

# Differentiation Rules!

**Julia:** Hmm...I don't think differentiation rules, it takes so long and I hate using that long limit definition!

**Dylan:** No no Julia, it's differentiation *rules*!

**Julia:** Ohhhh, that makes more sense!

## The Power Rule

**Julia:** I hate how long it takes to differentiate powers!

**Dylan:** Yeah, it takes forever! I feel like there was some sort of pattern to it though. I couldn't figure anything out though.

**James:** Sounds like you guys need my help again?

**Julia and Dylan:** Help us James!

**James:** There *is* a pattern! Check out this table I made!

$f(x)$	$\frac{d}{dx} f(x)$
$x^2$	$2x^1$
$x^3$	$3x^2$
$x^4$	$4x^3$

**Question 1** What pattern do you notice in James' table? Generalize this pattern in terms of  $x^n$ .

**Multiple Choice:**

- (a)  $n \cdot x^{n-1}$  ✓
- (b)  $n - 1 \cdot x^{n-1}$
- (c)  $n \cdot x^n$
- (d)  $n - 1 \cdot x^n$

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Learning outcomes:

**Question 2** Using the limit definition of a derivative, compute the derivative for  $x^5$

$$\frac{d}{dx}x^5 = \boxed{5x^4}$$

Notice that your answer fits the same pattern as before!

**Question 3** Use the power rule to differentiate the following functions.

$$f(x) = x^{10} \quad \frac{d}{dx}f(x) = \boxed{10x^9}$$

$$f(x) = 3x^2 \quad \frac{d}{dx}f(x) = \boxed{6x}$$

**Hint:** The value  $\frac{1}{x}$  can be represented by  $x^{-1}$ .

$$f(x) = \frac{5}{x} \quad \frac{d}{dx}f(x) = \boxed{-5x^{-2}}$$

## The Constant Rule

**Dylan:** Wow! That's neat!

**Julia:** I wish we could use rules like this all over the place though, it would really save me time.

**James:** There are plenty of places with rules like this! Why don't we look at a function like  $y = 3$ ?

Consider  $y = c$ , where  $c$  is some arbitrary constant.

**Question 4** Derive this function using the limit definition.

$$\frac{d}{dx}c = \boxed{0}$$

What does your answer mean?

**Free Response:**

**Question 5** Using what you found in the previous problem, compute the following derivatives:

$$f(x) = 2 \qquad \frac{d}{dx}f(x) = \boxed{0}$$

$$f(x) = 100 \qquad \frac{d}{dx}f(x) = \boxed{0}$$

$$f(x) = 0 \qquad \frac{d}{dx}f(x) = \boxed{0}$$

## The Constant Multiple Rule

**Julia:** James! Show us more! These things are going to save me so much time on my homework!

**James:** Alright alright, calm down Julia. We can look at a function like  $y = 3x$  next.

Consider  $y = k \cdot x$ , where  $k$  is some arbitrary constant.

**Question 6**  $\frac{d}{dx}(k \cdot x) = \boxed{k}$

What does your answer mean?

**Free Response:**

### Question 7

Using what you found in the previous problem, compute the following derivatives:

$$f(x) = 4x \qquad \frac{d}{dx}f(x) = \boxed{4}$$

$$f(x) = 10x \qquad \frac{d}{dx}f(x) = \boxed{10}$$

$$f(x) = \frac{1}{5}x \qquad \frac{d}{dx}f(x) = \boxed{\frac{1}{5}}$$

## The Sum and Difference Rules

**Dylan:** Wow, this stuff is awesome! Is there any way to put it all together?

Like, is there an easy way to tell what the derivative of  $f(x) = 3x + 4$  is?

**James:** There is Dylan!

**Question 8** Consider the differentiable functions  $f(x)$  and  $g(x)$ . We will define a function  $j(x) = f(x) + g(x)$ .

**Hint:** In  $j(x + h)$ , the  $(x + h)$  will replace  $x$  in the component functions as well.

Take the derivative of  $j(x)$  using the limit definition.

$$j'(x) = \boxed{\frac{j(x+h) - j(x)}{h}}$$

What does your answer mean?

**Free Response:**

**Question 9** Using what you found in the previous problem, compute the following derivatives:

$$f(x) = 3x^2 - 5x + 2, g(x) = x^2 + 3x \quad \frac{d}{dx}j(x) = \boxed{8x - 2}$$

$$f(x) = x^2 - 4x + 2, g(x) = -4x^2 + 3 \quad \frac{d}{dx}j(x) = \boxed{-6x - 4}$$

$$f(x) = 5x^3 + 3x, g(x) = 2x^2 - 13x \quad \frac{d}{dx}j(x) = \boxed{15x^2 + 4x - 10}$$

**Question 10** Julia wonders if a similar rule exists for  $m(x) = f(x) - g(x)$ . Using the limit definition of derivative, determine if there is a pattern. Then, if there is a rule, use it to solve the 1a, 1b, and 1c. If there is not, do them using the limit definition.

$$f(x) = 3x^2 - 5x + 2, g(x) = x^2 + 3x \quad \frac{d}{dx}m(x) = \boxed{4x - 8}$$

$$f(x) = x^2 - 4x + 2, g(x) = -4x^2 + 3 \quad \frac{d}{dx}m(x) = \boxed{10x - 4}$$

$$f(x) = 5x^3 + 3x, g(x) = 2x^2 - 13x \quad \frac{d}{dx}m(x) = \boxed{15x^2 - 4x + 16}$$

## In Summary

We've covered a lot of differentiation rules in this lab, to help you out we've made the following table.

**Question 11**

<i>Power Rule</i>	$\frac{d}{dx}x^n = \boxed{n} \boxed{*} \boxed{x^{n-1}}$ , where $n$ is any real number.
<i>Constant Rule</i>	$\frac{d}{dx}c = \boxed{0}$
<i>Constant Multiple Rule</i>	$\frac{d}{dx}(c \cdot f(x)) = \boxed{c} \boxed{*} \frac{d}{dx} \boxed{f(x)}$
<i>Sum Rule</i>	$\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx} \boxed{f(x)} \boxed{+} \frac{d}{dx} \boxed{g(x)}$
<i>Difference Rule</i>	$\frac{d}{dx}(f(x) - g(x)) = \frac{d}{dx} \boxed{f(x)} \boxed{-} \frac{d}{dx} \boxed{g(x)}$