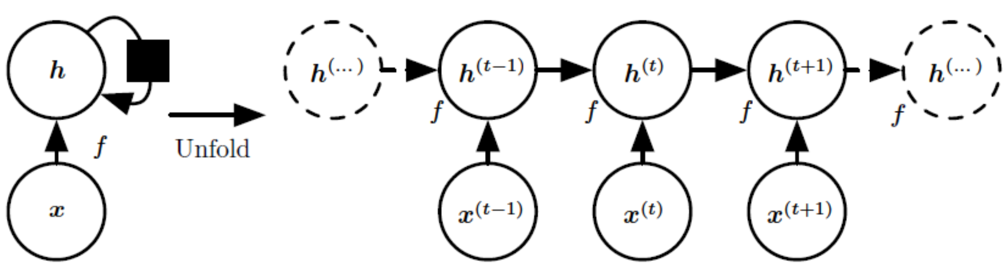
Lecture 10

Deep learning

1

RNN



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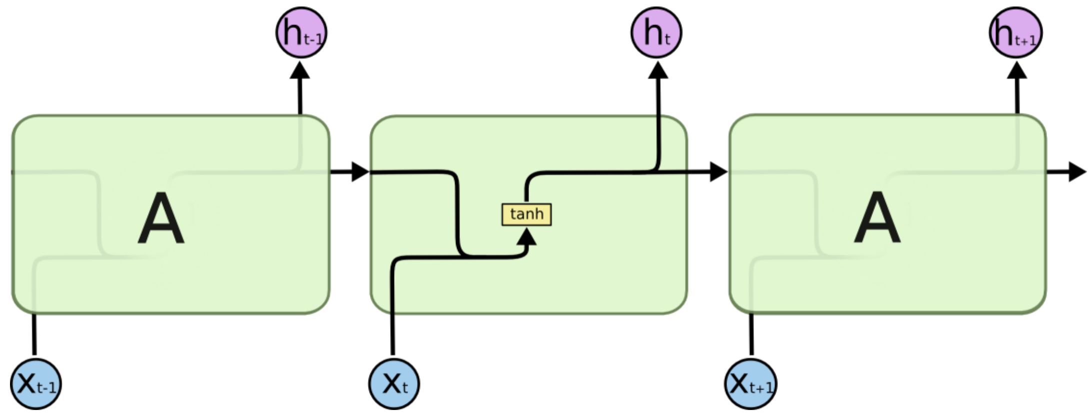
RNN



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Long Short-Term Memory and Gated Recurrent Units

- Gated RNNs (including LSTM and GRUs) are based on the idea of creating paths through time that have derivatives that neither vanish nor explode.
- Also, Gated RNNs learns what to *forget* and what to keep in an automated manner rather than doing it manually.
- LSTM recurrent networks have "LSTM cells" that have an internal recurrence (a self-loop), in addition to the outer recurrence of the RNN.
- Each cell has the same inputs and outputs as an ordinary recurrent network, but has more parameters and a system of gating units that controls the flow of information.

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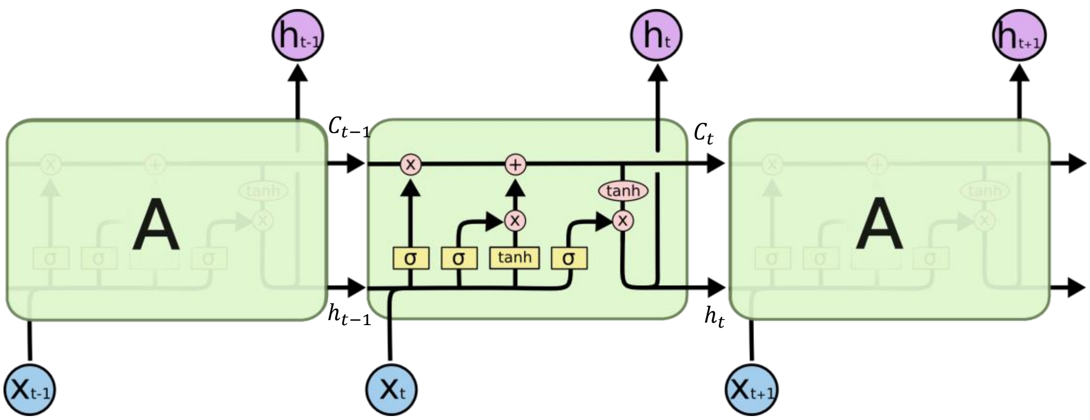
LSTM (Hochreiter & Schmidhuber (1997))



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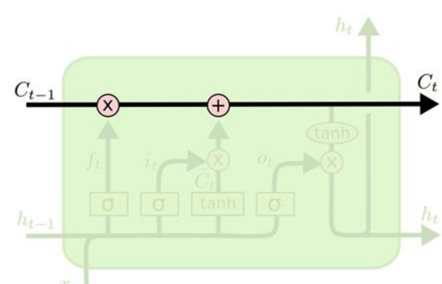
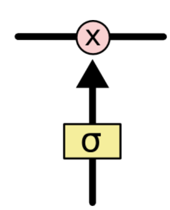
LSTM

- The key to LSTMs is the cell state, which the line that appears at the top of the diagram.
- The LSTM does have the ability to remove or add information to the cell states using a gated structure.
- Gates are a way to optionally let information through.
- They are composed out of a sigmoid neural net layer (outputs a number between zero and one that describes how much of each component should be let through) and a pointwise multiplication operation.



Cell state



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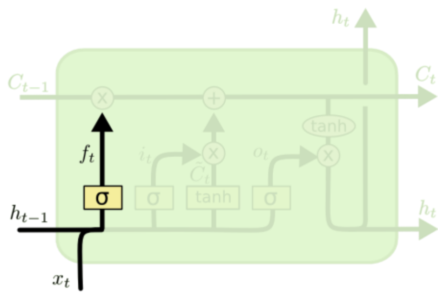
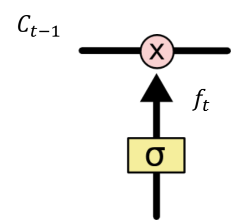
LSTM – Forget Gate (remove information from the cell state)

- Forget gate (f_t): a sigmoid gate that decides what information that we will throw away from the cell state.



$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] \ + \ b_f\right)$$



The sigmoid function produces f_t (a vector that has values between 0,1).

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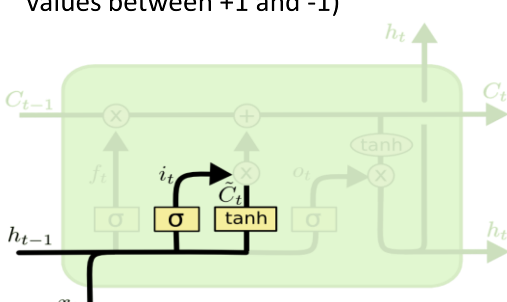
LSTM – Input Gate (Add information to the cell state)

- Input gate (i_t): a sigmoid gate that decides what new information we're going to store in the cell state.
- Two parts:
  - A sigmoid layer called the "input gate layer" decides which values we'll update (a vector that has values between 0,1 that will be multiplied by the tanh layer).
  - A tanh layer that creates a vector of new candidate values that can be added to the state (takes values between +1 and -1)



$$i_t = \sigma\left(W_i \cdot [h_{t-1}, x_t] \ + \ b_i\right)$$
$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] \ + \ b_C)$$

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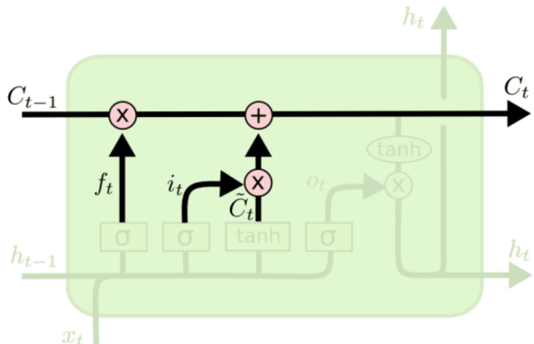
LSTM

- Combine the two forget and input gates to produce C_t



$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

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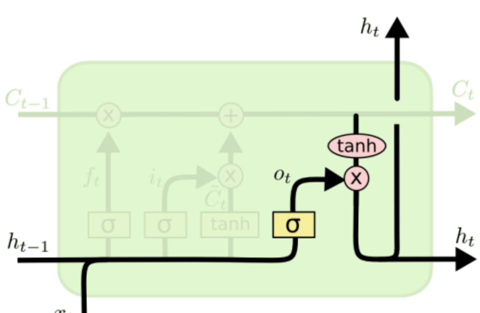
LSTM - Output Gate

- Output Gate (o_t): A sigmoid gate that decides what parts of the cell state we're going to output.
- Put the cell state through tanh and multiply it by the sigmoid gate



$$o_t = \sigma\left(W_o\ [h_{t-1}, x_t]\ +\ b_o\right)$$
$$h_t = o_t * \tanh\left(C_t\right)$$

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Variants on Long Short Term Memory

- Which pieces of the LSTM architecture are actually necessary?
- What other successful architectures could be designed that allow the network to dynamically control the time scale and forgetting behavior of different units?

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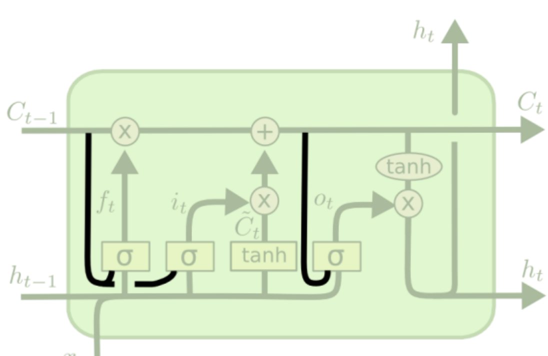
Variants on Long Short Term Memory

- Introduced by Gers & Schmidhuber (2000).
- Adding “peephole connections.” that allows the gate layers look at the cell state.



$$f_t = \sigma\left(W_f \cdot [\boldsymbol{C_{t-1}}, h_{t-1}, x_t] \;+\; b_f\right)$$
$$i_t = \sigma\left(W_i \cdot [\boldsymbol{C_{t-1}}, h_{t-1}, x_t] \;+\; b_i\right)$$
$$o_t = \sigma\left(W_o \cdot [\boldsymbol{C_t}, h_{t-1}, x_t] \;+\; b_o\right)$$

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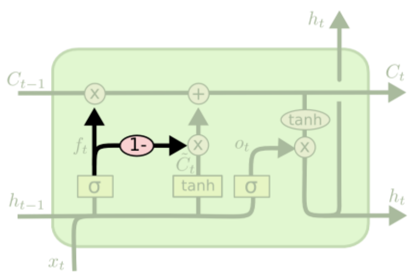
Variants on Long Short Term Memory

- Another variation is to use coupled forget and input gates.
- Instead of separately deciding what to forget and what we keep, we make those decisions together.
- We only forget when we're going to input something in its place. We only input new values to the state when we forget something older.



$$C_t = f_t * C_{t-1} + (1 - f_t) * \tilde{C}_t$$

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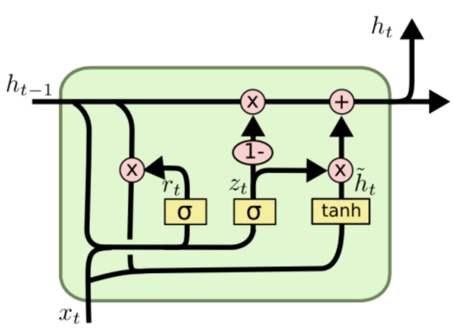
Gated Recurrent Units: introduced by Cho et al. (2014)

- Simpler than standard LSTM models
- It merges the cell state and hidden state and combines the forget and input gates into a single "update gate."



$$z_t = \sigma\left(W_z \cdot [h_{t-1}, x_t]\right)$$
$$r_t = \sigma\left(W_r \cdot [h_{t-1}, x_t]\right)$$
$$\tilde{h}_t = \tanh\left(W \cdot [r_t * h_{t-1}, x_t]\right)$$
$$h_t = (1 - z_t) * h_{t-1} + z_t * \tilde{h}_t$$

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Unsupervised Learning Algorithms

- Useful to learn useful properties about the structure of this dataset (e.g. learn the probability distribution that generated a dataset).
- Can be used in density function estimation or other tasks like denoising.
- Can perform other tasks such as clustering.

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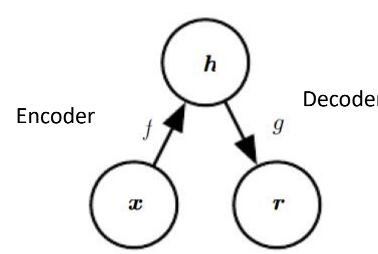
Autoencoders (Chapter 14)

- An **autoencoder** is a neural network that is trained to attempt to copy its input to its output.
- Internally, it has a hidden layer *h* that describes a **code** used to represent the input.
- The network may be viewed as consisting of two parts: an encoder function $h = f(x)$ and a decoder that produces a reconstruction $r = g(h)$.



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Autoencoders

- Autoencoders are restricted in ways that allow them to copy only approximately, and to copy only input that resembles the training data.
- Typically, we would like to prioritize learning some useful aspects of the data (e.g. if your input is a noisy data, you would like your autoencoder to learn how to recover the original data)
- If an autoencoder succeeds in simply learning to set $g(f(x)) = x$ everywhere, then it is not especially useful.
- Traditionally, autoencoders were used for dimensionality reduction or feature learning. Recently, theoretical connections between autoencoders and latent variable models have brought autoencoders to the forefront of generative modeling,

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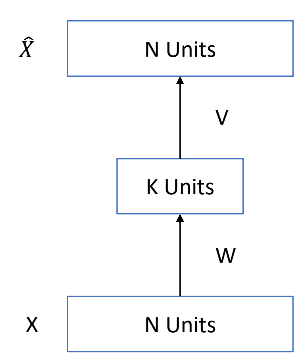
Autoencoders and PCA

- The simplest kind of autoencoder has one hidden layer, linear activations, and squared error loss

$$L(x,\hat{x}) = ||x - \hat{x}||^2$$

$$\hat{x} = \text{WVx (a linear function)}$$

- If K>=N, then we can choose WV that is the identity function
- If K<N, W maps x to a K-dimensional space, so it's doing dimensionality reduction
- The autoencoder should learn to choose the subspace which minimizes the squared distance from the data to the projections.
- Thus, it is equivalent to PCA which maximizes the variance of the projections.



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Difference between autoencoders and PCA

- In PCA, transformations are linear.
- When the decoder is linear and *L* is the mean squared error, an undercomplete autoencoder learns *to span the same subspace as PCA*.
- Autoencoders with nonlinear encoder functions *f* and nonlinear decoder functions *g* can learn a more powerful nonlinear generalization of PCA.
- If the encoder and decoder are allowed too much capacity, the autoencoder can learn to perform the copying task without extracting useful information about the distribution of the data.
- If the capacity of the autoencoder is allowed to become too great, an autoencoder can fail to learn anything useful about the dataset.
- Thus, *f* or *g,* in undercomplete autoencoders, typically has low capacity

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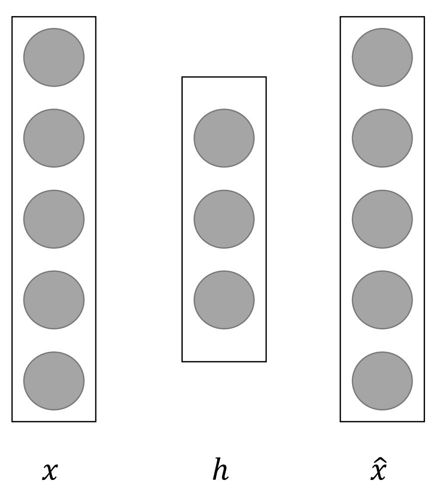
Undercomplete Autoencoders

- One way to obtain useful features from the autoencoder is to constrain *h* to have smaller dimension than *x*.
- An autoencoder whose code dimension is less than the input dimension is called **undercomplete**.
- Learning an undercomplete representation forces the autoencoder to capture the most salient (important) features of the training data.
- The learning process is described simply as minimizing a loss function

$$L(x, g(f(x)))$$

- where *L* is a loss function penalizing *g*(*f*(*x*)) is the decoder, f(x) is the encoder function.



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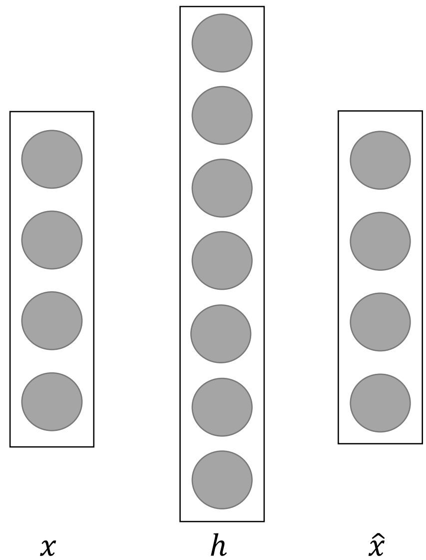
Overcomplete Autoencoders

- An autoencoder whose code dimension is greater than the input dimension is called **overcomplete.**
- In case of overcomplete autoencoder, even a linear encoder and linear decoder may learn to copy the input to the output without learning anything useful about the data distribution.
- Must be regularized



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Regularized Autoencoders

- Regularized autoencoders provide the ability to choose the decoder based on the complexity of distribution to be modeled.
- Rather than limiting the model capacity by keeping the encoder and decoder shallow and the code size small, regularized autoencoders use a loss function that encourages the model to have other properties besides the ability to copy its input to its output.
- These other properties include sparsity of the representation (sparse autoencoders), robustness to noise or to missing inputs (denoising autoencoders), and smallness of the derivative of the representation (Contractive autoencoders).
- A regularized autoencoder can be nonlinear and overcomplete but still learn something useful about the data distribution even if the model capacity is great enough to learn a trivial identity function.

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Sparse Autoencoders

- Typically used to learn features as a pre-processing for another task such as classification.
- A sparse autoencoder is simply an autoencoder whose training criterion involves a sparsity penalty $\Omega(h)$ on the code layer h, in addition to the reconstruction error:

$$L\left(x, g\left(f(x)\right)\right) + \Omega(h)$$

$$\Omega(h) = \lambda \sum_i |h_i|$$

- We can think of the penalty $\Omega(h)$ simply as a regularizer term added to a feedforward network whose primary task is to copy the input to the output (unsupervised learning objective) and possibly also perform some supervised task (with a supervised learning objective) that depends on these sparse features.

https://web.stanford.edu/class/cs294a/sparseAutoencoder_2011new.pdf

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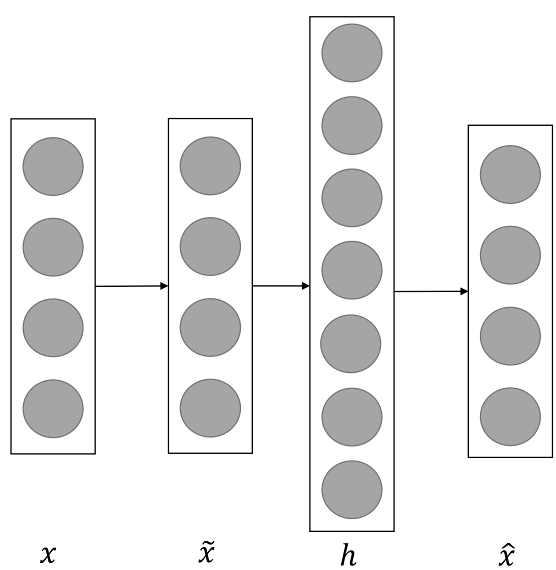
Denoising Autoencoders (DAE)

- Instead of feeding the original input x we feed a noise version of it $\tilde{x}$.
- A **denoising autoencoder** or DAE minimizes

$$L\left(x, g\left(f(\tilde{x})\right)\right)$$

- Reconstruction $\hat{x}$ is computed from the corrupted input $\tilde{x}$
- Loss function compares $\hat{x}$ reconstruction with the noiseless input x



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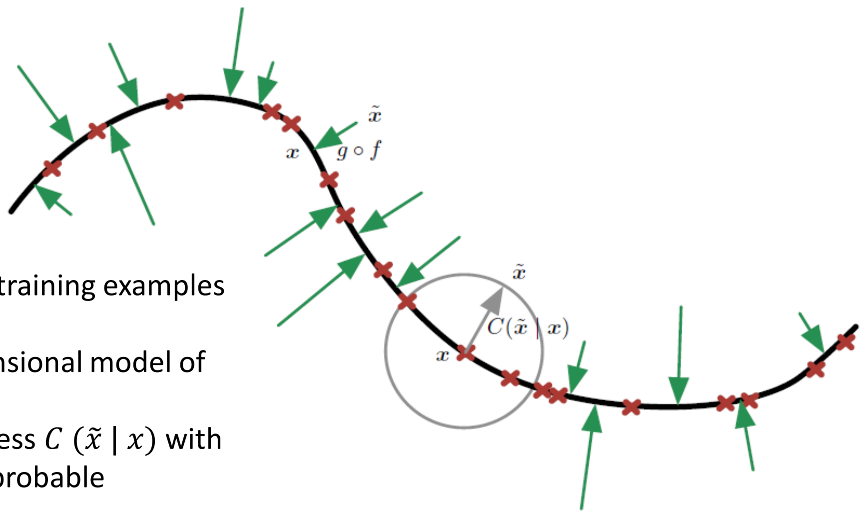
Denoising autoencoder



- Red crosses are the training examples **x.**
- Black line low-dimensional model of the autoencoder
- The corruption process $C\ (\tilde{x} \mid x)$ with a gray circle of equiprobable corruptions.

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Contractive Autoencoders (CAE)

- Contractive Autoencoders forces the model to learn a function that does not change much when *x* changes slightly

$$L\left(x, g\big(f(x)\big)\right) + \Omega(h, x)$$

$$\Omega(h, x) = \lambda \sum_i \|\nabla_x h_i\|^2$$

- Features that are sensitive to small changes in the inputs are penalized.
- The name **contractive** arises from the way that the CAE warps space.
- Specifically, because the CAE is trained to resist perturbations of its input, it is encouraged to map a neighborhood of input points to a smaller neighborhood of output points.

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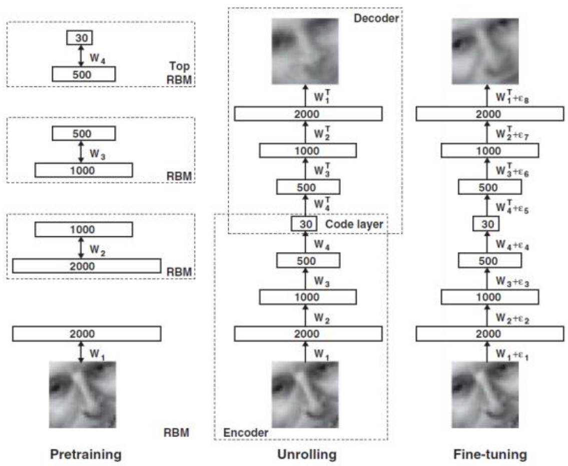
Deep Autoencoders

- “Reducing the Dimensionality of Data with Neural Networks” by G. E. Hinton* and R. R. Salakhutdinov



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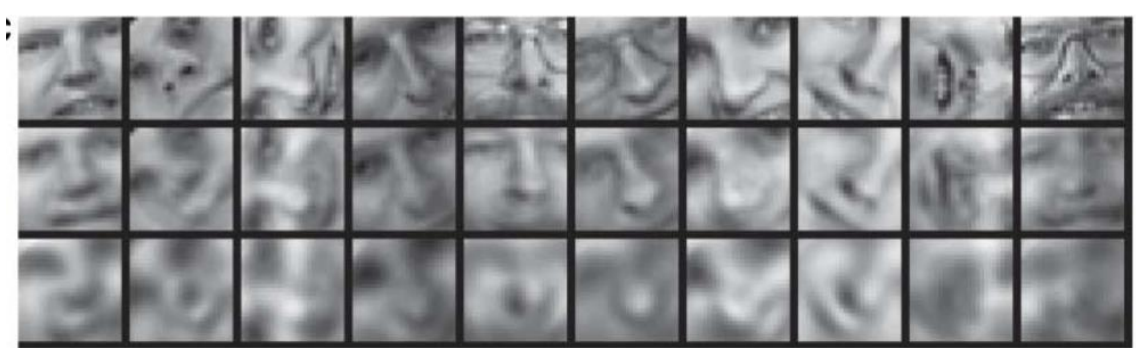
Deep Autoencoders

- “Reducing the Dimensionality of Data with Neural Networks” by G. E. Hinton* and R. R. Salakhutdinov



Top to bottom: 1) Random samples from the test data set; 2) reconstructions by the 30-dimensional autoencoder; 3) reconstructions by 30-dimensional PCA. The average squared errors are 126 and 135.

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