BACK PROPAGATION

Machine Learning

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Submitted To

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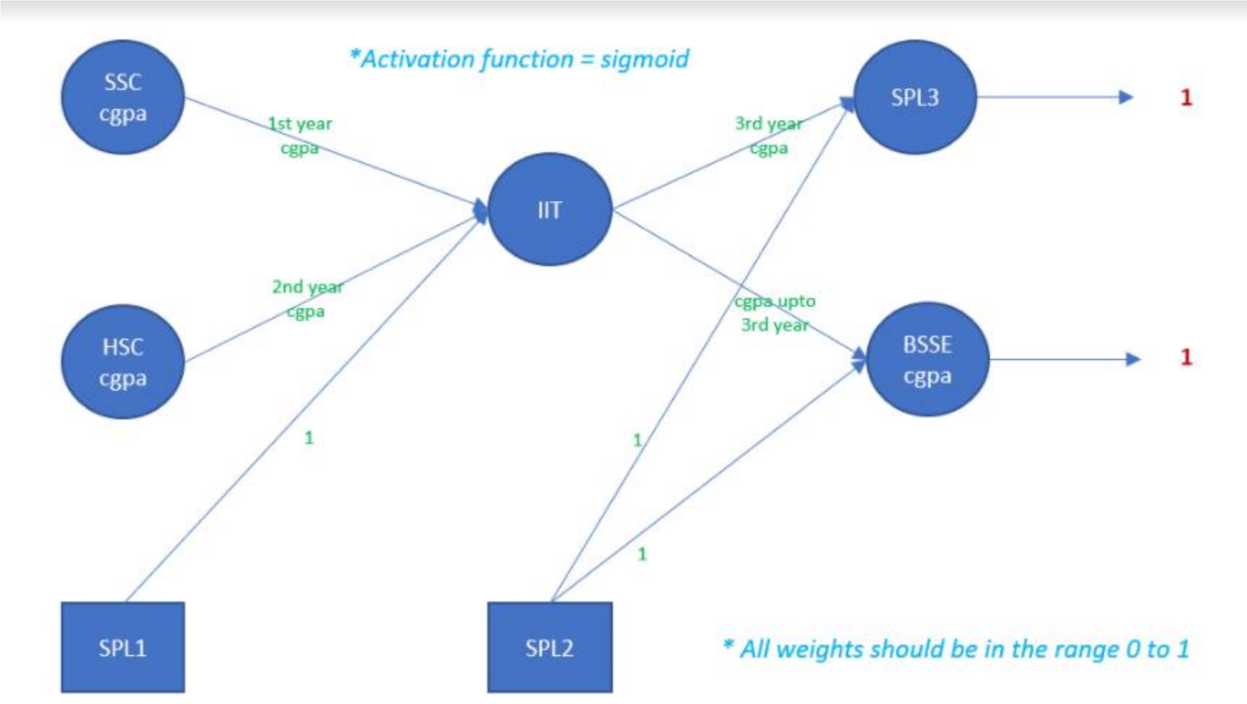
Problem

The problem is to update values for all parameters (weights and biases) using two iterations of back-propagation.



Solution

Let,

SSC cgpa = $\boldsymbol{i1}$ = 1, HSC cgpa = $\boldsymbol{i2}$ = 1,

SPL1 = $\boldsymbol{b1}$ = 0.94, SPL2 = $\boldsymbol{b2}$ = 1,

1st year cgpa = $\boldsymbol{w1}$ = 0.98, 2nd year cgpa = $\boldsymbol{w2}$ = 0.96, 3rd year cgpa = $\boldsymbol{w3}$ = 0.95,

Cgpa upto 3rd year = $\boldsymbol{w4}$ = 0.955

IIT = $\boldsymbol{h}$, SPL3 = $\boldsymbol{o1}$, BSSE cgpa = $\boldsymbol{o2}$

$\boldsymbol{target_{o1}}$ = 1, $\boldsymbol{target_{o2}}$ = 1

Learning rate, $\boldsymbol{\eta}$ = 0.01

Here, all the values have been converted in the range of 0 to 1.

1^{st} Iteration

Forward Pass

$\boldsymbol{net_h = i1 * w1 + i2 * w2 + b1 * 1} =$ (1 * 0.98) + (1 * 0.96) + (0.94 * 1) = 2.88

$$\boldsymbol{out_h = \frac{1}{1+e^{-net_h}}} = \frac{1}{1 + e^{-2.88}} = 0.947$$

$\boldsymbol{net_{o1} = out_h * w3 + b2 * 1}$ = 0.947 * 0.95 + 1 * 1 = 1.9

$\boldsymbol{net_{o2} = out_h * w4 + b2 * 1}$ = 0.947 * 0.955 + 1 * 1 = 1.904

$$\boldsymbol{out_{o1} = \frac{1}{1+e^{-net_{o1}}}} = \frac{1}{1 + e^{-1.9}} = 0.87$$

$$\boldsymbol{out_{o2} = \frac{1}{1+e^{-net_{o2}}}} = \frac{1}{1 + e^{-1.904}} = 0.87$$

Calculating Error

$$\boldsymbol{E_{o1} = (target_{o1} - out_{01})^2}$$

$$\boldsymbol{E_{o2} = (target_{o2} - out_{02})^2}$$

$$\begin{aligned}\boldsymbol{E_{total}} &= \boldsymbol{E_{01} + E_{o2}} \\ &= \boldsymbol{(target_{o1} - out_{01})^2 + (target_{o2} - out_{02})^2} \\ &= (1 - 0.87)^2 + (1 - 0.87)^2 = 0.0338\end{aligned}$$

Backward Pass

1. Adjusting w3:

Considering w3 to know how much a change in w3 affects the total error.

$$\boldsymbol{\frac{\partial E_{total}}{\partial w3} = \frac{\partial E_{total}}{\partial out_{o1}} * \frac{\partial out_{o1}}{\partial net_{o1}} * \frac{\partial net_{o1}}{\partial w3}}$$

Now,

$$\boldsymbol{\frac{\partial E_{total}}{\partial out_{o1}} = -2(target_{o1} - out_{o1})} = -2(1 - 0.87) = -0.26$$

$$\boldsymbol{\frac{\partial out_{o1}}{\partial net_{o1}} = out_{o1}(1 - out_{o1})} = 0.87(1 - 0.87) = 0.1131$$

$$\boldsymbol{\frac{\partial net_{o1}}{\partial w3} = out_h} = 0.947$$

$$\therefore \boldsymbol{\frac{\partial E_{total}}{\partial w3}} = (-0.26) * 0.1131 * 0.947 = -0.0278$$

So we get, $\boldsymbol{w3^+} = w3 - \eta * \frac{\partial E_{total}}{\partial w3} = 0.95 - 0.01 * (-0.0278) = 0.9502$

2. Adjusting w4:

Considering w4 to know how much a change in w4 affects the total error.

$$\frac{\partial \boldsymbol{E_{total}}}{\partial \boldsymbol{w4}} = \frac{\partial \boldsymbol{E_{total}}}{\partial \boldsymbol{out_{o2}}} * \frac{\partial \boldsymbol{out_{o2}}}{\partial \boldsymbol{net_{o2}}} * \frac{\partial \boldsymbol{net_{o2}}}{\partial \boldsymbol{w4}}$$

Now,

$$\frac{\partial \boldsymbol{E_{total}}}{\partial \boldsymbol{out_{o2}}} = -\boldsymbol{2}(\boldsymbol{target_{o2}} - \boldsymbol{out_{o2}}) = -2(1 - 0.87) = -0.26$$

$$\frac{\partial \boldsymbol{out_{o2}}}{\partial \boldsymbol{net_{o2}}} = \boldsymbol{out_{o2}}(\boldsymbol{1} - \boldsymbol{out_{o2}}) = 0.87(1 - 0.87) = 0.1131$$

$$\frac{\partial \boldsymbol{net_{o1}}}{\partial \boldsymbol{w4}} = \boldsymbol{out_h} = 0.947$$

$$\therefore \frac{\partial \boldsymbol{E_{total}}}{\partial \boldsymbol{w4}} = (-0.26) * 0.1131 * 0.947 = -0.0278$$

So we get, $\boldsymbol{w4^+} = w4 - \eta * \frac{\partial E_{total}}{\partial w4} = 0.955 - 0.01 * (-0.0278) = 0.9553$

3. Adjusting b2:

Considering b2 to know how much a change in b2 affects the total error.

$$\frac{\partial \boldsymbol{E_{total}}}{\partial \boldsymbol{b2}} = \frac{\partial \boldsymbol{E_{o1}}}{\partial \boldsymbol{b2}} + \frac{\partial \boldsymbol{E_{o2}}}{\partial \boldsymbol{b2}} = \left(\frac{\partial \boldsymbol{E_{o1}}}{\partial \boldsymbol{out_{o1}}} * \frac{\partial \boldsymbol{out_{o1}}}{\partial \boldsymbol{net_{o1}}} * \frac{\partial \boldsymbol{net_{o1}}}{\partial \boldsymbol{b2}}\right) + \left(\frac{\partial \boldsymbol{E_{o2}}}{\partial \boldsymbol{out_{o2}}} * \frac{\partial \boldsymbol{out_{o2}}}{\partial \boldsymbol{net_{o2}}} * \frac{\partial \boldsymbol{net_{o2}}}{\partial \boldsymbol{b2}}\right)$$

Now,

$$\frac{\partial \boldsymbol{E_{o1}}}{\partial \boldsymbol{out_{o1}}} = \frac{\partial(\boldsymbol{target_{o1}} - \boldsymbol{out_{o1}})}{\partial \boldsymbol{out_{o1}}} = -1$$

$$\frac{\partial \boldsymbol{out_{o1}}}{\partial \boldsymbol{net_{o1}}} = \boldsymbol{out_{o1}}(\boldsymbol{1} - \boldsymbol{out_{o1}}) = 0.1131$$

$$\frac{\partial \boldsymbol{net_{o1}}}{\partial \boldsymbol{b2}} = \frac{\partial(\boldsymbol{out_h} * \boldsymbol{w3} + \boldsymbol{b2} * \boldsymbol{1})}{\partial \boldsymbol{b2}} = 1$$

$$\frac{\partial \boldsymbol{E_{o2}}}{\partial \boldsymbol{out_{o2}}} = \frac{\partial(\boldsymbol{target_{o2}} - \boldsymbol{out_{o2}})}{\partial \boldsymbol{out_{o2}}} = -1$$

$$\frac{\partial \boldsymbol{out_{o2}}}{\partial \boldsymbol{net_{o2}}} = \boldsymbol{out_{o2}}(\boldsymbol{1} - \boldsymbol{out_{o2}}) = 0.1131$$

$$\frac{\partial \boldsymbol{net_{o2}}}{\partial \boldsymbol{b2}} = \frac{\partial(\boldsymbol{out_h} * \boldsymbol{w4} + \boldsymbol{b2} * \boldsymbol{1})}{\partial \boldsymbol{b2}} = 1$$

$$\therefore \frac{\partial \boldsymbol{E_{total}}}{\partial \boldsymbol{b2}} = (-1 * 0.1131 * 1) + (-1 * 0.1131 * 1) = -0.2262$$

So we get, $\boldsymbol{b2^+} = b2 - \eta * \frac{\partial E_{total}}{\partial b2} = 1 - 0.01 * (-0.2262) = 1.002$

4. Adjusting w1

Considering w1 to know how much a change in w1 affects the total error.

$$\frac{\partial E_{total}}{\partial w1} = \frac{\partial E_{total}}{\partial out_h} * \frac{\partial out_h}{\partial net_h} * \frac{\partial net_h}{\partial w1} = \left(\frac{\partial E_{o1}}{\partial out_h} + \frac{\partial E_{o2}}{\partial out_h}\right) * \frac{\partial out_h}{\partial net_h} * \frac{\partial net_h}{\partial w1}$$

Here,

$$\frac{\partial E_{o1}}{\partial out_h} = \frac{\partial E_{o1}}{\partial out_{o1}} * \frac{\partial out_{o1}}{\partial net_{o1}} * \frac{\partial net_{o1}}{\partial out_h} = -1 * 0.1131 * w3 = -0.1131 * 0.95 = -0.107$$

$$\frac{\partial E_{o2}}{\partial out_h} = \frac{\partial E_{o2}}{\partial out_{o2}} * \frac{\partial out_{o2}}{\partial net_{o2}} * \frac{\partial net_{o2}}{\partial out_h} = -1 * 0.1131 * w4 = -0.1131 * 0.955 = -0.108$$

$$\frac{\partial out_h}{\partial net_h} = out_h(1 - out_h) = 0.947(1 - 0.947) = 0.0502$$

$$\frac{\partial net_h}{\partial w1} = i1 = 1$$

$$\therefore \frac{\partial E_{total}}{\partial w1} = (-0.107 - 0.108) * 0.0502 * 1 = -0.011$$

So we get, $\boldsymbol{w1^+} = w1 - \eta * \frac{\partial E_{total}}{\partial w1} = 0.98 - 0.01 * (-0.011) = 0.98011$

5. Adjusting w2

Considering w2 to know how much a change in w2 affects the total error.

$$\frac{\partial E_{total}}{\partial w2} = \frac{\partial E_{total}}{\partial out_h} * \frac{\partial out_h}{\partial net_h} * \frac{\partial net_h}{\partial w2} = \left(\frac{\partial E_{o1}}{\partial out_h} + \frac{\partial E_{o2}}{\partial out_h}\right) * \frac{\partial out_h}{\partial net_h} * \frac{\partial net_h}{\partial w2}$$

Here,

$$\frac{\partial E_{o1}}{\partial out_h} = \frac{\partial E_{o1}}{\partial out_{o1}} * \frac{\partial out_{o1}}{\partial net_{o1}} * \frac{\partial net_{o1}}{\partial out_h} = -1 * 0.1131 * w3 = -0.1131 * 0.95 = -0.107$$

$$\frac{\partial E_{o2}}{\partial out_h} = \frac{\partial E_{o2}}{\partial out_{o2}} * \frac{\partial out_{o2}}{\partial net_{o2}} * \frac{\partial net_{o2}}{\partial out_h} = -1 * 0.1131 * w4 = -0.1131 * 0.955 = -0.108$$

$$\frac{\partial out_h}{\partial net_h} = out_h(1 - out_h) = 0.947(1 - 0.947) = 0.0502$$

$$\frac{\partial net_h}{\partial w2} = i2 = 1$$

$$\therefore \frac{\partial E_{total}}{\partial w2} = (-0.107 - 0.108) * 0.0502 * 1 = -0.011$$

So we get, $\boldsymbol{w2^+} = w2 - \eta * \frac{\partial E_{total}}{\partial w2} = 0.96 - 0.01 * (-0.011) = 0.96011$

6. Adjusting b1

Considering b1 to know how much a change in b1 affects the total error.

$$\frac{\partial \boldsymbol{E_{total}}}{\partial \boldsymbol{b1}} = \frac{\partial \boldsymbol{E_{total}}}{\partial \boldsymbol{out_h}} * \frac{\partial \boldsymbol{out_h}}{\partial \boldsymbol{net_h}} * \frac{\partial \boldsymbol{net_h}}{\partial \boldsymbol{b1}} = \left(\frac{\partial \boldsymbol{E_{o1}}}{\partial \boldsymbol{out_h}} + \frac{\partial \boldsymbol{E_{o2}}}{\partial \boldsymbol{out_h}}\right) * \frac{\partial \boldsymbol{out_h}}{\partial \boldsymbol{net_h}} * \frac{\partial \boldsymbol{net_h}}{\partial \boldsymbol{b1}}$$

Here,

$$\frac{\partial \boldsymbol{E_{o1}}}{\partial \boldsymbol{out_h}} = \frac{\partial \boldsymbol{E_{o1}}}{\partial \boldsymbol{out_{o1}}} * \frac{\partial \boldsymbol{out_{o1}}}{\partial \boldsymbol{net_{o1}}} * \frac{\partial \boldsymbol{net_{o1}}}{\partial \boldsymbol{out_h}} = -1 * 0.1131 * w3 = -0.1131 * 0.95 = -0.107$$

$$\frac{\partial \boldsymbol{E_{o2}}}{\partial \boldsymbol{out_h}} = \frac{\partial \boldsymbol{E_{o2}}}{\partial \boldsymbol{out_{o2}}} * \frac{\partial \boldsymbol{out_{o2}}}{\partial \boldsymbol{net_{o2}}} * \frac{\partial \boldsymbol{net_{o2}}}{\partial \boldsymbol{out_h}} = -1 * 0.1131 * w4 = -0.1131 * 0.955 = -0.108$$

$$\frac{\partial \boldsymbol{out_h}}{\partial \boldsymbol{net_h}} = \boldsymbol{out_h}(\boldsymbol{1} - \boldsymbol{out_h}) = 0.947(1 - 0.947) = 0.0502$$

$$\frac{\partial \boldsymbol{net_h}}{\partial \boldsymbol{b1}} = 1$$

$$\therefore \frac{\partial \boldsymbol{E_{total}}}{\partial \boldsymbol{b1}} = (-0.107 - 0.108) * 0.0502 * 1 = -0.0107$$

So we get, $\boldsymbol{b1}^{+} = b1 - \eta * \frac{\partial \boldsymbol{E_{total}}}{\partial \boldsymbol{b1}} = 0.94 - 0.01 * (-0.0107) = 0.9401$

1st Iteration Adjusted Values

After 1st iteration, we get the adjusted values as below:

SSC cgpa = ***i1*** = 1, HSC cgpa = ***i2*** = 1,

SPL1 = ***b1*** = 0.9401, SPL2 = ***b2*** = 1.002,

1st year cgpa = ***w1*** = 0.98011, 2nd year cgpa = ***w2*** = 0.96011, 3rd year cgpa = ***w3*** = 0.9502,

Cgpa upto 3rd year = ***w4*** = 0.9553

IIT = ***h***, SPL3 = ***o1***, BSSE cgpa = ***o2***

$\boldsymbol{target_{o1}}$ = 1, $\boldsymbol{target_{o2}}$ = 1

Learning rate, $\boldsymbol{\eta}$ = 0.01

These values will be used to adjust the weights and biases for 2nd iteration.

2^{nd} Iteration

Forward Pass

$\boldsymbol{net_h = i1 * w1 + i2 * w2 + b1 * 1} =$ (1 * 0.98011) + (1 * 0.96011) + (0.9401 * 1) = 2.88032

$\boldsymbol{out_h} = \frac{\boldsymbol{1}}{\boldsymbol{1 + e^{-net_h}}} = \frac{1}{1 + e^{-2.88032}} = 0.947$

$\boldsymbol{net_{o1} = out_h * w3 + b2 * 1}$ = 0.947 * 0.9502 + 1.002 * 1 = 1.902

$\boldsymbol{net_{o2} = out_h * w4 + b2 * 1}$ = 0.947 * 0.9553 + 1.002 * 1 = 1.906

$\boldsymbol{out_{o1}} = \frac{\boldsymbol{1}}{\boldsymbol{1+e^{-net_{o1}}}} = \frac{1}{1 + e^{-1.902}} = 0.8701$

$\boldsymbol{out_{o2}} = \frac{\boldsymbol{1}}{\boldsymbol{1 + e^{-net_{o2}}}} = \frac{1}{1 + e^{-1.906}} = 0.871$

Calculating Error

$\boldsymbol{E_{o1} = (target_{o1} - out_{01})^2}$

$\boldsymbol{E_{o2} = (target_{o2} - out_{02})^2}$

$$\begin{aligned}\boldsymbol{E_{total}} &= \boldsymbol{E_{01} + E_{o2}} \\ &= \boldsymbol{(target_{o1} - out_{01})^2 + (target_{o2} - out_{02})^2} \\ &= (1 - 0.8701)^2 + (1 - 0.871)^2 = 0.03\end{aligned}$$

Backward Pass

1. Adjusting w3:

Considering w3 to know how much a change in w3 affects the total error.

$$\boldsymbol{\frac{\partial E_{total}}{\partial w3} = \frac{\partial E_{total}}{\partial out_{o1}} * \frac{\partial out_{o1}}{\partial net_{o1}} * \frac{\partial net_{o1}}{\partial w3}}$$

Now,

$$\boldsymbol{\frac{\partial E_{total}}{\partial out_{o1}} = -2(target_{o1} - out_{o1})} = -2(1 - 0.8701) = -0.26$$

$$\boldsymbol{\frac{\partial out_{o1}}{\partial net_{o1}} = out_{o1}(1 - out_{o1})} = 0.8701(1 - 0.8701) = 0.113$$

$$\boldsymbol{\frac{\partial net_{o1}}{\partial w3} = out_h} = 0.947$$

$$\therefore \boldsymbol{\frac{\partial E_{total}}{\partial w3}} = (-0.26) * 0.113 * 0.947 = -0.03$$

So we get, $\boldsymbol{w3^+} = w3 - \eta * \frac{\partial E_{total}}{\partial w3} = 0.9502 - 0.01 * (-0.03) = 0.9505$

2. Adjusting w4:

Considering w4 to know how much a change in w4 affects the total error.

$$\frac{\partial \boldsymbol{E_{total}}}{\partial \boldsymbol{w4}} = \frac{\partial \boldsymbol{E_{total}}}{\partial \boldsymbol{out_{o2}}} * \frac{\partial \boldsymbol{out_{o2}}}{\partial \boldsymbol{net_{o2}}} * \frac{\partial \boldsymbol{net_{o2}}}{\partial \boldsymbol{w4}}$$

Now,

$$\frac{\partial \boldsymbol{E_{total}}}{\partial \boldsymbol{out_{o2}}} = -\boldsymbol{2}(\boldsymbol{target_{o2}} - \boldsymbol{out_{o2}}) = -2(1-0.871) = -0.258$$

$$\frac{\partial \boldsymbol{out_{o2}}}{\partial \boldsymbol{net_{o2}}} = \boldsymbol{out_{o2}}(\boldsymbol{1} - \boldsymbol{out_{o2}}) = 0.871(1-0.871) = 0.11$$

$$\frac{\partial \boldsymbol{net_{o1}}}{\partial \boldsymbol{w4}} = \boldsymbol{out_h} = 0.947$$

$$\therefore \frac{\partial \boldsymbol{E_{total}}}{\partial \boldsymbol{w4}} = (-0.258) * 0.11 * 0.947 = -0.0268$$

So we get, $\boldsymbol{w4^+} = w4 - \eta * \frac{\partial E_{total}}{\partial w4} = 0.9553 - 0.01 * (-0.0268) = 0.96$

3. Adjusting b2:

Considering b2 to know how much a change in b2 affects the total error.

$$\frac{\partial \boldsymbol{E_{total}}}{\partial \boldsymbol{b2}} = \frac{\partial \boldsymbol{E_{o1}}}{\partial \boldsymbol{b2}} + \frac{\partial \boldsymbol{E_{o2}}}{\partial \boldsymbol{b2}} = \left(\frac{\partial \boldsymbol{E_{o1}}}{\partial \boldsymbol{out_{o1}}} * \frac{\partial \boldsymbol{out_{o1}}}{\partial \boldsymbol{net_{o1}}} * \frac{\partial \boldsymbol{net_{o1}}}{\partial \boldsymbol{b2}}\right) + \left(\frac{\partial \boldsymbol{E_{o2}}}{\partial \boldsymbol{out_{o2}}} * \frac{\partial \boldsymbol{out_{o2}}}{\partial \boldsymbol{net_{o2}}} * \frac{\partial \boldsymbol{net_{o2}}}{\partial \boldsymbol{b2}}\right)$$

Now,

$$\frac{\partial \boldsymbol{E_{o1}}}{\partial \boldsymbol{out_{o1}}} = \frac{\partial(\boldsymbol{target_{o1}} - \boldsymbol{out_{o1}})}{\partial \boldsymbol{out_{o1}}} = -1$$

$$\frac{\partial \boldsymbol{out_{o1}}}{\partial \boldsymbol{net_{o1}}} = \boldsymbol{out_{o1}}(\boldsymbol{1} - \boldsymbol{out_{o1}}) = 0.113$$

$$\frac{\partial \boldsymbol{net_{o1}}}{\partial \boldsymbol{b2}} = \frac{\partial(\boldsymbol{out_h} * \boldsymbol{w3} + \boldsymbol{b2} * \boldsymbol{1})}{\partial \boldsymbol{b2}} = 1$$

$$\frac{\partial \boldsymbol{E_{o2}}}{\partial \boldsymbol{out_{o2}}} = \frac{\partial(\boldsymbol{target_{o2}} - \boldsymbol{out_{o2}})}{\partial \boldsymbol{out_{o2}}} = -1$$

$$\frac{\partial \boldsymbol{out_{o2}}}{\partial \boldsymbol{net_{o2}}} = \boldsymbol{out_{o2}}(\boldsymbol{1} - \boldsymbol{out_{o2}}) = 0.11$$

$$\frac{\partial \boldsymbol{net_{o2}}}{\partial \boldsymbol{b2}} = \frac{\partial(\boldsymbol{out_h} * \boldsymbol{w4} + \boldsymbol{b2} * \boldsymbol{1})}{\partial \boldsymbol{b2}} = 1$$

$$\therefore \frac{\partial \boldsymbol{E_{total}}}{\partial \boldsymbol{b2}} = (-1 * 0.113 * 1) + (-1 * 0.11 * 1) = -0.22$$

So we get, $\boldsymbol{b2^+} = b2 - \eta * \frac{\partial E_{total}}{\partial b2} = 1.002 - 0.01 * (-0.22) = 1.004$

4. Adjusting w1

Considering w1 to know how much a change in w1 affects the total error.

$$\frac{\partial \boldsymbol{E_{total}}}{\partial \boldsymbol{w1}} = \frac{\partial \boldsymbol{E_{total}}}{\partial \boldsymbol{out_h}} * \frac{\partial \boldsymbol{out_h}}{\partial \boldsymbol{net_h}} * \frac{\partial \boldsymbol{net_h}}{\partial \boldsymbol{w1}} = \left(\frac{\partial \boldsymbol{E_{o1}}}{\partial \boldsymbol{out_h}} + \frac{\partial \boldsymbol{E_{o2}}}{\partial \boldsymbol{out_h}}\right) * \frac{\partial \boldsymbol{out_h}}{\partial \boldsymbol{net_h}} * \frac{\partial \boldsymbol{net_h}}{\partial \boldsymbol{w1}}$$

Here,

$$\frac{\partial \boldsymbol{E_{o1}}}{\partial \boldsymbol{out_h}} = \frac{\partial \boldsymbol{E_{o1}}}{\partial \boldsymbol{out_{o1}}} * \frac{\partial \boldsymbol{out_{o1}}}{\partial \boldsymbol{net_{o1}}} * \frac{\partial \boldsymbol{net_{o1}}}{\partial \boldsymbol{out_h}} = -1 * 0.113 * w3 = -0.113 * 0.9502 = -0.11$$

$$\frac{\partial \boldsymbol{E_{o2}}}{\partial \boldsymbol{out_h}} = \frac{\partial \boldsymbol{E_{o2}}}{\partial \boldsymbol{out_{o2}}} * \frac{\partial \boldsymbol{out_{o2}}}{\partial \boldsymbol{net_{o2}}} * \frac{\partial \boldsymbol{net_{o2}}}{\partial \boldsymbol{out_h}} = -1 * 0.11 * w4 = -0.11 * 0.9553 = -0.105$$

$$\frac{\partial \boldsymbol{out_h}}{\partial \boldsymbol{net_h}} = \boldsymbol{out_h}(\boldsymbol{1} - \boldsymbol{out_h}) = 0.947(1 - 0.947) = 0.0502$$

$$\frac{\partial \boldsymbol{net_h}}{\partial \boldsymbol{w1}} = i1 = 1$$

$$\therefore \frac{\partial \boldsymbol{E_{total}}}{\partial \boldsymbol{w1}} = (-0.11 - 0.105) * 0.0502 * 1 = -0.01$$

So we get, $\boldsymbol{w1^+} = w1 - \eta * \frac{\partial \boldsymbol{E_{total}}}{\partial \boldsymbol{w1}} = 0.98011 - 0.01 * (-0.01) = 0.9802$

5. Adjusting w2

Considering w2 to know how much a change in w2 affects the total error.

$$\frac{\partial \boldsymbol{E_{total}}}{\partial \boldsymbol{w2}} = \frac{\partial \boldsymbol{E_{total}}}{\partial \boldsymbol{out_h}} * \frac{\partial \boldsymbol{out_h}}{\partial \boldsymbol{net_h}} * \frac{\partial \boldsymbol{net_h}}{\partial \boldsymbol{w2}} = \left(\frac{\partial \boldsymbol{E_{o1}}}{\partial \boldsymbol{out_h}} + \frac{\partial \boldsymbol{E_{o2}}}{\partial \boldsymbol{out_h}}\right) * \frac{\partial \boldsymbol{out_h}}{\partial \boldsymbol{net_h}} * \frac{\partial \boldsymbol{net_h}}{\partial \boldsymbol{w2}}$$

Here,

$$\frac{\partial \boldsymbol{E_{o1}}}{\partial \boldsymbol{out_h}} = \frac{\partial \boldsymbol{E_{o1}}}{\partial \boldsymbol{out_{o1}}} * \frac{\partial \boldsymbol{out_{o1}}}{\partial \boldsymbol{net_{o1}}} * \frac{\partial \boldsymbol{net_{o1}}}{\partial \boldsymbol{out_h}} = -1 * 0.113 * w3 = -0.113 * 0.9502 = -0.11$$

$$\frac{\partial \boldsymbol{E_{o2}}}{\partial \boldsymbol{out_h}} = \frac{\partial \boldsymbol{E_{o2}}}{\partial \boldsymbol{out_{o2}}} * \frac{\partial \boldsymbol{out_{o2}}}{\partial \boldsymbol{net_{o2}}} * \frac{\partial \boldsymbol{net_{o2}}}{\partial \boldsymbol{out_h}} = -1 * 0.11 * w4 = -0.1131 * 0.9553 = -0.105$$

$$\frac{\partial \boldsymbol{out_h}}{\partial \boldsymbol{net_h}} = \boldsymbol{out_h}(\boldsymbol{1} - \boldsymbol{out_h}) = 0.947(1 - 0.947) = 0.0502$$

$$\frac{\partial \boldsymbol{net_h}}{\partial \boldsymbol{w2}} = i2 = 1$$

$$\therefore \frac{\partial \boldsymbol{E_{total}}}{\partial \boldsymbol{w2}} = (-0.11 - 0.105) * 0.0502 * 1 = -0.01$$

So we get, $\boldsymbol{w2^+} = w2 - \eta * \frac{\partial \boldsymbol{E_{total}}}{\partial \boldsymbol{w2}} = 0.96011 - 0.01 * (-0.01) = 0.96021$

6. Adjusting b1

Considering b1 to know how much a change in b1 affects the total error.

$$\frac{\partial \boldsymbol{E_{total}}}{\partial \boldsymbol{b1}} = \frac{\partial \boldsymbol{E_{total}}}{\partial \boldsymbol{out_h}} * \frac{\partial \boldsymbol{out_h}}{\partial \boldsymbol{net_h}} * \frac{\partial \boldsymbol{net_h}}{\partial \boldsymbol{b1}} = \left(\frac{\partial \boldsymbol{E_{o1}}}{\partial \boldsymbol{out_h}} + \frac{\partial \boldsymbol{E_{o2}}}{\partial \boldsymbol{out_h}}\right) * \frac{\partial \boldsymbol{out_h}}{\partial \boldsymbol{net_h}} * \frac{\partial \boldsymbol{net_h}}{\partial \boldsymbol{b1}}$$

Here,

$$\frac{\partial \boldsymbol{E_{o1}}}{\partial \boldsymbol{out_h}} = \frac{\partial \boldsymbol{E_{o1}}}{\partial \boldsymbol{out_{o1}}} * \frac{\partial \boldsymbol{out_{o1}}}{\partial \boldsymbol{net_{o1}}} * \frac{\partial \boldsymbol{net_{o1}}}{\partial \boldsymbol{out_h}} = -1 * 0.113 * w3 = -0.113 * 0.9502 = -0.11$$

$$\frac{\partial \boldsymbol{E_{o2}}}{\partial \boldsymbol{out_h}} = \frac{\partial \boldsymbol{E_{o2}}}{\partial \boldsymbol{out_{o2}}} * \frac{\partial \boldsymbol{out_{o2}}}{\partial \boldsymbol{net_{o2}}} * \frac{\partial \boldsymbol{net_{o2}}}{\partial \boldsymbol{out_h}} = -1 * 0.11 * w4 = -0.1131 * 0.9553 = -0.105$$

$$\frac{\partial \boldsymbol{out_h}}{\partial \boldsymbol{net_h}} = \boldsymbol{out_h}(\boldsymbol{1} - \boldsymbol{out_h}) = 0.947(1 - 0.947) = 0.0502$$

$$\frac{\partial \boldsymbol{net_h}}{\partial \boldsymbol{b1}} = 1$$

$$\therefore \frac{\partial \boldsymbol{E_{total}}}{\partial \boldsymbol{b1}} = (-0.11 - 0.105) * 0.0502 * 1 = -0.011$$

So we get, $\boldsymbol{b1}^{+} = b1 - \eta * \frac{\partial \boldsymbol{E_{total}}}{\partial \boldsymbol{b1}} = 0.9401 - 0.01 * (-0.011) = 0.9402$

2nd Iteration Adjusted Values

After 2nd iteration, we get the adjusted values as below:

SPL1 = 0.9402

SPL2 = 1.004

1st year cgpa = 0.9802

2nd year cgpa = 0.96021

3rd year cgpa = 0.9505

Cgpa upto 3rd year = 0.96