Newton's Methods QUESTIONS

Introduction

Dylan: I'm so tired of having to solve roots by hand. It's a real drag.

Julia: Yeah, some of these roots are rough. I wish there was a better way!

James: There's always a better way!

Dylan and Julia: Show us!!!

James: Maybe you've heard of Sir Isaac Newton? He got tired of solving roots too, and made a whole method to approximate them!

Dylan: Wow! I'm just like him except worse in every way!

Newton's Method is a system of approximating roots of polynomials by using tangent lines from an initial estimate. While this method is extremely accurate when used properly, it is possible to have a very inaccurate estimate when used improperly.

Guided Example

In the following figure we have an initial guess x_0, then we have the blue tangent line with respect to the point x_0

Question 1 *What is the slope, in general, for the tangent line of* $y = f(x)$ *at* x_0*?*

$\boxed{f'(x_0)}$

What is the equation of the tangent line for the point $(x_0, f(x_0)$*? Please answer in slope-intercept form.*

$\boxed{y = f'(x)x_0 + b}$

How would you use the tangent line you found above to estimate the value of x_1*?*

Learning outcomes:

Free Response:

On Your Own

Question 2 *Consider the function* $f(x) = x^2 - 1$.

Graph of $x^2 - 1$

Find the tangent line at an initial estimate of $x_0 = 3$.

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Plot the tangent line and function on the same axes. Does the x-intercept of the tangent line seem more or less accurate than your initial estimate?

Multiple Choice:

(a) *More Accurate* ✓

(b) *Less Accurate*

What is the x-intercept of the tangent line?

$\boxed{I don't know}$

Continue this process until the x-intercepts change by less than .0001 on each interval.

Consider the function $g(x) = x^3 - 4x^2 - 1$.

Graph of $x^3 - 4x^2 - 1$

Using the same method as before, estimate a root of $g(x)$ using your own initial guess.

Explain why the function has only one solution with the help of a graph.

Graph of

Free Response:

Using $g(x)$ from the previous problem, use an initial guess of 2. After 5 iterations, what result do you get? $\boxed{A really bad one}$

Why is it important to use caution with Newton's method?

Free Response:

In Summary

Julia: Wow! Newton's Method is awesome!

Dylan: Yeah, it's way more accurate than just guessing! If you're too far off on that initial guess though...

James: Things can go downhill quickly. While Newton's Method can be handy, it's important to remember how important an accurate initial estimate is!

Dylan and Julia: Thanks James!