

Student's t Distribution

1. Overview

+

t Distribution

1. Overview →

What's it
used for?

2. Sampling Recap →

Put it into
context

3. Visual Comparison →

What does it
look like?

4. PDF and CDF Calculations →

Use it!

t Distribution

* t-test developed in 1908 by William Sealy Gosset



t Distribution

- * t-test developed in 1908 by William Sealy Gosset
- * Solved the problem of "small sample statistics"



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Underlying distribution is
NORMAL

Population standard
deviation unknown

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Sample size is too small for
C.L.T to apply

t Distribution



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NORMAL

Population standard
deviation unknown

Sample size is too small for
C.L.T to apply

- * The following measures would be t-distributed

$$\frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$\frac{b - \beta}{SE(b)}$$

$$\frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\left(\frac{s_1^2}{\sqrt{n_1}}\right) + \left(\frac{s_2^2}{\sqrt{n_2}}\right)}}$$

t Distribution

SAMPLING RECAP!

- * Take a sample of five observations
from a normally distributed population

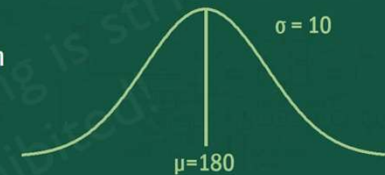
2. Sampling Recap

t Distribution

SAMPLING RECAP!

- * Take a sample of five observations
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Heights of female basketballers (cm)

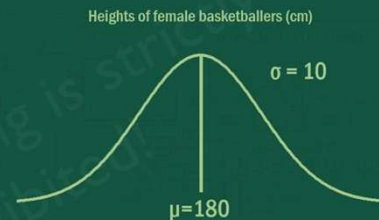


t Distribution

SAMPLING RECAP!

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[183 , 170 , 189 , 191 , 203]



t Distribution

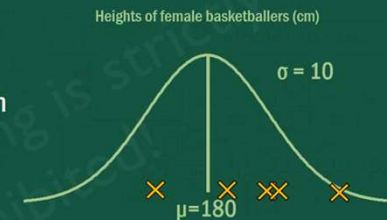
SAMPLING RECAP!

- * Take a sample of five observations from a normally distributed population

[183 , 170 , 189 , 191 , 203]

- * Find the average of that sample

$$\bar{X} = 187.2 \text{ cm}$$



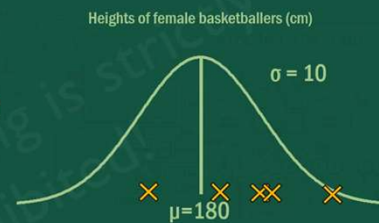
t Distribution

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t Distribution

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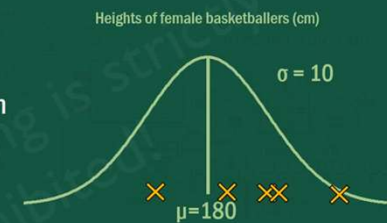
- * Take a sample of five observations from a normally distributed population

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- * How would such a sample mean (of size 5) be distributed?



t Distribution

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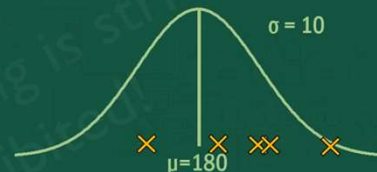
- * Find the average of that sample

$$\bar{X} = 187.2 \text{ cm}$$

- * How would such a sample mean (of size 5) be distributed?

$$\bar{x} \sim N\left(\mu, \left(\frac{\sigma}{\sqrt{n}}\right)^2\right) \quad \bar{x} \sim N\left(180, \left(\frac{10}{\sqrt{5}}\right)^2\right)$$

Heights of female basketballers (cm)



t Distribution

SAMPLING RECAP!

- * Imagine we are **TESTING** the population mean value of 180cm by using our sample

[183 , 170 , 189 , 191 , 203]

$$\bar{X} = 187.2 \text{ cm}$$

- * $H_0: \mu = 180$ $H_1: \mu \neq 180$

t Distribution

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$$\frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \sim z$$

t Distribution

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$$\frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \sim z \longrightarrow \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

t Distribution

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t Distribution

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 $\bar{X} = 187.2 \text{ cm}$

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$s = 12.05 \text{ cm}$

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t Distribution

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* Imagine we are **TESTING** the population mean value of 180cm by using our sample [183 , 170 , 189 , 191 , 203]
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t Distribution

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 $\bar{X} = 187.2 \text{ cm}$
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- * $H_0: \mu = 180 \quad H_1: \mu \neq 180$

$$\frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \sim Z \longrightarrow \frac{\bar{x} - \mu}{s / \sqrt{n}} \sim t_{n-1}$$

- * We need to adjust for the additional uncertainty around s .

t Distribution

SAMPLING RECAP!

- * Imagine we are **TESTING** the population mean value of 180cm by using our sample [183 , 170 , 189 , 191 , 203]
 $\bar{X} = 187.2 \text{ cm}$
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- * $H_0: \mu = 180 \quad H_1: \mu \neq 180$

$$\frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \sim Z \longrightarrow \frac{\bar{x} - \mu}{s / \sqrt{n}} \sim t_{n-1} \quad t = \frac{187.2 - 180}{12.05 / \sqrt{5}} \sim t_4$$

- * We need to adjust for the additional uncertainty around s .
- * The smaller the sample size, the more uncertain we are.

t Distribution

SAMPLING RECAP!

- * Imagine we are **TESTING** the population mean value of 180cm by using our sample [183 , 170 , 189 , 191 , 203]
 $\bar{X} = 187.2 \text{ cm}$
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- * $H_0: \mu = 180 \quad H_1: \mu \neq 180$

$$\frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \sim Z \longrightarrow \frac{\bar{x} - \mu}{s / \sqrt{n}} \sim t_{n-1}$$

- * We need to adjust for the additional uncertainty around s .
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3. Visualisation

t Distribution

Probability Distribution Function

$$-Z - t(df=4)$$



t Distribution

Probability Distribution Function

$$-Z - t(df=2)$$



t Distribution

Probability Distribution Function

$$-Z - t(df=1)$$



t Distribution

Probability Distribution Function

$$-Z - t(df=20)$$



t Distribution

Probability Distribution Function



$$PDF = \frac{\Gamma\left(\frac{n}{2}\right)}{\sqrt{(n-1)\pi} \Gamma\left(\frac{n-1}{2}\right)} \left(1 + \frac{x^2}{n-1}\right)^{-\frac{n}{2}}$$

t Distribution

Probability Distribution Function



=T.DIST(x,DF,FALSE)

FALSE means PDF
("Height!")

+

t Distribution

Probability Distribution Function



=T.DIST(x,DF,FALSE)

t Distribution

Cumulative Distribution Function



+

t Distribution

Cumulative Distribution Function



t Distribution

Cumulative Distribution Function



=T.DIST(x,DF,TRUE) →

TRUE means CDF
("Area!")

t Distribution

Cumulative Distribution Function



=T.DIST(x,DF,TRUE)

t Distribution

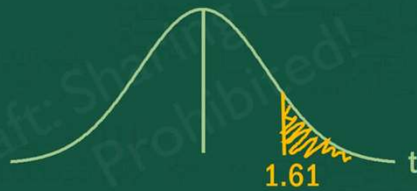
Cumulative Distribution Function

What proportion of the t-distribution (with 4 df) exists above $t=1.61$?

t Distribution

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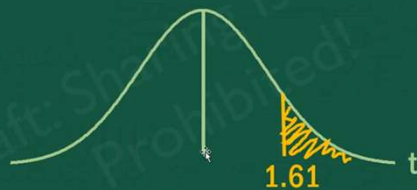


$$=1-T.DIST(1.61,4,TRUE)$$
$$=0.091$$

t Distribution

Cumulative Distribution Function

What proportion of the t-distribution (with 4 df) exists above $t=1.61$?



$$=1-T.DIST(1.61,4,TRUE)$$

t Distribution

Cumulative Distribution Function

What t statistic (with 4 df) provides 2.5% in the upper tail?



t Distribution

Cumulative Distribution Function

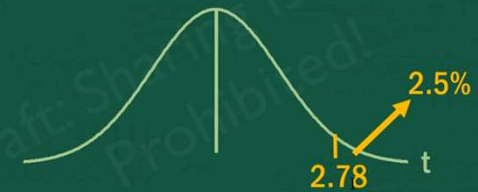
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t Distribution

Cumulative Distribution Function

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$$\begin{aligned} &= \text{T.INV}(0.975, 4) \\ &= 2.78 \end{aligned}$$

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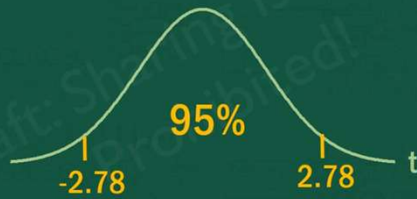


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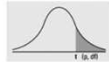
What t statistic (with 4 df) provides 2.5% in the upper tail?



$$=T.INV(0.975,4)$$
$$=2.78$$

THANK YOU!

Numbers in each row of the table are values on a t-distribution with (df) degrees of freedom for selected right-tail (greater-than) probabilities (p).



df/p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	638.6192
2	0.286675	0.816497	1.885618	2.919986	4.30205	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740097	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.729087	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44991	3.14267	3.70743	5.9588
7	0.263187	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396115	1.859248	2.30600	2.96440	3.35570	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.92144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795985	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	4.3178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08656	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07981	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319440	1.713872	2.06846	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317838	1.710882	2.06390	2.49216	2.79694	3.7454