ENGINEERING MATHEMATICS-I QUESTION BANK

Unit-I

S.No	Section-A	Marks
S.No	**Easy Type**	Marks
1	Find the rank of the matrix $A=\begin{pmatrix}1&3&4&3\\3&9&12&3\\1&3&4&1\end{pmatrix}$ using echelon form	2M
2	Solve the equations $x+2y+3z=0;\ 3x+4y+4z=0;\ 7x+10y+12z=0.$	2M
3	Find the eigen values of the matrix $\begin{pmatrix}5&4\\1&2\end{pmatrix}$.	2M
4	Find the sum and product of the eigen values of $\begin{bmatrix}3&1&4\\0&2&6\\0&0&5\end{bmatrix}$.	2M
5	If two eigen values of matrix $\begin{bmatrix}8&-6&2\\-6&7&-4\\2&-4&3\end{bmatrix}$ are 3 and 15 then find the third eigen value of matrix.	2M
S.No	**Moderate Type**	Marks
1	Verify Cayley-Himilton theorem for the matrix $\begin{pmatrix}1&4\\2&3\end{pmatrix}$.	2M
2	Determine the characteristic equation of the matrix $\begin{bmatrix}3&10&5\\-2&-3&-4\\3&5&7\end{bmatrix}$.	2M
3	Find the index and signature of the quadratic form $x_1^2+2x_2^2-3x_3^2$.	2M
4	Write the quadratic form corresponding to the symmetric matrix $\begin{bmatrix}0&1&1\\1&0&-1\\1&-1&0\end{bmatrix}$.	2M
5	Write the symmetric matrix associated with the quadratic form $x^2-y^2+2z^2+2xy-4yz+6xz$.	2M
	SECTION – B **Essay Questions**	
S.No	**Easy Questions**	Marks
1	(a) Test for consistency and solve $4x-2y+6z=8,\ x+y-3z=-1,\ 15x-3y+9z=21$. (b) Find the values of λ and μ for which the following system of equations	4M 4M

	$x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ has (i) no solution (ii) unique solution (iii) infinite solutions.	
2	Find the value of λ for which the following system of equations $3x - y + 4z = 3$, $x + 2y - 3z = -2$, $6x + 5y + \lambda z = -3$ will have infinite solutions and solve them for each value of λ.	8M
3	Solve the system of equations $x + y - 2z + 3w = 0$; $x - 2y + z - w = 0$; $4x + y - 5z + 8w = 0$; $5x - 7y + 2z - w = 0$.	8M
S.No	**Moderate Questions**	Marks
1	Find the eigen values and eigen vectors of $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.	8M
2	Using Cayley Hamilton theorem find the inverse of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Also find A^{-2}.	8M
3	Verify Cayley-Hamilton theorem and then find the inverse of $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$.	8M
S.No	**Typical Questions**	Marks
1	Reduce the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ to the diagonal form.	8M
2	Reduce the quadratic form $\mathbf{x^2 + 3y^2 + 3z^2 - 2yz}$ into canonical form and give the matrix of the transformation. Discuss the nature of the quadratic form.	8M
3	Reduce the quadratic form $\mathbf{3x^2 + 5y^2 + 3z^2 - 2yz + 2xz - 2xy}$ into canonical form and specify the matrix of the transformation. Discuss the nature of the quadratic form.	8M

Unit-II

	Section-A	
S.No	**Easy Type**	Marks
1	State Rolle's theorem.	2M
2	State Lagrange's mean value theorem.	2M
3	State Cauchy's mean value theorem.	2M
4	Using Rolle's theorem, find the value of 'c' for the function $f(x) = x^3 - 4x$ in the interval (-2, 2).	2M
5	Is Rolle's theorem applicable for the function $f(x) = \tan x$ in the interval $(0, \pi)$?	2M
S.No	**Moderate Type**	Marks
1	Is Lagrange's mean value theorem applicable for function $f(x) = \|x\|$ in the	2M

	interval (-1, 1)?	
2	Find the value 'c' of Lagrange's mean value theorem for $f(x)=x^2-3x+2$ in (-2,3).	2M
3	Determine the value 'c' of Cauchy's mean value theorem for the functions e^x and e^{-x} in the interval *(a, b).*	2M
4	Obtain the Maclaurin's series expansion of the function $f(x)=e^x$.	2M
5	Obtain the Maclaurin's series expansion of the function f(x) = sin x.	2M
	SECTION – B **Essay Questions**	
	Easy Questions	
1	a) Verify Rolle's theorem for $f(x)=(x-a)^m(x-b)^n$ in $[a,\ b]$, where m and n are positive integers. b) Using Rolle's theorem for $f(x)=x^{2m-1}(a-x)^{2n}$, find the value of x between 0 and a, where $f'(x)=0$.	4M 4M
2	a) Verify Rolle's theorem for $\frac{\sin x}{e^x}$ in $[0,\ \pi]$. b) In the Lagrange's mean value theorem, determine 'c' lying between a and b, if $f(x)=x(x-1)(x-2),$ where $a=0$ and $b=\frac{1}{2}$.	4M 4M
3	a) Verify Lagrange's mean value theorem and find the appropriate value of c for the function $f(x)=(x-1)(x-2)(x-3)$ in $(0,\ 4)$. b) Using Lagrange's mean value theorem, show that $x>\log(1+x)>\frac{x}{1+x}$, for $x>0$.	4M 4M
	Moderate Questions	
1	If $a<b$, prove that $\frac{b-a}{1+b^2}<\tan^{-1}b-\tan^{-1}a<\frac{b-a}{1+a^2}$ by using Lagrange's mean value theorem. Hence deduce the following i) $\frac{\pi}{4}+\frac{3}{25}<\tan^{-1}\left(\frac{4}{3}\right)<\frac{\pi}{4}+\frac{1}{6}$. ii) $\frac{5\pi+4}{20}<\tan^{-1}(2)<\frac{\pi+2}{4}$.	8M
2	a) If $f(x)=\sin^{-1}x,\ 0<a<b<1$, use mean value theorem to prove that $\frac{b-a}{\sqrt{(1-a^2)}}<\sin^{-1}b-\sin^{-1}a<\frac{b-a}{\sqrt{(1-b^2)}}$. b) Find '$c$' of Cauchy's mean value theorem for $f(x)=\sqrt{x}$ and $g(x)=\frac{1}{\sqrt{x}}$ in $(a,\ b)$, where $0<a<b$.	4M 4M
3	Verify Cauchy's mean value theorem for a) $f(x)=e^x$ and $g(x)=e^{-x}$ in $[a,\ b]$, where $0<a<b$. b) $f(x)=\sin x$ and $g(x)=\cos x$ in $\left[0,\ \frac{\pi}{2}\right]$.	4M 4M

	Typical Questions	
1	a) If x is positive, show that $x > \log(1+x) > x - \frac{x^2}{2}$. b) Using Taylor's theorem, prove that $x - \frac{x^3}{6} < \sin x < x - \frac{x^3}{6} + \frac{x^5}{120}$, for $x > 0$.	4M 4M
2	a) Verify Maclaurin's theorem for $f(x) = (1-x)^{\frac{5}{2}}$ with Lagrange's form of remainder up to 3 terms, where $x = 1$. b) Obtain the Taylors's series expansion of $\log_e x$ in powers of $(x-1)$.	4M 4M
3	Expand the following functions as Maclaurin's series a) $\log(1+x)$ b) $e^x \cos x$	4M 4M

Unit-III

	Section-A	
S.No	**Easy Type**	Marks
1	Find the first and second order partial derivatives of $x^3 + y^3 - 3axy$.	2M
2	If $u = e^{\frac{x}{y}}$ then find $xu_x + yu_y$.	2M
3	Find $\frac{du}{dt}$ if $u = x^2 y^3$ where $x = \log t$ and $y = e^t$.	2M
4	If $x = r\cos\theta,\ y = r\sin\theta$, z = z then find $\frac{\partial(x, y, z)}{\partial(r, \theta, z)}$.	2M
5	If $x = r\cos\theta,\ y = r\sin\theta$ then find $\frac{\partial(x, y)}{\partial(r, \theta)}$.	2M
S.No	**Moderate Type**	Marks
1	If $x = uv,\ y = \frac{u}{v}$ then find $\frac{\partial(x, y)}{\partial(u, v)}$	2M
2	Find the stationary points of $x^3 + y^3 - 3axy$	2M
3	If $f(x, y) = xy + (x - y)$ then find its stationary points	2M
4	If $u = x^2 y$ and $v = xy^2$ then find $\frac{\partial(u, v)}{\partial(x, y)}$.	2M
5	If $u = lx + my$ and $v = mx - ly$ find $\left(\frac{\partial u}{\partial x}\right)_y$ *and* $\left(\frac{\partial x}{\partial u}\right)_v$.	2M
	SECTION – B	
	Essay Questions	
	Easy Questions	
1	a. If $z(x+y) = x^2 + y^2$, show that $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$. b. Let $r^2 = x^2 + y^2 + z^2$ and $V = r^m$, prove that $V_{xx} + V_{yy} + V_{zz} = m(m+1)r^{m-2}$.	4M 4M

2	a. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$. b. If $v = x^y y^x$, prove that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = v(x + y + \log v)$.	4M 4M
3	a. If $u = \sin^{-1}(x - y),\ x = 3t$ and $y = 4t^3$, show that $\frac{du}{dt} = \frac{3}{\sqrt{(1-t^2)}}$. b. If u = $x^2 + y^2 + z^2$ and x = e^{2t}, y = e^{2t} cos 3t, z = e^{2t} sin 3t . Find $\frac{du}{dt}$ as a total derivative.	4M 4M
	Moderate Questions	
1	By the substitution $u = x^2 - y^2,\ v = 2xy,\ f(x, y) = \theta(u, v)$, show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 4(x^2 + y^2)\left(\frac{\partial^2\theta}{\partial u^2} + \frac{\partial^2\theta}{\partial v^2}\right)$	8M
2	a. If $x = r\cos\theta,\ y = r\sin\theta$, evaluate $\frac{\partial(r,\theta)}{\partial(x,y)}, \frac{\partial(x,y)}{\partial(r,\theta)}$ and prove that $\frac{\partial(r,\theta)}{\partial(x,y)} \cdot \frac{\partial(x,y)}{\partial(r,\theta)} = 1$. b. If $u = x^2 - 2y^2,\ v = 2x^2 - y^2$ where $x = r\cos\theta,\ y = r\sin\theta$, show that $\frac{\partial(u,v)}{\partial(r,\theta)} = 6r^3 \sin 2\theta$.	4M 4M
3	a. If $u = xyz,\ v = x^2 + y^2 + z^2,\ w = x + y + z$, find $\frac{\partial(x, y, z)}{\partial(u, v, w)}$. b. If $u = \frac{x+y}{1-xy},\ v = \tan^{-1}x + \tan^{-1}y$, find $\frac{\partial(u,v)}{\partial(x,y)}$. Are u and v functionally related. If so, find the relation.	4M 4M
	Typical Questions	
1	a. Discuss the maxima and minima of $f(x, y) = x^3 y^2 (1 - x - y)$. b. Discuss the maxima and minima of f(x,y) = $x^3 + y^3 - 3axy$	4M 4M
2	Find the volume of the greatest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.	8M
3	a. Find the dimensions of the rectangular box, open at the top, of maximum capacity whose surface is 432 sq.cms b. The sum of three numbers is constant. Prove that their product is maximum when they are equal	4M 4M

Unit-IV

SECTION – A		
S.No	**Easy Type**	Marks
1	Evaluate $\int_0^2 \int_0^x y\ dydx$	2M

2	Evaluate $\int_0^1\int_0^1 \frac{dxdy}{\sqrt{(1-x^2)(1-y^2)}}$	2M
3	Identify the limits of integration for $\iint_R f(x, y)\,dxdy$ where R is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$.	2M
4	Evaluate $\int_0^{\infty}\int_0^{\frac{\pi}{2}} e^{-r^2} r \; d\theta dr$.	2M
5	Identify the limits of integration for $\iint_R f(r,\theta)\,drd\,\theta$ over the cardiod $r = a(1-\cos\theta)$ above the initial line	2M
S.No	**Moderate Type**	Marks
1	Find the new limits of integration after changing the order of integration for $\int_{-a}^{a}\int_0^{\sqrt{a^2-y^2}} f(x, y)\,dxdy$.	2M
2	Find the new limits of integration after changing the order of integration for $\int_o^1\int_{x^2}^{x} f(x, y)\,dxdy$	2M
3	Convert into polar coordinates and then evaluate $\int_0^a\int_0^{\sqrt{(a^2-x^2)}} (x^2 + y^2)\,dydx$.	2M
4	The value of $\iint_R x^2 y^3\,dxdy$, where R is the region bounded by the rectangle $0 \le x \le 1$ and $0 \le y \le 3$.	2M
5	Identify the limits of integration for $\iint_R f(r,\theta)\,drd\,\theta$ over the region bounded by the circles $r = 2\sin\theta$ and $r = 4\sin\theta$.	2M
	SECTION-B **Essay Questions**	
	Easy Questions	
1	a. Evaluate $\int_0^1\int_x^{\sqrt{x}} (x^2 + y^2)\; dx\; dy$. b. Evaluate $\int_0^4\int_0^{x^2} e^{y/x}\; dy\; dx$.	4M 4M
2	a. Evaluate $\iint xy\; dx\; dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$. b. Evaluate $\iint (x+y)^2\; dx\; dy$ over the area bounded by ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.	4M 4M
3	a. By changing the order of integration, evaluate the integral $\int_0^1\int_0^{\sqrt{1-y^2}} x^3 y\; dx\; dy$. b. By changing the order of integration, evaluate the integral	4M 4M

	$\int_0^3 \int_1^{\sqrt{4-y}} (x+y)\, dx\, dy$.	
	Moderate Questions	
1	a. By changing the order of integration, evaluate the integral $\int_0^a \int_{x/a}^{\sqrt{x/a}} (x^2+y^2)\, dx\, dy$. b. By changing the order of integration, evaluate the integral $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dx\, dy$.	4M 4M
2	a. By changing the order of integration, evaluate the integral $\int_0^\infty \int_0^x x e^{-x^2/y}\, dx\, dy$. b. Show that $\iint r^2 \sin\theta\, dr\, d\theta = \frac{2a^3}{3}$ over the region R, where R is the semi-circle $r = 2a\cos\theta$ above the initial line	4M 4M
3	a. Evaluate $\iint \frac{r}{\sqrt{a^2+r^2}}\, dr\, d\theta$ over one loop of the lemniscates $r^2 = a^2 \cos 2\theta$. b. Evaluate $\iint r^3\, dr\, d\theta$ over the area bounded between the circles $r = 2\cos\theta$ and $r = 4\cos\theta$.	4M 4M
	Typical Questions	
1	a. Find the area lying inside the cardioid $r = a(1+\cos\theta)$ and outside the circle $r = a$. b. Find the area of the lemniscate $r^2 = a^2\cos 2\theta$.	4M 4M
2	a. Find the area common to the circles $r = a\cos\theta$ and $r = a\sin\theta$. b. Find the area lying inside the cardioid $r = 1+\cos\theta$ and outside the parabola $r(1+\cos\theta) = 1$.	4M 4M
3	a. Find the area lying between the parabola $y = 4x - x^2$ and the line y=x. b. Find the area enclosed by the curves $y = \frac{3x}{x^2+2}$ and $4y = x^2$.	4M 4M

Unit-V

	SECTION – A	
S.No	**Easy Type**	Marks

1	Evaluate $\int_0^1\int_1^2\int_2^3 x^2 y^3 z^2\, dx\, dy\, dz$.	2M
2	Evaluate $\int_0^1\int_0^1\int_0^1 (x^2 + y^2 + z^2)\, dz\, dy\, dx$	2M
3	If $\int_0^k\int_1^2\int_2^3 xyz\, dx\, dy\, dz = \frac{15}{8}$, find '$k$'.	2M
4	Transform the double integral $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\, dx\, dy$ into polar form.	2M
5	Express the triple integral $\iiint_R f(x, y, z)\, dx\, dy\, dz$ in Spherical Polar Coordinates.	2M
S.No	**Moderate Type**	Marks
1	Evaluate $\int_0^2\int_1^z\int_0^{yz} xyz\, dx\, dy\, dz$.	2M
2	The volume common to the cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$ is given by the double integral $2\int_{k_1}^{k_2}\int_{f(x)}^{g(x)} z\, dx\, dy$, find $k_1,\ k_2,\ f(x)$ and $g(x)$.	2M
3	Transform the triple integral $\int_0^1 \int_0^{\sqrt{(1-x^2)}} \int_{\sqrt{x^2+y^2}}^1 f(x, y, z)\, dx\, dy\, dx$ into spherical polar coordinates.	2M
4	Using the transformation $x + y = u,\ y = uv$ transform the double integral $\int_0^1\int_0^{1-x} e^{\frac{y}{(x+y)}}\, dy\, dx$ into uv-coordinate system.	2M
5	The volume bounded by the xy -plane, the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 3$ is given by $\iint_R (3 - x - y)\, dx\, dy$. Find x , y limits.	2M
	SECTION – B **Essay Questions**	
	Easy QUESTIONS	
1	Evaluate $\int_0^a\int_0^x\int_0^{x+y} e^{x+y+z}\, dx\, dy\, dz$.	8M
2	Evaluate $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z\ dz\, dx\, dy$.	8M
3	Evaluate $\iiint_V \frac{dx\, dy\, dz}{x^2 + y^2 + z^2}$ by changing into spherical co-ordinates, where V is the volume of the sphere $x^2 + y^2 + z^2 = a^2$.	8M
	Moderate Questions	

1	Evaluate $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}\,dxdy$ by changing into polar coordinates. Hence show that $\int_0^{\infty} e^{-x^2}\,dx = \sqrt{\frac{\pi}{2}}$	8M
2	Find the volume of the Ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.	8M
3	Find by triple integration, the volume of the sphere $x^2+y^2+z^2=a^2$.	8M
	Typical Questions	
1	Find the volume bounded by the xy-plane, the cylinder $x^2+y^2=1$ and the plane $x+y+z=3$.	8M
2	Find the volume common to the cylinders $x^2+y^2=a^2$ and $x^2+z^2=a^2$	8M
3	Find the volume of the portion of the sphere $x^2+y^2+z^2=a^2$ lying inside the cylinder $x^2+y^2=ay$.	8M

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