Implicit Differentiation - Finish solutions

Dylan: Woah! What's up with this?

Julia: I didn't know functions were explicit!

Dylan: The x and y are on the same side of the equation! I can't deal with this.

James: Functions can be explicit or implicit! And it not the way you're thinking Julia...

Introduction

So far we have dealt only with explicitly defined functions, where $y = f(x)$. Functions where there are both x and y on one side or both sides of the equation are called **implicit functions**.

Guided Example

Question 1 *Which of the following equations are defined implicitly?*

Select All Correct Answers:

(a)
$$y = x^2 + 5x - 7$$

(b)
$$y = \sin(x)$$

(c)
$$x^2 + y^2 = 1$$

✓

(d)
$$y = \sqrt{(x-3)}$$

Learning outcomes:

(e) $x^2y^3 + y = 5x + 8y$ ✓

Question 2 *Graph the following implicitly defined function below,*

$$x^2 + y^2 = 1$$

Graph of

Now, in the following sage cell, solve the function for y. For help using the solve command refer to the documentation here.

SAGE

```
x,y = var("x, y")
#eqn = x**2+y**2==1, this sets eqn to the unit circle
#use the solve command to solve eqn for y
```

Graph the two explicit equations on the same axis below.

Graph of

Which of the following are true?

Select All Correct Answers:

(a)

$$x^2 + y^2 = 1$$

is a function

(b)

$$-\sqrt{1 - x^2}$$

is a function ✓

(c)

$$\sqrt{1 - x^2}$$

is a function ✓

Question 3 *Using the functions you found, differentiate to find the slope of the tangent lines at the point* $(\frac{\sqrt{2}}{)}, (2\frac{\sqrt{2}}{2})$*. You may do this in the above sage cell or by hand. Slope of tangent line at* $(\frac{\sqrt{2}}{)}, (2\frac{\sqrt{2}}{2}) = \boxed{-1}$

Unfortunately not all implicit equations can be easily solved for y, which is why we use implicit differentiation!

Explanation. *Starting with*

$$x^2 + y^2 = 1$$

we apply the differential operator $\frac{d}{dx}$ *to both sides of the equation*

On Your Own

Consider the equation $y^4 + xy = x^3 - x + 2$.

Question 1 *Using the method shown in the previous section, evaluate the function for* y.

SAGE

```
x,y = var("x, y")
```

Does this equation look easy to differentiate?

$\boxed{No}$

Instead, let's treat our equation as an expression writing it instead as $y^4 + xy - x^3 + x - 2 = 0$ *Now consider* y *as* $y(x)$, *a function of* x, *and differentiate with respect to* x. *Each* y *term will gain* $\frac{dy}{dx}$. *Then, set the expression equal to zero, and solve for* $\frac{dy}{dx}$. *What does this represent?*

Free Response:

Question 2 *Using your result in the previous section, evaluate* $\frac{dy}{dx}$ *at* $x = 3$ *and* $x = 7$.

$x = 3 : \square$

$x = 7 : \square$

SAGE

Question 3

```
var("x, y")

```

Now use Sage Math to find the slope of $\sin(x^2) = \cos(xy^2)$ *at any point.* *Look here for information on implicit differentiation in Sage*

□

Perpendicular at a Point

Julia: Wow, implicit differentiation is rough.

Dylan: You're telling me... I've been doing this for hours! I wish we could at least do a little more with it if I have to learn it.

James: Did I hear that you guys want to know more about using implicit differentiation?

Julia and Dylan: James! Tell us more!

James: Alright guys, you can use implicit differentiation with implicit functions to tell if two functions are perpendicular at a point!

Julia: But how?

Dylan: Yeah, I don't see how that helps.

James: It's easy - all we have to do is see if the tangent lines are perpendicular at that point, and if they are, then so are the curves!

Question 4 *Graph* $3x - 2y + x^3 - x^2y = 0$ *and* $x^2 - 2x + y^2 - 3y = 0$ *on the same set of axes.*

Graph of

Do they look perpendicular anywhere? $\boxed{Yes}$

Question 5 ***Hint:*** *Remember, two perpendicular lines will have their slopes be the negative inverse of one another.*

Prove the two curves are (or are not) perpendicular at the origin.

Free Response:

In Summary

There are two main methods to solve implicit equations

(a) Solve for y and then differentiate.

(b) Treat y as $y(x)$ and differentiate for x, eventually solving for $\frac{dy}{dx}$ to give the value of the derivative at any point.