

# Rational Functions with Awful Questions

## Introduction

**James:** Hey guys, I slept through class yesterday... could you fill me in on what a rational function is?

**Julia:** See, class didn't make a lot of sense to me because I was thinking, "Functions can be rational?"

**Dylan:** They don't mean rational like me or you, Julia! It means *the function can be represented as a fraction where the numerator and denominator are both polynomials*.

**Julia and James:** Oh!

**Dylan:** Rational functions are pretty neat, because they can have two different types of discontinuities!

**Altogether:** LET'S DIVE IN!

## Guided Example

Consider the function  $f(x) = \frac{(x-2)(x+4)}{(x-3)(x+3)(x+4)}$

Graph the function using your favorite CAS system. Depending on the CAS you use, you may need to research how to show discontinuities in a graph. To do this, simply Google "CAS show discontinuities", where *CAS* is the name of whatever CAS you are using. At the time this document was written, Desmos did not include discontinuities by default, and thus, a Desmos powered graph has not been provided within this activity.

**Question 1** Describe the graph. What strange things do you notice?

**Free Response:**

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Learning outcomes:

The "hole" present in the graph is called a **removable discontinuity**.

The curve which goes vertical is called a **vertical asymptote**, another type of discontinuity.

## On Your Own

Find and report the discontinuities in the following functions:

$$a(x) = \frac{x^2 + 1}{x - 2}$$

$$b(x) = \frac{x^2 - 5x + 7}{x^2 - x - 6}$$

$$c(x) = \frac{x^2 - x}{x}$$

$$d(x) = \frac{x^2 - 5x + 7}{x^3 - 6x^2 + 8x - 3}$$

$$f(x) = \frac{2x^2 + 5}{x^2 - 25}$$

$$g(x) = \frac{x^3 - x^2 - 15x - 9}{x + 3}$$

$$h(x) = \frac{1}{3x^2 - x}$$

**Question 2** How can you tell if a rational function has a vertical asymptote or a removable discontinuity?

**Free Response:**

How can you find these discontinuities?

**Free Response:**

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## In Summary

**James:** These functions are pretty neat! What were they called again?

*Rational Functions with Awful Questions*

**Dylan:** They're called **rational functions**, *fractions where the numerator and denominator are both polynomials!*

**Julia:** So, when exactly does a **vertical asymptote** occur?

**James:** I know this one! **Vertical asymptotes** *occur at points where the denominator of the function will be zero, but the numerator is non-zero!*

**Julia:** That makes sense! But when do removable discontinuities occur then?

**Dylan:** **Removable discontinuities** *occur where the numerator and denominator are both zero.*