

Derivative

Julia: Ah, this sucks!

Dylan: What's up?

Julia: I'm supposed to find the slope of a parabola at a point, and I'm not sure how!

Dylan: Well if we had two points we could make a secant line to approximate it!

Julia: Secant line? What's that?

Dylan: A *secant line* is just a line which connects two points on a function!

Julia: But isn't the *tangent* line one that skims a curve at one point? So the slope of the tangent line is the slope at that point! See?

$$\text{Graph of } f(x) = x^2, g(x) = 2(x - 1) + 1$$

Dylan: Well do you know how to find the equation for a line with just one point?

Julia: ...

James: Come on guys we can approximate the tangent line using the secant line!

Altogether: Let's dive in!

Guided Example

Consider the function

$$f(x) = x^2$$

$$\text{Graph of } f(x) = x^2, g(x) = 2(x - 1) + 1$$

Question 1 Find the slope of the secant line between $x = 2$ and $x = 7$.

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Does this seem to be a good approximation for the slope of the tangent line at $x = 2$?

Learning outcomes:

Multiple Choice:

- (a) Yes
- (b) No ✓

Dylan thinks we can solve the problem by just picking something closer than 7.
Find the slope of the secant line between $x = 2$ and $x = 3$.

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Is this a good approximation for the slope of the tangent line at $x = 2$?

Multiple Choice:

- (a) Yes
- (b) No ✓

Is it better than the last attempt?

Multiple Choice:

- (a) Yes ✓
- (b) No

Julia: Dylan, this still isn't a great approximation...

Dylan: Well, I think we need to get even closer. Like, infinitesimally close!
But how would we do that....

James: You guys need some help?

Julia and Dylan: James! How do we find the slope of a line at a point?

James: It isn't too tough! Before, you were considering a certain point as your comparison. What if instead, you used the point you want to evaluate at plus something really small? Let's call it h .

Question 2 How can you make the h in

$$\frac{f(2+h) - f(2)}{(2+h) - 2}$$

approach 0?

Multiple Choice:

- (a) Use $\lim_{h \rightarrow 0}$. ✓
- (b) Use $\lim_{h \rightarrow \infty}$.
- (c) Divide the fraction by h .
- (d) Pick a function $f(x)$ so that $f(2)$ is 0.

Using the method you determined, approximate the slope of the tangent line at the point $x=2$.

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James: Want to know something really cool?

Julia and Dylan: What James?

James: The function we just discovered is how you determine a function's derivative! Using that process, you can find the instantaneous rate of change at any point on a function!

Julia and Dylan: Wow! So cool!

On Your Own

Using what you've learned, find the derivative of the following functions at the given point.

Question 1 $g(x) = x^2 + 1, x = 2$

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$$h(x) = \frac{1}{x}, x = 2$$

-0.25

$$f(x) = 3x^2 + 4x + 2, x = -1$$

-2

$$f(t) = \sqrt{t^2 + 1}, x = 3$$

$\frac{3}{\sqrt{10}}$

$$f(x) = x + x^{-1}, x = 4$$

$$\boxed{3.9375}$$

By replacing the point in our formula for the derivative with x , we may determine the derivative at any point on the function. Determine the derivative for the following functions.

Question 2 $m(x) = x^3$

$$\boxed{3x^2}$$

$$n(x) = 3x + 2$$

$$\boxed{3}$$

$$f(x) = 4 - x^2$$

$$\boxed{-2x}$$

$$f(x) = 12 + 7x$$

$$\boxed{7}$$

$$f(t) = \frac{4}{t+1}$$

$$\boxed{\frac{-4}{(x+1)^2}}$$

In Summary

Julia: So why is it called a secant line?

James: It comes from the Latin word *secare*, which means 'to cut'.

Dylan: Ohh, I get it now! Because a secant line is a line that 'cuts' a function!

In this lab we've (hopefully) learned the function for finding a derivative using the limit as h approaches 0. We also learned what secant and tangent lines are. For your convenience, the important definitions are listed below.

Definition 1. A *secant line* is any line that connects any two points on a curve.

Definition 2. A *tangent line* is a line which intersects a differentiable curve at a point where the slope of the curve equals the slope of the line.

Definition 3. The ***derivative*** of $f(x)$ at a is defined by the following limit:

$$\left[\frac{d}{dx} f(x) \right]_{x=a} = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}.$$