Derivative

Julia: Ah, this sucks!

Dylan: What's up?

Julia: I'm supposed to find the slope of a parabola at a point, and I'm not sure how!

Dylan: Well, what if we just make a secant line on the function?

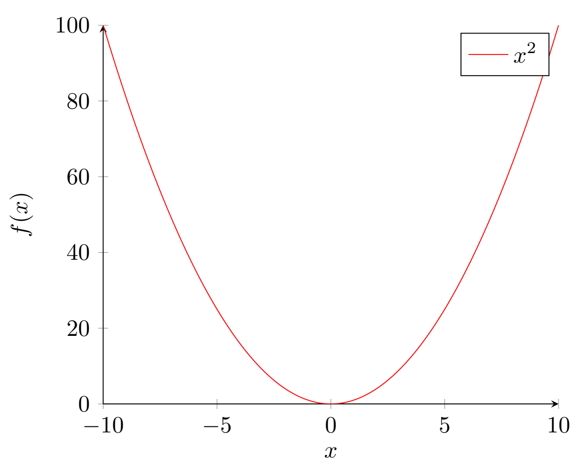
Julia: Secant line? What's that?

Dylan: *A secant line is just a line which connects two points on a function*!

Guided Example

Consider the function

$$f(x) = x^2$$



Question 1 *Find the slope between $x = 2$ and $x = 7$. Does this seem to be a good approximation for the rate of change at $x = 2$? Why or why not?* $\boxed{9}$

Question 2 *Dylan thinks we can solve the problem by just picking something closer than 10. What is the slope between $x = 2$ and $x = 3$?* $\boxed{5}$

Julia: Dylan, this still isn't a great approximation...

Dylan: Well, I think we need to get even closer. Like, infinitesimally close! But how would we do that....

James: You guys need some help?

Julia and Dylan: James! How do we find the slope of a line at a point?

James: It isn't too tough! Before, you were considering a certain point as your comparison. What if instead, you used the point you want to evaluate at plus something really small? Let's call it h.

How can you make the h in

$$\frac{f(a+h)-f(a)}{(a+h)-a}$$

become a value closer and closer to zero when we evaluate it? Using the method you determined in the previous question, approximate the rate of change at the point x=2.

James: The value at that point is the slope of the tangent line!

Dylan: What's a tangent line?

James: *A tangent line is a line which intersects a differentiable curve at a point where the slope of the curve equals the slope of the line.*. Want to know something really cool?

Julia and Dylan: What James?

James: The function you just discovered is how you determine a function's derivative! Using that process, you can find the rate of change at any point on a function!

Julia and Dylan: Wow! So cool!

On Your Own

Using what you've learned, find the derivative of the following functions at the given point.

Question 1 $g(x) = x^5 - 5x^4 - x^2 + 2x + 1$, $x = 2$ $\boxed{-82}$

Question 2 $h(x) = \dfrac{1}{x}$, $x = 2$ $\boxed{-0.25}$

By replacing the a in our formula for the derivative with x, we may determine the derivative at any point on the function. Determine the derivative for the following functions.

Question 3 $m(x) = x^3$ $\boxed{3x^2}$

Question 4 $n(x) = 3x + 2$ $\boxed{3}$

In Summary

Julia: So why is it called a secant line?

James: It comes from the Latin word secare which means to cut.

Dylan: Ohh, I get it now! Because a secant line is any line that connects two points on a function!

In this lab we've (hopefully) learned the function for finding a derivative using the limit as h approaches 0. We also learned what secant and tangent lines are. For your convenience, the important definitions are listed below.

Definition 1. *A* ***secant line*** *is any line that connects any two points on a curve.*

Definition 2. *A* ***tangent line*** *is a line which intersects a differentiable curve at a point where the slope of the curve equals the slope of the line.*

Definition 3. *The* ***derivative*** $f'(a)$ *is defined by the following limit:*

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$