غير مسموح بتصوير هذي النوتات

إلا من خلال معهد GSI

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معهد الدراسات العالمية للتدريب الأهلي

Statistics I – IE 230

Lecture 05

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Basic Mechanics ME 270 & ME 274 – Mechanics of Material NUCL 273 & ME 323
Thermodynamics ME200/ENGR 200 – Statistics IE 230 & IE 330 – Quality Control IE 530 – Operations Research IE 335

Binomial Distribution

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x} \qquad x = 0, 1, \ldots, n$$

If X is a binomial random variable with parameters p and n,

$$\mu = E(X) = np \qquad \text{and} \qquad \sigma^2 = V(X) = np(1-p)$$

Example (01) – Binomial Distribution

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Five basketball players have a probability to be with high weight with probability 0.4. Assume players are independent

a) What is the type of the distribution?

Binomial

b) What are the parameters of the distribution

$n = 5$

$p = 0.4$

$B(5, 0.4)$

c) Calculate mean, variance and standard deviation

$\mu = E(x) = np = 2$

$\sigma^2 = V(x) = np(1-p) = 1.2$

$\sigma = \sqrt{1.2} = 1.1$

d) Write probability distribution table (pmf)

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$X = 0, 1, 2, 3, 4, 5$

X : binomial random variable

$$f(0) = C_0^5 (0.4)^0 (0.6)^5 = 0.078$$

$$f(1) = C_1^5 (0.4)^1 (0.6)^4 = 0.259$$

$$f(2) = C_2^5 (0.4)^2 (0.6)^3 = 0.346$$

$$f(3) = C_3^5 (0.4)^3 (0.6)^2 = 0.230$$

$$f(4) = C_4^5 (0.4)^4 (0.6)^1 = 0.077$$

$$f(5) = C_5^5 (0.4)^5 (0.6)^0 = 0.010$$

X	0	1	2	3	4	5
f(x)	0.078	0.259	0.346	0.230	0.077	0.010

Example (02) – Binomial Distribution

Multiple choice question with five answers. A test contains 15 questions.

a) What is the probability of answering exactly 3 correct questions?

(1) $n = 15$

(2) $P = \frac{1}{5} = 0.2$

(3) $P(x=3) = C_3^{15} (0.2)^3 (0.8)^{12} = 0.25$

b) What is the probability of answering more than 4 correct questions?

$$P(x > 4) = 1 - P(x \leq 4)$$

$$1 - [P(0) + P(1) + P(2) + P(3) + P(4)]$$

$$P(0) = C_0^{15} (0.2)^0 (0.8)^{15} = 0.035$$

$$P(1) = C_1^{15} (0.2)^1 (0.8)^{14} = 0.132$$

$$P(2) = C_2^{15} (0.2)^2 (0.8)^{13} = 0.231$$

$$P(3) = C_3^{15} (0.2)^3 (0.8)^{12} = 0.250$$

$$P(4) = C_4^{15} (0.2)^4 (0.8)^{11} = 0.188$$

$$P(x > 4) = 1 - [0.035 + 0.132 + 0.231 + 0.250 + 0.188] = 0.164$$

c) What is the probability of answering less than 12 correct questions?

$P(x<12)=1-P(x\geq 12)$

$=1-[f(12)+f(13)+f(14)+f(15)]$

$=1-[9.54\times10^{-7}+5.5\times10^{-8}+1.97\times10^{-9}+3.27\times10^{-11}]=0.99999$

d) A student get grade B if he answered 11 correct questions or more. What is the probability of getting grade B.

$P(x\geq 11)=f(11)+f(12)+f(13)+f(14)+f(15)$

$f(11)=$

$f(12)=$

$f(13)=$

$f(14)=$

$f(15)=$

$P(x\geq 11)=$

Example (03) – Binomial Distribution

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A box contains 12 balls, 5 are red and the remaining are green.
Five balls are selected with replacement.

a) What is the probability of getting two red balls and three green balls in the sample?

(1) $n = 5$

(2) $P_R = \frac{5}{12}$

(3) $P(x=2) = C^5_2 \left(\frac{5}{12}\right)^2 \left(\frac{7}{12}\right)^3$

b) What is the probability of getting at most three green balls

(1) $n = 5$

(2) $P_G = \frac{7}{12}$

(3) $P(x \leq 3) = P(0) + P(1) + P(2) + P(3)$

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c) What is the probability of getting at least two red balls in the sample?

① $n = 5$

② $P_R = \frac{5}{12}$

③ $P(X \geq 2) = 1 - P(X < 2)$
$= 1 - [P(0) + P(1)]$

d) What is the probability of not getting any red balls in the sample?

$P(X = 0)$

$= C_0^5 \left(\frac{5}{12}\right)^0 \left(\frac{7}{12}\right)^5$

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Example (04) – Binomial Distribution

A traffic light is red with probability 0.3 and the green and yellow are equally likely

a) If you passed the traffic lights on 15 days, what is the probability it is green on exactly 5 times

(1) $n = 15$

(2) $P_G = 0.35$

(3) $P(X=5) = C^{15}_{5} (0.35)^{5} (0.65)^{10}$

b) If you passed the traffic lights on 10 days, what is the probability it is red more than 12 times?

(1) $n = 10$

(2) $P_R = 0.3$

(3) $P(X > 12) = P(13) + P(14) + P(15)$

c) If you passed the traffic lights on 20 days, what is the probability it is yellow at least half of the times?

1) $n = 20$

2) $P_y = 0.35$

3) $P(X \geq 10) = \sum_{x=10}^{20} C_x^{20} (0.35)^x (0.65)^{20-x}$

$$\boxed{P(X \geq 10) = 0.122}$$

d) What is the mean and variance of the traffic light to be green?

1) $n = 20$

2) $P_G = 0.35$

3) $\mu = np = (20)(0.35) = 7$

4) $\sigma^2 = np(1-p) = (20)(0.35)(0.65) = 4.55$

Example (05) – Negative Distribution

A shooter is trying to shot a rabbit. The rabbit must be shot exactly twice to be killed. If the probability to shot a rabbit is 0.25.

a) What is the probability that the second shot will hit the rabbit in the fifth time?

$r = 2$

$x = 5$

$p = 0.25$

$$P(x=5) = C_1^4 (0.25)^2 (0.75)^3 = 0.1055$$

b) What is the probability that the second shot will hit the rabbit in the seventh time?

$$P(x=7) = C_1^6 (0.25)^2 (0.75)^5 = 0.089$$

c) What is the probability that the shooter will need at least four times to hit the second shot?

$x = 4$

$r = 2$

$p = 0.25$

$P(x \geq 4) = 1 - P(x < 4) = 1 - [P(x=3) + P(x=2)]$

$P(x=2) = C_1^1 \, 0.25^2 (0.75)^0$ 0.0625

$P(x=3) = C_1^2 \, 0.25^2 (0.75)^1 = 0.09375$

$P(x \geq 4) = 1 - [0.09375 + 0.0625] = 0.844$

d) What is the probability that the shooter will need at most four times to hit the second shot?

$P(x \leq 4) = P(x=2) + P(x=3) + P(x=4)$

$P(x=2) = 0.0625$

$P(x=3) = 0.09375$

$P(x=4) = C_1^3 \, 0.25^2 \cdot 0.75^2 = 0.1054$

$P(x \leq 4) = 0.0625 + 0.09375 + 0.1054$

$\boxed{P(x \leq 4) = 0.262}$

Example (06) – Geometric Distribution

A car is not working properly. The driver is trying to turn the switch on. The probability the car will work is 0.3.

a) What is the probability that the driver will turn on the car on the fifth trial?

$$P(X=5) = (0.3)(0.7)^4 =$$

b) What is the probability that the driver will turn on the car on the eighth trial?

$$P(X=8) = (0.3)(0.7)^7 =$$

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c) What is the probability that the driver will need at least four trials to turn on the switch?

$$P(X \geq 4) = 1 - P(X < 4)$$

$$= 1 - [P(X=1) + P(X=2) + P(X=3)]$$

$$P(X=1) = (0.3)(0.7)^0 = 0.3$$

$$P(X=2) = (0.3)(0.7)^1 = 0.21$$

$$P(X=3) = (0.3)(0.7)^2 = 0.147$$

$$P(X \geq 4) = 1 - [0.3 + 0.21 + 0.147] =$$

d) What is the probability that the driver will need at most five trials to turn on the switch?

$$P(X \leq 5) = P(X=1) + P(X=2) + P(X=3)$$

$$+ P(X=4) + P(X=5)$$

$$P(X=1) = 0.3$$

$$P(X=2) = 0.21$$

$$P(X=3) = 0.147$$

$$\boxed{P(X \leq 5) = 0.83193}$$

$$P(X=4) = (0.3)(0.7)^3 = 0.1029$$

$$P(X=5) = (0.3(0.7)^4 = 0.07203$$

Example (07) – Geometric Distribution

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You are playing the game of luck. You have probability to win, lose & even with 0.2, 0.5 & 0.3

a) What is the probability that you will have the first win on the sixth game?

$P_W = 0.2$

$P_L = 0.5$

$P_e = 0.3$

$P(x=6) = (0.2)(0.8)^5 = 0.0655$

b) What is the probability that you will have the first lose on the fifth game?

$P(x=5) = (0.5)(0.5)^4 = 0.03125$

c) What is the probability that you will have the second even on the fourth game?

$$P(x=4) = C_1^3 (0.3)^2 (0.7)^2 = 0.1323$$

d) What is the probability that you will have the third win on the sixth game?

$$P(x=6) = C_2^5 (0.2)^3 (0.8)^4 = 0.0327$$

Hyper Geometric Distribution

لازم يعطي N, n, K

لازم without replacemt

العدد الكلي N

العينة $n \rightarrow$ draw - examine - test - inspect - sample - choose - select

K $\rightarrow$ النوع الي نبيه

$$f(x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}} \qquad x = 0, 1, 2, \cdots, n$$

$$E(x) = np = n\left[\frac{K}{N}\right] \qquad \boxed{p = \frac{K}{N}}$$

$$V(x) = np(1-p)\left[\frac{N-n}{N-1}\right]$$

Problem (   )

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A batch contains 36 bacteria cells and 12 of the cells are not capable of cellular replication. Suppose you examine three bacteria cells selected at random, without replacement. → n

(a) What is the probability mass function of the number of cells in the sample that can replicate?

(b) What are the mean and variance of the number of cells in the sample that can replicate?

(c) What is the probability that at least one of the selected cells cannot replicate?

36

36	
24	12

capable
able to
can

not capable
not able to
can not
can't

$n = 3$

Hypergeometric

a) $f(x) = \dfrac{\binom{24}{x}\binom{12}{3-x}}{\binom{36}{3}}$

$\boxed{K = 24}$

$x = 0, 1, 2, 3$

b) $E(x) = np = n\left(\frac{k}{N}\right)$

$k = 24$ can

$E(x) = (3)\left[\frac{24}{36}\right]$

$V(x) = np(1-p)\left[\frac{N-n}{N-1}\right]$ $p = \frac{k}{N}$

---

c) $P(X \geq 1) = 1 - P(X < 1) = 1 - P(X=0)$

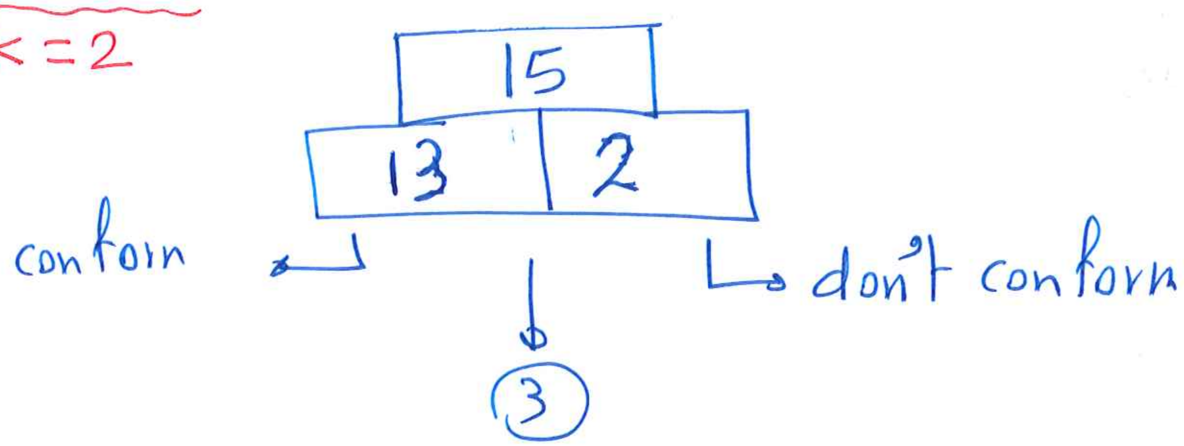
$\boxed{k = 12} \to$ can't

$$= 1 - \frac{\binom{12}{0}\binom{24}{3}}{\binom{36}{3}} = \boxed{}$$

Problem ( )

A shipment of chemicals arrives in 15 totes. Three of the totes are selected at random, without replacement, for an inspection of purity. If two of the totes do not conform to purity requirements, what is the probability that at least one of the nonconforming totes is selected in the sample?

$K=2$



$n=3$

a) $P(X \geq 1) = 1 - P(X < 1) = 1 - P(X=0)$

$\boxed{K=2}$

$$= 1 - \frac{\binom{2}{0}\binom{13}{3}}{\binom{15}{3}} = \boxed{}$$

Binomial يصير ← with replacemnt لو قال

$n=3$

$p = \left(\frac{K}{N}\right)$

Problem (   )

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An urn contains 18 red balls, 14 white balls, and 15 green balls. Select

a) If we selected 5 balls with replacement, what is the probability that there are 3 white balls in the sample
b) If we selected 7 balls without replacement, what is the probability that there are 3 red balls in the sample
c) If we selected balls with replacement, what is the probability the third ball is the first green ball
d) If we selected balls with replacement, what is the probability the seventh ball is the fourth white
e) If we selected two balls without replacement, event G1 indicates the first is green. Event G2 indicates that the second is green. Are these two events independent?
f) If we selected two balls with replacement, draw the tree diagram indicating the probability of each out come

a) with replacement : binomial

$n = 5$

$p = \frac{14}{47}$

$$P(X=3) = C^5_3 \cdot \left(\frac{14}{47}\right)^3 \left(\frac{33}{47}\right)^2$$

b) without replacement : hypergeometric

$N = 47$ $\quad k = 18$

$n = 7$

$$P(X=3) = \frac{C^{18}_3 \; C^{29}_4}{C^{47}_7}$$

c) with replacement : geometric

$r = 1$

$p = \frac{15}{47}$

$P(X=3) = \left(\frac{15}{47}\right)\left(\frac{32}{47}\right)^2$

d) with replacement : negative

$p = \frac{14}{47}$

$r = 4$

$P(X=7) = C^6_3 \cdot \left(\frac{14}{47}\right)^3 \left(\frac{33}{47}\right)^3$

e) without replacement → not independent

$P(G_1) = \frac{15}{47}$

$P(G_2) = \frac{14}{46} \cdot \frac{15}{47} + \frac{15}{46} \cdot \frac{32}{47}$

$P(G_2) = \frac{15}{47}$

$P(G_1 \cap G_2) = \frac{15}{47} \cdot \frac{14}{46} = \frac{105}{1081}$

not equal ∴ G_1 and G_2 not independent

Example (08) – Poisson distribution

Number of defects found on a pipe are distributed according to poison distribution with a mean number of 3.2 defects per meter.

a) What is the mean, variance & standard deviation?

$$E(x) = \mu = \lambda = 3.2$$

$$V(x) = \sigma^2 = \lambda = 3.2$$

$$\sigma = \sqrt{\lambda} = \sqrt{3.2} = 1.79$$

b) What is the probability there are three defects in four meters of pipes?

$$P(x=3) = \frac{e^{-(3.2)(4)} \cdot (3.2 * 4)^3}{3!}$$

$$P(x=3) = 9.65 \times 10^{-4}$$

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c) What is the probability there are 10 defects in sixth meters of pipes?

$$P(x=10) = \frac{e^{-(3.2\times 6)}\,(3.2\times 6)^{10}}{10!}$$

d) What is the probability there are no defects in two meters of pipes?

$$P(x=0) = \frac{e^{-(3.2\times 2)}\,(3.2\times 2)^{0}}{0!}$$

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e) What is the probability there are at least three defects in half meter of pipes?

$$P(X \geq 3) = 1 - P(X < 3) = 1 - [P(X=0) + P(X=1) + P(X=2)]$$

$$P(X=0) = \frac{e^{-(3.2 \times 0.5)} (3.2 \times 0.5)^0}{0!} =$$

$$P(X=1) = \frac{e^{-(3.2 \times 0.5)} (3.2 \times 0.5)^1}{1!} =$$

$$P(X=2) = \frac{e^{-(3.2 \times 0.5)} (3.2 \times 0.5)^2}{2!} =$$

f) What is the probability there are at most five defects in five meter of pipes?

$$P(X \leq 5) = \sum_{x=0}^{5} \left[\frac{e^{-(3.2 \times 5)} (3.2 \times 5)^x}{x!} \right] = 1.38 \times 10^{-3}$$

Problem

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The number of telephone calls that arrive at a phone exchange is often modeled as a Poisson random variable. Assume that on the average there are 10 calls per hour.

a. What is the probability that there are exactly 5 calls in one hour?
b. What is the probability that there are 3 or fewer calls in one hour?
c. What is the probability that there are exactly 15 calls in two hours?
d. What is the probability that there are exactly 5 calls in 30 minutes?

$\lambda = 10$ calls $\quad 1\,hr$

a) $P(X=5) = \dfrac{e^{-\lambda}\lambda^{X}}{X!}$

$\boxed{\lambda = 10 \quad 1\,hr}$

$$= \frac{e^{-10}(10)^5}{5!} = \boxed{}$$

b) $P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$\boxed{\lambda = 10 \quad 1\,hr}$

$$= \frac{e^{-10}(10)^0}{0!} + \frac{e^{-10}(10)^1}{1!} + \frac{e^{-10}10^2}{2!} + \frac{e^{-10}10^3}{3!}$$

c) $P(X=15) = \dfrac{e^{-20}20^{15}}{15!} =$

$\boxed{\begin{matrix}\lambda = 10 & 1\,hr \\ \lambda = 20 & 2\,h\end{matrix}}$

Problem (   )

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The number of flaws in bolts of cloth in textile manufacturing is assumed to be Poisson distributed with a mean of 0.1 flaw per square meter.

(a) What is the probability that there are two flaws in 1 square meter of cloth?

(b) What is the probability that there is one flaw in 10 square meters of cloth?

(c) What is the probability that there are no flaws in 20 square meters of cloth?

(d) What is the probability that there are at least two flaws in 10 square meters of cloth?

$\lambda = 0.1 \quad 1\,m^2$

$\lambda' = 1 \quad 10\,m^2$

$\lambda = 2 \quad 20\,m^2$

a) $P(X=2)$ $\qquad \lambda = 0.1$

$= \frac{e^{-0.1} \cdot 0.1^2}{2!} = \boxed{}$

b) $P(X=1)$ $\qquad \lambda = 1$

$= \frac{e^{-1} \cdot (1)^1}{1!} = \boxed{}$

c) $P(X=0)$ $\qquad \lambda = 2$

$= \frac{e^{-2} \cdot 2^0}{0!} = \boxed{}$

$\boxed{\lambda = 1}$

d) $P(X \geq 2) = 1 - P(X<2) = 1 - [P(X=0) + P(X=1)]$

Problem ( )

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The number of messages sent to a computer bulletin board is a Poisson random variable with a mean of five messages per hour.

(a) What is the probability that five messages are received in 1 hour?

(b) What is the probability that 10 messages are received in 1.5 hours?

(c) What is the probability that less than two messages are received in one-half hour?

$\lambda = 5 \quad 1\,hr$

$\lambda = 7.5 \quad 1.5\,hr$

a) $P(X=5) \qquad \lambda = 5$

b) $P(X=10) \qquad \lambda = 7.5$

c) $P(X<2) = P(X=0) + P(X=1) \qquad \boxed{\lambda = 2.5}$