Rational Functions

Introduction

James: Hey guys, I slept through class yesterday... could you fill me in on what a rational function is?

Julia: See, class didn't make a lot of sense to me because I was thinking, "Functions can be rational?"

Dylan: They don't mean rational like me or you, Julia! It means *the function can be represented as a fraction where the numerator and denominator are both polynomials.*

Julia and James: Oh!

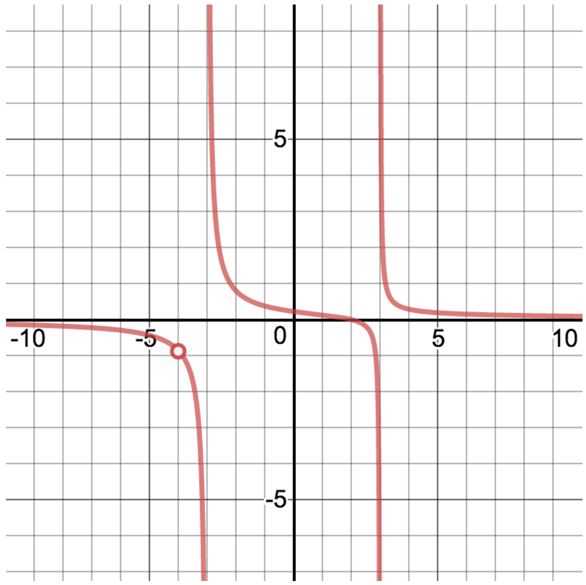
Dylan: Rational functions are pretty neat, because they can have two different types of discontinuities!

Altogether: LET'S DIVE IN!

Guided Example

Consider the function $f(x) = \dfrac{(x-2)(x+4)}{(x-3)(x+3)(x+4)}$

Learning outcomes:



Question 1 *Describe the graph. What strange things do you notice?*

Free Response:

The "hole" present in the graph is called a **removable discontinuity**.

The curve which goes vertical is called a **vertical asymptote**, another type of discontinuity.

On Your Own

Find and report the locations of discontinuities in the following functions, note that at this time Desmos does not show removable discontinuities. You will need to find those by hand:

Question 2 $a(x) = \dfrac{x^2+1}{x-2}$

Graph of

Select All Correct Answers:

(a) $x = 2$ ✓

(b) $x = -1$

(c) $x = 1$

(d) $x = 0$

(e) $x = 4$

(f) *None*

$b(x) = \dfrac{x^2 - 5x + 7}{x^2 - x - 6}$

Graph of

Select All Correct Answers:

(a) $x = 2$

(b) $x = -2$ ✓

(c) $x = 1$

(d) $x = 0$

(e) $x = 3$ ✓

(f) *None*

$c(x) = \dfrac{x^2 - x}{x}$

Graph of

Select All Correct Answers:

(a) $x = 2$

(b) $x = 5$

(c) $x = 1$

(d) $x = 0$ ✓

(e) $x = 3$

(f) *None*

$d(x) = \dfrac{x + 2}{x^2 - x - 6}$

Graph of

Select All Correct Answers:

(a) $x = -6$

(b) $x = 3$ ✓

(c) $x = 2$

(d) $x = 0$

(e) $x = -2$ ✓

(f) *None*

$f(x) = \dfrac{2x^2 + 5}{x^2 - 25}$

Graph of

Select All Correct Answers:

(a) $x = -1$

(b) $x = -5$ ✓

(c) $x = 10$

(d) $x = 0$

(e) $x = 5$ ✓

(f) *None*

$g(x) = \dfrac{x^3 - x^2 - 15x - 9}{x + 3}$

Graph of

Select All Correct Answers:

(a) $x = 12$

(b) $x = -4$

(c) $x = 3$

(d) $x = 5$

(e) $x = -3$

(f) *None* ✓

$h(x) = \dfrac{1}{3x^2 - x}$

Graph of

Select All Correct Answers:

(a) $x = -\sqrt{3}$

(b) $x = \dfrac{1}{3}$ ✓

(c) $x = 3$

(d) $x = 0$ ✓

(e) $x = -3$

(f) *None*

Question 3 *How can you tell if a rational function has a vertical asymptote or a removable discontinuity?*

Multiple Choice:

(a) *Vertical asymptotes occur where only the denominator approaches zero, and removable discontinuities*

occur where both the numerator and denominator approach zero. ✓

(b) *Vertical asymptotes occur where only the numerator approaches zero, and removable discontinuities*

occur where both the numerator and denominator approach zero.

(c) *Vertical asymptotes occur where both the numerator and denominator approach zero, and removable*

discontinuities occur where only the denominator approaches zero.

(d) *Vertical asymptotes occur where only the numerator approaches zero, and removable*

discontinuities occur where only the denominator approaches zero.

In Summary

James: These functions are pretty neat! What were they called again?

Dylan: They're called **rational functions**, *fractions where the numerator and denominator are both polynomials*!

Julia: So, when exactly does a **vertical asymptote** occur?

James: I know this one! **Vertical asymptotes** *occur at points where the denominator of the function will be zero, but the numerator is non-zero*!

Julia: That makes sense! But when do removable discontinuities occur then?

Dylan: **Removable discontinuities** *occur where the numerator and denominator are both zero.*