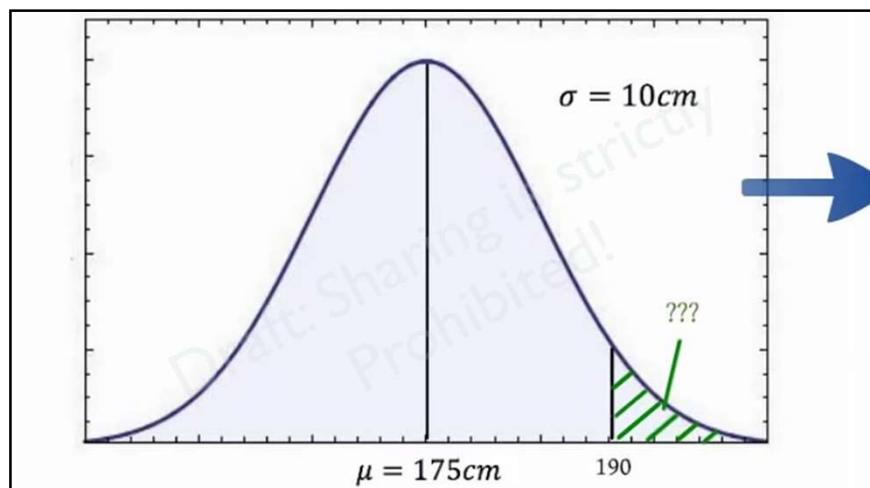


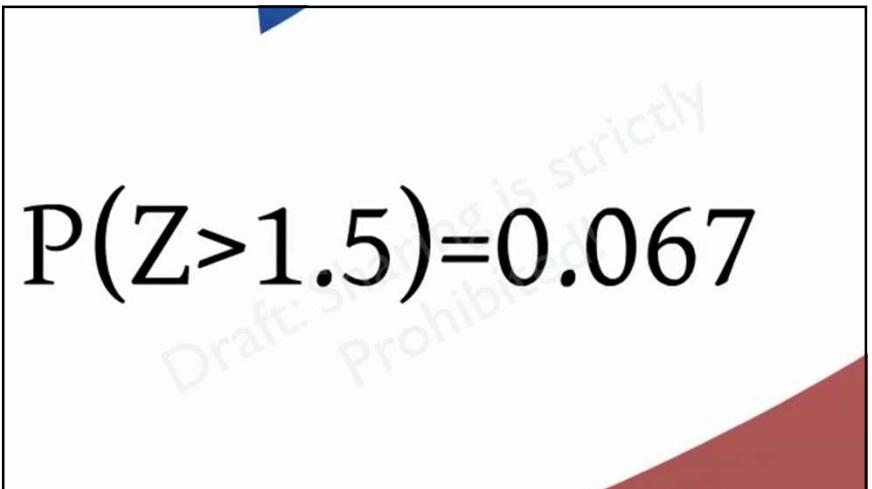
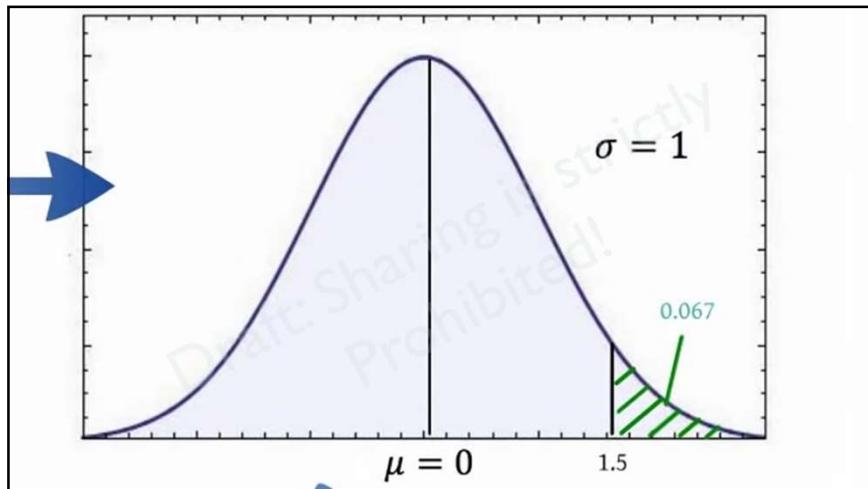
How to calculate p-values!!!

How to calculate p-values!!!

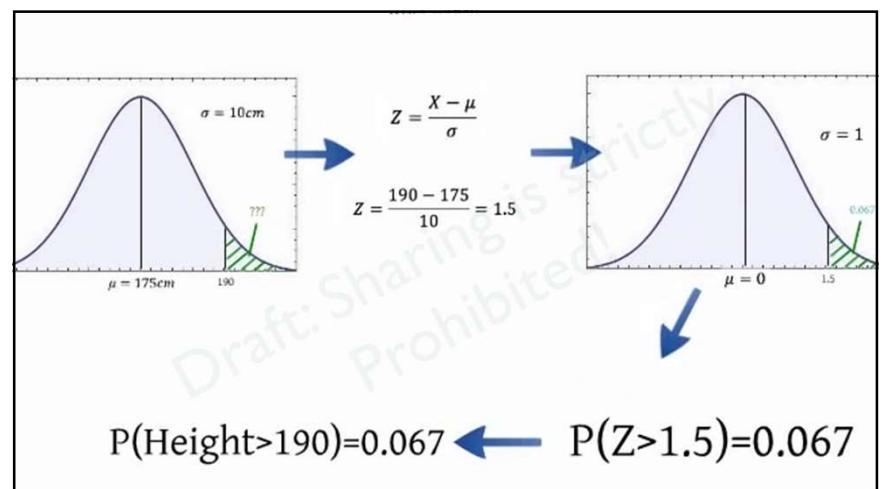
But, where do we use it?



$$Z = \frac{X - \mu}{\sigma}$$
$$Z = \frac{190 - 175}{10} = 1.5$$



$P(\text{Height} > 190) = 0.067$



Two-Sided p-values are the most common

One-Sided and **Two-Sided**

In contrast, **One-Sided p-values** are rarely used

One-Sided and Two-Sided

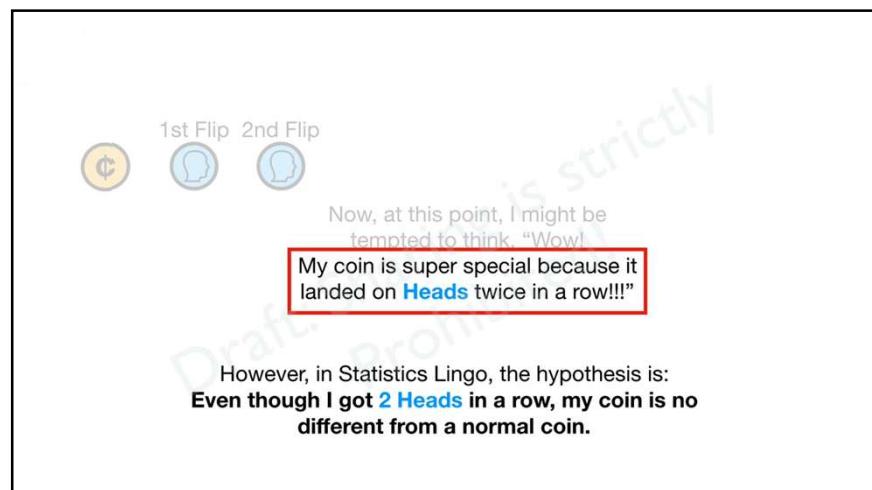
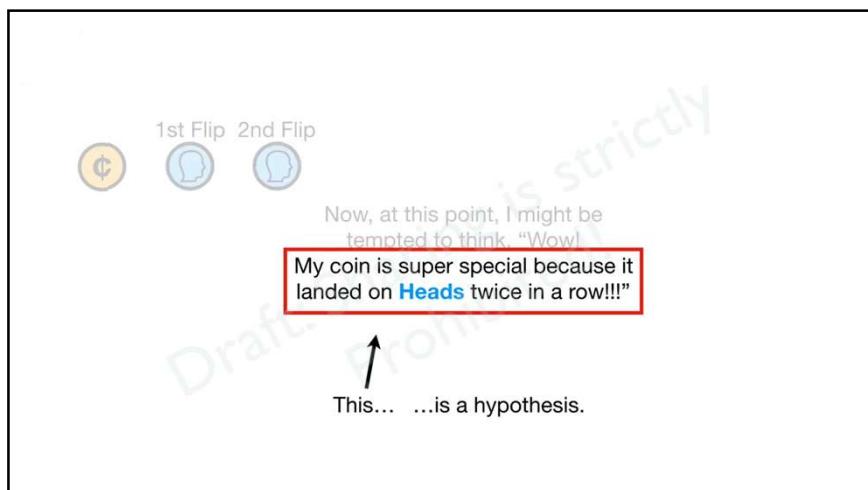
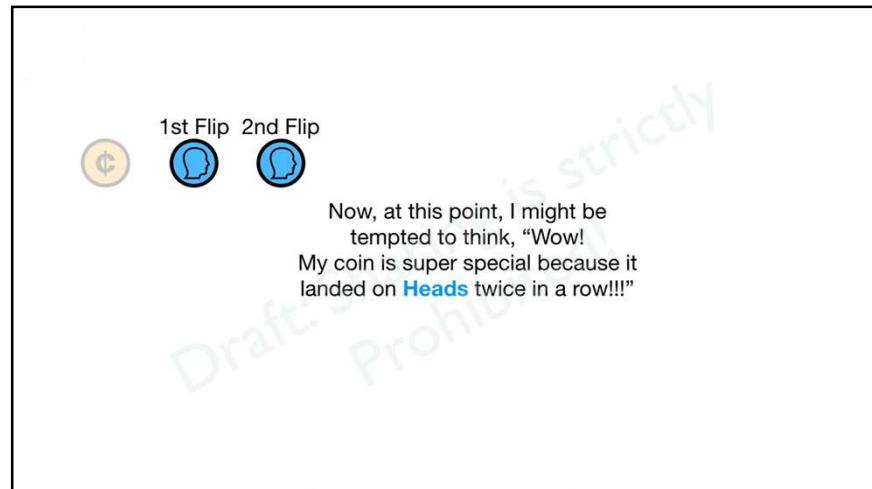
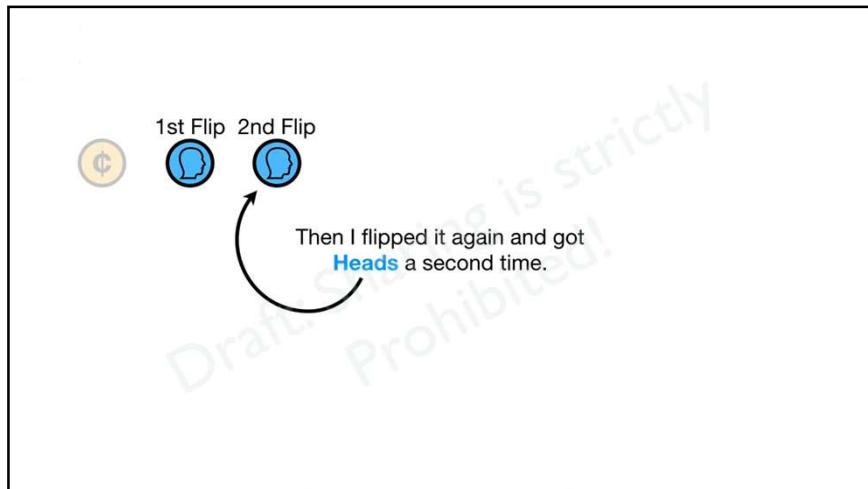


With that said, let's imagine I had a coin...



1st Flip

...and I flipped it once and got **Heads**.



1st Flip 2nd Flip

NOTE: Although we want to know if our coin is special...

My coin is super special because it landed on **Heads** twice in a row!!!

However, in Statistics Lingo, the hypothesis is:
Even though I got 2 Heads in a row, my coin is no different from a normal coin.

1st Flip 2nd Flip

...the Statistics Lingo version says the opposite, that our coin is the same as a **normal coin**.

My coin is super special because it landed on **Heads** twice in a row!!!

However, in Statistics Lingo, the hypothesis is:
Even though I got 2 Heads in a row, my coin is no different from a normal coin.

1st Flip 2nd Flip

Statisticians call this the **Null Hypothesis**, and a small **p-value** will tell us to reject it.

My coin is super special because it landed on **Heads** twice in a row!!!

However, in Statistics Lingo, the hypothesis is:
Even though I got 2 Heads in a row, my coin is no different from a normal coin.

1st Flip 2nd Flip

And if we reject this **Null Hypothesis**, we will know that our coin is special.

My coin is super special because it landed on **Heads** twice in a row!!!

However, in Statistics Lingo, the hypothesis is:
Even though I got 2 Heads in a row, my coin is no different from a normal coin.

1st Flip 2nd Flip

So let's test this hypothesis by calculating a **p-value**.

My coin is super special because it landed on **Heads** twice in a row!!!

However, in Statistics Lingo, the hypothesis is:
Even though I got 2 Heads in a row, my coin is no different from a normal coin.

p-values are determined by adding up probabilities, so let's start by figuring out the **probability** of getting **2 Heads** in a row.

1st Flip

0.5

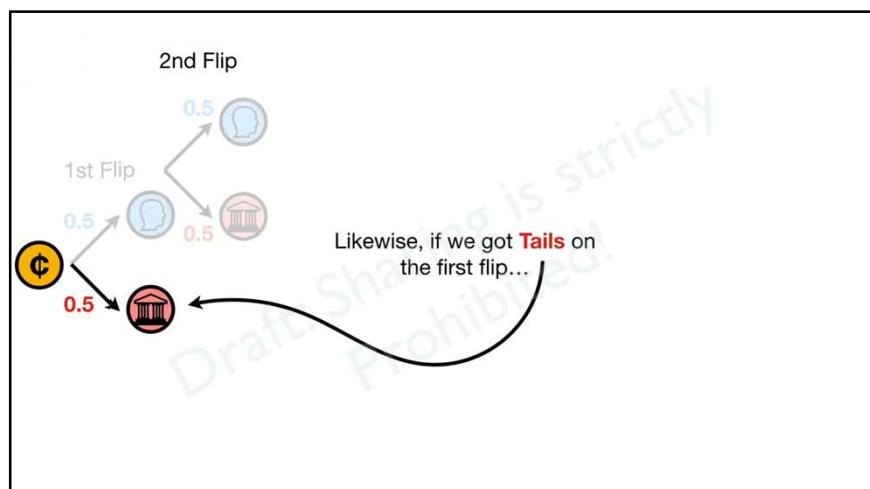
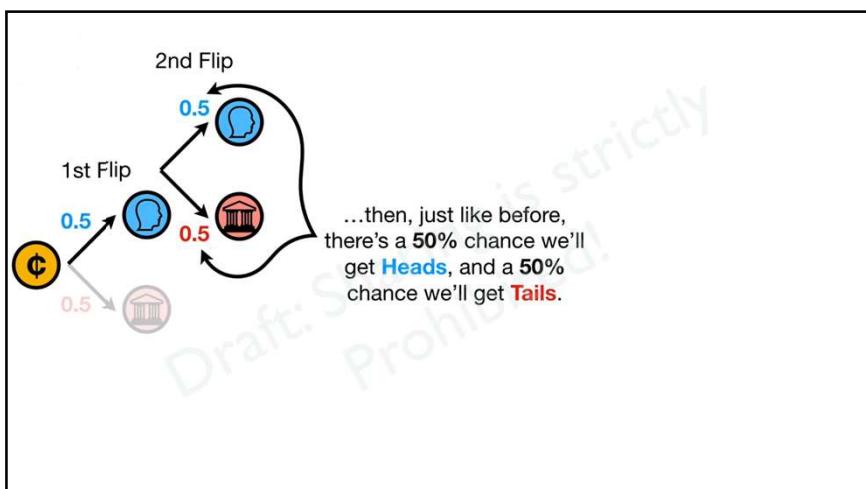
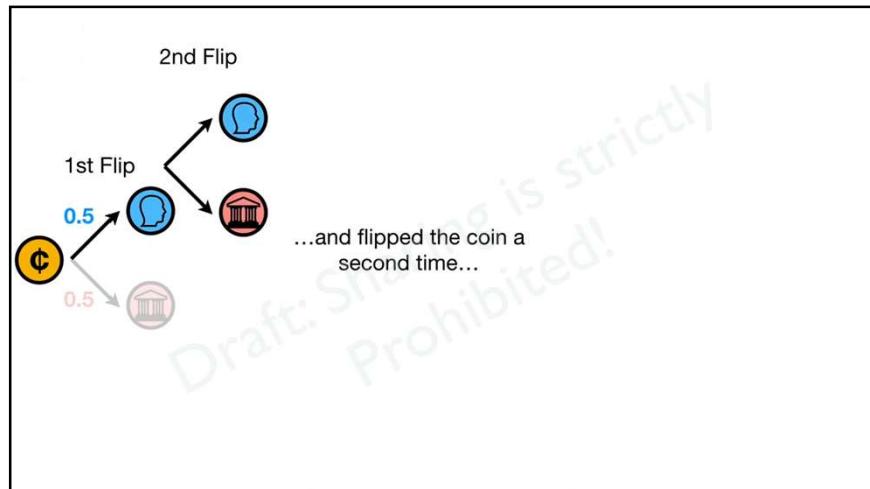
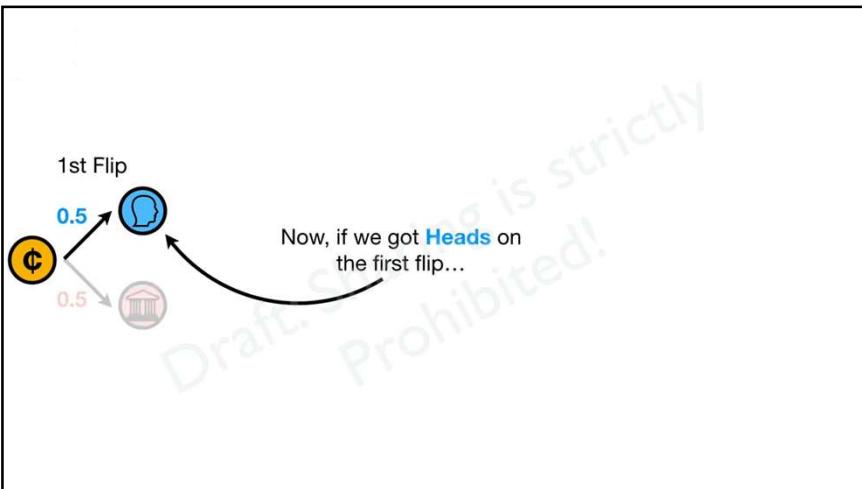
When we flip a normal, everyday coin, there's a **50%** chance we'll get **Heads**...

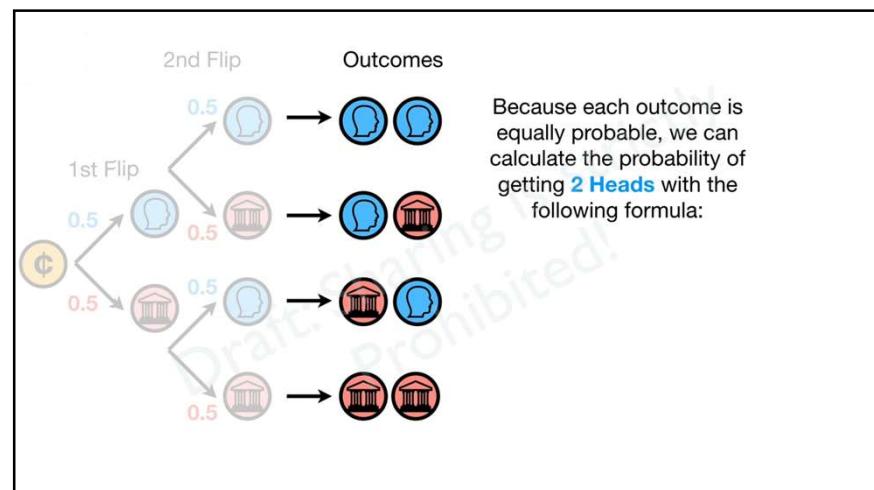
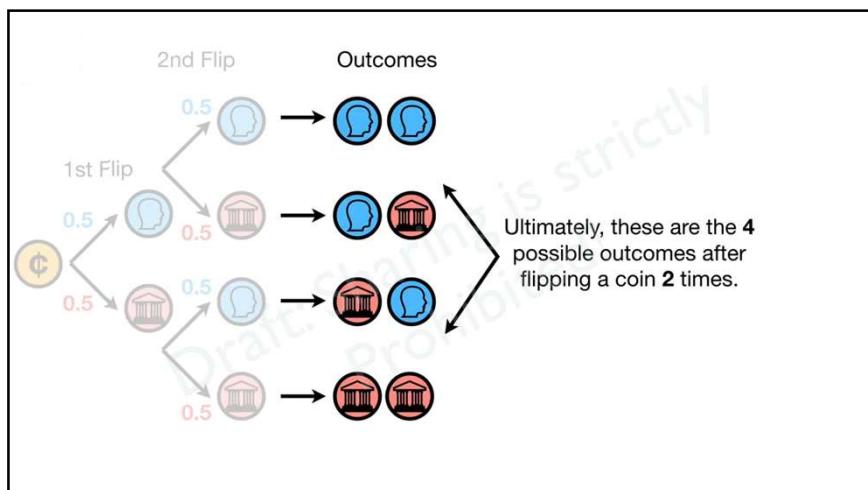
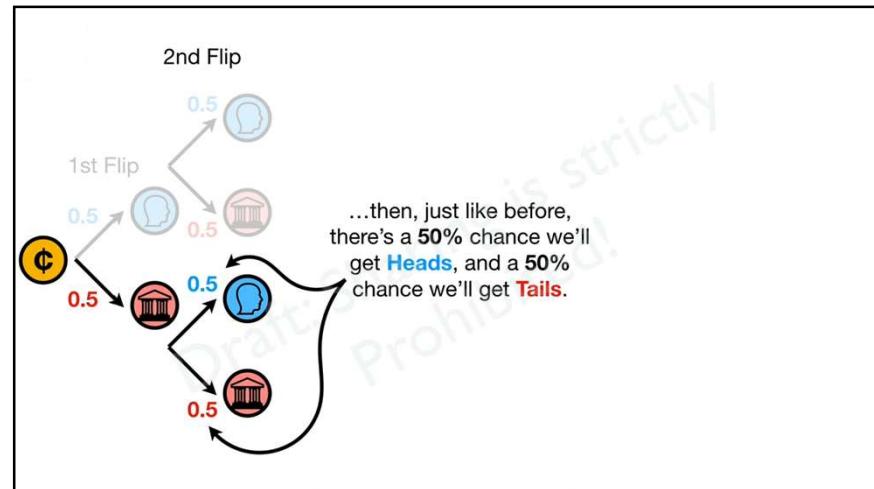
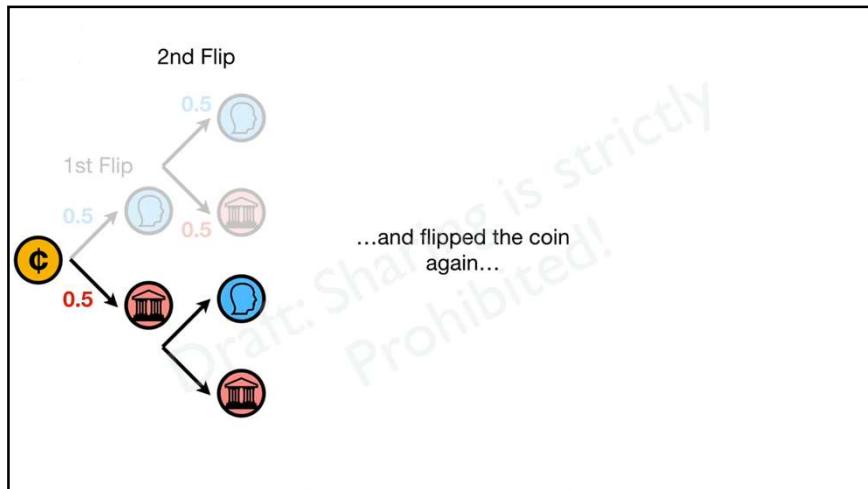
1st Flip

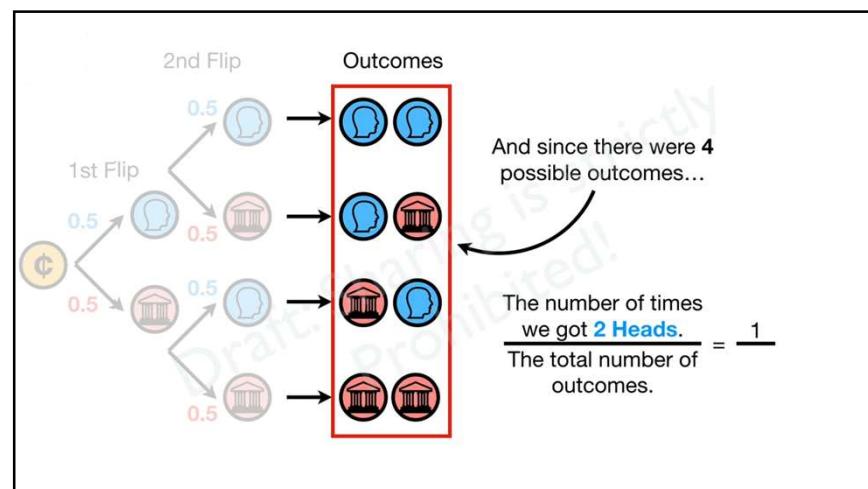
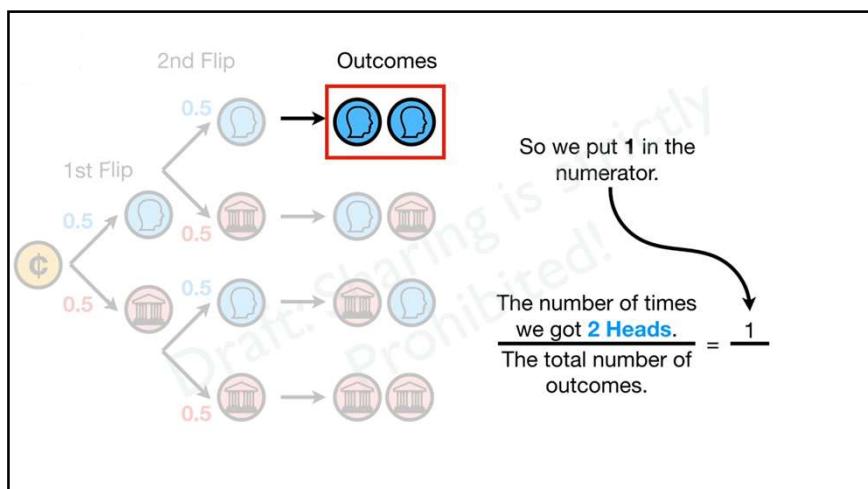
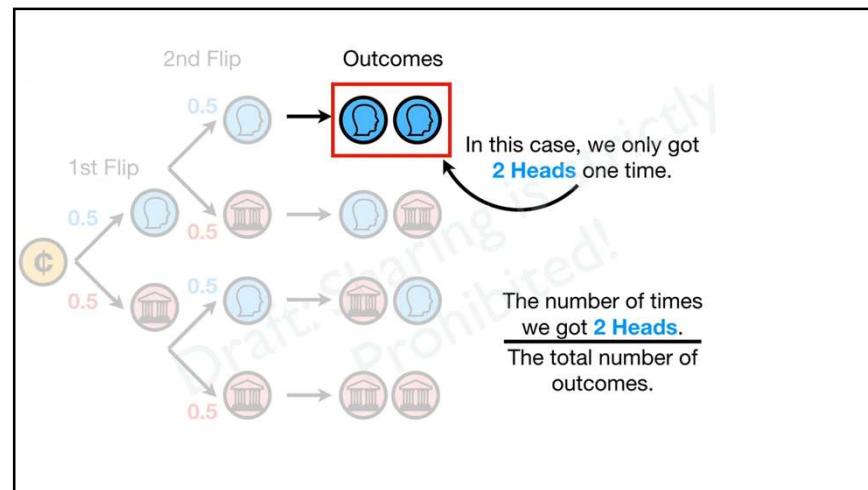
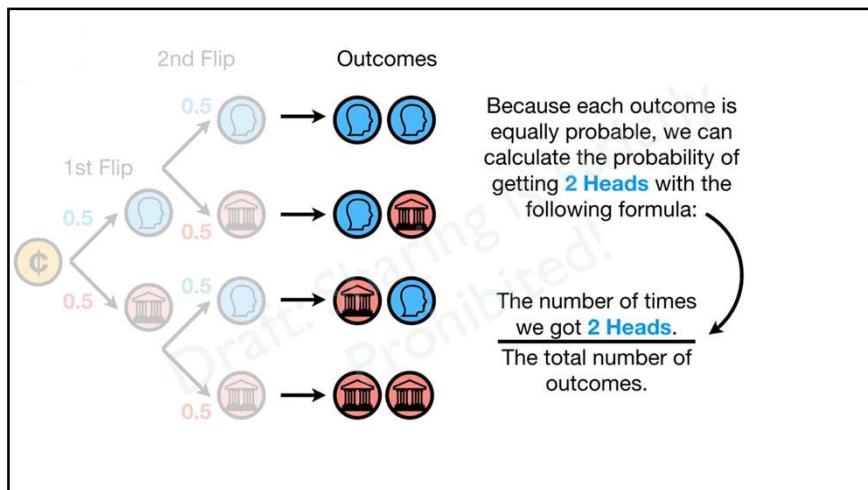
0.5

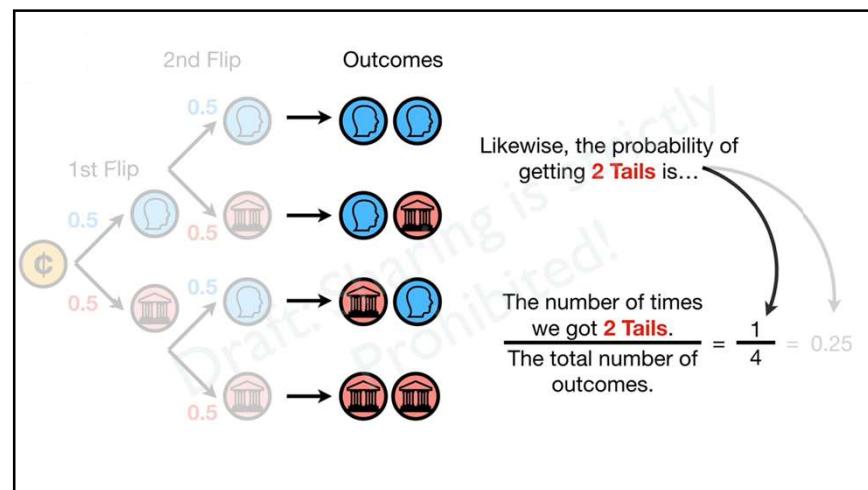
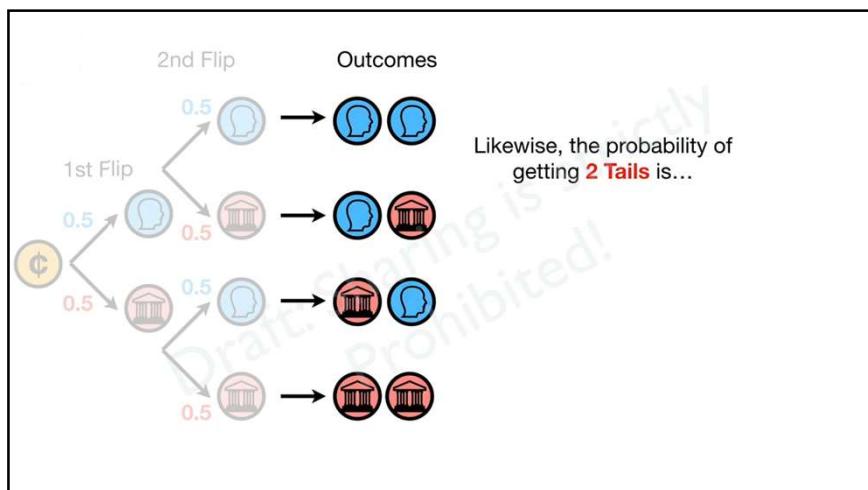
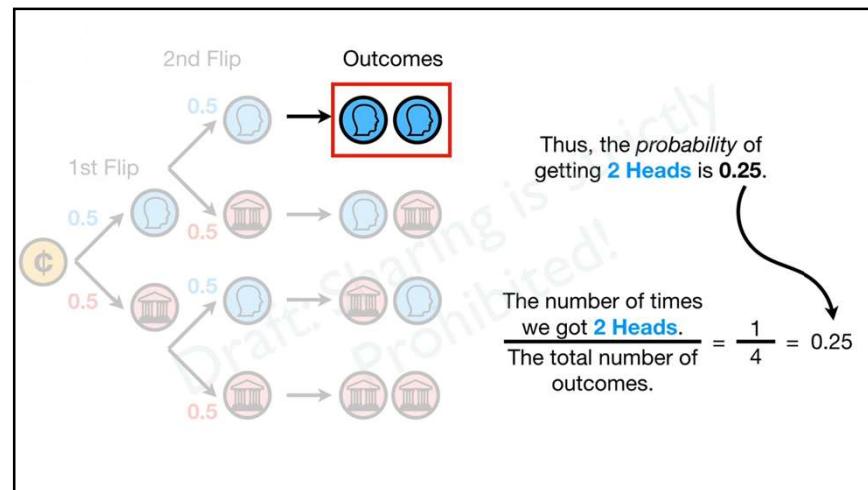
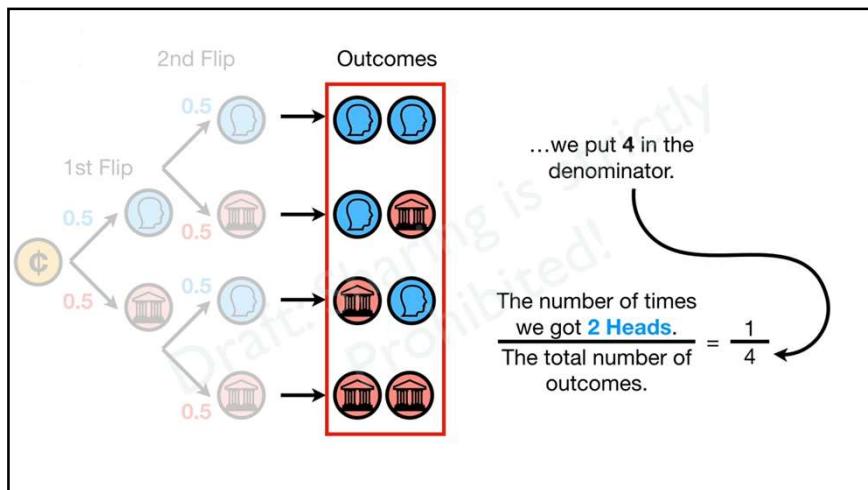
When we flip a normal, everyday coin, there's a **50%** chance we'll get **Heads**...

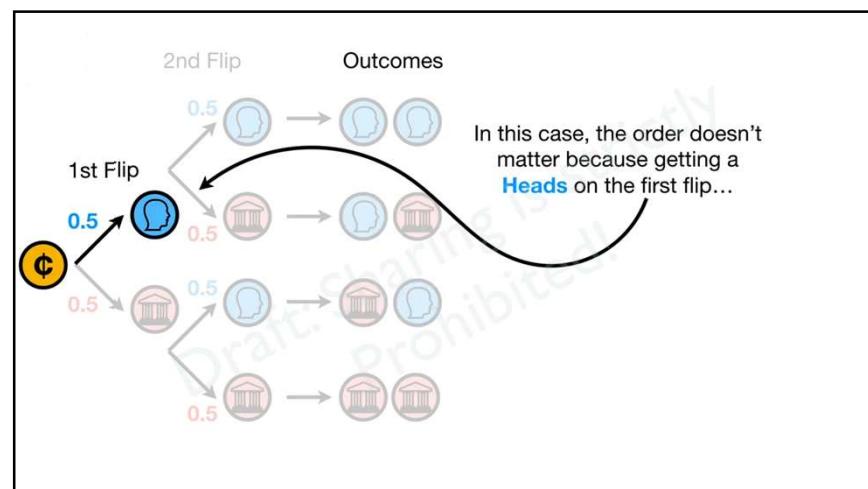
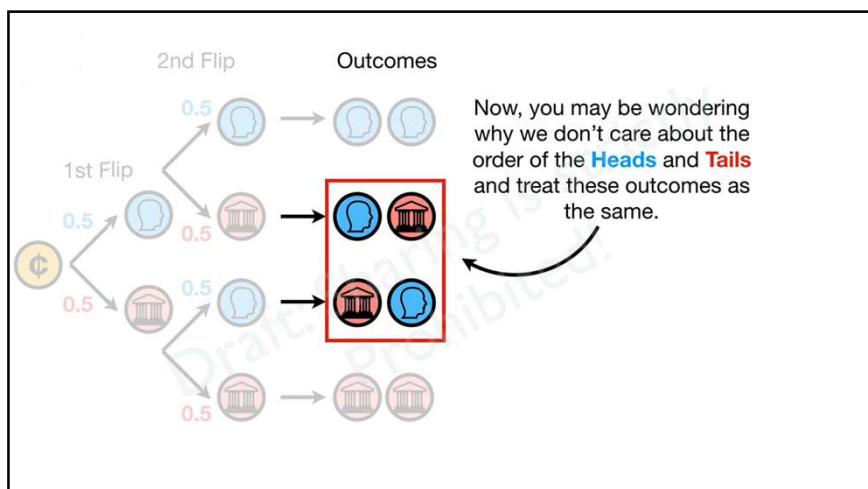
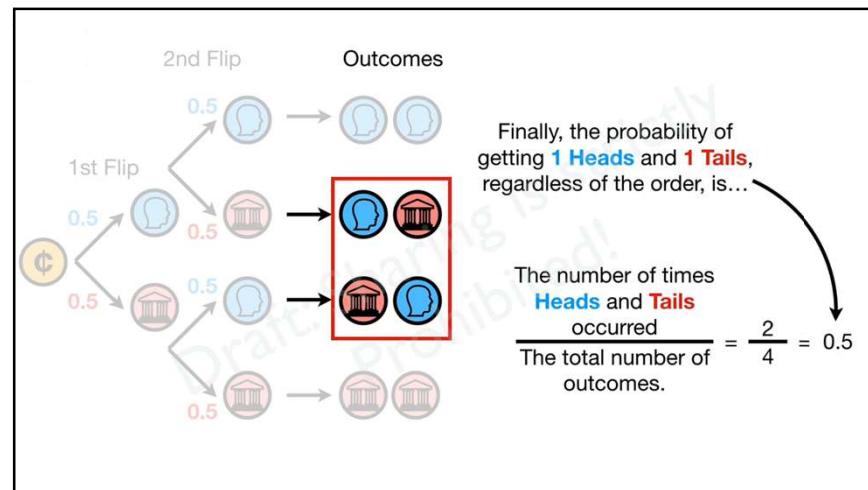
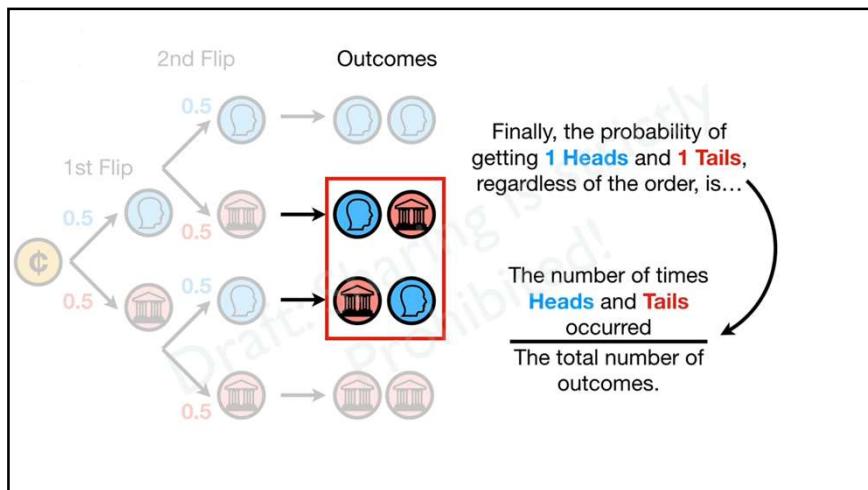
...and a **50%** chance we'll get **Tails**.

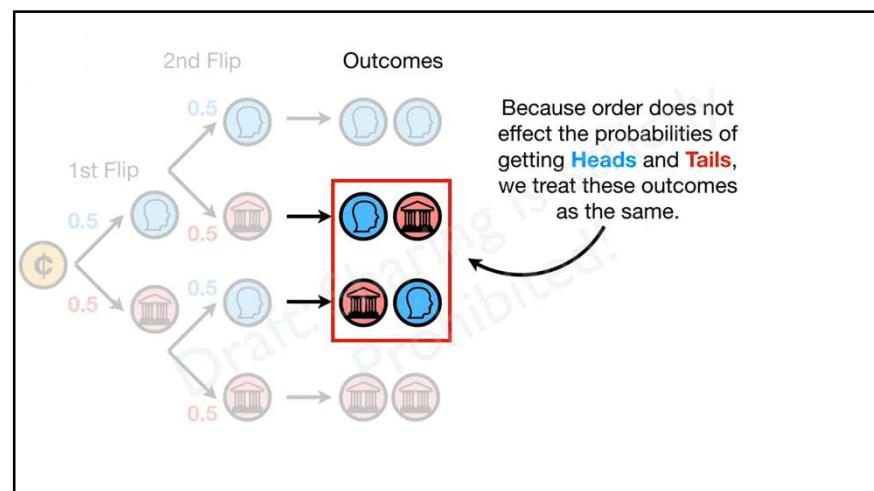
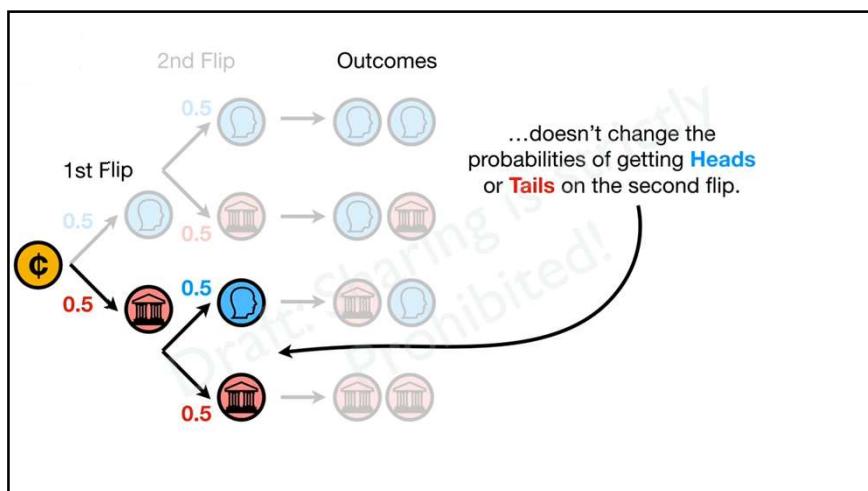
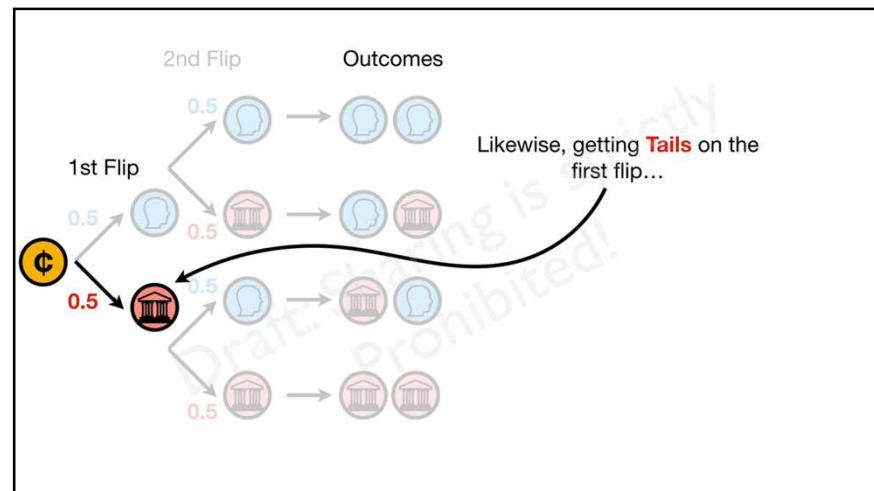
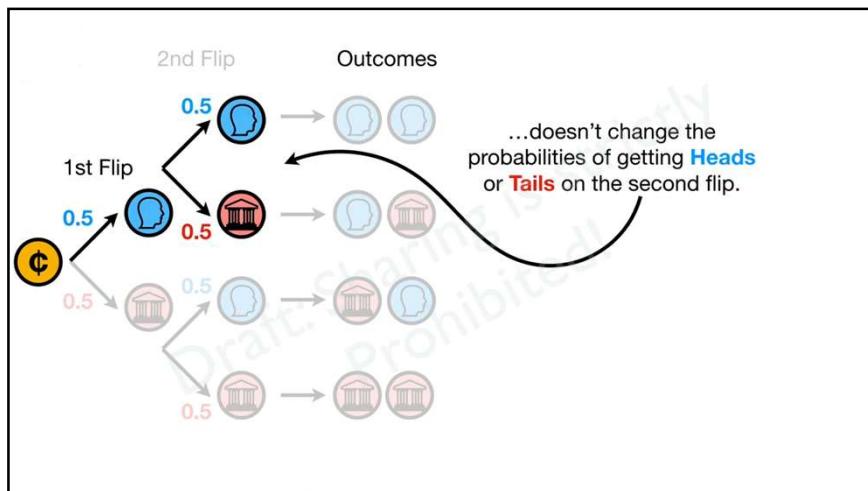


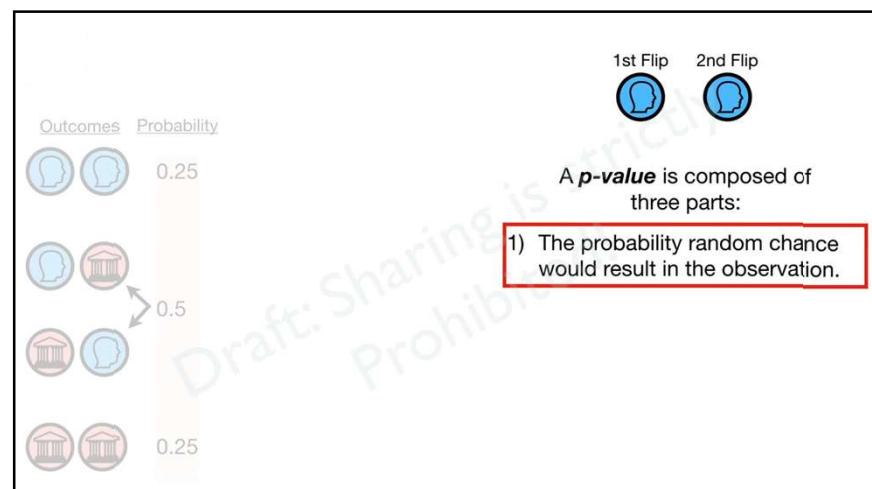
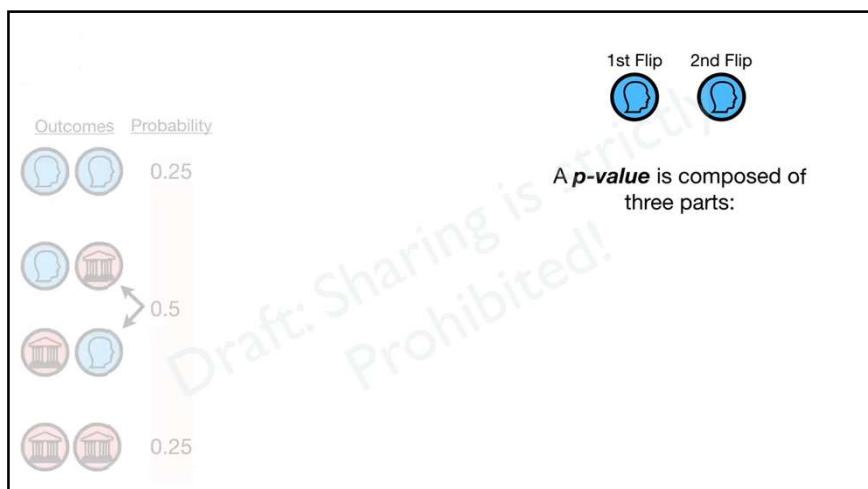
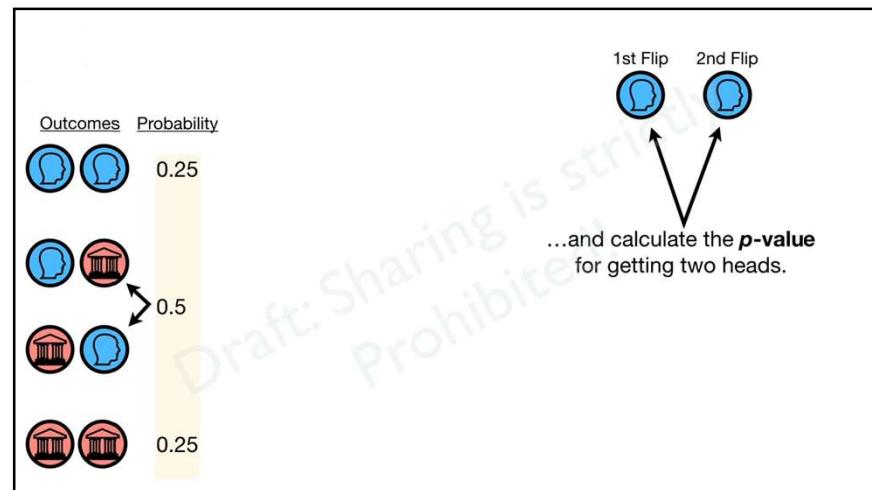
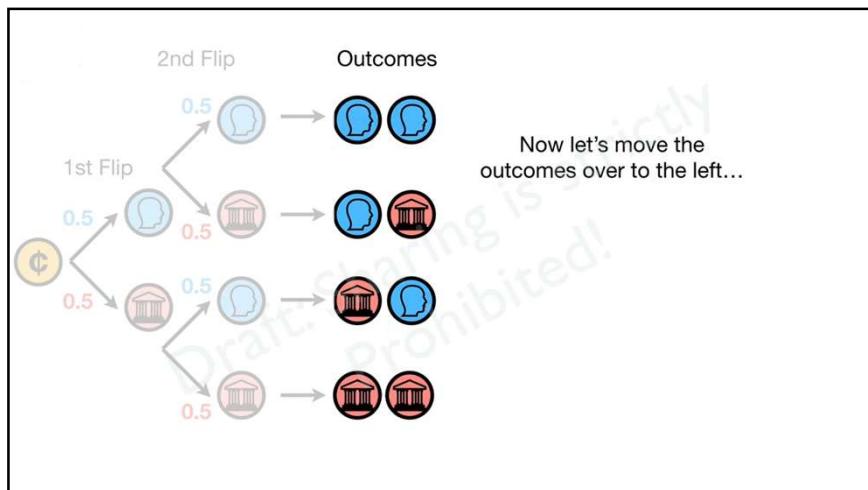


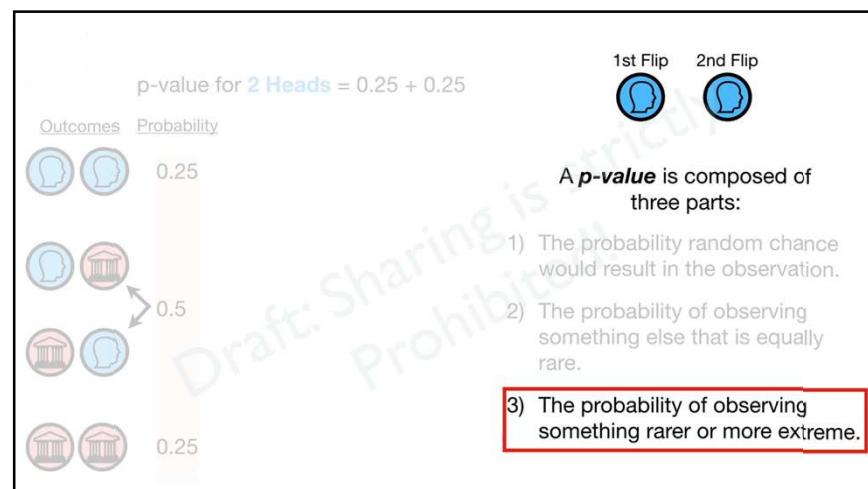
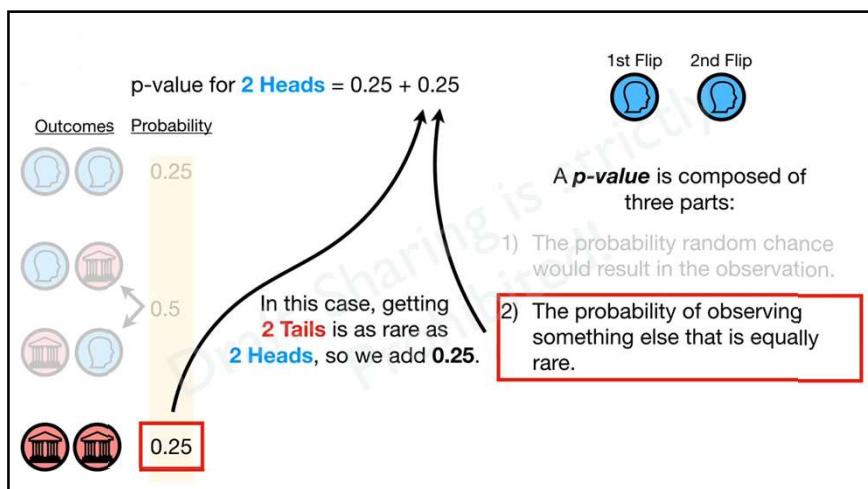
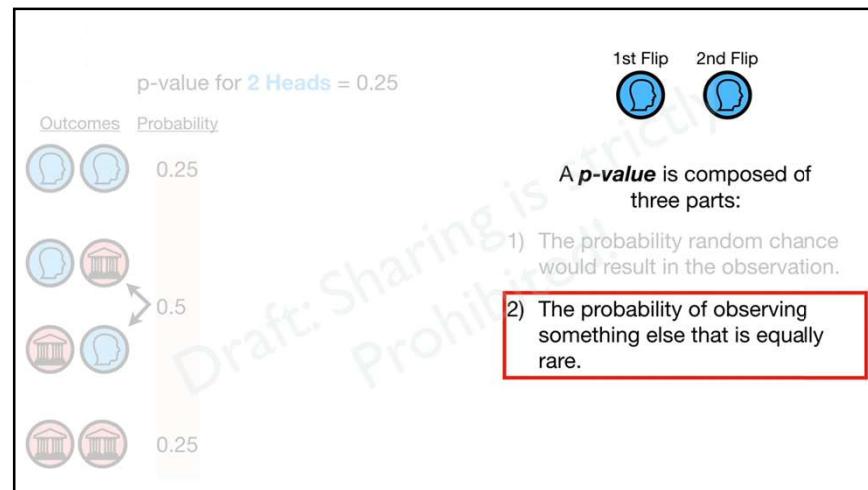
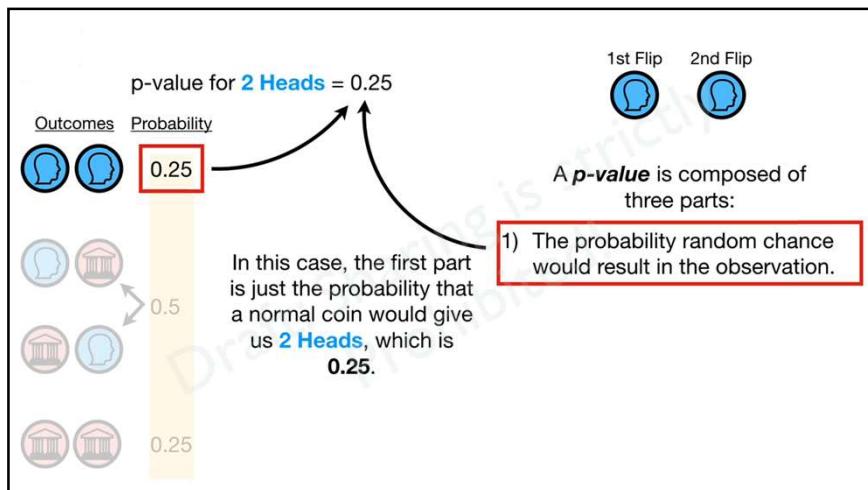












$p\text{-value for 2 Heads} = 0.25 + 0.25 + 0$

Outcomes	Probability
2 Heads	0.25
1 Head & 1 Tail	0.5
2 Tails	0.25

In this case, the third part is **0**, because no other outcomes are rarer than **2 Heads** or **2 Tails**.

A **p-value** is composed of three parts:

- 1) The probability random chance would result in the observation.
- 2) The probability of observing something else that is equally rare.
- 3) The probability of observing something rarer or more extreme.

$p\text{-value for 2 Heads} = 0.25 + 0.25 + 0$

Outcomes	Probability
2 Heads	0.25
1 Head & 1 Tail	0.5
2 Tails	0.25

Now we just add everything together...

A **p-value** is composed of three parts:

- 1) The probability random chance would result in the observation.
- 2) The probability of observing something else that is equally rare.
- 3) The probability of observing something rarer or more extreme.

$p\text{-value for 2 Heads} = 0.25 + 0.25 + 0 = 0.5$

Outcomes	Probability
2 Heads	0.25
1 Head & 1 Tail	0.5
2 Tails	0.25

...and the **p-value** for getting **2 Heads** = **0.5**.

A **p-value** is composed of three parts:

- 1) The probability random chance would result in the observation.
- 2) The probability of observing something else that is equally rare.
- 3) The probability of observing something rarer or more extreme.

$p\text{-value for 2 Heads} = 0.25 + 0.25 + 0 = 0.5$

Outcomes	Probability
2 Heads	0.25
1 Head & 1 Tail	0.5
2 Tails	0.25

Now remember, the reason we calculated the **p-value** was to test this hypothesis:

A **p-value** is composed of three parts:

- 1) The probability random chance would result in the observation.
- 2) The probability of observing something else that is equally rare.
- 3) The probability of observing something rarer or more extreme.

p-value for **2 Heads** = $0.25 + 0.25 + 0 = 0.5$

Outcomes	Probability
0.25	Now remember, the reason we calculated the p-value was to test this hypothesis:
0.5	Even though I got 2 Heads in a row, my coin is no different from a normal coin.
0.25	

1st Flip 2nd Flip

A p-value is composed of three parts:

- 1) The probability random chance would result in the observation.
- 2) The probability of observing something else that is equally rare.
- 3) The probability of observing something rarer or more extreme.

p-value for **2 Heads** = $0.25 + 0.25 + 0 = 0.5$

Outcomes	Probability
0.25	Typically, we only reject a hypothesis if the p-value is less than 0.05 ...
0.5	Even though I got 2 Heads in a row, my coin is no different from a normal coin.
0.25	

1st Flip 2nd Flip

A p-value is composed of three parts:

- 1) The probability random chance would result in the observation.
- 2) The probability of observing something else that is equally rare.
- 3) The probability of observing something rarer or more extreme.

p-value for **2 Heads** = $0.25 + 0.25 + 0 = 0.5$

Outcomes	Probability
0.25	...and since 0.5 > 0.05 , we fail to reject the hypothesis.
0.5	Even though I got 2 Heads in a row, my coin is no different from a normal coin.
0.25	

1st Flip 2nd Flip

A p-value is composed of three parts:

- 1) The probability random chance would result in the observation.
- 2) The probability of observing something else that is equally rare.
- 3) The probability of observing something rarer or more extreme.

p-value for **2 Heads** = $0.25 + 0.25 + 0 = 0.5$

Outcomes	Probability
0.25	In other words, the data, getting 2 Heads in a row, failed to convince us that our coin is special.
0.5	
0.25	

1st Flip 2nd Flip

A p-value is composed of three parts:

- 1) The probability random chance would result in the observation.
- 2) The probability of observing something else that is equally rare.
- 3) The probability of observing something rarer or more extreme.

p-value for 2 Heads = $0.25 + 0.25 + 0 = 0.5$

Outcomes	Probability
	0.25
	0.5
	0.25
	0.25

1st Flip 2nd Flip

A p-value is composed of three parts:

- 1) The probability random chance would result in the observation.
- 2) The probability of observing something else that is equally rare.
- 3) The probability of observing something rarer or more extreme.

NOTE: The probability of getting 2 Heads, 0.25, is different from the p-value for getting 2 Heads, 0.5.

p-value for 2 Heads = $0.25 + 0.25 + 0 = 0.5$

Outcomes	Probability
	0.25
	0.5
	0.25
	0.25

1st Flip 2nd Flip

A p-value is composed of three parts:

- 1) The probability random chance would result in the observation.
- 2) The probability of observing something else that is equally rare.
- 3) The probability of observing something rarer or more extreme.

This is because the p-value is the sum of three parts...

p-value for 2 Heads = $0.25 + 0.25 + 0 = 0.5$

Outcomes	Probability
	0.25
	0.5
	0.25
	0.25

1st Flip 2nd Flip

A p-value is composed of three parts:

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Outcomes	Probability
	0.25
	0.5
	0.25
	0.25

1st Flip 2nd Flip

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p-value for **2 Heads** = $0.25 + 0.25 + 0 = 0.5$

Outcomes	Probability
	0.25
	0.5
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	0.25

This is because the **p-value** is the sum of three parts...

A **p-value** is composed of three parts:

- 1) The probability random chance would result in the observation.
- 2) The probability of observing something else that is equally rare.
- 3) The probability of observing something rarer or more extreme.

1st Flip 2nd Flip

p-value for **2 Heads** = $0.25 + 0.25 + 0 = 0.5$

Outcomes	Probability
	0.25
	0.5
	0.25
	0.25

Now the question is, **“Why do we care about things that are equally rare or more extreme?”**

A **p-value** is composed of three parts:

- 1) The probability random chance would result in the observation.
- 2) The probability of observing something else that is equally rare.
- 3) The probability of observing something rarer or more extreme.

1st Flip 2nd Flip

p-value for **2 Heads** = $0.25 + 0.25 + 0 = 0.5$

Outcomes	Probability
	0.25
	0.5
	0.25
	0.25

In other words, why do we add **Parts 2 and 3** to the **p-value**?

A **p-value** is composed of three parts:

- 1) The probability random chance would result in the observation.
- 2) The probability of observing something else that is equally rare.
- 3) The probability of observing something rarer or more extreme.

1st Flip 2nd Flip

p-value for **2 Heads** = $0.25 + 0.25 + 0 = 0.5$

Outcomes	Probability
	0.25
	0.5
	0.25
	0.25

We add **Part 2**, the probability of something else that is equally rare, because although getting **2 Heads** might seem special, it doesn't seem as special when we know that other things are just as rare.

A **p-value** is composed of three parts:

- 1) The probability random chance would result in the observation.
- 2) The probability of observing something else that is equally rare.
- 3) The probability of observing something rarer or more extreme.

1st Flip 2nd Flip

For example, imagine giving a loved one a flower and saying, "This is the rarest flower of this species, none are equally as rare."



A **p-value** is composed of three parts:

- 1) The probability random chance would result in the observation.
- 2) The probability of observing something else that is equally rare.
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For example, imagine giving a loved one a flower and saying, "This is the rarest flower of this species, none are equally as rare."



A **p-value** is composed of three parts:

- 1) The probability random chance would result in the observation.
- 2) The probability of observing something else that is equally rare.
- 3) The probability of observing something rarer or more extreme.

Now imagine saying to your loved one, "This flower is equally as rare as all of these other flowers."



A **p-value** is composed of three parts:

- 1) The probability random chance would result in the observation.
- 2) The probability of observing something else that is equally rare.
- 3) The probability of observing something rarer or more extreme.

Now imagine saying to your loved one, "This flower is equally as rare as all of these other flowers."



A **p-value** is composed of three parts:

- 1) The probability random chance would result in the observation.
- 2) The probability of observing something else that is equally rare.
- 3) The probability of observing something rarer or more extreme.

In this case, your loved one might not think the flower is very special.



NOTE: Even though these flowers are different colors, just knowing that they are equally rare would be a bummer.

1st Flip 2nd Flip

A **p-value** is composed of three parts:

- 1) The probability random chance would result in the observation.
- 2) The probability of observing something else that is equally rare.
- 3) The probability of observing something rarer or more extreme.

1st Flip 2nd Flip

p-value for **2 Heads** = $0.25 + 0.25 + 0 = 0.5$

Outcomes	Probability
	0.25
	0.5
	0.25

A **p-value** is composed of three parts:

- 1) The probability random chance would result in the observation.
- 2) The probability of observing something else that is equally rare.
- 3) The probability of observing something rarer or more extreme.

Because a lot of equally rare things would make something less special, we add **Part 2** to the **p-value**.

p-value for **2 Heads** = $0.25 + 0.25 - 0 = 0.5$

Outcomes	Probability
	0.25
	0.5
	0.25

1st Flip 2nd Flip

A **p-value** is composed of three parts:

- 1) The probability random chance would result in the observation.
- 2) The probability of observing something else that is equally rare.
- 3) The probability of observing something rarer or more extreme.

And we add rarer things to the **p-value** for a similar reason.

1st Flip 2nd Flip

Going back to our flower example, imagine telling your loved one "This is the rarest flower of this species, none are rarer."



A **p-value** is composed of three parts:

- 1) The probability random chance would result in the observation.
- 2) The probability of observing something else that is equally rare.
- 3) The probability of observing something rarer or more extreme.

Going back to our flower example, imagine telling your loved one "This is the rarest flower of this species, none are rarer."



Again, there's a good chance your loved one would think that the flower was super special.

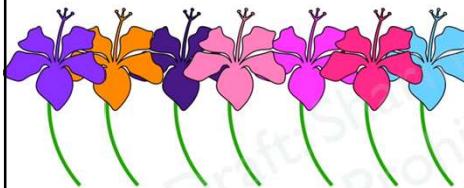
1st Flip 2nd Flip



A **p-value** is composed of three parts:

- 1) The probability random chance would result in the observation.
- 2) The probability of observing something else that is equally rare.
- 3) The probability of observing something rarer or more extreme.

Now imagine saying "There are a lot of flowers that are rarer than this one."



1st Flip 2nd Flip



A **p-value** is composed of three parts:

- 1) The probability random chance would result in the observation.
- 2) The probability of observing something else that is equally rare.
- 3) The probability of observing something rarer or more extreme.

Now imagine saying "There are a lot of flowers that are rarer than this one."



In this case, your loved one might not think the flower is very special.

1st Flip 2nd Flip



A **p-value** is composed of three parts:

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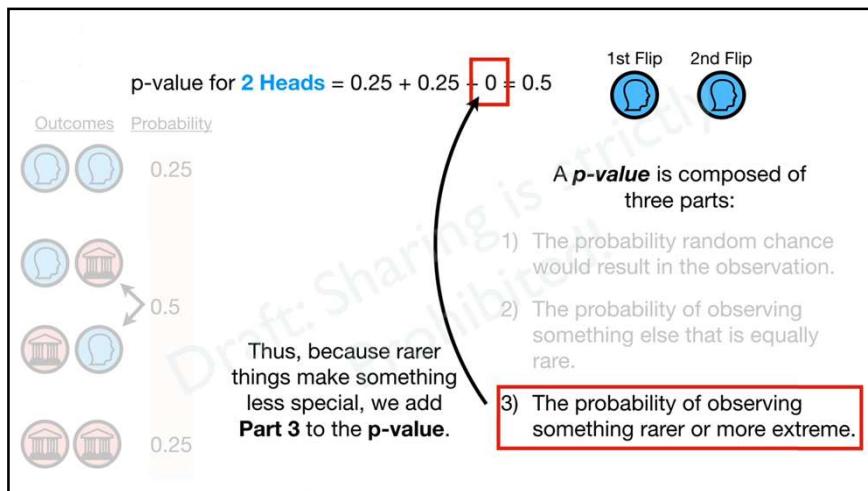
And like before, even though these flowers are all different colors, just knowing they are rarer would be a bummer.

1st Flip 2nd Flip

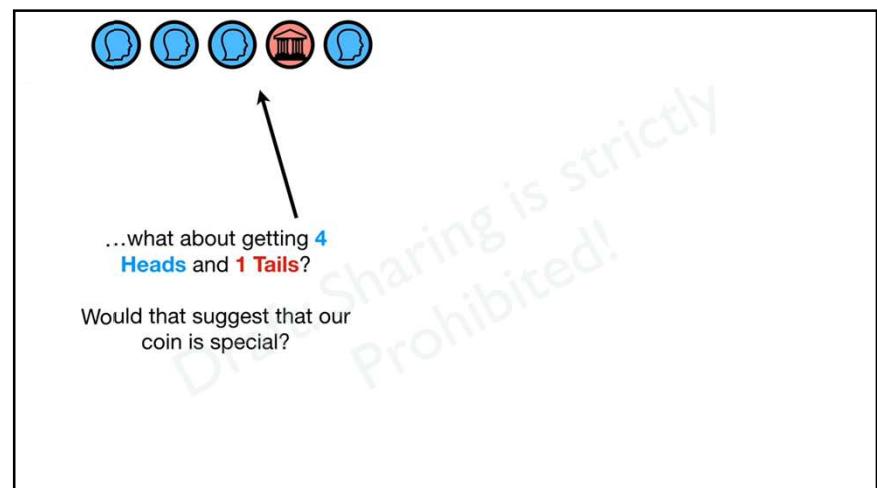
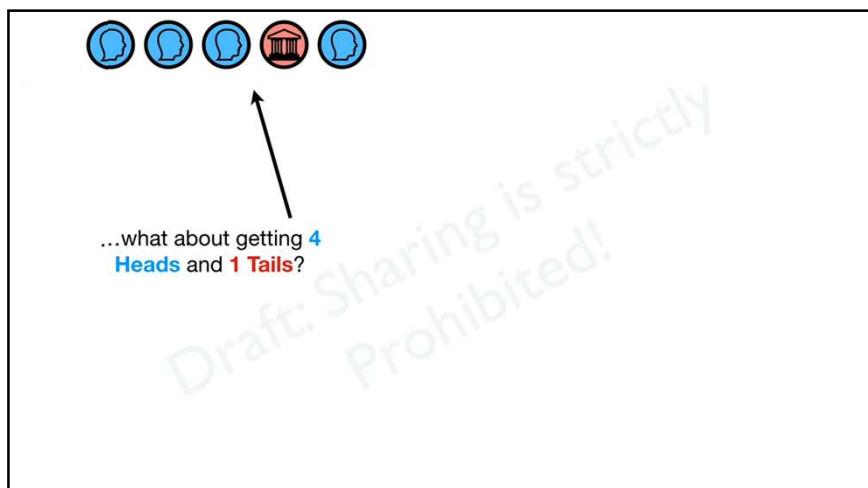


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- 2) The probability of observing something else that is equally rare.
- 3) The probability of observing something rarer or more extreme.



OK, now that we know that getting **2 Heads** in a row is not very special or statistically significant...





In other words, we can calculate a **p-value** to test this hypothesis:



In other words, we can calculate a **p-value** to test this hypothesis:

Even though I got 4 Heads and 1 Tails, my coin is no different from a normal coin.



Again, although we want to know if the coin is special, the **Null Hypothesis** focuses on a normal coin...

Even though I got 4 Heads and 1 Tails, my coin is no different from a normal coin.



...but if we get a small **p-value** and reject the **Null Hypothesis**, we will know that our coin *is* special.

Even though I got 4 Heads and 1 Tails, my coin is no different from a normal coin.



So let's calculate the **p-value** for getting **4 Heads** and **1 Tails**.

Even though I got **4 Heads** and **1 Tails**, my coin is no different from a normal coin.



First, we know that it is *possible* to flip a normal coin **5 times** and get **Heads** each time...



...so let's keep track of that with **5 Blue H's**.

HHHHH



We can also flip a coin **5 times** and get **4 Heads** and **1 Tails**.

T H H H H
H T H H H
H H T H H
H H H T H
H H H H T





NOTE: There are **5** different ways to get **4 Heads** and **1 Tails**, but we treat them all the same because the order of **Heads** and **Tails** doesn't matter.

HHHHH
TTHHH
HTTHH
HHTHH
HHHTH
HHHHT



Likewise, there are **10** ways that we can flip a coin and get **3 Heads** and **2 Tails**...

TTTHH
THTHH
THHTH
TTHHT
THHHT
HTHHH
HTTHH
HTHTH
HTHHT
HHHTH
HHHHT



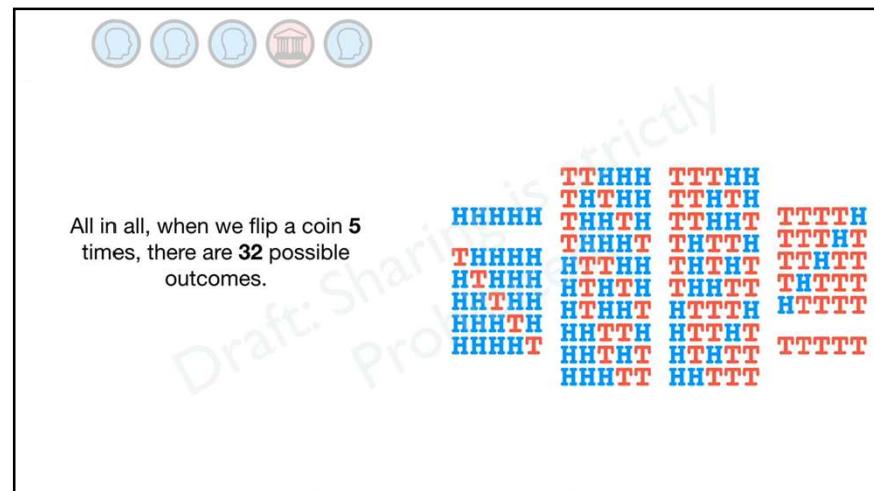
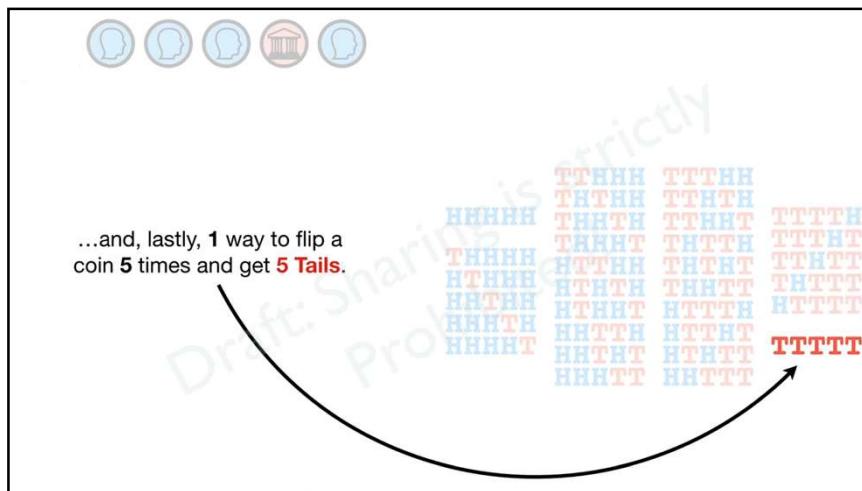
...and **10** ways to get **2 Heads** and **3 Tails**...

TTTHH TTTHH
THTHH TTHHT
THHHH THHTH
HTHHH HTTHH
HTHTH HTHTH
HHTHH HHTTH
HHHTH HTTHT
HHHTT HHTHT
HHHTT HHTTT
HHHTT HHHTT



...and **5** ways to get **1 Heads** and **4 Tails**...

TTTHH TTTHH
THTHH TTHHT
THHHH THHTH
HTHHH HTTHH
HTHTH HTHTH
HHTHH HHTTH
HHHTH HTTHT
HHHTT HHTHT
HHHTT HHTTT
HHHTT HHHTT





The **p-value** for getting **4 Heads** and **1 Tails** is...

- 1) The probability we randomly get **4 Heads** and **1 Tails**:

$$\frac{5}{32}$$

Since **5** of the **32** outcomes had **4 Heads** and **1 Tails**.

HHHHH	TTHHH	TTTTH	
THTHH	TTHTH	TTHTH	TTTHT
THHTH	TTHHT	TTHTT	TTHTT
THHHT	THTHT	TTHTT	TTHTT
HHTHH	HTHTH	HTHTT	HTHTT
HHTHT	HHTTH	HTTHT	HTTHT
HHTTH	HHTHT	HTHTT	HTHTT
HHTTT	HHTHT	HTHTT	HTHTT
HHHTH	HHTHT	HTHTT	HTHTT
HHHTT	HHTHT	HTHTT	HTHTT
HHHTT	HHTHT	HTHTT	HTHTT



The **p-value** for getting **4 Heads** and **1 Tails** is...

- 1) The probability we randomly get **4 Heads** and **1 Tails**:

$$\frac{5}{32} +$$

- 2) The probability we randomly get something else that is equally rare:

HHHHH	TTHHH	TTTTH	
TTHHH	THHHT	TTHHT	TTTHT
THHHT	THHTH	THHTT	THHTT
HHTHH	HTHTH	HTHTT	HTHTT
HHTHT	HHTTH	HTHTT	HTHTT
HHTTH	HHTHT	HTHTT	HTHTT
HHTTT	HHTHT	HTHTT	HTHTT
HHHTH	HHTHT	HTHTT	HTHTT
HHHTT	HHTHT	HTHTT	HTHTT
HHHTT	HHTHT	HTHTT	HTHTT



The **p-value** for getting **4 Heads** and **1 Tails** is...

- 1) The probability we randomly get **4 Heads** and **1 Tails**:

$$\frac{5}{32} + \frac{5}{32} \leftarrow$$

TTTTH			
TTHHT	TTTTH	TTTTH	
TTTHT	TTTTH	TTTTH	
TTHTT	TTTTH	TTTTH	
HTTHH	HTTHT	HTTHT	TTTHT
HTHTH	HTTHT	HTTHT	TTTHT
HTHTH	HTTHT	HTTHT	TTTHT
HTHTH	HTTHT	HTTHT	TTTHT
HTHTH	HTTHT	HTTHT	TTTHT
HTHTH	HTTHT	HTTHT	TTTHT
HHHTH	HHHTH	HHHTH	HHHTH
HHHTT	HHHTH	HHHTH	HHHTH
HHHTT	HHHTH	HHHTH	HHHTH
HHHTT	HHHTH	HHHTH	HHHTH
HHHTT	HHHTH	HHHTH	HHHTH
HHHTT	HHHTH	HHHTH	HHHTH

Since **5** of the **32** outcomes has **1 Head** and **4 Tails**.

- 2) The probability we randomly get something else that is equally rare:



The **p-value** for getting **4 Heads** and **1 Tails** is...

- 1) The probability we randomly get **4 Heads** and **1 Tails**:

$$\frac{5}{32} + \frac{5}{32} +$$

- 2) The probability we randomly get something else that is equally rare:

TTTTH			
THHHT	TTTTH	TTTTH	TTTHT
THHTH	TTTTH	TTTTH	TTTHT
HHTHH	HTTHT	HTTHT	HTTHT
HHTHT	HTTHT	HTTHT	HTTHT
HHTTH	HTTHT	HTTHT	HTTHT
HHTTT	HTTHT	HTTHT	HTTHT
HHHTH	HHHTH	HHHTH	HHHTH
HHHTT	HHHTH	HHHTH	HHHTH
HHHTT	HHHTH	HHHTH	HHHTH
HHHTT	HHHTH	HHHTH	HHHTH
HHHTT	HHHTH	HHHTH	HHHTH
HHHTT	HHHTH	HHHTH	HHHTH

- 3) The probability we randomly get something rarer or more extreme:



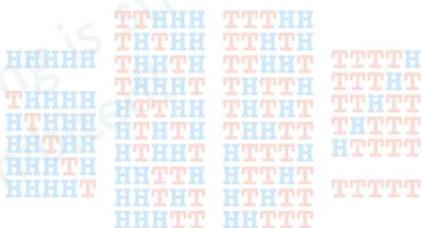
The **p-value** for getting **4 Heads** and **1 Tails** is...

- 1) The probability we randomly get **4 Heads** and **1 Tails**: $\frac{5}{32} + \frac{5}{32} + \frac{2}{32}$
- 2) The probability we randomly get something else that is equally rare: 
- 3) The probability we randomly get something rarer or more extreme:

Because both **5 Heads** and **5 Tails** only occurred once each, they are rarer than **4 Heads** and **1 Tails**.



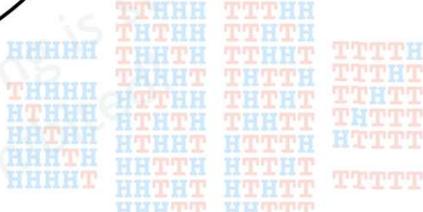
The **p-value** for getting **4 Heads** and **1 Tails** is...

$$\frac{5}{32} + \frac{5}{32} + \frac{2}{32} = 0.375$$




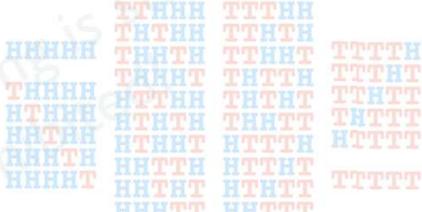
Even though I got **4 Heads** and **1 Tails**, my coin is no different from a normal coin.

Again, we typically only reject the **Null Hypothesis** if the **p-value** is less than **0.05**...

$$\frac{5}{32} + \frac{5}{32} + \frac{2}{32} = 0.375$$




In other words, the data, getting **4 Heads** and **1 Tails**, did not convince us that our coin was special.

$$\frac{5}{32} + \frac{5}{32} + \frac{2}{32} = 0.375$$


With coin tosses, it's pretty easy to calculate **probabilities** and **p-values** because it's pretty easy to list all of the possible outcomes.

HHHHH	TTHHH	TTTHH	TTTHT	TTTTH
TTHHH	THTHH	TTHTH	TTTHT	TTHTH
HTHHH	HHTHH	HTHTH	HTTHT	HTHTT
HHTHH	HHTTH	HHTHT	HHTTT	HHTTT
HHHTH	HHHTT	HHTHT	HHTTT	HTTTT
HHHHT	HHHTH	HHTTH	HHTHT	TTTTT
HHHTT	HHHTH	HHTHT	HHTTT	

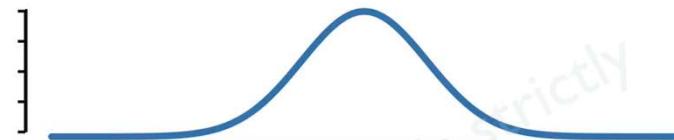
But what if we wanted to calculate **probabilities** and **p-values** for how tall or short people are?

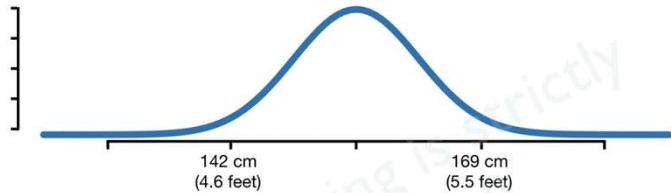


In theory, we could try to list every single possible value for height.

152.4 cm	152.9 cm	153.4 cm	etc...
152.5 cm	153.0 cm	153.5 cm	...
152.6 cm	153.1 cm	153.6 cm	etc...
152.7 cm	153.2 cm	153.6 cm	...
152.8 cm	153.3 cm	153.8 cm	etc...

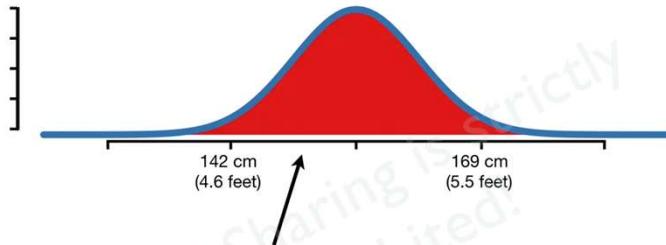
However, in practice, when we calculate **probabilities** and **p-values** for something continuous, like **Height**, we usually use something called a *statistical distribution*.



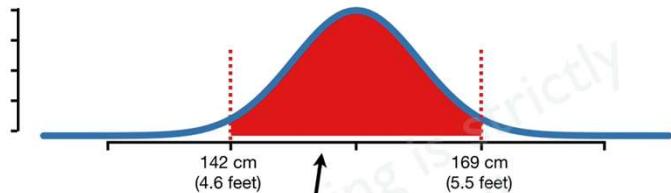


Here we have a distribution of height measurements from Brazilian women between **15** and **49** years old taken in **1996**.

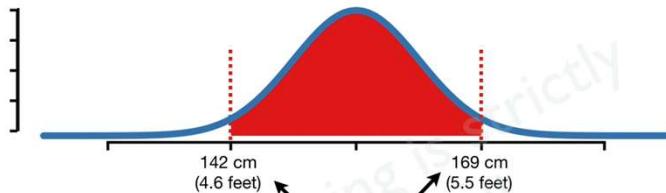
Data from Height of Nations: A Socioeconomic Analysis of Cohort Differences and Patterns among Women in 54 Low-to-Middle-Income Countries, Subramanian, Ozaltin and Finlay (2011)



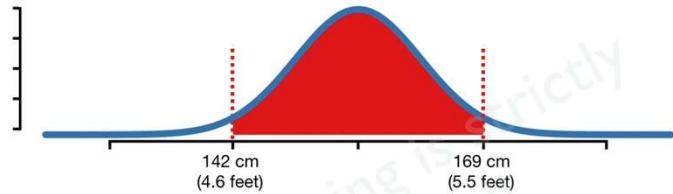
The **red area** under the curve indicates the probability that a person's height will be within a range of possible values.



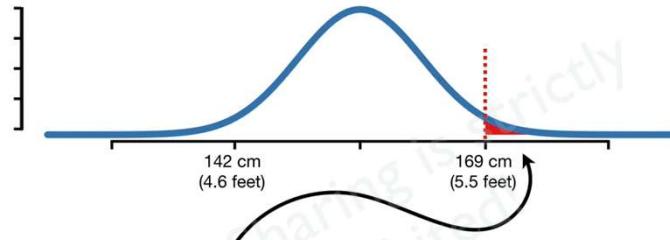
For example, **95%** of the area under the curve is between **142** and **169**...



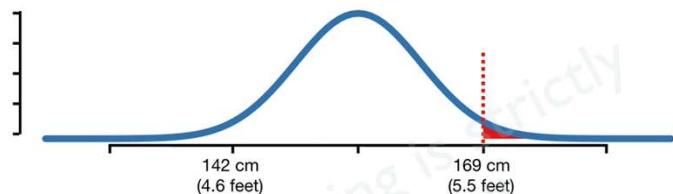
...and that means that **95%** of the Brazilian women were between **142** and **169** cm tall.



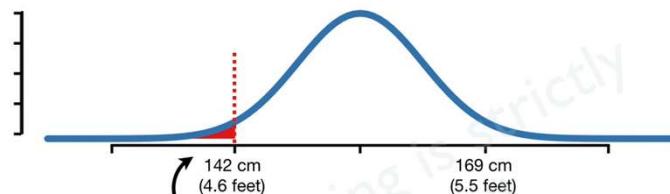
In other words, there is a **95%** probability that each time we measure a Brazilian woman, their height will be between **142** and **169** cm.



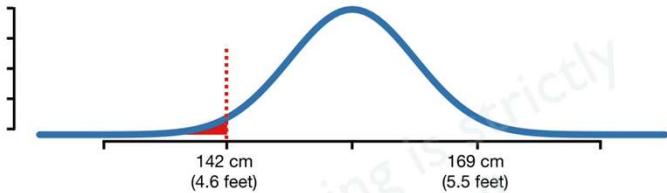
2.5% of the total area under the curve is greater than **169**.



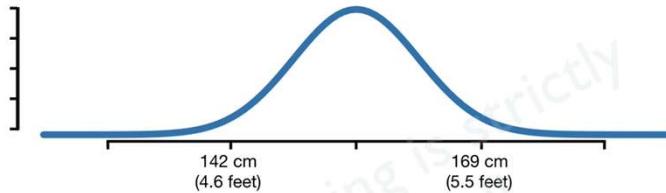
And that means there is a **2.5%** probability that each time we measure a Brazilian woman, their height will be *greater than* **169** cm.



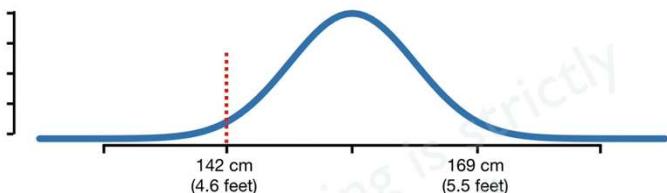
Likewise, **2.5%** of the total area under the curve is less than **142**.



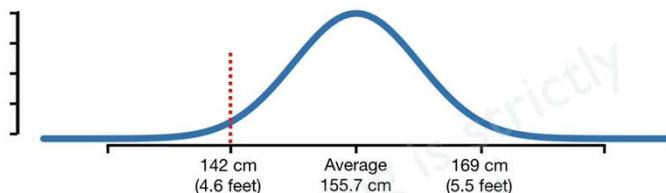
Thus, there is a **2.5%** probability that each time we measure a Brazilian woman, their height will be *less* than **142 cm**.



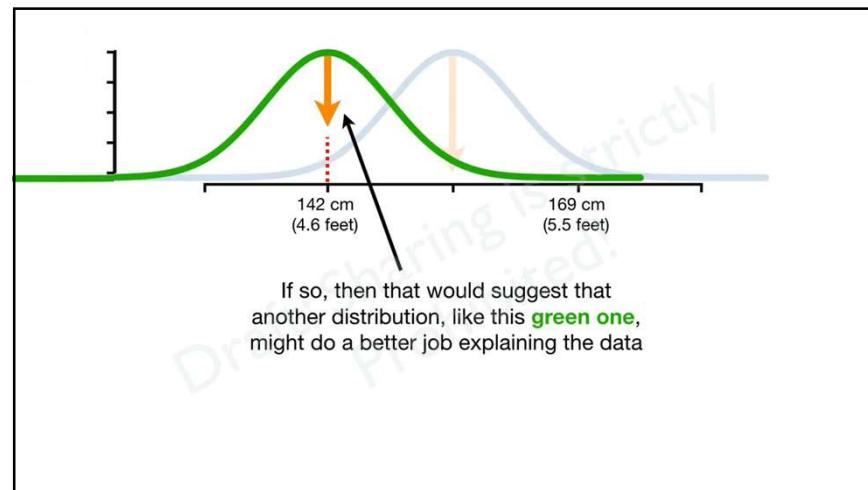
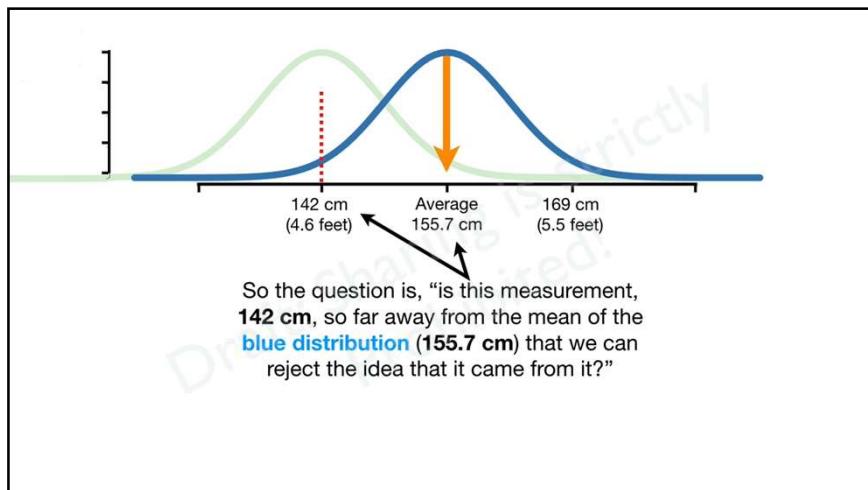
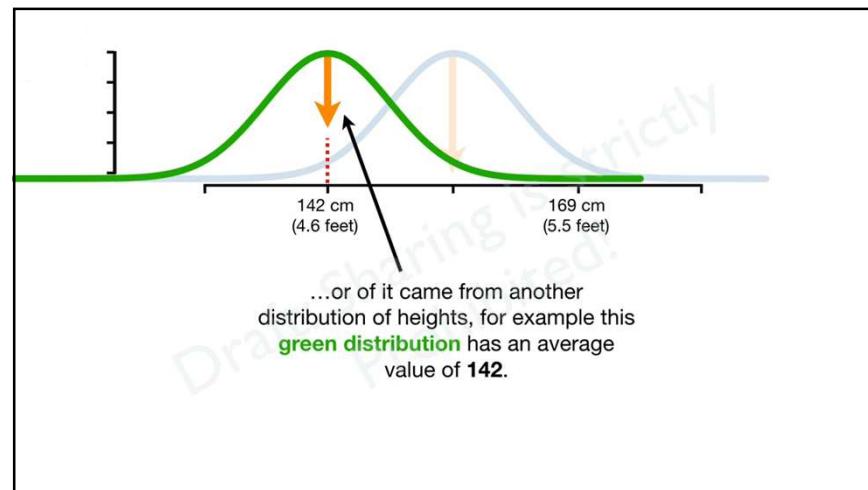
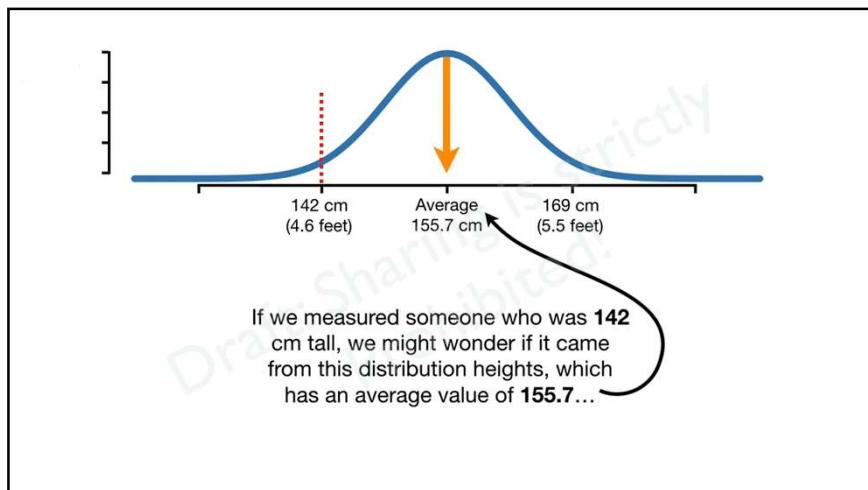
To calculate **p-values** with a distribution, you add up the percentages of area under the curve.

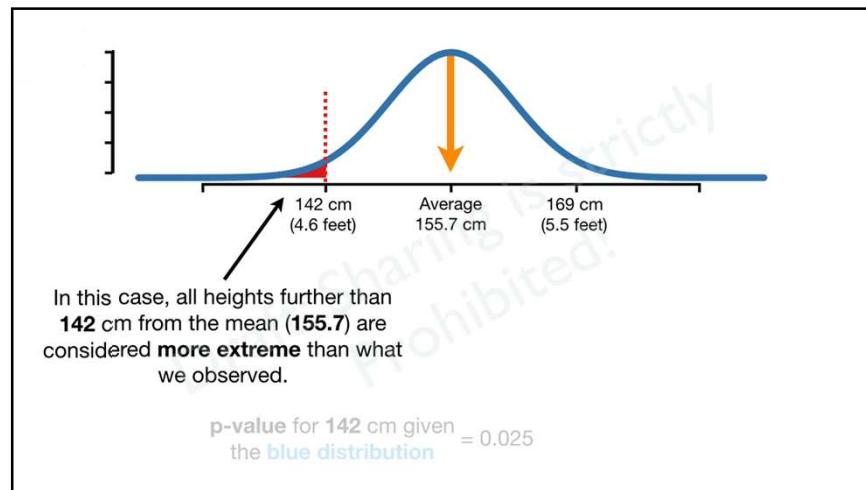
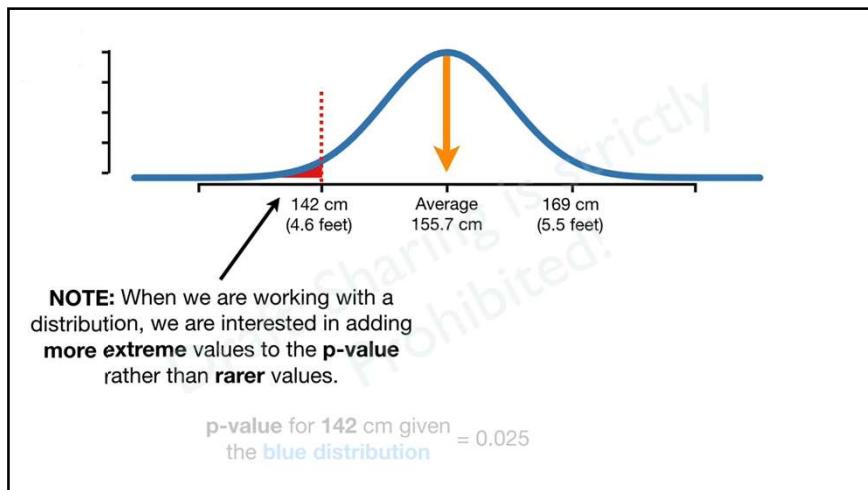
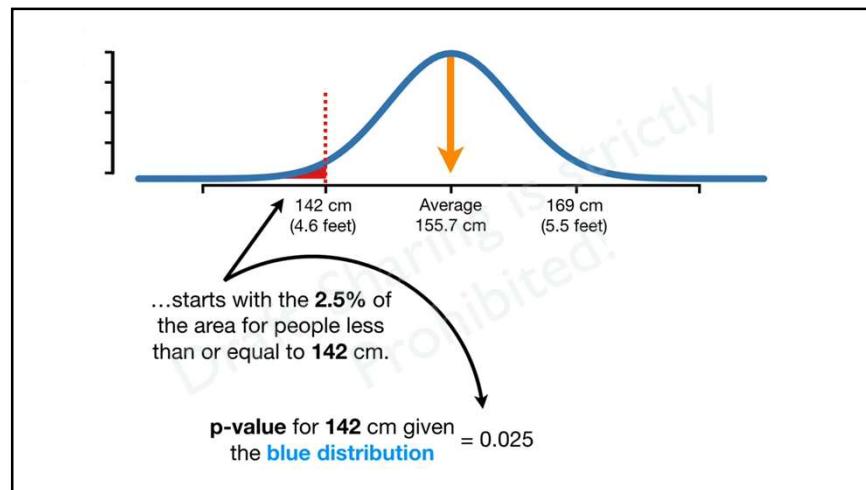
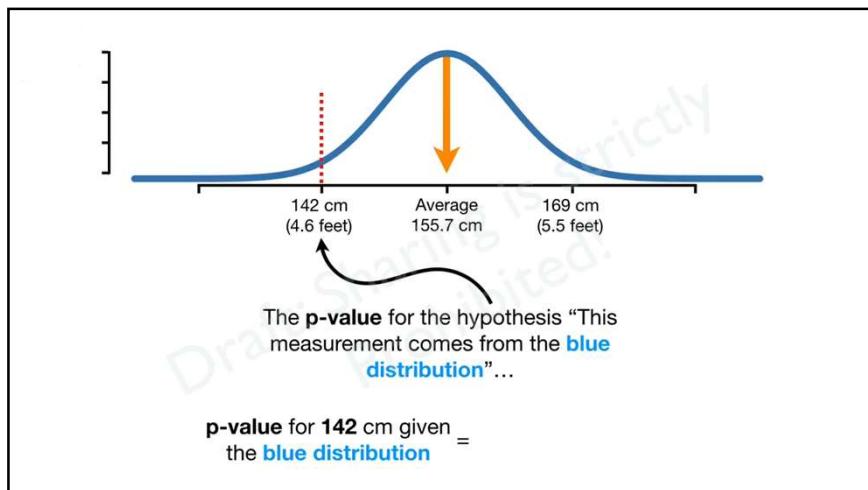


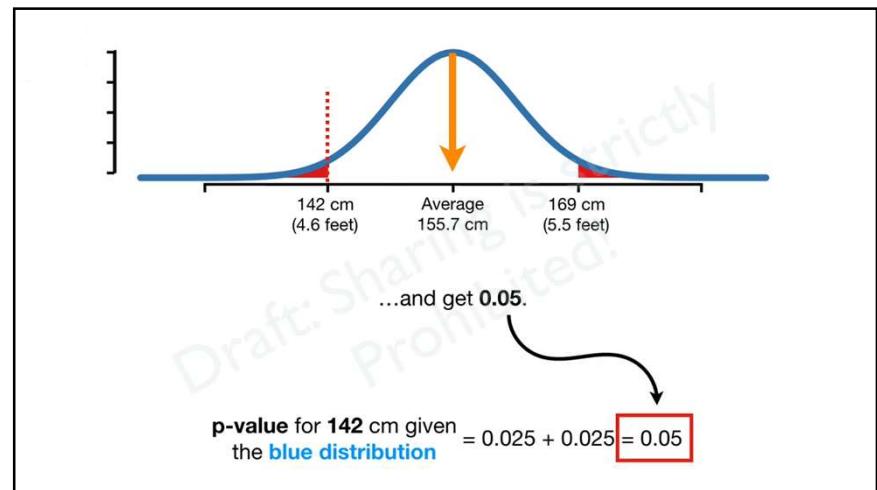
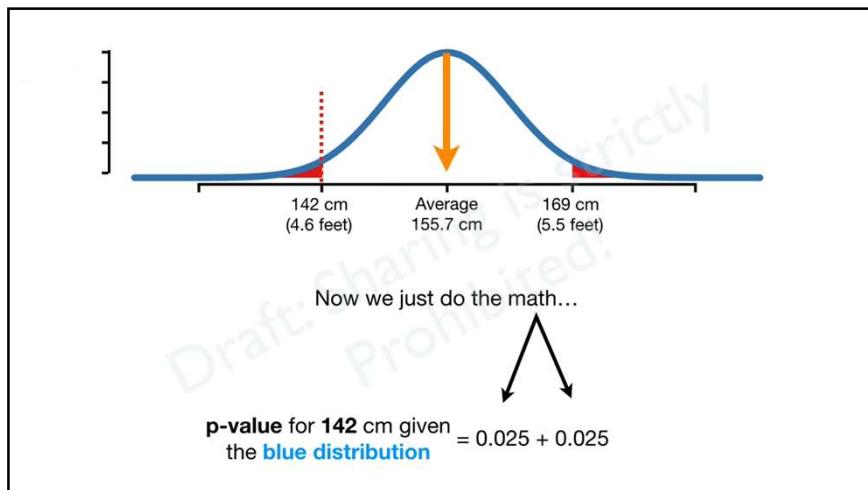
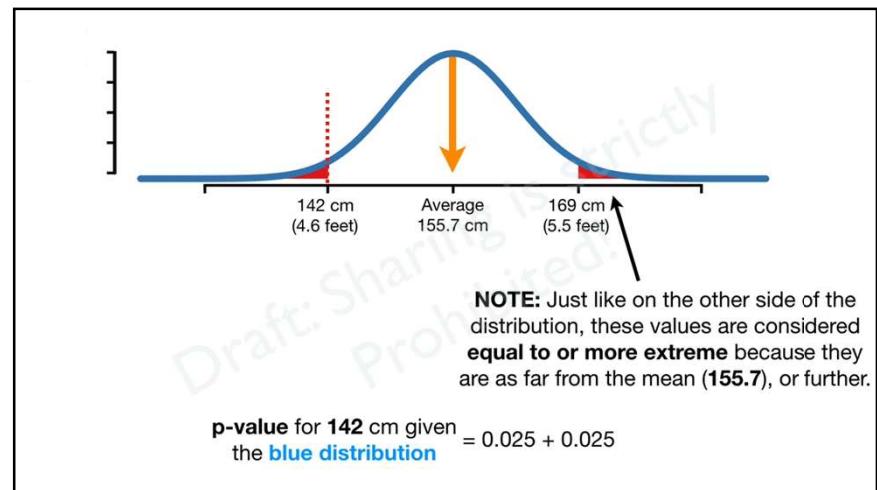
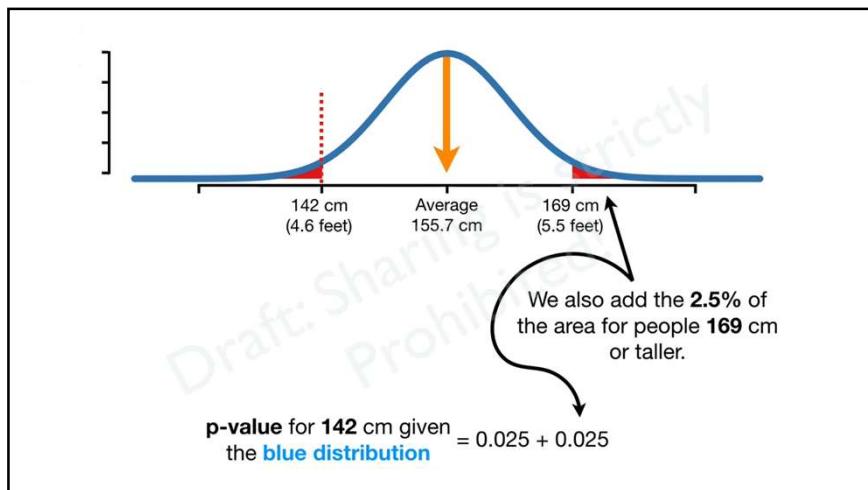
For example, imagine we measured someone who was **142 cm** tall.

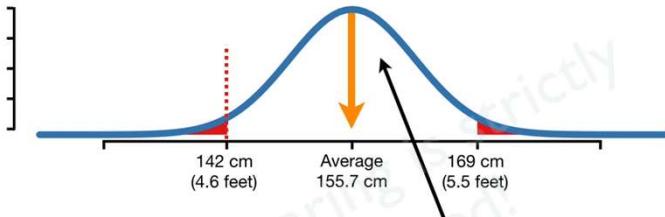


If we measured someone who was **142 cm** tall, we might wonder if it came from this distribution heights, which has an average value of **155.7...**



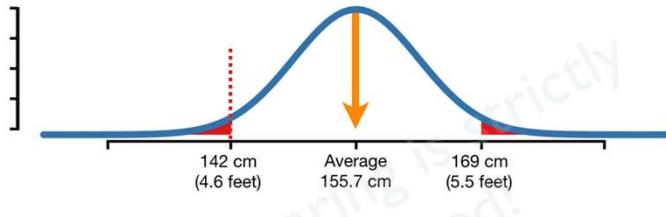






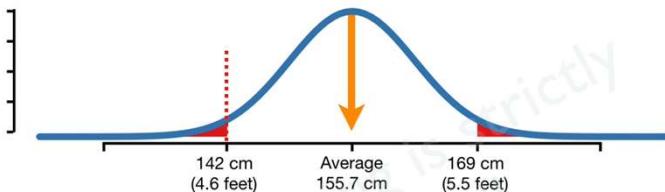
So the **p-value** for the hypothesis “Someone 142 cm tall could come from the **blue distribution**” is **0.05**.

$$\text{p-value for } 142 \text{ cm given the blue distribution} = 0.025 + 0.025 = 0.05$$



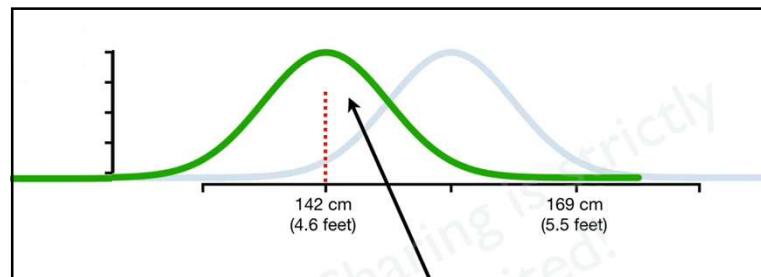
And since the cutoff for significance is usually **0.05**, we would say...

$$\text{p-value for } 142 \text{ cm given the blue distribution} = 0.025 + 0.025 = 0.05$$



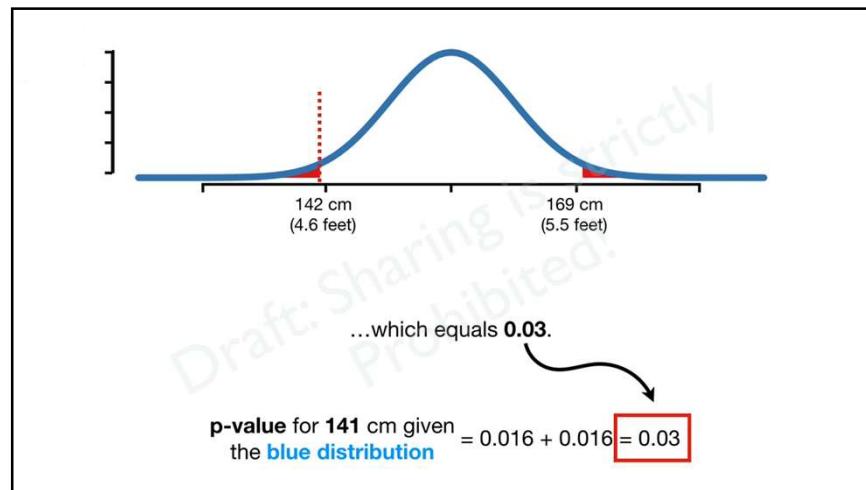
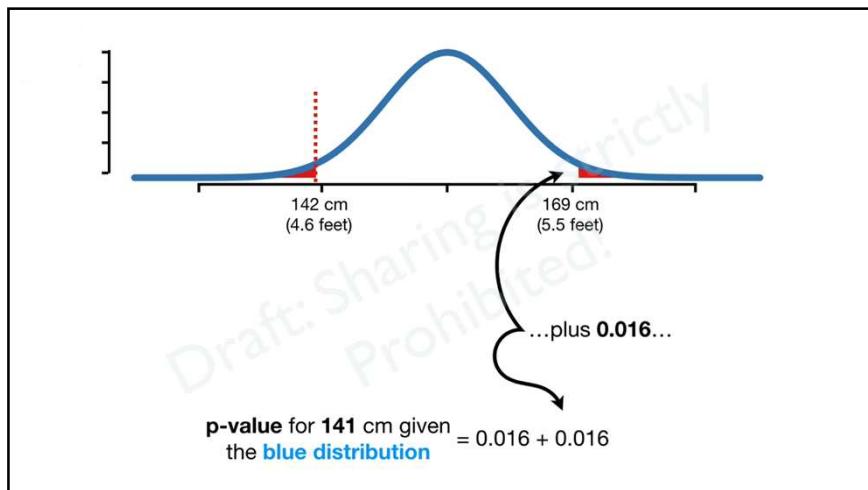
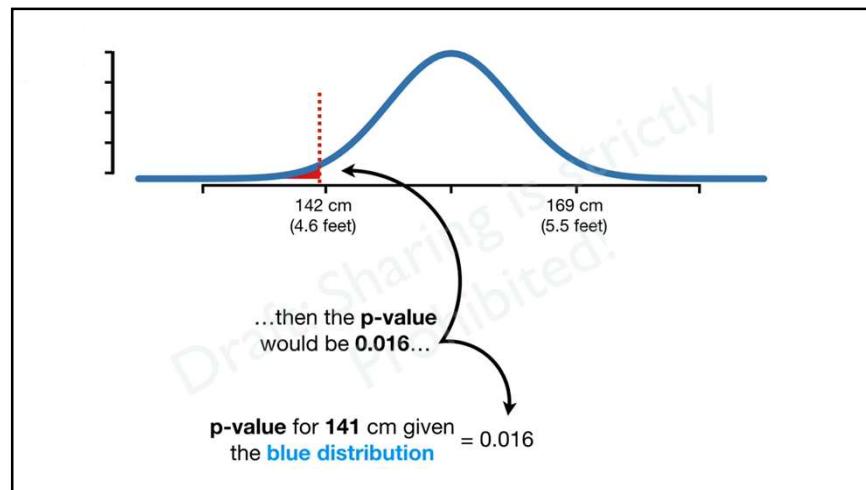
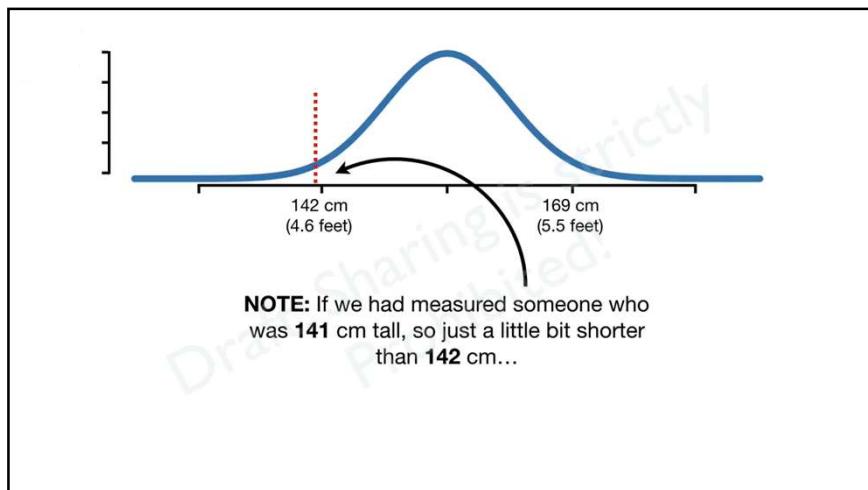
“Hmmm. Maybe it could come from this distribution, maybe not. It’s hard to tell since the **p-value** is right on the borderline.”

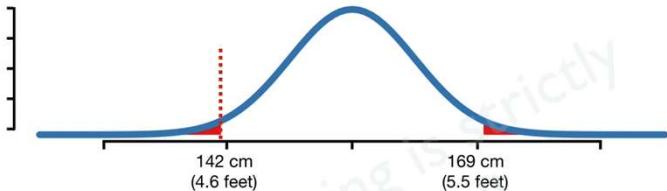
$$\text{p-value for } 142 \text{ cm given the blue distribution} = 0.025 + 0.025 = 0.05$$



...or maybe they come from this distribution.
The data are inconclusive.

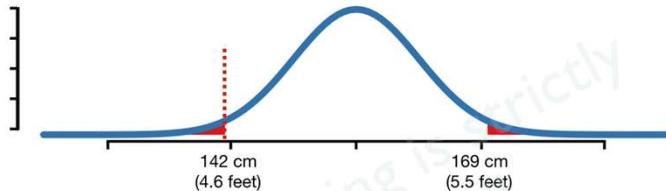
$$\text{p-value for } 142 \text{ cm given the blue distribution} = 0.025 + 0.025 = 0.05$$





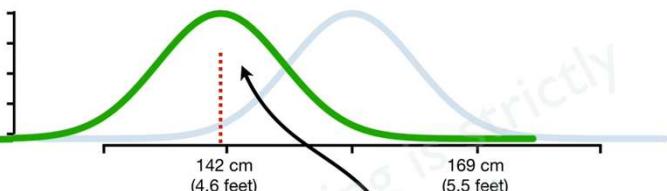
And since $0.03 < 0.05$, the standard threshold, we can reject the hypothesis that, given the **blue distribution**, it is normal to measure someone 141 cm tall.

$$\text{p-value for } 141 \text{ cm given the blue distribution} = 0.016 + 0.016 = 0.03$$



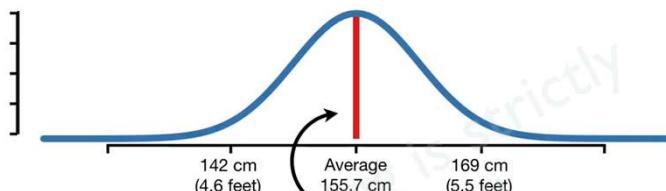
Thus, we will conclude that it's pretty special to measure someone that short.

$$\text{p-value for } 141 \text{ cm given the blue distribution} = 0.016 + 0.016 = 0.03$$

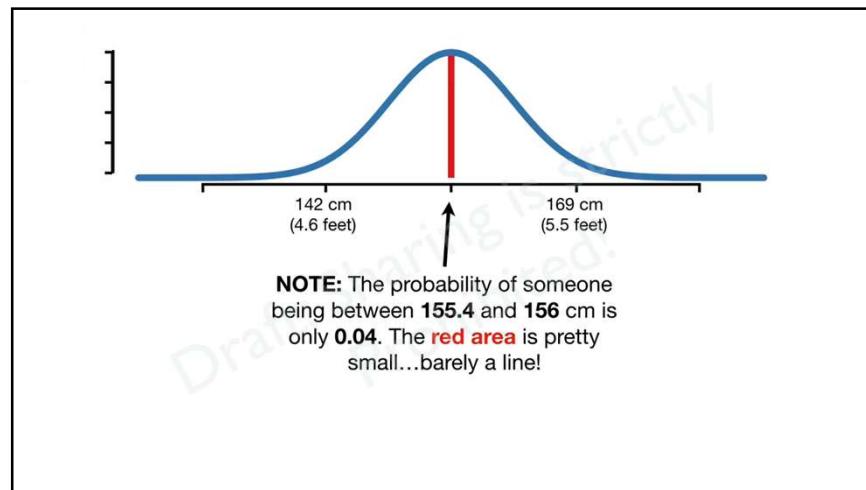
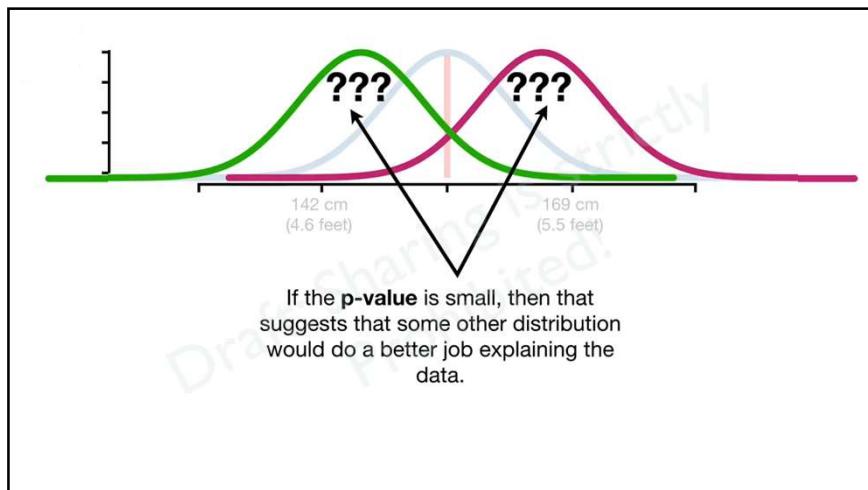
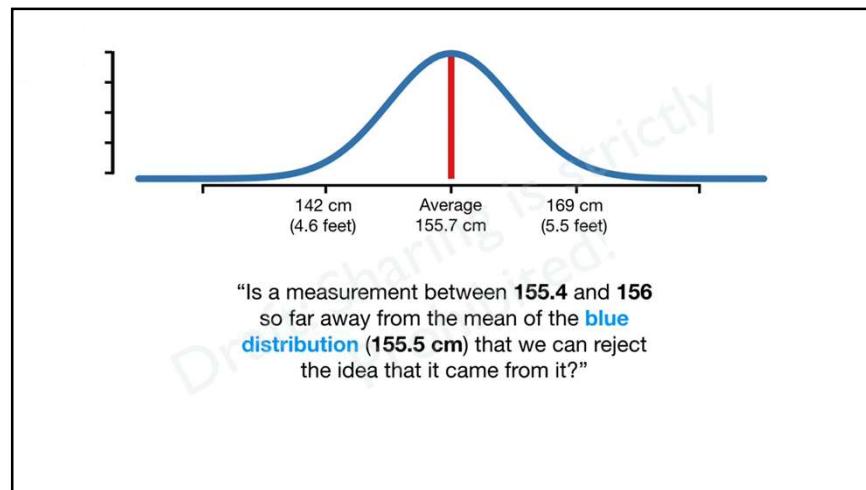
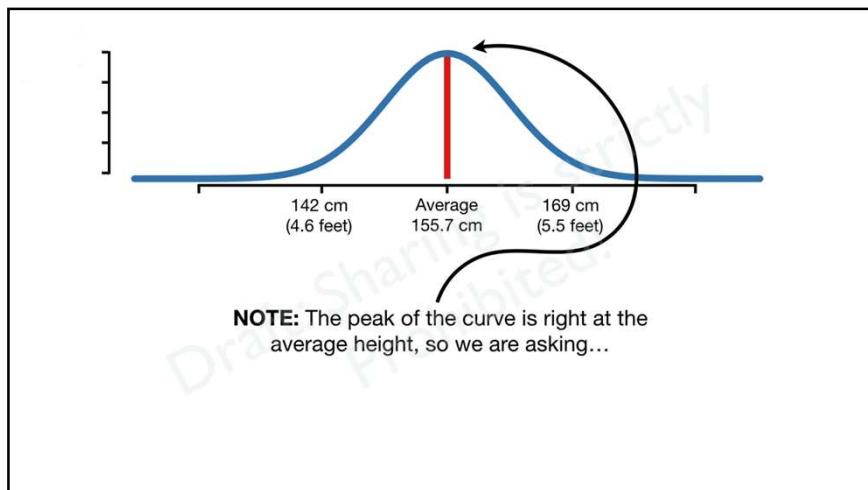


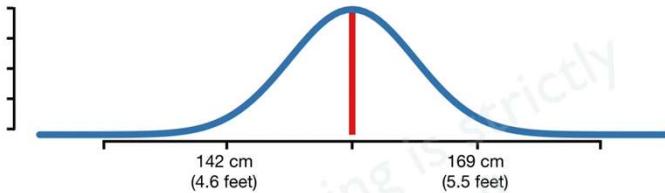
And that suggests that a different distribution of heights makes more sense.

$$\text{p-value for } 141 \text{ cm given the blue distribution} = 0.016 + 0.016 = 0.03$$



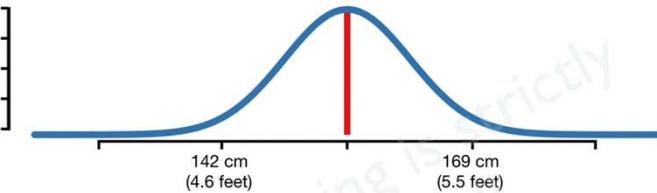
Now, what if we measured someone who is between 155.4 and 156 cm tall?





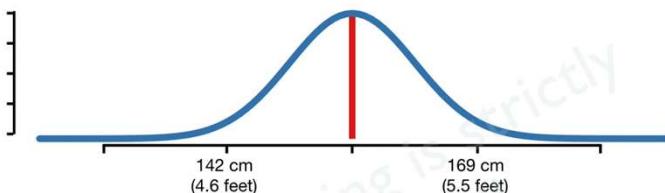
So **0.04** is the first part of calculating the **p-value**, since, given this distribution of heights, that is the probability that we would randomly measure someone in this range of values.

**p-value for between
155.4 and 156 cm given = 0.04
the blue distribution**



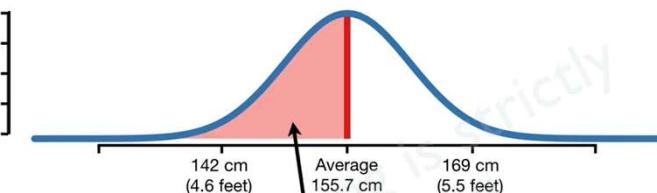
So **0.04** is the first part of calculating the **p-value**, since, given this distribution of heights, that is the probability that we would randomly measure someone in this range of values.

**p-value for between
155.4 and 156 cm given = 0.04
the blue distribution**



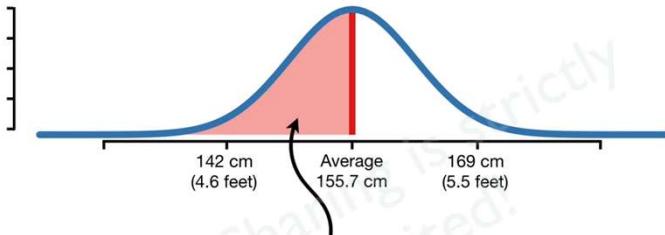
...now we need to figure out the **more extreme** parts.

**p-value for between
155.4 and 156 cm given = 0.04
the blue distribution**



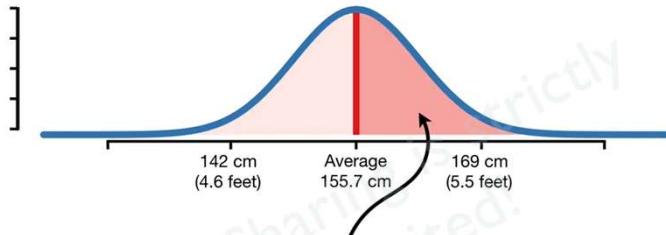
On the left side, all of the heights **< 155.4** are further from the mean (155.7), thus, they are all **more extreme**.

**p-value for between
155.4 and 156 cm given = 0.04
the blue distribution**



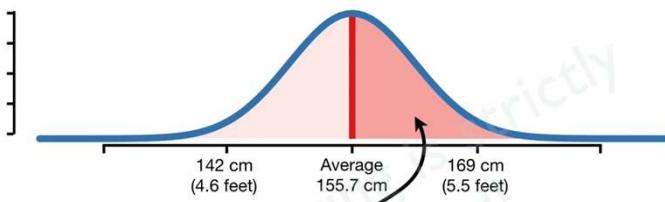
And because the **48%** of the area under the curve is for heights **< 155.4**, we add **0.48** to the **p-value**.

p-value for between
155.4 and 156 cm given = 0.04
the **blue distribution**



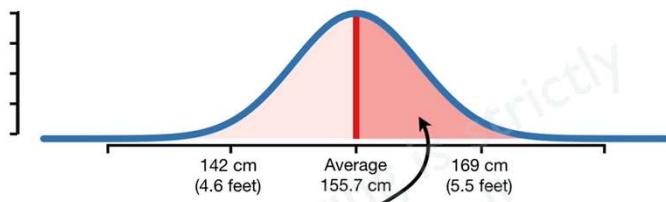
On the right side, all of the heights **> 156** are further from the mean (155.7), thus, they are all **more extreme**.

p-value for between
155.4 and 156 cm given = 0.04 + 0.48
the **blue distribution**



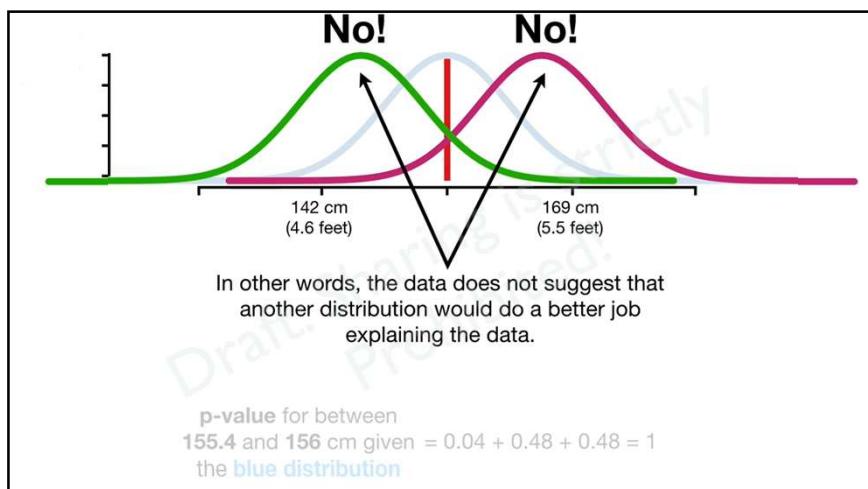
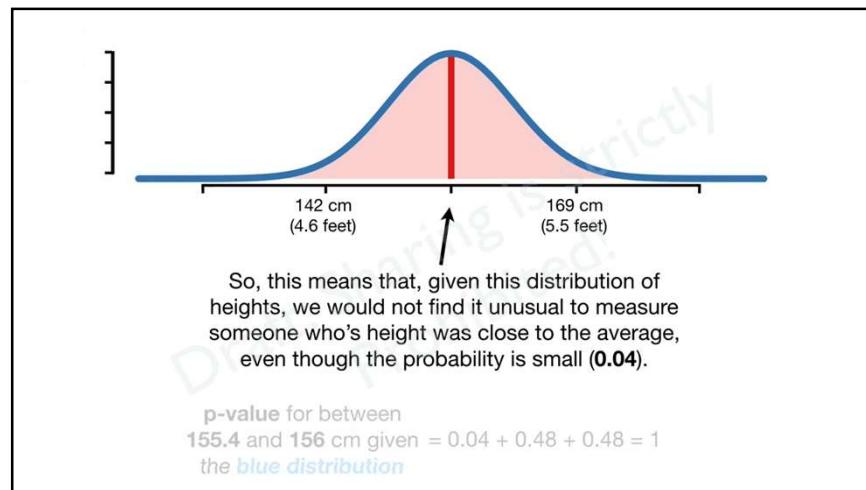
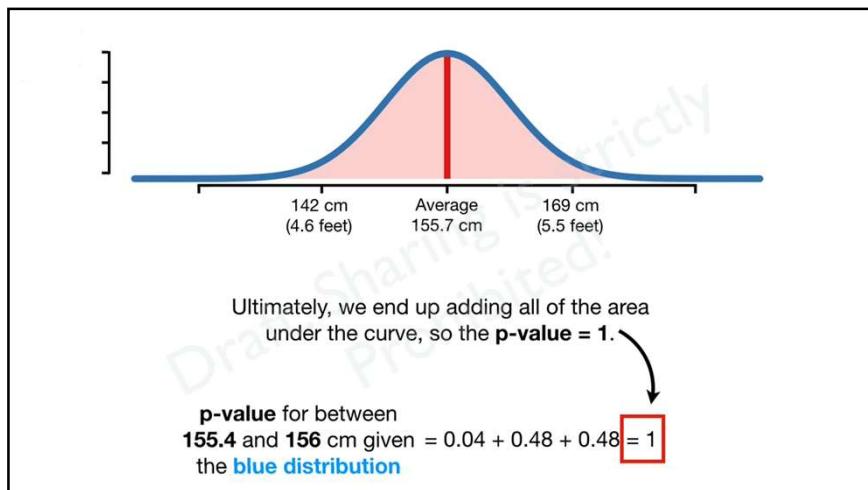
And because the **48%** of the area under the curve is for heights **> 156**, we add **0.48** to the **p-value**.

p-value for between
155.4 and 156 cm given = 0.04 + 0.48
the **blue distribution**



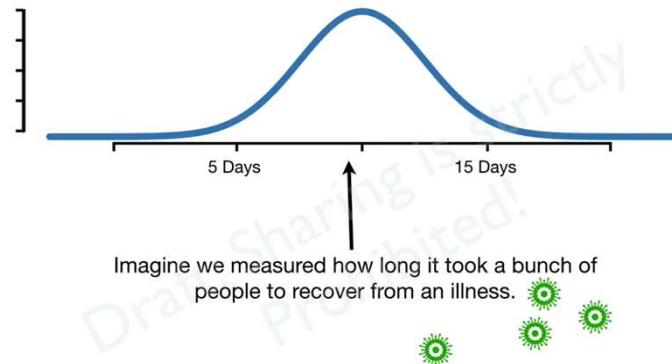
And because the **48%** of the area under the curve is for heights **> 156**, we add **0.48** to the **p-value**.

p-value for between
155.4 and 156 cm given = 0.04 + 0.48 + 0.48 =
the **blue distribution**

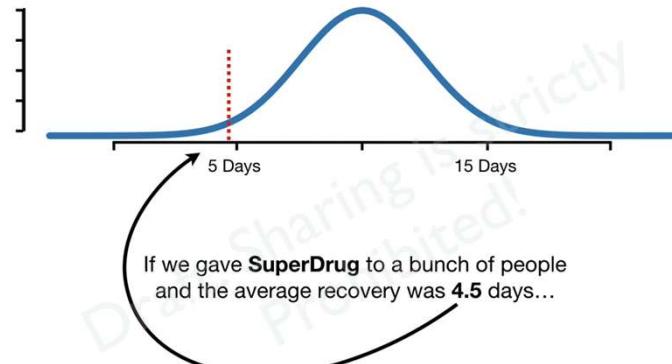


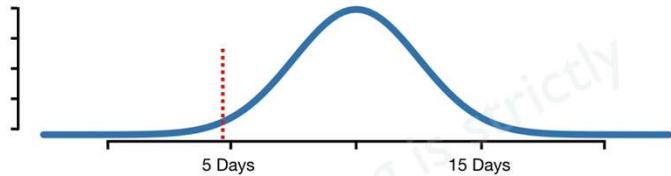
So far all we've only talked about **2-Sided p-values**.

Now I'll give you an example of a **One-Sided p-value** and tell you why it has the potential to be dangerous.

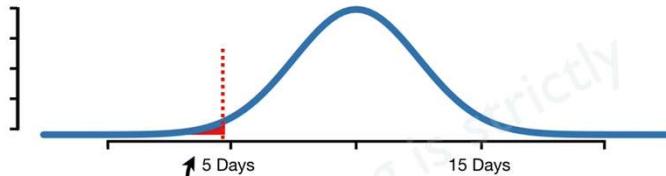


Now imagine we created a new drug, **SuperDrug**, and wanted to see if it helped people recover in fewer days.



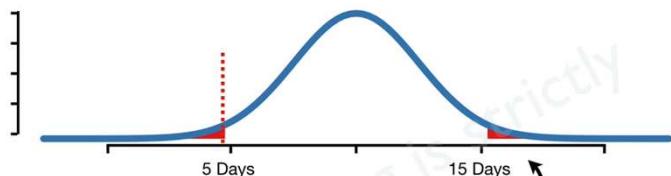


...then a **Two-Sided p-value**, like the ones we've been computing all along, would be...



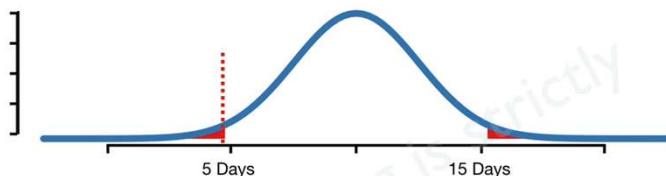
...the sum of *this* area under the curve, **0.016...**

Two-Sided p-value for 4.5 days = 0.016



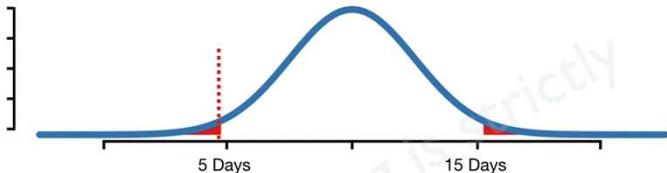
...plus *this* area under the curve, **0.016...**

Two-Sided p-value for 4.5 days = $0.016 + 0.016$



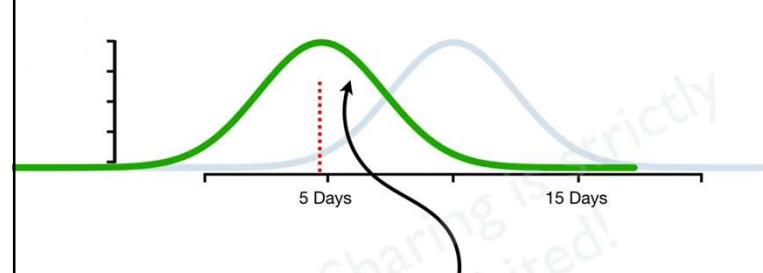
...and the total is **0.03**.

Two-Sided p-value for 4.5 days = $0.016 + 0.016 = 0.03$



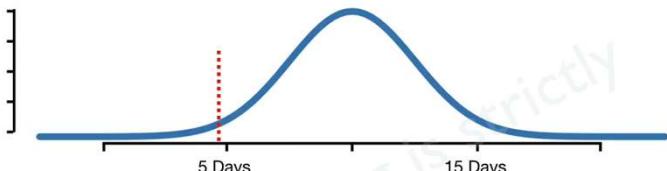
And since $0.03 < 0.05$, the **Two-Sided p-value** tells us that, given this distribution of recovery times, **SuperDrug** did something unusual.

$$\text{Two-Sided p-value for 4.5 days} = 0.016 + 0.016 = 0.03$$

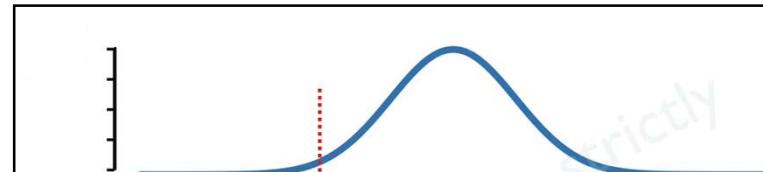


...and that suggests that some other distribution does a better job explaining the data.

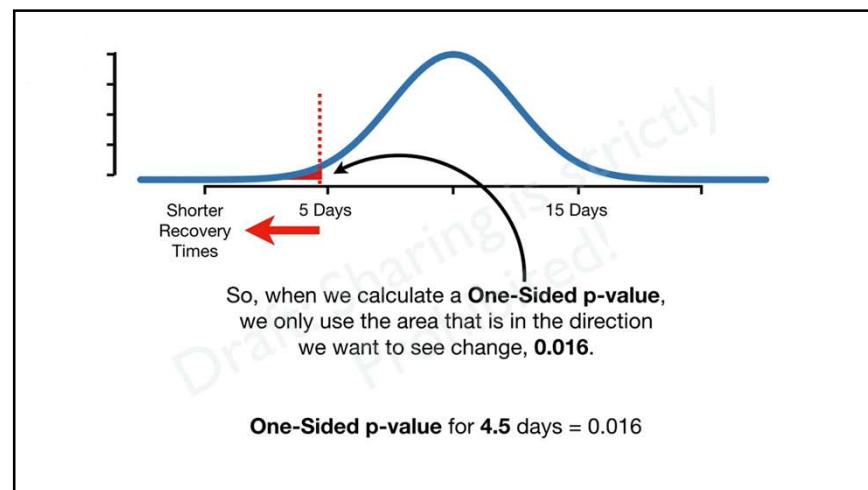
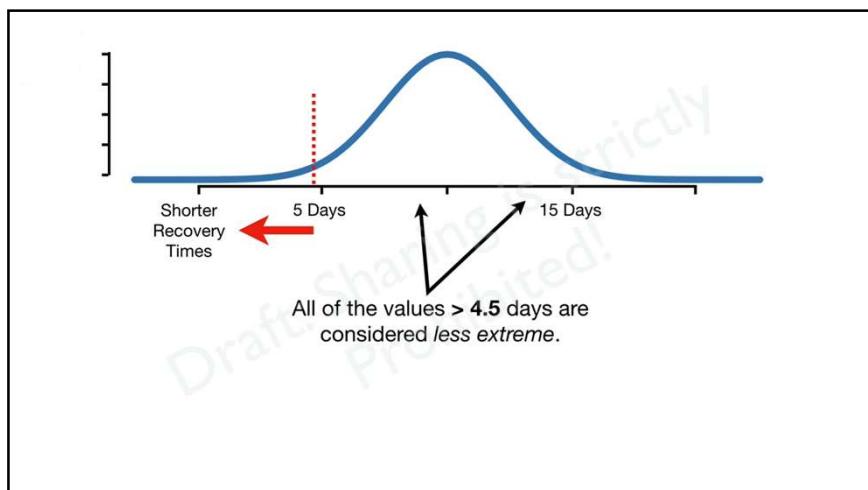
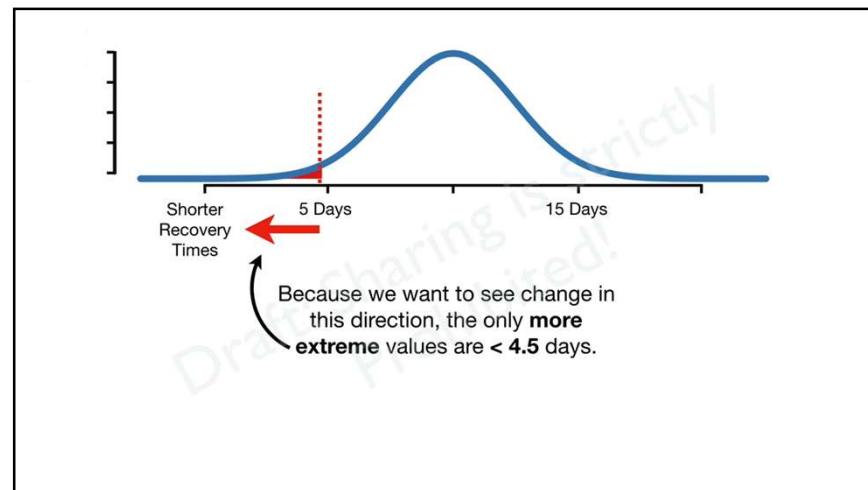
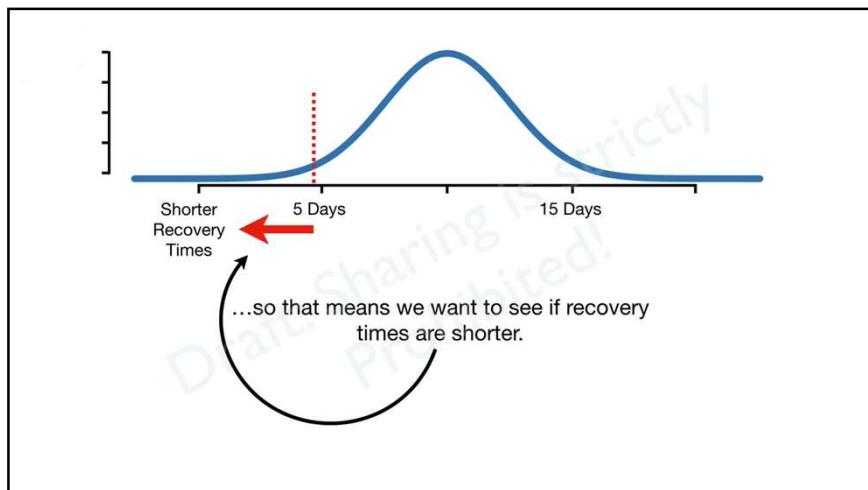
$$\text{Two-Sided p-value for 4.5 days} = 0.016 + 0.016 = 0.03$$

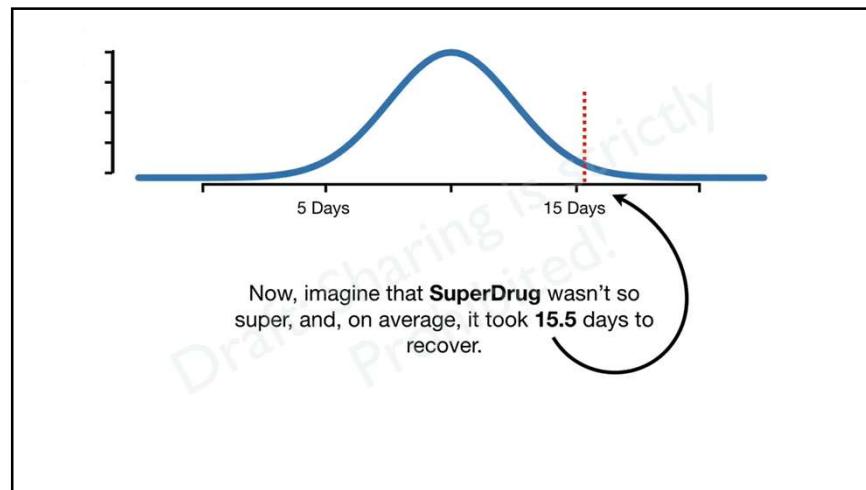
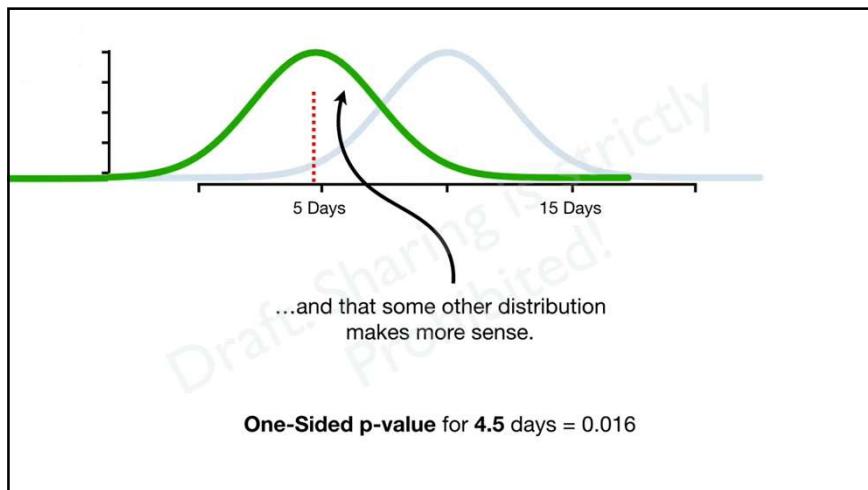
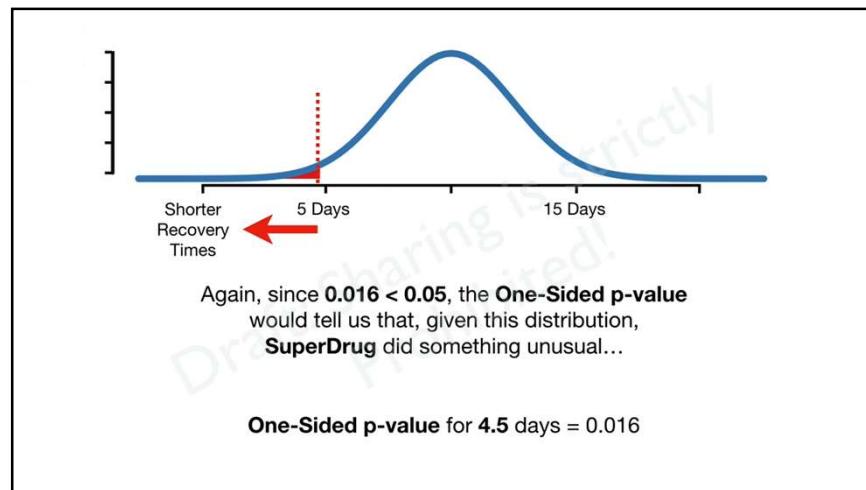
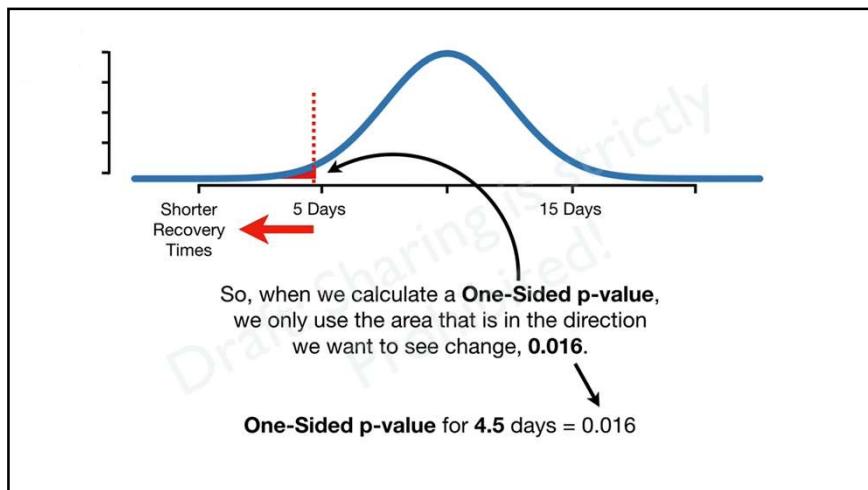


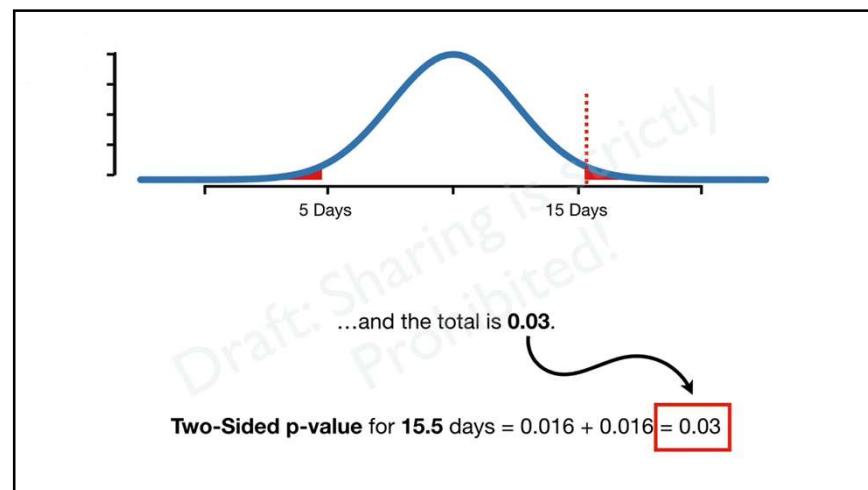
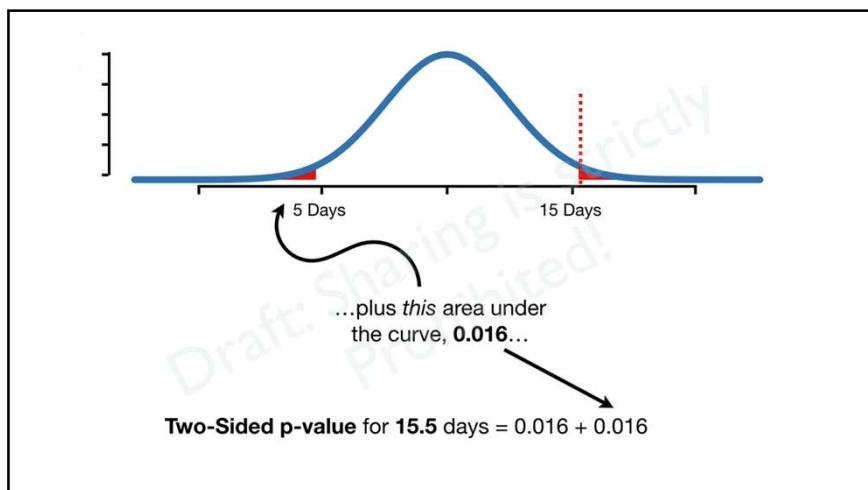
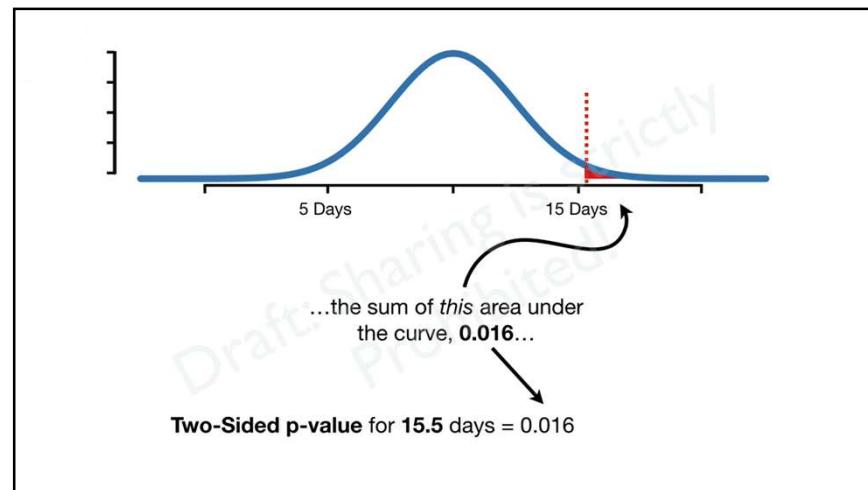
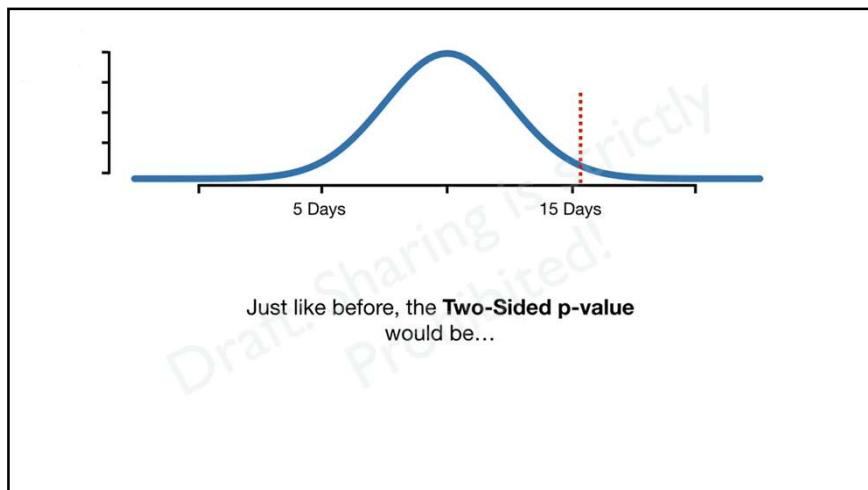
For a **One-Sided p-value**, the first thing we do is decide which direction we want to see change in.

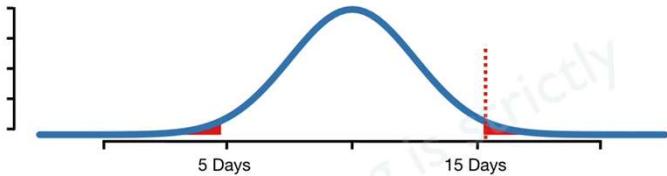


In this case, we'd like **SuperDrug** to shorten the time it takes to recover from the illness...

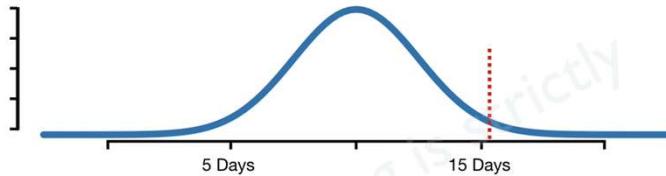




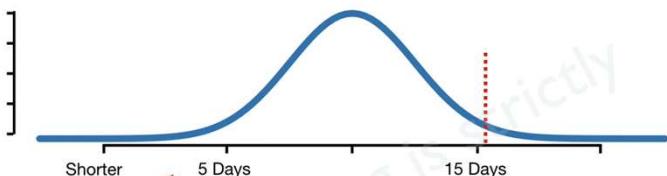




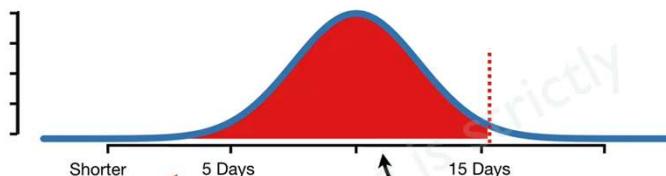
In other words, regardless of whether **SuperDrug** is super and makes things better, or if it is not so super and makes things worse, a **Two-Sided p-value** will detect something unusual happened.



For a **One-Sided p-value**, the first thing we do is decide which direction we want to see change in.

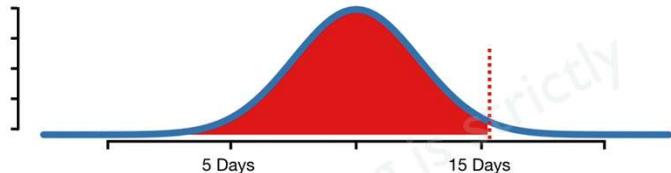


...and just like before, that means we want to see if recovery times are shorter.



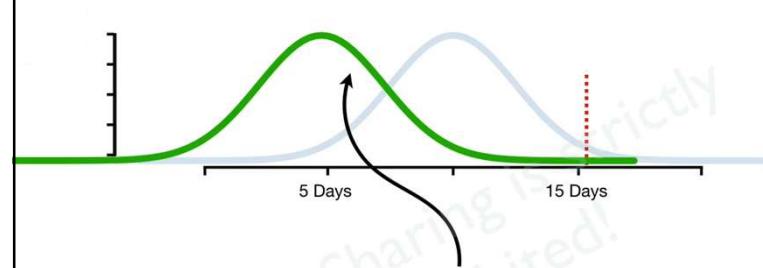
So the **One-Sided p-value** is this huge area, **0.98**, because it is **more extreme** in the direction we want to see change.

One-Sided p-value for 15.5 days = 0.98



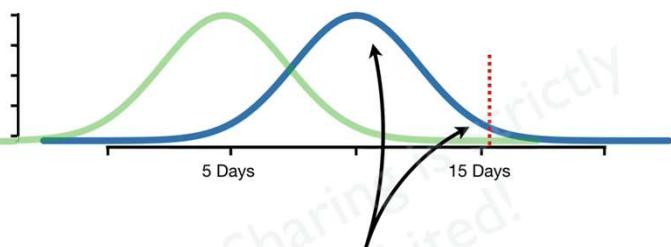
And since $0.98 > 0.05$, the **One-Sided p-value** would not detect that **SuperDrug** was doing anything unusual.

One-Sided p-value for 15.5 days = 0.98



In other words, the **One-Sided p-value** is only looking to see if a distribution to the left of the original mean makes more sense...

One-Sided p-value for 15.5 days = 0.98



...and since the observation is on the right side of the mean, we fail to reject the hypothesis that the original distribution makes sense.

One-Sided p-value for 15.5 days = 0.98



And since failing to detect that **SuperDrug** is making things worse would be bad, **One-Sided p-values** are tricky and should be avoided, or only be used by experts who really know what they are doing.

In summary, a **p-value** is composed of three parts:

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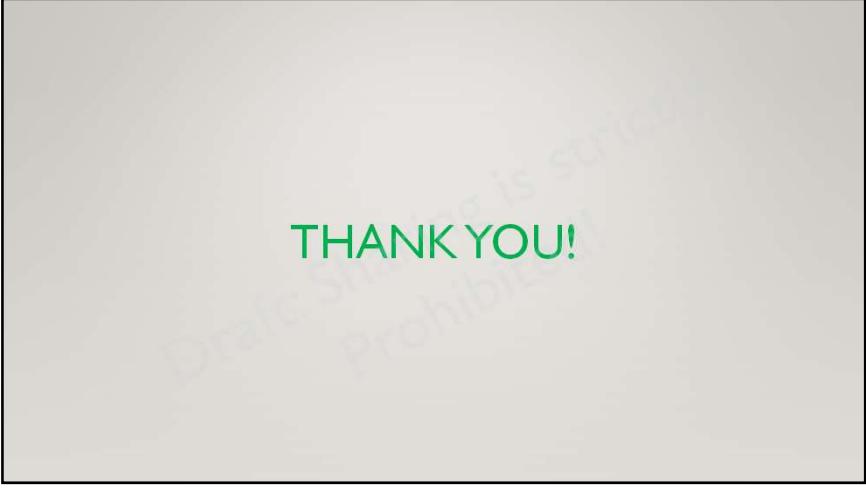
- 1) The probability random chance would result in the observation.

In summary, a **p-value** is composed of three parts:

- 1) The probability random chance would result in the observation.
- 2) The probability of observing something else that is equally rare.

In summary, a **p-value** is composed of three parts:

- 1) The probability random chance would result in the observation.
- 2) The probability of observing something else that is equally rare.
- 3) The probability of observing something rarer or more extreme.



THANK YOU!