# Newton's Methods

## Introduction

**Dylan:** I'm so tired of having to solve roots by hand. It's a real drag.

**Julia:** Yeah, some of these roots are rough. I wish there was a better way!

**James:** There's always a better way!

**Dylan and Julia:** Show us!!!

**James:** Maybe you've heard of Sir Isaac Newton? He got tired of solving roots too, and made a whole method to approximate them!

**Dylan:** Wow! I'm just like him except worse in every way!

Newton's Method is a system of approximating roots of polynomials by using tangent lines from an initial estimate. While this method is extremely accurate when used properly, it is possible to have a very inaccurate estimate when used improperly.

## Guided Example

In the following figure we have an initial guess $x_0$, then we have the blue tangent line with respect to the point $x_0$

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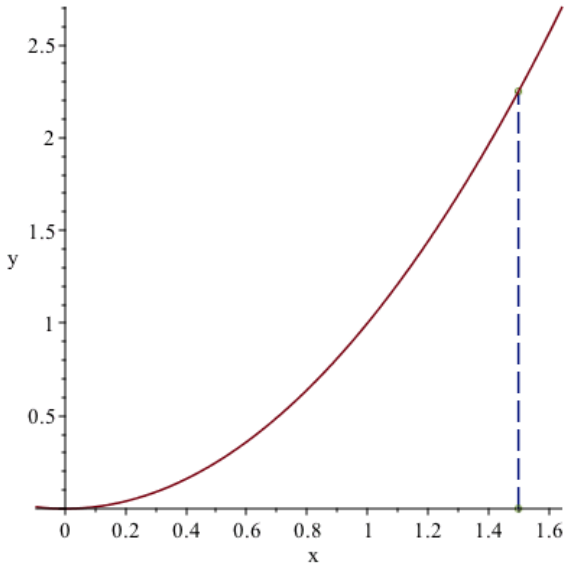
Learning outcomes:




**Question 1** *What is the slope, in general, for the tangent line of* $y = f(x)$ *at* $x_0$*?*

***Multiple Choice:***

(a) $f'(x_0)$ ✓

(b) $f'(x)$

(c) $f(x)$

(d) $f(f(x))$

*What is the equation of the tangent line for the point* $(x_0, f(x_0)$*? Please answer in slope-intercept form.*

***Multiple Choice:***

(a) $y = f(x) \cdot x + b$

(b) $y = f'(x_0) \cdot x_0 + b$ ✓

(c) $y = f'(x) \cdot b + x_0$

(d) $y = f'(x) \cdot b + f(x)$

*How would you use the tangent line you found above to estimate the value of* $x_1$*?*

***Free Response:***

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## On Your Own

**Question 2** *Consider the function* $f(x) = x^2 - 1$.

*Graph of* $x^2 - 1$

*Find the equation of the tangent line at an initial estimate of* $x_0 = 3$.

$y = \square$

*Plot the tangent line and function on the same axes. Does the x-intercept of the tangent line seem more or less accurate than your initial estimate?*

***Multiple Choice:***

(a) *More Accurate* ✓

(b) *Less Accurate*

*What is the x-intercept of the tangent line?*

$\boxed{Idon'tknow}$

*Continue this process until the x-intercepts change by less than .0001 on each iteration. How many iterations did this take?*

$\square$

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**Question 3** *Consider the function* $g(x) = x^3 - 4x^2 - 1$.

*Graph of* $x^3 - 4x^2 - 1$

*Explain why the function has only one solution with the help of a graph.*

*Graph of*

***Free Response:***

*Using* $g(x)$ *from the previous problem, use an initial guess of 2. After 5 iterations, what result do you get?* $\boxed{Areallybadone}$

*Why is it important to use caution with Newton's method?*

***Free Response:***

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**Question 4** *Consider the function* $h(x) = 4x^3 - 12x^2 + 2x + 1$.

*Graph of* $4x^3 - 12x^2 + 2x + 1$

*Use an initial guess of* $x = 3$ *to estimate a root of* $h(x)$. *What do you find?*

□

*Look at the graph, and attempt to estimate another root using* $x = 0$. *Did you find the root to the right or the left of this point?*

***Multiple Choice:***

(a) *Left* ✓

(b) *Right*

*Increment the initial guess by* 0.02 *and use Newton's method until you find the other root. What value of* $x$ *is the first to work?* □

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# In Summary

**Julia:** Wow! Newton's Method is awesome!

**Dylan:** Yeah, it's way more accurate than just guessing! If you're too far off on that initial guess though...

**James:** Things can go downhill quickly. While Newton's Method can be handy, it's important to remember how important an accurate initial estimate is!

**Dylan and Julia:** Thanks James!