Implicit Differentiation

Dylan: Woah! What's up with this?

Julia: I didn't know functions were explicit!

Dylan: The x and y are on the same side of the equation! I can't deal with this.

James: Functions can be explicit or implicit! And it not the way you're thinking Julia...

Introduction

So far we have dealt only with explicitly defined functions, where $y = f(x)$. Functions where there are both x and y on one side or both sides of the equation are called implicit functions.

Guided Example

Question 1 *Give an example of an explicit function and an implicit function, making sure your implicit function is not easily solvable for y.*

Free Response:

Question 2 *Take the implicit function you defined in part one, and graph it. What do you notice?*

Graph of

Free Response:

Now, using your CAS, solve the function for y, and graph the resulting equation(s). What do you notice?

SAGE

```
define(['canvas'],function(canvas) {alert("hi")
    var ctx = canvas.getContext('2d');
    ctx.fillRect(0, 0, 200, 200);
    ctx.clearRect(30, 30, 140, 140);
    ctx.strokeRect(45, 45, 110, 110);
});
```

Using the function(s) you found, find the slope of the tangent lines at a point with two values on your graph.

Implicit Differentiation Using Substitution

Consider the equation $-x^2 * y^3 + y^5 - 32 = 0$.

(a) Using the method shown in the previous section, evaluate the function for y. Does this equation look easy to differentiate?

(b) Instead, let's treat our equation as an expression - because it equals zero, we don't have to worry about moving anything over. Now consider y as $y(x)$, a function of x, and differentiate with respect to x. Each y term will gain $\frac{dy}{dx}$. Then, set the expression equal to zero, and solve for $\frac{dy}{dx}$. What does this represent?

(c) Using your result in the previous section, evaluate $\frac{dy}{dx}$ at $x = 3$ and $x = 7$.

(d) Using the same strategy, find the slope of $\sin(x^2) = \cos(xy^2)$ at any point.

Perpendicular at a Point

Julia: Wow, implicit differentiation is rough.

Dylan: You're telling me... I've been doing this for hours! I wish we could at least do a little more with it if I have to learn it.

James: Did I hear that you guys want to know more about using implicit differentiation?

Julia and Dylan: James! Tell us more!

James: Alright guys, you can use implicit differentiation with implicit functions to tell if two functions are perpendicular at a point!

Julia: But how?

Dylan: Yeah, I don't see how that helps.

James: It's easy - all we have to do is see if the tangent lines are perpendicular at that point, and if they are, then so are the curves!

(a) Graph $3x - 2y + x^3 - x^2y = 0$ and $x^2 - 2x + y^2 - 3y = 0$ on the same set of axes. Do they look perpendicular anywhere?

(b) Prove the two curves are (or are not) perpendicular at the origin. *Remember, two perpendicular lines will have their slopes be the negative inverse of one another.*

In Summary

There are two main methods to solve implicit equations

(a) Solve for y and then differentiate.

(b) Treat y as $y(x)$ and differentiate for x, eventually solving for $\frac{dy}{dx}$ to give the value of the derivative at any point.