

Differentiation Rules!

Julia: Hmm...I don't think differentiation rules, it takes so long and I hate using that long limit definition!

Dylan: No no Julia, it's differentiation *rules*!

Julia: Ohhhh, that makes more sense!

The Power Rule

Julia: I hate how long it takes to differentiate powers!

Dylan: Yeah, it takes forever! I feel like there was some sort of pattern to it, but I couldn't figure anything out.

James: Sounds like you guys need my help again?

Julia and Dylan: Help us James!

James: There *is* a pattern! Check out this table I made!

$f(x)$	$\frac{d}{dx}f(x)$
x^2	$2x^1$
x^3	$3x^2$
x^3	$4x^3$

Question 1 What pattern do you notice in James' table? Generalize this pattern in terms of x^n .

Multiple Choice:

- (a) $n \cdot x^{n-1}$ ✓
- (b) $n - 1 \cdot x^{n-1}$
- (c) $n \cdot x^n$
- (d) $n - 1 \cdot x^n$

Learning outcomes:

Feedback (correct): Congrats! You've found what's known as the **Power Rule**!

Definition 1. The **derivative** of $f(x)$ at a is defined by the following limit:

$$\left[\frac{d}{dx} f(x) \right]_{x=a} = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}.$$

Question 2 Using the limit definition of a derivative, compute the derivative for x^3 .

$$\frac{d}{dx} x^3 = \boxed{3x^2}$$

Feedback (correct): Notice that your answer fits the same pattern as before!

Question 3 Use the power rule to differentiate the following functions.

$$f(x) = x^{10} \quad \frac{d}{dx} f(x) = \boxed{10x^9}$$

$$f(x) = 3x^2 \quad \frac{d}{dx} f(x) = \boxed{6x}$$

Hint: The value $\frac{1}{x}$ can be represented by x^{-1} .

$$f(x) = \frac{5}{x} \quad \frac{d}{dx} f(x) = \boxed{-5x^{-2}}$$

The Constant Rule

Dylan: Wow! That's neat!

Julia: I wish we could use rules like this all over the place though, it would really save me time.

James: There are plenty of places with rules like this! Why don't we look at a function like $y = 3$?

Consider $y = c$, where c is some arbitrary constant.

Question 4

Differentiate this function using the limit definition.

$$\frac{d}{dx}c = \boxed{0}$$

What can you generalize about the derivative of $y = c$ based on this?

Multiple Choice:

(a) $\frac{d}{dx}c = 2c$

(b) $\frac{d}{dx}c = 0$ ✓

(c) $\frac{d}{dx}c = x$

(d) $\frac{d}{dx}c = \frac{c}{2}$

Feedback (correct): Congrats! You've found what's known as the **Constant Rule!**

Question 5 Using what you found in the previous question, compute the following derivatives:

$$f(x) = 2 \quad \frac{d}{dx}f(x) = \boxed{0}$$

$$f(x) = 100 \quad \frac{d}{dx}f(x) = \boxed{0}$$

$$f(x) = 0 \quad \frac{d}{dx}f(x) = \boxed{0}$$

The Constant Multiple Rule

Julia: James! Show us more! These things are going to save me so much time on my homework!

James: Alright alright, calm down Julia. We can look at a function like $y = 3x$ next.

Consider $y = kx$, where k is some arbitrary constant.

Differentiation Rules!

Question 6 Differentiate this function using the limit definition: $\frac{d}{dx}(kx) = \boxed{k}$

What can you generalize about the derivative of $y = kx$ based on this?

Multiple Choice:

(a) $\frac{d}{dx}kx = 2k$

(b) $\frac{d}{dx}kx = kx^2$

(c) $\frac{d}{dx}kx = k \checkmark$

(d) $\frac{d}{dx}kx = x$

Feedback (correct): Congrats! You've found what's known as the **Constant Multiple Rule**!

Question 7

Using what you found in the previous problem, compute the following derivatives:

$$f(x) = 4x \quad \frac{d}{dx}f(x) = \boxed{4}$$

$$f(x) = 10x \quad \frac{d}{dx}f(x) = \boxed{10}$$

$$f(x) = \frac{1}{5}x \quad \frac{d}{dx}f(x) = \boxed{\frac{1}{5}}$$

The Sum and Difference Rules

Dylan: Wow, this stuff is awesome! Is there any way to put it all together?

Like, is there an easy way to tell what the derivative of $f(x) = 3x + 4$ is?

James: There is Dylan!

Question 8 Consider the differentiable functions $f(x)$ and $g(x)$. Let $F(x) = f(x) + g(x)$, use the limit definition to find $F'(x)$.

$$\begin{aligned}
F'(x) &= \lim_{h \rightarrow 0} \frac{F(x+h) - F(x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{[f(x+h) + g(x+h)] - [f(x) + g(x)]}{h} \\
&= \lim_{h \rightarrow 0} \left[\frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h} \right] \\
&= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} + \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h}
\end{aligned}$$

What can you generalize based on this?

Multiple Choice:

- (a) $(f(x) + g(x))' = f'(x) - g'(x)$
- (b) $(f(x) + g(x))' = g'(x) \cdot f(x) + f'(x)$
- (c) $(f(x) + g(x))' = f(x) \cdot g'(x) - g(x) \cdot f'(x)$
- (d) $(f(x) + g(x))' = f'(x) + g'(x) \checkmark$

Feedback (correct): Congrats! You've found what's known as the **Sum Rule**!

Question 9 Using what you found in the previous problem, compute the following derivatives where $F(x) = f(x) + g(x)$:

$f(x) = 3x^2 - 5x + 2, g(x) = x^2 + 3x$	$\frac{d}{dx} F(x) = \boxed{8x - 2}$
$f(x) = x^2 - 4x + 2, g(x) = -4x^2 + 3$	$\frac{d}{dx} F(x) = \boxed{-6x - 4}$
$f(x) = 5x^3 + 3x, g(x) = 2x^2 - 13x$	$\frac{d}{dx} F(x) = \boxed{15x^2 + 4x - 10}$

Question 10 Julia wonders if a similar rule exists for $m(x) = f(x) - g(x)$. Using the limit definition of derivative, determine if there is a pattern. Then, if there is a rule, use it to solve the following problems. If there is not, do them using the limit definition.

$f(x) = 3x^2 - 5x + 2, g(x) = x^2 + 3x$	$\frac{d}{dx} m(x) = \boxed{4x - 8}$
$f(x) = x^2 - 4x + 2, g(x) = -4x^2 + 3$	$\frac{d}{dx} m(x) = \boxed{10x - 4}$

Differentiation Rules!

$$f(x) = 5x^3 + 3x, \quad g(x) = 2x^2 - 13x \qquad \frac{d}{dx}m(x) = \boxed{15x^2 - 4x + 16}$$

In Summary

We've covered a lot of differentiation rules in this lab, to help you out we've made the following table for you to fill out.

Question 11

Power Rule	$\frac{d}{dx}x^n = \boxed{n} \cdot \boxed{x^{(n-1)}}$
Constant Rule	$\frac{d}{dx}c = \boxed{0}$
Constant Multiple Rule	$\frac{d}{dx}c \cdot f(x) = \boxed{c} \cdot \frac{d}{dx}\boxed{f(x)}$
Sum Rule	$\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}\boxed{f(x)} + \frac{d}{dx}\boxed{g(x)}$
Difference Rule	$\frac{d}{dx}(f(x) - g(x)) = \frac{d}{dx}\boxed{f(x)} - \frac{d}{dx}\boxed{g(x)}$