

# Differentiation Rules!

**Julia:** Hmm...I don't think differentiation rules, it takes so long and I hate using that long limit definition!

**Dylan:** No no Julia, it's differentiation *rules*!

**Julia:** Ohhhh, that makes more sense!

## The Power Rule

**Julia:** I hate how long it takes to differentiate powers!

**Dylan:** Yeah, it takes forever! I feel like there was some sort of pattern to it though. I couldn't figure anything out though.

**James:** Sounds like you guys need my help again?

**Julia and Dylan:** Help us James!

**James:** There *is* a pattern! Check out this table I made!

$f(x)$	$f'(x)$
$x^2$	$2x^1$
$x^3$	$3x^2$
$x^4$	$4x^3$

**Question 1** What pattern do you notice in James' table?

**Free Response:**

Generalize this pattern in terms of  $x^n$

$$\frac{d}{dx}x^n = \boxed{n * x^{(n-1)}}$$

**Question 2** Using the limit definition of a derivative, compute the derivative for  $x^5$

$$\frac{d}{dx}x^5 = \boxed{5x^4}$$

Learning outcomes:

### The Constant Rule

**Dylan:** Wow! That's neat!

**Julia:** I wish we could use rules like this all over the place though, it would really save me time.

**James:** There are plenty of places with rules like this! Why don't we look at a function like  $y = 3$ ?

Consider  $y = c$ , where  $c$  is some arbitrary constant.

**Question 3** Derive this function using the limit definition. What does your answer mean?

$$\frac{d}{dx}c = \boxed{0}$$

**Free Response:**

**Question 4** Using what you found in the previous problem, compute the following derivatives:

$$\frac{d}{dx}2 = \boxed{0}$$

$$\frac{d}{dx}100 = \boxed{0}$$

$$\frac{d}{dx}0 = \boxed{0}$$

### The Constant Multiple Rule

**Julia:** James! Show us more! These things are going to save me so much time on my homework!

**James:** Alright alright, calm down Julia. We can look at a function like  $y = 3x$  next.

Consider  $y = kx$ , where  $k$  is some arbitrary constant.

**Question 5**  $\frac{\partial y}{\partial x}(k * x) = \boxed{k}$

What does your answer mean?

**Free Response:** This is the model solution

### Question 6

Using what you found in the previous problem, compute the following derivatives:

$$\frac{\partial y}{\partial x}(4 * x) = \boxed{4} \quad \frac{\partial y}{\partial x}(10 * x) = \boxed{10} \quad \frac{\partial y}{\partial x}\left(\frac{1}{5} * x\right) = \boxed{\frac{1}{5}}$$

### The Sum and Difference Rules

**Dylan:** Wow, this stuff is awesome! Is there any way to put it all together? Like, is there an easy way to tell what the derivative of  $f(x) = 3x + 4$  is?

**James:** There is Dylan!

Consider the differentiable functions  $f(x)$  and  $g(x)$ . We will define a function  $j(x) = f(x) + g(x)$ .

- (a) Take the derivative of  $j(x)$  using the limit definition. What does your answer mean? *Hint: In  $j(x+h)$ , the  $(x+h)$  will replace  $x$  in the component functions as well.*

$$j'(x) = \boxed{\frac{j(x+h) - j(x)}{h}}$$

- (b) Using what you found in the previous problem, compute the following derivatives:

(c)  $f(x) = 3x^2 - 5x + 2$ ,  $g(x) = x^2 + 3x$

$$j'(x) = \boxed{8x - 2}$$

(d)  $f(x) = x^2 - 4x + 2$ ,  $g(x) = -4x^2 + 3$

$$j'(x) = \boxed{-6x - 4}$$

(e)  $f(x) = 5x^3 + 3x$ ,  $g(x) = 2x^2 - 13x$

$$j'(x) = \boxed{15x^2 + 4x - 10}$$

- (f) Julia wonders if a similar rule exists for  $j(x) = f(x) - g(x)$ . Using the limit definition of derivative, determine if there is a pattern. Then, if there is a rule, use it to solve the 1a, 1b, and 1c. If there is not, do them using the limit definition.

*Differentiation Rules!*

(g)  $f(x) = 3x^2 - 5x + 2$ ,  $g(x) = x^2 + 3x$

$$j'(x) = \boxed{4x - 8}$$

(h)  $f(x) = x^2 - 4x + 2$ ,  $g(x) = -4x^2 + 3$

$$j'(x) = \boxed{-10x - 4}$$

(i)  $f(x) = 5x^3 + 3x$ ,  $g(x) = 2x^2 - 13x$

$$j'(x) = \boxed{15x^2 - 4x + 16}$$

**In Summary**

We've covered a lot of differentiation rules in this lab, to help you out we've made the following table.

Power Rule	$\frac{d}{dx}(x^n) = n * x^{(n-1)}$ , where $n$ is any real number besides 0.
Constant Rule	$\frac{d}{dx}(c) = 0$
Constant Multiple Rule	$\frac{d}{dx}(cf(x)) = c \frac{d}{dx}f(x)$
Sum Rule	$\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$
Difference Rule	$\frac{d}{dx}(f(x) - g(x)) = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$