.333	0.462

Notes: This table reports the Sharpe ratios (anlzd.) for various strategies applied to different country equity markets evaluated over the period from 1981-01 to 2018-12 for most countries—first year of data (1980) is used to define the SLOW signal prior to the evaluation period. Static-speed strategy weights are formed according to  $w_t(a) = (1 - a)w_{\text{SLOW},t} + aw_{\text{FAST},t}$  for speed parameter  $a \in [0, 1]$ , where  $w_{\text{SLOW},t}$  equals +1 if the trailing 12-month return (arithmetic average monthly return) is non-negative and otherwise equals −1; and  $w_{\text{FAST},t}$  equals +1 if the trailing 1-month return is nonnegative and otherwise equals −1. MED is the strategy for which  $a = \frac{1}{2}$ . Monthly strategy returns are formed as  $r_{t+1}(a) = w_t(a)r_{t+1}$ , where  $r_{t+1}$  is the excess equity return for the country. Excess returns are obtained via Datastream. See Internet Appendix B for data details. Green-highlighted values are larger than the average of SLOW and FAST values.

which corresponds to strategies that tilt to SLOW after Corrections and to FAST after Rebounds.

To summarize, state-dependent speed pairs that yield the highest performance tend to be those that are relatively slow after Corrections and relatively fast after Rebounds. Moreover, speed pairs in this region are closer to

optimal, and Sharpe ratio performance in this region is less sensitive to estimation error.

### 5.3. Market-cycle patterns

The conclusion from our state-dependent speed analysis—elect slower-speed momentum after Correc-

**Table 9**

Sharpe ratios of the dynamic-speed strategy in international markets.

Country	Sharpe Ratio (anlzd.)					DYN ( $\hat{a}_{Co}$ , $\hat{a}_{Re}$ )
	$a = 0.00$ SLOW	0.25	0.50 MED	0.75	1.00 FAST	
AUS	0.363	0.412	0.415	0.346	0.256	0.516 (0.00, 1.00)
AUT	0.422	0.560	0.670	0.694	0.647	0.729 (0.60, 0.76)
BEL	0.678	0.718	0.679	0.547	0.392	0.815 (0.28, 1.00)
CAN	0.405	0.545	0.652	0.644	0.564	0.650 (0.61, 1.00)
CHE	0.644	0.774	0.832	0.750	0.605	0.956 (0.00, 0.94)
DEU	0.459	0.479	0.437	0.326	0.208	0.535 (0.00, 0.61)
DNK	0.599	0.662	0.650	0.531	0.385	0.557 (0.11, 1.00)
ESP	0.147	0.140	0.112	0.066	0.024	0.202 (0.00, 0.80)
FIN	0.604	0.703	0.699	0.513	0.320	0.787 (0.23, 0.75)
FRA	0.424	0.495	0.505	0.405	0.284	0.622 (0.39, 0.87)
GBR	0.303	0.219	0.067	−0.097	−0.205	0.477 (0.00, 1.00)
HKG	0.297	0.453	0.597	0.632	0.585	0.545 (0.25, 1.00)
IRL	0.624	0.694	0.697	0.604	0.474	0.721 (0.49, 0.68)
ITA	0.363	0.339	0.272	0.170	0.073	0.363 (0.00, 0.00)
JPN	0.519	0.566	0.559	0.478	0.370	0.519 (0.00, 0.00)
NLD	0.397	0.356	0.222	0.036	−0.095	0.461 (0.00, 0.80)
NOR	0.394	0.527	0.624	0.599	0.508	0.580 (0.59, 1.00)
SGP	0.288	0.392	0.473	0.470	0.415	0.531 (0.61, 1.00)
SWE	0.587	0.685	0.688	0.519	0.336	0.782 (0.14, 0.58)
USA	0.595	0.624	0.570	0.421	0.265	0.670 (0.00, 0.83)

Notes: This table reports the Sharpe ratios (anlzd.) for various strategies applied to different country equity markets evaluated over the 15-year period from 2004-01 to 2018-12. Static-speed strategy weights are formed according to  $w_t(a) = (1 - a)w_{SLOW,t} + aw_{FAST,t}$  and DYN is the dynamic (state-dependent) speeds strategy as described in the notes of Table 6. Monthly strategy returns are formed as  $r_{t+1}(a) = w_t(a)r_{t+1}$ , where  $r_{t+1}$  is the excess equity market factor return for the country. Excess returns are obtained via Datastream. See Internet Appendix B for data details. Green-highlighted values are at least as large as the maximum static-speed values and yellow-highlighted values are at least as large as the maximum of SLOW and FAST values.

tion months and faster-speed momentum after Rebound months—is reinforced by market cycle patterns, as those reported in Table 7.

Table 7 reports for the 50-year evaluation period the relative frequency of each market state in the subsequent month (first four columns) given the market state in the preceding month (rows). It also reports the relative frequency of positive and negative returns in the subsequent month (Up and Down states, respectively, in the last two columns) given the market state in the preceding month (rows). First, Corrections tend to revert to Bulls. Given the current month is a Correction state, then most of the time (61.2%) the next month is a Bull month and an Up month. If we view a Correction state as an alarm that uptrend could be turning to downtrend, then this alarm tends to be a false alarm. The fast strategy takes a short position after Corrections, which is a bad bet. Therefore, adopting a slower-speed momentum position after Corrections is a better bet. Second, the market is more likely to go up after Rebounds. Given the current month is a Rebound state, then most of the time (55.6%) the next month is an Up month. If we view a Rebound state as an alarm that downtrend could be turning to uptrend, then regardless of the accuracy of this alarm, FAST takes a long position, which is a good bet. Therefore, adopting a faster-speed momentum position after Rebounds is more effective than adopting slower speeds after Rebounds.

## 6. Evidence from other equity markets

The conclusion to elect intermediate-speed momentum strategies from our static-speed analysis using U.S. equity data in Section 4 carries over to international equity markets. Table 8 reports the Sharpe ratios for static-speed momentum strategies applied in 20 country equity markets and evaluated since 1981.<sup>36</sup> For all countries in the table, as predicted by Proposition 4, the Sharpe ratio for MED is higher than the average of the Sharpe ratios of SLOW and FAST.

Table 9 reports the Sharpe ratios for static-speed and dynamic-speed momentum strategies applied in these 20 equity markets and evaluated over the last 15 years. Here, we use the earlier part of the data sample before 2004 to estimate the state-conditional speeds of the dynamic strategy used in the evaluation period (similarly to Section 5) for each country. As before, if either  $\hat{a}_{Co}$  or  $\hat{a}_{Re}$  fall outside the unit interval  $[0,1]$ , then we set its value to the nearest endpoint, 0 or 1. Because the estimation period for four states (Bull, Correction, Bear, Rebound) is relatively short (20 years of data split four ways), the estimates are rela-

<sup>36</sup> Our data sample begins in 1980 for most countries. See Internet Appendix B for data details. Data in the first year (1980) is used to define the SLOW (12-month) signal for the first strategy return in 1981-01.

tively noisy, and the above restriction comes into play for approximately half of the speed estimates in Table 9.

With few exceptions, the recommendation from our state-dependent speed analysis on U.S. equity in Section 5 carries over to international equity markets: employ slower-speed momentum after Corrections and faster-speed momentum after Rebounds. For most countries, the Sharpe ratio of the DYN strategy is higher than the highest static-speed strategy (highlighted green in Table 9). The countries for which their DYN Sharpe ratios are not greater than or equal to the highest static speed are Canada (CAN), Denmark (DNK), Hong Kong (HKG), Japan (JNP), and Norway (NOR). Of these, all but Denmark and Hong Kong have DYN Sharpe ratios at least as high as the maximum of SLOW and FAST. Nevertheless, even for Denmark and Hong Kong, their DYN Sharpe ratios are higher than the average Sharpe ratios of the SLOW and FAST strategies and higher than the Sharpe ratios of some static-speed strategies.

## 7. Conclusion

We develop a model of time-varying trend to study the dynamics of elementary slow (12-month) and fast (1-month) monthly TS momentum strategies. We use its predictions for the performance of these strategies to estimate the level of persistence in expected returns and noise in realized returns of the U.S. stock market. We find relatively high persistence and high noise, properties that make the union of information in slow and fast momentum signals relevant for detecting trends and momentum turning points.

Accordingly, we use the agreement or disagreement between the uptrend or downtrend indications of slow and fast momentum to characterize four market cycles: Bull, Correction, Bear, and Rebound. Bulls and Bears proxy for uptrend and downtrend, respectively, while Corrections and Rebounds proxy for turning points in trend. We find that these market cycles, despite being simple in construction, possess predictive information for stock market returns and have close connections to the macroeconomy and the business cycle. Bear states, which predict negative expected returns, tend to concentrate early in recessions and coincide with bad macro news, which is difficult to rationalize under the prevailing view of countercyclical risk premia. Bull and Rebound states, which predict positive expected returns, tend to concentrate late in recessions, consistent with a countercyclical view. All four market states can occur in any phase of the business cycle, however, which suggests a role for the sense of cyclicity itself to vary over time in models of time-varying risk premia.

We apply the predictive potential of our market cycles to develop investable strategies with positive unconditional alpha. Intermediate-speed strategies, formed by blending slow and fast TS momentum, vary the exposure to the good bets associated with uptrend (Bull) or downtrend (Bear) phases and the bad bets associated with turning points (Correction or Rebound). For the U.S. stock market, we find that intermediate-speed strategies empirically exhibit many advantages over slow and fast strategies, in-

cluding higher Sharpe ratios, drawdowns that are less severe, skewness that is more positive, and higher significance of alphas. We provide analytical results that generalize our findings for Sharpe ratios and skewness to any market.

We find that both market timing and volatility timing play important but overlooked roles for TS momentum. In particular, we provide a novel model-free decomposition of momentum's alpha into the sum of market-timing and volatility-timing components. Market timing, the focus of recent TS momentum studies, reflects the covariance between strategy positions and subsequent market returns; in contrast, volatility timing reflects the covariance between strategy positions and subsequent squared market return deviations from their mean. For the U.S. stock market, we estimate about two-thirds of TS momentum's alpha over our 50-year evaluation period is attributable to market timing, and the remaining one-third is attributable to volatility timing.

We derive a dynamic momentum strategy that varies its speeds based on market cycles so to maximize its Sharpe ratio. We implement an investable version and find modest, but consistent improvements, when compared to static strategies, in out-of-sample risk-adjusted performance. We document that these results, with few exceptions, hold up across international equity markets.

Our framework is purposely simple. Based on the sign of past returns, our elementary strategies will either buy or sell. That is, we intentionally use binary signals that do not take into account the magnitude of past returns. Not only does it allow us to appropriately relate our work to previous literature, it allows us to look at four market states and analytically derive the moments of various trading strategies. Further, we are able to tackle a heretofore unanswered question: Is it possible to dynamically adjust momentum speed to improve performance characteristics over static strategies? Building a richer model that takes return magnitudes into account is a topic of on-going research.

## Declaration of Competing Interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

Campbell Harvey reports financial support was provided by Research Affiliates.

## Data availability

Data and code are available at <https://data.mendeley.com/datasets/k59vgvhhkg/1>.

## Acknowledgments

Nikolai Roussanov was the editor for this article. We thank the editor as well as two anonymous referees for their comments. We appreciate the comments of Rob Arnott, Ravi Jagannathan, Juhani Linnainmaa, conference participants at the American Finance Association Annual Meeting, and seminar participants at Research Affiliates.

The initial version of the paper was written when Goulding and Mazzoleni were employees of Research Affiliates. The views expressed in this article are those of the authors and do not necessarily reflect the positions of STRS Ohio. We thank Kay Jaitly for editorial assistance. We thank Rahul Jindal for excellent research assistance. We also thank Ashish Garg, who was a collaborator on an early version of this paper.

## Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:[10.1016/j.jfineco.2023.05.007](https://doi.org/10.1016/j.jfineco.2023.05.007).

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