

# Differentiation Rules!

**Julia:** Hmm...I don't think differentiation rules, it takes so long and I hate using that long limit definition!

**Dylan:** No no Julia, it's differentiation *rules*!

**Julia:** Ohhhh, that makes more sense!

## The Power Rule

**Julia:** I hate how long it takes to differentiate powers!

**Dylan:** Yeah, it takes forever! I feel like there was some sort of pattern to it, but I couldn't figure anything out.

**James:** Sounds like you guys need my help again?

**Julia and Dylan:** Help us James!

**James:** There *is* a pattern! Check out this table I made!

$f(x)$	$\frac{d}{dx}f(x)$
$x^2$	$2x^1$
$x^3$	$3x^2$
$x^3$	$4x^3$

**Question 1** What pattern do you notice in James' table? Generalize this pattern in terms of  $x^n$ .

**Multiple Choice:**

- (a)  $n \cdot x^{n-1}$  ✓
- (b)  $n - 1 \cdot x^{n-1}$
- (c)  $n \cdot x^n$
- (d)  $n - 1 \cdot x^n$

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Learning outcomes:

**Feedback (correct):** Congrats! You've found what's known as the **Power Rule**!

**Definition 1.** The **derivative** of  $f(x)$  at  $a$  is defined by the following limit:

$$\left[ \frac{d}{dx} f(x) \right]_{x=a} = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}.$$

**Question 2** Using the limit definition of a derivative, compute the derivative for  $x^3$ .

$$\frac{d}{dx} x^3 = \boxed{3x^2}$$

**Feedback (correct):** Notice that your answer fits the same pattern as before!

**Question 3** Use the power rule to differentiate the following functions.

$$f(x) = x^{10} \quad \frac{d}{dx} f(x) = \boxed{10x^9}$$

$$f(x) = 3x^2 \quad \frac{d}{dx} f(x) = \boxed{6x}$$

**Hint:** The value  $\frac{1}{x}$  can be represented by  $x^{-1}$ .

$$f(x) = \frac{5}{x} \quad \frac{d}{dx} f(x) = \boxed{-5x^{-2}}$$

## The Constant Rule

**Dylan:** Wow! That's neat!

**Julia:** I wish we could use rules like this all over the place though, it would really save me time.

**James:** There are plenty of places with rules like this! Why don't we look at a function like  $y = 3$ ?

Consider  $y = c$ , where  $c$  is some arbitrary constant.

**Question 4**

Differentiate this function using the limit definition.

$$\frac{d}{dx}c = \boxed{0}$$

What can you generalize about the derivative of  $y = c$  based on this?

**Multiple Choice:**

- (a)  $\frac{d}{dx}c = 2c$
- (b)  $\frac{d}{dx}c = 0$  ✓
- (c)  $\frac{d}{dx}c = x$
- (d)  $\frac{d}{dx}c = \frac{c}{2}$

**Feedback (correct):** Congrats! You've found what's known as the **Constant Rule!**

**Question 5** Using what you found in the previous question, compute the following derivatives:

$$f(x) = 2 \quad \frac{d}{dx}f(x) = \boxed{0}$$

$$f(x) = 100 \quad \frac{d}{dx}f(x) = \boxed{0}$$

$$f(x) = 0 \quad \frac{d}{dx}f(x) = \boxed{0}$$

## The Constant Multiple Rule

**Julia:** James! Show us more! These things are going to save me so much time on my homework!

**James:** Alright alright, calm down Julia. We can look at a function like  $y = 3x$  next.

Consider  $y = kx$ , where  $k$  is some arbitrary constant.

## Differentiation Rules!

**Question 6** Differentiate this function using the limit definition:  $\frac{d}{dx}(kx) = \boxed{k}$

What can you generalize about the derivative of  $y = kx$  based on this?

**Multiple Choice:**

(a)  $\frac{d}{dx}kx = 2k$

(b)  $\frac{d}{dx}kx = kx^2$

(c)  $\frac{d}{dx}kx = k \checkmark$

(d)  $\frac{d}{dx}kx = x$

**Feedback (correct):** Congrats! You've found what's known as the **Constant Multiple Rule**!

### Question 7

Using what you found in the previous problem, compute the following derivatives:

$$f(x) = 4x \quad \frac{d}{dx}f(x) = \boxed{4}$$

$$f(x) = 10x \quad \frac{d}{dx}f(x) = \boxed{10}$$

$$f(x) = \frac{1}{5}x \quad \frac{d}{dx}f(x) = \boxed{\frac{1}{5}}$$

## The Sum and Difference Rules

**Dylan:** Wow, this stuff is awesome! Is there any way to put it all together?

Like, is there an easy way to tell what the derivative of  $f(x) = 3x + 4$  is?

**James:** There is Dylan!

**Question 8** Consider the differentiable functions  $f(x)$  and  $g(x)$ . Let  $F(x) = f(x) + g(x)$ , use the limit definition to find  $F'(x)$ .

$$\begin{aligned}
F'(x) &= \lim_{h \rightarrow 0} \frac{F(x+h) - F(x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{[f(x+h) + g(x+h)] - [f(x) + g(x)]}{h} \\
&= \lim_{h \rightarrow 0} \left[ \frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h} \right] \\
&= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} + \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h}
\end{aligned}$$

What can you generalize based on this?

**Multiple Choice:**

- (a)  $(f(x) + g(x))' = f'(x) - g'(x)$
- (b)  $(f(x) + g(x))' = g'(x) + f'(x)$
- (c)  $(f(x) + g(x))' = f(x) \cdot g'(x) - g(x) \cdot f'(x)$
- (d)  $(f(x) + g(x))' = f'(x) + g'(x) \checkmark$

**Feedback (correct):** Congrats! You've found what's known as the **Sum Rule**!

**Question 9** Using what you found in the previous problem, compute the following derivatives where  $j(x) = f(x) + g(x)$ :

$f(x) = 3x^2 - 5x + 2, g(x) = x^2 + 3x$	$\frac{d}{dx}j(x) = \boxed{8x - 2}$
$f(x) = x^2 - 4x + 2, g(x) = -4x^2 + 3$	$\frac{d}{dx}j(x) = \boxed{-6x - 4}$
$f(x) = 5x^3 + 3x, g(x) = 2x^2 - 13x$	$\frac{d}{dx}j(x) = \boxed{15x^2 + 4x - 10}$

**Question 10** Julia wonders if a similar rule exists for  $m(x) = f(x) - g(x)$ . Using the limit definition of derivative, determine if there is a pattern. Then, if there is a rule, use it to solve the 1a, 1b, and 1c. If there is not, do them using the limit definition.

$f(x) = 3x^2 - 5x + 2, g(x) = x^2 + 3x$	$\frac{d}{dx}m(x) = \boxed{4x - 8}$
$f(x) = x^2 - 4x + 2, g(x) = -4x^2 + 3$	$\frac{d}{dx}m(x) = \boxed{10x - 4}$

## Differentiation Rules!

$$f(x) = 5x^3 + 3x, \quad g(x) = 2x^2 - 13x \qquad \frac{d}{dx}m(x) = \boxed{15x^2 - 4x + 16}$$

## In Summary

We've covered a lot of differentiation rules in this lab, to help you out we've made the following table for you to fill out.

### Question 11

Power Rule	$\frac{d}{dx}x^n = \boxed{n} \cdot \boxed{x^{(n-1)}}$
Constant Rule	$\frac{d}{dx}c = \boxed{0}$
Constant Multiple Rule	$\frac{d}{dx}c \cdot f(x) = \boxed{c} \cdot \frac{d}{dx}\boxed{f(x)}$
Sum Rule	$\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}\boxed{f(x)} + \frac{d}{dx}\boxed{g(x)}$
Difference Rule	$\frac{d}{dx}(f(x) - g(x)) = \frac{d}{dx}\boxed{f(x)} - \frac{d}{dx}\boxed{g(x)}$