Derivative

Julia: Ah, this sucks!

Dylan: What's up?

Julia: I'm supposed to find the slope of a parabola at a point, and I'm not sure how!

Dylan: Well, what if we just make a secant line on the function?

Julia: Secant line? What's that?

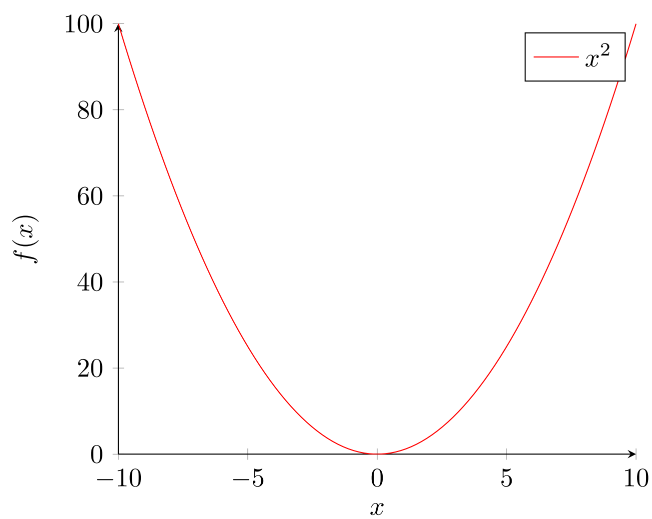
Dylan: *A secant line is just a line which connects two points on a function*!

Guided Example

Using your favorite computer algebra system, answer the following questions. Include an image of your graphs along with your answers, so the instructor may see your progress in your lab submission.

Consider the function

$$f(x) = x^2$$



(a) Find the slope between $x = 2$ and $x = 7$. Does this seem to be a good approximation for the rate of change at $x = 2$? Why or why not?

(b) Dylan thinks we can solve the problem by just picking something closer than 10. Now find the slope between $x = 2$ and $x = 3$. Is this a good approximation? Is it better than the last attempt?

Julia: Dylan, this still isn't a great approximation...

Dylan: Well, I think we need to get even closer. Like, infinitesimally close! But how would we do that....

James: You guys need some help?

Julia and Dylan: James! How do we find the slope of a line at a point?

James: It isn't too tough! Before, you were considering a certain point as your comparison. What if instead, you used the point you want to evaluate at plus something really small? Let's call it h.

(c) How can you make the h in

$$\frac{f(a+h)-f(a)}{(a+h)-a}$$

become a value closer and closer to zero when we evaluate it?

(d) Using the method you determined in the previous question, approximate the rate of change at the point x=2.

James: The value at that point is the slope of the tangent line!

Dylan: What's a tangent line?

James: *A tangent line is a line which intersects a differentiable curve at a point where the slope of the curve equals the slope of the line.*. Want to know something really cool?

Julia and Dylan: What James?

James: The function you just discovered is how you determine a function's derivative! Using that process, you can find the rate of change at any point on a function!

Julia and Dylan: Wow! So cool!

On Your Own

Using what you've learned, find the derivative of the following functions at the given point.

(a) $g(x) = x^5 - 5x^4 - x^2 + 2x + 1$, $x = 2$

(b) $h(x) = \dfrac{1}{x}$, $x = 2$

By replacing the a in our formula for the derivative with x, we may determine the derivative at any point on the function. Determine the derivative at any point for the following functions.

(a) $m(x) = x^3$

(b) $n(x) = 3x + 2$

In Summary

Julia: So why is it called a secant line?

James: It comes from the Latin word secare which means to cut.

Dylan: Ohh, I get it now! Because a secant line is any line that connects two points on a function!

In this lab we've (hopefully) learned the function for finding a derivative using the limit as h approaches 0. We also learned what secant and tangent lines are. For your convenience, the important definitions are listed below.

Definition 1. *A **secant line** is any line that connects any two points on a curve.*

Definition 2. *A **tangent line** is a line which intersects a differentiable curve at a point where the slope of the curve equals the slope of the line.*

Definition 3. *The **derivative** $f'(a)$ is defined by the following limit:*

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$