CSE 604: Artificial Intelligence
Quiz 2 Solutions
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Full Marks: 50
Time: 50 minutes

Problem I. Logic [12 Marks]

Consider a vocabulary with the following symbols:

Occupation (p, o): Predicate. Person p has occupation o.
Customer (p1, p2): Predicate. Person p1 is a customer of person p2.
Boss (p1, p2): Predicate. Person p1 is a boss of person p2.

Doctor, **Surgeon**, **Lawyer**, **Actor** : Constants denoting occupations.
Shuvra, **Himel**: Constants denoting people.

Use the given symbols to write the following assertions in **first order logic**.

a. Himel is an actor, but he also holds another job

Occupation (Himel, Actor) ∧ ∃o [o ≠ Actor ∧ Occupation (Himel, o)]

b. Some surgeons do not have a boss

∃p Occupation (p, Surgeon) ∧ ¬∃p2 Boss (p2, p)

c. Shuvra does not have a doctor

¬∃p Occupation (p, Doctor) ∧ Customer (Shuvra, p)

d. Himel has a boss who has a lawyer

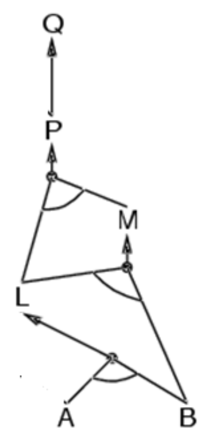
∃p Boss (p, Himel) ∧ [∃p2 Occupation (p2, Lawyer) ∧ Customer (p, p2)]

Problem II. Logical Inference [12 Marks]

The following propositional logic facts (sentences) are given in Horn form:

- $P \Rightarrow Q$
- $(L \wedge M) \Rightarrow P$
- $(B \wedge L) \Rightarrow M$
- $(A \wedge B) \Rightarrow L$
- A
- B

i. Use **forward chaining** to prove that the sentence **Q is true** [4 marks]
Ans: Q is True



ii. Convert each of these sentences to Conjunctive Normal Form (**CNF**) [4 marks]

1. $\neg P \vee Q$
2. $\neg L \vee \neg M \vee P$
3. $\neg B \vee \neg L \vee M$
4. $\neg A \vee \neg B \vee L$
5. A
6. B

iii. Now use **resolution** on the CNF clauses to prove that **Q is true** [4 marks]

7. $\neg Q$ $(\neg\alpha)$

Resolve 1 and 7: 8. $\neg P$

Resolve 2 and 8: 9. $\neg L \vee \neg M$

Resolve 3 and 9: 10. $\neg B \vee \neg L$

Resolve 4 and 10: 11. $\neg A \vee \neg B$

Resolve 5 and 11: 12. $\neg B$

Resolve 6 and 12: <empty> So Q is True

Problem III. Quantifying Uncertainty [10 Marks]

1. State whether the following statements are true or false. **Explain your answer**. [6 marks]

   a. If $P(a \mid b, c) = P(a)$, then $P(b \mid c) = P(b)$

      FALSE.
      If $P(a \mid b, c) = P(a)$, that means a is independent of b and c. But that says nothing about the independence of b with regards to c, which is expressed by $P(b \mid c) = P(b)$.

   b. If $P(a \mid b) = P(a)$, then $P(a \mid b, c) = P(a \mid c)$

      TRUE.

      If a is independent of b, then a given b and c is the same as a given just c.

2. A doctor knows that the disease dengue causes the patient to have a severe headache 70% of the time. The doctor also knows some unconditional facts: the prior probability that a patient has dengue is 1/50, and the prior probability that any patient has a severe headache is 5%. Under these circumstances, what is the probability that a patient, who comes in with a severe headache, has dengue? [4 marks]

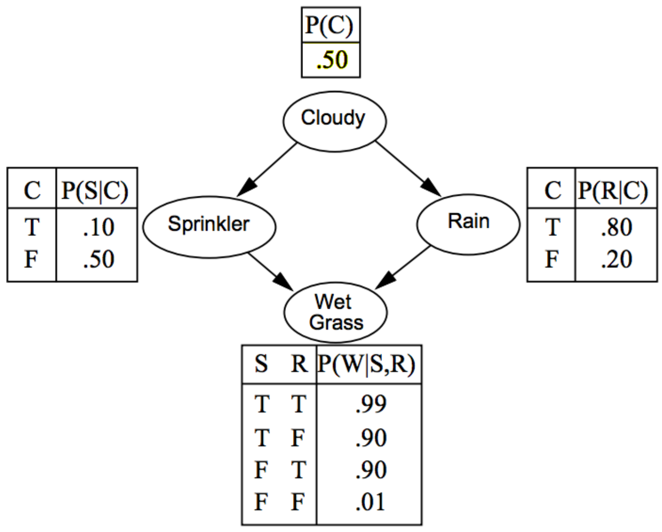
   P(dengue | headache) = P (headache | dengue) * P (dengue) / P(headache)

   = 0.7 * 0.02 / 0.05

   = 0.28

Problem IV. Probabilistic Reasoning [16 Marks]

A simple Bayesian Network, with corresponding conditional probability tables, is given below:



From this Bayesian network, answer the following questions:

1. Which pairs of variables in this network are conditionally independent of each other given other variables? State your answers in the following format: "X and Y are cond. ind. given Z". Mention all such pairs [4 marks]

   Sprinkler and Rain are cond. ind., given Cloudy

   Cloudy and Wet Grass are cond. ind., given Sprinkler

   Cloudy and Wet Grass are cond. ind., given Rain

2. What is the probability that it is cloudy, it is raining, the sprinkler is off, and the grass is wet? (Just write the corresponding values, you do not need a calculator) [4 marks]

   P(c, r, s', w) = P(w | s', r) P(s' | c) P(r | c) P(c) = 0.90 * 0.90 * 0.80 * 0.50 = 0.324

3. What is the probability that it is raining, given the grass is wet? (Expand your equation up to the point where the rest of the calculation becomes obvious) [8 marks]

   P(r | w) = P(r, w) / P(w)

   Lets evaluate them separately:

   P(r, w) = P(r, w, s', c') + P(r, w, s', c) + P(r, w, s, c') + P(r, w, s, c)

   P(w) = P(w, c', s', r') + P(w, c', s', r) + ….. (all 8 combinations)

   Now evaluate each of the terms similar to Part 2 of this question