

# Newton's Methods QUESTIONS

## Introduction

**Dylan:** I'm so tired of having to solve roots by hand. It's a real drag.

**Julia:** Yeah, some of these roots are rough. I wish there was a better way!

**James:** There's always a better way!

**Dylan and Julia:** Show us!!!

**James:** Maybe you've heard of Sir Isaac Newton? He got tired of solving roots too, and made a whole method to approximate them!

**Dylan:** Wow! I'm just like him except worse in every way!

Newton's Method is a system of approximating roots of polynomials by using tangent lines from an initial estimate. While this method is extremely accurate when used properly, it is possible to have a very inaccurate estimate when used improperly.

## Guided Example

In the following figure we have an initial guess  $x_0$ , then we have the blue tangent line with respect to the point  $x_0$

**Question 1** What is the slope, in general, for the tangent line of  $y = f(x)$  at  $x_0$ ?

$f'(x_0)$

What is the equation of the tangent line for the point  $(x_0, f(x_0))$ ? Please answer in slope-intercept form.

$y = f'(x)x_0 + b$

How would you use the tangent line you found above to estimate the value of  $x_1$ ?

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Learning outcomes:

**Free Response:**

**On Your Own**

**Question 2** Consider the function  $f(x) = x^2 - 1$ .

Graph of  $x^2 - 1$

Find the tangent line at an initial estimate of  $x_0 = 3$ .

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Plot the tangent line and function on the same axes. Does the x-intercept of the tangent line seem more or less accurate than your initial estimate?

**Multiple Choice:**

- (a) More Accurate ✓
- (b) Less Accurate

What is the x-intercept of the tangent line?

Continue this process until the x-intercepts change by less than .0001 on each interval.

Consider the function  $g(x) = x^3 - 4x^2 - 1$ .

Graph of  $x^3 - 4x^2 - 1$

Using the same method as before, estimate a root of  $g(x)$  using your own initial guess.

Explain why the function has only one solution with the help of a graph.

Graph of

**Free Response:**

Using  $g(x)$  from the previous problem, use an initial guess of 2. After 5 iterations, what result do you get?

Why is it important to use caution with Newton's method?

**Free Response:**

## **In Summary**

**Julia:** Wow! Newton's Method is awesome!

**Dylan:** Yeah, it's way more accurate than just guessing! If you're too far off on that initial guess though...

**James:** Things can go downhill quickly. While Newton's Method can be handy, it's important to remember how important an accurate initial estimate is!

**Dylan and Julia:** Thanks James!