

Newton's Methods

Introduction

Dylan: I'm so tired of having to solve roots by hand. It's a real drag.

Julia: Yeah, some of these roots are rough. I wish there was a better way!

James: There's always a better way!

Dylan and Julia: Show us!!!

James: Maybe you've heard of Sir Isaac Newton? He got tired of solving roots too, and made a whole method to approximate them!

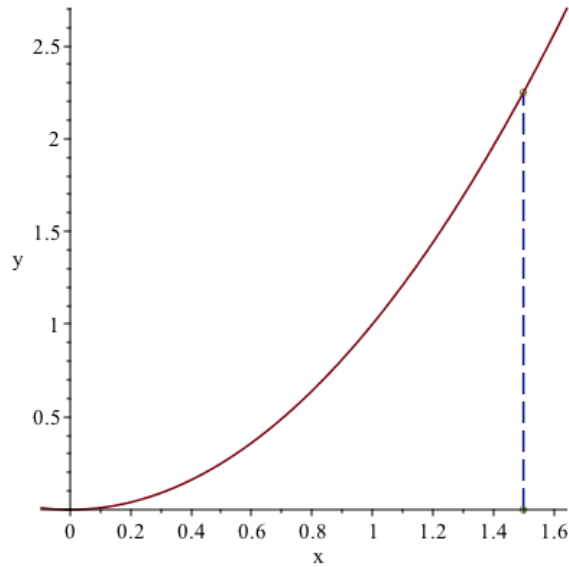
Dylan: Wow! I'm just like him except worse in every way!

Newton's Method is a system of approximating roots of polynomials by using tangent lines from an initial estimate. While this method is extremely accurate when used properly, it is possible to have a very inaccurate estimate when used improperly.

Guided Example

In the following figure we have an initial guess x_0 , then we have the blue tangent line with respect to the point x_0

Learning outcomes:



Question 1 What is the slope, in general, for the tangent line of $y = f(x)$ at x_0 ?

Multiple Choice:

- (a) $f'(x_0)$ ✓
- (b) $f'(x)$
- (c) $f(x)$
- (d) $f(f(x))$

What is the equation of the tangent line for the point $(x_0, f(x_0))$? Please answer in slope-intercept form.

Multiple Choice:

- (a) $y = f(x) \cdot x + b$
- (b) $y = f'(x_0) \cdot x_0 + b$ ✓
- (c) $y = f'(x) \cdot b + x_0$
- (d) $y = f'(x) \cdot b + f(x)$

How would you use the tangent line you found above to estimate the value of x_1 ?

Free Response:

On Your Own

Question 2 Consider the function $f(x) = x^2 - 1$.

Graph of $x^2 - 1$

Find the equation of the tangent line at an initial estimate of $x_0 = 3$.

$y = \square$

Plot the tangent line and function on the same axes. Does the x-intercept of the tangent line seem more or less accurate than your initial estimate?

Multiple Choice:

- (a) More Accurate ✓
- (b) Less Accurate

What is the x-intercept of the tangent line?

Idon'tknow

Continue this process until the x-intercepts change by less than .0001 on each iteration. How many iterations did this take?

☐

Question 3 Consider the function $g(x) = x^3 - 4x^2 - 1$.

Graph of $x^3 - 4x^2 - 1$

Explain why the function has only one solution with the help of a graph.

Graph of

Free Response:

Using $g(x)$ from the previous problem, use an initial guess of 2. After 5 iterations, what result do you get? Areallybadone

Why is it important to use caution with Newton's method?

Free Response:

Question 4 Consider the function $h(x) = 4x^3 - 12x^2 + 2x + 1$.

Graph of $4x^3 - 12x^2 + 2x + 1$

Use an initial guess of $x = 3$ to estimate a root of $h(x)$. What do you find?

☐

Look at the graph, and attempt to estimate another root using $x = 0$. Did you find the root to the right or the left of this point?

Multiple Choice:

(a) Left ✓

(b) Right

Increment the initial guess by 0.02 and use Newton's method until you find the other root. What value of x is the first to work? ☐

In Summary

Julia: Wow! Newton's Method is awesome!

Dylan: Yeah, it's way more accurate than just guessing! If you're too far off on that initial guess though...

James: Things can go downhill quickly. While Newton's Method can be handy, it's important to remember how important an accurate initial estimate is!

Dylan and Julia: Thanks James!