Logarithms

Introduction

Dylan: $\log_b(x)$? Why are they talking about trees on this paper?

Julia: Well, that doesn't seem quite right... it probably isn't talking about forests or anything. Is it?

James: Come on you guys, the lecture *just* went over this! It's the inverse exponential function!

Examining Log Rules

Dylan: Alright, so it isn't about trees, and maybe I wasn't paying attention during the lecture. So, what do I need to know before I do the lab?

Julia: Well, at least you're admitting it! I think I remember us going over a few rules for logarithms, but I can't quite seem to remember how they went...

James: Let's do a refresher then!

For the following multiple choice questions, you'll be given the left hand side of the equation. Match it up with the right hand side!

Question 1 $log_b\left(\frac{x}{y}\right)$

Multiple Choice:

(a) $\log_b(y) - \log_b(x)$

(b) $\dfrac{\log_b(x)}{\log_b(y)}$

(c) $\log_b(x) - \log_b(y)$ ✓

Learning outcomes:
Author(s):

(d) $\dfrac{\log_b(y)}{\log_b(x)}$

$log_b(x * y)$

Multiple Choice:

(a) $\log_b(y) \div \log_b(x)$

(b) $\log_b(x) \cdot \log_b(y)$

(c) $\log_b(y) \cdot \log_b(x)$

(d) $\log_b(x) + \log_b(y)$ ✓

$log_b(x^y)$

Multiple Choice:

(a) $y \cdot log_b(x)$ ✓

(b) $x \cdot \log_b(y)$

(c) $\log_b(x^y)$

(d) $\log_b(y^x)$

Beyond the Basics

James: Now that we've gotten past the basic stuff, let's talk about the meaty stuff - calculus with logs!

Dylan: I'm not going to like this, am I?

Julia: Sometimes you're way too into this James...

Example 1. *Lets work together to determine the value of* $\dfrac{d}{dx}\ln(x)$

Explanation. *First, lets think about a number that might make it easier for us to determine the value.*

Multiple Choice:

(a) x^2

(b) x

(c) e^x ✓

(d) π

Feedback (attempt): *We want e^x because it is its own derivative - that will make our differentiation more than a bit easier!*

Now, lets consider a general equation $y = \ln(e^y)$.

What is the derivative of the left hand side? $\frac{d}{dy}y = \boxed{1}$

On the right hand side, we apply the chain rule to see

$$\frac{d}{dy}\ln(e^y) = \frac{dx}{dy} \cdot \frac{d}{dx}\ln(x), \text{ where } x = e^y$$

Now, because we know that $\frac{d}{dy}e^y = e^y$, we can change our $\frac{dx}{dy} \cdot \frac{d}{dx}\ln(x) = 1$ to what?

Multiple Choice:

(a) x

(b) $\frac{d}{dz} = x$

(c) $\frac{1}{\frac{d}{dx}}$

(d) $x * \frac{d}{dx}\ln(x)$ ✓

Feedback (attempt): *And thus, we see that we have $\frac{d}{dx}\ln(x) = \frac{1}{x}$. So in general, the derivative of $\ln(x)$ is $\frac{1}{x}$!*

Dylan: Well, that was quite a bit James!

James: Its good to know!

Julia: Do you think you could give us a little practice James? I wanna be sure I understand how to use it to get an A on this coming exam!

James: It would be my pleasure!

Practice with Logarithmic Differentiation

Take the derivative of the following functions, without using any technology (except to enter your answer!).

Problem 2 $\ln(\ln(x)^3) =$ $\boxed{\dfrac{3}{x \cdot \ln(x)}}$

Problem 3 $\ln(\cot(x)) =$ $\boxed{-\csc(x) \cdot \sec(x)}$

Now, use your knowledge of Sage (and if necessary, a quick glance back at the Intro to Sage lab) to evaluate the following derivatives at the indicated point.

SAGE

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Problem 4 $\ln(\sin(\cos(\ln(x))))$ *at* $x = 10$

$$\boxed{\frac{(\frac{-1}{10} \cdot \cos(\cos(\log(10))) \cdot \sin(\log(10)))}{\sin(\cos(\log(10)))}}$$

Problem 5 $16^{\ln(\csc(x)^2)}$ *at* $x = 1$

$$\boxed{-3 \cdot \ln(16) \cdot \cot(1) \cdot 16^{3\ln(\csc(1))}}$$

Finally, we'll look at the general form of the derivative of a logarithmic function with any base b. Here's a hint to get you started:

Use your knowledge of $\dfrac{d}{dx}\ln(x)$ and the change of base formula, $\log_b(x) = \dfrac{\ln(x)}{\ln(b)}$ to find the derivative for $\log_b(x)$.

Problem 6 *What is* $\dfrac{d}{dx}\log_b(x)$*?* $\boxed{\dfrac{1}{x \cdot \ln(b)}}$