

## Implicit Differentiation

**Dylan:** Woah! What's up with this?

**Julia:** I didn't know functions were explicit!

**Dylan:** The  $x$  and  $y$  are on the same side of the equation! I can't deal with this.

**James:** Functions can be explicit or implicit! And it not the way you're thinking Julia...

### Introduction

So far we have dealt only with explicitly defined functions, where  $y = f(x)$ . Functions where there are both  $x$  and  $y$  on one side or both sides of the equation are called implicit functions.

### Guided Example

**Question 1** Give an example of an explicit function and an implicit function, making sure your implicit function is not easily solvable for  $y$ .

**Free Response:**

**Question 2** Take the implicit function you defined in part one, and graph it. What do you notice?

Graph of

**Free Response:**

Now, using your CAS, solve the function for  $y$ , and graph the resulting equation(s). What do you notice?

```
1 Desmos.enabledFeatures === {
2   GraphingCalculator: false,
3   FourFunctionCalculator: false,
4   ScientificCalculator: true
5 }
```

SAGE

Graph of

Using the function(s) you found, find the slope of the tangent lines at a point with two values on your graph.

### Implicit Differentiation Using Substitution

Consider the equation  $-x^2 * y^3 + y^5 - 32 = 0$ .

- (a) Using the method shown in the previous section, evaluate the function for  $y$ . Does this equation look easy to differentiate?
- (b) Instead, let's treat our equation as an expression - because it equals zero, we don't have to worry about moving anything over. Now consider  $y$  as  $y(x)$ , a function of  $x$ , and differentiate with respect to  $x$ . Each  $y$  term will gain  $\frac{dy}{dx}$ . Then, set the expression equal to zero, and solve for  $\frac{dy}{dx}$ . What does this represent?
- (c) Using your result in the previous section, evaluate  $\frac{dy}{dx}$  at  $x = 3$  and  $x = 7$ .
- (d) Using the same strategy, find the slope of  $\sin(x^2) = \cos(xy^2)$  at any point.

### Perpendicular at a Point

**Julia:** Wow, implicit differentiation is rough.

**Dylan:** You're telling me... I've been doing this for hours! I wish we could at least do a little more with it if I have to learn it.

**James:** Did I hear that you guys want to know more about using implicit differentiation?

**Julia and Dylan:** James! Tell us more!

**James:** Alright guys, you can use implicit differentiation with implicit functions to tell if two functions are perpendicular at a point!

**Julia:** But how?

**Dylan:** Yeah, I don't see how that helps.

**James:** It's easy - all we have to do is see if the tangent lines are perpendicular at that point, and if they are, then so are the curves!

- (a) Graph  $3x - 2y + x^3 - x^2y = 0$  and  $x^2 - 2x + y^2 - 3y = 0$  on the same set of axes. Do they look perpendicular anywhere?

- (b) Prove the two curves are (or are not) perpendicular at the origin.  
*Remember, two perpendicular lines will have their slopes be the negative inverse of one another.*

### **In Summary**

There are two main methods to solve implicit equations

- (a) Solve for  $y$  and then differentiate.
- (b) Treat  $y$  as  $y(x)$  and differentiate for  $x$ , eventually solving for  $\frac{dy}{dx}$  to give the value of the derivative at any point.