

# Implicit Differentiation - Finish solutions

**Dylan:** Woah! What's up with this?

**Julia:** I didn't know functions were explicit!

**Dylan:** The  $x$  and  $y$  are on the same side of the equation! I can't deal with this.

**James:** Functions can be explicit or implicit! And it not the way you're thinking Julia...

## Introduction

So far we have dealt only with explicitly defined functions, where  $y = f(x)$ . Functions where there are both  $x$  and  $y$  on one side or both sides of the equation are called implicit functions.

## Guided Example

**Question 1** Give an example of an explicit function and an implicit function, making sure your implicit function is not easily solvable for  $y$ .

**Free Response:**

**Question 2** Take the implicit function you defined in part one, and graph it. What do you notice?

Graph of

**Free Response:**

Learning outcomes:

**Question 3** Now, in the following sage cell, solve the function for  $y$ , and graph the resulting equation(s). What do you notice? For help using sage refer to the [documentation](#) here.

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SAGE

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```

1 x,y = var("x, y")
2 #eqn = x**2+y**2==1, this would set eqn to the unit circle
3 #replace it with your equation then use the solve command to #solve for y
4

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**Free Response:**

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**Question 4** Using the function(s) you found, find the slope of the tangent lines at a point with two values on your graph.

**Free Response:**

## Implicit Differentiation Using Substitution

Consider the equation  $-x^2 \cdot y^3 + y^5 - 32 = 0$ .

**Question 1** Using the method shown in the previous section, evaluate the function for  $y$ . Does this equation look easy to differentiate? No

Instead, let's treat our equation as an expression - because it equals zero, we don't have to worry about moving anything over. Now consider  $y$  as  $y(x)$ , a function of  $x$ , and differentiate with respect to  $x$ . Each  $y$  term will gain  $\frac{dy}{dx}$ . Then, set the expression equal to zero, and solve for  $\frac{dy}{dx}$ . What does this represent?

**Free Response:**

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**Question 2** Using your result in the previous section, evaluate  $\frac{dy}{dx}$  at  $x = 3$  and  $x = 7$ .

$x = 3$  : ☐

$x = 4$  : ☐

**Question 3** Using the same strategy, find the slope of  $\sin(x^2) = \cos(xy^2)$  at any point.

☐

## Perpendicular at a Point

**Julia:** Wow, implicit differentiation is rough.

**Dylan:** You're telling me... I've been doing this for hours! I wish we could at least do a little more with it if I have to learn it.

**James:** Did I hear that you guys want to know more about using implicit differentiation?

**Julia and Dylan:** James! Tell us more!

**James:** Alright guys, you can use implicit differentiation with implicit functions to tell if two functions are perpendicular at a point!

**Julia:** But how?

**Dylan:** Yeah, I don't see how that helps.

**James:** It's easy - all we have to do is see if the tangent lines are perpendicular at that point, and if they are, then so are the curves!

**Question 4** Graph  $3x - 2y + x^3 - x^2y = 0$  and  $x^2 - 2x + y^2 - 3y = 0$  on the same set of axes.

Graph of

Do they look perpendicular anywhere?

**Question 5 Hint:** Remember, two perpendicular lines will have their slopes be the negative inverse of one another.

Prove the two curves are (or are not) perpendicular at the origin.

**Free Response:**

## **In Summary**

There are two main methods to solve implicit equations

- (a) Solve for  $y$  and then differentiate.
- (b) Treat  $y$  as  $y(x)$  and differentiate for  $x$ , eventually solving for  $\frac{dy}{dx}$  to give the value of the derivative at any point.