Forecasting the General Index of Dhaka Stock Exchange

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Forecasting the General Index of Dhaka Stock Exchange

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Abstract

The Dhaka Stock Exchange (DSE) is an emerging stock exchange located in the capital city of Bangladesh. This present study focuses on finding a predictive model for the DSE general index. According to the Box-Jenkins methodology, ARIMA (2, 2, 1) model was found well fitted from a set of different possible ARIMA models. But the diagnostic tests such as ACF plot of residuals, standardized residual plot, shows that our model forecasts mean of the series pretty good though, we need to consider the volatility of the series to get the more accurate forecast of the data set. Conditional variance model, eGARCH (1, 1) was found as the best fits to our DSE data. The R package rugarch is used to fit the model.

Keywords: DSE, General Index, ARIMA, Volatility, ARCH, GARCH

JEL: G17: C580

1. Introduction

Financial markets first came to prominence during the 17th century at the start of the industrial revolution. The first financial markets came about in London. The stock exchange provides a marketplace where shares can be bought and sold. The main function of stock exchanges is to promote tile movement of capital across the Region, to increase investment opportunities and encourage optimum financing for firms irrespective of where the entity resides. The stock exchange performs various functions simultaneously for the growth and development of the economy. Among the developing countries, the contribution of the capital market has lately been recognized.

Dhaka Stock Exchange is committed to becoming a world-class Stock Exchange with unique investment opportunities for local as well as foreign investors in a fast developing market. The development of the capital market is crucial for capital accumulation, an efficient allocation of resources and the promotion of economic growth. Capital markets of different countries of the world collapsed in the face of global recession in the recent past, the capital markets of Bangladesh remained

quite buoyant at that time. Given the growing number of ordinary investors in capital markets, the limited supply of securities and investors' expectations for more profit at times made the market volatile. Nevertheless, various steps have been taken to maintain market stability and to establish a transparent and vibrant capital market while deepening it. The role of a stock market index is to measure changes in the value of specific groups of stocks and help measure changes in the entire market. Indexes can provide a quick snapshot to see how a specific group of stocks performs compared to other groups of stocks. The market index can be defined as an aggregate value produced by combining several stocks or other investment vehicles together and expressing their total values against a base value from a specific date. Dhaka Stock Exchange publishes three share indices by following Index Calculation Algorithm published by International Organization of Securities Exchange Commissions (IOSCO) to mark its overall market activities: DSI (all shares) DS20 and DGEN indexes. Among them, the general index (DGEN) is commonly used by the stakeholders of the stock market. To make this index values more useful for investors to track changes in market values over long periods of time, we have tried to find a suitable model to predict those in advance.

2. Literature Review

Bangladesh capital market is one of the smallest in Asia but the third largest in the south Asia region. The capital market in Bangladesh is still at a developing stage. The stock market has a birth story of its own. Before Independence, Dhaka was the capital of East Pakistan and was ruled by the Pakistan government as a result of Partition of Indian subcontinent took place in 1947 from the United Kingdom. Dhaka Stock Exchange Limited was incorporated under the company's act. 1913 named as the East Pakistan stock exchange association ltd on 28.04.1954. As a public company on 23.06.1962, the name was revised to East Pakistan stock exchange ltd. Again on 14.05.1964, the name of East Pakistan stock exchange limited was changed to "Dhaka stock exchange ltd". Although incorporated in 1954, the formal trading was started in 1956 and the Dhaka stock exchange (DSE) was shifted to Motijheel, the heart of the capital city of Bangladesh Dhaka in 1959. DSE is the biggest stock market of Bangladesh and in 2015, the combined market capitalization of listed companies on the Dhaka Stock exchange bourse stood at over \$40 billion.

Although the stock market is much more dynamic than the indexes suggest, along with the fact that there are different ways to calculate the indexes, causing calculation bias, the stock market indexes are useful in a number of ways to stock investors. First, the market indexes provide a historical perspective of stock market performance, giving investors more insight into their investment decisions. Investors who do not know which individual stocks to invest in can use indexing as a method of choosing their stock investments. By wanting to match the performance of the market, investors can invest in index mutual funds or index exchange-traded funds (ETFs) that track the performance of the indexes with which they are aligned. This form of investing gives investors the opportunity to do as well as the markets and not significantly underperform the markets. The second benefit of stock market indexes is that they provide a yardstick with which investors can compare the performance of their individual stock portfolios. Individual investors with professionally managed portfolios can use the indexes to determine how well their managers are doing in managing their money. The third major use of stock market indexes is as a forecasting tool. Studying the historical performance of the stock market indexes, you can forecast trends in the market. Consequently, the market indexes provide investors with a useful tool for forecasting trends in the market.

There is a plenty of scholarly papers featuring on the Dhaka Stock Exchange. Basher et al. (2007) empirically examined the time-varying risk-return relationship and the impact of institutional factors such as circuit breaker on volatility for the emerging equity market of Bangladesh using daily and weekly stock returns. They found that the DSE equity returns showed negative skewness, excess kurtosis, and deviation from normality. The returns displayed significant serial correlation suggesting stock market inefficiency. The results also showed a significant relationship between conditional volatility and stock returns, but the risk-return parameter was found to be sensitive to the choice of

samples and frequencies of data. Hossain and Kamal (2010) found that unidirectional causality prevailed between stock market development and economic growth in the Bangladesh economy. They also identified that both the variables stock market development and economic growth share the same stochastic trend in Bangladesh economy. Rahman and Moazzem (2011) identify the causal relationship between decisions taken by the regulatory authority and market volatility. Ali (2011) investigated the long-run equilibrium, short-run dynamics adjustment as well as a causal relationship between DSE allshare price index and macroeconomic variables of the consumer price index (CPI), GDP, foreign remittances and import payment. Cointegration among the variables was found significant and VECM estimated that the system corrects its previous period's level of disequilibrium by 5.98 percent per month. Zaman(2012) determined dividend policy and return on assets for 25 out of 30 Dhaka Stock Exchange-listed private commercial banks in Bangladesh during January 2006 - December 2010. She found that a negative correlation exists between the profitability of commercial banks and its respective dividend policy in 2006 but the correlation becomes positive from 2007 onwards. She also showed that, with time, the variation in dividend policies can be strongly explained by variation in their respective profitability. Hossain and Nasrin (2012) revealed that the most important factors influencing retail investors in equity market of Bangladesh are company specific attributes/reputation, net asset value, and accounting information. The study also examined that Demographic characteristics of sample respondents such as gender, age, occupation, income, education, and experience also has a significant influence on equity of shares in the market. Ahmad et al. (2012) examined and compared the relationship between stock market development and economic growth of Bangladesh and Pakistan in terms of size (market capitalization), liquidity (total value of stocks traded and stock turnover ratio) and volume (total number of companies listed in the stock exchange of each of the country). Their analysis showed that Pakistan stock markets contribute to the economic growth in terms of the large size of its stock market whereas economic growth in terms of the liquidity of its stock market. Kumar et al. (2012) discussed the relationship between the equity market and economic development in the context of Bangladesh. Uddin et al. (2013) put a great stride to identify what determines the share prices of stock market focusing exclusively on the financial sector of Bangladesh. Their findings show that Earnings per share (EPS), Net asset value (NAV), Net profit after tax (NPAT) and Price earnings ratio (P/E) have a strong relationship with stock prices. Huda (2013)implemented the factor analysis over the period 2000-2011 on Dhaka Stock Exchange (DSE) & Chittagong Stock Exchange (CSE) data and modeled that Turnover of capital market largely depends on four indicators i.e. No. of Listed Securities, Initial Public Offering (IPOs), Market Capitalization, Issued Capital in the both stock markets in Bangladesh during the study period. Afroze (2013) found indicators for measuring money the indicators for measuring the performance of Dhaka Stock Exchange Limited for the fiscal years 2006 to 2010. Islam et al. (2014) compare the volatility of price between Dhaka stock exchange (DSE) and Chittagong Stock Exchange (CSE) and find that CSE is more volatile than DSE. They have also mentioned that the general price is less volatile than CSE30 and DSE20 which means that the top 20 and 30 securities influence the whole market. Abedin et al. (2015) found the significant month of the year effect presents in DSE in their study over the period 2000 to 2012. As a result, investors can outperform the market and this is against in principle of market efficiency. Hasan (2015) applied daily return data for the three stock indices of Dhaka Stock Exchange such as DSI (from 02 January 1993 to 27 January 2013) with a total of 4823 daily return observations, DGEN(from 01 January 2002 to 31 July 2013) with a total of 2903 daily return observations, and DSE-20 (from 01 January 2001 to 27 January 2013) with a total of 3047 daily return observations and found that all the return series do not follow the random walk model, and thus the Dhaka Stock Exchange is inefficient in weak form. Mazumder (2015) revealed that stock markets have made a substantial contribution to the economic development of Bangladesh. Hossain et al. (2015) modeled the direct impact of a stock trade, invested stock capital, stock volume, current market value, and DSE general on DSE prices for the period from June 2004 to July 2013 as the basis on a daily scale by applying vector autoregressive (VAR) models. Royand and Ashrafuzzama(2015) failed to predict stock price suitably

and but found an unusual difference lying between intrinsic value, determined by multiple models, and the actual price of the stocks.

3. Past Researches Using ARIMA, ARCH and GARCH Model

ARIMA model has been applied in various sectors at the national and international level. In various sectors like production estimation (Mandal, 2006), price estimation (Raymond, 1997 & Nochai, 2006), Market forecasting (Parish Jr, 2006) etc. ARIMA model has been applied. Shitan et al. (2012) discussed Seasonal ARIMA modeling on Bangladesh Export Values.

Since its inception, GARCH model is been widely used around the world to model the volatility of financial time series data. AL-Loughani and Chappell (2001) examined The Kuwait stock exchange index for evidence e of a day-of-the-week effect. They also confirmed that a nonlinear GARCH (1, 1) model provide s a good explanation of the data and allows identification and modeling of the day-ofthe-week effect. Hansen and Lunde(2005) tried 330 ARCH-type models in terms of their ability to describe the conditional variance and compared out-of-sample using DM-\$ data and IBM return data. They evidenced that a GARCH (1,1) is outperformed by more sophisticated models in the context of analysis of exchange rates, whereas the GARCH(1,1) inferior to models that can accommodate a leverage effect in analysis of IBM returns. Magnus and Fosu (2006) forecasted volatility (conditional variance) on the Ghana Stock Exchange using a random walk (RW), GARCH(1,1), EGARCH(1,1), and TGARCH(1,1) models. They found the GARCH (1,1) model outperformed the other models under the assumption that the innovations follow a normal distribution. Alexander and Lazar (2006) analyzed the general normal mixture GARCH (1, 1) model which can capture time variation in both conditional skewness and kurtosis. They concluded that for modeling exchange rates, generalized two-component normal mixture GARCH(1,1) models perform three or more components, and better than symmetric and skewed Student's f-GARCH models. Brewer et al. (2007) investigated the interest rate sensitivity of monthly stock based on a generalized autoregressive conditionally heteroscedastic in the mean (GARCH-M) model. Results based on data for the period 1975 through 2000 indicate that life insurer equity sensitive to the long-term interest that interest sensitivity varies across sub-period and across risk-based and size-based portfolios. Alberg et al. (2008) produced a comprehensive empirical analysis of the mean return and conditional variance of the Tel Aviv Stock Exchange (TASE) indices using various GARCH models. Their results showed that the asymmetric GARCH model with fat-tailed densities improves overall estimation for measuring conditional variance. The EGARCH model using a skewed Student-t distribution is the most successful for forecasting TASE indices.

Akgul and Sayyan (2008) tried to compare stable, integrated and long-memory generalized autoregressive conditional heteroscedasticity (GARCH) models in forecasting the volatility of returns in the Turkish foreign exchange market for the period 1990–2005 and for the sub-period that covers the floating exchange rate regime 2001–2005. In the first period, they found that long-memory GARCH specifications capture the temporal pattern of volatility for returns in US and Canadian dollars against Turkish lira. Their result also confirms that when long memory, asymmetry and power terms in the conditional variance are employed, together with the skewed and leptokurtic conditional distribution (of innovations), the most accurate out-of-sample volatility is produced for the first and sub-period. Bonilla et al (2011) applied Hinich portmanteau bi-correlation test to detect for the adequacy of using GARCH (Generalized Autoregressive Conditional Heteroscedasticity) as the data-generating process to model conditional volatility of stock market index rates of return in 13 emerging economies and suggested that policymakers should use caution when using autoregressive models for policy analysis and forecast because the inadequacy of GARCH models has strong implications for the pricing of stock index options, portfolio selection, and risk management. Especially, measures of spillover effects and output volatility may not be accurate when using GARCH models to evaluate economic policy. Panait and Slavescu (2012) applied data mining to compare the volatility structure of high (daily) and low (weekly, monthly) frequencies for seven Romanian companies traded on Bucharest Stock Exchange and three market indices, during 1997-2012. For each of the 10-time series and three frequencies, they modeled a GARCH-in-mean model and we got that persistence is more present in the daily returns as compared with the weekly and monthly series. They also concluded that showed that GARCH-in-mean was well fitted on the weekly and monthly time series but behaved less well on the daily time series. Zakaria et al (2012) employed different univariate specifications of the Generalized Autoregressive Conditional Heteroscedastic (GARCH) model, including both symmetric and asymmetric models for two African exchanges; Khartoum Stock Exchange, KSE (from Sudan) and Cairo and Alexandria Stock Exchange, CASE (from Egypt). Their results show that the conditional variance (volatility) is an explosive process for the KSE index returns series, while it is quite persistent for the CASE index returns series. Furthermore, the asymmetric GARCH models find significant evidence for asymmetry in stock returns in the two markets, confirming the presence of leverage effect in the returns series. Padilla and Ortega(2013) discussed the application of the generalized autoregressive conditional heteroscedasticity models (GARCH) in order to forecast the variance and return of the IPC, the EMBI, the weighted-average government funding rate, the fix exchange rate and the Mexican oil reference, as important tools for investment decisions from 2005 to 2011. Nkoro and Uko (2013) studied the impact of domestic macroeconomic variables on Nigeria's stock market returns, using Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model and annual data (1985-2009). The results revealed that, out of the six macroeconomic variables employed, inflation, government expenditure, index of manufacturing output and, interest rate, exert strong significant influence on stock returns. Inflation and government expenditure have a positive significant impact, while the index of manufacturing output and interest rate has a negative significant impact. On the other hand, money supply and foreign exchange rate exert no significant influence on stock returns in Nigeria. The time-varying volatility of Nigeria's stock market returns is moderately persistent. Olugbode et al. (2014) examined the sensitivity of 31 UK non-financial industries to exchange and interest rate exposure from 1990 to 2006 using first-order autoregressive exponential GARCH-in-mean (eGARCH-M) model. We found that the stock returns of UK industries are more affected by long-term interest rate risk than exchange rate risk and short-term interest rate risk. Ramadan (2014) used daily return for the Market Capitalization Weighted Price Index of the Jordanian stock market for the period from the first trading day on the year 2000 to the day of the year 2013 of Amman Stock Exchange the relationship between risk and return. The full period was separated into two periods: before the global financial crisis (BGFC) and during the global financial crisis (DGFC), to show if there is any impact of the financial crisis on the relationship between the risk and return. The conditional relationship and return at the FULL period did the trade-off theory, has been no statistically significant effect of risk, measured by volatility, on the market return during the FULL period, nor in the Before Global Financial Crisis (BGFC) period. Kalyanaraman (2014) estimated the conditional volatility of Saudi stock market by applying AR (1)-GARCH (1, 1) model to the daily stock returns data spanning from August 1, 2004, to October 31, 2013. The result showed that a linear symmetric GARCH (1, 1) model is adequate to estimate the volatility of the stock market of the country. They also concluded that Saudi stock market returns are characterized by volatility clustering and follow a non-normal distribution time and the market has persistence and are predictable varying volatility.

Everywhere in the world, stock exchange creates a value of money by moving the fund as a continuous supply to the economy. At present may, people are considering investment as their career. So, the stock exchange of Bangladesh could be a great source of employment. As the process of trading security is too much easy, people from different segment always have an opportunity to invest in the market. But this limited knowledge about investment is share market causes a great loss to many people in Bangladesh in recent time. The Stock market has to ensure the efficiency and security of the investment for the people who are trading in the market. The background of the present research is to help the decision makers to ensure a valid forecast of the market index so that the traders can understand the market's overall condition and feel confident while trading in the market.

4. Data

The whole sample consists of the daily DSE general index from 1/3/2010 to 7/31/2013, for a total of 860 observations.

5. Econometric Methodology

In real life, most time-series data or stochastic processes are non-stationary and the means of these processes are not constant through different time lags. The study focuses on to develop an appropriate model to forecast the daily index of DSE which is indeed a non-stationary time series. The univariate ARIMA model which is basically an extrapolation method for forecasting is applied in some cases on data which show evidence of non-stationarity. The ARIMA model is, in theory, the most general class popularized models forecasting time series and was first Jenkins. ARIMA (p, d, q)Completely ignores independent variables and assumes that past values of the series plus previous error terms contain information for the purposes of forecasting. The integers refer to the Autoregressive (AR), Integrated (I) and Moving Average (MA) parts of the dataset respectively. Evidence of non-stationarity in a time series data can be stationarized by transformations such as differencing and logging.

The ARIMA model is used to deal with a univariate time series data and it is function of autoregression (AR) and moving average (MA) model. The process of AR depends on a weighted sum of its past values and a random disturbance term while the process of MA model depends on a weighted sum of current and lagged random disturbances. If a time series is not stationary, it can be differenced (integrated) once or more to become stationary. Therefore, the stationary process of ARIMA model is a combination of both lagged from past values and random disturbances, as well as a current disturbance term. The Auto-Regressive Integrated Moving Average (ARIMA) model can be written as:A process $\{X_t\}$ is said to be an ARIMA (p,d,q) if $\{(1\text{-B})^d\,X_t\,\}$ is a causal ARMA(p ,q) . We write the model as:

$$\emptyset(B) (1-B)^d X_t = \theta(B) Z_t, \quad \{Z_t\} \sim WN(0,\sigma^2)$$

The process is stationary if and only if d=0. Differencing X_t 'd' times, results in an ARMA (p, q) with $\emptyset(B)$ and $\theta(B)$ as AR & MA polynomials. Box and Jenkins (1976) methodology of Univariate ARIMA model is considered the most flexible method and is used by a numbers of researchers for forecasting time series data. Their proposed methodology of ARIMA model consists of four steps i.e. Identification, Estimation, Diagnostics, and Forecast, which are applied in the first part of our research.

In conventional time series and econometric models, the variance of the disturbance term is assumed to be constant and we can forecast the series by selecting the best ARIMA model for the mean However, volatility is in practice the most common phenomenon where the conditional variance of the time series varies over time such as many economic and financial time series exhibit periods of unusually high volatility followed by periods of relative tranquility. The two-pass method like ARIMA model is not able to handle non-normal error distributions. In such situations, the assumption of constant variance is inappropriate. Engle (1982), Bollerslev (1986) and others developed a class of models that address such concerns and also allow for modeling both the level (the first moment) and the variance (the second moment) of a process. The first moment equation can be a non-seasonal ARIMA, seasonal ARIMA, or dynamic regression model whereas the second moment equation that is used to model volatility in financial applications are widely known as various GARCH extensions.

Engle (1982) first proposed the autoregressive conditional heteroscedasticity (ARCH) to model this kind of changing variance. More specifically, in these processes, the variance of the error displays autoregressive behavior – certain successive periods demonstrate large error variance while certain others show small error variance. The ARCH method essentially regresses the conditional variance, also known as *conditional volatility*, on the squared returns or observations from past lags. Hence, ARCH is usually specified with the help of the lags being used for the covariates. So ARCH (q) will be represented as:

$$u_t = \sigma_t \varepsilon_t \sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i u_{t-i}^2$$

The error terms are also referred to as *innovations* in the time-series data. Often the variance of the current innovation is related to the squares of the previous innovations. Bollerslev (1986) proposed an extension of the ARCH type models in order to allow longer memory and a more flexible lag structure, called GARCH which introduces p lags of conditional variance by allowing the conditional variance to follow an ARMA process. So combining with ARCH, this becomes GARCH (p, q).

$$u_t = \sigma_t \varepsilon_t \sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i u_{t-i}^2 + \sum_{j=1}^p \delta_j \sigma_{t-j}^2$$

In this research we applied the simplest The Garch (1, 1) model which can be expressed as:

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

Here, the variance (σ_t) is a function of an intercept (ω) , a shock from the prior period (α) and the variance from last period (β) . The autoregressive root that governs the persistence of the shocks of the volatility is the sum of $\alpha + \beta$. Linear GARCH models all allow prior shocks to have a symmetric affect on (σ_t) . Non-linear models allow for asymmetric shocks to volatility.

5.1. IGARCH (Engle and Bollerslev 1986)

In financial time series, the conditional volatility (σ_t^2) is often persistent and therefore may cause a unit root [If $\alpha + \beta = 1$, the return process is a random walk in previous equation of GARCH (1, 1)] condition in the model such a special case of a GARCH model is referred to as Integrated GARCH (Engle and Bollerslev 1986). An IGARCH (1, 1) model can be written as

$$u_t = \sigma_t \varepsilon_t \sigma_t^2 = \alpha_0 + (1 - \beta) \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

The key feature of IGARCH model is the long memory or persistence of shocks on the volatility.

5.2. EGARCH (Nelson (1991)

This model allows for asymmetry in the effect of the shocks. Positive and negative returns can impact the volatility in different ways. An EGARCH (1, 1) model can be written as

$$\log(\sigma_t^2) = \omega + \beta \log(\sigma_{t-1}^2) + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \alpha \frac{|\varepsilon_{t-1}|}{\sigma_{t-1}}$$

Since we model the log (σ_t^2) , then even if the parameters are negative, σ_t^2 will be positive. We can account for the leverage effect by using EGARCH.

The Generalised Error Distribution (GED) and the reparametrised Johnson SU distribution (JSU) were found as the best model for the innovation ε_t .

5.3. Generalised Error Distribution (Nelson, 1991)

$$f(t) = \nu \exp\left(-\frac{1}{2} \left| \frac{t}{\lambda \sigma_*} \right| \right) \left[2^{\frac{\nu+1}{\nu}} \Gamma(1/\nu) \lambda \sigma_t \right]^{-1}$$

where v is the tail-thickness parameter and $\lambda = \left[\frac{\Gamma(1/v)}{2^{2/v}\Gamma_3/v}\right]^{0.5}$. When v=2, the distribution becomes standard normal. For v<2, the distribution has thicker tails than the normal distribution. The conditional kurtosis is given by $(\Gamma(1/v)\Gamma(5/v))/(\Gamma(1/v))^2$.

5.4. Johnson's SU Distribution (Johnson, 1954)

$$f(t) = \frac{\delta}{\lambda \sqrt{1 + \left(\frac{t - \xi}{\lambda}\right)^2}} \phi \left[\gamma + \delta \sinh^{-1} \left(\frac{t - \xi}{\lambda}\right) \right]$$

Where, ϕ is the density function of N(0,1) ζ and λ both are positive as well as location and scale parameters respectively, γ and δ can be interpreted as a skewness and kurtosis parameter. The Johnson SU distribution is in fact most relevant for financial applications, since it can fit data with leptokurtic and skewed distribution. Despite this flexibility, the JSU distribution has however the disadvantage that it is not guaranteed to exist for any set of mean, variance, skewness and kurtosis.

The Standard GARCH model (Bollerslev, 1986), The GJR GARCH model (Glosten et al. 1993), The Component Standard GARCH model (Lee and Engle, 1999) were also used in search of the best model for our dataset. A range of univariate distributions including the Normal (Gauss, 1809), Generalized Error (Nelson, 1991), Student's t (Gosset,1908) and their skew variants ('snorm', 'sged' and 'sstd') the Generalized Hyperbolic (Barndorff el. 1978), Normal Inverse Gaussian (Barndorff el. 1977) and Johnson's reparametrized SU (Johnson, 1954) distribution were used for fitting the standardized innovations. Finally, theAIC(Akaike, 1974),AICc (Cavanaugh, 1997), BIC (Akaike, 1979),HQIC (Hannan and Quinn, 1978) and SIC(Shibata, 1989) information criteria were applied to select the best model.

6. Results and Discussions

The whole dataset of 860 observations (Daily General Index of DSE from 1/3/2010 to 7/31/2013) was divided into two different sets; Training Dataset (contains first 759 observations) and Test dataset (contains last 101 observations). At first, we tried some exploratory analysis to get an overview of our dataset. From Table-01, we can see that Training Data significantly differs from Test dataset with respect to mean, Standard deviation, quartile deviation and other descriptive measures but has almost unique features like the whole dataset. Both the datasets are positively skewed as well as follows the leptokurtic distribution. From Figure-01 and 02, we can see that most of the observations of our test dataset vary from 5000 to 7000 and the overall distribution is a bimodal distribution showing that the dataset will not be normally distributed as well has no consistent pattern over the entire study period. Figure-03 tells us that the time series dataset does not follow the normal distribution and we can guess that the dataset will not be having the white noise properties as well as.

After the exploring the descriptive features of our dataset (Training dataset) we can see that the dataset is non-stationary over the entire study period (see Figure-04) and it also gives us an overall view of the volatility existing within the dataset. The Autocorrelation Function (ACF) plot (Figure-05) shows high correlation at different lags of the dataset. Also the Partial Autocorrelation Function(PACF) plot(Figure-06) shows high correlation at lags one; which means that we need to take first difference of dataset and check out the time plot whether it becomes stationary series or not. The figure-07 reveals that the dataset has got about to be stationary while there is some high inconsistency of the dataset in between observation 200 to 300. The ACF plot (Figure-08) at first difference still shows a high correlation at lag one and also the PACF plot (Figure-09) shows correlations at different lags outside the control limits. To overcome the uncertainty about the stationarity of the dataset we applied various unit root tests such as ADF (Dickey and Fuller, 1979) test, PP (Phillips and Perron, 1988) test, KPSS (Kwiatkowski et al. 1992) test and ZA (Zivot et al. 1992) test. The tests results (Table-02) reveals at various levels of significance (one percent, five percent, and ten percent) that the

dataset has a unit root at first difference, that is the dataset is still non-stationary and it requires further transformations to make it stationary. From the time plot (Figure-10) of the dataset at the second difference, the dataset appears to be stationary except some inconsistency in between observation 200 to 300 (which is happening due to unusual observations originated from the economic recession in the economy of Bangladesh during that time period). The ACF plot of the same series in Figure-11 shows that no significant correlation at any lags of the dataset. Also, the PACF plot (Figure-12) shows a wave of decaying correlations at various lags just from the first lag of the dataset. The unit different root tests results (Table-03) at one percent, five percent and ten percent level of significance shows that the dataset has no unit root at the second difference; that is the dataset has finally got long awaited stationarity after the second difference. The dataset herewith confirms the property of a white noise series for fitting the Autoregressive Integrated Moving Average (ARIMA) model and the order of integration for our dataset is two (that is difference parameter in ARIMA model, d=2).

Picking up the most appropriate order for the autoregressive part and the moving average part respectively of an ARIMA model is a challenging decision as per the proposed methodology of ARIMA modeling by Box and Jenkins. We have tried different combinations of orders for the Autoregressive part and Moving average part of the ARIMA model on trial and error basis(Table-04) recording their information criteria values; Akaike Information Criteria(AIC), Corrected AIC (AICc), and Bayesian Information Criteria (BIC). Based on the lowest AIC values and BIC values we can think of two possible combinations of ARIMA models; ARIMA (2, 2, 1) and ARIMA (0, 2, 1) respectively. Table-05 in the appendix section gives the lower Mean Error(ME), Root Mean Square Error(RMSE), Mean Percentage Error(MPE), Mean Absolute Percentage Error(MAPE), and Correlation at Lag one for ARIMA model (2, 2, 1) and lower Mean Absolute Error(MAE) and Mean Absolute Square Error(MASE) for ARIMA model (0, 2, 1). So we can now suggest ARIMA model (2, 2, 1) to fit for our daily General index data with confidence. The fitted ARIMA model (2, 2, 1) model is described in Table-06. The model states that the index at present time depends on its one period past observation by 1.9977, depends on its two periods past observation by 1.0223, depends on its three periods past observation by 0.1475, depends on its four periods past observation by 0.0849 and also depends on its one period past error terms by 0.9958; whereas, the errors of the model are independent and identically distributed with mean zero and constant variance 15441. The time plot (Figure-13) of forecasted values for the series shows the movement of the mean of the series along the confidence interval and it is found to be pretty consistent downward moving series. The residual plot (Figure-14) of the fitted model shows that the standardized residuals are not randomly distributed (rather gathering inconsistently around some data points). The ACF plot of residuals from the same figure is also showing high correlation at different lags; that is, it violates the assumption of white series. Finally, the p-value plots confirm the pitfalls of ARIMA model fitting to this data showing p-value more than outside the control limit. Now we can guess that the dataset has time-varying variance (volatility feature) which is not incorporated by the ARIMA model as it only models the mean of a given time series data. As our proposed ARIMA model is not capable to capture all the features of the data series, the dataset itself requires a more advanced model to forecast the series and the existing volatility within the dataset. So we have to try the time series model that captures the volatility of a time series data, such as Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model to find the most suitable model to fit our daily general index data of DSE.

The univariate GARCH specification in rugarch package allows us to implements a rich set of univariate GARCH models such as, The standard GARCH (sGARCH), The integrated GARCH (iGARCH), The exponential GARCH (eGARCH), The GJR-GARCH (gjrGARCH), The Component sGARCH (csGARCH) models against a range of univariate distributions including the Normal ('norm'), Generalized Error ('ged'), Student ('std') and their skew variants ('snorm', 'sged' and 'sstd') the Generalized Hyperbolic ('ghyp'), Normal Inverse Gaussian ('nig') and Johnson's reparametrized SU ('jsu') distribution for fitting the standardized innovations along with the mean model, ARIMA(2,2,1) proposed by the Box-Jenkins methodology. The Akaike (AIC), Bayesian (BIC), Hannan-Quinn

(HQIC) and Shibata (SIC) information criteria were applied to enable us selecting the model by penalizing overfitting at different rates to compares the empirical distribution of the standardized residuals with the theoretical ones from the chosen density for the conditional variance. The various specifications of possible estimated GARCH models along with their information criteria values were listed from Table-7A to Table-7E. Based on Akaike (AIC), Bayesian (BIC), Hannan-Quinn (HQIC) and Shibata (SIC) information criteria the best possible four models are enlisted in Table-7F. After that, finally we narrowed down our focus on The integrated GARCH (iGARCH) model and The exponential GARCH (eGARCH) model against the Generalized Error ('ged') and the Johnson's reparametrized SU ('jsu') distribution respectively along with the mean model, ARIMA(2,2,1) to fit the volatility of our daily index data. The summary of these two models is given in Table-7G.

Clearly, the eGARCH model with its specifications in Table-7G best fits our dataset. The sum of the coefficient of α and β less than one in case of both GARCH model for their specific conditional distribution. The ARCH effect denoted by the alpha value is found significant for both models whereas the garch effect was found significant for eGARCH mode only. Nevertheless, the estimate of α is smaller than the estimate of β in both cases that is to show negative shocks haven't a larger effect on conditional volatility than positive shocks of the same magnitude. The negative value of α shows that the volatility will be decaying in the future as the relation between past error and past volatility is negative. The positive dependency of present volatility on the past volatility is reflected by the positive beta value. In eGARCH model, $\gamma > 0$ the news impact is asymmetric on the other words bad news increase volatility. In the eGARCH model positive and significant leverage effect parameter indicating the absence of the leverage effect in data. That is a positive shock of our dataset will have more effect on its volatility compared to the negative shocks. Shape parameter for Johnson's SU Distribution is more than two and statistically significant which indicate that index's distribution is leptokurtic and the significant negative value of the skewness parameter confirms that they are left tailed, that is GARCH residuals still tend to be heavy-tailed.

The plot of Data Series superimposed on two conditional volatility (Figure-15), the plot of series by 1%Value at Risk (with unconditional mean) (Figure-16), Conditional SD (vs |returns|) plot (Figure-17), the plot of empirical density for standardized residuals (Figure-18), the QQ-Plot of Standardized Residuals (Figure-19), the ACF plot of Squared Standardized Residuals (Figure-20) also tell us that the eGARCH model best fits our dataset.

Forecast in GARCH models is critically dependent on the expected value of the innovations and hence the density is chosen. One step ahead forecasts are based on the value of the previous data, while n-step ahead (n>1) are based on the unconditional expectation of the models. The Figure-21 shows the plot of forecasted values by our eGARCH model with specifications. The plot shows the movement of mean value as well as a confidence interval of variance for our index numbers.

We carried out a Simulation of eGARCH using its parameters (Figure-22). The plot shows us that the distribution of the actual index is almost similar with that of the simulated distribution whereas the existing conditional variance within the dataset has got slightly different distribution than the simulated conditional variance but has an almost similar shape of distribution though.

We have addressed the issue of parameter uncertainty by the generation of a Monte Carlo experiment by simulating and fitting a model multiple times (N=1000). Figure-23 shows simulated parameter density of a set of parameters from our eGARCH mode. The actual parameter values are pretty consistent with the simulated parameter distribution. Figure-24 shows the simulated eGARCH stats plot. Half-life, unconditional variance, unconditional mean, and sample skewness have got the leptokurtic distribution whereas sample kurtosis has got the right-tailed as well as leptokurtic distribution as the size of N goes up. Figure-25 shows the Root Mean Square Error (RMSE) Rate of Change of true versus estimated parameters in relation to the data size (square root (N)). The Root Mean Squared Error (RMSE) of parameters in relation to the data size is pretty downward slopping except for alpha, beta, omega, and gamma.

7. Conclusion

Individual measures such as market indexes, of the market are convenient indicators or gauges of the stock market. These market indexes are convenient gauges of the stock market that also indicate the direction of the market over a period of time. By using these market indexes, we can compare how well individual stocks and mutual funds have performed against comparable market indicators for the same period. Our study have found that, Exponential GARCH (1, 1) as the conditional variance model taking into consideration the mean model ARIMA (2,2,1) and the Johnson's SU distribution for the model errors density was found as the most competent model to predict the volatility of index data and forecast future values for over the entire study period. Our effort to find a suitable predicating model for our DSE general index suffers from the limitation that a good forecasting technique for a situation may become an inappropriate technique for a different situation. The validation of the particular model must be examined as time changes.

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Appendix-A

Table-01: Summary on Dataset

Compl	lete Data	aset(Dail	ly Genera	al Index	of DSE	from 1/3/2	2010 to 7	7/31/2013)				Complete Dataset(Daily General Index of DSE from 1/3/2010 to 7/31/2013)								
Sam ple size	Mea n	SD	First Quar tile	Medi an	Third Quar tile	Trem. Mean	MA D	Min	Max	Rang e	Skew	Kurt	SE							
860	5415. 03	1186. 5	4385	5347. 68	6178	5301.5 7	5301. 57	1337.2 4	8918.5 1	5308. 08	0.71	-0.04	40. 46							
Traini	Training Dataset(Daily General Index of DSE from 1/3/2010 to 2/28/2013)																			
Sam ple size	Mea n	SD	First Quar tile	Medi an	Third Quar tile	Trim Mean	MA D	Min	Max	Rang e	Skew	Kurt	SE							
759	5594. 33	1142. 93	4584	5524. 11	6260	5484.6 1	1231. 4	3616.2 4	8918.5 1	5302. 27	0.68	-0.02	41. 49							
Test I)ataset(I	Daily Ge	neral In	dex of D	SE from	3/3/2013	to 7/31/2	2013)												
Sam ple size	Mea n	SD	First Quar tile	Medi an	Third Quar tile	Trim med Mean	MA D	Minim um	Maxim um	Rang e	Skewn ess	Kurt osis	SE							
101	4067. 65	341.3 9	3739	3976. 9	4359	4046.2	459.4 7	3610.4 3	4775.2	1164. 77	0.35	-1.15	33. 97							

Table-02: Unit root test Summary at first difference

	Result of Unit root	Test on Daily (General Index of DSE at first	difference
Name of the Test	Null Hypothesis	Value of Test Statistics	Critical Value	Decision
Augmented	H _o : There is a unit	-2.8857	1pct 5pct 10pct	There is a unit root.
Dickey-Fuller	root in the process.	3.3013	tau3 -3.96 -3.41 -3.12	(<u>Time series</u> is non- <u>stationary</u>)
Test		4.9373	phi2 6.09 4.68 4.03	
			phi3 8.27 6.25 5.34	
Phillips-Perron	H _o : There is a unit	-2.8749	1pct 5pct 10pct	There is a unit root.
Test	root in the process.		Z-tau -3.97 -3.42 -3.13	(<u>Time series</u> is non- <u>stationary</u>)
Kwiatkowski–	H _o : There is no unit	0.4703	1pct 5pct 10pct	There is a unit root.
Phillips-Schmidt-	root in the process		tau 0.216 0.146 0.119	(<u>Time series</u> is non- <u>stationary</u>)
Shin Test				
Zivot-Andrews	H _o : There is a unit	-3.5765	1pct 5pct 10pct	There is a unit root.
Test	root in the process.		t-Stat -4.93 -4.42 -4.11	(<u>Time series</u> is non- <u>stationary</u>)

 Table-03:
 Unit root test Summary at second difference

·	Result of Unit root	Test on Daily G	eneral Index of DSE at secon	nd difference
Name of the Test	Null Hypothesis	Value of Test Statistics	Critical Value	Decision
Augmented	H _o : There is a unit	-20.9454	1pct 5pct 10pct	There is no unit root.
Dickey-Fuller	root in the process.	146.237	tau3 -3.96 -3.41 -3.12	(<u>Time series</u> is <u>Stationary</u>)
Test		219.355	phi2 6.09 4.68 4.03	
			phi3 8.27 6.25 5.34	
Phillips-Perron	H _o : There is a unit	-26.9919	1pct 5pct 10pct	There is no unit root.
Test	root in the process.		Z-tau -3.97 -3.42 -3.13	(<u>Time series</u> is <u>Stationary</u>)
Kwiatkowski-	H _o : There is no unit	0.0883	1pct 5pct 10pct	There is no unit root.
Phillips-	root in the process		tau 0.216 0.146 0.119	(<u>Time series</u> is <u>Stationary</u>)
Schmidt–Shin				
Test				
Zivot-Andrews	H _o : There is a unit	-21.0477	1pct 5pct 10pct	There is no unit root.
Test	root in the process.		t-Stat -4.93 -4.42 -4.11	(Time series is Stationary)

Table-04: Selection of parameter of ARIMA model

	Se	lection of approp	oriate ARIMA m	odel (p, d=2, q)		
			Order of Autore	gressive Part (p)	
Order of Moving Average Part (q)	0	1	2	3	4	5
0	AIC=9963.76	AIC=9797.76	AIC=9704.90	AIC=9638.42	AIC=9614.07	AIC=9592.38
	AICc=9963.76	AICc=9797.78	AICc=9704.93	AICc=9638.47	AICc=9614.15	AICc=9592.49
	BIC=9968.39	BIC=9807.02	BIC=9718.79	BIC=9656.94	BIC=9637.22	BIC=9620.15
1	AIC=9464.04	AIC=9465.70	AIC=9462.27	AIC=9463.20	AIC=9465.03	AIC=9465.86
	AICc=9464.06	AICc=9465.73	AICc=9462.32	AICc=9463.28	AICc=9465.14	AICc=9466.01
	BIC=9473.30	BIC=9479.58	BIC=9480.79	BIC=9486.34	BIC=9492.80	BIC=9498.26
2	AIC=9465.63	AIC=9466.78	AIC=9463.31	AIC=9462.76	AIC=9464.66	AIC=9468.13
	AICc=9465.66	AICc=9466.83	AICc=9463.39	AICc=9462.87	AICc=9464.81	AICc=9468.32
	BIC=9479.51	BIC=9485.30	BIC=9486.46	BIC=9490.53	BIC=9497.06	BIC=9505.16
3	AIC=9462.65	AIC=9463.44	AIC=9465.15	AIC=9464.63	AIC=9466.46	AIC=9468.37
	AICc=9462.70	AICc=9463.52	AICc=9465.26	AICc=9464.78	AICc=9466.65	AICc=9468.61
	BIC=9481.17	BIC=9486.59	BIC=9492.93	BIC=9497.04	BIC=9503.49	BIC=9510.03
4	AIC=9462.93	AIC=9464.25	AIC=9465.71	AIC=9466.33	AIC=9467.43	AIC=9466.09
	AICc=9463.01	AICc=9464.36	AICc=9465.86	AICc=9466.52	AICc=9467.67	AICc=9466.39
	BIC=9486.08	BIC=9492.03	BIC=9498.12	BIC=9503.36	BIC=9509.10	BIC=9512.39
5	AIC=9464.66	AIC=9466.88	AIC=9466.44	AIC=9467.52	AIC=9472.02	AIC=9471.22
	AICc=9464.77	AICc=9467.03	AICc=9466.64	AICc=9467.76	AICc=9472.31	AICc=9471.58
	BIC=9492.43	BIC=9499.29	BIC=9503.48	BIC=9509.18	BIC=9518.31	BIC=9522.15

 Table-05:
 Selection of parameter of ARIMA model AICc versus BIC

Which model should we consider?									
ARIMA(0,2,1)	ARIMA(0,2,1) ME RMSE MAE MPE MAPE MASE ACF1								
(BIC=9473.30)	-6.565205	124.5624	81.4507	-0.1284181	1.481932	0.9900136	0.01757632		
ARIMA(2,2,1)	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1		
(AICc=9462.32)	-6.315149	124.0967	81.6454	-0.1430352	1.468295	0.9923801	0.006784007		

Table-06: Fitted ARIMA model

ARIMA (2,2,1)	First Autoregressive p	Second Autoregressive pa	First Moving Average pa
(standard error of parameters)	0.0223 (0.0364)	-0.0849 (0.0364)	-0.9958(0.0057)
$Y_t = 1.9777 Y_{t-1}$	$+ 1.0223Y_{t-2} + 0.1475Y_{t-3}$	$+ 0.0849Y_{t-4} + 0.9958 e_{t-1}e$	$_{t} \sim IID(0, 15441)$

Table-07-A: Selection of GARCH model

GARCH Model		Mean Model: ARIMA(2,2,1) Variance Model : eGARCH					
GARCH Model		Informati	on Criteria		Distribution		
	Akaike	Bayes	Shibata	Hannan-Quinn			
GARCH(1,1)	12.195	12.244	12.195	12.214			
GARCH(1,2)	12.185	12.240	12.184	12.206	Normal		
GARCH(2,1)	12.103	12.164	12.103	12.127	Normai		
GARCH(2,2)	12.056	12.123	12.055	12.082			
GARCH(1,1)	11.891	11.946	11.891	11.912			
GARCH(1,2)	12.185	12.246	12.185	12.208	Skew-Normal		
GARCH(2,1)	11.893	11.961	11.893	11.919	Skew-Normai		
GARCH(2,2)	12.053	12.126	12.052	12.081			
GARCH(1,1)	11.886	11.941	11.885	11.907			
GARCH(1,2)	11.886	11.948	11.886	11.910	Chardent t		
GARCH(2,1)	11.887	11.955	11.887	11.913	Student-t		
GARCH(2,2)	11.890	11.963	11.889	11.918			

GARCH Model		Mean Model: ARIMA(2,2,1) Variance Model : eGARCH Information Criteria					
GARCH Model		Distribution					
	Akaike	Bayes	Shibata	Hannan-Quinn			
GARCH(1,1)	11.883	11.944	11.883	11.906			
GARCH(1,2)	11.884	11.951	11.883	11.910	Skew-Student-t		
GARCH(2,1)	11.884	11.958	11.884	11.912	Skew-Student-t		
GARCH(2,2)	11.887	11.966	11.886	11.917			
GARCH(1,1)	11.883	11.938	11.883	11.904			
GARCH(1,2)	12.084	12.145	12.084	12.108	Generalized error		
GARCH(2,1)	12.032	12.100	12.032	12.058	Generalized error		
GARCH(2,2)	12.008	12.081	12.008	12.036			
GARCH(1,1)	11.882	11.943	11.882	11.906			
GARCH(1,2)	12.087	12.154	12.086	12.112	Skew- Generalized		
GARCH(2,1)	11.884	11.958	11.884	11.913	error		
GARCH(2,2)	12.010	12.089	12.009	12.040			
GARCH(1,1)	12.092	12.153	12.091	12.115			
GARCH(1,2)	11.882	11.949	11.882	11.908	Inverse Gaussian		
GARCH(2,1)	12.034	12.108	12.034	12.063	iliverse Gaussian		
GARCH(2,2)	12.014	12.093	12.013	12.044			
GARCH(1,1)	12.095	12.162	12.095	12.121			
GARCH(1,2)	12.091	12.164	12.090	12.119	Generalized		
GARCH(2,1)	12.036	12.116	12.036	12.067	Hyperbolic		
GARCH(2,2)	12.014	12.099	12.013	12.047			
GARCH(1,1)	11.881	11.942	11.881	11.905			
GARCH(1,2)	11.882	11.950	11.882	11.908	I. l ? CII		
GARCH(2,1)	11.883	11.956	11.882	11.911	Johnson' SU		
GARCH(2,2)	11.885	11.965	11.885	11.916			

Table-07-B: Selection of GARCH model

a.p.		Mean Model: Variance Mod	ARIMA(2,2,1) el: sGARCH		Conditional		
GARCH Model	_	Information Criteria					
	Akaike	Bayes	Shibata	Hannan-Quinn			
GARCH(1,1)	11.917	11.960	11.917	11.933			
GARCH(1,2)	11.914	11.962	11.913	11.932	Name of		
GARCH(2,1)	11.920	11.969	11.919	11.938	Normal		
GARCH(2,2)	11.916	11.971	11.916	11.937			
GARCH(1,1)	11.912	11.961	11.912	11.931			
GARCH(1,2)	11.908	11.963	11.908	11.930	Skew-Normal		
GARCH(2,1)	11.914	11.970	11.914	11.936	Skew-Normai		
GARCH(2,2)	11.911	11.972	11.910	11.934			
GARCH(1,1)	11.902	11.951	11.902	11.921			
GARCH(1,2)	11.901	11.956	11.900	11.922	Student-t		
GARCH(2,1)	11.905	11.960	11.905	11.926	Student-t		
GARCH(2,2)	11.903	11.964	11.903	11.927			
GARCH(1,1)	11.901	11.956	11.900	11.922			
GARCH(1,2)	11.899	11.960	11.899	11.922	Skew-Student-t		
GARCH(2,1)	11.903	11.964	11.903	11.927	Skew-Student-t		
GARCH(2,2)	11.901	11.969	11.901	11.927			
GARCH(1,1)	11.899	11.948	11.899	11.918			
GARCH(1,2)	11.898	11.953	11.898	11.919	Generalized error		
GARCH(2,1)	11.902	11.957	11.902	11.923	Generalized error		
GARCH(2,2)	11.900	11.962	11.900	11.924			
GARCH(1,1)	11.900	11.955	11.900	11.921			
GARCH(1,2)	11.899	11.960	11.898	11.922	Skew- Generalized		
GARCH(2,1)	11.903	11.964	11.902	11.926	error		
GARCH(2,2)	11.901	11.968	11.901	11.927			

GARCH Model		Conditional			
Grifferi Model		Informati	on Criteria		Distribution
	Akaike	Bayes	Shibata	Hannan-Quinn	
GARCH(1,1)	11.899	11.954	11.899	11.920	
GARCH(1,2)	11.897	11.958	11.897	11.921	Inverse Gaussian
GARCH(2,1)	11.901	11.963	11.901	11.925	iliveise Gaussiali
GARCH(2,2)	11.899	11.967	11.899	11.925	
GARCH(1,1)	11.902	11.963	11.901	11.925	
GARCH(1,2)	11.901	11.968	11.900	11.927	Generalized
GARCH(2,1)	11.904	11.971	11.903	11.930	Hyperbolic
GARCH(2,2)	11.902	11.975	11.901	11.930	
GARCH(1,1)	11.899	11.954	11.899	11.920	
GARCH(1,2)	11.897	11.959	11.897	11.921	Johnson' SU
GARCH(2,1)	11.902	11.963	11.902	11.925	Johnson SU
GARCH(2,2)	11.900	11.967	11.899	11.926	

Table-07-C: Selection of GARCH model

		Mean Model:	ARIMA(2,2,1)		
a		Variance Mode			Conditional
GARCH Model		Distribution			
	Akaike	Bayes	Shibata	Hannan-Quinn	1
GARCH(1,1)	11.914	11.951	11.914	11.928	
GARCH(1,2)	11.911	11.954	11.911	11.927	Normal
GARCH(2,1)	11.917	11.960	11.917	11.933	Normai
GARCH(2,2)	11.913	11.962	11.913	11.932	
GARCH(1,1)	11.909	11.952	11.909	11.926	
GARCH(1,2)	11.906	11.955	11.906	11.925	Cl N 1
GARCH(2,1)	11.912	11.961	11.912	11.931	Skew-Normal
GARCH(2,2)	11.908	11.963	11.908	11.929	
GARCH(1,1)	11.900	11.942	11.900	11.916	
GARCH(1,2)	11.898	11.947	11.898	11.917	C. I.
GARCH(2,1)	11.902	11.951	11.902	11.921	Student-t
GARCH(2,2)	11.900	11.955	11.900	11.922	
GARCH(1,1)	11.898	11.947	11.898	11.917	
GARCH(1,2)	11.896	11.951	11.896	11.917	Skew-Student-t
GARCH(2,1)	11.901	11.956	11.900	11.922	
GARCH(2,2)	11.899	11.960	11.898	11.922	
GARCH(1,1)	11.897	11.939	11.896	11.913	
GARCH(1,2)	11.895	11.944	11.895	11.914	C 1: 1
GARCH(2,1)	11.899	11.948	11.899	11.918	Generalized error
GARCH(2,2)	11.898	11.953	11.898	11.919	
GARCH(1,1)	11.897	11.946	11.897	11.916	
GARCH(1,2)	11.896	11.951	11.896	11.917	Skew- Generalized
GARCH(2,1)	11.900	11.955	11.900	11.921	error
GARCH(2,2)	11.899	11.960	11.898	11.922	
GARCH(1,1)	11.896	11.945	11.896	11.915	
GARCH(1,2)	11.894	11.949	11.894	11.916	
GARCH(2,1)	11.899	11.954	11.899	11.920	Inverse Gaussian
GARCH(2,2)	11.897	11.958	11.896	11.920	
GARCH(1,1)	11.899	11.954	11.898	11.920	
GARCH(1,2)	11.898	11.959	11.898	11.922	Generalized
GARCH(2,1)	11.901	11.962	11.901	11.925	Hyperbolic
GARCH(2,2)	11.899	11.966	11.899	11.925	- 4
GARCH(1,1)	11.897	11.946	11.896	11.915	
GARCH(1,2)	11.895	11.950	11.894	11.916	
GARCH(2,1)	11.900	11.955	11.899	11.921	Johnson' SU
GARCH(2,2)	11.897	11.958	11.897	11.921	

Table-07-D: Selection of GARCH model

		Mean Model : Variance Mode	ARIMA(2,2,1)		Conditional	
GARCH Model		Information Criteria				
	Akaike	Bayes	Shibata	Hannan-Quinn	Distribution	
GARCH(1,1)	11.947	12.002	11.946	11.968		
GARCH(1,2)	11.923	11.984	11.922	11.946	N. 1	
GARCH(2,1)	11.951	12.012	11.951	11.975	Normal	
GARCH(2,2)	11.962	12.029	11.961	11.988		
GARCH(1,1)	11.951	12.012	11.951	11.975		
GARCH(1,2)	11.953	12.021	11.953	11.979	Cl N 1	
GARCH(2,1)	11.949	12.016	11.949	11.975	Skew-Normal	
GARCH(2,2)	11.947	12.021	11.947	11.976		
GARCH(1,1)	11.921	11.982	11.920	11.944		
GARCH(1,2)	11.923	11.991	11.923	11.949	Student-t	
GARCH(2,1)	11.924	11.997	11.923	11.952	Student-t	
GARCH(2,2)	11.960	12.028	11.960	11.986		
GARCH(1,1)	11.960	12.028	11.960	11.986		
GARCH(1,2)	11.919	11.992	11.919	11.947	Skew-Student-t	
GARCH(2,1)	11.919	11.992	11.918	11.947	Skew-Student-t	
GARCH(2,2)	11.908	11.987	11.907	11.939		
GARCH(1,1)	11.907	11.968	11.907	11.931		
GARCH(1,2)	11.921	11.988	11.921	11.947	Generalized error	
GARCH(2,1)	11.925	11.992	11.924	11.951	Generalized error	
GARCH(2,2)	11.908	11.981	11.908	11.936		
GARCH(1,1)	11.919	11.987	11.919	11.945		
GARCH(1,2)	18.604	18.677	18.604	18.632	Skew- Generalized	
GARCH(2,1)	11.927	12.001	11.927	11.955	error	
GARCH(2,2)	11.909	11.988	11.908	11.939		
GARCH(1,1)	11.917	11.984	11.917	11.943		
GARCH(1,2)	11.918	11.991	11.917	11.946	Inverse Gaussian	
GARCH(2,1)	11.907	11.981	11.907	11.936	iliveise Gaussiali	
GARCH(2,2)	11.924	12.003	11.923	11.954		
GARCH(1,1)	11.920	11.993	11.919	11.948		
GARCH(1,2)	11.906	11.986	11.906	11.937	Generalized	
GARCH(2,1)	11.920	12.000	11.920	11.951	Hyperbolic	
GARCH(2,2)	11.909	11.994	11.908	11.942		
GARCH(1,1)	11.919	11.987	11.919	11.945		
GARCH(1,2)	11.915	11.989	11.915	11.943	Johnson' SU	
GARCH(2,1)	11.920	11.993	11.919	11.948	JUHISUH SU	
GARCH(2,2)	11.906	11.986	11.906	11.937		

Table-07-E: Selection of GARCH model

GARCH Model		Conditional Distribution				
	Akaike	Bayes	Shibata	Hannan-Quinn	=	
GARCH(1,1)	11.904	11.953	11.904	11.923		
GARCH(1,2)	11.902	11.957	11.902	11.924	Normal	
GARCH(2,1)	11.904	11.965	11.903	11.927	Normal	
GARCH(2,2)	11.906	11.974	11.906	11.932		
GARCH(1,1)	11.899	11.954	11.899	11.920		
GARCH(1,2)	11.898	11.959	11.897	11.921	Skew-Normal	
GARCH(2,1)	11.900	11.967	11.900	11.926	Skew-Norman	
GARCH(2,2)	11.903	11.976	11.902	11.931		
GARCH(1,1)	11.892	11.948	11.892	11.914		
GARCH(1,2)	11.893	11.954	11.892	11.916	Student-t	
GARCH(2,1)	11.894	11.961	11.894	11.920	Student-t	
GARCH(2,2)	11.897	11.970	11.896	11.925		

GARCH Model		Conditional					
GARCH Model		Information Criteria					
	Akaike	Bayes	Shibata	Hannan-Quinn			
GARCH(1,1)	11.890	11.952	11.890	11.914			
GARCH(1,2)	11.891	11.958	11.890	11.917	Skew-Student-t		
GARCH(2,1)	11.893	11.966	11.892	11.921	Skew-Student-t		
GARCH(2,2)	11.895	11.975	11.895	11.926			
GARCH(1,1)	11.889	11.944	11.889	11.910			
GARCH(1,2)	11.889	11.950	11.889	11.913	Generalized error		
GARCH(2,1)	11.891	11.958	11.890	11.917	Generalized error		
GARCH(2,2)	11.893	11.967	11.893	11.922			
GARCH(1,1)	11.889	11.950	11.889	11.913			
GARCH(1,2)	11.889	11.957	11.889	11.915	Skew- Generalized		
GARCH(2,1)	11.891	11.964	11.891	11.919	error		
GARCH(2,2)	11.894	11.973	11.893	11.924			
GARCH(1,1)	11.889	11.950	11.888	11.912			
GARCH(1,2)	11.889	11.956	11.889	11.915	Inverse Gaussian		
GARCH(2,1)	11.891	11.964	11.890	11.919	iliverse Gaussian		
GARCH(2,2)	11.894	11.973	11.893	11.925			
GARCH(1,1)	11.891	11.958	11.891	11.917			
GARCH(1,2)	11.891	11.965	11.891	11.920	Generalized		
GARCH(2,1)	11.893	11.973	11.893	11.924	Hyperbolic		
GARCH(2,2)	11.896	11.982	11.895	11.929			
GARCH(1,1)	11.889	11.950	11.889	11.913			
GARCH(1,2)	11.889	11.957	11.889	11.915	Johnson' SU		
GARCH(2,1)	11.892	11.965	11.891	11.920	Johnson SU		
GARCH(2,2)	11.894	11.974	11.894	11.925			

Table-07-F: Selection of the Best GARCH model

Information Criteria	Best GARCH model				
Akaike	Mean Model:	ARIMA(2,2,1)			
[7A]	Variance Model:	eGARCH(1,1)			
	Conditional Distribution:	Johnson's SU Distribution			
Bayes	Mean Model:	ARIMA(2,2,1)			
[7C]	Variance Model:	iGARCH(1,1)			
	Conditional Distribution:	Generalized Error Distribution			
Shibata	Mean Model:	ARIMA(2,2,1)			
[7A]	Variance Model:	eGARCH(1,1)			
	Conditional Distribution:	Johnson's SU Distribution			
Hannan-Quinn	Mean Model:	ARIMA(2,2,1)			
[7A]	Variance Model:	eGARCH(1,1)			
	Conditional Distribution:	Generalized Error Distribution			

Table-07-G: Selection of the Best GARCH model

GARCH Model: eGARCH(1,1)				GARCH Model: iGARCH(1,1)					
Mean Model: ARIMA(2,0,1)				Mean Model: ARIMA(2,0,1)					
Distribution: Johnson's SU Distribution				Distribution: Generalized Error Distribution					
Parameters	Estimate	Std. Error	t value	Pr(> t)	Parameters	Estimate	Std. Error	t value	Pr (> t)
mu	-0.079855	0.059782	-1.33578	0.181621	mu	0.062389	0.087870	0.71002	0.477692
ar1	-0.023521	0.035048	-0.67111	0.502149	ar1	-0.020015	0.038424	-0.52090	0.602439
ar2	-0.076442	0.037015	-2.06515	0.038909	ar2	-0.063262	0.039724	-1.59253	0.111267
ma1	-0.952008	0.016558	-57.49660	0.000000	ma1	-0.964468	0.005130	-188.00479	0.000000
omega	0.238747	0.008339	28.62905	0.000000	omega	156.964211	74.158592	2.11660	0.034294
alpha1	-0.101680	0.021480	-4.73364	0.000002	alpha1	0.175535	0.033447	5.24806	0.000000
beta1	0.973688	0.000073	13382.76	0.000000	beta1	0.824465	NA	NA	NA
gamma1	0.265833	0.007444	35.71069	0.000000	shape	1.449352	0.110145	13.15861	0.000000
skew	-0.522605	0.237621	-2.19932	0.027855					
shape	2.407700	0.161749	14.88537	0.000000					

Appendix-B

Figure-01: Histogram of Daily General Index of DSE from 1/3/2010 to 7/31/2013

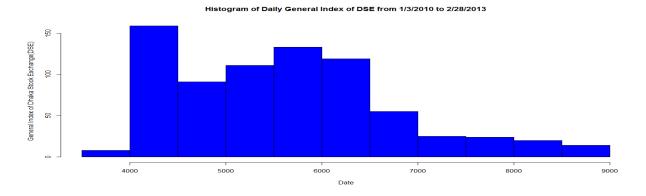


Figure-02: Distribution of Daily General Index of DSE from 1/3/2010 to 7/31/2013

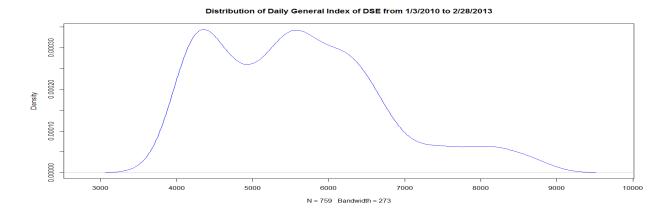


Figure-03: Quantile plot of Daily General Index of DSE from 1/3/2010 to 7/31/2013

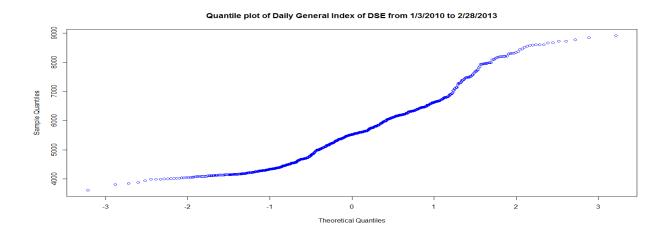


Figure-04: Time plot of Daily General Index of DSE from 1/3/2010 to 7/31/2013

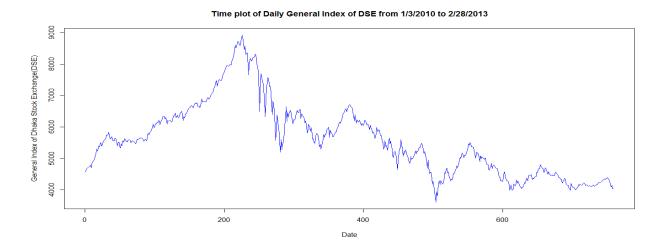


Figure-05: ACF plot of Daily General Index of DSE from 1/3/2010 to 7/31/2013.

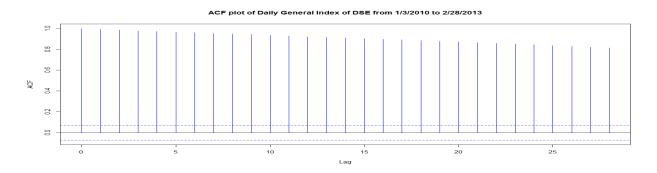


Figure-06: PACF plot of Daily General Index of DSE from 1/3/2010 to 7/31/2013.

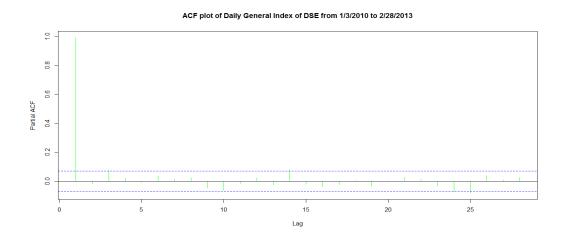


Figure-07: Time plot of Daily General Index of DSE at first difference

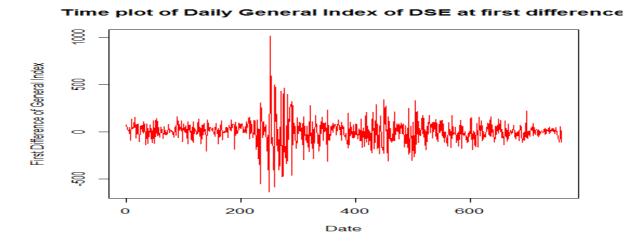


Figure-08: ACF plot of Daily General Index of DSE at first difference

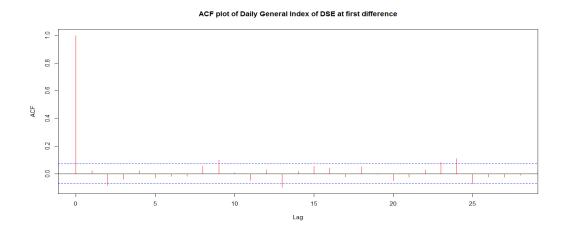


Figure-09: PACF plot of Daily General Index of DSE at first difference

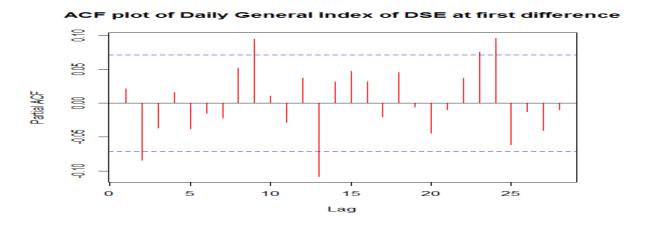


Figure-10: Time plot of Daily General Index of DSE at second difference

Time plot of Daily General Index of DSE at second different

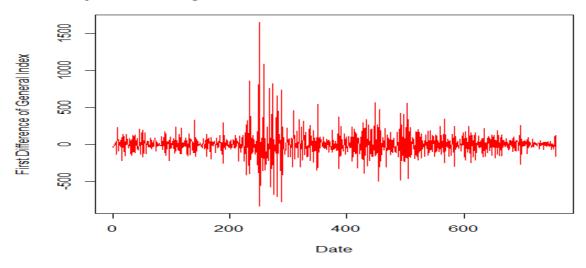


Figure-11: ACF plot of Daily General Index of DSE at second difference

ACF plot of Daily General Index of DSE at second difference

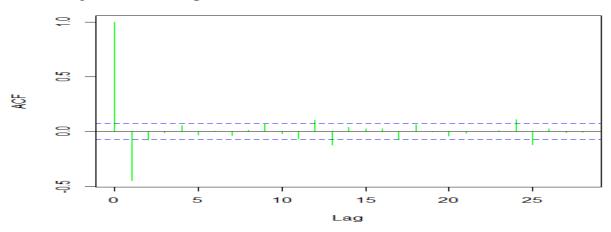


Figure-12: PACF plot of Daily General Index of DSE at second difference

PACF plot of Daily General Index of DSE at second differen

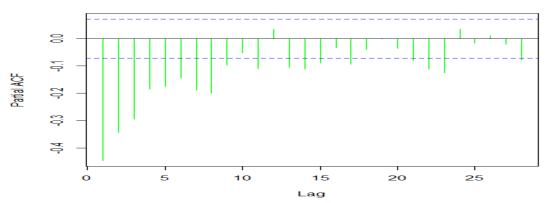


Figure-13: Forecasted Daily General Index of DSE by ARIMA (2, 2, 1) Model

Time plot of Forsecated and original Daily General Index of I

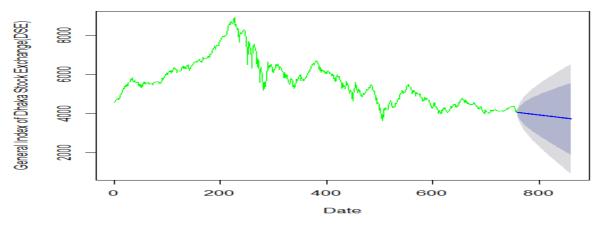


Figure-14: Residual plot of ARIMA (2,2,1) Model

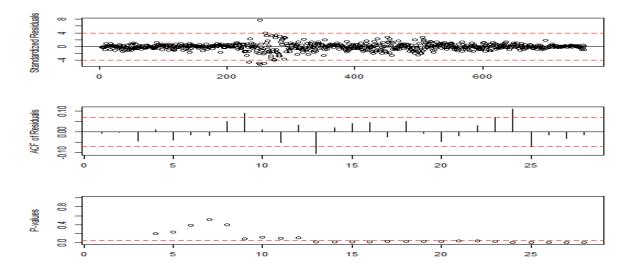


Figure-15: Series with 2 Conditional SD Superimposed for eGARCH (left) and iGARCH(right)

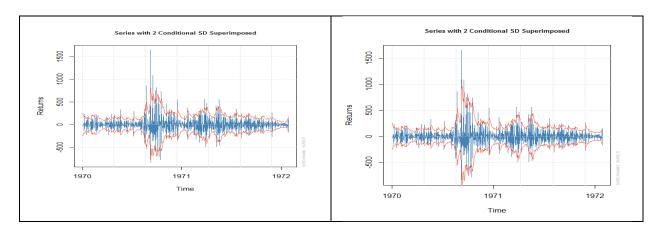


Figure-16: Series with 1% VaR Limits for eGARCH (left) and iGARCH (right)

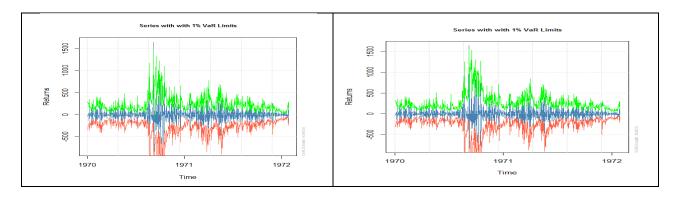


Figure-17: Conditional SD (vs |returns|) for eGARCH (left) and iGARCH (right)

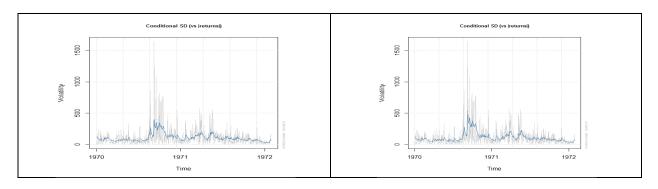


Figure-18: empirical density of standardized residuals for eGARCH (left) and iGARCH (right)

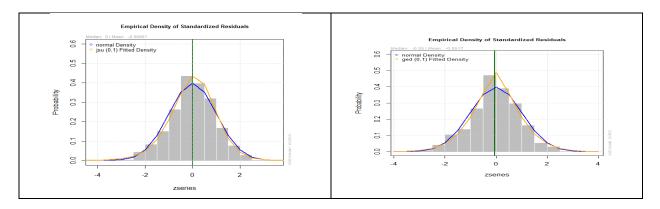


Figure-19: QQ-Plot of Standardized Residuals for eGARCH (left) and iGARCH (right)

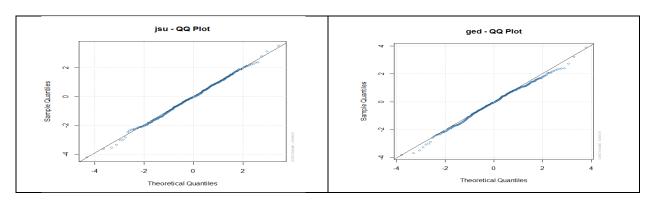


Figure-20: ACF of standardized residuals for eGARCH (left) and iGARCH (right)

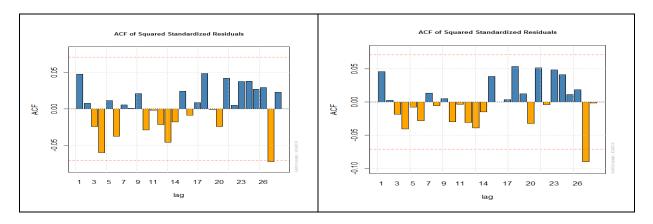


Figure- 21: forecasted series and volatility limit value by eGARCH (1, 1)

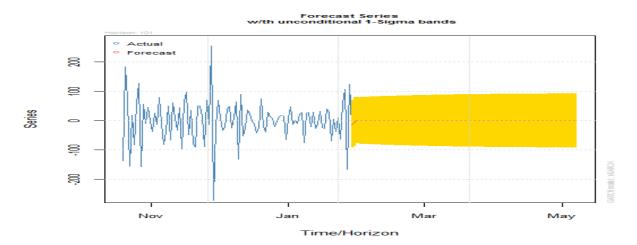


Figure-22: Simulated eGARCH model

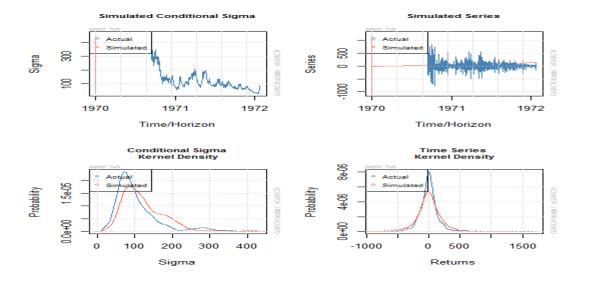


Figure-23: Simulated density plot of eGARCH model

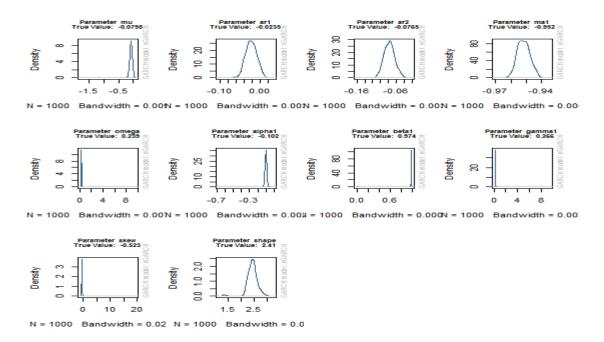


Figure-24: Simulated eGARCH model statsplot

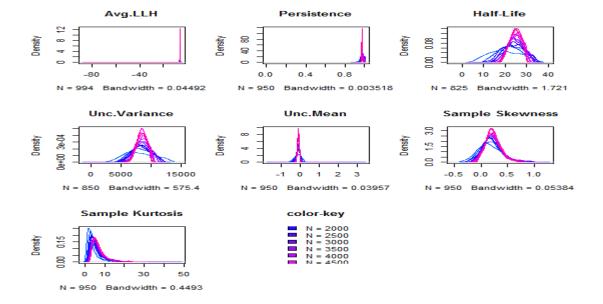


Figure-25: Plot of the Root Mean Square Error (RMSE) Rate of Change of true versus estimated parameters in relation to the data size

