

## Differentiation Rules! Again!

**Julia:** You know, some of those rules we learned were pretty useful, but some of these derivatives still suck! There **HAS** to be a better way!

**Dylan:** I'm sure there is, and I'm sure I know who could help us!

**James:** Did I hear my name?

**Dylan:** Not yet!

**Julia:** James!

**James:** There are more rules for differentiation that can make your life just a little bit easier!

### The Product Rule

**James:** From the last time we did this, what rule do you think would exist for the product of two functions?

**Julia:** Well, last time we added or subtracted the derivative of both functions, so I bet we multiply the derivative of both!

**Dylan:** Let's check!

Consider the functions  $f(x) = 2x$  and  $g(x) = 3x^3 + x^2$ .

Graph of  $2x, 3x^3 + x^2$

**Question 1** Use Julia's guess to find the derivative of  $f(x) * g(x)$ .

Use the limit definition of the derivative to find the derivative of  $f(x) * g(x)$ .

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Was Julia right?

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Learning outcomes:

**Julia:** Darn! It didn't work!

**Dylan:** It must be a little harder than that...

**James:** That's right Dylan, but it is easier than the limit definition! All we have to do is use

$$\frac{d}{dx} f(x) * g(x) = f(x) * g'(x) + f'(x) * g(x).$$

This is called the **Product Rule**.

**Question 2** Using the Product Rule, derive the products of the following functions:

$$f(x) = \sin(x) + x^2, g(x) = 3x^3 + x$$

□

$$f(x) = \cos(x) + 4x, g(x) = 3x^2 + x$$

□

$$f(x) = x^2, g(x) = 3x^3 - 3x$$

□

$$f(x) = x^7, g(x) = 2x^3$$

□

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### The Quotient Rule

**Dylan:** Wow! That's gonna save a ton of time with products! Is there anything like it we can do with quotients?

**James:** There is! It's even called the **Quotient Rule**!

**Julia:** I bet it's a pain too though, just like the product rule.

**James:** Well, why don't you try using your intuition first rather than guessing?

**Dylan:** Alright, well, I guess I would divide the derivative of the numerator by the derivative of the denominator.

**Question 3** Consider the functions  $f(x) = x^3$  and  $g(x) = \cos(x)$ .

Graph of  $x^3, \cos(x)$

Use Dylan's guess to find the derivative of  $\frac{f(x)}{g(x)}$ .

## Differentiation Rules! Again!

$$\boxed{3x^2/\sin(x)}$$

Use the limit definition of the derivative to find the derivative of  $\frac{f(x)}{g(x)}$ .

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Was Dylan right?

☐ No

**Julia:** I knew it! It's never that easy!

**James:** Now calm down Julia, this rule is worse than the last one, but it's much better than going through by hand:

$$\frac{d}{dx} * \frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}.$$

**Question 4** Using the Product Rule, derive the products of the following functions:

$$f(x) = \sin(x) + x^2, g(x) = 3x^3 + x$$

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$$f(x) = \cos(x) + 4x, g(x) = 3x^2 + x$$

☐

$$f(x) = x^2, g(x) = 3x^3 - 3x$$

☐

$$f(x) = x^7, g(x) = 2x^3$$

☐

## The Chain Rule

**James:** There's one last rule to learn today; the **Chain Rule**.

**Dylan:** That rule sounds pretty cool! When do we use it though? I thought we already covered the functions we need to know...

**Julia:** Yeah, what else is there?

**James:** We use the chain rule in composition of functions, like when we have  $\sin(2x) - 2x$  is a function, and so is  $\sin()$ !

**Julia:** And how bad is the rule?

*Differentiation Rules! Again!*

**James:** This one is a little more tricky -

$$\frac{d}{dx}f(g(x)) = f'(g(x)) * g'(x).$$

**Dylan and Julia:** That's so gross.

**James:** Well, let's give it a try and see if you like it more than the limit definition!

**Question 5** Consider  $f(x) = \cos(x)$  and  $g(x) = x^3$

*Graph of  $\cos(x), x^3$*

*Using the limit definition of derivative, evaluate the derivative of  $f(g(x))$ .*

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*Now, evaluate the same limit using the chain rule. Was it any better?*

**Question 6** *Using the Chain Rule, derive the compositions  $f(g(x))$  for the following functions:*

$f(x) = 3x + x^2, g(x) = x^4 + 7x$

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$f(x) = \cos(x), g(x) = \sin(x)$

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$f(x) = x^2 - 5x, g(x) = \sqrt{x+3}$

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$f(x) = x^7, g(x) = \sin(x) - x^3 + 3$

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