Derivative

Julia: Ah, this sucks!

Dylan: What's up?

Julia: I'm supposed to find the slope of a parabola at a point, and I'm not sure how!

Dylan: Well if we had two points we could make a secant line to approximate it!

Julia: Secant line? What's that?

Dylan: A *secant line* is just a line which connects two points on a function!

Julia: But isn't the *tangent* line one that skims a curve at one point? So the slope of the tangent line is the slope at that point! See?

Graph of $f(x) = x^2, g(x) = 2(x - 1) + 1$

Dylan: Well do you know how to find the equation for a line with just one point?

Julia: ...

James: Come on guys we can approximate the tangent line using the secant line!

Altogether: Let's dive in!

Guided Example

Consider the function $f(x) = x^2$. In green is $g(x) = 4x - 4$ the tangent line at the point $x = 2$. Thus the slope of the tangent line at $x = 2$ is 4.

Graph of $f(x) = x^2, g(x) = 4x - 4$

Question 1 *Find the slope of the secant line between $x = 2$ and $x = 7$.*

$\boxed{9}$

Does this seem to be a good approximation for the slope of the tangent line at $x = 2$?

Learning outcomes:

Multiple Choice:

(a) *Yes*

(b) *No* ✓

Dylan thinks we can solve the problem by just picking something closer than 7. Find the slope of the secant line between $x = 2$ and $x = 3$.

$\boxed{5}$

Is this a good approximation for the slope of the tangent line at $x = 2$?

Multiple Choice:

(a) *Yes*

(b) *No* ✓

Is it better than the last attempt?

Multiple Choice:

(a) *Yes* ✓

(b) *No*

Julia: Dylan, this still isn't a great approximation...

Dylan: Well, I think we need to get even closer. Like, infinitesimally close! But how would we do that....

James: You guys need some help?

Julia and Dylan: James! How do we find the slope of a line at a point?

James: It isn't too tough! Before, you were considering a certain point as your comparison. What if instead, you used the point you want to evaluate at plus something really small? Let's call it h.

Question 2 *How can you make the h in*

$$\frac{f(2+h) - f(2)}{(2+h) - 2}$$

approach 0?

Multiple Choice:

(a) *Use* $\lim_{h\to 0}$. ✓

(b) *Use* $\lim_{h\to\infty}$.

(c) *Divide the fraction by* h.

(d) *Pick a function* $f(x)$ *so that* $f(2)$ *is 0.*

Using the method you determined, approximate the slope of the tangent line at the point x=2.

$\boxed{4}$

James: Want to know something really cool?

Julia and Dylan: What James?

James: The function we just discovered is how you determine a function's derivative! Using that process, you can find the instantaneous rate of change at any point on a function!

Julia and Dylan: Wow! So cool!

On Your Own

Using what you've learned, find the derivative of the following functions at the given point.

Question 1 ***Hint:*** *Remember you can enter* $\sqrt{x}$ *as either*

`sqrt(...) or (...)^(1/2)`

$g(x) = x^2 + 1$, $x = 2$

$\boxed{4}$

$h(x) = \dfrac{1}{x}$, $x = 2$

$\boxed{-0.25}$

$f(x) = 3x^2 + 4x + 2$, $x = -1$

$\boxed{-2}$

$f(x) = \sqrt{x^2+1}$, $x = 3$

$\boxed{\dfrac{3}{\sqrt{10}}}$

$f(x) = x + x^{-1}$,$x = 4$

$\boxed{15/16}$

By replacing the point in our formula for the derivative with x, we may determine the derivative at any point on the function. Determine the derivative for the following functions.

Question 2 $m(x) = x^3$

$\boxed{3x^2}$

$n(x) = 3x + 2$

$\boxed{3}$

$f(x) = 4 - x^2$

$\boxed{-2x}$

$f(x) = 12 + 7x$

$\boxed{7}$

$f(x) = \dfrac{4}{x+1}$

$\boxed{\dfrac{-4}{(x+1)^2}}$

In Summary

Julia: So why is it called a secant line?

James: It comes from the Latin word secare, which means 'to cut'.

Dylan: Ohh, I get it now! Because a secant line is a line that 'cuts' a function!

In this lab we've (hopefully) learned the function for finding a derivative using the limit as h approaches 0. We also learned what secant and tangent lines are. For your convenience, the important definitions are listed below.

Definition 1. *A **secant line** is any line that connects any two points on a curve.*

Definition 2. *A **tangent line** is a line which intersects a differentiable curve at a point where the slope of the curve equals the slope of the line.*

Definition 3. *The **derivative** of $f(x)$ at a is defined by the following limit:*

$$\left[\frac{d}{dx}f(x)\right]_{x=a} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}.$$