Transformations of Functions

SAGE

```
caseInsensitive = function(a,b) {
   return a.toLowerCase() == b.toLowerCase();
 };
```

Julia: Ugh!

Dylan: What's up Julia?

Julia: I have these functions I have to graph, and they're *so* close to functions I know really well, but they're a little bit different and it makes it so I have to calculate a bunch of points before I can confidently graph it!

James: Sounds like you could use some help Julia!

Julia and Dylan: James!

James: There are a ton of ways to transform functions, so let's get going and look at how we can modify our favorite functions!

Introduction

While you work with many different functions, there are a few basic types of functions. These include polynomials, rational functions, trigonometric functions, exponential functions, and logarithmic functions. In this lab we will explore different variations on these basic functions called **transformations**.

Guided Example

Consider the function $f(x) = x^2$.

Graph of $f(x) = x^2$

Learning outcomes:

Question 1 *On the same axis graph $g(x) = f(x) + 2$. What change happened from $f(x)$ to $g(x)$?*

The graph shifted $\boxed{2}$ *units* $\boxed{up}$.

What can you infer about $f(x) - 2$?

The graph would shift $\boxed{2}$ *units* $\boxed{down}$.

Consider the function $f(x+2) = (x+2)^2$. How do you think this graph will be different from the graph of $f(x)$?

Free Response:

Graph the function $f(x+2)$ in the desmos window above, was your prediction correct? What can you infer about the function $f(x-2)$? Graph this function to verify your prediction.

Free Response:

What rule can you write about a general function $f(x+c)$ where c is a positive constant? The function will shift $\boxed{c}$ *units* $\boxed{left}$

Why do you think the graph moves in the direction it does when using the rule you determined in the last question? Hint: Think about the x-intercept and how it changes when you add or subtract a constant from the x value

Free Response:

How do you think the graph of $f(x) = x^2$ be affected when you multiply the whole function by some constant c?

Free Response:

Graph the function $c \cdot f(x)$ for the following values of $c = 2, \frac{1}{2}, -2, \frac{-1}{2}$

Graph of

Describe what is happening to the function based on the value of c, what can you generalize from this? It may be helpful to make a table with the x and y values to understand why this change happens.

Free Response:

On your own

Question 2 *Using $f(x) = x^2$ as your base function create a new function that will shift the graph up 4 units, to the right 3 units, reflect it across the x-axis and stretch it vertically by a factor of 2 and graph it below*

Graph of

Graph the function $f(2x)$

Graph of

What constant does this stretch or compress x^2 by?

$\boxed{1/2}$

Graph $f(2x+6)$ on the same axis above, what transformation occurred?

Free Response:

Note the following expansion of the general function $f(x) = (ax+b)^2$:

$$f(x) = (ax+b)^2 = \left(a\left(x + \frac{b}{a}\right)\right)^2 = a^2\left(x + \frac{b}{a}\right)^2$$

From this expansion, how is a function in the form $f(x) = (ax+b)^2$ being shifted and stretched/compressed in terms of a and b?

Free Response:

In Summary

For the following questions, pick in which way the general graph $f(x)$ would change under certain transformations.

Question 3

$$c \cdot f(x)$$

When $c > 1$

Multiple Choice:

(a) *Shrink $f(x)$ vertically by c*

(b) *Stretch $f(x)$ vertically by c* ✓

(c) *Shrink $f(x)$ horizontally by c*

(d) *Stretch $f(x)$ horizontally by c*

(e) *Flip $f(x)$ over the x axis*

When $c < -1$

Multiple Choice:

(a) *Flip $f(x)$ over the x axis*

(b) *Shrink $f(x)$ horizontally by c*

(c) *Flip $f(x)$ over the y axis and stretch horizontally by c*

(d) *Flip $f(x)$ over the x axis and stretch vertically by c* ✓

(e) *Flip $f(x)$ over the x axis and stretch horizontally by c*

When $0 < c < 1$

Multiple Choice:

(a) *Stretch $f(x)$ horizontally by c*

(b) *Shrink $f(x)$ vertically by c* ✓

(c) *Shrink $f(x)$ horizontally by c*

(d) *Stretch $f(x)$ horizontally by c*

(e) *Flip $f(x)$ over the x axis*

Question 4

$$f(x + c)$$

When $c > 0$

Multiple Choice:

(a) *Shift $f(x)$ left by $|c|$.* ✓

(b) *Flip $f(x)$ over the x-axis.*

(c) *Shift $f(x)$ right by $|c|$*

(d) *Flip $f(x)$ over the x-axis and shift it up by $|c|$.*

(e) *No change occurs to $f(x)$.*

When $c < 0$

Multiple Choice:

(a) *Shift $f(x)$ left by $|c|$.*

(b) *Flip $f(x)$ over the x-axis.*

(c) *Shift $f(x)$ right by $|c|$* ✓

(d) *Flip $f(x)$ over the x-axis and shift it up by $|c|$.*

(e) *No change occurs to $f(x)$.*

When $c = 0$

Multiple Choice:

(a) *Shift $f(x)$ left by $|c|$.*

(b) *Flip $f(x)$ over the x-axis.*

(c) *Shift $f(x)$ right by $|c|$*

(d) *Flip $f(x)$ over the x-axis and shift it up by $|c|$.*

(e) *No change occurs to $f(x)$.* ✓

Question 5

$$f(x) + c$$

When $c > 0$

Multiple Choice:

(a) *Shift $f(x)$ down by $|c|$.*

(b) *Stretch $f(x)$ vertically by $|c|$.*

(c) *Flip $f(x)$ over the x-axis.*

(d) *Shift $f(x)$ up by $|c|$.* ✓

(e) *No change will occur.*

When $c = 0$

Multiple Choice:

(a) *Shift $f(x)$ down by $|c|$.*

(b) *Stretch $f(x)$ vertically by $|c|$.*

(c) *Flip $f(x)$ over the x-axis.*

(d) *Shift $f(x)$ up by $|c|$.*

(e) *No change will occur.* ✓

When $c < 0$

Multiple Choice:

(a) *Shift $f(x)$ down by $|c|$.* ✓

(b) *Stretch $f(x)$ vertically by $|c|$.*

(c) *Flip $f(x)$ over the x-axis.*

(d) *Shift $f(x)$ up by $|c|$.*

(e) *No change will occur.*