

# Derivative

**Julia:** Ah, this sucks!

**Dylan:** What's up?

**Julia:** I'm supposed to find the slope of a parabola at a point, and I'm not sure how!

**Dylan:** Well, what if we just make a secant line on the function?

**Julia:** Secant line? What's that?

**Dylan:** *A secant line is just a line which connects two points on a function!*

## Guided Example

Consider the function

$$f(x) = x^2$$

**Question 1** Find the slope between  $x = 2$  and  $x = 7$ .

*Does this seem to be a good approximation for the rate of change at  $x = 2$ ?*

*Dylan thinks we can solve the problem by just picking something closer than 10. What is the slope between  $x = 2$  and  $x = 3$ ?*

*Is this a good approximation?*

*Is it better than the last attempt?*

**Julia:** Dylan, this still isn't a great approximation...

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Learning outcomes:

**Dylan:** Well, I think we need to get even closer. Like, infinitesimally close!  
But how would we do that....

**James:** You guys need some help?

**Julia and Dylan:** James! How do we find the slope of a line at a point?

**James:** It isn't too tough! Before, you were considering a certain point as your comparison. What if instead, you used the point you want to evaluate at plus something really small? Let's call it  $h$ .

**Question 2** *How can you make the  $h$  in*

$$\frac{f(a+h) - f(a)}{(a+h) - a}$$

*become a value closer and closer to zero when we evaluate it?*

**Hint:** *You'll need to use a limit here!*

**Free Response:**

*Using the method you determined, approximate the rate of change at the point  $x=2$ .*

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**James:** The value at that point is the slope of the tangent line!

**Dylan:** What's a tangent line?

**James:** *A tangent line is a line which intersects a differentiable curve at a point where the slope of the curve equals the slope of the line..* Want to know something really cool?

**Julia and Dylan:** What James?

**James:** The function you just discovered is how you determine a function's derivative! Using that process, you can find the rate of change at any point on a function!

**Julia and Dylan:** Wow! So cool!

## On Your Own

Using what you've learned, find the derivative of the following functions at the given point.

**Question 1**  $g(x) = x^5 - 5x^4 - x^2 + 2x + 1, x = 2$

$h(x) = \frac{1}{x}, x = 2$

By replacing the  $a$  in our formula for the derivative with  $x$ , we may determine the derivative at any point on the function. Determine the derivative for the following functions.

**Question 2**  $m(x) = x^3$

$n(x) = 3x + 2$

## In Summary

**Julia:** So why is it called a secant line?

**James:** It comes from the Latin word *secare* which means to cut.

**Dylan:** Ohh, I get it now! Because a secant line is any line that connects two points on a function!

In this lab we've (hopefully) learned the function for finding a derivative using the limit as  $h$  approaches 0. We also learned what secant and tangent lines are. For your convenience, the important definitions are listed below.

**Definition 1.** A *secant line* is any line that connects any two points on a curve.

**Definition 2.** A *tangent line* is a line which intersects a differentiable curve at a point where the slope of the curve equals the slope of the line.

**Definition 3.** The *derivative*  $f'(a)$  is defined by the following limit:

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$