BACK PROPAGATION

Report on Back Propagation

Course Name: CSE-837 Machine Learning

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BSSE 0917

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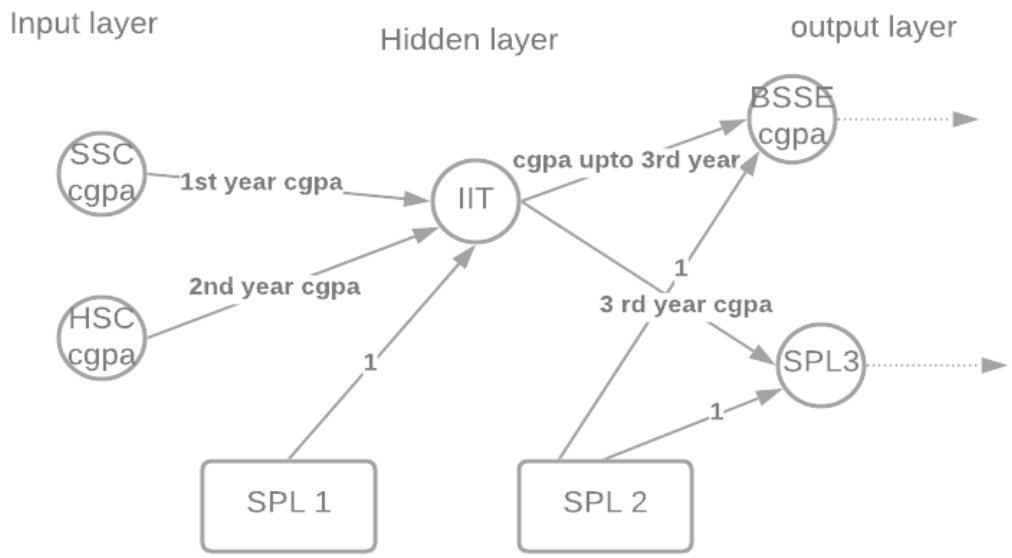
Contents

Back Propagation



Problem Statement:

Calculate the updated value for all parameters (weights and biases) two times using two iterations of back propagation

Calculations

Let,

Here ***i***= iteration, ***b***=bias, ***w***=weight, ***o***= output, ***h***= hidden layer

Weight and bias	**Expression**	**Value**
1st year CGPA	w1	3.56/4=0.89
2nd year CGPA	w2	3.40/4=0.85
3rd year CGPA	w3	3.46/4=0.87
upto 3rd year CGPA	w4	0.852
SPL1	b1	3.5/4=0.88
SPL2	b2	3.75/4=0.9375

SSC CGPA = ***i1*** = 1, HSC CGPA = ***i2***=1

IIT = ***h***, SPL3 = ***o1***, BSSE CGPA = ***o2***

Output,

$Actual_{o1}$= 1, $Actual_{o2}$ = 1
Learning rate, η = 0.01
Here, all the values have been converted in the range of 0 to 1.

1st iteration

Forward Pass

net_h / net_{IIT} = (SSC CGPA* 1st year CGPA + HSC CGPA*2nd year CGPA+SPL1*1)

$= i1 * w1 + i2 * w2 + b1 * 1$ = (1 * 0.89) + (1 * 0.85) + (0.88 * 1) = 2.62

$out_h = \frac{1}{1+e^{-neth}} = \frac{1}{1+e^{-2.62}} = 0.932$

$net_{o1} = out_h * w3 + b2 * 1$ = 0.932 * 0.86 + 0.9375* 1 = 1.73902

$out_{o1} = \frac{1}{1+e^{-net_{o1}}} = \frac{1}{1+e^{-1.73902}} = 0.85$

$net_{o2} = out_h * w4 + b2 * 1$ = 0.932* 0.852+ 0.9375 * 1 = 1.73156

$out_{o2} = \frac{1}{1+e^{-net_{o2}}} = \frac{1}{1+e^{-1.73156}} = 0.85$

Error Calculation:

E $_{total}$ = $\Sigma(Actual\mathbf{output} - Desired\mathbf{output})$^2

$= (Actual_{o1}\text{-}out_{o1})^2 + (Actual_{o2}\text{-}out_{o2})^2$

$= (1\text{-}0.85)^2 + (1\text{-}0.85)^2$

$= 0.045$

Backward Pass

Adjusting 3rd year CGPA(w3):

Considering w3 to know how much a change in w3 affects the total error.

$$\frac{\partial E_{total}}{\partial w3} = \frac{\partial E_{total}}{\partial out_{o1}} * \frac{\partial out_{o1}}{\partial net_{o1}} * \frac{\partial net_{o1}}{\partial w3}$$

Now,

$\frac{\partial Etotal}{\partial outo1} = -2(Actual_{o1} - out_{o1}) = -2(1 - 0.85) = -0.30$

$\frac{\partial outo1}{\partial neto1} = outo1(1 - outo1) = 0.85(1 - 0.85) = 0.1275$

$\frac{\partial neto1}{\partial w3} = outh = 0.932$

So,

$\frac{\partial Etotal}{\partial w3}$ = (-0.30) *0.1275*0.932= -0.0356

So, we get, $w3+ = w3- \eta * \frac{\partial Etotal}{\partial w3}$

= 0.87 – 0.01*(-0.0356) = 0.8703

Adjusting up to 3rd year CGPA(w4):

Considering w4 to know how much a change in w4 affects the total error.

$$\frac{\partial Etotal}{\partial w4} = \frac{\partial Etotal}{\partial outo2} * \frac{\partial outo2}{\partial neto2} * \frac{\partial neto2}{\partial w4}$$

$\frac{\partial Etotal}{\partial outo2}$ = -2 ($Actual_{o2} – out_{o2}$) = -2(1 - 0.85) = -0.30

$\frac{\partial outo2}{\partial neto2}$ = $out_{o2}(1 – out_{o2}) = 0.85(1 - 0.85) = 0.1275$

$\frac{\partial neto2}{\partial w4} = out_h = 0.932$

$\frac{\partial Etotal}{\partial w4}$ = (-0.30) *0.1275*0.932= -0.0356

So, we get, $w4+ = w4- \eta * \frac{\partial Etotal}{\partial w4}$

= 0.852 – 0.01*(-0.0356) = 0.8523

Adjusting bias SPL2(b2)

Considering w4 to know how much a change in w4 affects the total error.

$\frac{\partial Etotal}{\partial b2} = \frac{\partial Eo1}{\partial b2} + \frac{\partial Eo2}{\partial b2} = (\frac{\partial Eo1}{\partial outo1} * \frac{\partial outo1}{\partial neto1} * \frac{\partial neto1}{\partial b2}) + (\frac{\partial Eo2}{\partial outo2} * \frac{\partial outo2}{\partial neto2} * \frac{\partial neto2}{\partial b2})$

$\frac{\partial Eo1}{\partial outo1}$= -1 $\frac{\partial Eo2}{\partial outo2}$= -1

$\frac{\partial outo1}{\partial neto1} = outo1(1 - outo1) = 0.85(1 - 0.85) = 0.1275$ $\frac{\partial outo2}{\partial neto2} = outo2(1 - outo2) = 0.1275$

$\frac{\partial neto1}{\partial b2}$= 1 $\frac{\partial neto2}{\partial b2}$ =1

$\frac{\partial Etotal}{\partial b2}$ = -0.255

So, we get, $b2^{+} = b2 - \eta * \frac{\partial Etotal}{\partial b2}$

= 0.9375 – 0.01*(-0.255) = 0.8703 = 0.94005

Adjusting 1st year CGPA(w1)

Considering w1 to know how much a change in w1 affects the total error.

$\frac{\partial Etotal}{\partial w1} = \frac{\partial Etotal}{\partial outh} * \frac{\partial outh}{\partial neth} * \frac{\partial neth}{\partial w1} = (\frac{\partial Eo1}{\partial outh} + \frac{\partial Eo2}{\partial outh}) * \frac{\partial outh}{\partial neth} * \frac{\partial neth}{\partial w1}$

$\frac{\partial Eo1}{\partial outh} = \frac{\partial Eo1}{\partial outo1} * \frac{\partial outo1}{\partial neto1} * \frac{\partial neto1}{\partial outh}$ = (-1) *0.1275*w3 = -0.1275* 0.87 = -0.1089

$\frac{\partial Eo2}{\partial outh} = \frac{\partial Eo2}{\partial outo2} * \frac{\partial outo2}{\partial neto2} * \frac{\partial neto2}{\partial outh}$= (-1) *0.1275*w4= -0.1275*0.852= -0.1086

$\frac{\partial outh}{\partial neth}$= $outh(1 - outh)$ = 0.932*(1-0.932) =0.063

$\frac{\partial neth}{\partial w1} = i1$ = 1

Now,

$\frac{\partial Etotal}{\partial w1}$ = -0.0137

So, we get, $w1^{+}$ =w1- $\eta * \frac{\partial Etotal}{\partial w1}$

= 0.89 – 0.01*(-0.0137) = 0.890137

Adjusting 2nd year CGPA (w2)

Considering w2 to know how much a change in w2 affects the total error.

$\frac{\partial Etotal}{\partial w2} = \frac{\partial Etotal}{\partial outh} * \frac{\partial outh}{\partial neth} * \frac{\partial neth}{\partial w2} = (\frac{\partial Eo1}{\partial outh} + \frac{\partial Eo2}{\partial outh}) * \frac{\partial outh}{\partial neth} * \frac{\partial neth}{\partial w2}$

$\frac{\partial Eo1}{\partial outh} = \frac{\partial Eo1}{\partial outo1} * \frac{\partial outo1}{\partial neto1} * \frac{\partial neto1}{\partial outh}$ = (-1) *0.1275*w3 = -0.1275* 0.87 = -0.1089

$\frac{\partial Eo2}{\partial outh} = \frac{\partial Eo2}{\partial outo2} * \frac{\partial outo2}{\partial neto2} * \frac{\partial neto2}{\partial outh}$ = (-1) *0.1275*w4= -0.1275*0.852= -0.1086

$\frac{\partial outh}{\partial neth} = outh\ (1 - outh) = 0.932*(1-0.932) = 0.063$

$\frac{\partial neth}{\partial w2} = i2 = 1$

Now,

$\frac{\partial Etotal}{\partial w2}$ = -0.0137

So, we get, $w2+ = w2- \eta * \frac{\partial Etotal}{\partial w2}$

= 0.85 – 0.01*(-0.0137) = 0.850137

Adjusting bias SPL1(b1)

Considering b1 to know how much a change in b1 affects the total error.

$\frac{\partial Etotal}{\partial b1} = \frac{\partial Etotal}{\partial outh} * \frac{\partial outh}{\partial neth} * \frac{\partial neth}{\partial b1} = (\frac{\partial Eo1}{\partial outh} + \frac{\partial Eo2}{\partial outh}) * \frac{\partial outh}{\partial neth} * \frac{\partial neth}{\partial b1}$

$\frac{\partial Eo1}{\partial outh} = \frac{\partial Eo1}{\partial outo1} * \frac{\partial outo1}{\partial neto1} * \frac{\partial neto1}{\partial outh}$ = (-1) *0.1275*w3 = -0.1275* 0.87 = -0.1089

$\frac{\partial Eo2}{\partial outh} = \frac{\partial Eo2}{\partial outo2} * \frac{\partial outo2}{\partial neto2} * \frac{\partial neto2}{\partial outh}$ = (-1) *0.1275*w4= -0.1275*0.852= -0.1086

$\frac{\partial outh}{\partial neth} = outh\ (1 - outh) = 0.932*(1-0.932) = 0.063$

$\frac{\partial neth}{\partial b1} = 1$

Now,

$\frac{\partial Etotal}{\partial b1}$ = -0.0137

So, we get, $b1+ = b1- \eta * \frac{\partial Etotal}{\partial b1}$

= 0.88– 0.01*(-0.0137) = 0.880137

After 1st iteration adjusted values are –

Weight and bias	Expression	Value
1st year CGPA	w1	0.890137
2nd year CGPA	w2	0.850137
3rd year CGPA	w3	0.8703
upto 3rd year CGPA	w4	0.8523
SPL1	b1	0.880137
SPL2	b2	0.94005

SSC CGPA = $\boldsymbol{i1}$ = 1, HSC CGPA = $\boldsymbol{i2}$ = 1,

IIT = $\boldsymbol{h}$, SPL3 = $\boldsymbol{o1}$, BSSE CGPA = $\boldsymbol{o2}$
$\boldsymbol{Actual_{o1}}$ = 1, $\boldsymbol{Actual_{o2}}$ = 1
Learning rate, $\boldsymbol{\eta}$ = 0.01
These values will be used to adjust the weights and biases for 2nd iteration.

2nd iteration

Forward Pass

$\boldsymbol{net_h}$ = (SSC CGPA* 1st year CGPA + HSC CGPA*2nd year CGPA+SPL1*1)

$= \boldsymbol{i1} * \boldsymbol{w1} + \boldsymbol{i2} * \boldsymbol{w2} + \boldsymbol{b1} * \boldsymbol{1}$ = (1 * 0. 890137) + (1 * 0. 850137) + (0. 880137 * 1) = 2.6204

$\boldsymbol{out_h} = \frac{1}{1+e^{-neth}} = \frac{1}{1+e^{-2.6204}} = 0.932163$

$\boldsymbol{net_{o1}} = \boldsymbol{out_h} * \boldsymbol{w3} + \boldsymbol{b2} * \boldsymbol{1}$ = 0.932 * 0.8703 + 0.94005* 1 = 1.7513

$\boldsymbol{out_{o1}} = \frac{1}{1+e^{-\text{net}_{o1}}} = \frac{1}{1+e^{-1.7513}} = 0.852$

$\boldsymbol{net_{o2}} = \boldsymbol{out_h} * \boldsymbol{w4} + \boldsymbol{b2} * \boldsymbol{1}$ = 0.8523* 0.852+ 0.94005* 1 = 1.67

$\boldsymbol{out_{o2}} = \frac{1}{1+e^{-\text{net}_{o2}}} = \frac{1}{1+e^{-1.67}} = 0.84$

Error Calculation:

E $_{\text{total}}$ = $\Sigma(Actual\boldsymbol{output} - Desired\boldsymbol{output})$^2

$= (\text{Actual}_{\boldsymbol{o1}}\text{-out}_{\boldsymbol{o1}})^2 + (\text{Actual}_{\boldsymbol{o2}}\text{-out}_{\boldsymbol{o2}})^2$

$= (1\text{-}0.852)^2 + (1\text{-}0.84)^2$

=0.0475

Backward Pass

Adjusting 3rd year CGPA (w3)

Considering w3 to know how much a change in w3 affects the total error.

$$\frac{\partial E_{total}}{\partial w3} = \frac{\partial E_{total}}{\partial out_{o1}} * \frac{\partial out_{o1}}{\partial net_{o1}} * \frac{\partial net_{o1}}{\partial w3}$$

Now,

$\frac{\partial Etotal}{\partial out1} = -2(Actual_{o1} - out_{o1}) = -2(1 - 0.852) = -0.30$

$\frac{\partial out1}{\partial net1} = out1(1 - out1) = 0.852(1 - 0.852) = 0.1261$

$\frac{\partial net1}{\partial w3} = outh = 0.932163$

So,

$\frac{\partial Etotal}{\partial w3}$ = (-0.30) *0.1261*0.932163= -0.0353

So, we get, $w3+ = w3 - \eta * \frac{\partial Etotal}{\partial w3}$

= 0.8703 – 0.01*(-0.0353) = 0.8707

Adjusting up to 3rd year CGPA (w4)

Considering w4 to know how much a change in w4 affects the total error.

$$\frac{\partial Etotal}{\partial w4} = \frac{\partial Etotal}{\partial out2} * \frac{\partial out2}{\partial net2} * \frac{\partial net2}{\partial w4}$$

$\frac{\partial Etotal}{\partial out2}$ = -2 ($Actual_{o2}$ – $out2$) = -2(1 - 0.84) = -0.32

$\frac{\partial out2}{\partial net2}$ = $out2(1 - out2) = 0.84(1 - 0.84) = 0.1344$

$\frac{\partial net2}{\partial w4} = outh = 0.932163$

$\frac{\partial Etotal}{\partial w4}$ = (-0.32) *0.1344*0.932153= -0.0401

So, we get, $\boldsymbol{w}4+ = \boldsymbol{w}4- \boldsymbol{\eta}*\frac{\partial Etotal}{\partial w4}$

= 0.8523 – 0.01*(-0.0401) = 0.8527

Adjusting bias SPL2(b2)

Considering wb2 to know how much a change in b2 affects the total error.

$$\frac{\partial Etotal}{\partial b2} = \frac{\partial Eo1}{\partial b2} + \frac{\partial Eo2}{\partial b2} = \left(\frac{\partial Eo1}{\partial outo1} * \frac{\partial outo1}{\partial neto1} * \frac{\partial neto1}{\partial b2}\right) + \left(\frac{\partial Eo2}{\partial outo2} * \frac{\partial outo2}{\partial neto2} * \frac{\partial neto2}{\partial b2}\right)$$

$\frac{\partial Eo1}{\partial outo1}$= -1

$\frac{\partial outo1}{\partial neto1} = outo1(1 - outo1) = 0.1261$

$\frac{\partial neto1}{\partial b2}$= 1

$\frac{\partial Eo2}{\partial outo2}$= -1

$\frac{\partial outo2}{\partial neto2} = outo2(1 - outo2) = 0.1344$

$\frac{\partial neto2}{\partial b2}$ =1

$\frac{\partial Etotal}{\partial b2}$ = -0.2605

So, we get, $\mathbf{b2}+ = \mathbf{b2}- \boldsymbol{\eta}*\frac{\partial Etotal}{\partial b2}$

= 0.94005 – 0.01*(-0.2605) = 0.94266

Adjusting 1st year CGPA (w1)

Considering w1 to know how much a change in w1 affects the total error.

$$\frac{\partial Etotal}{\partial w1} = \frac{\partial Etotal}{\partial outh} * \frac{\partial outh}{\partial neth} * \frac{\partial neth}{\partial w1} = \left(\frac{\partial Eo1}{\partial outh} + \frac{\partial Eo2}{\partial outh}\right) * \frac{\partial outh}{\partial neth} * \frac{\partial neth}{\partial w1}$$

$\frac{\partial Eo1}{\partial outh} = \frac{\partial Eo1}{\partial outo1} * \frac{\partial outo1}{\partial neto1} * \frac{\partial neto1}{\partial outh}$ = (-1) *0.1261*w3 = -0.1261* 0.8703 = -0.1097

$\frac{\partial Eo2}{\partial outh}=\frac{\partial Eo2}{\partial outo2} * \frac{\partial outo2}{\partial neto2} * \frac{\partial neto2}{\partial outh}$= (-1) *0.1344*w4= -0.1344*0.8523= -0.1145

$\frac{\partial outh}{\partial neth}= outh\,(1 - outh) = 0.932*(1-0.932) =0.063$

$\frac{\partial neth}{\partial w1}=i1 = 1$

Now,

$\frac{\partial Etotal}{\partial w1}$ = -0.0141

So, we get, $\mathbf{w1}+ = w1 - \eta * \frac{\partial Etotal}{\partial w1}$

$= 0.890137 - 0.01*(-0.0141) = 0.8903$

Adjusting 2nd year CGPA(w2)

Considering w2 to know how much a change in w2 affects the total error.

$$\frac{\partial Etotal}{\partial w2} = \frac{\partial Etotal}{\partial outh} * \frac{\partial outh}{\partial neth} * \frac{\partial neth}{\partial w2} = \left(\frac{\partial Eo1}{\partial outh} + \frac{\partial Eo2}{\partial outh}\right) * \frac{\partial outh}{\partial neth} * \frac{\partial neth}{\partial w2}$$

$$\frac{\partial Eo1}{\partial outh} = \frac{\partial Eo1}{\partial outo1} * \frac{\partial outo1}{\partial neto1} * \frac{\partial neto1}{\partial outh} = (-1) * 0.1261 * w3 = -0.1261 * 0.8703 = -0.1097$$

$$\frac{\partial Eo2}{\partial outh} = \frac{\partial Eo2}{\partial outo2} * \frac{\partial outo2}{\partial neto2} * \frac{\partial neto2}{\partial outh} = (-1) * 0.1344 * w4 = -0.1344 * 0.8523 = -0.1145$$

$$\frac{\partial outh}{\partial neth} = out_h (1 - out_h) = 0.932*(1-0.932) = 0.063$$

$$\frac{\partial neth}{\partial w2} = i2 = 1$$

Now,

$$\frac{\partial Etotal}{\partial w2} = -0.0141$$

So, we get, $\mathbf{w2}+ = w2 - \eta * \frac{\partial Etotal}{\partial w2}$

$= 0.850137 - 0.01*(-0.0141) = 0.8503$

Adjusting bias SPL2(b1)

Considering b1 to know how much a change in b1 affects the total error.

$$\frac{\partial Etotal}{\partial b1} = \frac{\partial Etotal}{\partial outh} * \frac{\partial outh}{\partial neth} * \frac{\partial neth}{\partial b1} = \left(\frac{\partial Eo1}{\partial outh} + \frac{\partial Eo2}{\partial outh}\right) * \frac{\partial outh}{\partial neth} * \frac{\partial neth}{\partial b1}$$

$$\frac{\partial Eo1}{\partial outh} = \frac{\partial Eo1}{\partial outo1} * \frac{\partial outo1}{\partial neto1} * \frac{\partial neto1}{\partial outh} = (-1) * 0.1261 * w3 = -0.1261 * 0.8703 = -0.1097$$

$$\frac{\partial Eo2}{\partial outh} = \frac{\partial Eo2}{\partial outo2} * \frac{\partial outo2}{\partial neto2} * \frac{\partial neto2}{\partial outh} = (-1) * 0.1344 * w4 = -0.1344 * 0.8523 = -0.1145$$

$$\frac{\partial outh}{\partial neth} = out_h (1 - out_h) = 0.932*(1-0.932) = 0.063$$

$$\frac{\partial neth}{\partial b1} = 1$$

Now,

$\frac{\partial Etotal}{\partial b1}$ = -0.0141

So, we get, $\mathbf{b1}$+ =b1- $\eta * \frac{\partial Etotal}{\partial b1}$

= 0.880137– 0.01*(-0.0141) = 0.8803

After 2nd iteration adjusted values are -

Weight and bias	**Expression**	**Value**
1st year CGPA	w1	0.8903
2nd year CGPA	w2	0.8503
3rd year CGPA	w3	0.8707
upto 3rd year CGPA	w4	0.8527
SPL1	b1	0.8803
SPL2	b2	0.9427