

# Transformations of Functions

**Julia:** Ugh!

**Dylan:** What's up Julia?

**Julia:** I have these functions I have to graph, and they're *so* close to functions I know really well, but they're a little bit different and it makes it so I have to calculate a bunch of points before I can confidently graph it!

**James:** Sounds like you could use some help Julia!

**Julia and Dylan:** James!

**James:** There are a ton of ways to transform functions, so let's get going and look at how we can modify our favorite functions!

## Introduction

While you work with many different functions, there are only a few basic types of functions. These include polynomials, rational functions, trigonometric functions, exponential functions, and logarithmic functions. In this lab we will explore different variations on these basic functions called **transformations**.

## Guided Example

Consider the function  $f(x) = x^2$ .

Graph of  $f(x) = x^2$

**Question 1** On the same axis graph  $g(x) = f(x) + 2$ , what change happened from  $f(x)$  to  $g(x)$ ?

The graph shifted  units .

What can you infer about  $f(x) - 2$ ?

The graph would shift  units .

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Learning outcomes:

Consider the function  $f(x + 2)$ , or  $(x + 2)^2$ . How do you think this graph will be different from the graph of  $f(x)$ ?

**Free Response:**

Graph the function  $f(x + 2)$ , was your prediction correct? What can you infer about the function  $f(x - 2)$ ? Graph this function to verify your prediction.

**Free Response:**

What rule can you write about a general function  $f(x + c)$  where  $c$  is a positive constant? The function will shift  $c$  units *left*

Why do you think the graph moves in the direction it does when using the rule you determined in the last question? *Hint: Think about the x-intercept and how it changes when you add or subtract a constant from the x value*

**Free Response:**

How do you think the graph of  $f(x)$  be affected when you multiply the whole function by some constant  $c$ ? Graph the function for the following values of  $c = 2, \frac{1}{2}, -2, -\frac{1}{2}$

Graph of

**Free Response:**

Describe what is happening to the function based on the value of  $c$ , what can you generalize from this? It may be helpful to make a table with the x and y values to understand why this change happens.

**Free Response:**

## On your own

**Question 2** Using  $g(x) = x^2$  as your base function create a new function that will shift the graph up 4 units, to the right 3 units, reflect it across the x-axis and stretch it vertically by a factor of 2 and graph it below

Graph of

Graph the function  $g(2x)$

Graph of

What constant does this stretch or compress  $x^2$  by?

$\boxed{1/c}$

Graph  $g(2x + 6)$  on the same axis above, what transformation occurred?

**Free Response:**

Note the following expansion of the general function  $f(x) = (ax + b)^2$ :

$$f(x) = (ax + b)^2 = \left( a \left( x + \frac{b}{a} \right) \right)^2 = a^2 \left( x + \frac{b}{a} \right)^2$$

From this expansion, how is a function in the form  $f(x) = (ax + b)^2$  being shifted and stretched/compressed in terms of  $a$  and  $b$ ?

**Free Response:**

## In Summary

For the following questions, pick in which way the general graph  $f(x)$  would change under certain transformations.

### Question 3

$$c \cdot f(x)$$

When  $c > 1$

**Multiple Choice:**

- (a) Shrink  $f(x)$  vertically by  $c$
- (b) Stretch  $f(x)$  vertically by  $c$  ✓
- (c) Shrink  $f(x)$  horizontally by  $c$
- (d) Stretch  $f(x)$  horizontally by  $c$
- (e) Flip  $f(x)$  over the  $x$  axis

When  $c < -1$

**Multiple Choice:**

- (a) Flip  $f(x)$  over the  $x$  axis
- (b) Shrink  $f(x)$  horizontally by  $c$
- (c) Flip  $f(x)$  over the  $y$  axis and stretch horizontally by  $c$
- (d) Flip  $f(x)$  over the  $x$  axis and stretch vertically by  $c$  ✓
- (e) Flip  $f(x)$  over the  $x$  axis and stretch horizontally by  $c$

When  $0 < c < 1$

**Multiple Choice:**

- (a) Stretch  $f(x)$  horizontally by  $c$
  - (b) Shrink  $f(x)$  vertically by  $c$  ✓
  - (c) Shrink  $f(x)$  horizontally by  $c$
  - (d) Stretch  $f(x)$  horizontally by  $c$
  - (e) Flip  $f(x)$  over the  $x$  axis
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**Question 4**

$$f(x + c)$$

When  $c > 0$

**Multiple Choice:**

- (a) Shift  $f(x)$  left by  $|c|$ . ✓
- (b) Flip  $f(x)$  over the  $x$ -axis.
- (c) Shift  $f(x)$  right by  $|c|$
- (d) Flip  $f(x)$  over the  $x$ -axis and shift it up by  $|c|$ .
- (e) No change occurs to  $f(x)$ .

When  $c < 0$

**Multiple Choice:**

- (a) Shift  $f(x)$  left by  $|c|$ .

- (b) Flip  $f(x)$  over the x-axis.
- (c) Shift  $f(x)$  right by  $|c|$  ✓
- (d) Flip  $f(x)$  over the x-axis and shift it up by  $|c|$ .
- (e) No change occurs to  $f(x)$ .

When  $c = 0$

**Multiple Choice:**

- (a) Shift  $f(x)$  left by  $|c|$ .
  - (b) Flip  $f(x)$  over the x-axis.
  - (c) Shift  $f(x)$  right by  $|c|$
  - (d) Flip  $f(x)$  over the x-axis and shift it up by  $|c|$ .
  - (e) No change occurs to  $f(x)$ . ✓
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**Question 5**

$$f(x) + c$$

When  $c > 0$

**Multiple Choice:**

- (a) Shift  $f(x)$  down by  $|c|$ .
- (b) Stretch  $f(x)$  vertically by  $|c|$ .
- (c) Flip  $f(x)$  over the x-axis.
- (d) Shift  $f(x)$  up by  $|c|$ . ✓
- (e) No change will occur.

When  $c = 0$

**Multiple Choice:**

- (a) Shift  $f(x)$  down by  $|c|$ .
- (b) Stretch  $f(x)$  vertically by  $|c|$ .

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- (c) *Flip  $f(x)$  over the x-axis.*
- (d) *Shift  $f(x)$  up by  $|c|$ .*
- (e) *No change will occur. ✓*

When  $c < 0$

**Multiple Choice:**

- (a) *Shift  $f(x)$  down by  $|c|$ . ✓*
  - (b) *Stretch  $f(x)$  vertically by  $|c|$ .*
  - (c) *Flip  $f(x)$  over the x-axis.*
  - (d) *Shift  $f(x)$  up by  $|c|$ .*
  - (e) *No change will occur.*
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