1. (8.3) Yes, assuming we have at least one object in our in our world, then that object is equal to itself.

2. (8.6)

   (a) valid

   (b) valid

   (c) valid

3. (8.9)

   (a) Paris and Marseilles are both in France.

      i. 2

      ii. 1

      iii. 3

   (b) There is a country that borders both Iraq and Pakistan.

      i. 1

      ii. 3

      iii. 2

      iv. 2

   (c) All countries that border Ecuador are in South America.

      i. 1

      ii. 1

      iii. 3

      iv. 3

   (d) No region in South America borders any region in Europe

      i. 1

      ii. 1

      iii. 3

      iv. 2

   (e) No two adjacent countries have the same map color.

      i. 1

      ii. 3

      iii. 3

      iv. 2

4. (8.10)

   (a) $Occupation(Emily, Surgeon) \vee Occupation(Emily, Lawyer)$

   (b) $\exists o \; (o \neq Actor) \wedge Occupation(Joe, Actor) \wedge Occupation(Joe, o)$

   (c) $\forall p \; Occupation(p, Surgeon) \Rightarrow Occupation(p, Doctor)$

(d) $\neg\exists p\ Occupation(p, Lawyer) \wedge Customer(Joe, p)$

(e) $\exists p\ Boss(p, Emily) \wedge Occupation(p, Lawyer)$

(f) $\exists p1\ Occupation(p1, Lawyer) \wedge \forall p2\ Customer(p2, p1) \Rightarrow Occupation(p2, Doctor)$

(g) $\forall p1\ Occupation(p1, Surgeon) \Rightarrow \exists p2\ Occupation(p2, Lawyer) \wedge Customer(p1, p2)$

5. Assuming predicates $Parent(p, q)$ and $Female(p)$ and constants $Joan$ and $Kevin$, with the obvious meanings, express each of the following sentences in first-order logic. (You may use the abbreviateion $\exists^1$ to mean "there exists exactly one.")

(a) $\exists d\ Female(d) \wedge Parent(Joan, d)$

(b) $\exists^1 d\ Female(d) \wedge Parent(Joan, d)$

(c) $[\exists^1 d\ Parent(Joan, d)] \wedge \forall d\ Parent(Joan, d) \Rightarrow Female(d)$

(d) $\exists^1 d\ Parent(Joan, d) \wedge Parent(Kevin, d)$

(e) $[\exists d\ Parent(Joan, d) \wedge Parent(Kevin, d)] \wedge \neg\exists c Parent(Joan, c) \wedge \neg Parent(Kevin, c)$

6. (8.20)

(a) $\exists y\ ((1 + 1) \times y = x)$

(b) $\neg\exists y, z\ (1 < y) \wedge (1 < z) \wedge (y \times z = x)$

(c)

$$\begin{aligned} &\forall x\ [\exists y\ ((1 + 1) \times y = x)] \\ \Rightarrow &\exists p, q\ [\neg\exists y, z\ (1 < y) \wedge (1 < z) \wedge [y \times z = p \vee y \times z = q]] \wedge (p + q = x) \end{aligned}$$