Differentiation Rules!

Julia: Hmm...I don't think differentiation rules, it takes so long and I hate using that long limit definition!

Dylan: No no Julia, it's differentiation *rules*!

Julia: Ohhhh, that makes more sense!

The Power Rule

Julia: I hate how long it takes to differentiate powers!

Dylan: Yeah, it takes forever! I feel like there was some sort of pattern to it though. I couldn't figure anything out though.

James: Sounds like you guys need my help again?

Julia and Dylan: Help us James!

James: There *is* a pattern! Check out this table I made!

$f(x)$	$f'(x)$
x^2	$2x^1$
x^3	$3x^2$
x^4	$4x^3$

Question 1 *What pattern do you notice in James' table?*

Free Response:

Generalize this pattern in terms of x^n

$$\frac{d}{dx}x^n = \boxed{n * x^(n-1)}$$

Question 2 *Using the limit definition of a derivative, compute the derivative for* x^5

$$\frac{d}{dx}x^5 = \boxed{5x^4}$$

Learning outcomes:

The Constant Rule

Dylan: Wow! That's neat!

Julia: I wish we could use rules like this all over the place though, it would really save me time.

James: There are plenty of places with rules like this! Why don't we look at a function like $y = 3$?

Consider $y = c$, where c is some arbitrary constant.

Question 3 *Derive this function using the limit definition. What does your answer mean?*

$\frac{dy}{dx}(y = c) = \boxed{0}$

Free Response:

Question 4 *Using what you found in the previous problem, compute the following derivatives:*

$x = 2 \quad \frac{dy}{dx} y = c = \boxed{0}$

$x = 100 \frac{\partial}{\partial}(y = c) = \boxed{0}$

$\frac{\partial}{\partial x = 0}(y = c) = \boxed{0}$

The Constant Multiple Rule

Julia: James! Show us more! These things are going to save me so much time on my homework!

James: Alright alright, calm down Julia. We can look at a function like $y = 3x$ next.

Consider $y = kx$, where k is some arbitrary constant.

Question 5 $\frac{\partial y}{\partial x}(k * x) = \boxed{k}$

What does your answer mean?

Free Response: *This is the model solution*

Question 6

Using what you found in the previous problem, compute the following derivatives:

$\frac{\partial y}{\partial x}(4 * x) = \boxed{4}$ $\frac{\partial y}{\partial x}(10 * x) = \boxed{10}$ $\frac{\partial y}{\partial x}(\frac{1}{5} * x) = \boxed{\frac{1}{5}}$

The Sum and Difference Rules

Dylan: Wow, this stuff is awesome! Is there any way to put it all together? Like, is there an easy way to tell what the derivative of $f(x) = 3x + 4$ is?

James: There is Dylan!

Consider the differentiable functions $f(x)$ and $g(x)$. We will define a function $j(x) = f(x) + g(x)$.

(a) Take the derivative of $j(x)$ using the limit definition. What does your answer mean? *Hint: In $j(x+h)$, the $(x+h)$ will replace x in the component functions as well.*

$j'(x) = \boxed{\frac{\text{J}(x+h) - j(x)}{h}}$

(b) Using what you found in the previous problem, compute the following derivatives:

(c) $f(x) = 3x^2 - 5x + 2$, $g(x) = x^2 + 3x$

$j'(x) = \boxed{8x - 2}$

(d) $f(x) = x^2 - 4x + 2$, $g(x) = -4x^2 + 3$

$j'(x) = \boxed{-6x - 4}$

(e) $f(x) = 5x^3 + 3x$, $g(x) = 2x^2 - 13x$

$j'(x) = \boxed{15x^2 + 4x - 10}$

(f) Julia wonders if a similar rule exists for $j(x) = f(x) - g(x)$. Using the limit definition of derivative, determine if there is a pattern. Then, if there is a rule, use it to solve the 1a, 1b, and 1c. If there is not, do them using the limit definition.

(g) $f(x) = 3x^2 - 5x + 2$, $g(x) = x^2 + 3x$

$j'(x) = \boxed{4x - 8}$

(h) $f(x) = x^2 - 4x + 2$, $g(x) = -4x^2 + 3$

$j'(x) = \boxed{-10x - 4}$

(i) $f(x) = 5x^3 + 3x$, $g(x) = 2x^2 - 13x$

$j'(x) = \boxed{15x^2 - 4x + 16}$

In Summary

We've covered a lot of differentiation rules in this lab, to help you out we've made the following table.

Power Rule	$\frac{d}{dx}(x^n) = n * x^{(n-1)}$,where n is any real number besides 0.
Constant Rule	$\frac{d}{dx}(c) = 0$
Constant Multiple Rule	$\frac{d}{dx}(cf(x)) = c\frac{d}{dx}f(x)$
Sum Rule	$\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$
Difference Rule	$\frac{d}{dx}(f(x) - g(x)) = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$