Practical Discrete Mathematics (CSE 1402)

MINOR ASSIGNMENT-2: BOOLEAN LOGICS, TRUTH-TABLE, MATHEMATICAL INDUCTION PRINCIPLE

1. Verify whether the following two logical expressions are equivalent using Python and the `sympy` library:

$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$$

   Hint: Use the function `simplify_logic` from the `sympy` library.

2. Write a Python program to generate the truth table for the following Boolean function:

$$F(A, B, C) = (A \vee B) \wedge (\neg C)$$

3. Write a Python program to generate the truth table for the following Boolean function without using the `sympy` library:

$$F(A, B, C) = (A \vee B) \oplus (B \wedge C)$$

4. Write a Python program to simplify the following Boolean expression using the `sympy` library:

$$F(A, B, C) = (A \wedge B) \vee (\neg A \wedge C) \vee (B \wedge C)$$

5. Write a Python program to check whether the following logical statement is a tautology:

$$(P \vee Q) \vee \neg P$$

6. Find the Disjunctive Normal Form (DNF) and Conjunctive Normal Form (CNF) of the Boolean function:

$$F(A, B, C) = (A \wedge B) \vee (\neg A \wedge C) \vee (B \wedge C)$$

7. A Boolean function F is self-dual if:

$$F(A, B, C, ...) = \neg F(\neg A, \neg B, \neg C, ...)$$

   Check whether the function

$$F(A, B) = A \vee B$$

   is self-dual using Python.

8. Write a Python function to prove that the product of an even integer and any integer is even.

9. Prove by induction that the sum of the first n odd numbers is equal to n^2, i.e.,

$$1 + 3 + 5 + \cdots + (2n - 1) = n^2.$$

10. Given the following truth table:

A	B	C	$F(A, B, C)$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

    Write a Python program to construct a simplified Boolean expression that represents this function.

11. Given the following truth table for a Boolean function $F(A, B, C)$, determine its **Conjunctive Normal Form (CNF)**.

A	B	C	$F(A, B, C)$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

    Write a Python program to:

    (a) Identify the rows where $F(A, B, C) = 0$.

    (b) Construct the CNF expression using these rows.

    (c) Output the simplified CNF form.

12. Implement a function that checks whether the sum of two even numbers is always even.

13. Write a function that verifies whether the sum of three consecutive integers is always a multiple of 3.

14. Create a function that verifies whether the product of two odd numbers is always odd.

15. Write a function that assumes the existence of a smallest positive rational number and leads to a contradiction.

16. Create a function that proves an odd number cannot be equal to twice an integer.

17. Write a function that checks whether the sum of a rational number and an irrational number is always irrational.

18. Write a Python program that demonstrates proof by induction for the sum of the first n natural numbers:

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}.$$

19. The sum of the first $(n+1)$ powers of 2 is claimed to be:

$$1 + 2 + 4 + \cdots + 2^n = 2^n$$

for all $n \geq 0$.

Write a Python program to verify whether this formula holds for different values of n.

20. The sum of the reciprocals of the first n natural numbers is claimed to be:

$$1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} = \ln(n)$$

for all $n \geq 1$.

Write a Python program to verify whether this formula holds for different values of n.