Rational Functions with Awful Questions

Introduction

James: Hey guys, I slept through class yesterday... could you fill me in on what a rational function is?

Julia: See, class didn't make a lot of sense to me because I was thinking, "Functions can be rational?"

Dylan: They don't mean rational like me or you, Julia! It means *the function can be represented as a fraction where the numerator and denominator are both polynomials.*

Julia and James: Oh!

Dylan: Rational functions are pretty neat, because they can have two different types of discontinuities!

Altogether: LET'S DIVE IN!

Guided Example

Consider the function $f(x) = \dfrac{(x-2)(x+4)}{(x-3)(x+3)(x+4)}$

Graph the function using your favorite CAS system. Depending on the CAS you use, you may need to research how to show discontinuities in a graph. To do this, simply Google "*CAS* show discontinuities", where *CAS* is the name of whatever CAS you are using. At the time this document was written, Desmos did not include discontinuities by default, and thus, a Desmos powered graph has not been provided within this activity.

Question 1 *Describe the graph. What strange things do you notice?*

Free Response:

Learning outcomes:

The "hole" present in the graph is called a **removable discontinuity**.

The curve which goes vertical is called a **vertical asymptote**, another type of discontinuity.

On Your Own

Find and report the discontinuities in the following functions:

$$a(x) = \frac{x^2 + 1}{x - 2}$$

$$b(x) = \frac{x^2 - 5x + 7}{x^2 - x - 6}$$

$$c(x) = \frac{x^2 - x}{x}$$

$$d(x) = \frac{x^2 - 5x + 7}{x^3 - 6x^2 + 8x - 3}$$

$$f(x) = \frac{2x^2 + 5}{x^2 - 25}$$

$$g(x) = \frac{x^3 - x^2 - 15x - 9}{x + 3}$$

$$h(x) = \frac{1}{3x^2 - x}$$

Question 2 *How can you tell if a rational function has a vertical asymptote or a removable discontinuity?*

Free Response:

How can you find these discontinuities?

Free Response:

In Summary

James: These functions are pretty neat! What were they called again?

Dylan: They're called **rational functions**, *fractions where the numerator and denominator are both polynomials*!

Julia: So, when exactly does a **vertical asymptote** occur?

James: I know this one! **Vertical asymptotes** *occur at points where the denominator of the function will be zero, but the numerator is non-zero*!

Julia: That makes sense! But when do removable discontinuities occur then?

Dylan: Removable discontinuities *occur where the numerator and denominator are both zero.*