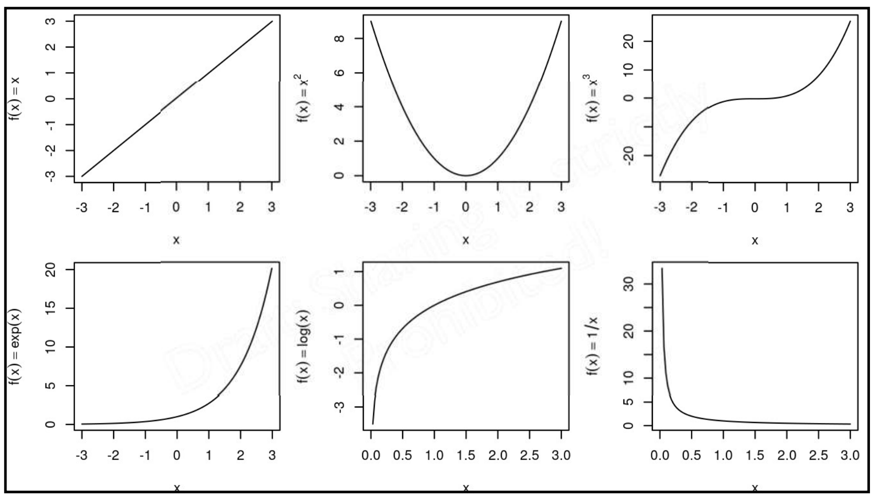
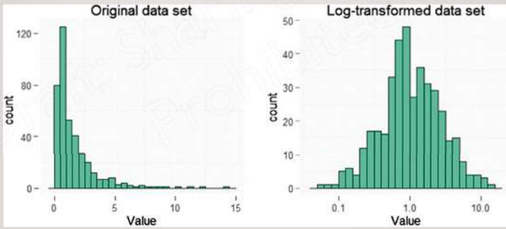
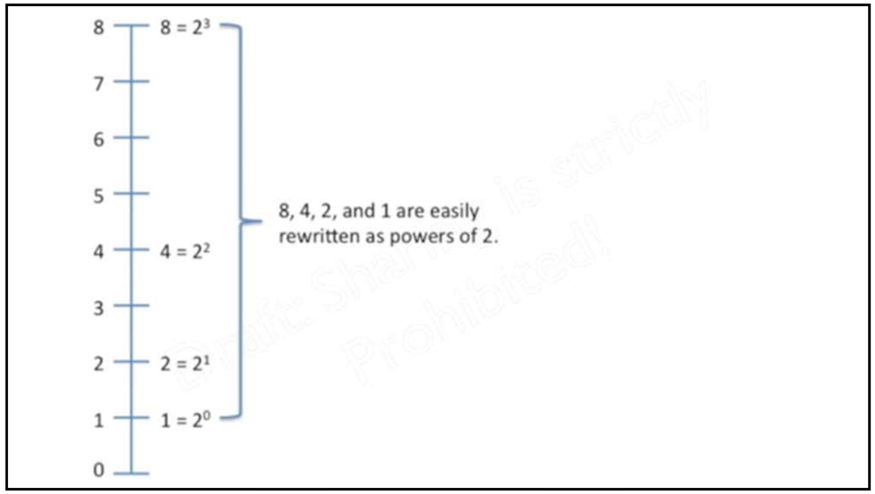
Functional Form & Transformation

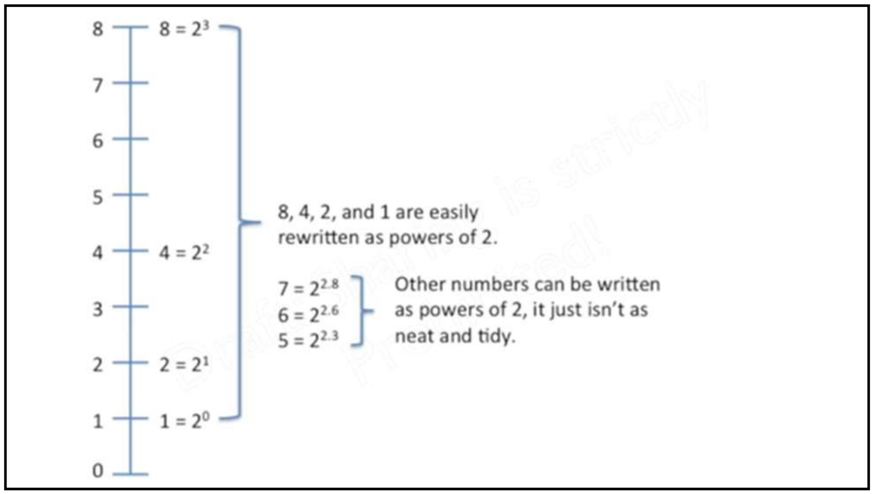


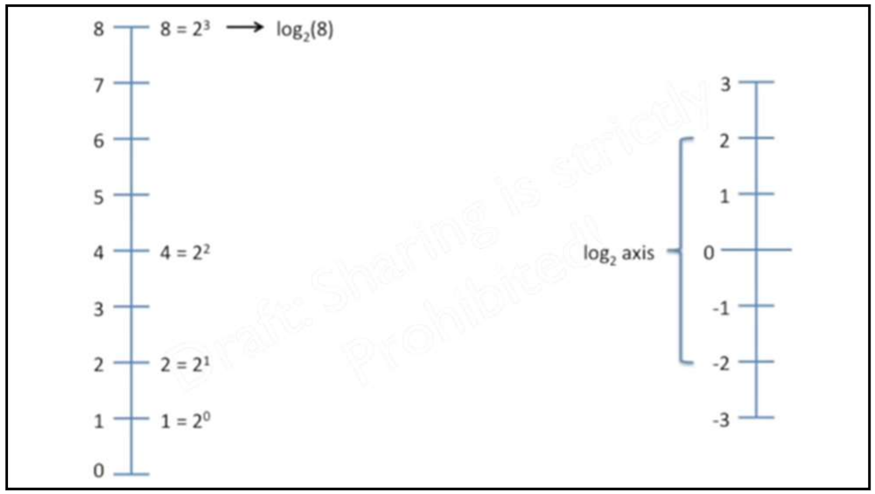
Log Transformation

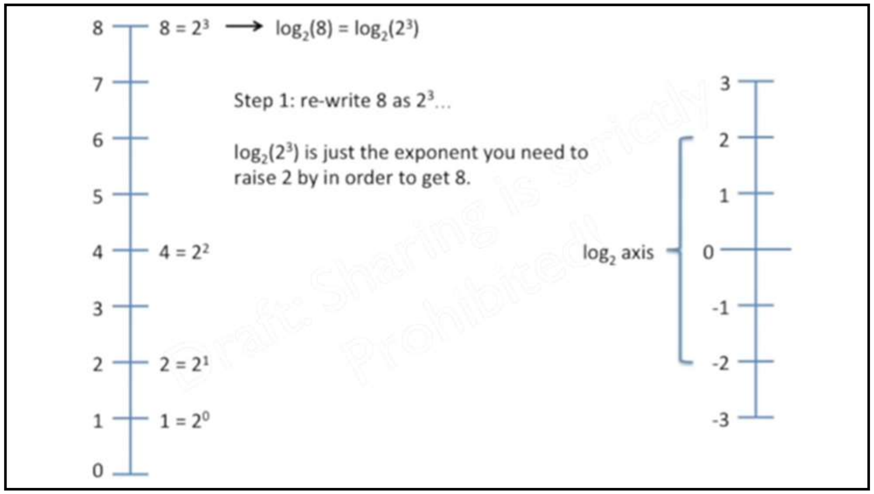
- Used mostly because of skewed distribution. Logarithm naturally reduces the dynamic range of a variable so that the differences are preserved while the scale is not that dramatically skewed.
- Imagine some people got 100,000,000 loan and some got 10000 and some 0. Any feature scaling will probably put 0 and 10000 so close to each other as the biggest number anyway pushes the boundary. Logarithm solves the issue.

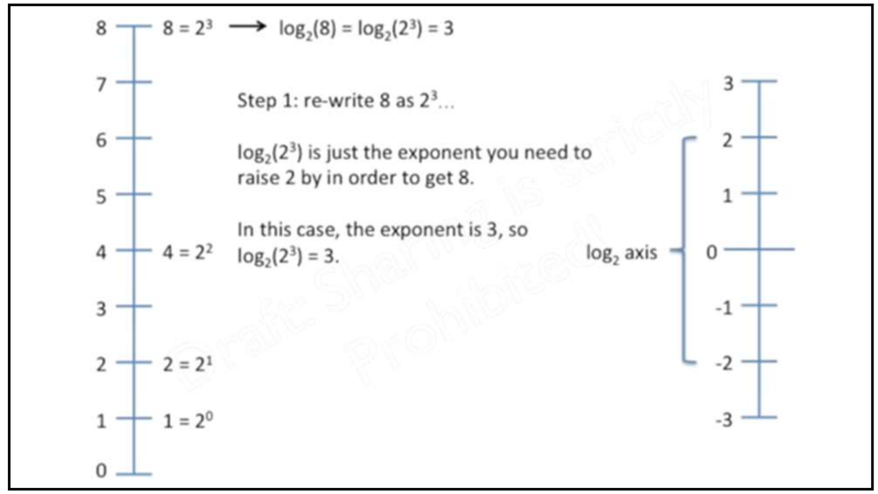


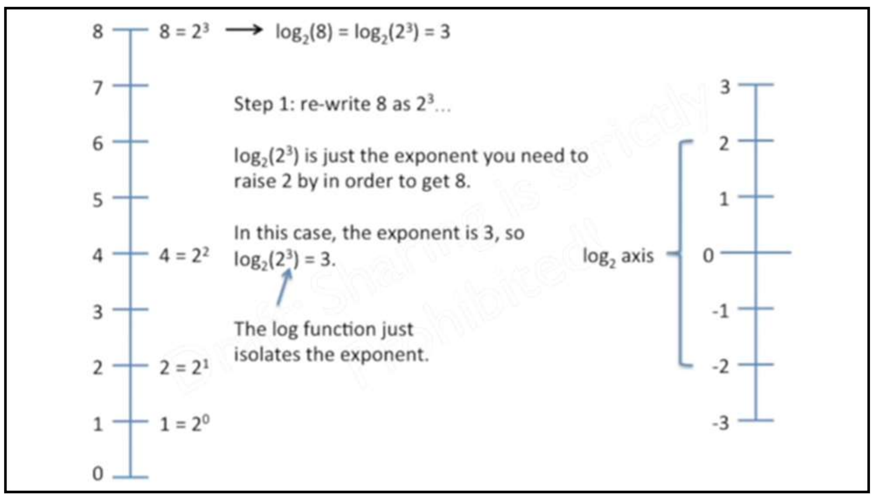


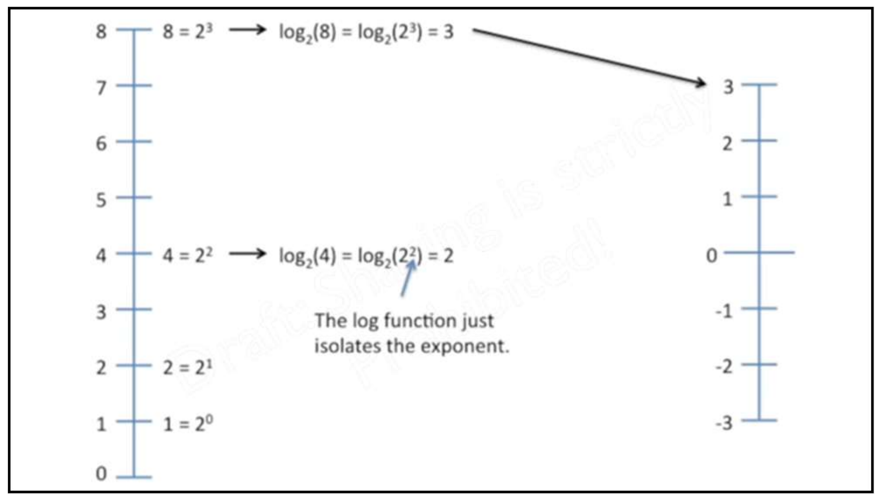


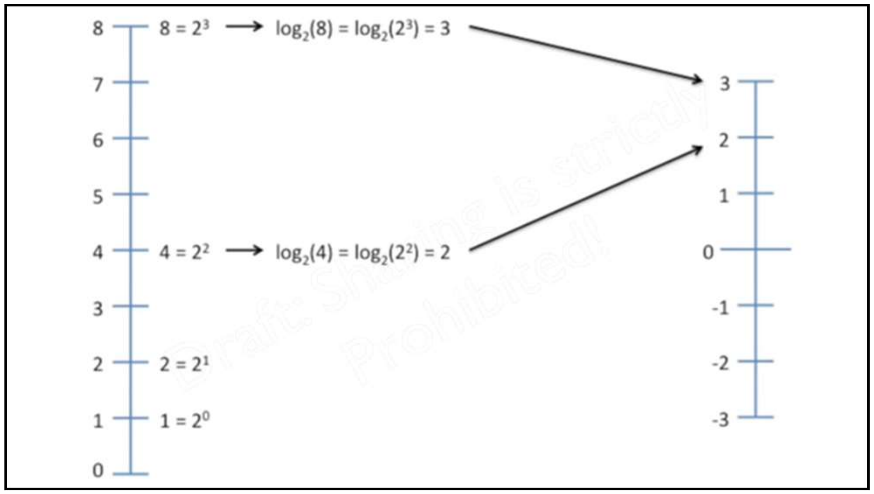


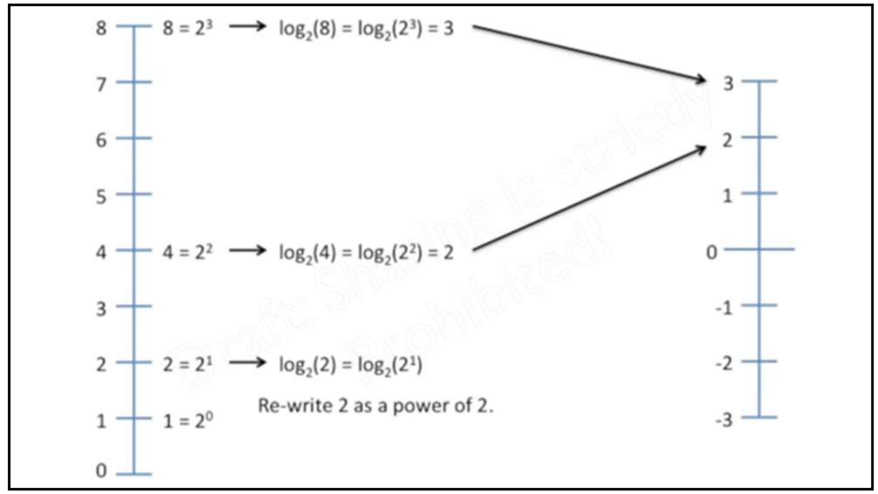

8 = 2^3 → log_2(8) = log_2(2^3)
Step 1: re-write 8 as 2^3…
log_2(2^3) is just the exponent you need to raise 2 by in order to get 8.
4 = 2^2
2 = 2^1
1 = 2^0
log_2 axis
8
7
6
5
4
3
2
1
0
3
2
1
0
-1
-2
-3

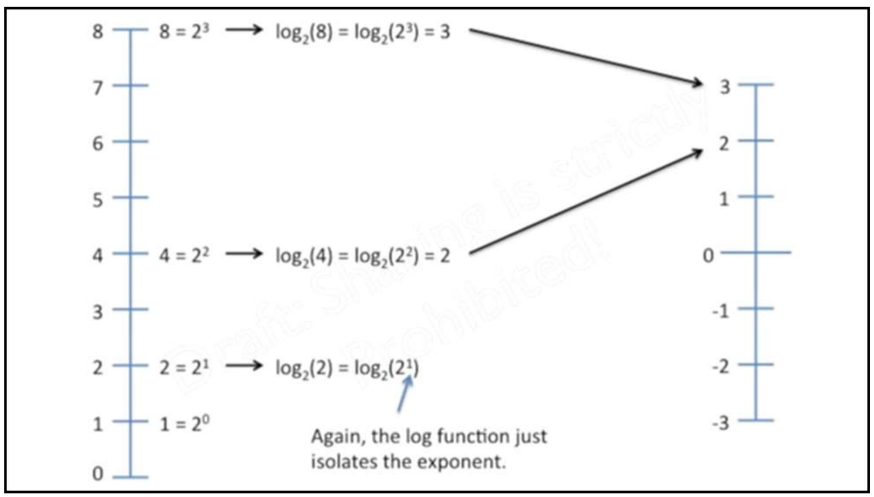

8 = 2^3 → log_2(8) = log_2(2^3) = 3
Step 1: re-write 8 as 2^3…
log_2(2^3) is just the exponent you need to raise 2 by in order to get 8.
In this case, the exponent is 3, so log_2(2^3) = 3.
4 = 2^2
2 = 2^1
1 = 2^0
log_2 axis

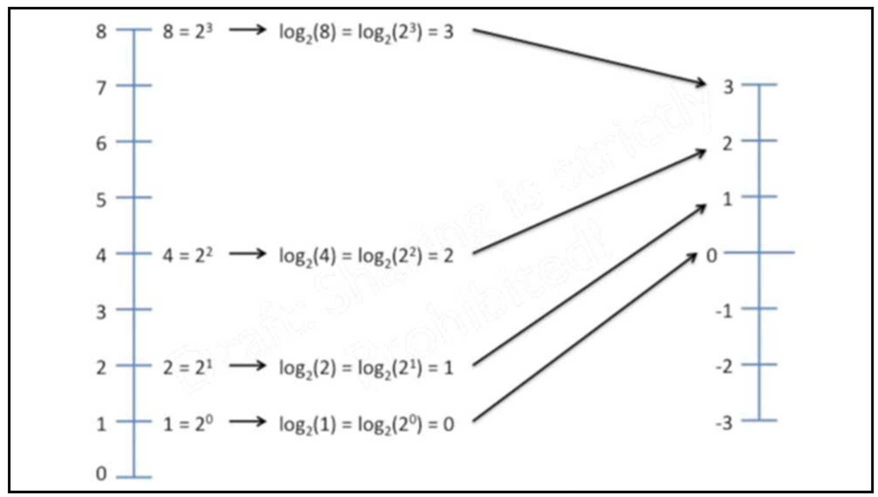

8 = 2^3 → log_2(8) = log_2(2^3) = 3
Step 1: re-write 8 as 2^3…
log_2(2^3) is just the exponent you need to raise 2 by in order to get 8.
In this case, the exponent is 3, so log_2(2^3) = 3.
The log function just isolates the exponent.
4 = 2^2
2 = 2^1
1 = 2^0
log_2 axis

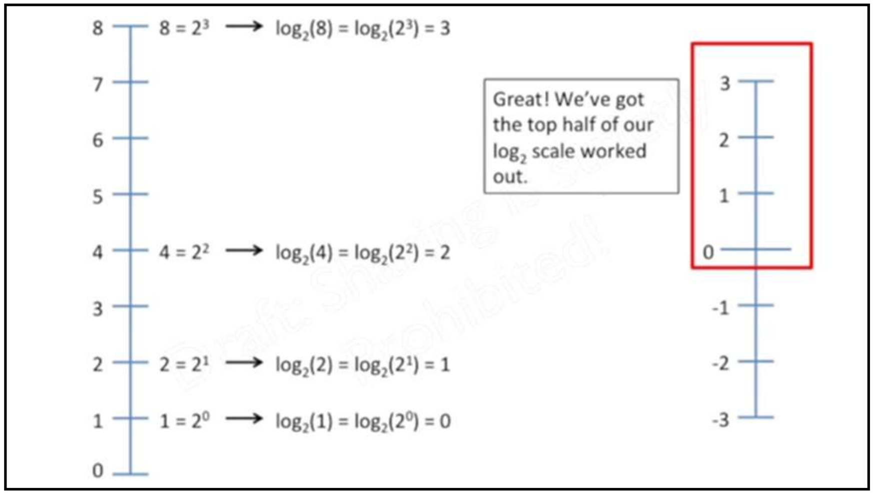

8 = 2^3 → log_2(8) = log_2(2^3) = 3
4 = 2^2 → log_2(4) = log_2(2^2) = 2
The log function just isolates the exponent.
2 = 2^1
1 = 2^0

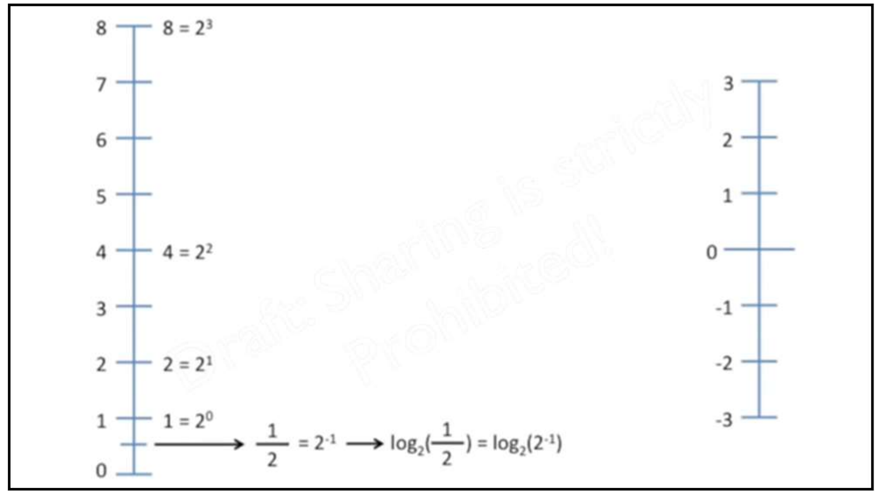

$8 = 2^3 \longrightarrow \log_2(8) = \log_2(2^3) = 3$
$4 = 2^2 \longrightarrow \log_2(4) = \log_2(2^2) = 2$
$2 = 2^1$
$1 = 2^0$

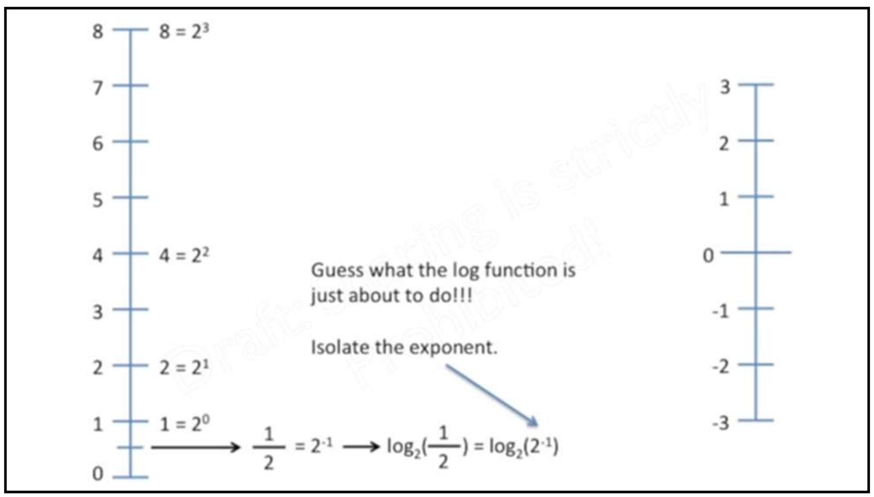

$8 = 2^3 \longrightarrow \log_2(8) = \log_2(2^3) = 3$
$4 = 2^2 \longrightarrow \log_2(4) = \log_2(2^2) = 2$
$2 = 2^1 \longrightarrow \log_2(2) = \log_2(2^1)$
Re-write 2 as a power of 2.
$1 = 2^0$

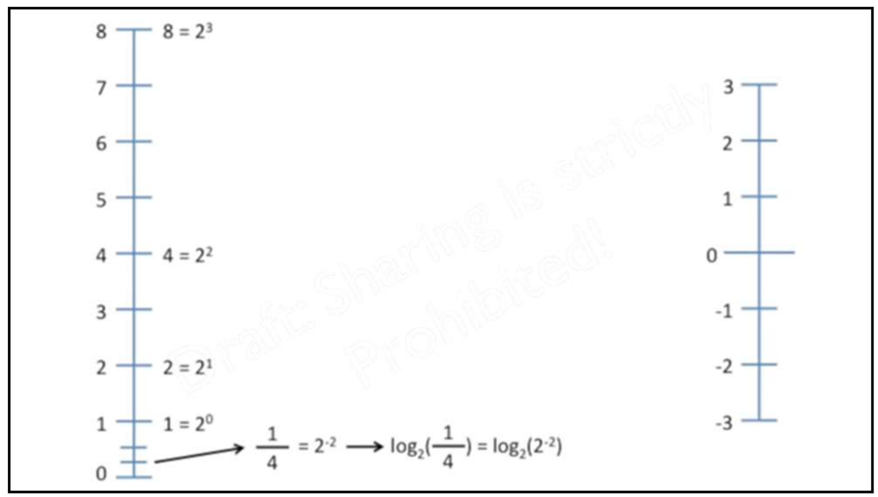

$8 = 2^3 \longrightarrow \log_2(8) = \log_2(2^3) = 3$
$4 = 2^2 \longrightarrow \log_2(4) = \log_2(2^2) = 2$
$2 = 2^1 \longrightarrow \log_2(2) = \log_2(2^1)$
$1 = 2^0$
Again, the log function just isolates the exponent.

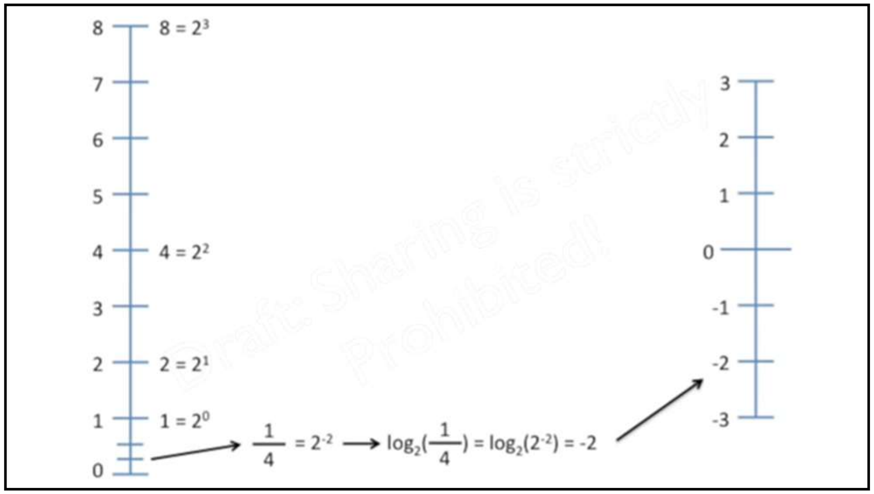

$8 = 2^3 \longrightarrow \log_2(8) = \log_2(2^3) = 3$
$4 = 2^2 \longrightarrow \log_2(4) = \log_2(2^2) = 2$
$2 = 2^1 \longrightarrow \log_2(2) = \log_2(2^1) = 1$
$1 = 2^0 \longrightarrow \log_2(1) = \log_2(2^0) = 0$

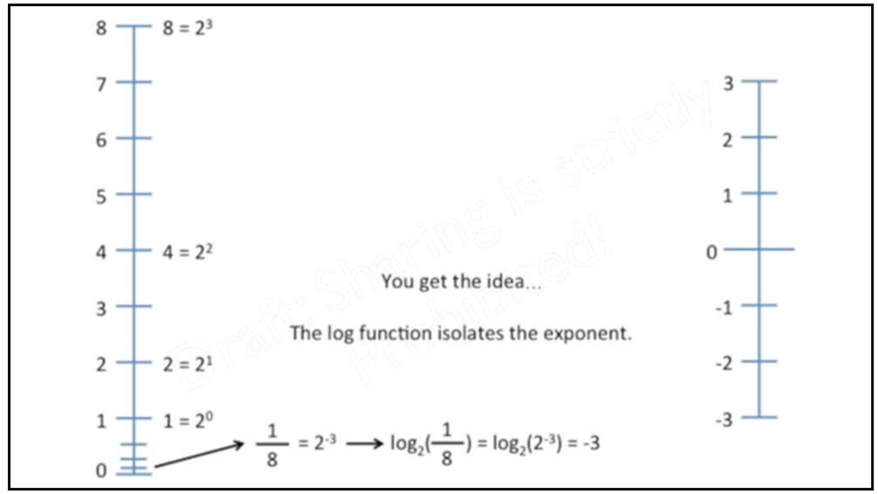

8 = 2^3 ⟶ log_2(8) = log_2(2^3) = 3
4 = 2^2 ⟶ log_2(4) = log_2(2^2) = 2
2 = 2^1 ⟶ log_2(2) = log_2(2^1) = 1
1 = 2^0 ⟶ log_2(1) = log_2(2^0) = 0
Great! We've got the top half of our log_2 scale worked out.
8 7 6 5 4 3 2 1 0
3 2 1 0 -1 -2 -3

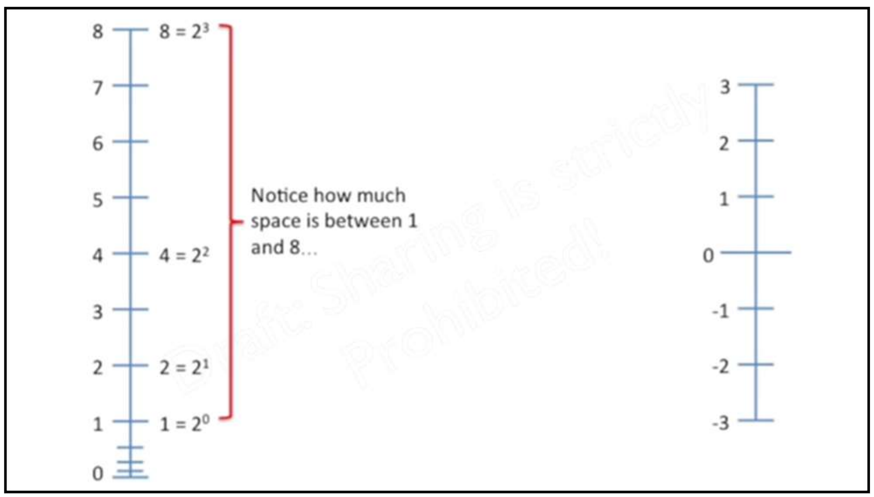

8 = 2^3
4 = 2^2
2 = 2^1
1 = 2^0
1/2 = 2^-1 ⟶ log_2(1/2) = log_2(2^-1)
8 7 6 5 4 3 2 1 0
3 2 1 0 -1 -2 -3

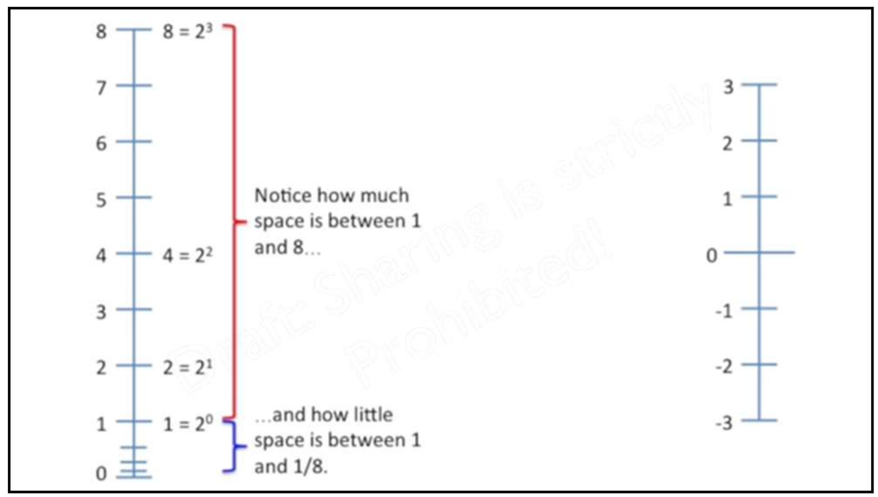

8 = 2^3
4 = 2^2
2 = 2^1
1 = 2^0
Guess what the log function is just about to do!!!
Isolate the exponent.
1/2 = 2^-1 ⟶ log_2(1/2) = log_2(2^-1)
8 7 6 5 4 3 2 1 0
3 2 1 0 -1 -2 -3

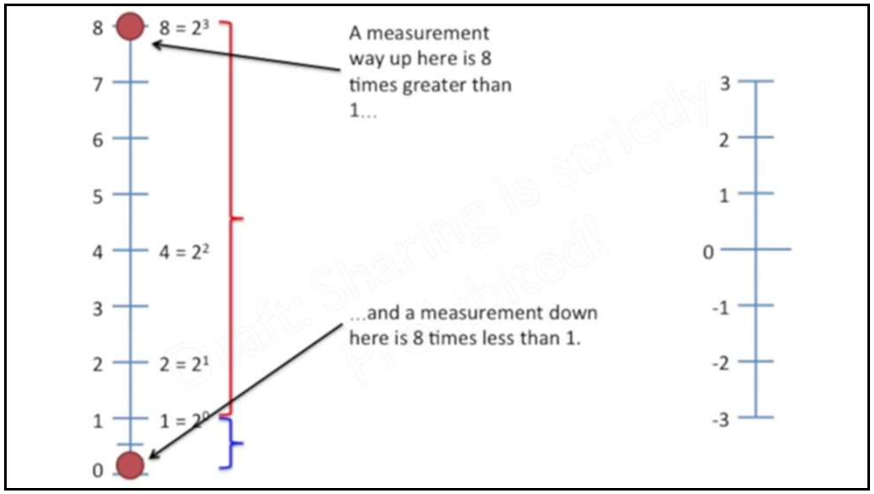

8 = 2^3
4 = 2^2
2 = 2^1
1 = 2^0
1/4 = 2^-2 ⟶ log_2(1/4) = log_2(2^-2)
8 7 6 5 4 3 2 1 0
3 2 1 0 -1 -2 -3

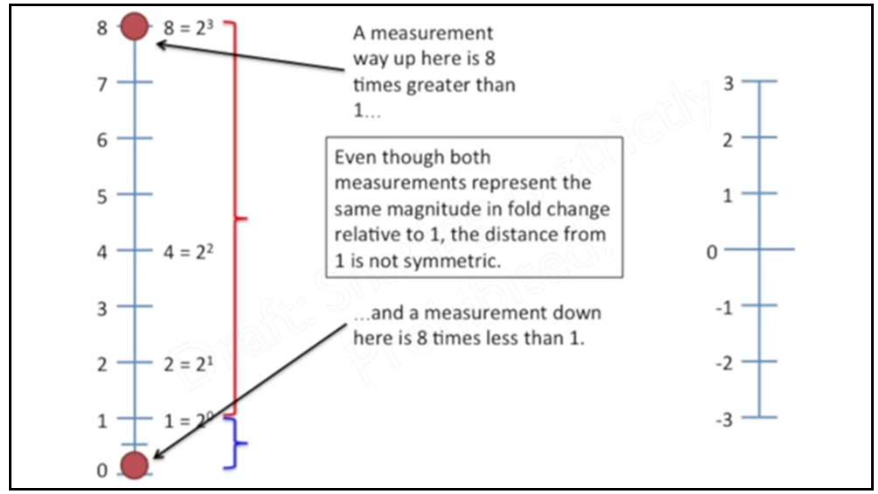

8 = 2³
4 = 2²
2 = 2¹
1 = 2⁰
1/4 = 2⁻² ⟶ log₂(1/4) = log₂(2⁻²) = -2
0 1 2 3 4 5 6 7 8
-3 -2 -1 0 1 2 3

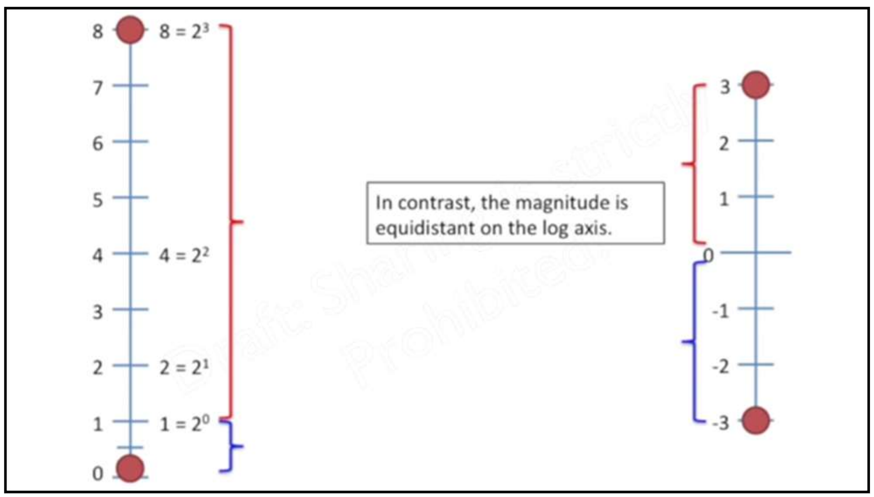

8 = 2³
4 = 2²
2 = 2¹
1 = 2⁰
You get the idea…
The log function isolates the exponent.
1/8 = 2⁻³ ⟶ log₂(1/8) = log₂(2⁻³) = -3
0 1 2 3 4 5 6 7 8
-3 -2 -1 0 1 2 3

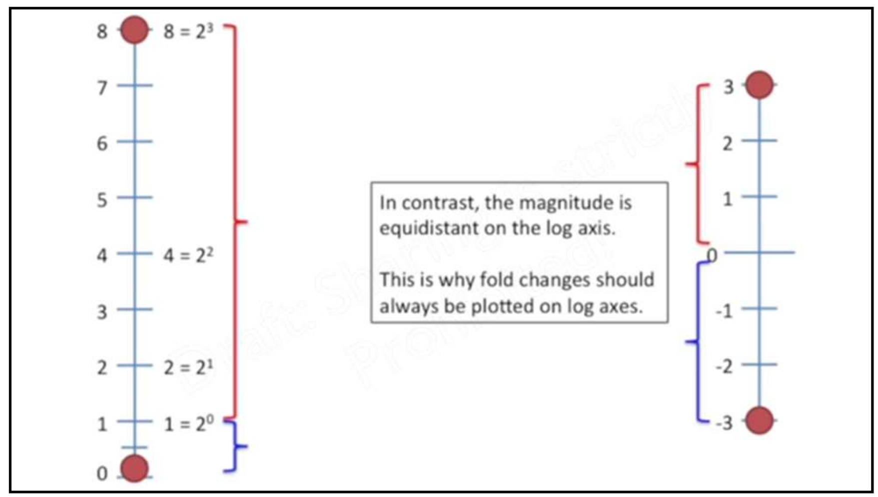

8 = 2³
4 = 2²
2 = 2¹
1 = 2⁰
Notice how much space is between 1 and 8…
0 1 2 3 4 5 6 7 8
-3 -2 -1 0 1 2 3

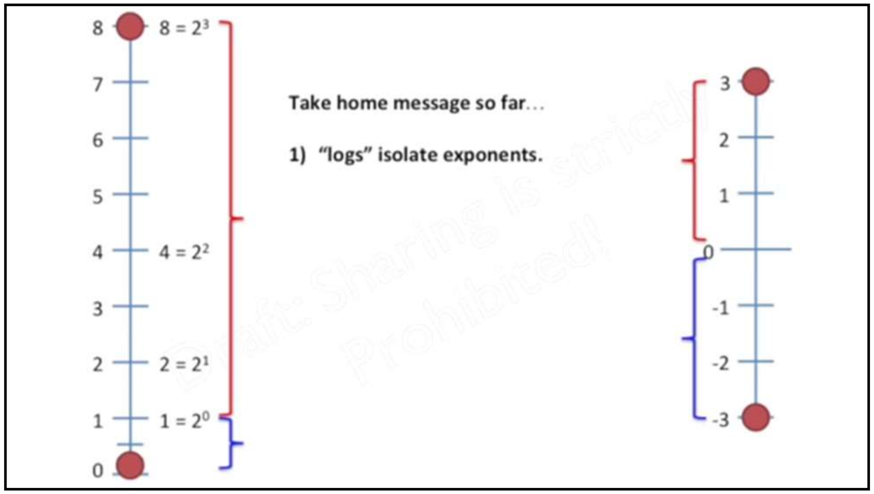

8 = 2³
4 = 2²
2 = 2¹
1 = 2⁰
Notice how much space is between 1 and 8…
…and how little space is between 1 and 1/8.
0 1 2 3 4 5 6 7 8
-3 -2 -1 0 1 2 3

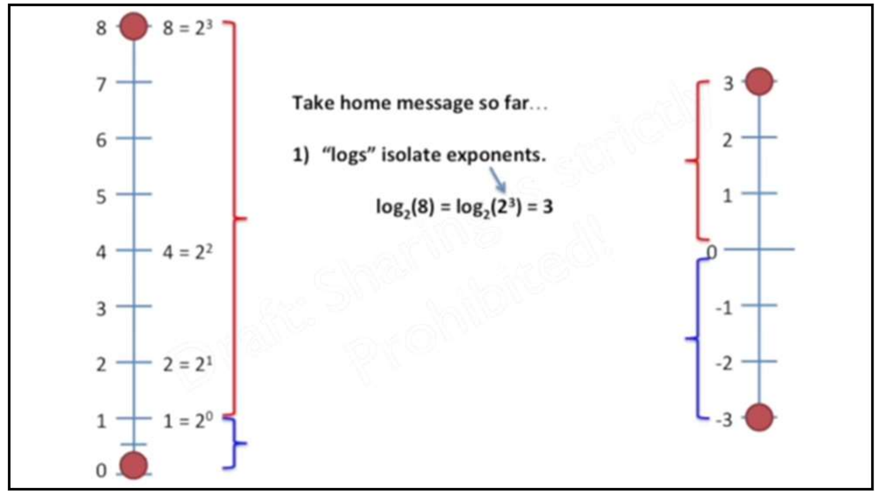

8 = 2³
4 = 2²
2 = 2¹
1 = 2⁰
A measurement way up here is 8 times greater than 1…
…and a measurement down here is 8 times less than 1.

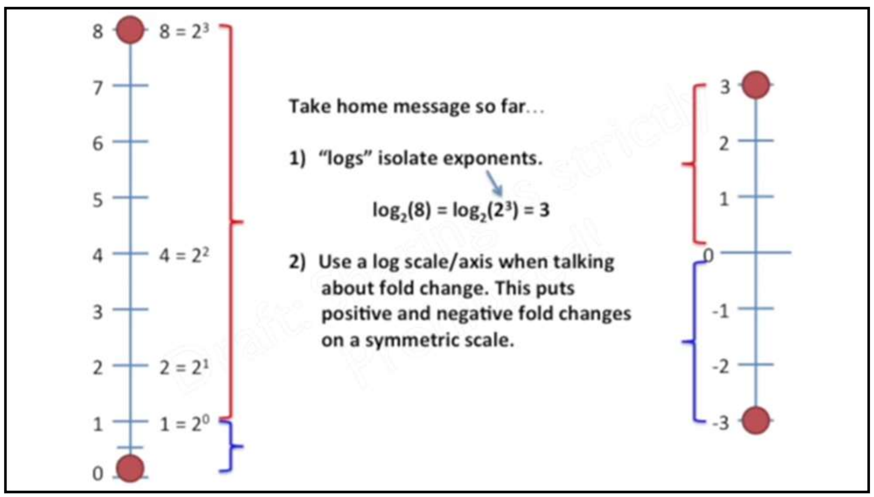

8 = 2³
4 = 2²
2 = 2¹
1 = 2⁰
A measurement way up here is 8 times greater than 1…
Even though both measurements represent the same magnitude in fold change relative to 1, the distance from 1 is not symmetric.
…and a measurement down here is 8 times less than 1.

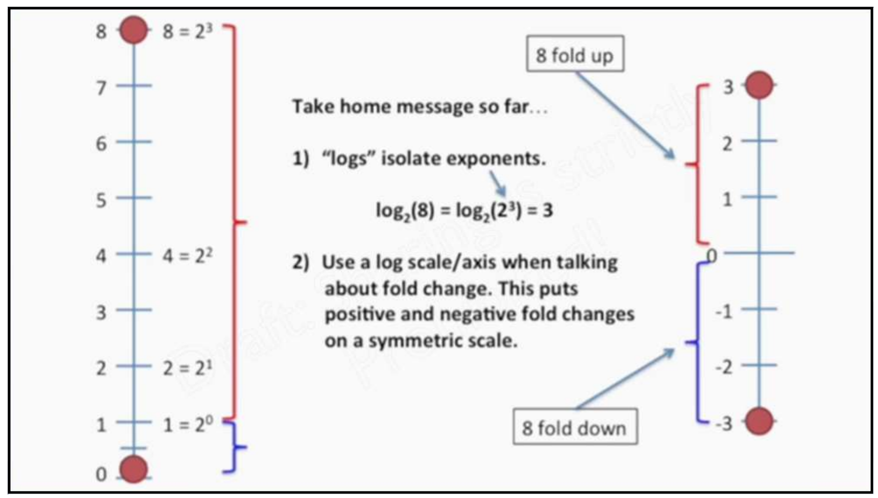

8 = 2³
4 = 2²
2 = 2¹
1 = 2⁰
In contrast, the magnitude is equidistant on the log axis.


8 = 2³
4 = 2²
2 = 2¹
1 = 2⁰
In contrast, the magnitude is equidistant on the log axis.
This is why fold changes should always be plotted on log axes.


8 = 2^3
4 = 2^2
2 = 2^1
1 = 2^0
Take home message so far…
1) "logs" isolate exponents.


8 = 2^3
4 = 2^2
2 = 2^1
1 = 2^0
Take home message so far…
1) "logs" isolate exponents.
$\log_2(8) = \log_2(2^3) = 3$


8 = 2^3
4 = 2^2
2 = 2^1
1 = 2^0
Take home message so far…
1) "logs" isolate exponents.
$\log_2(8) = \log_2(2^3) = 3$
2) Use a log scale/axis when talking about fold change. This puts positive and negative fold changes on a symmetric scale.


8 = 2^3
4 = 2^2
2 = 2^1
1 = 2^0
8 fold up
Take home message so far…
1) "logs" isolate exponents.
$\log_2(8) = \log_2(2^3) = 3$
2) Use a log scale/axis when talking about fold change. This puts positive and negative fold changes on a symmetric scale.
8 fold down

Regression

Functional Form & Transformations

Part a) Dealing with numerical variables

* Non-linear relationships (squared, inverse etc)
* Logarithms

Regression

Functional Form & Transformations

Part a) Dealing with numerical variables

* Non-linear relationships (squared, inverse etc)
* Logarithms

Part b) Dealing with categorical X variables

* Dummy variables
* Interaction variables

Regression

Functional Form & Transformations

Part a) Dealing with numerical variables

* Non-linear relationships (squared, inverse etc)
* Logarithms

Part b) Dealing with categorical X variables

* Dummy variables
* Interaction variables

Part c) Dealing with categorical Y variables

* Logit models

Regression

Dataset:

Jaybob's Used Car Sales (jaybob.csv)

Variables:

"Price" - advertised sale price ($AUD)
"Age" - model age (yrs)
"Odometer" - odometer reading ('000 kms)
"Pink slip" - presence of RWC (1= yes, 0=no)
"Sold" - whether car sold (1=yes, 0=no)

Regression

Dataset:

Jaybob's Used Car Sales (jaybob.csv)

	A	B	C	D	E	F
1	Car ID	Price	Age	Odometer	Pink slip	Sold?
2	1	$ 1,000	28	30.298	1	1
3	2	$ 9,000	40	19.647	1	0
4	3	$ 500	58	170.270	0	1
5	4	$ 3,000	12	68.394	1	1
6	5	$ 9,500	3	11.662	0	0
7	6	$ 1,500	23	87.973	0	0
8	7	$ 4,000	4	3.496	1	0
9	8	$ 2,000	13	40.986	1	1
10	9	$ 2,500	5	21.098	1	1

Numerical variables

Model 1

$Price_i = \beta_0 + \beta_1 Age_i + \beta_2 Odometer_i + \varepsilon_i$

Numerical variables

Model 1

$Price_i = \beta_0 + \beta_1 Age_i + \beta_2 Odometer_i + \varepsilon_i$

DV: Price	Coef	SE	t	P-value
Intercept	4615.901	792.153	5.83	0.0000
Age	98.922	29.974	3.30	0.0014
Odometer	-23.029	6.284	-3.66	0.0004

$R^2 = 0.171$

Numerical variables

Model 1

$Price_i = \beta_0 + \beta_1 Age_i + \beta_2 Odometer_i + \varepsilon_i$

DV: Price	Coef	SE	t	P-value
Intercept	4615.901	792.153	5.83	0.0000
Age	98.922	29.974	3.30	0.0014
Odometer	-23.029	6.284	-3.66	0.0004

$R^2 = 0.171$

$\widehat{Price_i} = 4615.9 + 98.9(Age_i) - 23.0(Odometer_i)$

Numerical variables

Model 1

$$Price_i = \beta_0 + \beta_1 Age_i + \beta_2 Odometer_i + \varepsilon_i$$

DV: Price	Coef	SE	t	P-value
Intercept	4615.901	792.153	5.83	0.0000
Age	98.922	29.974	3.30	0.0014
Odometer	-23.029	6.284	-3.66	0.0004

$R^2 = 0.171$

$$\widehat{Price}_i = 4615.9 + 98.9(Age_i) - 23.0(Odometer_i)$$

For every additional year in age, the car can be expected to increase in price by $98.92, on average, holding odometer constant.

Numerical variables

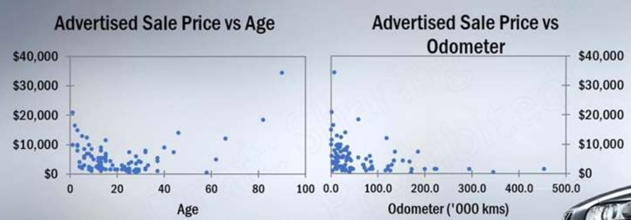
Check scatter plots!

Numerical variables

Check scatter plots!

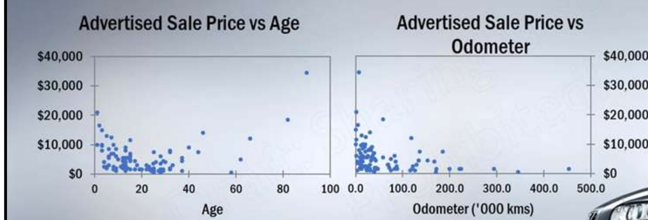
Numerical variables

Check scatter plots!



Numerical variables

Check scatter plots!



$$Price_i = \beta_0 + \beta_1 Age_i + \beta_2 Age_i^2 + \beta_3\left(\frac{1}{Odometer_i}\right) + \varepsilon_i$$

Numerical variables

Model 2

$$Price_i = \beta_0 + \beta_1(Age_i) + \beta_2(Age_i)^2 + \beta_2\left(\frac{1}{Odometer_i}\right) + \varepsilon_i$$

	A	B	C	D	E	F	G	H
1	Car ID	Price	Age	Age2	Odometer	1/Odometer	Pink slip	Sold?
2	1	$ 1,000	28	784	30.298	0.033	1	1
3	2	$ 9,000	40	1600	19.647	0.051	1	0
4	3	$ 500	58	3364	170.270	0.006	0	1
5	4	$ 3,000	12	144	68.394	0.015	1	1
6	5	$ 9,500	3	9	11.662	0.086	0	0
7	6	$ 1,500	23	529	87.973	0.011	0	0
8	7	$ 4,000	4	16	3.496	0.286	1	0
9	8	$ 2,000	13	169	40.986	0.024	1	1
10	9	$ 2,500	5	25	21.098	0.047	1	1

Numerical variables

Model 2

$$Price_i = \beta_0 + \beta_1(Age_i) + \beta_2(Age_i)^2 + \beta_2\left(\frac{1}{Odometer_i}\right) + \varepsilon_i$$

Numerical variables

Model 2

$$Price_i = \beta_0 + \beta_1(Age_i) + \beta_2(Age_i)^2 + \beta_2\left(\frac{1}{Odometer_i}\right) + \varepsilon_i$$

DV: Price	Coef	SE	t	P-value
Intercept	8809.879	806.803	10.92	0.0000
Age	-429.664	56.393	-7.62	0.0000
Age2	7.318	0.735	9.96	0.0000
1/Odometer	1942.243	676.217	2.87	0.0050

$R^2 = 0.585$

$$\widehat{Price_i} = 8809.9 - 429.7(Age_i) + 7.3(Age_i)^2 + 1942.2\left({}^{1}/_{Odometer_i}\right)$$

Numerical variables

Logarithms

Numerical variables

Logarithms

*** What is a logarithm?**

Numerical variables

Logarithms

*** What is a logarithm?**

A way of scaling skewed data

Numerical variables

Logarithms

*** What is a logarithm?**

A way of scaling skewed data

Original scale	Log scale (BASE 10)
10	Log(10) = 1
100	Log(100) = 2
1,000	Log(1,000) = 3
10,000	Log(10,000) = 4

Numerical variables

Logarithms

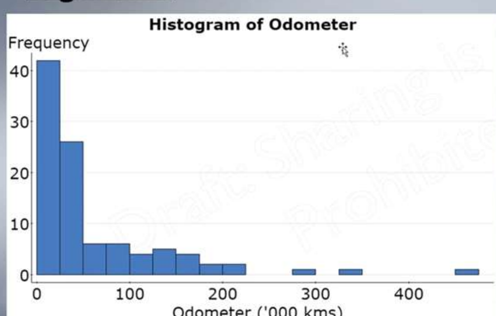
*** What is a logarithm?**

A way of scaling skewed data

Original scale	Log scale (BASE e)
2.718...	Ln(2.718...) = 1
7.389...	Ln(7.389...) = 2
20.086...	Ln(20.086...) = 3
54.598...	Ln(54.598...) = 4

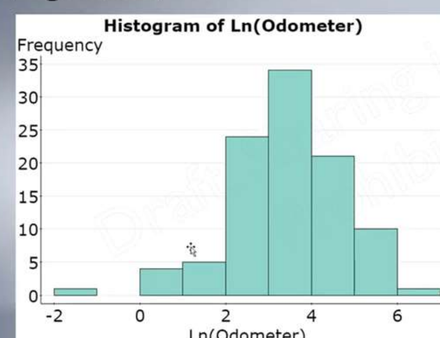
Numerical variables

Logarithms



Numerical variables

Logarithms



Numerical variables

Model 3

$$Price_i = \beta_0 + \beta_1(Age_i) + \beta_2(Age_i)^2 + \beta_3 Ln(Odometer_i) + \varepsilon_i$$

Numerical variables

Model 3

$$Price_i = \beta_0 + \beta_1(Age_i) + \beta_2(Age_i)^2 + \beta_3 Ln(Odometer_i) + \varepsilon_i$$

DV: Price	Coef	SE	t	P-value
Intercept	11863.069	940.941	12.61	0.0000
Age	-365.576	58.864	-6.21	0.0000
Age2	6.628	0.749	8.85	0.0000
Ln(Odometer)	-1079.375	272.050	-3.97	0.0001

$R^2 = 0.613$

$$\widehat{Price}_i = 11863.1 - 365.6\,(Age_i) + 6.63\,(Age_i)^2 - 1079.4\,Ln(Odometer_i)$$

Numerical variables

Model 3

$$\widehat{Price}_i = 11863.1 - 365.6\,(Age_i) + 6.63\,(Age_i)^2 - 1079.4\,Ln(Odometer_i)$$

Numerical variables

Model 3

$$\widehat{Price}_i = 11863.1 - 365.6\,(Age_i) + 6.63\,(Age_i)^2 - 1079.4\,Ln(Odometer_i)$$

A 1 unit increase in the natural log of the odometer reading decreases the price by $1079.40, on average, holding age constant

Numerical variables

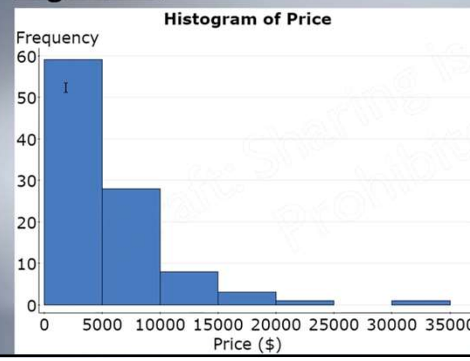
Model 3

$$\widehat{Price_i} = 11863.1 - 365.6\,(Age_i) + 6.63\,(Age_i)^2 - 1079.4\,Ln(Odometer_i)$$

A 1% increase in the odometer reading decreases the price by:
1079.4/100 = $10.79
...on average, holding age constant.

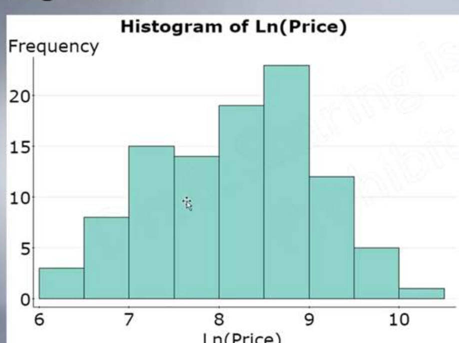
Numerical variables

Logarithms



Numerical variables

Logarithms



Numerical variables

Model 4

$$Ln(Price_i) = \beta_0 + \beta_1(Age_i) + \beta_2(Age_i)^2 + \beta_2 Ln(Odometer_i) + \varepsilon_i$$

Numerical variables

Model 4

$$Ln(Price_i) = \beta_0 + \beta_1(Age_i) + \beta_2(Age_i)^2 + \beta_2 Ln(Odometer_i) + \varepsilon_i$$

DV: Ln(Price)	*Coef*	*SE*	*t*	*P-value*
Intercept	9.392	0.211	44.47	0.0000
Age	-0.054	0.013	-4.12	0.0001
Age2	0.001	0.000	5.00	0.0000
Ln(Odometer)	-0.197	0.061	-3.23	0.0017

$\mathrm{R}^2 = 0.362$

Numerical variables

Model 4

$$Ln(Price_i) = \beta_0 + \beta_1(Age_i) + \beta_2(Age_i)^2 + \beta_2 Ln(Odometer_i) + \varepsilon_i$$

DV: Ln(Price)	*Coef*	*SE*	*t*	*P-value*
Intercept	9.392	0.211	44.47	0.0000
Age	-0.054	0.013	-4.12	0.0001
Age2	0.001	0.000	5.00	0.0000
Ln(Odometer)	-0.197	0.061	-3.23	0.0017

$\mathrm{R}^2 = 0.362$

$$Ln(Price_i) = 9.392 - 0.054\,(Age_i) + 0.001\,(Age_i)^2 - 0.197\,Ln(Odometer_i)$$

Numerical variables

Model 4

$$Ln(\widehat{Price}_i) = 9.392 - 0.054\,(Age_i) + 0.001\,(Age_i)^2 - 0.197\,Ln(Odometer_i)$$

Numerical variables

Model 4

$$Ln(\widehat{Price}_i) = 9.392 - 0.054\,(Age_i) + 0.001\,(Age_i)^2 - 0.197\,Ln(Odometer_i)$$

A 1% increase in the odometer reading decreases the price by 0.197%, on average, holding age constant

Part (b) - Categorical X variables

Categorical X variables

Binary variables

Pink Slip = 1 if car has roadworthy certificate
= 0 otherwise

Categorical X variables

Binary variables

Pink Slip = 1 if car has roadworthy certificate
= 0 otherwise

$Price_i = \beta_0 + \beta_1(Pink\ Slip_i) + \varepsilon_i$

Categorical X variables

Binary variables

Pink Slip = 1 if car has roadworthy certificate
= 0 otherwise

$Price_i = \beta_0 + \beta_1(Pink\ Slip_i) + \varepsilon_i$

DV: Price	Coef	SE	t	P-value
Intercept	3978.3	1056.29	3.76625	0.00028
Pink slip	1625.6	1203.76	1.35047	0.17998

$\widehat{Price_i} = 3978 + 1626\ (Pink\ Slip_i)$

Categorical X variables

Binary variables

Pink Slip = 1 if car has roadworthy certificate

= 0 otherwise

$$\widehat{Price_i} = 3978 + 1626\,(Pink\ Slip_i)$$

A car with a pink slip would command a sale price $1,626 more than a car without a pink slip, on average.

Categorical X variables

Model 5

$$Ln(Price_i) = \beta_0 + \beta_1(Age_i) + \beta_2(Age_i)^2 + \beta_3 Ln(Odometer_i) + \beta_4(Pink\ Slip_i) + \varepsilon_i$$

DV: Ln(Price)	Coef	SE	t	P-value
Intercept	9.237	0.276	33.51	0.0000
Age	-0.052	0.014	-3.78	0.0003
Age2	0.001	0.000	4.72	0.0000
Ln(Odometer)	-0.198	0.061	-3.24	0.0016
Pink slip	0.156	0.178	0.87	0.3846

$R^2 = 0.367$

Categorical X variables

Model 5

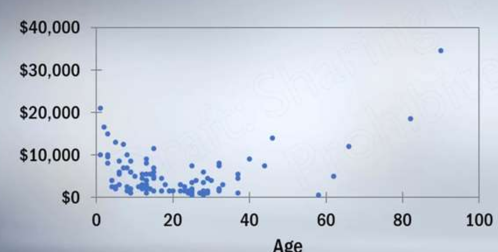
$$Ln(\widehat{Price_i}) = 9.237 - 0.052(Age_i) + 0.001(Age_i)^2 - 0.198\,Ln(Odometer_i) + 0.156(Pink\ Slip_i)$$

A car with a pink slip would command a sale price 15.6% higher than a car without a pink slip, on average, holding all other variables constant.

Categorical X variables

Multi-level categorical variables

Advertised Sale Price vs Age



Categorical X variables

Multi-level categorical variables

Categorical X variables

Multi-level categorical variables

AgeCat = 1 if age <= 5
= 2 if 5 < age <= 15
= 3 if 15 < age <= 35
= 4 if age > 35

Categorical X variables

Multi-level categorical variables

AgeCat = 1 if age <= 5
= 2 if 5 < age <= 15
= 3 if 15 < age <= 35
= 4 if age > 35

$$Ln(Price_i) = \beta_0 + \beta_1(Age_i) + \beta_2(Age_i)^2$$
$$+ \beta_3 Ln(Odometer_i) + \beta_4(Pink\ Slip_i) + \varepsilon_i$$

Categorical X variables

Multi-level categorical variables

AgeCat = 1 if age <= 5
= 2 if 5 < age <= 15
= 3 if 15 < age <= 35
= 4 if age > 35

$$Ln(Price_i) = \beta_0 + \beta_1(AgeCat_i)$$
$$+ \beta_2 Ln(Odometer_i) + \beta_3(Pink\ Slip_i) + \varepsilon_i$$

Categorical X variables

Multi-level categorical variables

AgeCat1 = 1 if age <= 5
= 0 otherwise

Categorical X variables

Multi-level categorical variables

AgeCat1 = 1 if age <= 5
= 0 otherwise

AgeCat2 = 1 if 5 < age <= 15
= 0 otherwise

Categorical X variables

Multi-level categorical variables

AgeCat1 = 1 if age <= 5
= 0 otherwise

AgeCat2 = 1 if 5 < age <= 15
= 0 otherwise

AgeCat3 = 1 if 15 < age <= 35
= 0 otherwise

Categorical X variables

Multi-level categorical variables

AgeCat1 = 1 if age <= 5
= 0 otherwise

AgeCat2 = 1 if 5 < age <= 15
= 0 otherwise

AgeCat3 = 1 if 15 < age <= 35
= 0 otherwise

AgeCat4 = 1 if age > 35
= 0 otherwise

Categorical X variables

Multi-level categorical variables

AgeCat1 = 1 if age <= 5
= 0 otherwise

AgeCat2 = 1 if 5 < age <= 15
= 0 otherwise

AgeCat3 = 1 if 15 < age <= 35
= 0 otherwise

AgeCat4 = 1 if age > 35
= 0 otherwise

$$Ln(Price_i) = \beta_0 + \beta_1(AgeCat1_i) + \beta_2(AgeCat2_i) + \beta_3(AgeCat3_i) + \beta_4(AgeCat4_i) + \beta_5 Ln(Odometer_i) + \beta_6(Pink\ Slip_i) + \varepsilon_i$$

Categorical X variables

Multi-level categorical variables

$$Ln(Price_i) = \beta_0 + \beta_1(AgeCat1_i) + \beta_2(AgeCat2_i) + \beta_3(AgeCat3_i) + \beta_4(AgeCat4_i) + \beta_5 Ln(Odometer_i) + \beta_6(Pink\ Slip_i) + \varepsilon_i$$

BUT!

$$AgeCat1_i = 1 - AgeCat2_i - AgeCat3_i - AgeCat4_i$$

Categorical X variables

Multi-level categorical variables

$$Ln(Price_i) = \beta_0 + \beta_1(AgeCat1_i) + \beta_2(AgeCat2_i) + \beta_3(AgeCat3_i) + \beta_4(AgeCat4_i) + \beta_5 Ln(Odometer_i) + \beta_6(Pink\ Slip_i) + \varepsilon_i$$

BUT!

$$AgeCat1_i = 1 - AgeCat2_i - AgeCat3_i - AgeCat4_i$$

Dummy variable TRAP

Categorical X variables

Model 6

$$Ln(Price_i) = \beta_0 + \beta_1(AgeCat2_i) + \beta_2(AgeCat3_i) + \beta_3(AgeCat4_i) + \beta_4 Ln(Odometer_i) + \beta_5(Pink\ Slip_i) + \varepsilon_i$$

Categorical X variables

Model 6

$$\widehat{Ln(Price_i)} = 8.948 - 0.129(AgeCat2_i) - 0.733(AgeCat3_i) + 0.474(AgeCat4_i) - 0.225Ln(Odometer_i) + 0.344(Pink\ Slip_i)$$

DV: Ln(Price)	*Coef*	*SE*	*t*	*P-value*
Intercept	8.948	0.279	32.04	0.0000
AgeCat2	-0.129	0.247	-0.52	0.6024
AgeCat3	-0.733	0.262	-2.80	0.0061
AgeCat4	0.474	0.326	1.45	0.1500
Ln(Odometer)	-0.225	0.062	-3.64	0.0004
Pink slip	0.344	0.174	1.98	0.0502

Categorical X variables

Model 6

$$\widehat{Ln(Price_i)} = 8.948 - 0.129(AgeCat2_i) - 0.733(AgeCat3_i) + 0.474(AgeCat4_i) - 0.225Ln(Odometer_i) + 0.344(Pink\ Slip_i)$$

On average, holding all other variables constant, a car in age category 2 will command a price 12.9% lower than a car in age category 1.

Categorical X variables

Model 6

$$\widehat{Ln(Price_i)} = 8.948 - 0.129(AgeCat2_i) - 0.733(AgeCat3_i) + 0.474(AgeCat4_i) - 0.225Ln(Odometer_i) + 0.344(Pink\ Slip_i)$$

On average, holding all other variables constant, a car in age category 4 will command a price 47.4% higher than a car in age category 1.

Categorical X variables

Model 6

$$\widehat{Ln(Price_i)} = 8.948 - 0.129(AgeCat2_i) - 0.733(AgeCat3_i) + 0.474(AgeCat4_i) - 0.225Ln(Odometer_i) + 0.344(Pink\ Slip_i)$$

On average, holding all other variables constant, a car in age category 3 will command a price 73.3% lower than a car in age category 1.

Interaction terms

Interaction terms

Intuition

Build a model to explain the salary of all of Google's employees

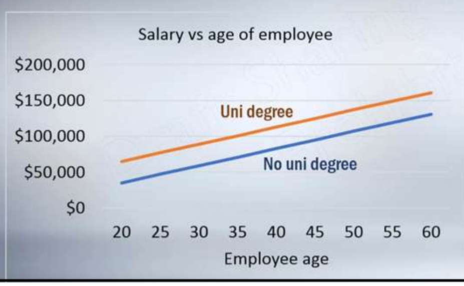
$$Salary_i = \beta_0 + \beta_1(Employee\ Age_i) + \beta_2(Uni\ degree_i) + \varepsilon_i$$

Interaction terms

Intuition

Build a model to explain the salary of all of Google's employees

$$Salary_i = \beta_0 + \beta_1(Employee\ Age_i) + \beta_2(Uni\ degree_i) + \varepsilon_i$$

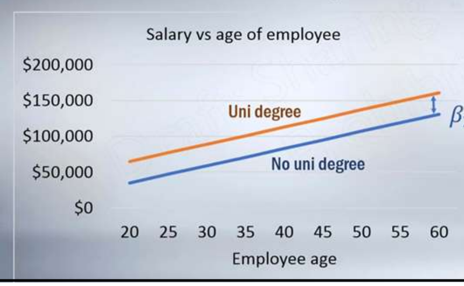


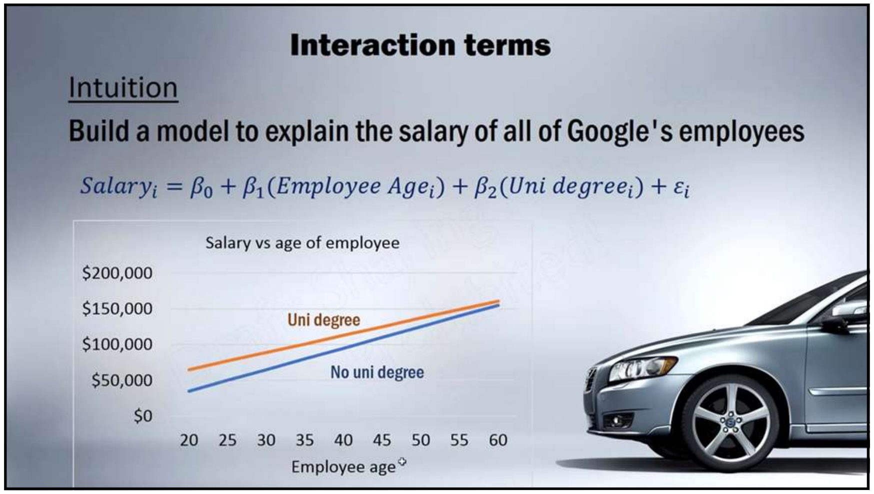
Interaction terms

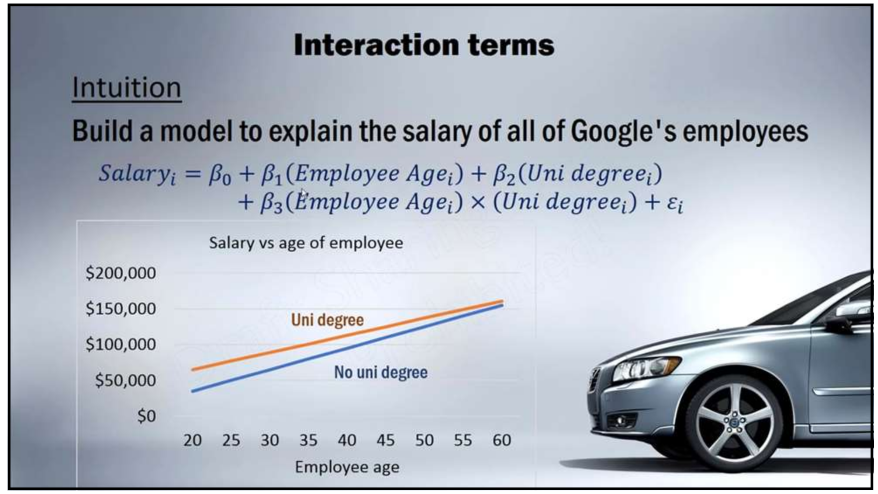
Intuition

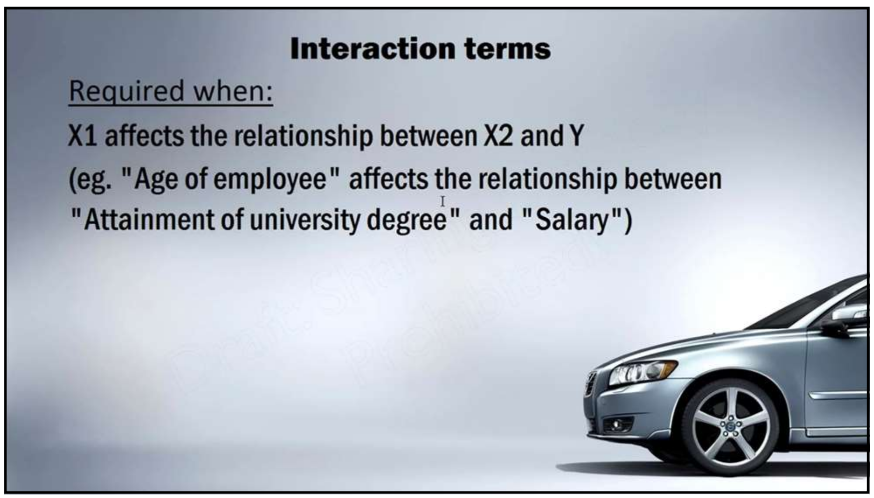
Build a model to explain the salary of all of Google's employees

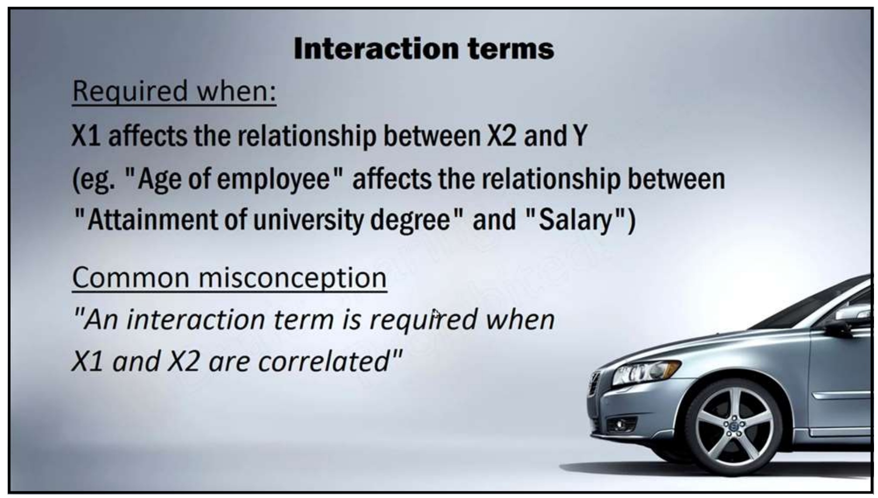
$$Salary_i = \beta_0 + \beta_1(Employee\ Age_i) + \beta_2(Uni\ degree_i) + \varepsilon_i$$




Interaction terms
Intuition
Build a model to explain the salary of all of Google's employees
$Salary_i = \beta_0 + \beta_1(Employee\ Age_i) + \beta_2(Uni\ degree_i) + \varepsilon_i$
Salary vs age of employee
$200,000
$150,000
$100,000
$50,000
$0
20 25 30 35 40 45 50 55 60
Employee age
Uni degree
No uni degree


Interaction terms
Intuition
Build a model to explain the salary of all of Google's employees
$Salary_i = \beta_0 + \beta_1(Employee\ Age_i) + \beta_2(Uni\ degree_i) + \beta_3(Employee\ Age_i) \times (Uni\ degree_i) + \varepsilon_i$
Salary vs age of employee
$200,000
$150,000
$100,000
$50,000
$0
20 25 30 35 40 45 50 55 60
Employee age
Uni degree
No uni degree


Interaction terms
Required when:
X1 affects the relationship between X2 and Y
(eg. "Age of employee" affects the relationship between "Attainment of university degree" and "Salary")


Interaction terms
Required when:
X1 affects the relationship between X2 and Y
(eg. "Age of employee" affects the relationship between "Attainment of university degree" and "Salary")
Common misconception
"An interaction term is required when X1 and X2 are correlated"

Interaction terms

Model 7

$$Ln(Price_i) = \beta_0 + \beta_1(AgeCat2_i) + \beta_2(AgeCat3_i) + \beta_3(AgeCat4_i) + \beta_4 Ln(Odometer) + \beta_5(Pink\ Slip_i) + \beta_6(Pink\ Slip_i) \times (AgeCat4_i)$$

DV: Ln(Price)	Coef	SE	t	P-value
Intercept	9.125	0.274	33.28	0.0000
AgeCat2	-0.181	0.238	-0.76	0.4495
AgeCat3	-0.800	0.252	-3.18	0.0020
AgeCat4	-0.390	0.424	-0.92	0.3595
Ln(Odometer)	-0.209	0.059	-3.53	0.0007
Pink slip	0.123	0.182	0.68	0.5005
Pink slip X AgeCat4	1.371	0.453	3.02	0.0032

Interaction terms

Model 7

$$Ln(\widehat{Price_i}) = 9.125 - 0.181(AgeCat2_i) - 0.800(AgeCat3_i) - 0.390 \times (AgeCat4_i) - 0.209 Ln(Odometer) + 0.123(Pink\ Slip_i) + 1.371(Pink\ Slip_i) \times (AgeCat4_i)$$

Interpret the coefficient of Pink slip:

Interaction terms

Model 7

$$Ln(\widehat{Price_i}) = 9.125 - 0.181(AgeCat2_i) - 0.800(AgeCat3_i) - 0.390 \times (AgeCat4_i) - 0.209 Ln(Odometer) + 0.123(Pink\ Slip_i) + 1.371(Pink\ Slip_i) \times (AgeCat4_i)$$

Interpret the coefficient of Pink slip:

For models less than (or equal to) 35 years old, attaining a pink slip increases the price by an average of 12.3% , holding all else constant...

Interaction terms

Model 7

$$Ln(\widehat{Price_i}) = 9.125 - 0.181(AgeCat2_i) - 0.800(AgeCat3_i) - 0.390 \times (AgeCat4_i) - 0.209 Ln(Odometer) + 0.123(Pink\ Slip_i) + 1.371(Pink\ Slip_i) \times (AgeCat4_i)$$

Interpret the coefficient of Pink slip:

... BUT for models older than 35 years, attaining a pink slip increases the price by an average of 149.4% , holding all else constant.

Interaction terms

REVISION QUESTION

Using model 7, find the expected sale price of my 1974 Datsun 120Y Coupe with 290,000km on the odometer and a road worthy certificate.

Interaction terms

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Using model 7, find the expected sale price of my 1974 Datsun 120Y Coupe with 290,000km on the odometer and a road worthy certificate.

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Ln(Odometer)	-0.209	0.059	-3.53	0.0007
Pink slip	0.123	0.182	0.68	0.5005
Pink slip X AgeCat4	1.371	0.453	3.02	0.0032

Interaction terms

$$\widehat{Ln(Price_i)} = 9.125 - 0.181(AgeCat2_i) - 0.800(AgeCat3_i)$$
$$-0.390\,(AgeCat4_i) - 0.209Ln(Odometer) + 0.123(Pink\ Slip_i)$$
$$+1.371(Pink\ Slip_i) \times (AgeCat4_i)$$

Interaction terms

$$\widehat{Ln(Price_i)} = 9.125 - 0.181(AgeCat2_i) - 0.800(AgeCat3_i)$$
$$-0.390\ (AgeCat4_i) - 0.209Ln(Odometer) + 0.123(Pink\ Slip_i)$$
$$+1.371(Pink\ Slip_i) \times (AgeCat4_i)$$

$$\widehat{Ln(Price_i)} = 9.125 - 0.390\ - 0.209Ln(290) + 0.123 + 1.371$$

Interaction terms

$$\widehat{Ln(Price_i)} = 9.125 - 0.181(AgeCat2_i) - 0.800(AgeCat3_i)$$
$$-0.390\ (AgeCat4_i) - 0.209Ln(Odometer) + 0.123(Pink\ Slip_i)$$
$$+1.371(Pink\ Slip_i) \times (AgeCat4_i)$$

$$\widehat{Ln(Price_i)} = 9.125 - 0.390\ - 0.209Ln(290) + 0.123 + 1.371$$

$$\widehat{Ln(Price_i)} = 9.044$$

Interaction terms

$$\widehat{Ln(Price_i)} = 9.125 - 0.181(AgeCat2_i) - 0.800(AgeCat3_i)$$
$$-0.390\ (AgeCat4_i) - 0.209Ln(Odometer) + 0.123(Pink\ Slip_i)$$
$$+1.371(Pink\ Slip_i) \times (AgeCat4_i)$$

$$\widehat{Ln(Price_i)} = 9.125 - 0.390\ - 0.209Ln(290) + 0.123 + 1.371$$

$$\widehat{Ln(Price_i)} = 9.044$$

$$\widehat{Price_i} = e^{9.044}$$

Interaction terms

$$\widehat{Ln(Price_i)} = 9.125 - 0.181(AgeCat2_i) - 0.800(AgeCat3_i)$$
$$-0.390\ (AgeCat4_i) - 0.209Ln(Odometer) + 0.123(Pink\ Slip_i)$$
$$+1.371(Pink\ Slip_i) \times (AgeCat4_i)$$

$$\widehat{Ln(Price_i)} = 9.125 - 0.390\ - 0.209Ln(290) + 0.123 + 1.371$$

$$\widehat{Ln(Price_i)} = 9.044$$

$$\widehat{Price_i} = e^{9.044}$$

$$\widehat{Price_i} = \$8,468$$

THANK YOU!