```
caseInsensitive = function(a,b) {
    return a.toLowerCase() == b.toLowerCase();
};
```

Julia: Ugh!

Dylan: What's up Julia?

Julia: I have these functions I have to graph, and they're *so* close to functions I know really well, but they're a little bit different and it makes it so I have to calculate a bunch of points before I can confidently graph it!

James: Sounds like you could use some help Julia!

Julia and Dylan: James!

James: There are a ton of ways to transform functions, so let's get going and look at how we can modify our favorite functions!

Introduction

While you work with many different functions, there are a few basic types of functions. These include polynomials, rational functions, trigonometric functions, exponential functions, and logarithmic functions. In this lab we will explore different variations on these basic functions called **transformations**.

Guided Example

Consider the function $f(x) = x^2$.

Graph of
$$f(x) = x^2$$

Learning outcomes:

Question 1 On the same axis graph g(x) = f(x) + 2. What change happened from f(x) to g(x)?

The graph shifted $\boxed{2}$ units \boxed{up}

What can you infer about f(x) - 2?

The graph would shift $\boxed{2}$ units \boxed{down}

Consider the function $f(x+2) = (x+2)^2$. How do you think this graph will be different from the graph of f(x)?

Free Response:

Graph the function f(x+2) in the desmos window above, was your prediction correct? What can you infer about the function f(x-2)? Graph this function to verify your prediction.

Free Response:

What rule can you write about a general function f(x+c) where c is a positive constant? The function will shift c units e

Why do you think the graph moves in the direction it does when using the rule you determined in the last question? Hint: Think about the x-intercept and how it changes when you add or subtract a constant from the x value

Free Response:

How do you think the graph of $f(x) = x^2$ be affected when you multiply the whole function by some constant c?

Free Response:

Graph the function $c \cdot f(x)$ for the following values of $c = 2, \frac{1}{2}, -2, \frac{-1}{2}$

Graph of

Describe what is happening to the function based on the value of c, what can you generalize from this? It may be helpful to make a table with the x and y values to understand why this change happens.

Free Response:

On your own

Question 2 Using $f(x) = x^2$ as your base function create a new function that will shift the graph up 4 units, to the right 3 units, reflect it across the x-axis and stretch it vertically by a factor of 2 and graph it below

Graph of

Graph the function f(2x)

Graph of

What constant does this stretch or compress x^2 by?

1/2

Graph f(2x+6) on the same axis above, what transformation occurred?

Free Response:

Note the following expansion of the general function $f(x) = (ax + b)^2$:

$$f(x) = (ax + b)^2 = \left(a\left(x + \frac{b}{a}\right)\right)^2 = a^2\left(x + \frac{b}{a}\right)^2$$

From this expansion, how is a function in the form $f(x) = (ax+b)^2$ being shifted and stretched/compressed in terms of a and b?

Free Response:

In Summary

For the following questions, pick in which way the general graph f(x) would change under certain transformations.

Question 3

 $c \cdot f(x)$

When c > 1

Multiple Choice:

(a) Shrink f(x) vertically by c

- (b) Stretch f(x) vertically by $c \checkmark$
- (c) Shrink f(x) horizontally by c
- (d) Stretch f(x) horizontally by c
- (e) Flip f(x) over the x axis

When c < -1

Multiple Choice:

- (a) Flip f(x) over the x axis
- (b) Shrink f(x) horizontally by c
- (c) Flip f(x) over the y axis and stretch horizontally by c
- (d) Flip f(x) over the x axis and stretch vertically by $c \checkmark$
- (e) Flip f(x) over the x axis and stretch horizontally by c

When 0 < c < 1

Multiple Choice:

- (a) Stretch f(x) horizontally by c
- (b) Shrink f(x) vertically by $c \checkmark$
- (c) Shrink f(x) horizontally by c
- (d) Stretch f(x) horizontally by c
- (e) Flip f(x) over the x axis

Question 4

f(x+c)

When c > 0

Multiple Choice:

- (a) Shift f(x) left by |c|.
- (b) Flip f(x) over the x-axis.

- (c) Shift f(x) right by |c|
- (d) Flip f(x) over the x-axis and shift it up by |c|.
- (e) No change occurs to f(x).

When c < 0

Multiple Choice:

- (a) Shift f(x) left by |c|.
- (b) Flip f(x) over the x-axis.
- (c) Shift f(x) right by $|c| \checkmark$
- (d) Flip f(x) over the x-axis and shift it up by |c|.
- (e) No change occurs to f(x).

When c = 0

Multiple Choice:

- (a) Shift f(x) left by |c|.
- (b) Flip f(x) over the x-axis.
- (c) Shift f(x) right by |c|
- (d) Flip f(x) over the x-axis and shift it up by |c|.
- (e) No change occurs to f(x).

Question 5

f(x) + c

When c > 0

Multiple Choice:

- (a) Shift f(x) down by |c|.
- (b) Stretch f(x) vertically by |c|.
- (c) Flip f(x) over the x-axis.

- (d) Shift f(x) up by |c|.
- (e) No change will occur.

When c = 0

Multiple Choice:

- (a) Shift f(x) down by |c|.
- (b) Stretch f(x) vertically by |c|.
- (c) Flip f(x) over the x-axis.
- (d) Shift f(x) up by |c|.
- (e) No change will occur. ✓

When c < 0

Multiple Choice:

- (a) Shift f(x) down by |c|.
- (b) Stretch f(x) vertically by |c|.
- (c) Flip f(x) over the x-axis.
- (d) Shift f(x) up by |c|.
- (e) No change will occur.