# Logarithms

## Introduction

**Dylan:** $\log_b(x)$? Why are they talking about trees on this paper?

**Julia:** Well, that doesn't seem quite right... it probably isn't talking about forests or anything. Is it?

**James:** Come on you guys, the lecture *just* went over this! It's the inverse exponential function!

## Examining Log Rules

**Dylan:** Alright, so it isn't about trees, and maybe I wasn't paying attention during the lecture. So, what do I need to know before I do the lab?

**Julia:** Well, at least you're admitting it! I think I remember us going over a few rules for logarithms, but I can't quite seem to remember how they went...

**James:** Let's do a refresher then!

For the following multiple choice questions, you'll be given the left hand side of the equation. Match it up with the right hand side!

**Question 1** $log_b\left(\frac{x}{y}\right)$

***Multiple Choice:***

(a) $\log_b(y) - \log_b(x)$

(b) $\dfrac{\log_b(x)}{\log_b(y)}$

(c) $\log_b(x) - \log_b(y)$ ✓

---

Learning outcomes:
Author(s):

(d) $\dfrac{\log_b(y)}{\log_b(x)}$

$log_b(x * y)$

***Multiple Choice:***

(a) $\log_b(y) \div \log_b(x)$

(b) $\log_b(x) \cdot \log_b(y)$

(c) $\log_b(y) \cdot \log_b(x)$

(d) $\log_b(x) + \log_b(y)$ ✓

$log_b(x^y)$

***Multiple Choice:***

(a) $y \cdot log_b(x)$ ✓

(b) $x \cdot \log_b(y)$

(c) $\log_b(x^y)$

(d) $\log_b(y^x)$

---

# Beyond the Basics

**James:** Now that we've gotten past the basic stuff, let's talk about the meaty stuff - calculus with logs!

**Dylan:** I'm not going to like this, am I?

**Julia:** Sometimes you're way too into this James...

**Example 1.** *Lets work together to determine the value of* $\dfrac{d}{dx}\ln(x)$

**Explanation.** *First, lets think about a number that might make it easier for us to determine the value.*

***Multiple Choice:***

(a) $x^2$

(b) $x$

(c) $e^x$ ✓

(d) $\pi$

***Feedback (attempt):*** *We want $e^x$ because it is its own derivative - that will make our differentiation more than a bit easier!*

*Now, lets consider a general equation $y = \ln(e^y)$.*

*What is the derivative of the left hand side?* $\frac{d}{dy}y = \boxed{1}$

*On the right hand side, we apply the chain rule to see*

$$\frac{d}{dy}\ln(e^y) = \frac{dx}{dy} \cdot \frac{d}{dx}\ln(x), \text{ where } x = e^y$$

*Now, because we know that $\frac{d}{dy}e^y = e^y$, we can change our $\frac{dx}{dy} \cdot \frac{d}{dx}\ln(x) = 1$ to what?*

***Multiple Choice:***

(a) $x$

(b) $\frac{d}{dz} = x$

(c) $\frac{1}{\frac{d}{dx}}$

(d) $x * \frac{d}{dx}\ln(x)$ ✓

***Feedback (attempt):*** *And thus, we see that we have $\frac{d}{dx}\ln(x) = \frac{1}{x}$. So in general, the derivative of $\ln(x)$ is $\frac{1}{x}$!*

**Dylan:** Well, that was quite a bit James!

**James:** Its good to know!

**Julia:** Do you think you could give us a little practice James? I wanna be sure I understand how to use it to get an A on this coming exam!

**James:** It would be my pleasure!

# Practice with Logarithmic Differentiation

Take the derivative of the following functions, without using any technology (except to enter your answer!).

**Problem 2** $\ln(\ln(x)^3) = \boxed{\dfrac{3}{x \cdot \ln(x)}}$

**Problem 3** $\ln(\cot(x)) = \boxed{-\csc(x) \cdot \sec(x)}$

Now, use your knowledge of Sage (and if necessary, a quick glance back at the Intro to Sage lab) to evaluate the following derivatives at the indicated point.

SAGE

```
1
```

**Problem 4** $\ln(\sin(\cos(\ln(x))))$ *at* $x = 10$

$\boxed{-1/10 \cdot \sin \cdot \ln(10) \cdot \cot(\cos(\ln(10)))}$

**Problem 5** $16^{\ln(\csc(x)^2)}$ *at* $x = 1$

$\boxed{-3 \cdot \ln(16) \cdot \cot(1) \cdot 16^{3\ln(\csc(1))}}$

Finally, we'll look at the general form of the derivative of a logarithmic function with any base $b$. Here's a hint to get you started:

Use your knowledge of $\dfrac{d}{dx}\ln(x)$ and the change of base formula, $\log_b(x) = \dfrac{\ln(x)}{\ln(b)}$ to find the derivative for $\log_b(x)$.

**Problem 6** *What is* $\dfrac{d}{dx}\log_b(x)$*?* $\boxed{\dfrac{1}{x \cdot \ln(b)}}$